Temperature dependence of dim 6 gluon operators and their effects on charmonium sumrules

[Phys.Rev. D93 (2016) no.1, 016001 H.Kim, K.Morita and S.H.Lee]

HyungJoo Kim (Yonsei Univ.)

Collaborators : Su Houng Lee, Kenji Morita

HIM meeting 2016

Contents

QCD sum rules in heavy quark system

- **Correlation function**

$$\Pi^{\Gamma}(q) = i \int d^4 x\, e^{iqx} \langle T\{ j^{\Gamma}(x)\, j^{\Gamma}(0)\} \rangle$$

- **OPE part**

In the large $Q^2 (= -q^2)$ region, we can expand the product of two current operator to single local operator.

$$i \int dx\, e^{iqx}\, T(j_{\Gamma}(x) j_{\Gamma}(0)) = C_I^{\Gamma} I + \sum_n C_n^{\Gamma}(q) O_n$$

I (unit operator)	$d = 0$,
$O_m = m\bar{q}q$,	$d = 4$,
$O_G = G^a_{\mu\nu} G^a_{\mu\nu}$,	$d = 4$,
$O_{\Gamma} = \bar{q}\Gamma_1 q \bar{q}\Gamma_2 q$,	$d = 6$,
$O_{\sigma} = m\bar{q}\sigma_{\mu\nu} \frac{\lambda^a}{2} q G^a_{\mu\nu}$,	$d = 6$,
$O_f = f_{abc} G^a_{\mu\nu} G^b_{\nu\gamma} G^c_{\gamma\mu}$,	$d = 6$,

In heavy quark system,

$$\langle 0|\bar{h}h|0\rangle = -\frac{1}{12\, m_h} \left\langle 0 \left| \frac{\alpha_s}{\pi} G^a_{\mu\nu} G^a_{\mu\nu} \right| 0 \right\rangle + \ldots$$

$$\Pi^{\Gamma}_{\mathrm{OPE}}(q) = C_I^{\Gamma} + C_{G^2}^{\Gamma} \langle g^2 G^a_{\mu\nu} G^a_{\mu\nu} \rangle + C_{G^3}^{\Gamma} \langle g^3 f^{abc} G^a_{\mu\alpha} G^b_{\alpha\beta} G^c_{\beta\mu} \rangle + \ldots$$

QCD sum rules in heavy quark system

- **Phenomenological part**

By dispersion relation, $\Pi(q)$ is related to its imaginary part and imaginary part is related to spectral density.

$$\Pi^{\Gamma}_{\mathrm{Phen}}(q) = \frac{1}{\pi}\int_0^{\infty} \frac{\mathrm{Im}\Pi^{\Gamma}(s)}{s-q^2}\,\mathrm{ds} \qquad \mathrm{Im}\Pi^{\Gamma}(s) = f_R\,\delta(s-m_R) + \theta(s-s_0)\,\mathrm{Im}\Pi^{\Gamma,\mathrm{pert}}(s)$$

(resonance) (continuum)

- **QCD sum rules in heavy quark system**

$$\Pi^{\Gamma}_{\mathrm{OPE}}(q) = \Pi^{\Gamma}_{\mathrm{Phen}}(q)$$

$$C^{\Gamma}_{I} + C^{\Gamma}_{G^2}\left\langle g^2\, G^{a}_{\mu\nu}\, G^{a}_{\mu\nu}\right\rangle + C^{\Gamma}_{G^3}\left\langle g^3 f^{\mathrm{abc}}\, G^{a}_{\mu\alpha}\, G^{b}_{\alpha\beta}\, G^{c}_{\beta\mu}\right\rangle + \ldots = \frac{1}{\pi}\int_0^{\infty}\frac{\mathrm{Im}\Pi^{\Gamma}(s)}{s-q^2}\,\mathrm{ds}$$

We get information about properties of hadrons in terms of QCD variables and condensates.

Charmonium and Sequential scenario

- **Charmonium**

mesons formed by a bound state of a charm quark and a charm anti-quark

P	ηc(1S)	2980.3±1.2 MeV
V	J/ψ(1S)	3096.916±0.011 MeV
S	χc0(1P)	3414.75±0.31 MeV
A	χc1(1P)	3556.20±0.09 MeV

- **Sum rule in Vacuum**(Shifman, Reinders et al)

Mass of charmonium is very well described by QCD sum rule including dim4 gluon condensates.

- **Sum rule at Finite T**

Using Lattice results of dim4 gluon condensates, temperature dependences of some charmoniums are calculated using QCD sum rule.

- **Sequential dissociation scenario**

Lattice QCD expects that the charmonium ground state(S-wave) J/Ψ can survive in a QGP up to 1.2Tc while the excited states(P-wave) χ_c and Ψ' are dissociated just above Tc.

● Goals

- 1. Calculate all Wilson coefficients of gluon operators upto dim6.
- 2. Estimate the temperature dependence of dim6 gluon condensates.
- 3. Investigate temperature behavior of charmonium near Tc.

Dimension 6 gluon operators

Dim 6 gluon operators can be made up of

1) $3 \times G_{\mu\nu}$

2) $2 \times D_\mu$ and $2 \times G_{\mu\nu}$

Among them, 2-scalar*, 3-twist4, and 1-twist2 are independent by index symmetry and Bianchi identity. (twist#, #=dimension-spin)

Scalar : $f^{abc} G^a_{\mu\nu} G^b_{\mu\alpha} G^c_{\nu\alpha}$, $G^a_{\mu\alpha} G^a_{\nu\alpha;\nu\mu}$

ex) $O_{\mu\nu}|_{twist4} = \frac{1}{2}(O_{\mu\nu} + O_{\nu\mu}) - \frac{1}{4} g_{\mu\nu} O_{\mu\mu}$

Twist4 : $f^{abc} G^a_{\mu\alpha} G^b_{\alpha\beta} G^c_{\beta\nu}$, $G^a_{\mu\kappa} G^a_{\nu\lambda;\lambda\kappa}$, $G^a_{\mu\lambda} G^a_{\lambda\kappa;\kappa\nu}$

Twist2 : $G^a_{\mu\kappa} G^a_{\nu\kappa;\alpha\beta}$

symmetric and traceless

Notation) $G^a_{\mu\nu} G^a_{\kappa\lambda;\alpha\beta} \equiv G^a_{\mu\nu} D_\beta D_\alpha G^a_{\kappa\lambda}$

* S. Narison and R. Tarrach, Phys. Lett. B **125**, 217 (1983)

Dimension 6 gluon operators

Especially, in the heavy quark system(charmonium)

Scalar : $f^{abc} G^a_{\mu\nu} G^b_{\mu\alpha} G^c_{\nu\alpha}$, $G^a_{\mu\alpha;} G^a_{\nu\alpha;\nu\mu} (= j^a_\mu j^a_\mu)$

Twist4 : $f^{abc} G^a_{\mu\alpha} G^b_{\alpha\beta} G^c_{\beta\nu}$, $G^a_{\mu\kappa} G^a_{\nu\lambda;\lambda\kappa} (= j^a_\mu j^a_\nu)$, $G^a_{\mu\lambda} G^a_{\lambda\kappa;\kappa\nu} (= G^a_{\mu\lambda;\nu} j^a_\lambda)$

Twist2 : $G^a_{\mu\kappa} G^a_{\nu\kappa;\alpha\beta}$

Heavy quark system ex) $\langle \bar{h}h \rangle \simeq \frac{1}{m_h} \left\langle \frac{\alpha_s}{\pi} G^a_{\mu\nu} G^a_{\mu\nu} \right\rangle$

Scalar : $f^{abc} G^a_{\mu\nu} G^b_{\mu\alpha} G^c_{\nu\alpha} \rightarrow fG^3$

Twist4 : $f^{abc} G^a_{\mu\alpha} G^b_{\alpha\beta} G^c_{\beta\nu} \rightarrow G_3$

Twist2 : $G^a_{\mu\kappa} G^a_{\nu\kappa;\alpha\beta} \rightarrow G_4$

---

where,

$$\langle g^3 f^{abc} G^a_{\mu\alpha} G^b_{\alpha\beta} G^c_{\beta\nu} \rangle = G_3 \left(u_\mu u_\nu - \frac{1}{4} g_{\mu\nu} \right)$$

$$\left\langle \frac{\alpha_s}{\pi} G^a_{\mu\kappa} G^a_{\nu\kappa;\alpha\beta} \right\rangle = G_4 \left(u_\mu u_\nu u_\alpha u_\beta + \frac{1}{48} (g_{\mu\nu} g_{\alpha\beta} + g_{\mu\alpha} g_{\nu\beta} + g_{\mu\beta} g_{\nu\alpha}) - \frac{1}{8} (u_\mu u_\nu g_{\alpha\beta} + u_\mu u_\beta g_{\alpha\nu} + u_\nu u_\alpha g_{\mu\beta} + (\mu \leftrightarrow \alpha, \nu \leftrightarrow \beta)) \right)$$

$u_\mu = (1, 0, 0, 0)$: medium four velocity

Field Representations

- dimension 4

There are two independent dim 4 gluon operators and they can be represented by color E and B fields,

scalar : $G^a_{\mu\nu} G^a_{\mu\nu} \Rightarrow 2\left(B^2 - E^2\right)$

twist2 : $G_2 \Rightarrow -\frac{2}{3}\left(E^2 + B^2\right)$ where $\left\langle \frac{\alpha_s}{\pi} G^a_{\mu\alpha} G^a_{\nu\alpha} \right\rangle = \left(u_\mu u_\nu - \frac{1}{4} g_{\mu\nu}\right) G_2$

$$G^a_{\mu\nu} = \begin{bmatrix} 0 & E^a_x & E^a_y & E^a_z \\ -E^a_x & 0 & B^a_z & -B^a_y \\ -E^a_y & -B^a_z & 0 & B^a_x \\ -E^a_z & B^a_y & -B^a_x & 0 \end{bmatrix}$$

cf) $$\begin{bmatrix} 0 & -E_x/c & -E_y/c & -E_z/c \\ E_x/c & 0 & -B_z & B_y \\ E_y/c & B_z & 0 & -B_x \\ E_z/c & -B_y & B_x & 0 \end{bmatrix} = F^{\mu\nu}$$

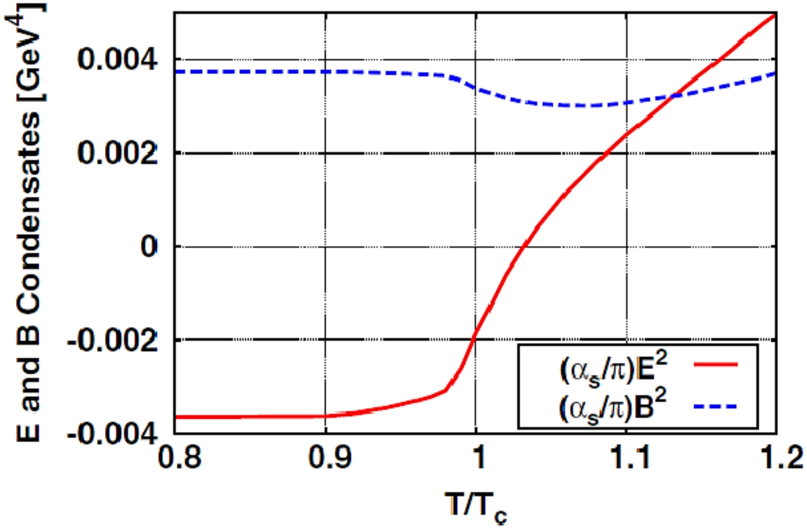
- dimension 6

scalar : $fG^3 \Rightarrow f^{abc}\left(3\, B^a \cdot \left(E^b \times E^c\right) - B^a \cdot \left(B^b \times B^c\right)\right)$

twist4 : $G_3 \Rightarrow \frac{1}{3} f^{abc}\left(B^a \cdot \left(E^b \times E^c\right) + B^a \cdot \left(B^b \times B^c\right)\right)$

twist2 : G_4 cannot be represented by E and B fields.

Temperature dependence of BEE and BBB

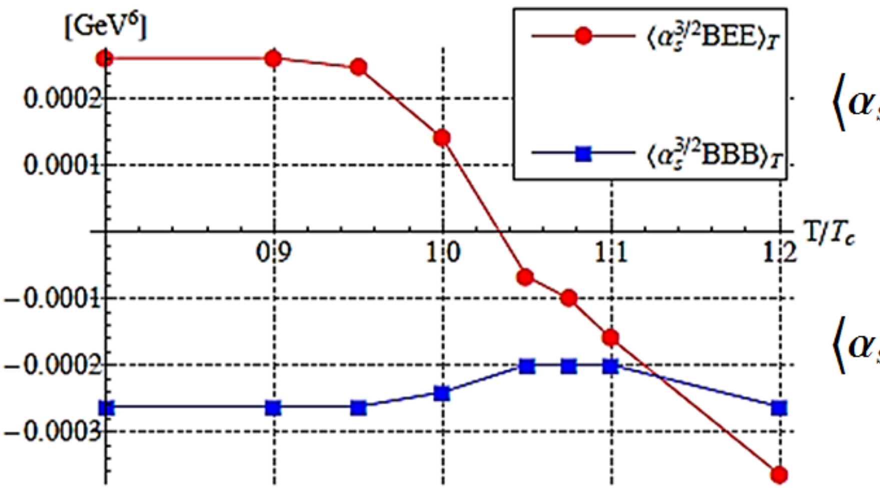


We know the T dependence of $\left\langle \frac{\alpha_s}{\pi} E^2 \right\rangle_T$ and $\left\langle \frac{\alpha_s}{\pi} B^2 \right\rangle_T$ from lattice calculations.**

**Phys. Rev. D79, 011501(2009) S.H.Lee and K.Morita

- **Assumption**

We assume that fields are isotropic and their angular correlations can be neglected.



$$\langle \alpha_s{}^{3/2}\, B\, E\, E \rangle_T \Rightarrow \langle \alpha_s{}^{3/2}\, B\, E\, E \rangle_{T=0} \frac{\left\langle \frac{\alpha_s}{\pi} B^2 \right\rangle_T^{1/2} \left\langle \frac{\alpha_s}{\pi} E^2 \right\rangle_T}{\left\langle \frac{\alpha_s}{\pi} B^2 \right\rangle_{T=0}^{1/2} \left\langle \frac{\alpha_s}{\pi} E^2 \right\rangle_{T=0}}$$

$$\langle \alpha_s{}^{3/2}\, B\, B\, B \rangle_T \Rightarrow \langle \alpha_s{}^{3/2}\, B\, B\, B \rangle_{T=0} \frac{\left\langle \frac{\alpha_s}{\pi} B^2 \right\rangle_T^{3/2}}{\left\langle \frac{\alpha_s}{\pi} B^2 \right\rangle_{T=0}^{3/2}}$$

Temperature dependence of G_4

- **Assumption**

We estimate order of ratio between G2 and G4 using gluon distribution function $G(x, \mu)$*** and thermal gluon mass $m_G(T)$****.

$$G_{2n} \simeq -(-i)^{2n-2} A^G_{2n} \quad \text{where} \quad A^G_{2n} = 2 \int dx\, x^{n-1}\, G(x, \mu^2)$$

Assume temperature behavior of G4 as,

$$\frac{G_4}{G_2} \simeq -m_G^2 \frac{A_4}{A_2}$$

where $A_4 \sim 0.02$

$A_2 \sim 0.9$

$m_G \sim 0.6\,\text{GeV}$ near T_c

*** P.R.L. 82 (1999) F.Klingl, S.S.Kim, S.H.Lee, P.Morath, and W.Weise
**** Phys.Rev. C57 1879 (1998) P.Levai and U.W.Heinz

Applications to Charmonium Sumrules

$$\Pi^{\Gamma}(q) = i\int d^4x\, e^{iqx} \left\langle T\left\{ j^{\Gamma}(x)\, j^{\Gamma}(0) \right\}\right\rangle$$

Dim6 Twist operators' Wilson coefficients

$$= C_{\text{pert}} + C_{G^2}\left\langle G^2 \right\rangle + C_{G^3}\left\langle \text{f}G^3 \right\rangle + C_{G_2}\langle G_2\rangle + \boxed{C_{G_3}\langle G_3\rangle + C_{G_4}\langle G_4\rangle} \text{ : upto dim 6}$$

where $j^{\Gamma} = \bar{c}\,\Gamma c$ and $\Gamma = I(\chi_{c0}),\ i\gamma_5(\eta_c),\ \gamma_\mu(J/\Psi),\ \left(q_\mu q_\nu / q^2 - g_{\mu\nu}\right)\gamma^\nu\gamma_5(\chi_{c1})$

$^1S_0$	ηc	$j^P = \bar{c}i\gamma_5 c$	2980.3±1.2 MeV
$^3S_1$	J/ψ	$j^V = \bar{c}\gamma_\mu c$	3096.916±0.011 MeV
$^3P_0$	χc0	$j^S = \bar{c}c$	3414.75±0.31 MeV
$^3P_1$	χc1	$j^A = \bar{c}(q_\mu q_\nu/q^2 - g_{\mu\nu})\gamma_\nu\gamma_5 c$	3510.66±0.07 MeV

- **Try1 : Moment sum rule** (SVZ/RRY)
- **Try2 : Perturbed Borel sum rule**(Bertlmann) for simple calculation
- **Try3 : Borel sum rule**

Moment sum rule

- **Moments $M_n(Q^2)$**

To pick out the lowest lying resonance, we define moments M_n.

$$\Pi^\Gamma(q) = i \int d^4 x\, e^{iqx} \langle T \{ j^\Gamma(x)\, j^\Gamma(0) \} \rangle$$

$$M_n^\Gamma(Q^2) \equiv \frac{1}{n!} \left(-\frac{d}{d\,Q^2} \right)^n \Pi^\Gamma(Q^2) \qquad (Q^2 = -q^2)$$

$$m_\Gamma^2 = \min \left\{ \frac{M_{n-1}^\Gamma(Q^2)}{M_n^\Gamma(Q^2)} - Q^2 \right\}$$

- **Parameters******

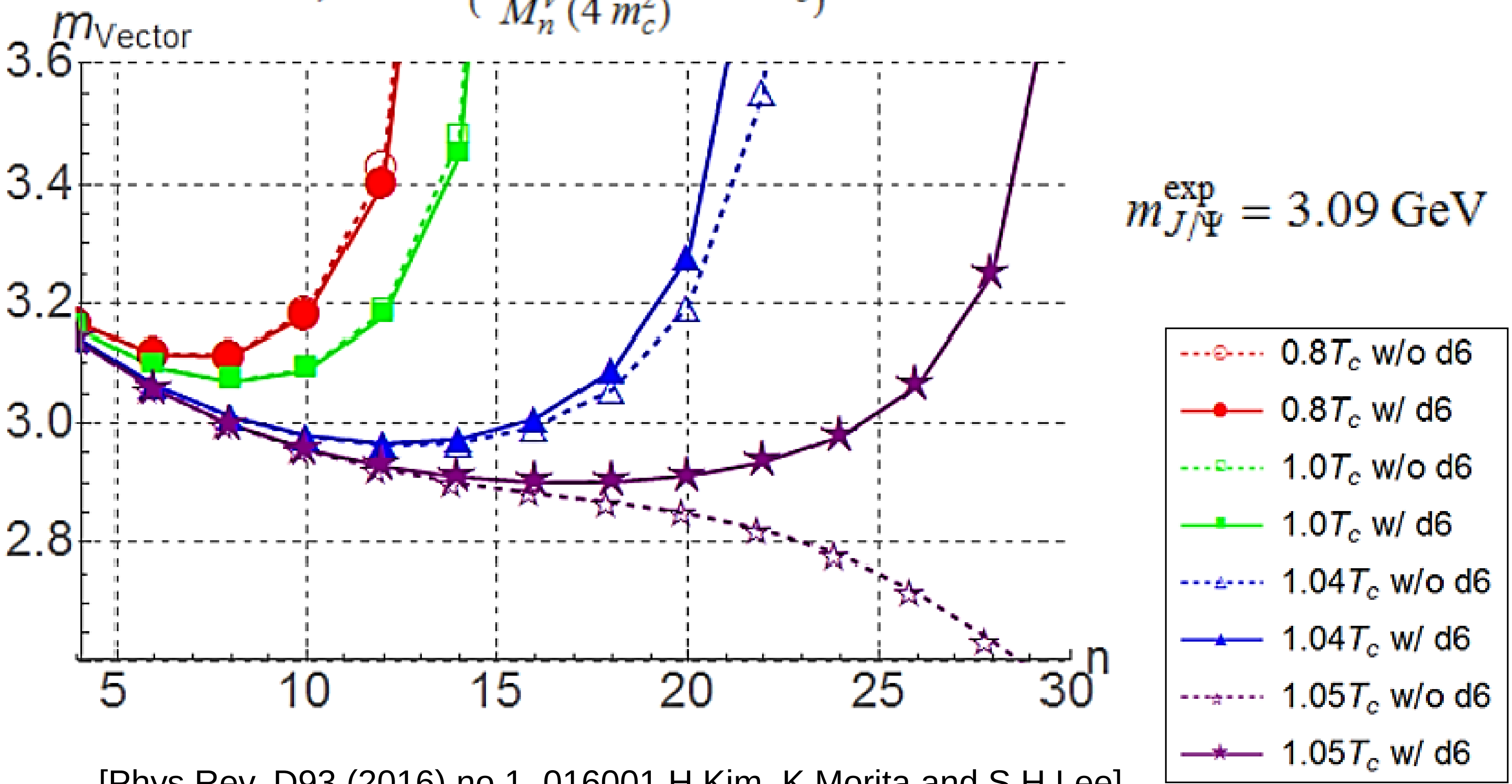
$$\left(\xi = \frac{Q^2}{4\, m_c^2} \right)$$

S waves($\xi = 1$)	P waves($\xi = 2.5$)
$m_c = 1.23$ GeV	$m_c = 1.21$ GeV
$\alpha_s = 0.21$	$\alpha_s = 0.17$
$\sqrt{s_0} = 3.8$ GeV	$\sqrt{s_0} = 3.8$ GeV

***** Phys.Rept. 127 (1985) Reinders, Rubinstein, Yazaki

Moment sum rule Results(Vector)

$$m_V^2 = \min\left\{\frac{M_{n-1}^V(4\,m_c^2)}{M_n^V(4\,m_c^2)} - 4\,m_c^2\right\}$$

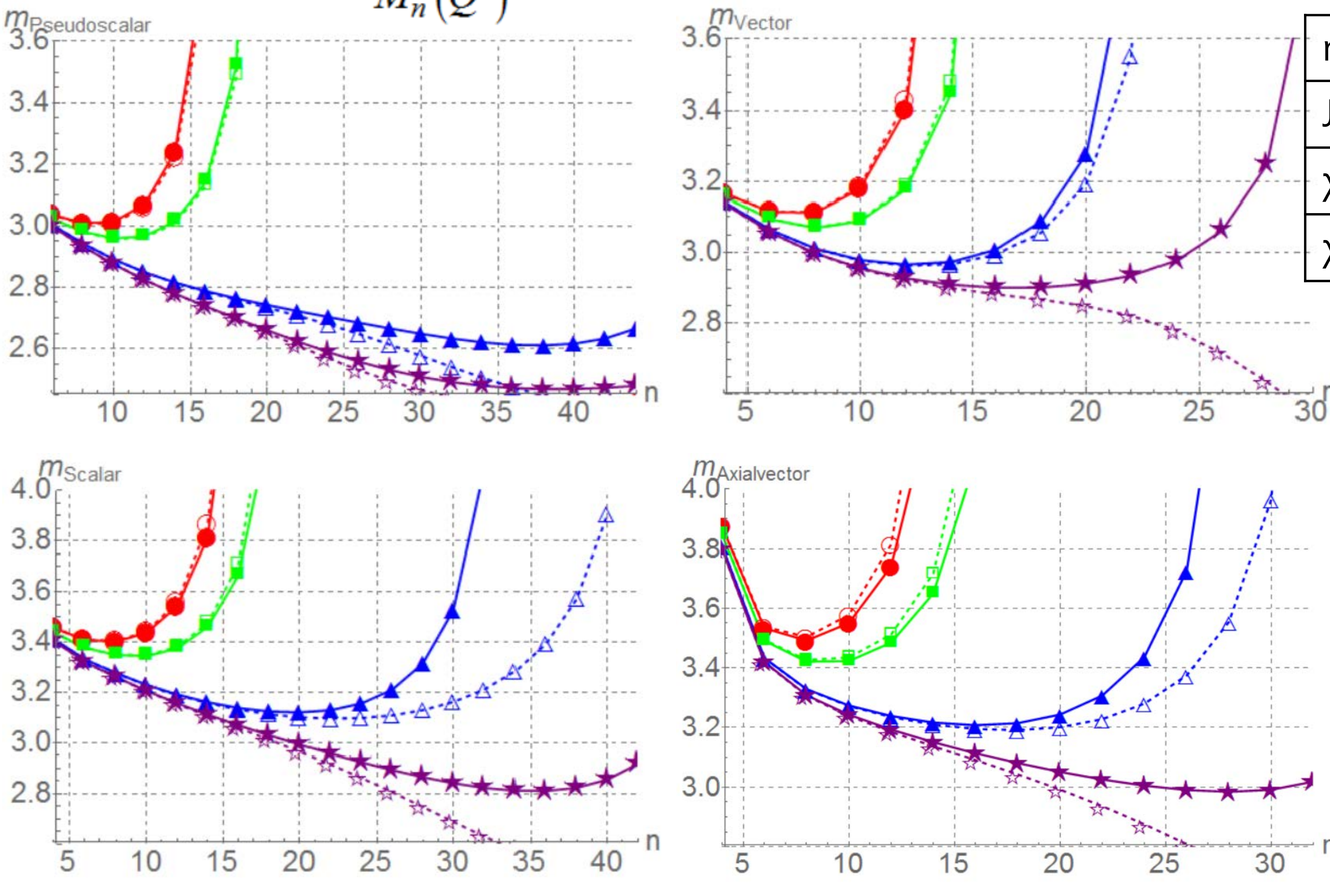


$$m_{J/\Psi}^{\exp} = 3.09\ \text{GeV}$$

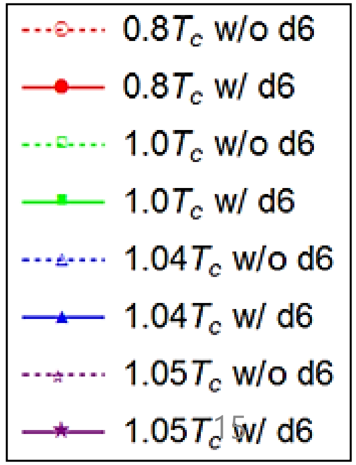
[Phys.Rev. D93 (2016) no.1, 016001 H.Kim, K.Morita and S.H.Lee]

Moment sum rule Results

$$m_\Gamma^2 = \min\left\{ \frac{M_{n-1}^\Gamma(Q^2)}{M_n^\Gamma(Q^2)} - Q^2 \right\}$$



ηc (P)	2.98 GeV
J/ψ (V)	3.09 GeV
χc0 (S)	3.41 GeV
χc1 (A)	3.51 GeV



Criterion to determine valid n region

At low n, the whole spectrum(res+cont) contributes and condensate corrections are too small.

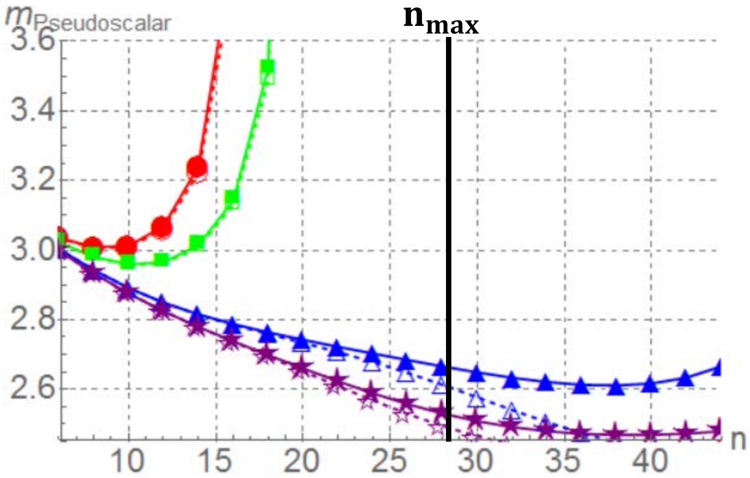
At large n, the ground state dominant but condensate correction become very large.

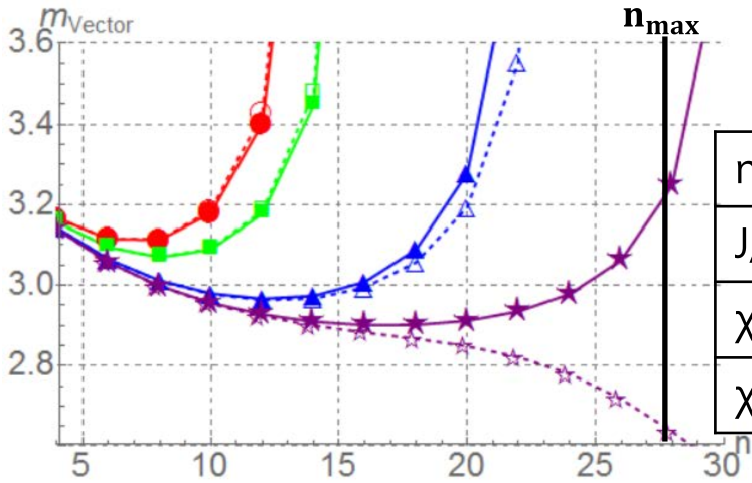
- n_{min} : $$\frac{\text{continuum contribution from } M_n^\Gamma}{\text{total perturbative part from } M_n^\Gamma} \leq 0.3$$

- n_{max} : $$\frac{\text{dim4 contribution from } M_n^\Gamma}{\text{total } M_n^\Gamma} \leq 0.3$$

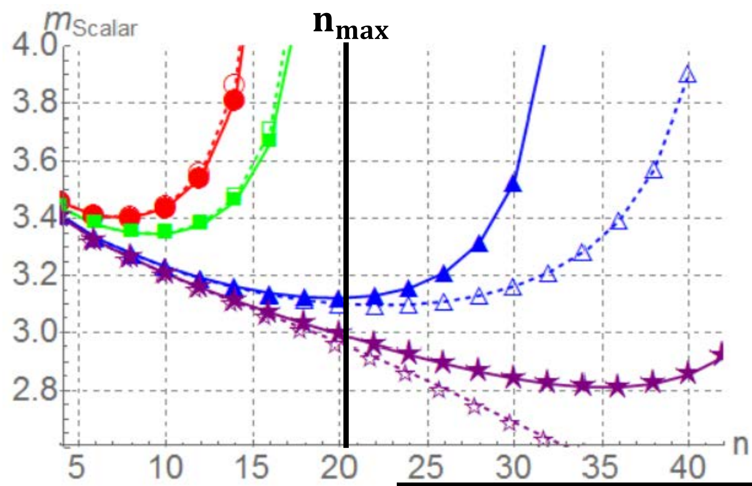
$$\frac{\text{dim6 contribution from } M_n^\Gamma}{\text{total } M_n^\Gamma} \leq 0.1$$

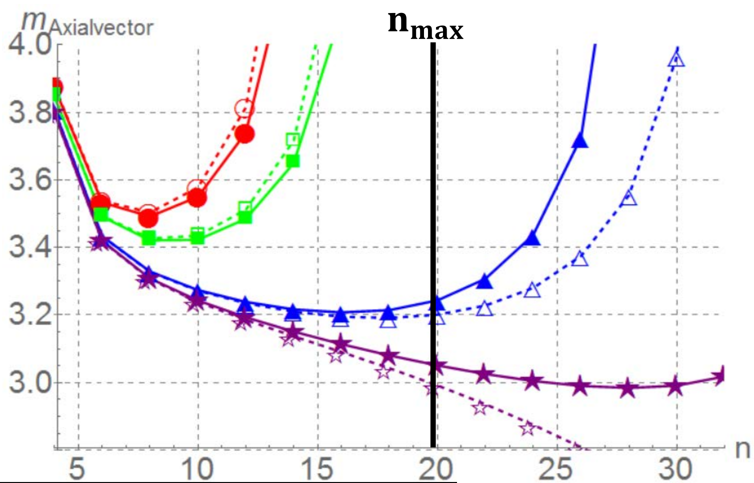
Moment sum rule Results

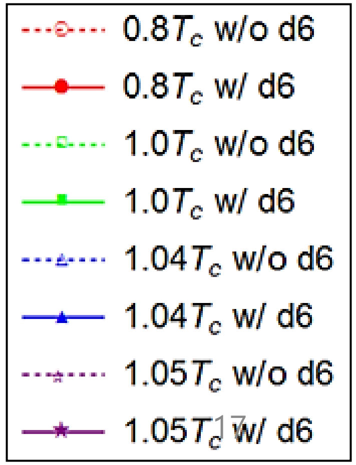




ηc (P)	2.98 GeV
J/ψ (V)	3.09 GeV
χc0 (S)	3.41 GeV
χc1 (A)	3.51 GeV







	Psuedo	Vector	Scalar	Axial
dim4	1.03 Tc	1.04 Tc	1.04 Tc	1.04 Tc
dim6	1.03 Tc	1.05 Tc	1.04 Tc	1.04 Tc

Borel sum rule

- **Exponential Moments $\boldsymbol{M(\sigma)}$** (Borel transformed moments)

$$\Pi^{\Gamma}(q) = i \int d^4 x\, e^{iqx} \left\langle T\left\{ j^{\Gamma}(x)\, j^{\Gamma}(0) \right\} \right\rangle$$

$$M(\sigma) \equiv \lim_{\substack{n, Q^2 \to \infty \\ Q^2/n \to \sigma}} \frac{(Q^2)^{n+1}\pi}{n!} \left(-\frac{d}{dQ^2} \right)^n \Pi^{\Gamma}(Q^2)$$

$$m_{\Gamma}^2 \simeq \min\left\{ -\frac{M'(\sigma)}{M(\sigma)} \right\} \quad (\text{in the valid } \sigma \text{ region})$$

- **Advantages of Borel sum rule**

OPE side : convergence of the OPE ⬆

Phenomenological side : lowest resonance’s contribution ⬆

- **parameters**

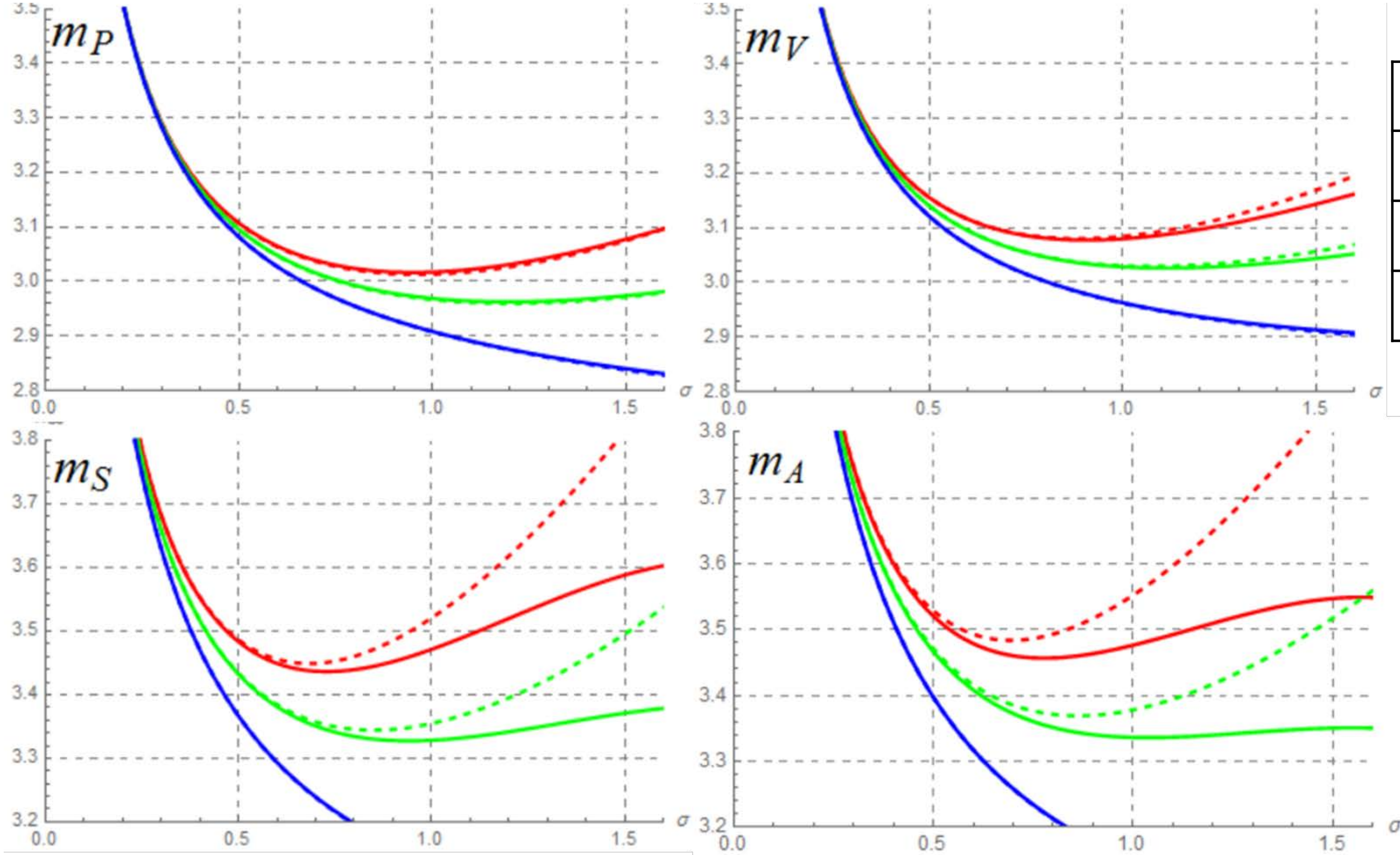
For both S and P states, we use same parameters and less than Moment SR.

$$m_c = 1.27\,\text{GeV}, \;\; \alpha_s = 0.3, \;\; \sqrt{s_0} = 3.6\,\text{GeV}$$

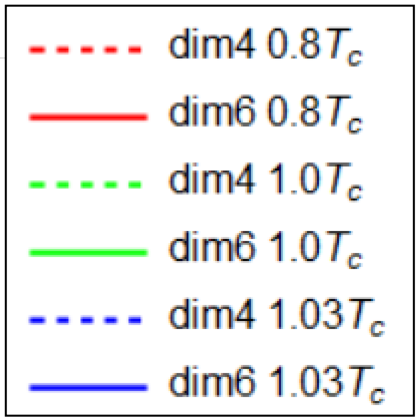
Perturbed Borel sum rule Results

$$M(\sigma) = e^{-4m^2\sigma}\,\pi A(\sigma)\left[1 + \alpha_s\, a(\sigma) + b(\sigma)\left\langle g^2 G^2\right\rangle + c(\sigma)\left\langle g^3\, fG^3\right\rangle + \ldots\right]$$

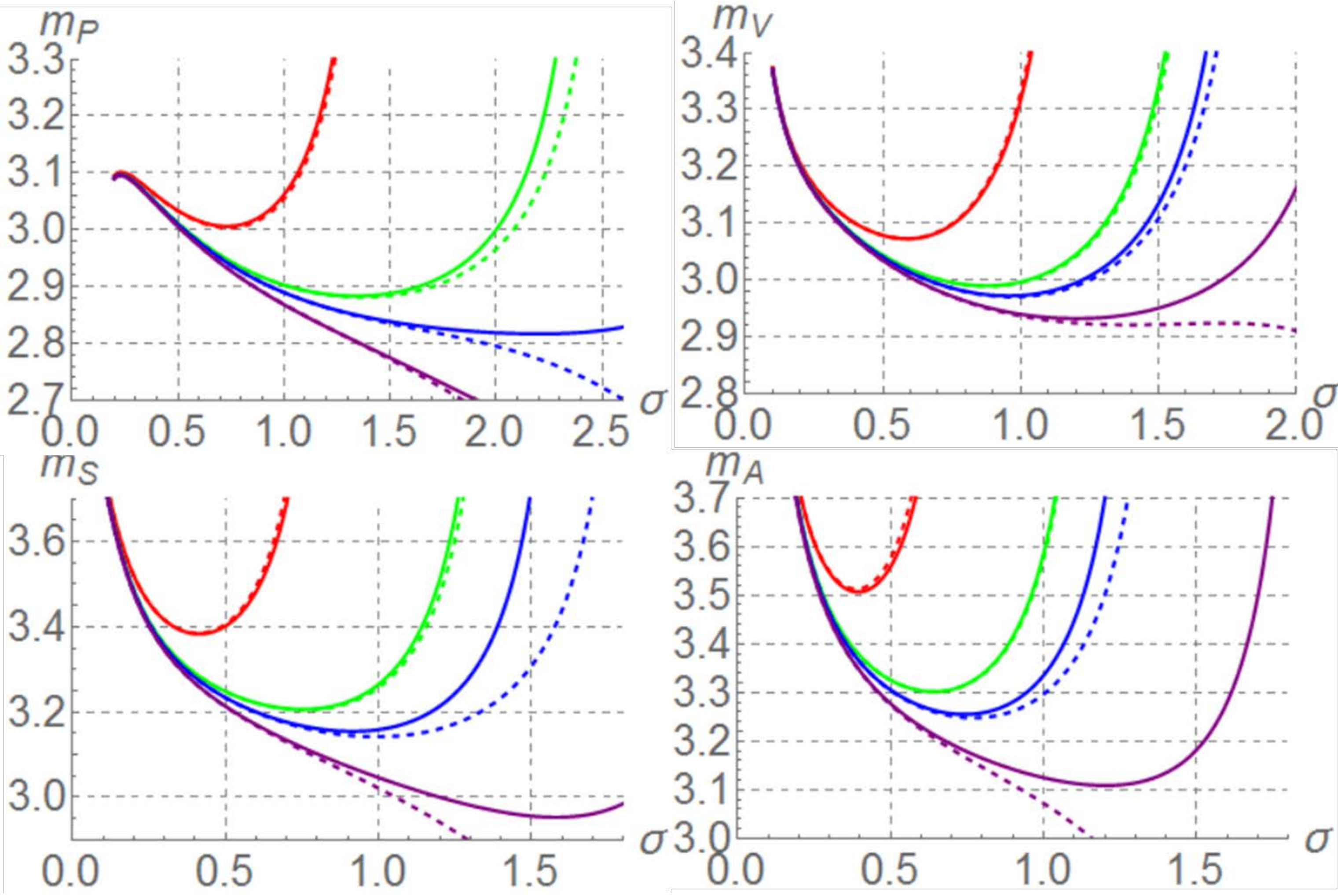
$$R(\sigma) = -\frac{M'(\sigma)}{M(\sigma)} = -\frac{d}{d\sigma}\log M[\sigma] \simeq \frac{d}{d\sigma}\log[1+f(\sigma)] \simeq \frac{d\,f(\sigma)}{d\sigma}$$



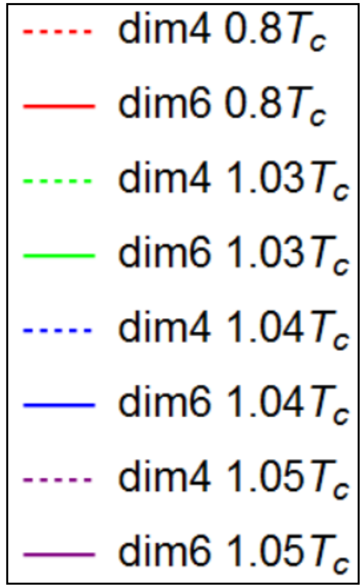
ηc (P)	2.98 GeV
J/ψ (V)	3.09 GeV
χc0 (S)	3.41 GeV
χc1 (A)	3.51 GeV



Borel sum rule Results

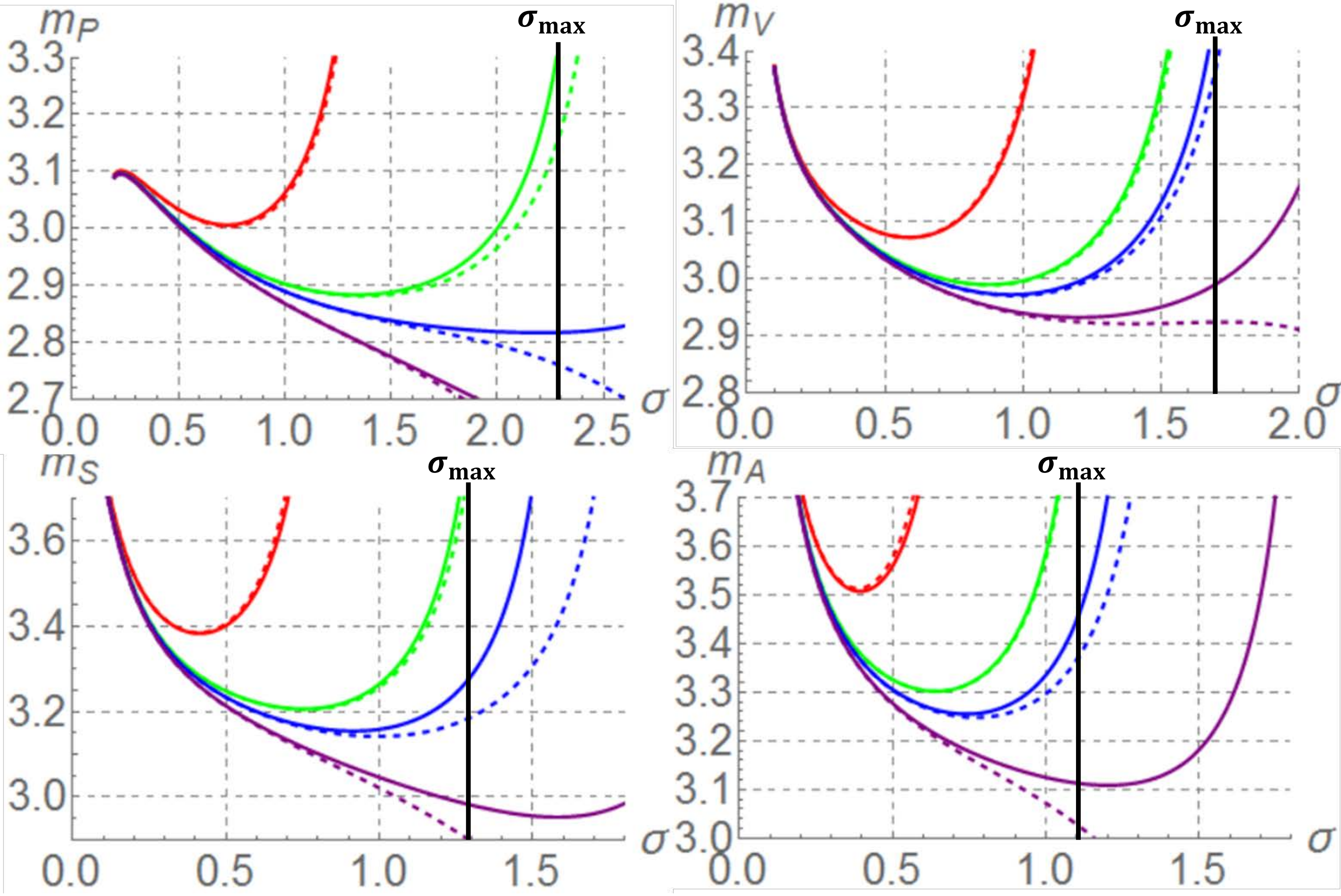


ηc (P)	2.98 GeV
J/ψ (V)	3.09 GeV
χc0 (S)	3.41 GeV
χc1 (A)	3.51 GeV

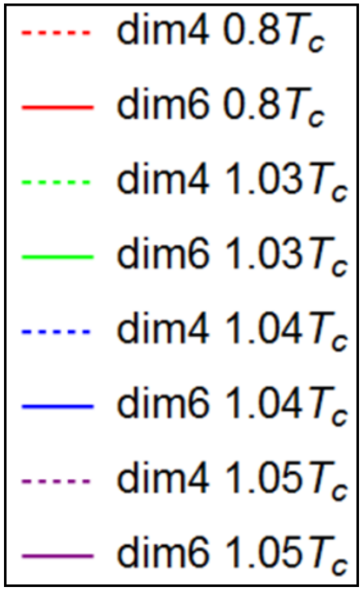


	Psuedo	Vector	Scalar	Axial
$m_{0.8T_c}$	3.0 GeV	3.08 GeV	3.39 GeV	3.50 GeV

Borel sum rule Results



ηc (P)	2.98 GeV
J/ψ (V)	3.09 GeV
χc0 (S)	3.41 GeV
χc1 (A)	3.51 GeV



	Psuedo	Vector	Scalar	Axial
dim6	1.04 Tc	1.05 Tc	1.04 Tc	1.04 Tc

Conclusions

- We calculated and completed OPE for heavy quark S, P, V, and A currents upto dimension 6 with nonzero spin operators.
- We estimate temperature dependence of dimension 6 gluon condensates based on the temperature dependence of dimension 4 E and B condensates extracted from lattice gauge theory.
- We improved the previous QCD sumrules for charmonium near Tc based on dimension 4 operators, by including contribution of dimension 6 operators.
- All sum rule method well describe mass of charmonium at vacuum.
- Moment sumrule improved vector current's stability upto 1.05 Tc but pseudo scalar current looks too unstable in this sum rule.
- Perturbed borel sumrule is not good to investigate T dependence.
- We found an enhanced stability from Borel sumrule at T=1.05Tc for the vector current and T=1.04Tc for others. We could not find remarkable difference between S and P states.

Thank you for your listening!!