Heavy-Ion Meeting, Nov.25, 2016
Rare Isotope Science Project,
Institute for Basic Science, Daejeon, Korea

SINAP

Nuclear transport and Esym probes in heavy-ion collisions

Jun Xu (徐骏)
Shanghai Institute of Applied Physics, CAS

content

- Esym and isospin effective mass splitting
- Esym probes in heavy-ion collisions
  - probes at subsaturation densities
    - Metastable n/p ratio or t/^{3}He ratio
    - Isoscaling
    - isospin diffusion
    - np flow splitting
  - probes at suprasaturation densities
    - Preequilibrium n/p ratio or t/^{3}He ratio
    - π^-/π^+ ratio
    - K^0/K^+ ratio
    - Preequilibrium np flow splitting
- Transport comparison project
  - Heavy-ion calculation
  - Box calculation

Part I
Esym and probes in HIC

Nuclear symmetry energy

EOS for infinite nuclear matter (parabolic approximation):

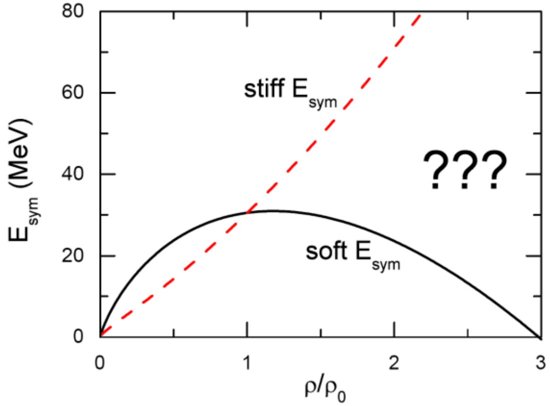
$$E(\rho,\delta) = E_0(\rho) + \boxed{E_{sym}(\rho)}\delta^2 + O(\delta^4), \delta = \frac{\rho_n - \rho_p}{\rho}$$

isospin asymmetry

Our knowledge about E0 and Esym:

$E_0(0.16fm^{-3})$ ~ -16 MeV (from Liquid-Drop)
other densities from GMR, kaon production, flow…

Esym($0.16fm^{-3}$) ~ 30 MeV (from Liquid-Drop)
other densities from Esym probes…



$$E_{sym}(\rho) = E_{sym}(\rho_0) + L\chi + \frac{K_{sym}}{2!}\chi^2 + O(\chi^3) \quad \chi = (\rho - \rho_0)/3\rho_0$$

Slope parameter: $$L = 3\rho_0 \left.\frac{dE_{sym}}{d\rho}\right|_{\rho=\rho_0}$$

Curvature parameter: $$K_{sym} = 9\rho_0^2 \left.\frac{d^2E_{sym}}{d\rho^2}\right|_{\rho=\rho_0}$$

Related to incompressibility of asymmetric nuclear matter:

$$K_{sat} = K_0 + \left(K_{sym} - 6L - \frac{J_0}{K_0}\right)\delta^2 + O\left(\delta^2\right)$$

Symmetry potential

Mean-field potential:

$$U_\tau = \frac{\delta \varepsilon_p}{\delta \rho_\tau} \qquad U_\tau = U_0 \pm U_{sym}\delta + U_{sym}^{(2)}\delta^2 + O(\delta^3)$$

Neutron: +
Proton: -

Relation with symmetry energy (from Hugenholtz-Van Hove(HVH) theorem):

$$E_{sym}(\rho) = \frac{1}{2}U_{sym}(\rho, p_F) + \frac{1}{6}\left.\frac{\partial[t(p)+U_0(\rho,p)]}{\partial p}\right|_{p=p_F} \cdot p_F$$

Single-particle kinetic energy:

$$t(p) = p^2/2m$$

$$L(\rho) = \frac{3}{2}U_{sym}(\rho, p_F) + 3U_{sym}^{(2)}(\rho, p_F) + \left.\frac{\partial U_{sym}}{\partial p}\right|_{p=p_F} \cdot p_F + \frac{1}{6}\left.\frac{\partial[t(p)+U_0(\rho,p)]}{\partial p}\right|_{p=p_F} \cdot p_F + \frac{1}{6}\left.\frac{\partial^2[t(p)+U_0(\rho,p)]}{\partial p^2}\right|_{p=p_F} \cdot p_F^2$$

C. Xu and B.A. Li, PRC, 2010

For momentum-independent potential: $U_{sym} = 2E_{sym}^{pot}$

$$E_{sym} = E_{sym}^{kin} + E_{sym}^{pot}$$

$$E_{sym}^{kin}(\rho_0) \sim 12.5 MeV\,(SRC\,?)$$

Nucleon effective mass

P-mass: $$\frac{\tilde{m}_\tau^*}{m} = \left[1 + \frac{m}{p}\frac{\partial U_\tau(p,\varepsilon_\tau(p))}{\partial p}\right]^{-1}$$

E-mass: $$\frac{\overline{m}_\tau^*}{m} = 1 - \frac{\partial U_\tau(p,\varepsilon_\tau(p))}{\partial \varepsilon_\tau}$$

$\tau = n, p$

Z.X. Li, Nucl. Phys. Rev. (2014)

Dirac mass: $m^*_{Dirac,\tau} = m + \Sigma_\tau^s$ Σ_τ^s: scalar self-energy

Skyrme-Hartree-Fock: non-relativistic, momentum-dependent potential
Relativistic mean-field: relativistic, meson exchange, Schrödinger equivalent potential

Comparison between **non-relativistic mass** with **relativistic mass**

Lorentz effective mass:

$$m^*_{Lorentz,\tau} = m\left(1 - \frac{dU_{SEP,\tau}}{dE_\tau}\right) = (E_\tau - \Sigma_\tau^0)\left(1 - \frac{d\Sigma_\tau^0}{dE_\tau}\right) - (m + \Sigma_\tau^s)\frac{d\Sigma_\tau^s}{dE_\tau} + m - E_\tau = m^*_{Landau,\tau} + m - E_\tau$$

M. Jaminon and C. Mahaux, PRC (1989); B.A. Li, L.W. Chen, and C.M Ko, Phys. Rep. (2008)

Landau effective mass: $m^*_{Landau,\tau} = p\frac{dp}{dE_\tau}$ density of states

$$m^*_{Landau,\tau} = (E_\tau - \Sigma_\tau^0)\left(1 - \frac{d\Sigma_\tau^0}{dE_\tau}\right) - (m + \Sigma_\tau^s)\frac{d\Sigma_\tau^s}{dE_\tau}$$

Σ_τ^0: time component of vector self-energy

Isospin effective mass splitting

$$E(\rho,\delta) = E_0(\rho) + E_{sym}(\rho)\delta^2 + O(\delta^4), \delta = \frac{\rho_n - \rho_p}{\rho}$$

Isospin dynamics in nuclear reactions

$$\frac{d\vec{p}}{dt} = -\nabla U_\tau$$

→ **Symmetry energy/potential**

$$\frac{d\vec{r}}{dt} = \frac{\vec{p}}{m} + \nabla_p U_\tau = \frac{\vec{p}}{m_\tau^*}$$

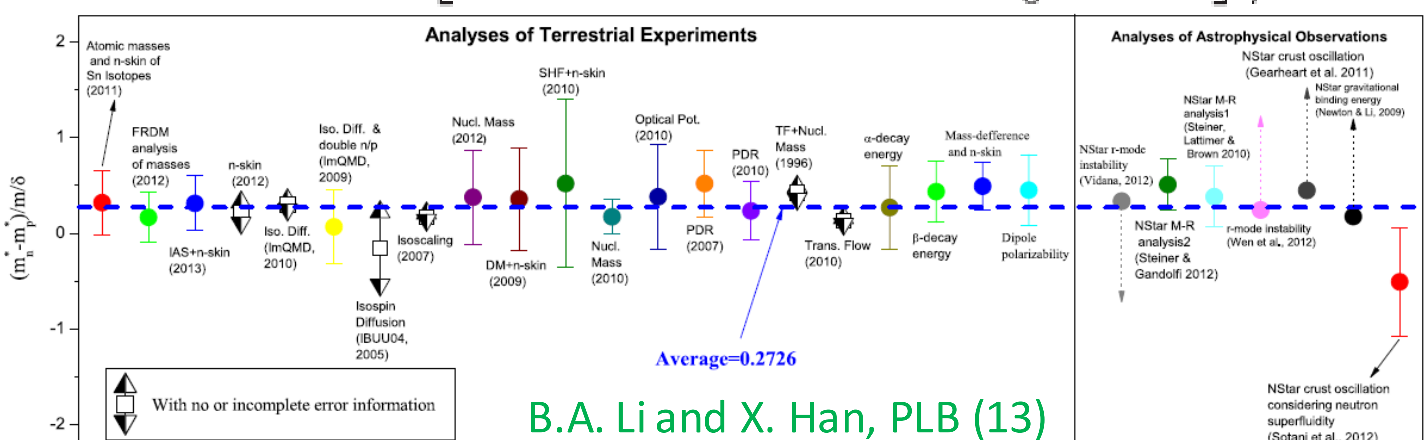
→ **Effective mass** $\frac{m_\tau^*}{m} = \left[1 + \frac{m}{p}\frac{dU_\tau(p)}{dp}\right]^{-1}, \tau = n, p$

(non-relativistic p-mass)

$$\varepsilon_\tau = \frac{p^2}{2m} + U_\tau(p) \sim \frac{p^2}{2m_\tau^*} + U_\tau^{MID}$$

Related to symmetry energy

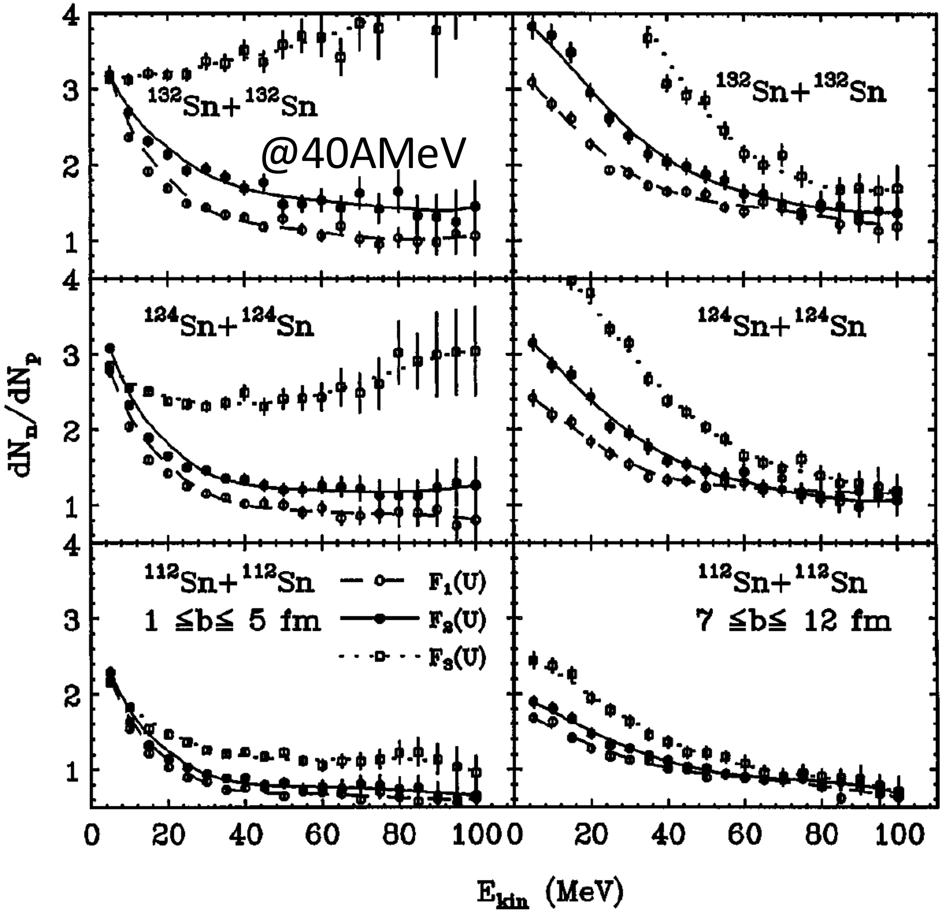
$$m^*_{n-p}(\rho_0,\delta) \approx \delta \cdot \left[3E_{\rm sym}(\rho_0) - L(\rho_0) - \frac{1}{3}\frac{m}{m_0^*}E_F(\rho_0)\right] \Big/ \left[E_F(\rho_0)\cdot(m/m_0^*)^2\right]$$



B.A. Li and X. Han, PLB (13)

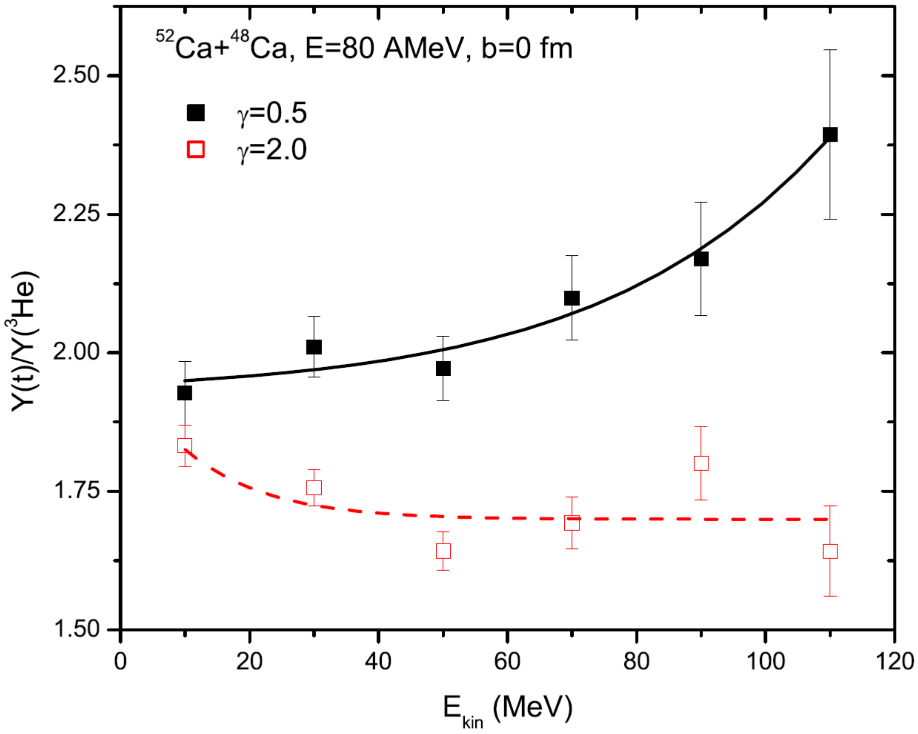
Metastable n/p or t/^{3}He ratio: probes of Esym at subsaturation densities

F_1、 F_2、 F_3 representing 3 parameterizations for Esym from stiff to soft



B.A. Li , C.M. Ko, and Z.Z. Ren, PRL, 1997

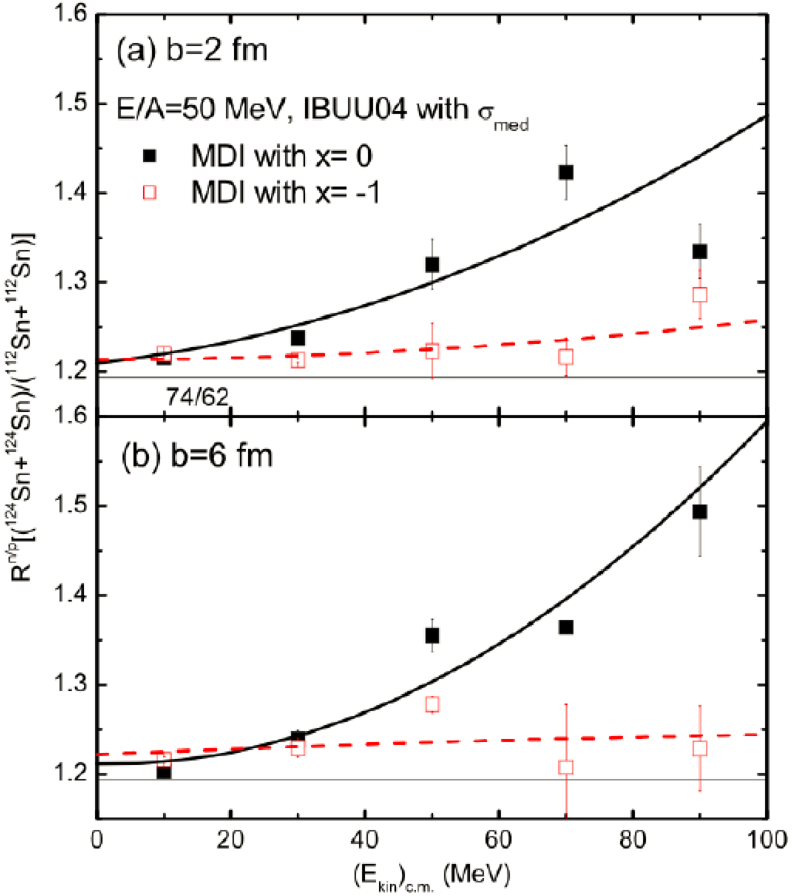
$$E_{sym}(\rho) = E_{sym}(\rho_0)(\rho/\rho_0)^{\gamma}$$



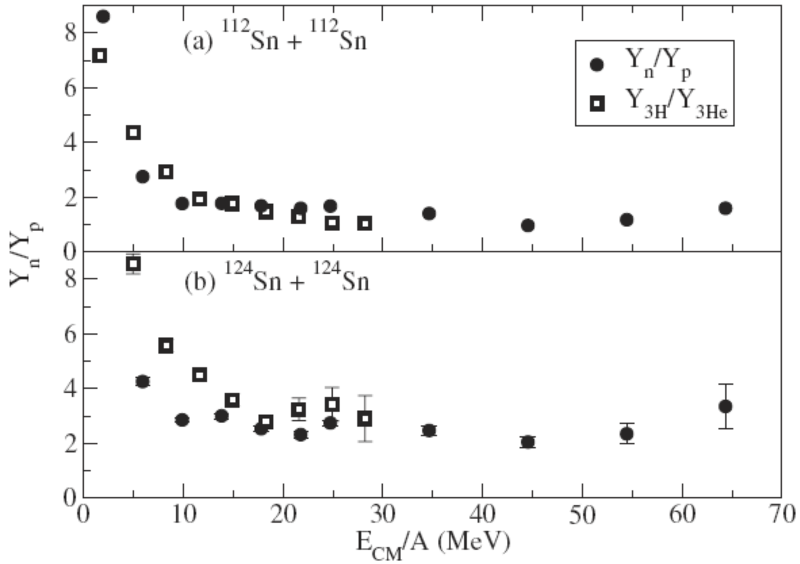
L.W. Chen, C.M. Ko, and B.A. Li, PRC, 2003

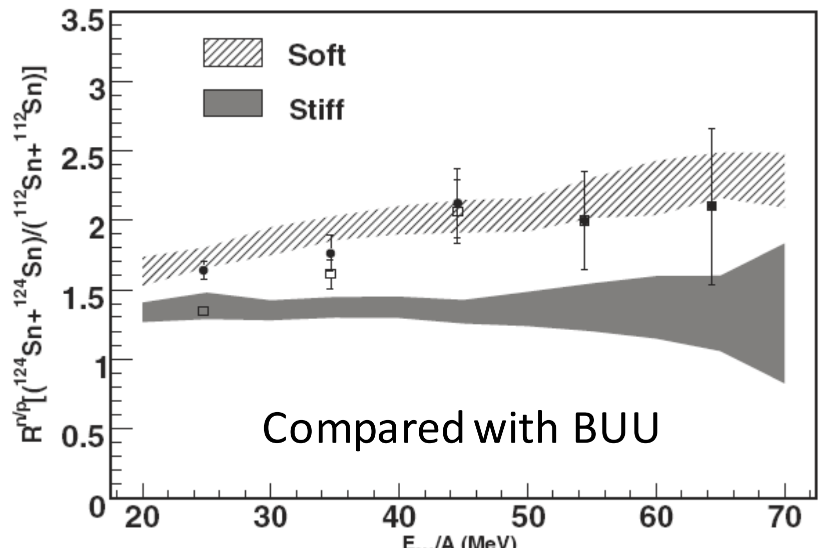
Further studies of n/p ratio

Propose to measure double n/p ratio
due to the low efficiency for neutron detecting



B.A. Li et al., PLB, 2006

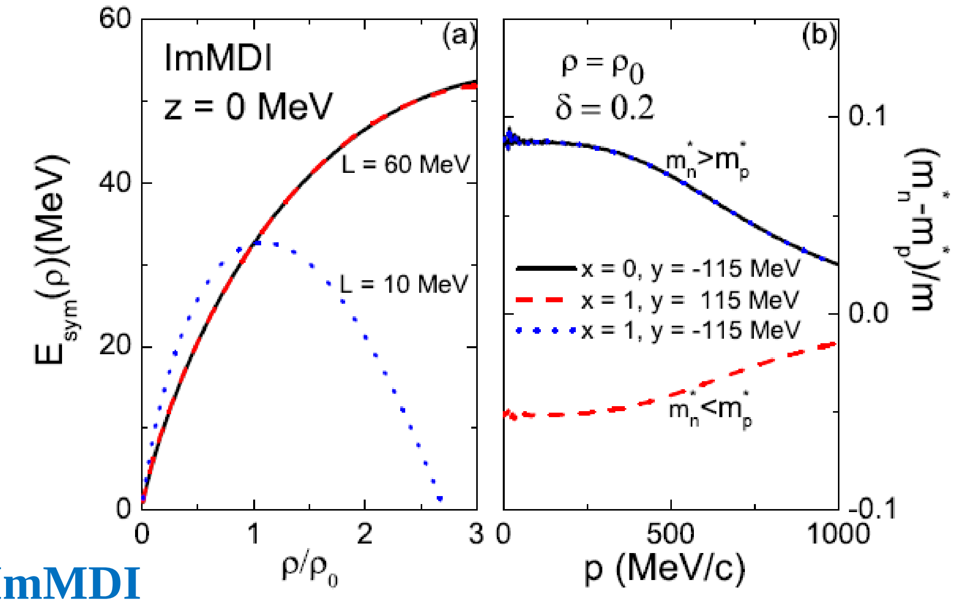




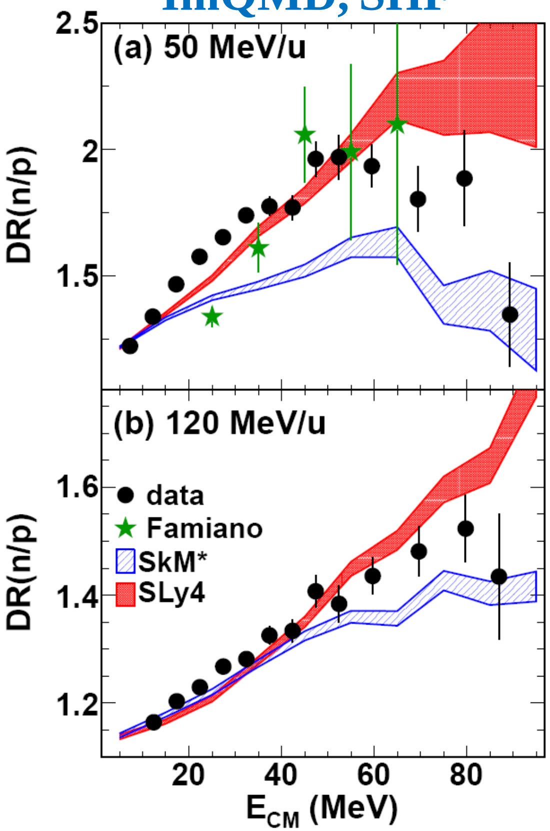
Experimental data from NSCL/MSU
M.A. Famiano et al., PRL, 06

Skyrme	S_0(MeV)	L (MeV)	m_n^*/m_n	m_p^*/m_p
SLy4	32	46	0.68	0.71
SkM*	30	46	0.82	0.76

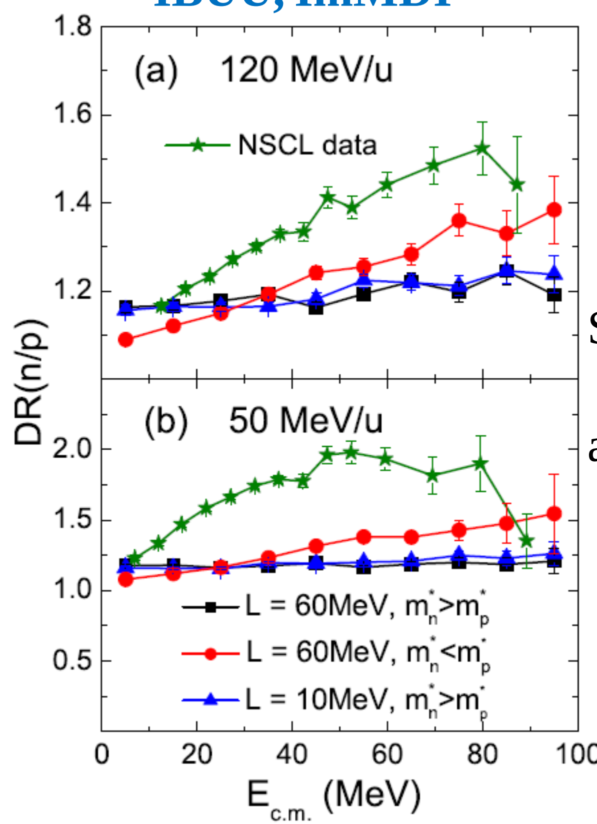
D.D.S. Coupland et al., arXiv:1406.4546



ImQMD, SHF



IBUU, ImMDI

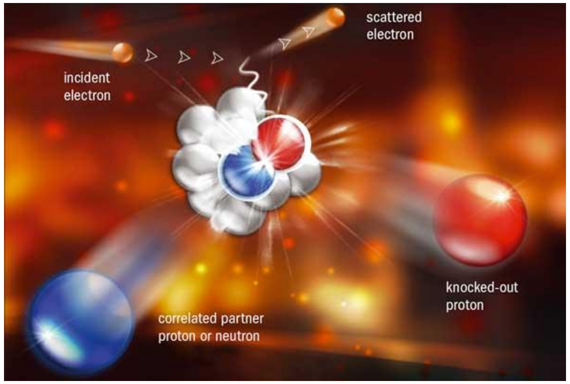


$$\mathrm{DR(n/p)} = \frac{[Y(n)/Y(p)]_{^{124}\mathrm{Sn}+^{124}\mathrm{Sn}}}{[Y(n)/Y(p)]_{^{112}\mathrm{Sn}+^{112}\mathrm{Sn}}}$$

Still below the NSCL/MSU data no matter how the symmetry energy and effective mass splitting is adjusted.

H.Y. Kong, Y. Xia, JX*, L.W. Chen, B.A. Li, and Y.G. Ma PRC 91, 047601 (2015)

DR(n/p) and short-range correlation



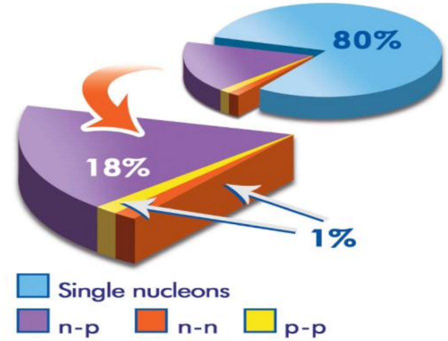


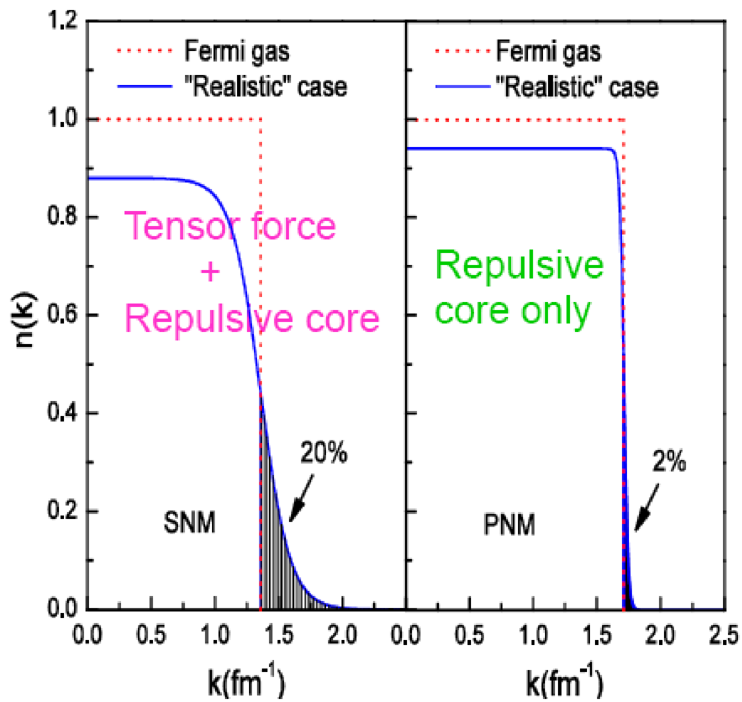
Figure 3: The average fraction of nucleons in the various initial state configurations of ^{12}C.

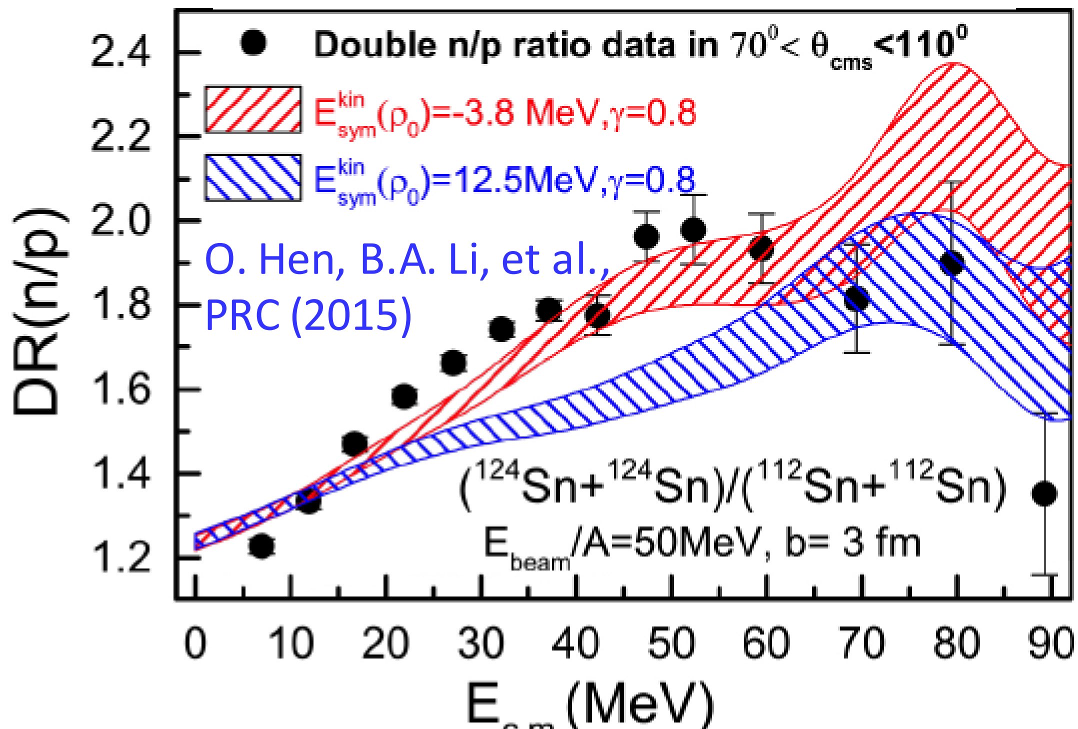
$$E_{kin} = \alpha \int_0^\infty \frac{\hbar^2 k^2}{2m} n(k) k^2 dk$$

$$E^{kin}_{sym} = E^{kin}_{PNM} - E^{kin}_{SNM} < 0$$

$$E^{pot}_{sym}(\rho) = [E_{sym}(\rho_0) - \eta E^{kin}_{sym}(\rho_0)|_{\rm FG}](\rho/\rho_0)^\gamma$$

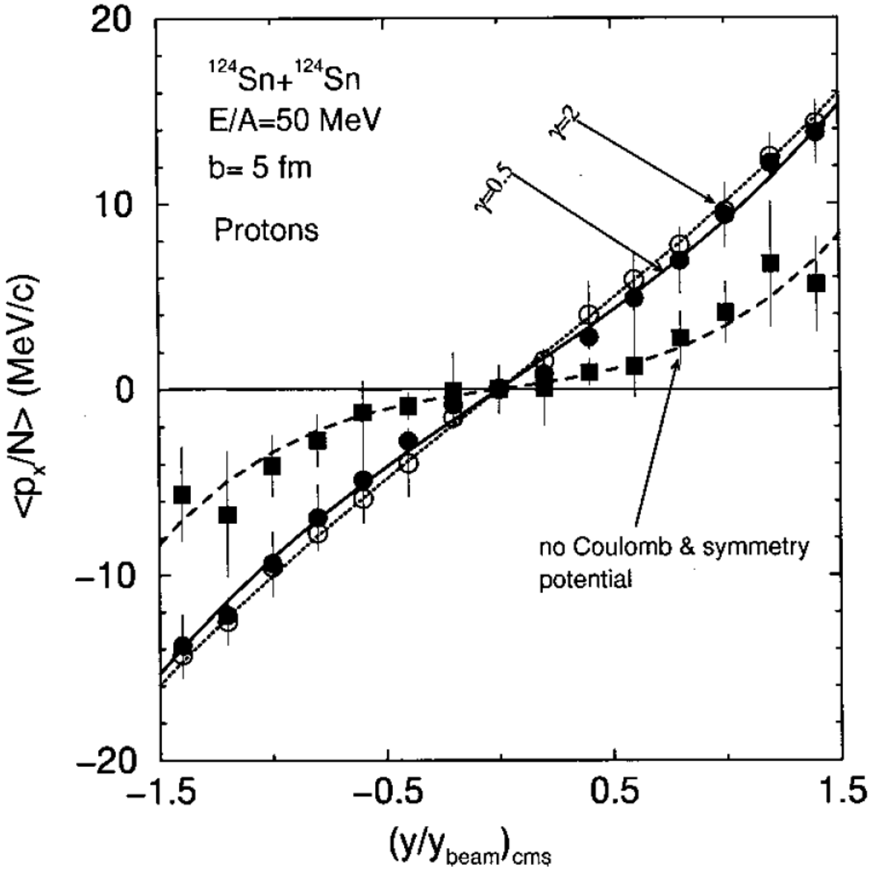
$$V^{n/p}_{\rm sym}(\rho,\delta) = [E_{sym}(\rho_0) - \eta E^{kin}_{sym}(\rho_0)|_{\rm FG}](\rho/\rho_0)^\gamma \times [\pm 2\delta + (\gamma - 1)\delta^2],$$

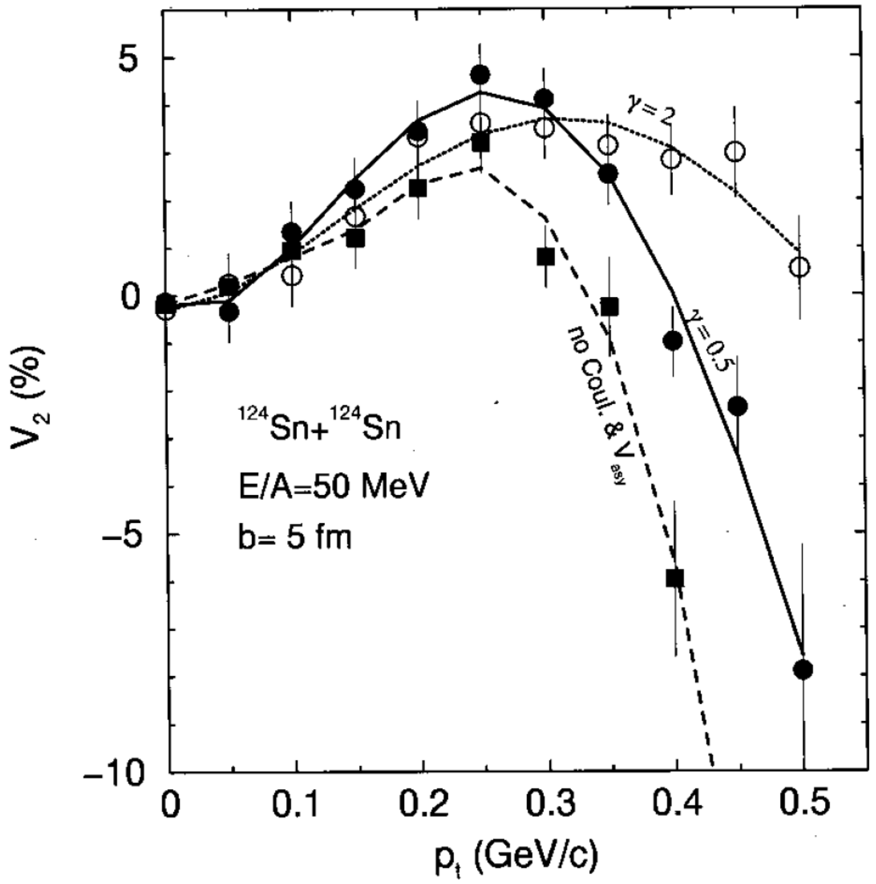




Neutron-proton collective flow: probes of Esym at subsaturation densities

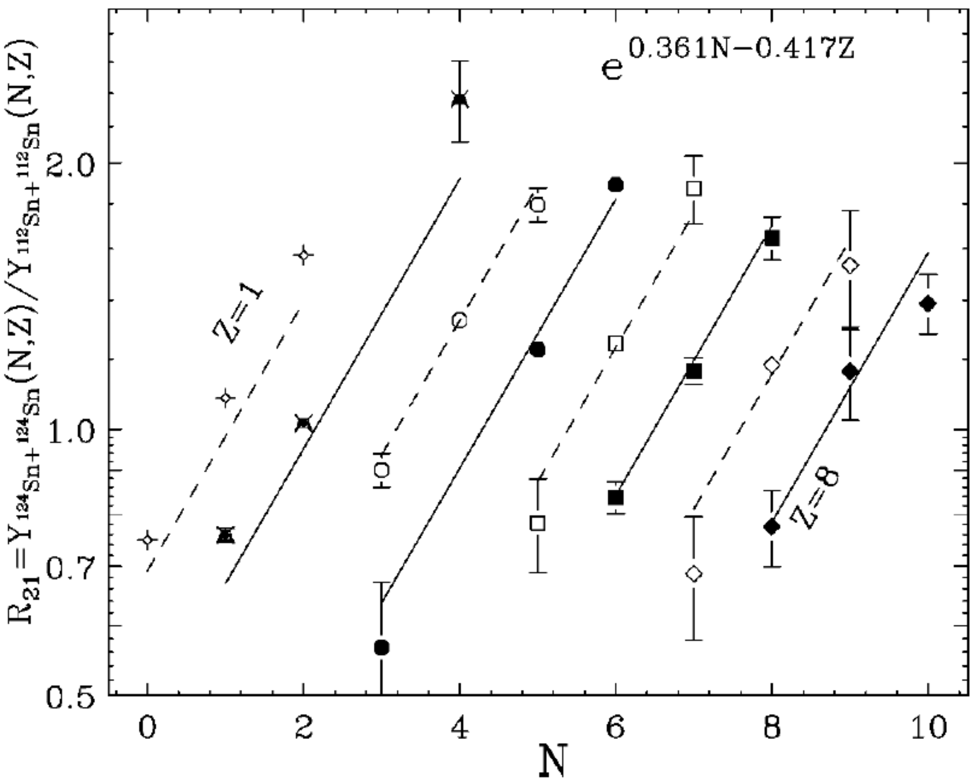
$$E_{sym}(\rho) = E_{sym}(\rho_0)(\rho/\rho_0)^{\gamma}$$





B.A. Li, A.T. Sustich, and B. Zhang, PRC, 2001

Isoscaling of produced fragments: probe of Esym at subsaturation densities



M.B. Tsang et al., PRC, 2001
M.B. Tsang et al., PRL, 2001

Ratios of produced isotopes or isotones Satisfy the exponential relation

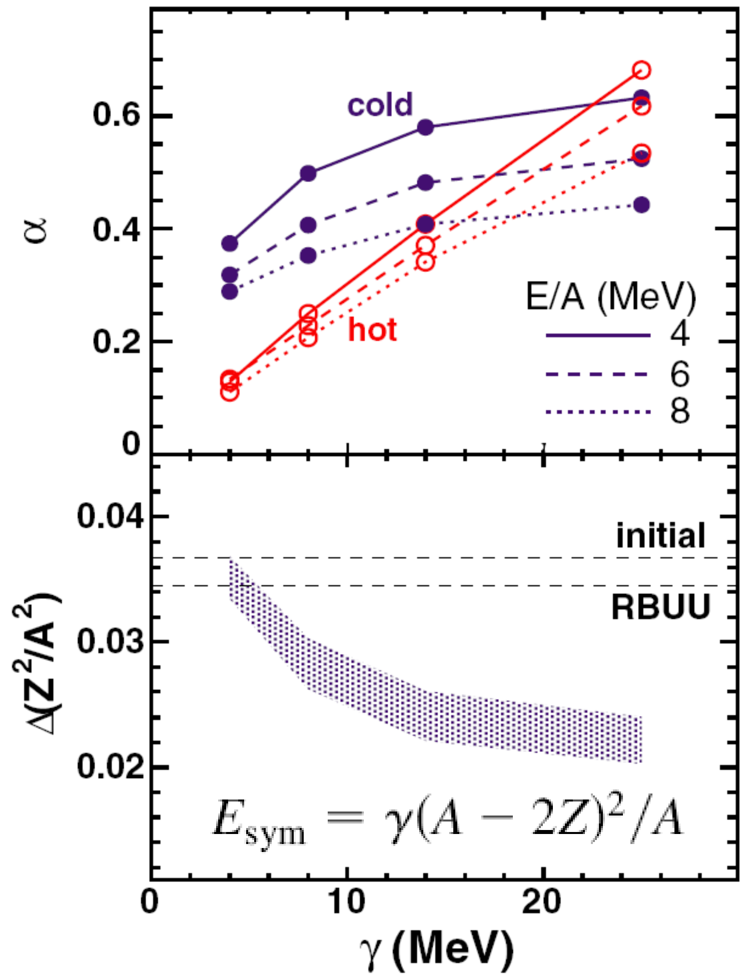
$$R_{12}(N,Z)=Y_2(N,Z)/Y_1(N,Z)\propto \exp(\alpha N+\beta Z)$$

N: neutron number， *Z*: proton number， Y_1 and Y_2: multiplicities in ^{124}Sn+^{124}Sn and ^{112}Sn+^{112}Sn reactions， α and β: isoscaling parameters

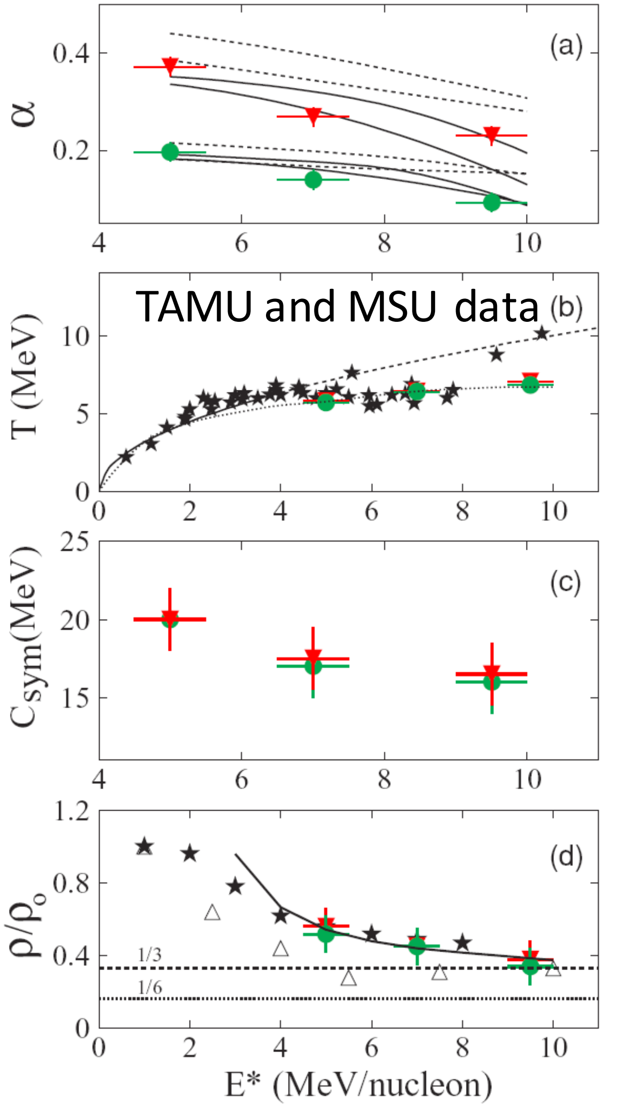
$$\alpha=\frac{4E_{sym}(\rho,T)}{T}\left[\left(\frac{Z_1}{A_1}\right)^2-\left(\frac{Z_2}{A_2}\right)^2\right]$$

Z_1 and Z_2: neutron number of fragments in ^{124}Sn+^{124}Sn and ^{112}Sn+^{112}Sn reactions;
A_1 and A_2 : mass number of fragments in ^{124}Sn+^{124}Sn and ^{112}Sn+^{112}Sn reactions

Further analysis of isoscaling

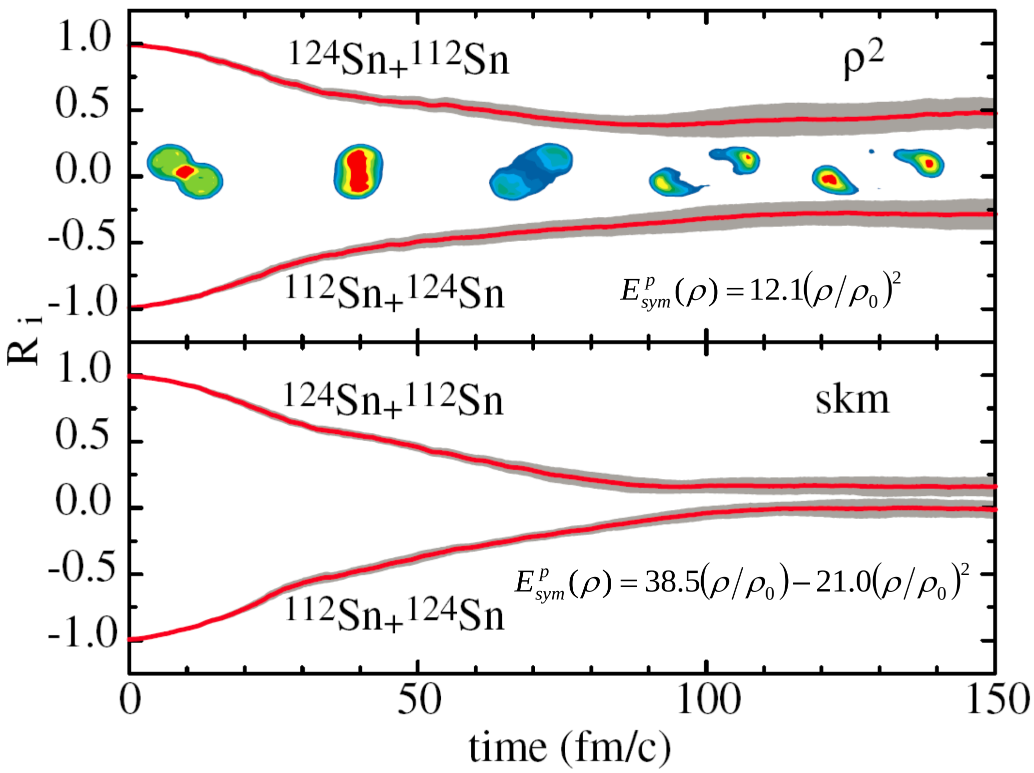


INDRA and ALADIN Collaboration, PRL, 2005



D.V. Shetty, S.J. Yennello and G.A. Souliotis, PRC, 2007

Isospin diffusion: probe of Esym at subsaturation densities



Isospin diffusion coefficient

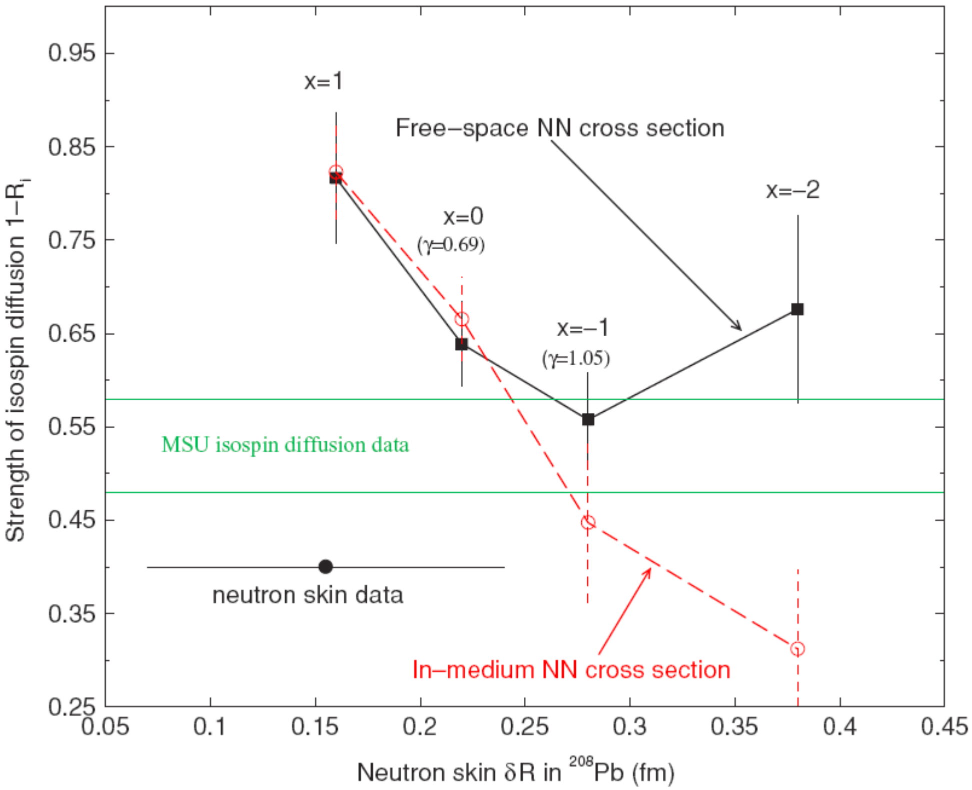
$$R_i = \frac{2x - x_{124+124} - x_{112+112}}{x_{124+124} - x_{112+112}}$$

x can be any isospin-sensitive tracer. It can be the isoscaling coefficient α, or the average isospin asymmetry of reaction residues.

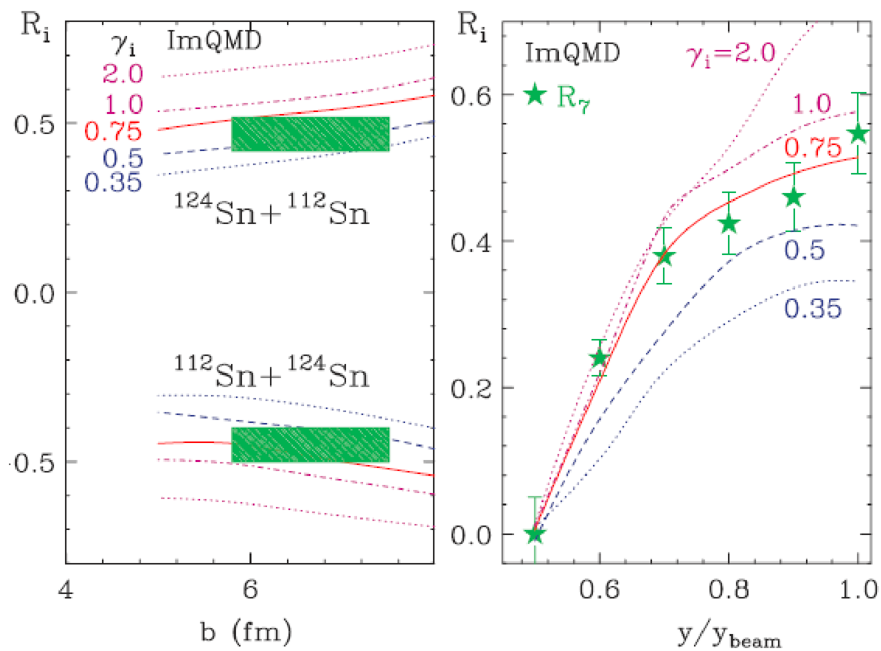
For ^{124}Sn+^{124}Sn reaction, $R_i=1$,
For ^{112}Sn+^{112}Sn reaction, $R_i=-1$

M.B. Tsang et al., PRL, 2004

Further analysis of isospin diffusion



$$S(\rho) = \frac{C_{s,k}}{2}\left(\frac{\rho}{\rho_0}\right)^{2/3} + \frac{C_{s,p}}{2}\left(\frac{\rho}{\rho_0}\right)^{\gamma_i}$$



Isospin diffusion data MSU2004
Compared with IBUU

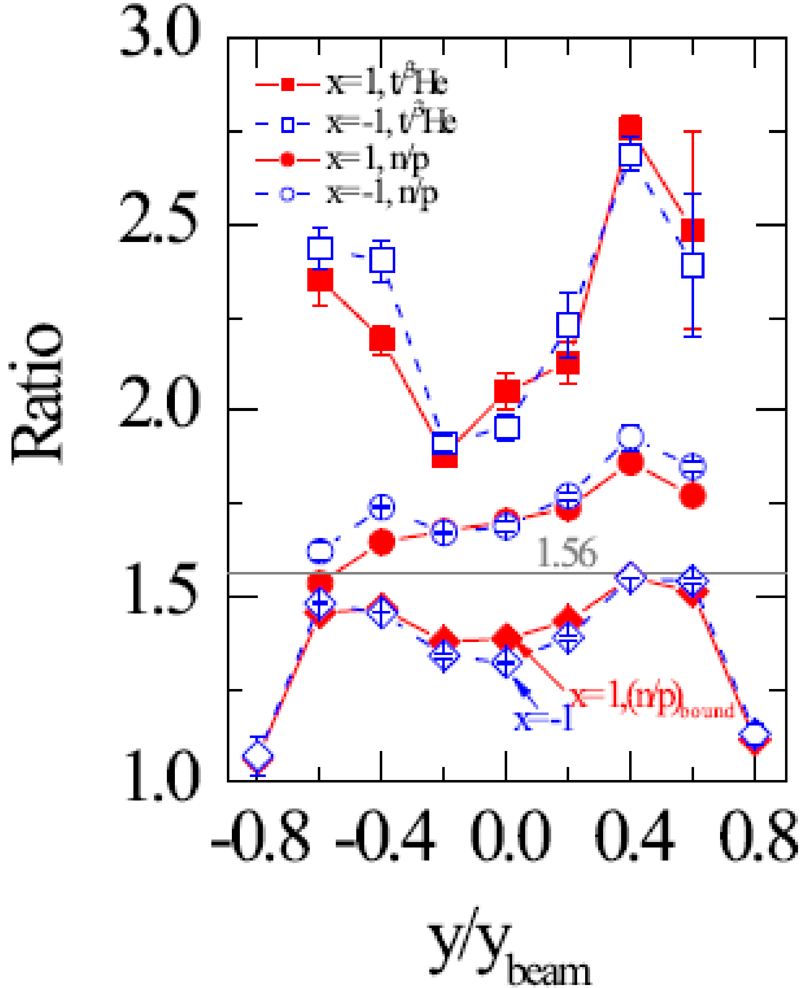
L.W. Chen, C.M. Ko, and B.A. Li, PRL, 2005;
B.A. Li and L.W. Chen, PRC, 2005

New isospin diffusion data
Compared with ImQMD

M.B. Tsang et al., PRL, 2009

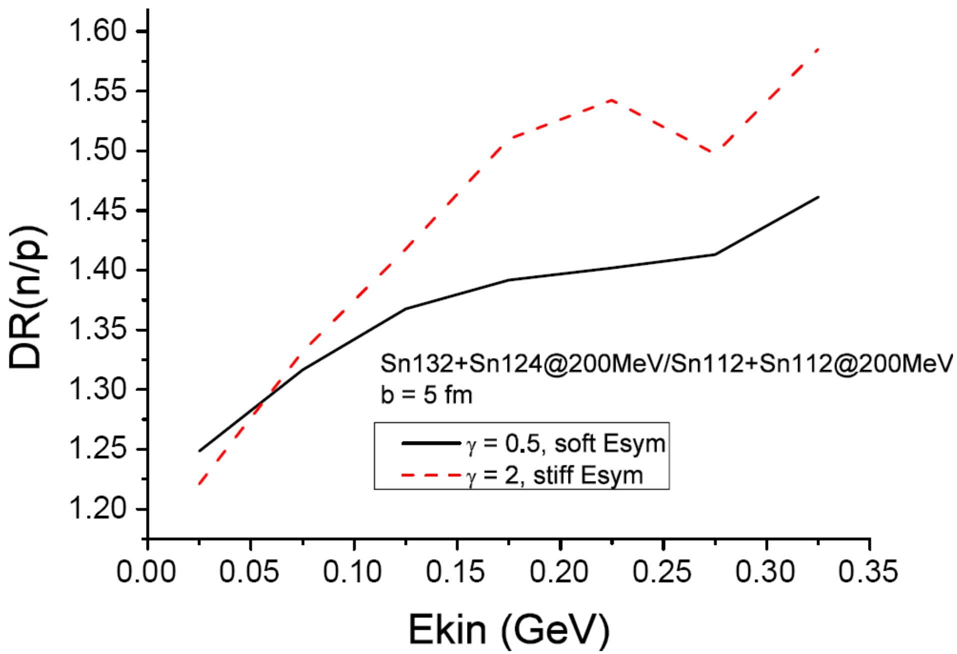
Preequilibrium n/p ratio or t/^{3}He ratio: probe of Esym at suprasaturation densities

Sn132+Sn124@400MeV



G.C. Yong et al., PRC, 2009

$$E_{sym}(\rho) = E_{sym}(\rho_0)(\rho/\rho_0)^{\gamma}$$



Jun Xu, Kyungil Kim, and Youngman Kim, here, 2016

π^-/π^+ ratio: probe of Esym at suprasaturation densities

$$n + n \longrightarrow p + \Delta^-$$

$$\Delta^- \longrightarrow n + \pi^-$$

$$n + p \xrightarrow{1/2} p + \Delta^0$$

$$\Delta^0 \xrightarrow{2/3} n + \pi^0$$

$$\Delta^0 \xrightarrow{1/3} p + \pi^-$$

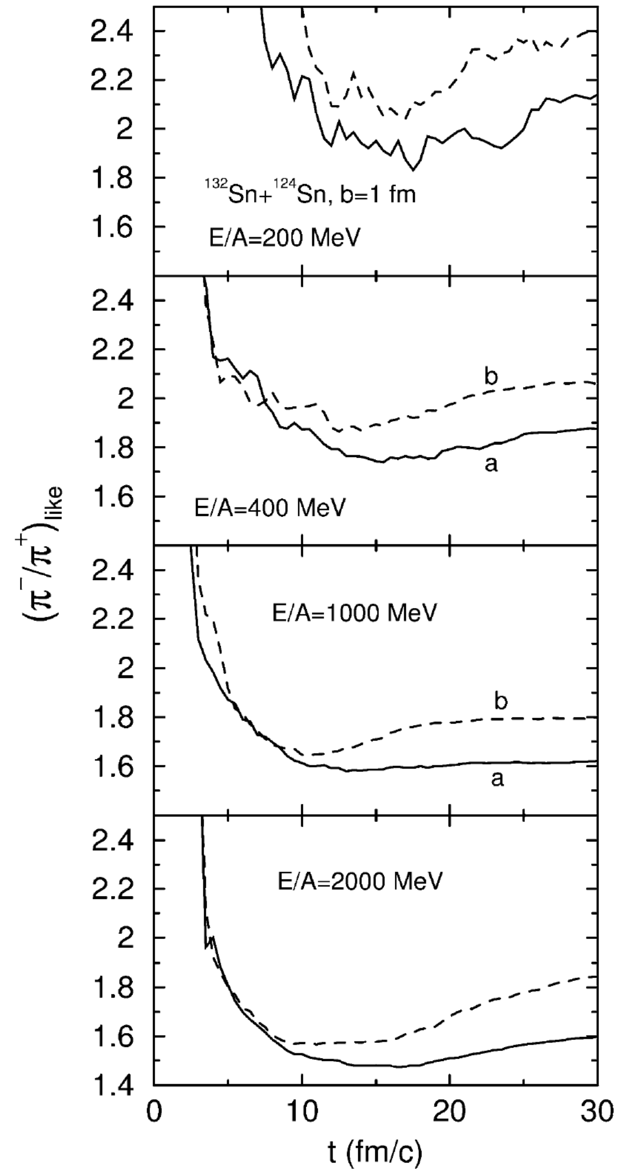
$$n + p \xrightarrow{1/2} n + \Delta^+$$

$$\Delta^+ \xrightarrow{2/3} p + \pi^0$$

$$\Delta^+ \xrightarrow{1/3} n + \pi^+$$

$$p + p \longrightarrow n + \Delta^{++}$$

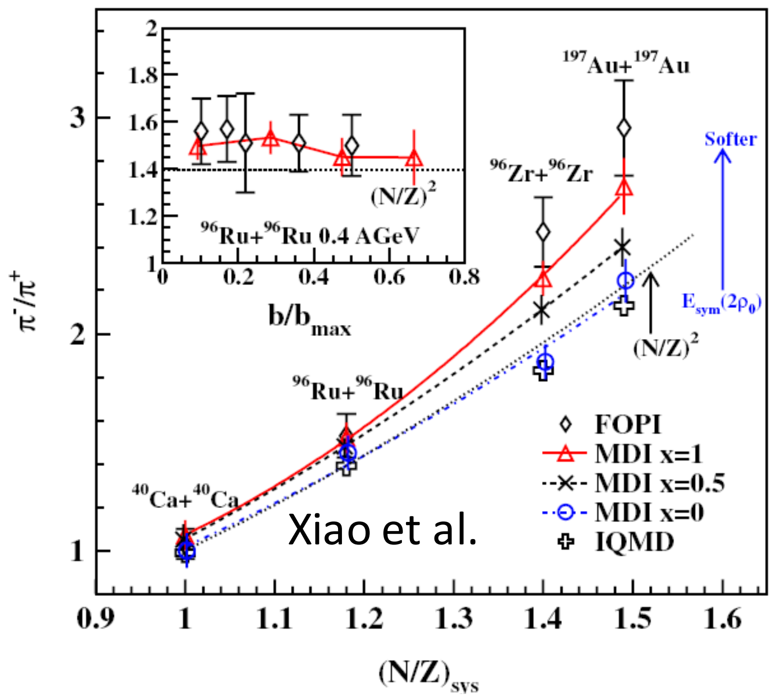
$$\Delta^{++} \longrightarrow p + \pi^+$$

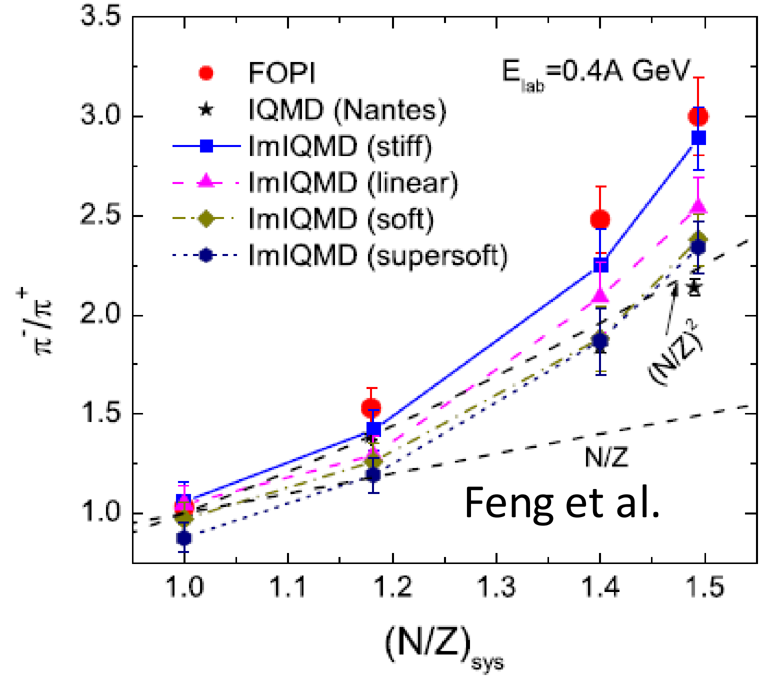


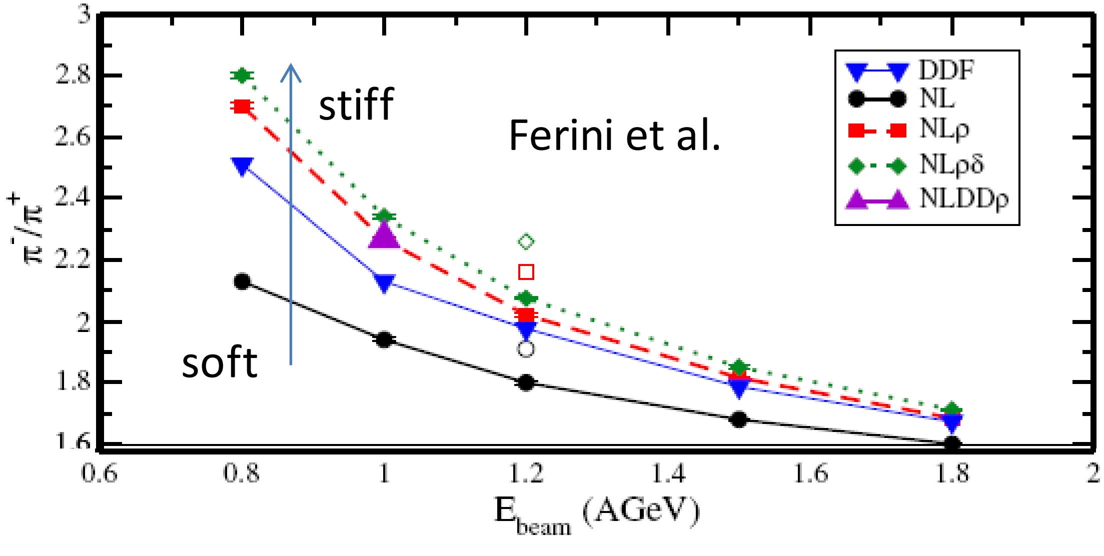
A stiffer Esym leads to a less neutron-rich high-density phase and a smaller π^-/π^+ ratio.

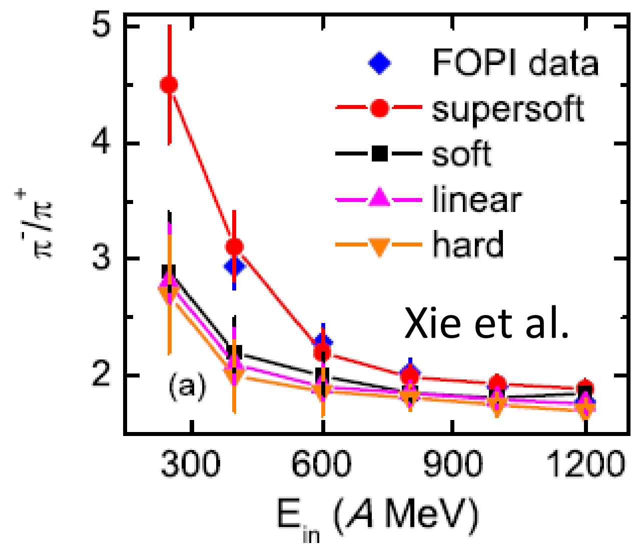
B.A. Li, NPA, 2002

Studies on π^-/π^+ ratio and Esym

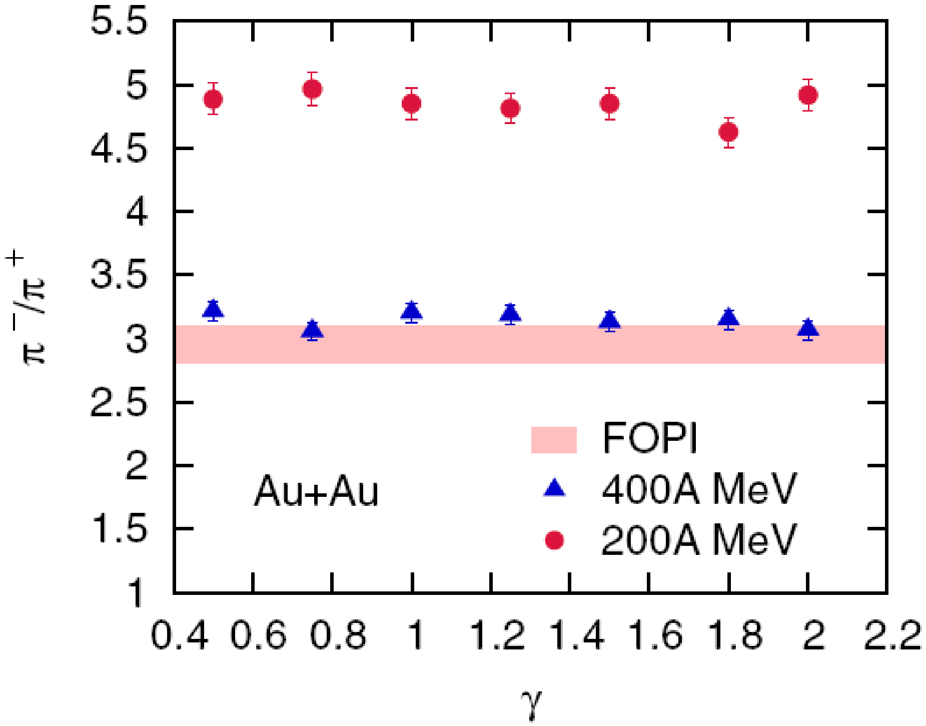








Pion mean-field effects



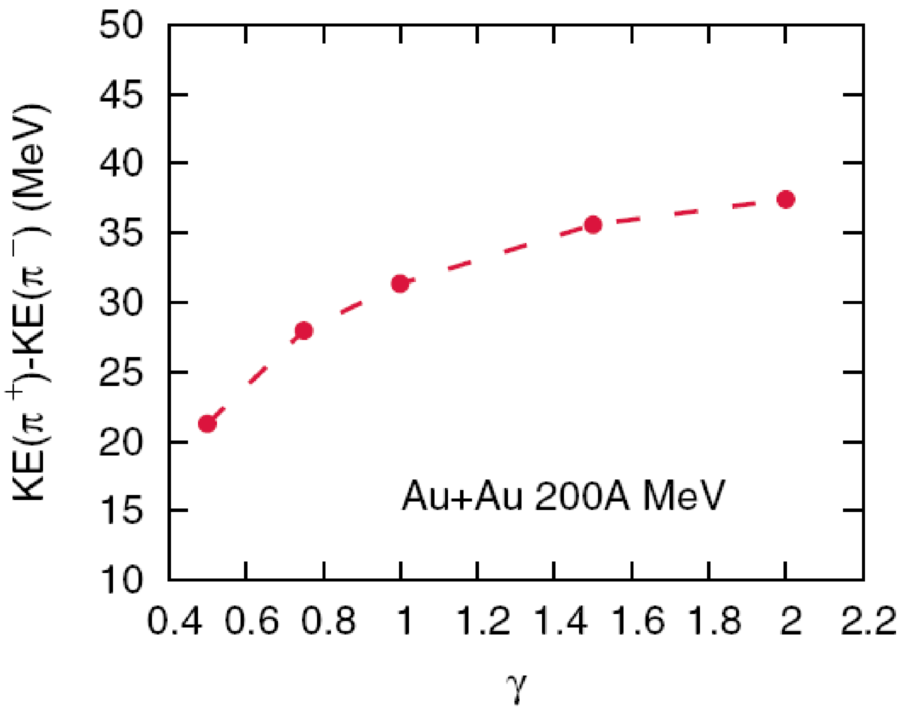
π^-/π^+ ratio is insensitive to Esym
Kinetic energy difference of π^+ and π^- is sensitive to Esym

Potential part of Esym:

$$S_{\text{int0}}(\rho) = S_{\text{int0}}\left(\frac{\rho}{\rho_0}\right)^{\gamma}$$

Effective pion s-wave potential:

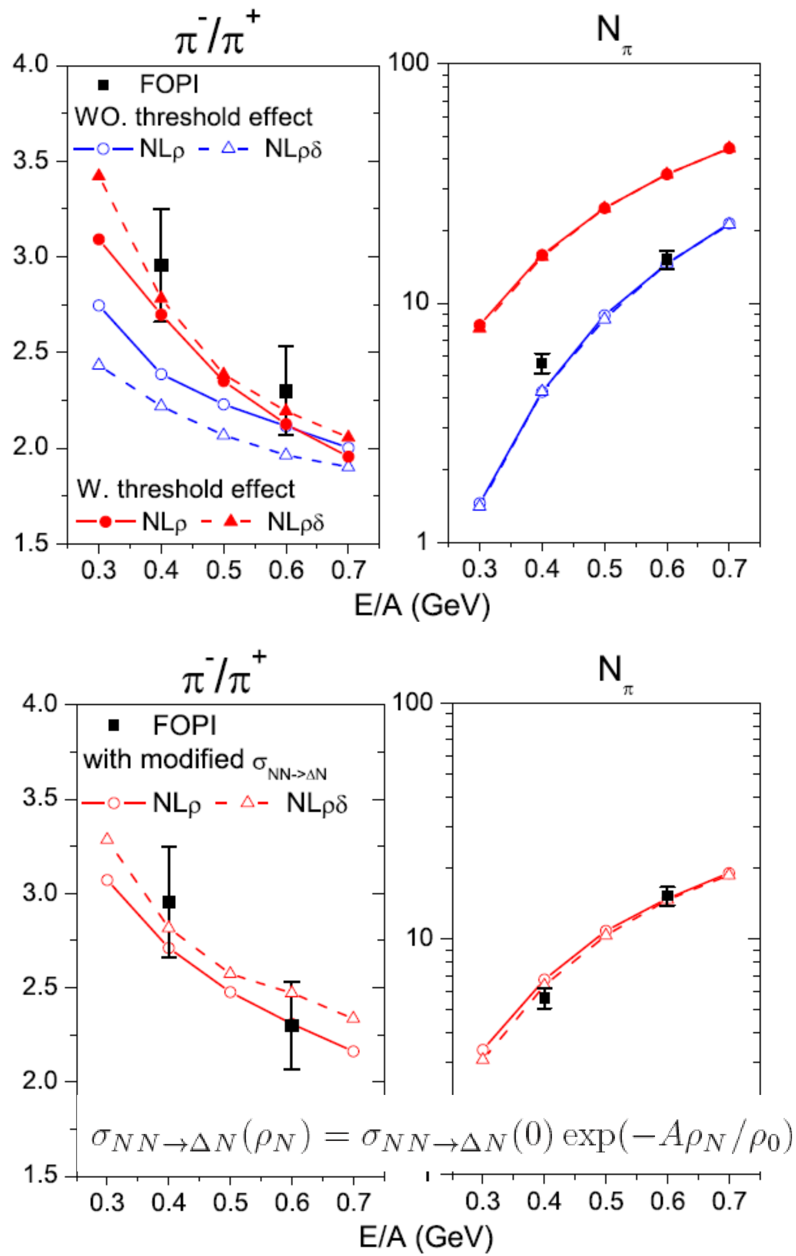
$$U_{\pi^\pm} = \mp 8\, S_{\text{int0}}\, \rho_T\, \frac{\rho^{\gamma-1}}{\rho_0^{\gamma}}$$



J. Hong and P. Danielewicz, PRC, 2014

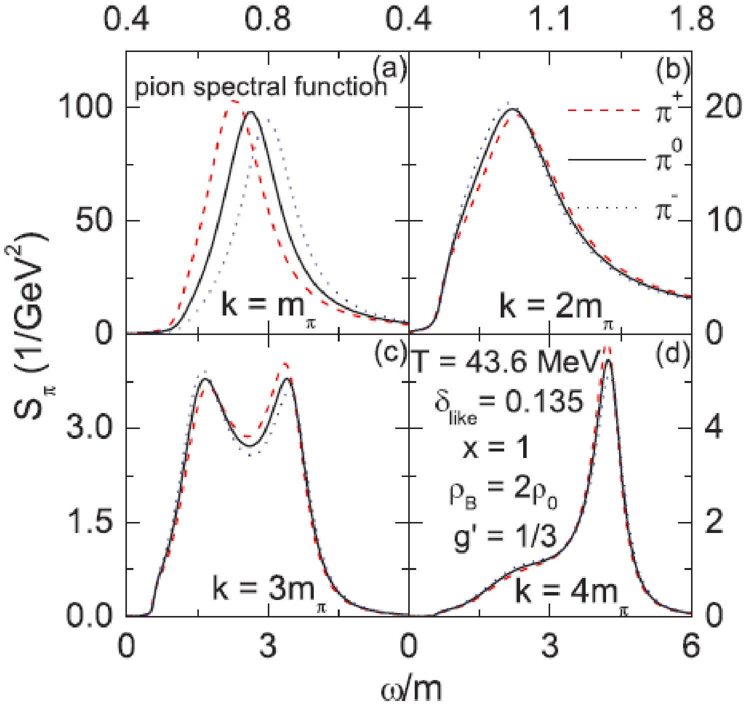
Pion threshold effect

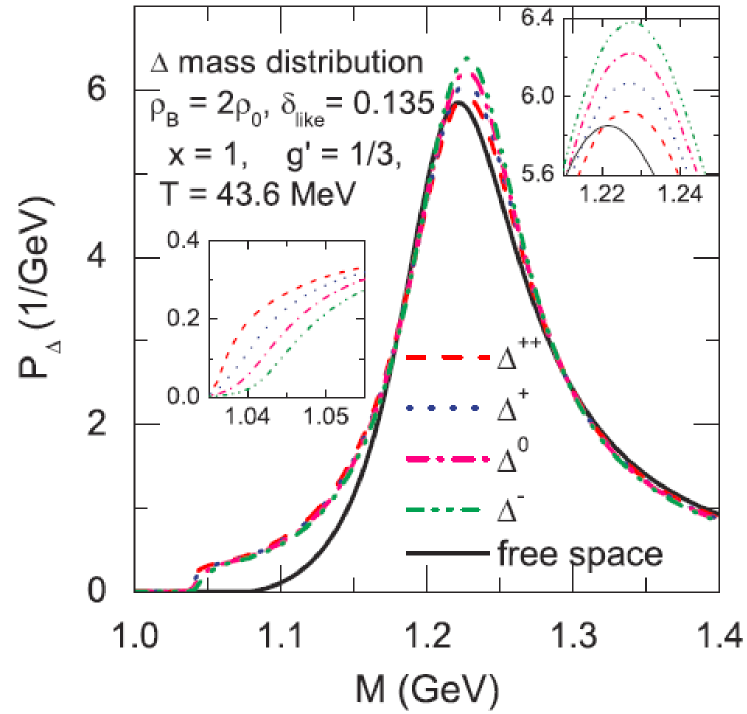
scattering	$\Sigma_1^s + \Sigma_2^s - \Sigma_3^s - \Sigma_4^s$	$\Sigma_1^\mu + \Sigma_2^\mu - \Sigma_3^\mu - \Sigma_4^\mu$
elastic		
$NN \to NN$	0	0
$N\Delta \to N\Delta$	0	0
$\Delta\Delta \to \Delta\Delta$	0	0
inelastic		
$pp \to n\Delta^{++}$	$-2g_\delta\delta_3$	$2g_\rho\rho_3^\mu$
$pp \to p\Delta^{+}$	$-(2/3)g_\delta\delta_3$	$(2/3)g_\rho\rho_3^\mu$
$pn \to n\Delta^{+}$	$-(2/3)g_\delta\delta_3$	$(2/3)g_\rho\rho_3^\mu$
$pn \to p\Delta^{0}$	$(2/3)g_\delta\delta_3$	$-(2/3)g_\rho\rho_3^\mu$
$nn \to n\Delta^{0}$	$(2/3)g_\delta\delta_3$	$-(2/3)g_\rho\rho_3^\mu$
$nn \to p\Delta^{-}$	$2g_\delta\delta_3$	$-2g_\rho\rho_3^\mu$
decay	$\Sigma_1^s - \Sigma_2^s$	$\Sigma_1^\mu - \Sigma_2^\mu$
$\Delta^{++} \to p\pi^{+}$	0	0
$\Delta^{+} \to p\pi^{0}$	$(2/3)g_\delta\delta_3$	$-(2/3)g_\rho\rho_3^\mu$
$\Delta^{+} \to n\pi^{+}$	$-(4/3)g_\delta\delta_3$	$(4/3)g_\rho\rho_3^\mu$
$\Delta^{0} \to p\pi^{-}$	$(4/3)g_\delta\delta_3$	$-(4/3)g_\rho\rho_3^\mu$
$\Delta^{0} \to n\pi^{0}$	$-(2/3)g_\delta\delta_3$	$(2/3)g_\rho\rho_3^\mu$
$\Delta^{-} \to n\pi^{-}$	0	0



T. Song and C.M. Ko, arXiv: 1403.7363[nucl-th]

Pion s-wave and p-wave interaction





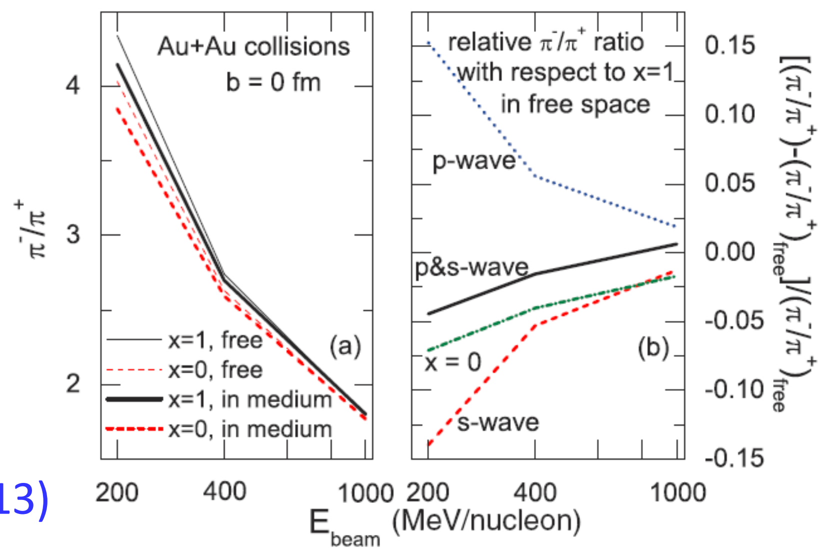
Softening of pion spectral function

$$s-wave:$$

$$N+N \rightarrow N+N+\pi$$

$$m_{\pi^-}\uparrow, m_{\pi^+}\downarrow, \pi^-/\pi^+\downarrow$$

J. Xu et al., PRC 81, 024910 (2010); PRC 87, 067601 (2013)



Softeining of Δ mass distribution

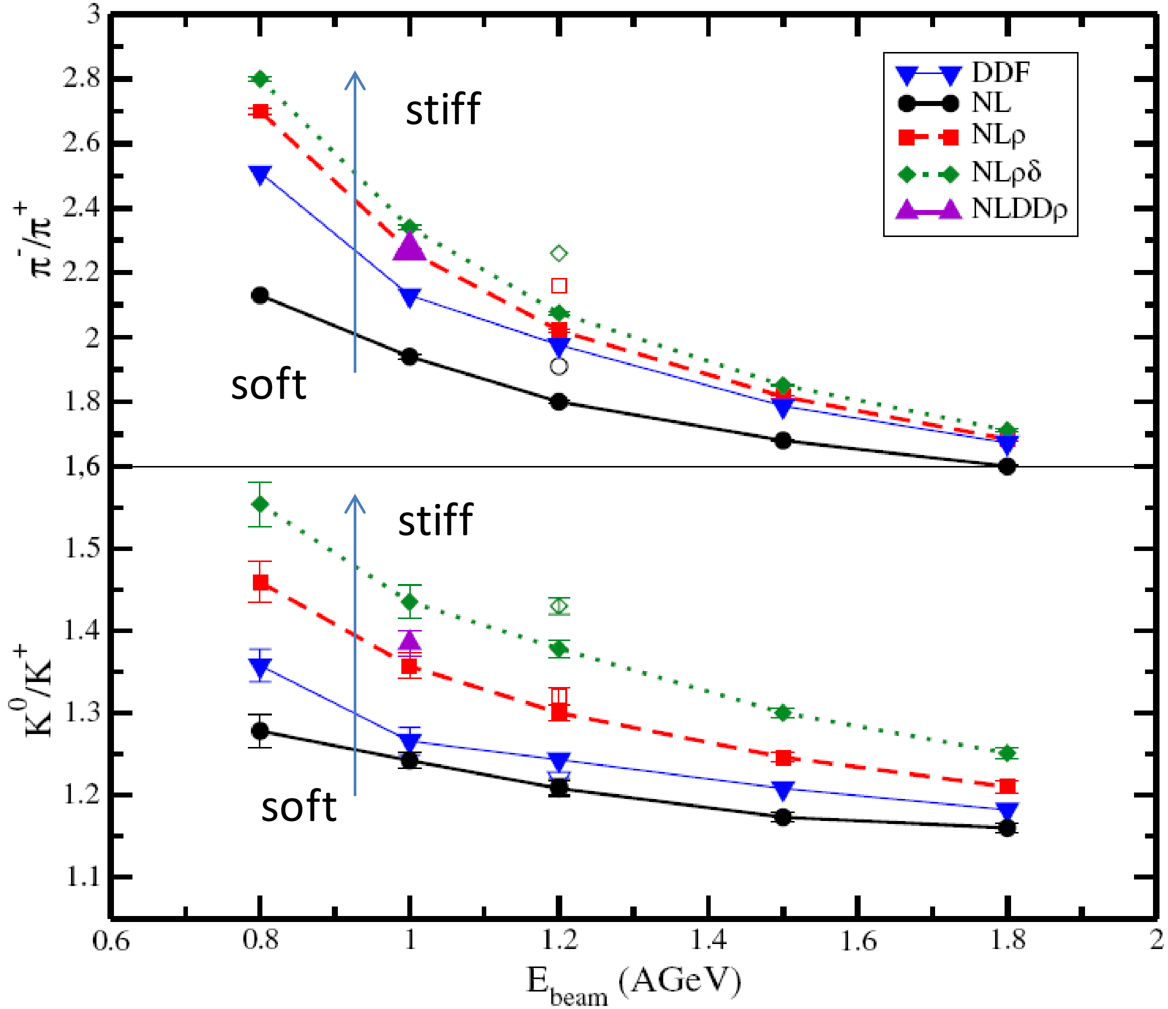
$$p-wave:$$

$$N+N \rightarrow N+\Delta$$

$$\Delta \rightarrow N+\pi$$

$$m_{\pi^-}\downarrow, m_{\pi^+}\uparrow, \pi^-/\pi^+\uparrow$$

K^0/K^+ ratio: probe of Esym at suprasaturation densities



G. Ferini et al., PRL, 2006

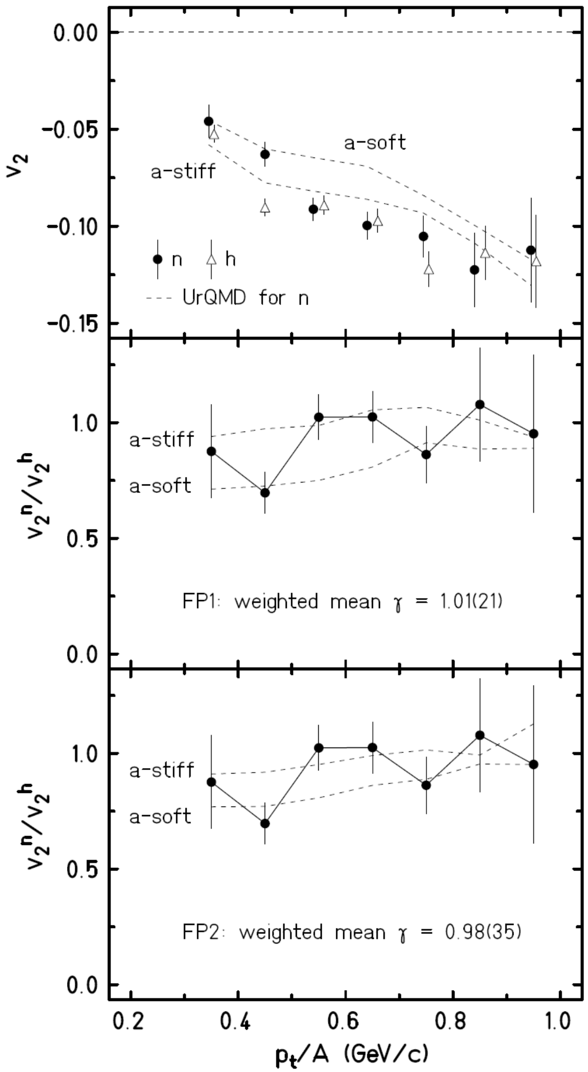
$$n + \pi^- \rightarrow \Sigma^- + K^0$$

$$p + \pi^+ \rightarrow \Sigma^+ + K^+$$

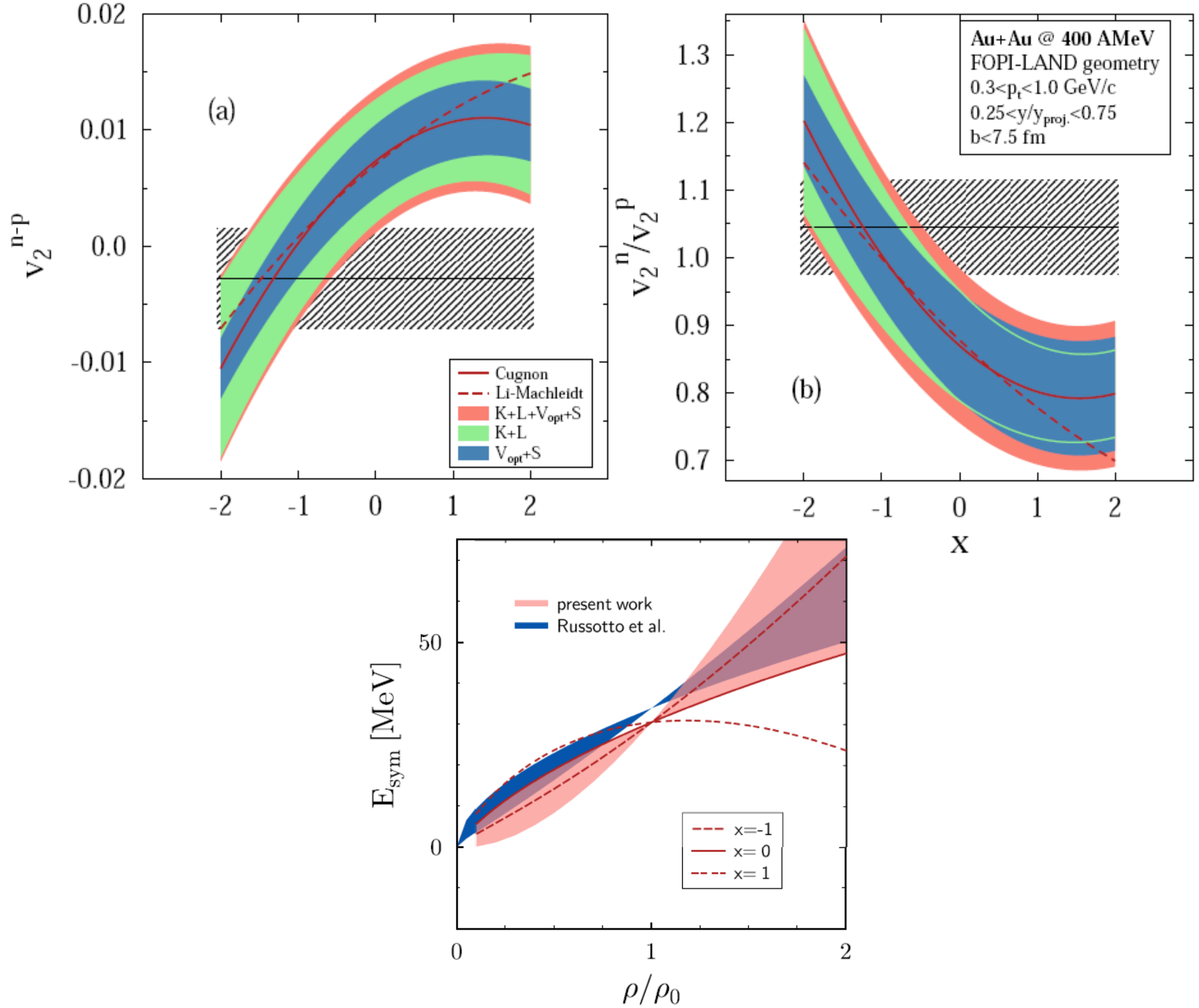
DDF, NL … : different parameterizations of RMF, corresponding to different Esym

A more neutron-rich high-density phase leads to more K^0, and generally a larger K^0/K^+ ratio (other effects? threshold, Kaon potential).

Preequilibrium np flow splitting: probe of Esym at suprasaturation densities

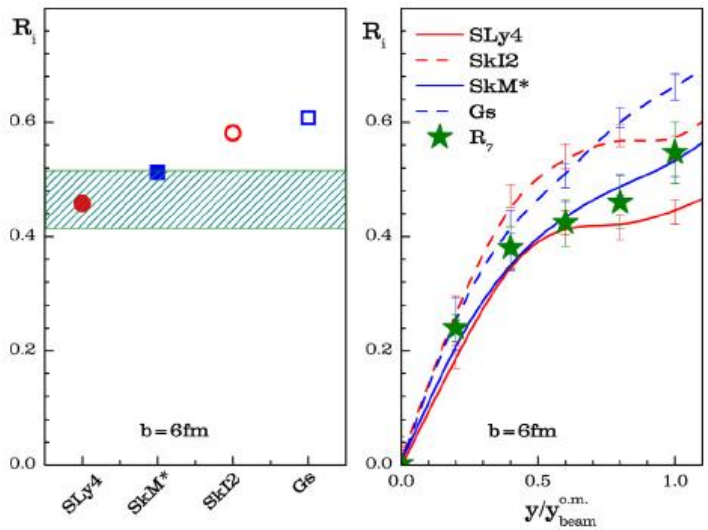


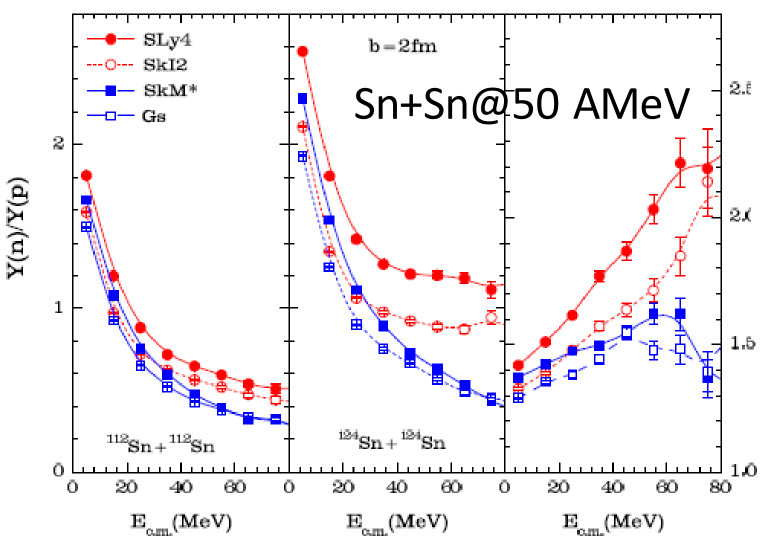
P. Russotto et al., PLB, 2011



M.D. Cozma et al., PRC, 2013

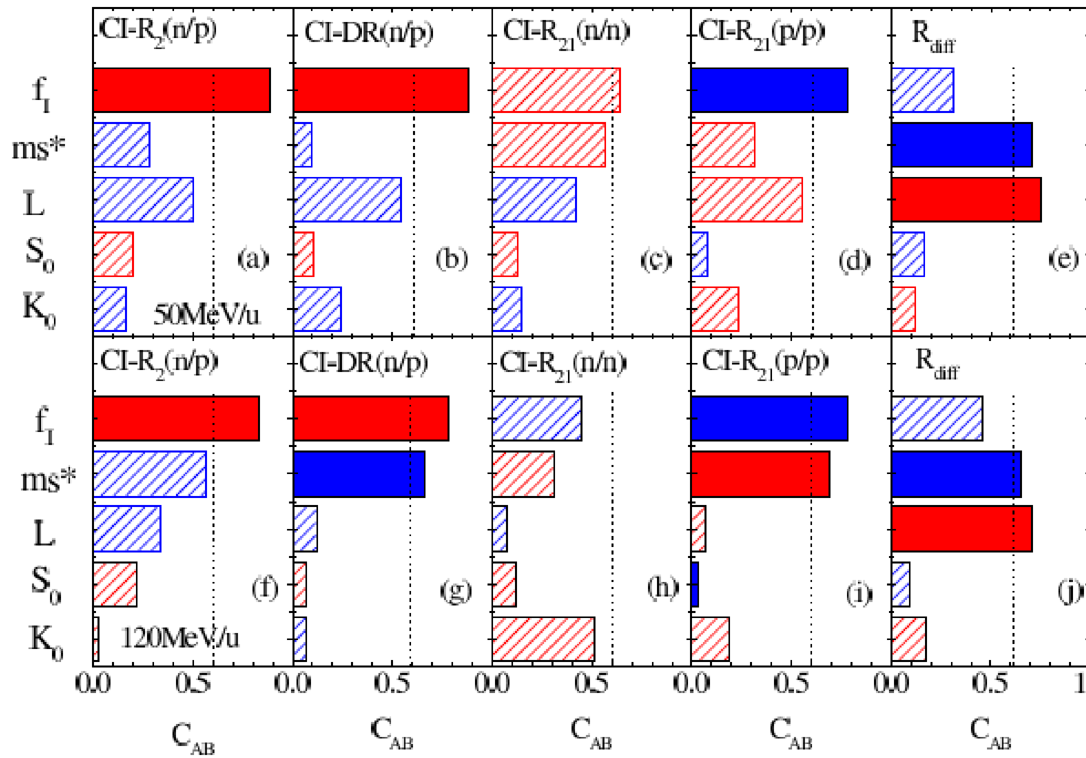
More detailed studies on effects from Esym and mv*





Y.X. Zhang, M.B. Tsang, Z.X. Li, and H. Liu, PLB, 2014

$$f_I = \frac{1}{2\delta}\left(\frac{m}{m_n^*} - \frac{m}{m_p^*}\right) = \frac{m}{m_s^*} - \frac{m}{m_v^*}$$



Covariance analysis

$$C_{AB} = \frac{cov(A,B)}{\sigma(A)\sigma(B)}$$

Blue:
negative correlation
Red:
Positive correlation

Y.X. Zhang, M.B. Tsang, Z.X. Li, and H. Liu, arXiv: 1507.06718 [nucl-th]

Part II
transport comparison project

PHYSICAL REVIEW C **93**, 044609 (2016)

Understanding transport simulations of heavy-ion collisions at 100A and 400A MeV: Comparison of heavy-ion transport codes under controlled conditions

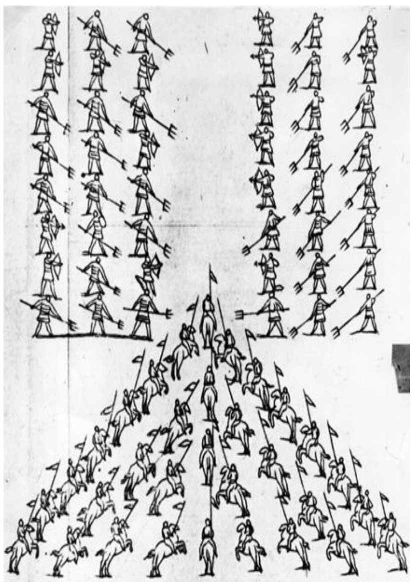
Jun Xu,[1,*] Lie-Wen Chen,[2,†] ManYee Betty Tsang,[3,‡] Hermann Wolter,[4,§] Ying-Xun Zhang,[5,‖] Joerg Aichelin,[6] Maria Colonna,[7] Dan Cozma,[8] Pawel Danielewicz,[3] Zhao-Qing Feng,[9] Arnaud Le Fèvre,[10] Theodoros Gaitanos,[11] Christoph Hartnack,[6] Kyungil Kim,[12] Youngman Kim,[12] Che-Ming Ko,[13] Bao-An Li,[14] Qing-Feng Li,[15] Zhu-Xia Li,[5] Paolo Napolitani,[16] Akira Ono,[17] Massimo Papa,[18] Taesoo Song,[19] Jun Su,[20] Jun-Long Tian,[21] Ning Wang,[22] Yong-Jia Wang,[15] Janus Weil,[19] Wen-Jie Xie,[23] Feng-Shou Zhang,[24] and Guo-Qiang Zhang[1]

Nuclear EOS, E_{sym}

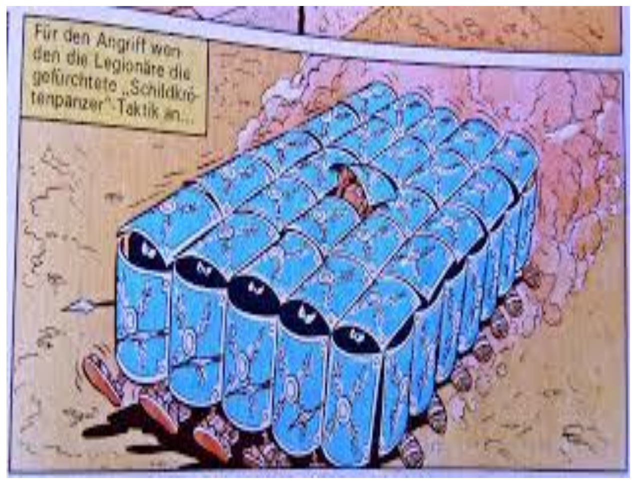
How reliable?

Heavy-ion experiments ⟷ **Mean-field potential**

Transport simulations



Initialization



Mean Field

NN scatterings

theoretical uncertainties?

Compare results from various models

Images from
H. Wolter

History of Transport Model Comparison Project

- **Trento I (2004):** energy 1-2 GeV/A, emphasis on particle production π,K

mean field and Pauli blocking not quite so important

Summary Published in **J.Phys. G 31, 741 (2005)**

- **Trento II (2009):** energy 100, 400 MeV/A

Uncertainties not well understood, results not published

- Shanghai (2014): Mainly 100 AMeV, also 400 AMeV

observables: stopping, flow

Nonobservables: stability, scattering and Pauli blocking rate

Results published in **Phys. Rev. C 93, 044609 (2016)**

- **Zoom(?) (2016): box calculation (Cascade, Vlasov, pion production)**

Participating Codes

Boltzmann-Uehling-Uhlenbeck(BUU)-type models (9)

BUU-type	code correspondents	energy range	reference
BLOB	P.Napolitani,M.Colonna	$0.01 \sim 0.5$	[19]
GIBUU-RMF	J.Weil	$0.05 \sim 40$	[20]
GIBUU-Skyrme	J.Weil	$0.05 \sim 40$	[20]
IBL	W.J.Xie,F.S.Zhang	$0.05 \sim 2$	[21]
IBUU	J.Xu,L.W.Chen,B.A.Li	$0.05 \sim 2$	[11, 22]
pBUU	P.Danielewicz	$0.01 \sim 12$	[23]
RBUU	K. Kim,Y.Kim,T.Gaitanos	$0.05 \sim 2$	[24]
RVUU	T.Song,G.Q.Li,C.M.Ko	$0.05 \sim 2$	[25]
SMF	M.Colonna,P.Napolitani	$0.01 \sim 0.5$	[26]

In GeV

Find representative references for each code in
Phys. Rev. C 93, 044609 (2016), arXiv: 1603:08149 [nucl-th]

Participating Codes

Quantum-Molecular-Dynamics(QMD)-type models(9)

QMD-type	code correspondents	energy range	reference
AMD	A.Ono	$0.01 \sim 0.3$	[27]
IQMD-BNU	J.Su,F.S.Zhang	$0.05 \sim 2$	[28]
IQMD	C.Hartnack,J.Aichelin	$0.05 \sim 2$	[29, 30]
CoMD	M.Papa	$0.01 \sim 0.3$	[31]
ImQMD-CIAE	Y.X.Zhang,Z.X.Li	$0.02 \sim 0.4$	[32]
IQMD-IMP	Z.Q.Feng	$0.01 \sim 10$	[33]
IQMD-SINAP	G.Q.Zhang	$0.05 \sim 2$	[34]
TuQMD	D.Cozma	$0.1 \sim 2$	[35]
UrQMD	Y.J.Wang,Q.F.Li	$0.05 \sim 200$	[36, 37]

In GeV

ImQMD-GXNU: low-energy fusion reaction

Find representative references for each code in
Phys. Rev. C 93, 044609 (2016), arXiv: 1603:08149 [nucl-th]

A taste of BUU-type models

BUU equation: $$\left(\frac{\partial}{\partial t}+\frac{\vec{p}}{m}\cdot\nabla_r-\nabla_r U\cdot\nabla_p\right)f(\vec{r},\vec{p};t)=I_{coll}[f;\sigma_{12}]$$

$$I_{coll} = \frac{1}{(2\pi)^6}\int dp_2 dp_3 d\Omega |v-v_2| \frac{d\sigma_{12}^{med}}{d\Omega}(2\pi)^3\delta(p+p_2-p_3-p_4) \times [f_3 f_4(1-f)(1-f_2)-f f_2(1-f_3)(1-f_4)]$$

test-particle (TP) method: parallel events

C.Y. Wong, PRC 25, 1460 (1982); G.F. Bertsch and S. Das Gupta, Phys. Rep. 160, 189 (1988).

Point particle or finite size (triangular, Gaussian)

$$f(\vec{r},\vec{p};t)=\frac{1}{N_{TP}}\sum_{i=1}^{N_{TP}A} g(\vec{r}-\vec{r}_i(t))\tilde{g}(\vec{p}-\vec{p}_i(t))$$

Equation of motion from pseudoparticle method:

$$d\vec{r}_i/dt=\nabla_{\vec{p}_i}H; \qquad d\vec{p}_i/dt=-\nabla_{\vec{r}_i}H.$$

A taste of QMD-type models

Total wave function as products of single-particle wave function:

$$\Psi(\vec{r}_1, ..., \vec{r}_N; t) = \Pi \phi_i(\vec{r}_i; t),$$

$$\phi_i(\vec{r}_i; t) = \frac{1}{(2\pi)^{3/4}(\Delta x)^{3/2}} \exp\left[-\frac{(\vec{r}_i - \vec{R}_i(t))^2}{(2\Delta x)^2} + i\vec{r}_i \cdot \vec{P}_i(t)\right]$$

Wigner function (phase-space distribution):

$$f_i(\vec{r}, \vec{p}) = \frac{1}{(\pi\hbar)^3} \exp\left[-\frac{\left(\vec{r} - \vec{R}_i(t)\right)^2}{2(\Delta x)^2}\right] \exp\left[-\frac{\left(\vec{p} - \vec{P}_i(t)\right)^2 \cdot 2(\Delta x)^2}{\hbar^2}\right]$$

Canonical equation of motion:

$$d\vec{R}_i / dt = \nabla_{\vec{p}_i} H, \qquad d\vec{P}_i / dt = -\nabla_{\vec{r}_i} H$$

Ch. Hartnack et al., PRC 495, 303 (1989); J. Aichelin, Phys. Rep. 202, 233 (1988).

AMD and FMD: wave function antisymmetrized

Homework list for heavy-ion part

Mode B). Au+Au@100 AMeV   b = 7 and 20 fm

B.1) No Surface Term mode: Turn off the surface term in the mean field (e.g., the Yukawa interaction in the QMD-like models, the gradient term in the BUU-like models). Allow collisions between all nucleons (or TP).

B-Full: both mean field and NN scattering

B.2) Vlasov mode: Turn off all collisions and use mean field as in B.1 (no surface terms)..

B-Vlasov: only mean field

B.3) Cascade mode: Turn off all interaction potentials in B.1 mode

B-Cascade: only NN scattering

Mode D). Au+Au@400 AMeV   b = 7 fm

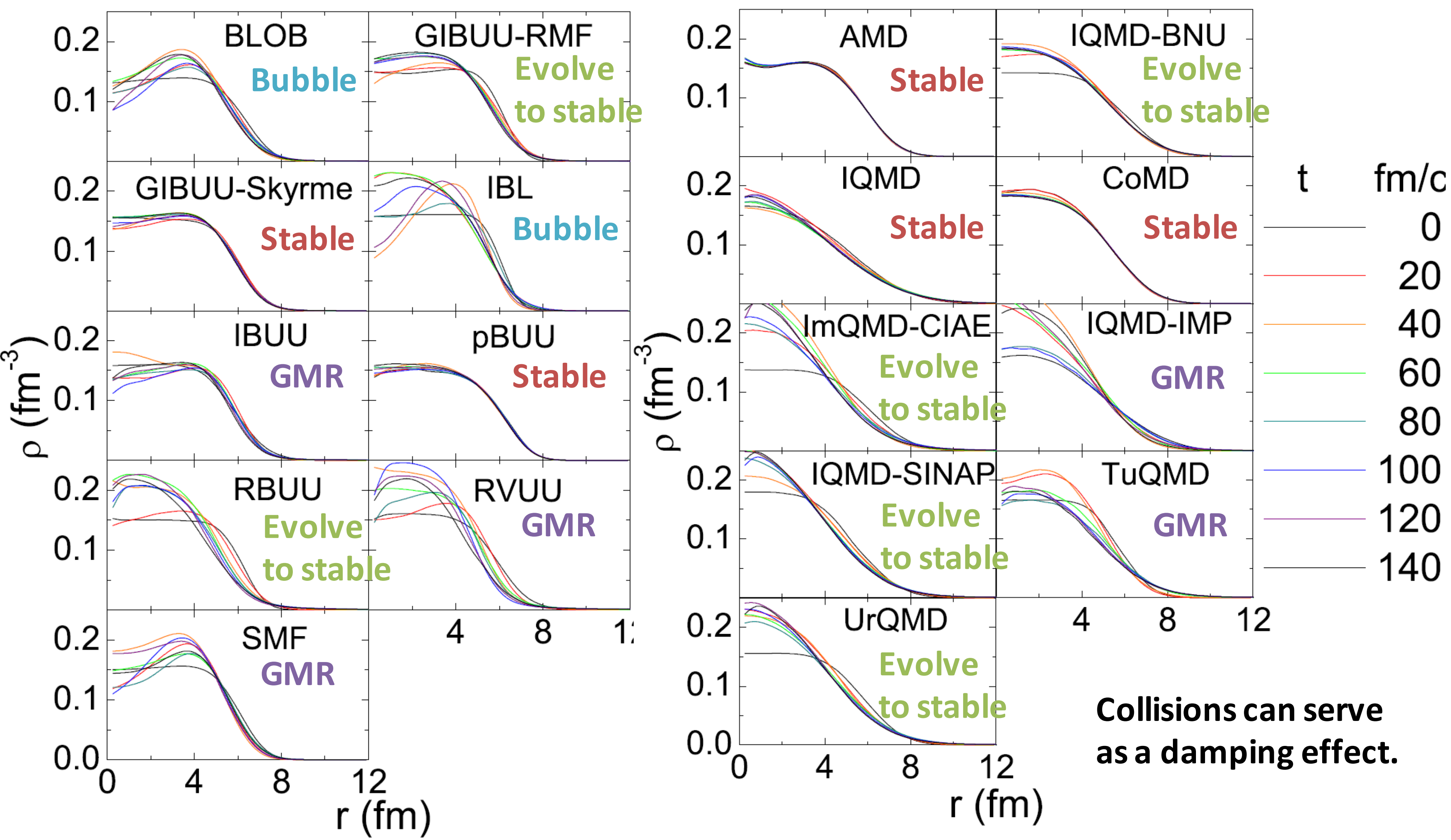
D-Full: 400AMeV, both mean field and NN scattering

Try to have:

Same reaction condition: initialization (space, momentum)
Same mean-field potential and constant isotropic NN scattering cross section

BUU: 100 test particles, 10 runs; QMD: 1000 events

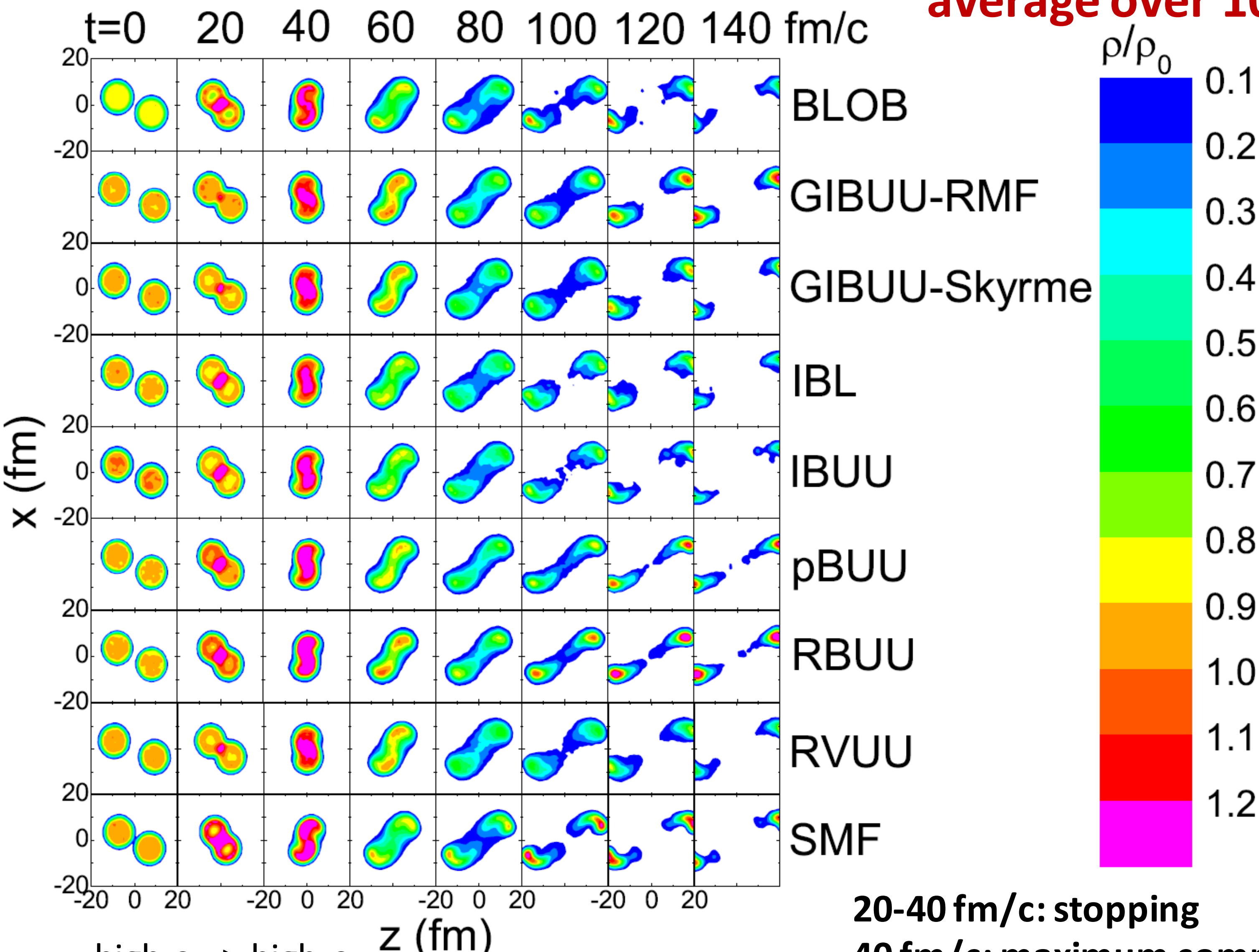
Stability (b=20 fm)



Collisions can serve as a damping effect.

Density evolution at b = 7 fm - BUU

average over 1000 events



high ρ => high ρ

Density => attempted collisions, mean field

Pauli blocking => successful collisions

20-40 fm/c: stopping

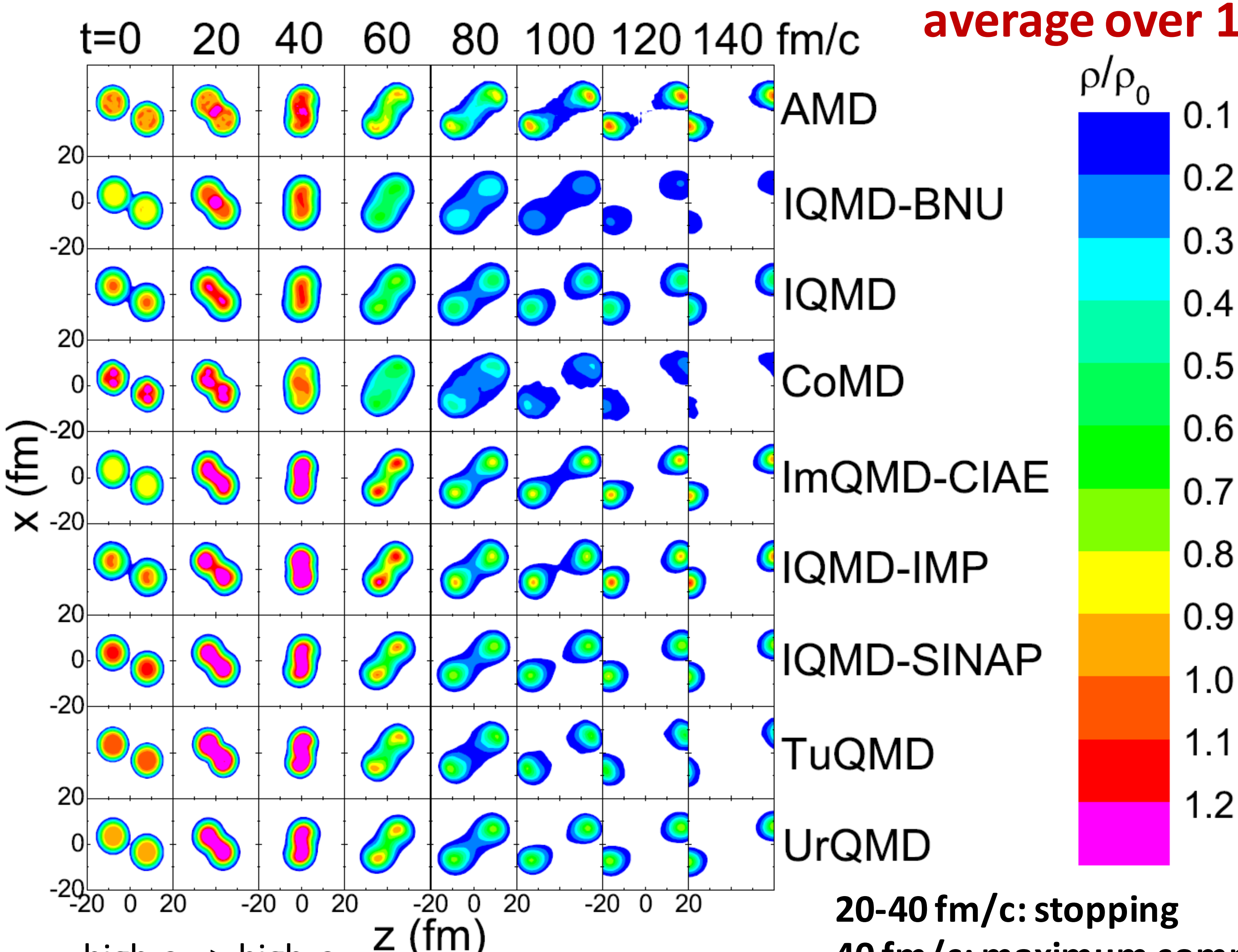
40 fm/c: maximum compression

60-80 fm/c: sideward flow developed

Around 100 fm/c: neck

Density evolution at b = 7 fm - QMD

average over 1000 events



high ρ => high ρ

Density => attempted collisions, mean field

Pauli blocking => successful collisions

20-40 fm/c: stopping

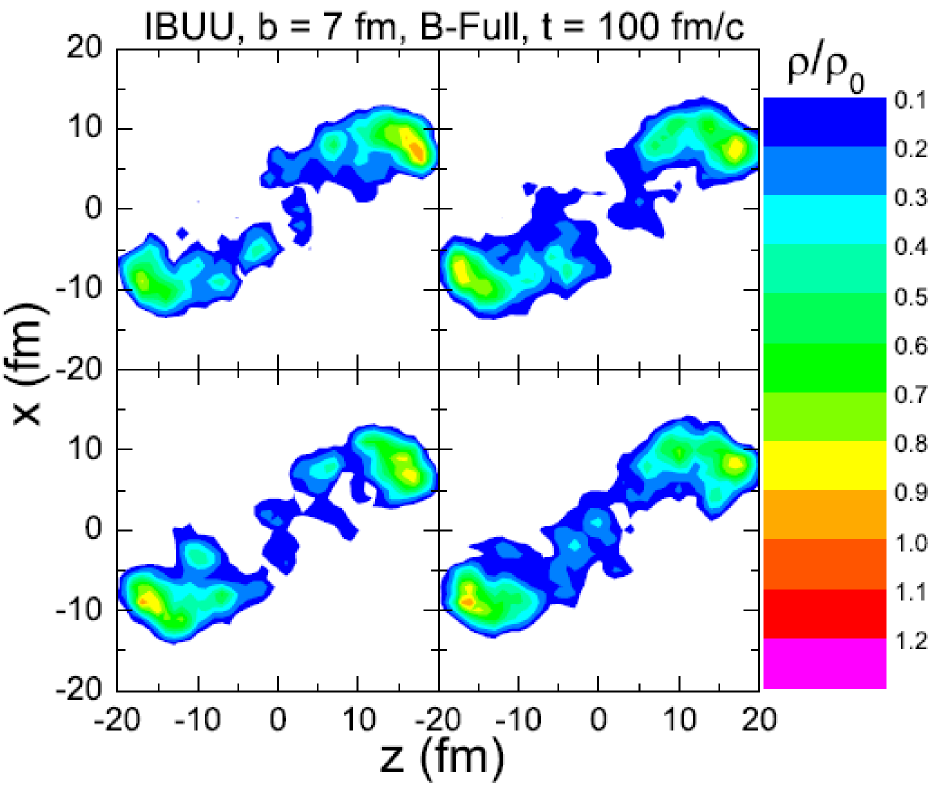
40 fm/c: maximum compression

60-80 fm/c: sideward flow developed

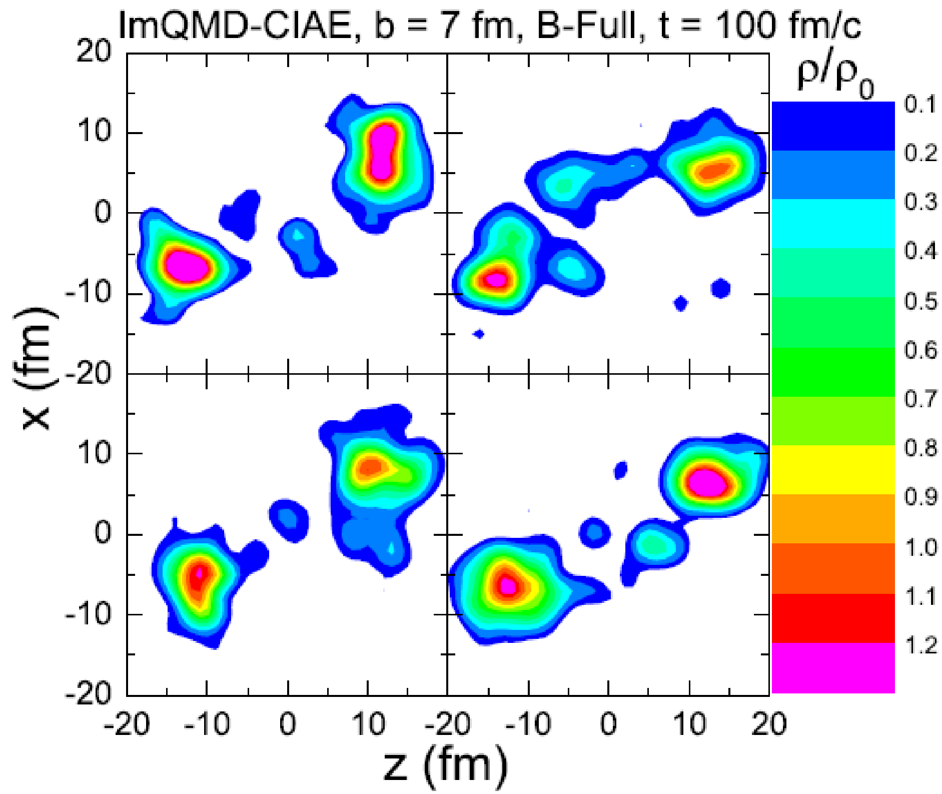
Around 100 fm/c: neck

Fluctuation in BUU and QMD

4 runs with 100 TPs per nucleon



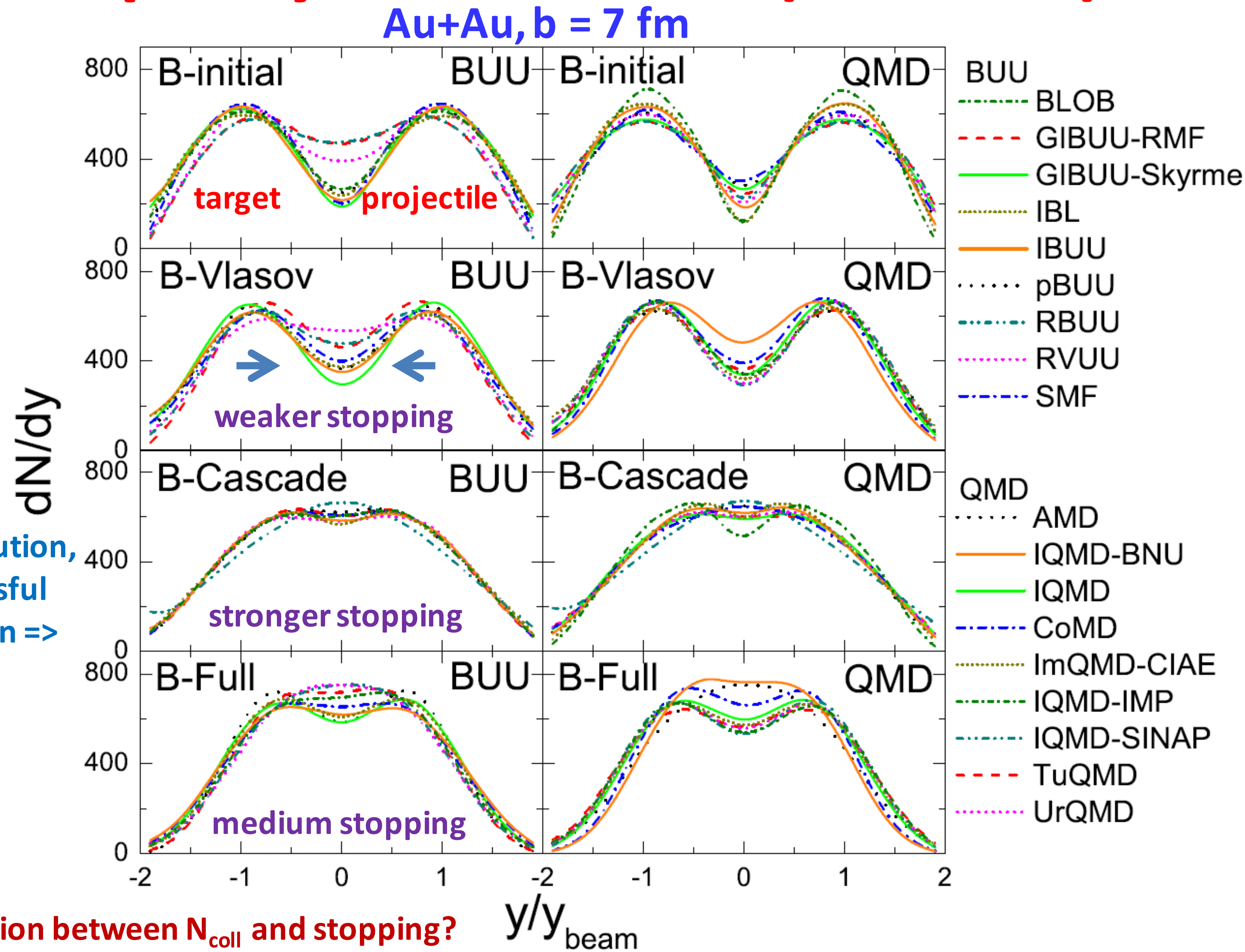
4 individual events



Fluctuation related to N_{TP}

Fluctuation related to the Gaussian width

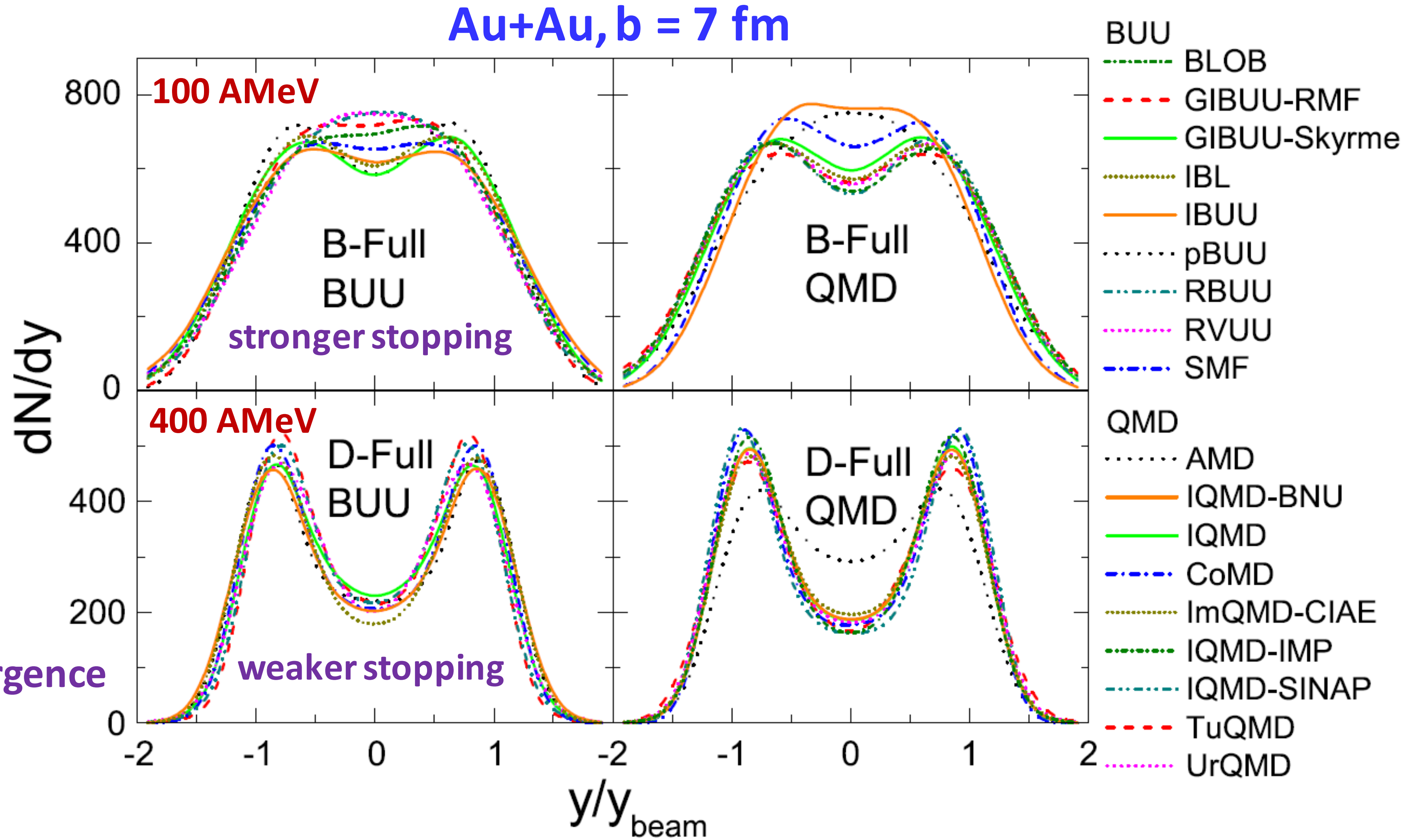
Rapidity distribution (B-mode)



Initial distribution, successful collision =>

Correlation between N_{coll} and stopping?

Rapidity distribution (Full mode)

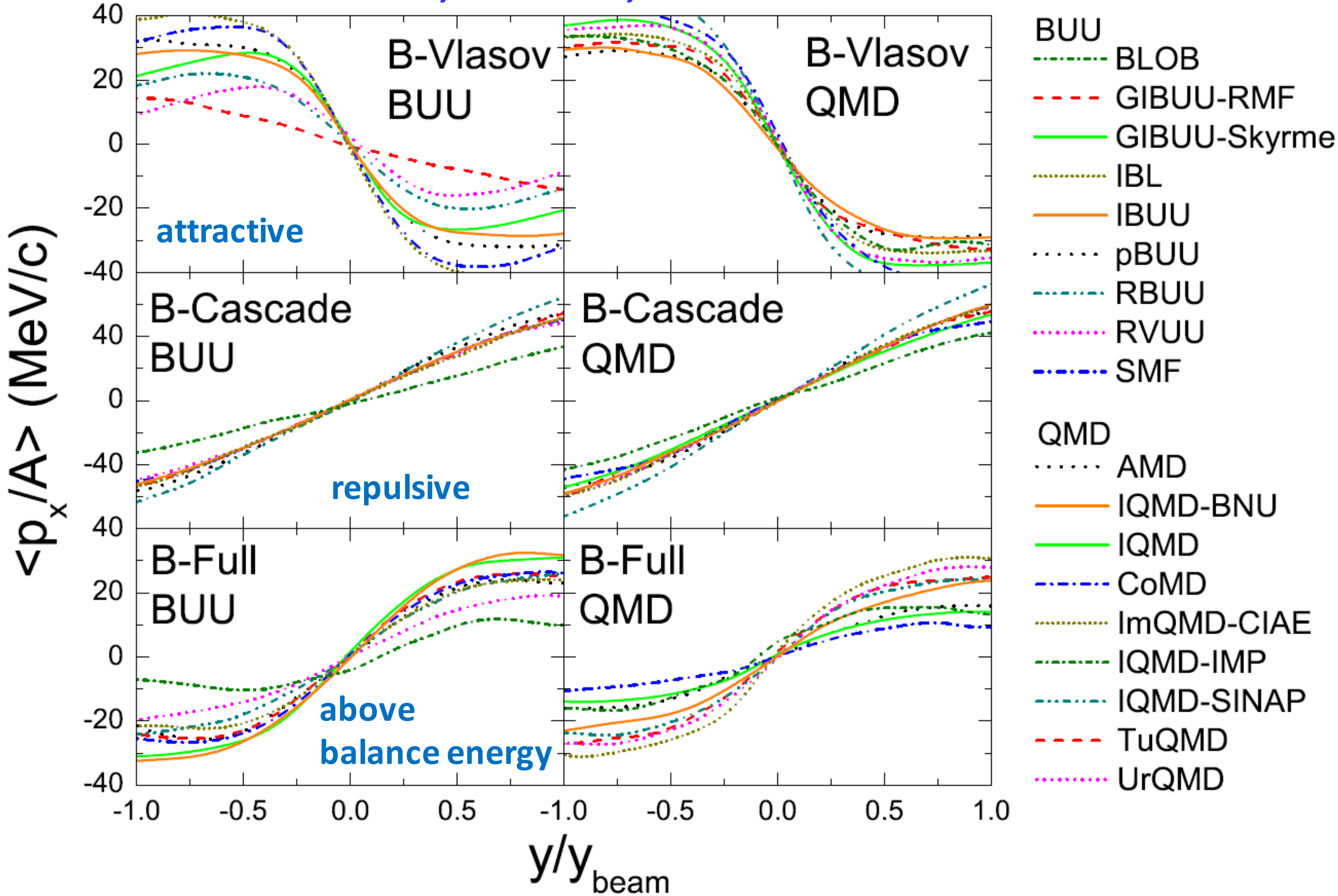


Good convergence

400AMeV: weaker Pauli blocking, less sensitive to initialization, good convergence of N_{coll} at larger $\sqrt{s}$

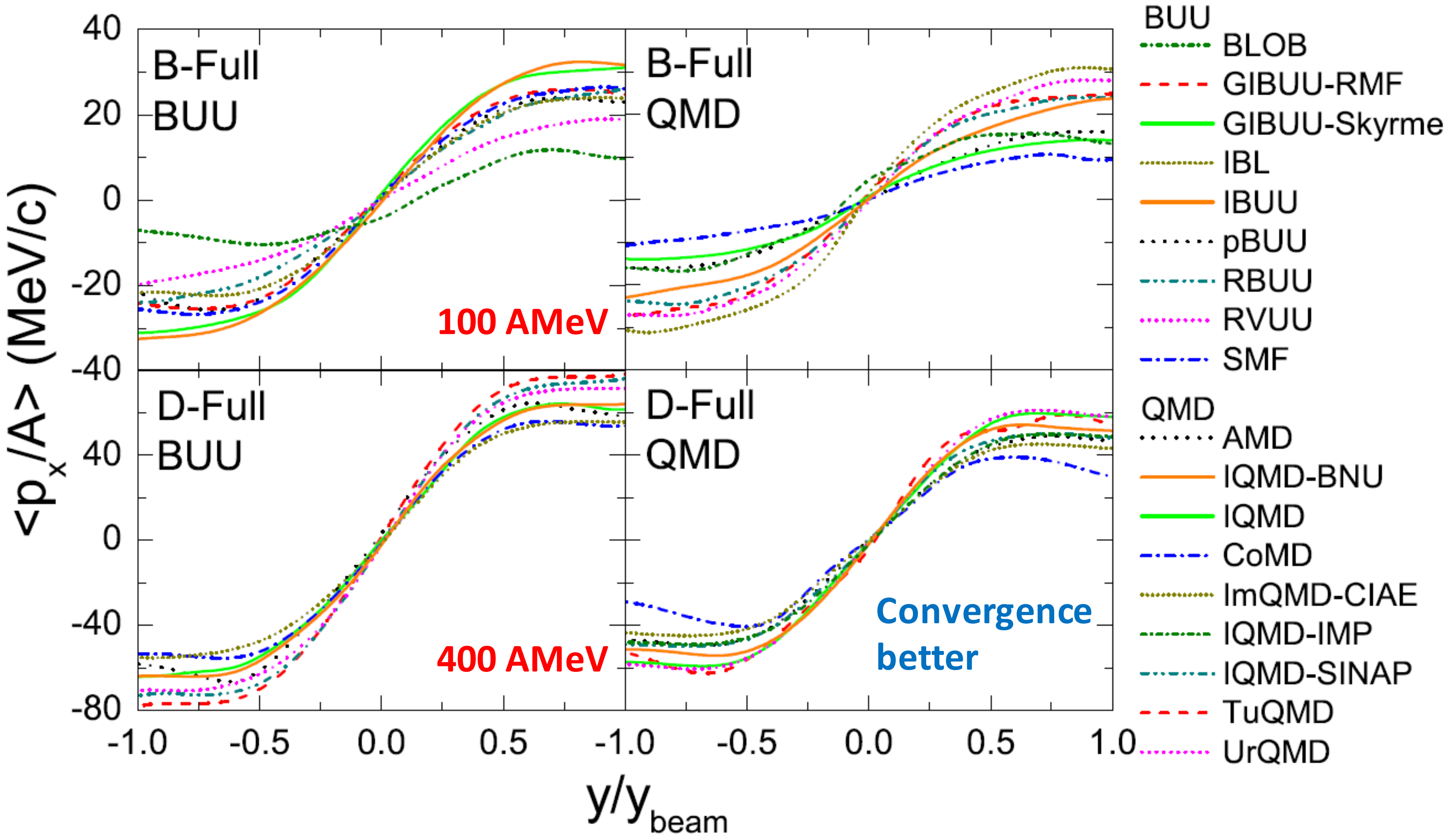
Transverse flow (B-mode)

Au+Au, b = 7 fm, 100 AMeV

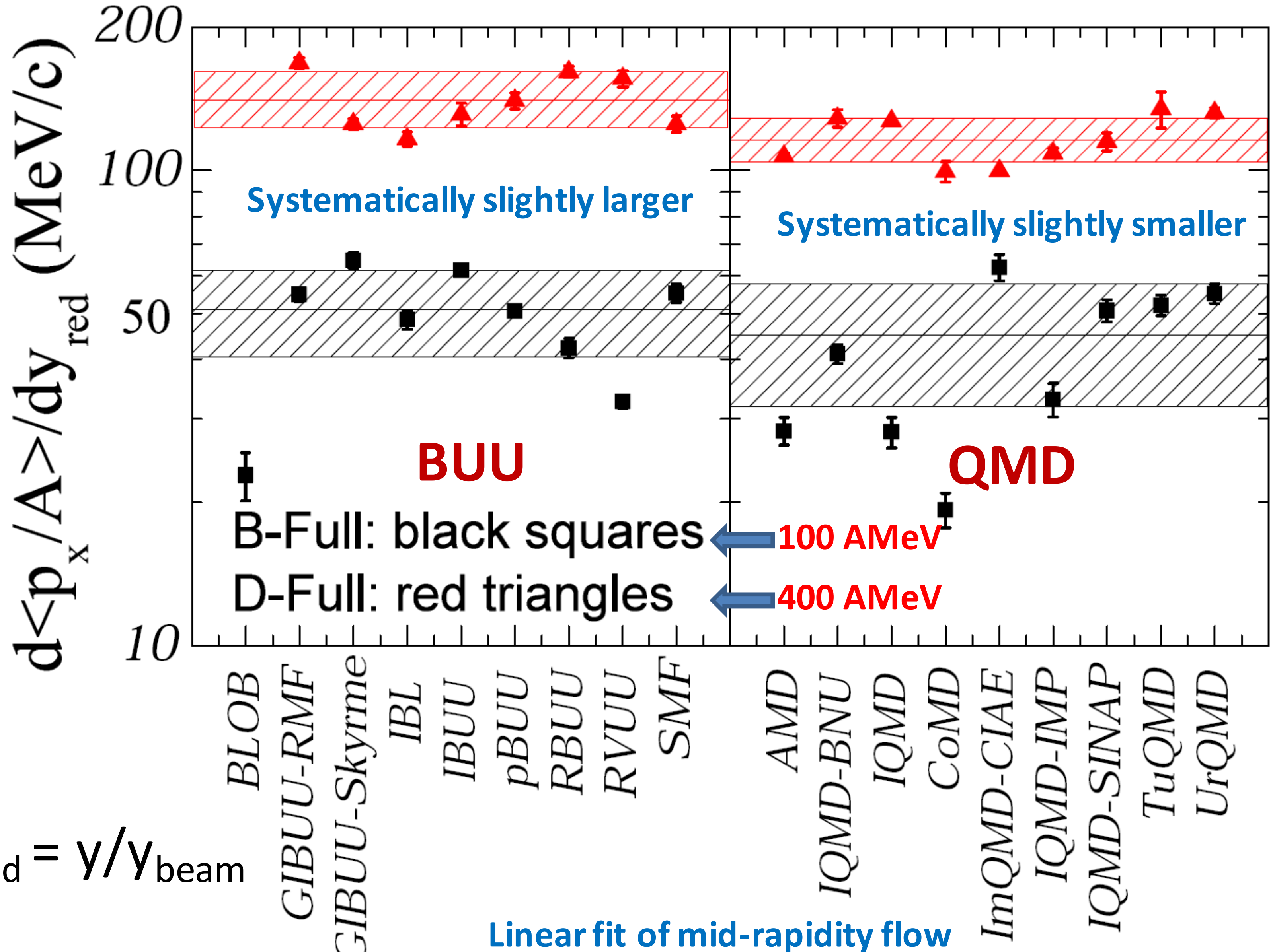


Transverse flow (Full-mode)

Au+Au, b = 7 fm



Uncertainties: initialization, Pauli blocking



$y_{red} = y/y_{beam}$

Theoretical uncertainties of flow parameter: about 30% at 100 AMeV, 13% at 400 AMeV

Transport2016 - box calculation

Common setup: L*L*L box with periodic boundary condition (L=20 fm)
1280 nucleons=> saturation density

Hw1: Cascade calculation, constant isotropic NN scattering cross section, uniform, Fermi Dirac

- CT0, CT5: T = 0 and 5 MeV without Pauli blocking
- CBOP1T0, CBOP1T5: T = 0 and 5 MeV with Pauli blocking in each code
- CBOP2T0, CBOP2T5: T = 0 and 5 MeV with ideal Pauli blocking

Hw2: Vlasov calculation, $\rho(z) = \rho_0 + a_\rho \sin(kz)$ $k = n\,2\pi/L$, n = 1 and 2

Hw3: pion production, N+N<->N+Δ, Δ<->N+π, π^-/π^+

(Phase I due date: Jan.27, 2017) **(Hw3 results available in 2017, π^-/π^+ theoretical error bar?)**

Results of Hw1 and Hw2 from **Yong-Jia Wang** and **Ying-Xun Zhang**

Initial configuration well controlled

Theoretical answer available

conclusions and outlook

- Nuclear symmetry energy and isospin splitting of the effective mass are both responsible for the isospin dynamics in intermediate-energy HIC.
- Transport comparison project can help to extract the theoretical error bar due to simulation.
- Box calculation is helpful in understanding the difference in mean field, scatterings, Pauli blocking, and particle production, eventually reducing the theoretical error bar.

An agreement in treating each part of the simulation

Towards an accurate description of heavy-ion collisions, and thus an accurate constraint on Esym!

Acknowledgements

Collaborators:

Lie-Wen Chen (Shang Jiaotong University)

Bao-An Li (Texas A&M University-Commerce)

Che Ming Ko (Texas A&M University)

Yu-Gang Ma (Shanghai Institute of Applied Physics)

Betty Tsang (Michigan State University)

Hermann Wolter (Universität München)

Ying-Xun Zhang (China Institute of Atomic Energy)

Yong-Jia Wang (Huzhou University)

Akira Ono (Tohoku University)

Maria Colonna (INFN-LNS)

Thanks for the invitation by Prof. Youngman Kim
to Rare Isotope Science Project, Institute for Basic Science

Announcement of Transport2017

Transport2017 to be held from March 27 to 30, 2017, in Michigan State University

Mainly focus on box calculation and Bayesian analysis

- Organizing Committee

Akira Ono (Co-Chair), Jun Xu (Co-Chair), Maria Colonna, Pawel Danielewicz, Betty Tsang, Hermann Wolter, Ying Xun Zhang, Yong Jia Wang

- Local Organizing Committee

Pawel Danielewicz, Genie Jhang, William Lynch, Pierre Morfouace, and Betty Tsang

Thank you!

xujun@sinap.ac.cn