

# Some Topics In Hot and Dense Matter in QCD

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# Expansion of nuclear (subatomic) physics, since late '70

Hadron dynamics

Nucleons, Nucl. force



Nuclear (hadronic) matter in extreme conditions; **high T and/or density**,  
**Eg. compact stars, early universe,**  
**rel. heavy-ion collisions**



**QCD:**

**governing the dynamics of quarks and gluons**

**Nuclear Physics**



Many-body physics of quarks,  
gluons and hadrons based on  
QCD

# Contents

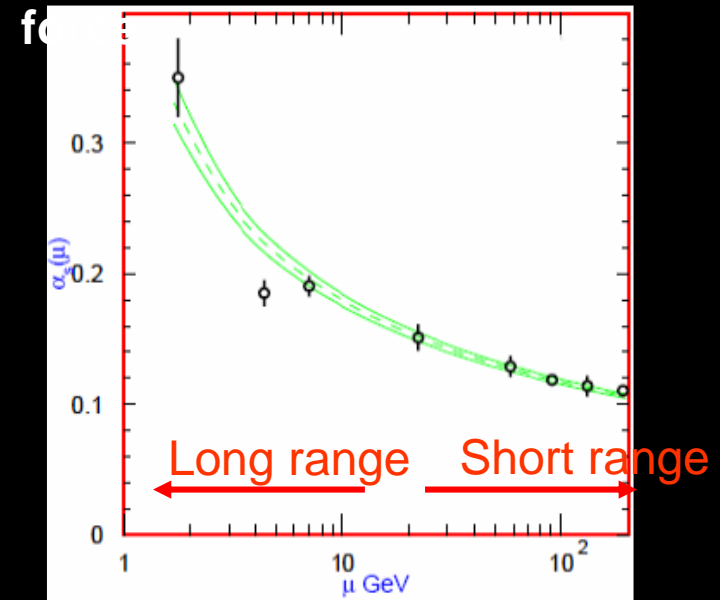
1. From nuclear physics to subatomic physics based on Quantum Chromo Dynamics (QCD)
2. Basic properties QCD
3. Chiral symmetry and axial anomaly
4. Possible change of QCD vacuum
5. Creating a system in which QCD vacuum is changed.
6. Expected phenomena associated with (partial) restoration of chiral symmetry
7. Effective restoration of axial symmetry
8. Baryon sector
9. Brief summary and concluding remarks

# QCD: Fundamental theory of the strong interaction



D. Gross, D. Politzer, F. Wilczek  
2004 Nobel prize

Strength of the



Y. Nambu



Color degrees of freedom,  
Having written down QCD for the first time

# (Classical) QCD Lagrangian

$$\mathcal{L}^{\text{cl}} = \bar{q}(i\gamma^\mu D_\mu - m)q - \frac{1}{4}F_{\mu\nu}^a F_a^{\mu\nu}$$

$$D_\mu = \partial_\mu - igt^a A_\mu^a$$

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + gf_{abc}A_\mu^b A_\nu^c$$


$$q = {}^t(u, d, s, c, b, t) \quad m = \text{diag}(m_u, m_d, m_s, \dots)$$

## Quantum theory: ' ,

Gauge fixing+ Fadeev-Popov ghost fields

Regularization with some scale  $\mu$

Independence of physical values of observables of  $\mu$

 Renormalization group equation , which in turn describes the scale dependence of observables.

# Chiral Transformation

$$\frac{1 - \gamma_5}{2} q_i \equiv q_{iL} \rightarrow L_{ij} q_{jL}, \quad (\text{left handed})$$

$$\frac{1 + \gamma_5}{2} q_i \equiv q_{iR} \rightarrow R_{ij} q_{jR}, \quad (\text{right handed})$$

$$L = \exp(i\boldsymbol{\theta}_L \cdot \boldsymbol{\lambda}/2) \equiv U(\boldsymbol{\theta}_L),$$

$$R = \exp(i\boldsymbol{\theta}_R \cdot \boldsymbol{\lambda}/2) \equiv U(\boldsymbol{\theta}_R),$$

$$\boldsymbol{\theta}_{L,R} \cdot \boldsymbol{\lambda} = \sum_{a=0}^8 \theta_{L,R}^a \lambda^a.$$

Chirality:

$$\gamma_5 q_L = -q_L, \quad \gamma_5 q_R = q_R.$$

For  $N_f = 3$ , the chiral transformation forms

$$U(3)_L \otimes U(3)_R$$

# Chiral Invariance of Classical QCD Lagrangian in the chiral limit ( $m=0$ )

$$\bar{q}\gamma^\mu q = \bar{q}_L\gamma^\mu q_L + \bar{q}_R\gamma^\mu q_R$$

$$\begin{aligned} \rightarrow & \bar{q}_L L^\dagger \gamma^\mu L q_L + \bar{q}_R R^\dagger \gamma^\mu R q_R \\ & = \bar{q}_L \gamma^\mu q_L + \bar{q}_R \gamma^\mu q_R \end{aligned}$$

invariant!

In the chiral limit ( $m=0$ ),

; Chiral invariant

$$D_\mu = \partial_\mu - i g t^a A_\mu^a$$



$$\mathcal{L}_0^{cl} = \bar{q}(i\gamma^\mu D_\mu - \cancel{m})q - \frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu} ; \text{Chiral invariant!}$$

# Special Chiral transformations

(i)  $\theta_L = \theta_R \equiv \alpha$



$$q \rightarrow U(\alpha)q, \quad \bar{q} \rightarrow \bar{q}U^\dagger(\alpha)$$

gauge transformation:  $U_V(N_f)$

generator;  $Q^a = \int d\mathbf{x} q^\dagger(x) \lambda^a / 2 q(x)$

(ii)  $\theta_L = -\theta_R \equiv -\beta$



$$q \rightarrow U(\beta\gamma_5)q, \quad \bar{q} \rightarrow \bar{q}U^\dagger(\beta\gamma_5)$$

Axial gauge transformation:

generator;  $Q_5^a = \int d\mathbf{x} q^\dagger(x) \lambda^a \gamma_5 q(x)$



# Current divergences and Quantum Anomalies

From Noether's theorem:

$$\partial_\mu(\bar{q}\gamma^\mu\lambda^a q) = i \sum_{i,j}^{N_f} \bar{q}_i(m_i - m_j)\lambda^a q_j \quad (a = 0 \sim N_f^2 - 1)$$

$i, j = u, d, s, \dots$

$$\partial_\mu(\bar{q}\gamma^\mu\gamma_5\lambda^a q) = i \sum_{i,j}^{N_f} \bar{q}_i(m_i + m_j)\gamma_5\lambda^a q_j \quad (a = 1 \sim N_f^2 - 1)$$

$$\partial_\mu(\bar{q}\gamma^\mu\gamma_5 q) = i \sum_i^{N_f} \bar{q}_i 2m_i \gamma_5 q_i + 2N_f \frac{g^2}{32\pi^2} F_{\mu\nu}^a \tilde{F}_a^{\mu\nu} \quad (\tilde{F}_a^{\lambda\rho} = \frac{1}{2}\epsilon^{\mu\nu\lambda\rho} F_{\mu\nu}^a)$$

Quantum effects!

Chiral Anomaly

Dilatation

$$\partial_\mu D^\mu = \Theta_{\mu}^{\mu} = (1 + \gamma_m) \sum_i^{N_f} \bar{q}_i m_i q_i + \frac{\beta}{2g} F_{\mu\nu}^a F_a^{\mu\nu}$$

Dilatation(scale) Anomaly

$\Theta_{\mu\nu}$ ; energy-momentum tensor of QCD

Some symmetries existing in the classical level are broken explicitly in the quantum level. Quantum Anomaly

# What is the matter?

According to modern QFT, the matter is an excited state of quantum fields.

The ground state is the vacuum.

**Def.**  $a_\alpha |0\rangle = 0 \iff |0\rangle$  ; the vacuum

The vacuum is defined through the annihilation operators of the Matter.

The matter and the vacuum are inter-determined.  
The modern theory of the matter is automatically the theory of the vacuum.

Determining what the matter is equivalent to determine the vacuum.

# Example from condensed matter physics:

**normal metal**

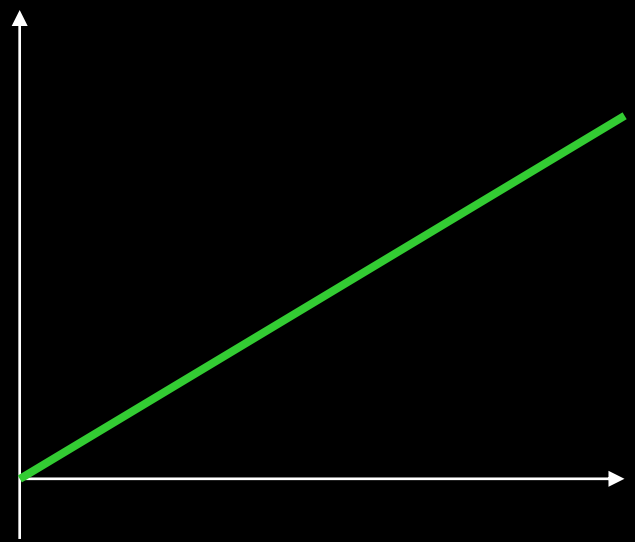
$$\langle \psi_{\uparrow} \psi_{\downarrow} \rangle = 0$$



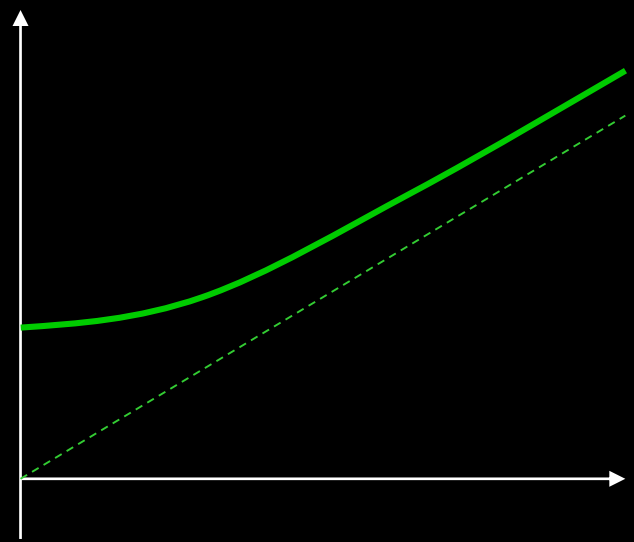
**Super conductor**

$$\langle \psi_{\uparrow} \psi_{\downarrow} \rangle \neq 0.$$

Particle number is not a good q. #.



**Gap**



~ dispersion relation of a relativistic particle with the mass .

**change in the vacuum**



**change in elementary excitations**

c.f. Particle number conservation  $\longleftrightarrow$  Gauge invariance;  $\psi \rightarrow e^{i\theta} \psi$

# Nonperturbative properties of QCD

## Gell-Mann-Oakes-Renner

$$f_\pi^2 m_\pi^2 \simeq -\hat{m} \langle \bar{u}u + \bar{d}d \rangle$$

$$\hat{m} = (m_u + m_d)/2$$

using

$$f_\pi = 93 \text{ MeV and } \hat{m}(1\text{GeV}) = (7 \pm 2) \text{ MeV,}$$

We have

$$\langle \bar{u}u \rangle \simeq \langle \bar{d}d \rangle \simeq [-(225 \pm 25) \text{ MeV}]^3 \quad \text{at } \mu^2 = 1\text{GeV}$$

Chiral symmetry is spontaneously broken in QCD vacuum.

QCD sum rules for heavy-quark systems,

$$\left\langle \frac{\alpha_s}{\pi} F_{\mu\nu}^a F_a^{\mu\nu} \right\rangle = (350 \pm 30 \text{ MeV})^4$$

# The notion of Spontaneous Symmetry Breaking

$Q^a$  the generators of a continuous transformation

$$\partial^\mu j_\mu^a = 0 \quad ; \quad j_\mu^a(x) \quad \text{Noether current} \quad Q^a = \int d\mathbf{x} j_0^a(x)$$

eg. Chiral transformation for  $SU_L(2) \otimes SU_R(2)$

$$Q_5^a = \int d\mathbf{x} \bar{q} \gamma^0 \gamma_5 \tau^a q / 2 \quad \text{Notice; } [iQ_5^a, \bar{q}(x) i \gamma_5 \tau^b q(x)] = -\delta^{ab} \bar{q}(x) q(x)$$

The two modes of symmetry realization in the vacuum  $|0\rangle$  :

a. Wigner mode

$$Q^a |0\rangle = 0 \quad \forall a$$

b. Nambu-Goldstone mode

$$Q^a |0\rangle \neq 0 \quad \exists a$$

The symmetry is spontaneously broken.

Now,  $\langle 0 | \bar{q} q | 0 \rangle = \langle 0 | [Q_5^a, \bar{q} \gamma_5 \tau^a q] | 0 \rangle$

$$\langle 0 | \bar{q} q | 0 \rangle \neq 0 \quad \xrightarrow{\quad \downarrow \quad} \quad Q_5^a |0\rangle \neq 0$$

Chiral symmetry is spontaneously broken!

# $U_A(1)$ Problem

# of the generators

$$G = U_L(3) \otimes U_R(3)$$

$$2 \times (8+1) = 18$$

$$H = U_V(1) \otimes SU_f(3)$$

$$1+8=9$$

$$\# \text{ of NG-bosons} = \dim G - \dim H = 18 - 9 = 9 \quad (?)$$

Nambu-Goldstone Theorem

★ Number of the lightest pseudo-scalar mesons

$$\pi^\pm, \pi^0(140) \quad K^\pm, K^0, \bar{K}^0(500) \quad \eta(550) \quad \ll \eta' (958)$$

$$3 + 4 + 1 = 8 \neq 9 !$$

Why is  $\eta'$  so massive ?

-----  $U_A(1)$  Problem

c.f. Without the anomaly,

$$m_{\eta_0} \leq \sqrt{3} m_\pi$$

-- S.Weinberg('75) --

$$\exists U_A(1) \text{ Anomaly } \partial_\mu (\bar{q} \gamma_\mu \gamma_5 q) = 2N_f \frac{g^2}{32\pi^2} F_{\mu\nu}^a \tilde{F}_a^{\mu\nu} \text{ Operator Equation!}$$

even in the chiral limit!  $\neq 0$  + Instantons

# The properties of QCD

- The particle picture and the laws governing them change according to the scales: in high resolution, almost massless quarks and gluons interacting weakly  
in low resolution, quarks and gluons can not be isolated because of the strong interaction (**confinement**)
- only hadrons are seen as elementary excitations in the low resolution or at low energies; hadrons would pop up as the resolution is lowered!
- The existence of the pion, the lightest hadrons, is related with the origin of the mass of nucleons and other hadrons (chiral transition; Y. Nambu)

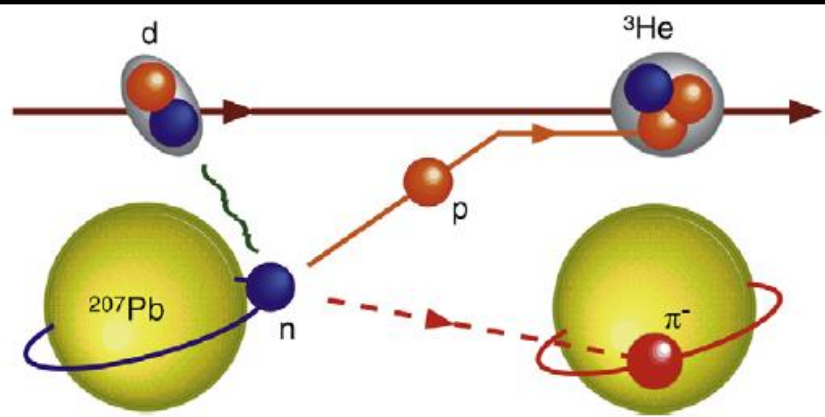
**Hadron/quark physics in the low-energy regime is a condensed matter physics of the vacuum (Y. Nambu; 1960-1961)**

**High temperature and density act as hard scales so that the phase transition of the QCD vacuum occur.**

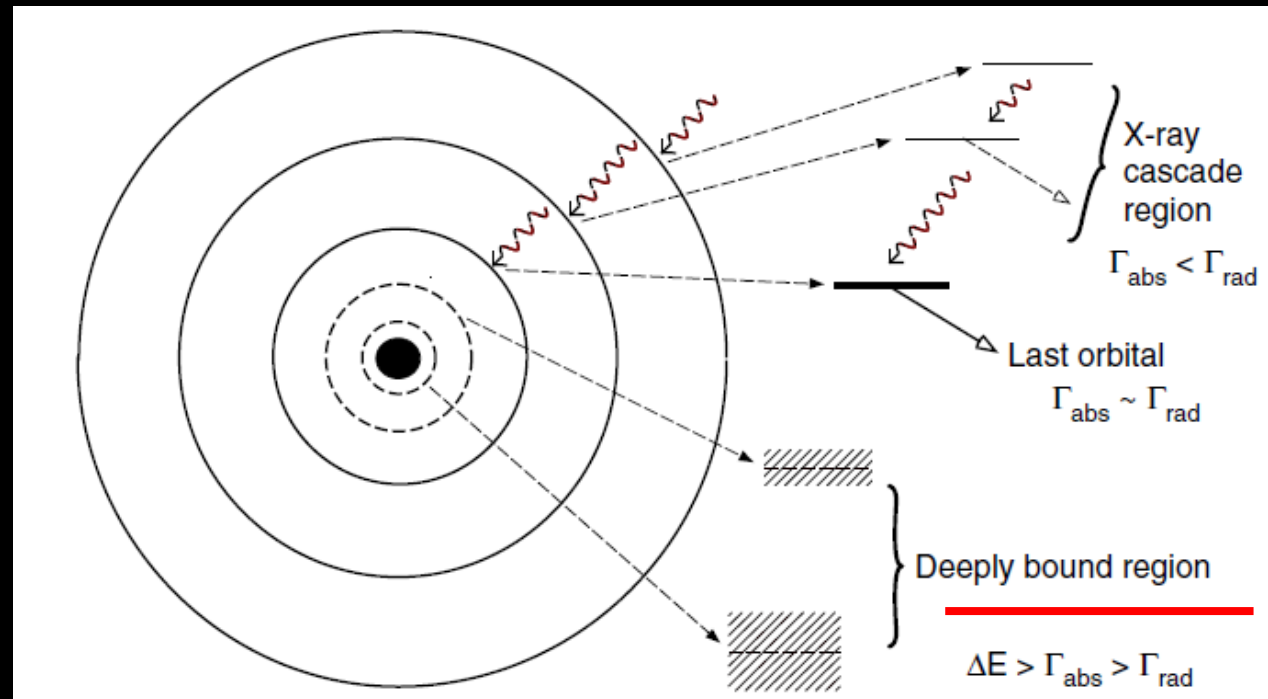
Realizing an environment  
where QCD vacuum is changed!

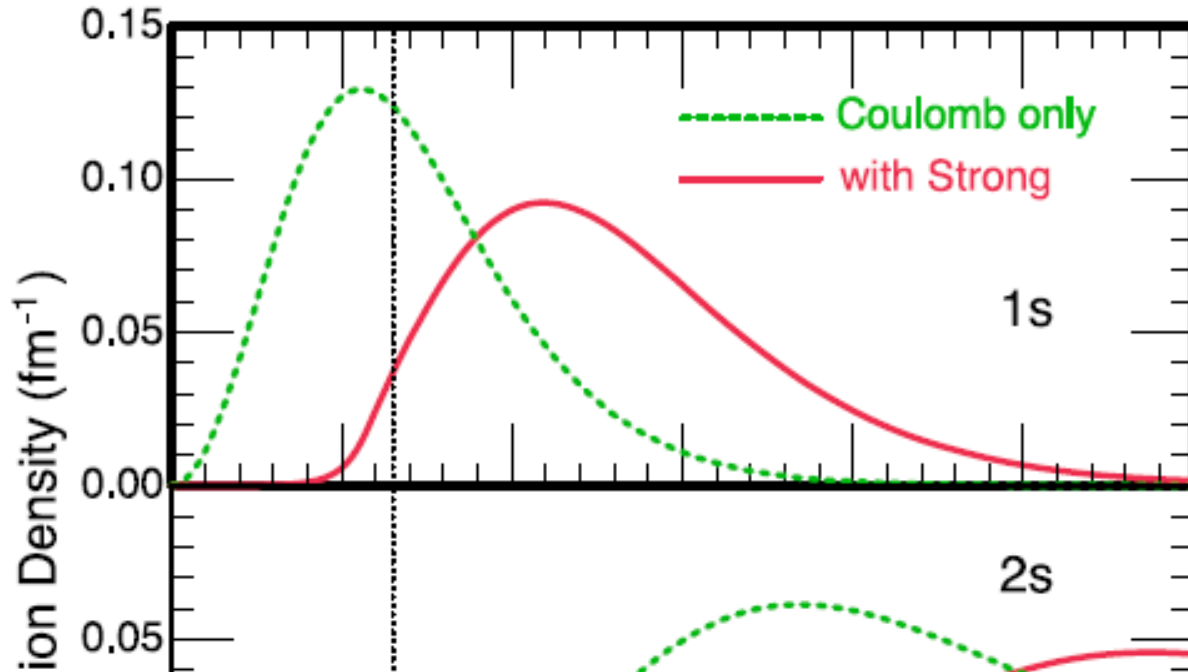
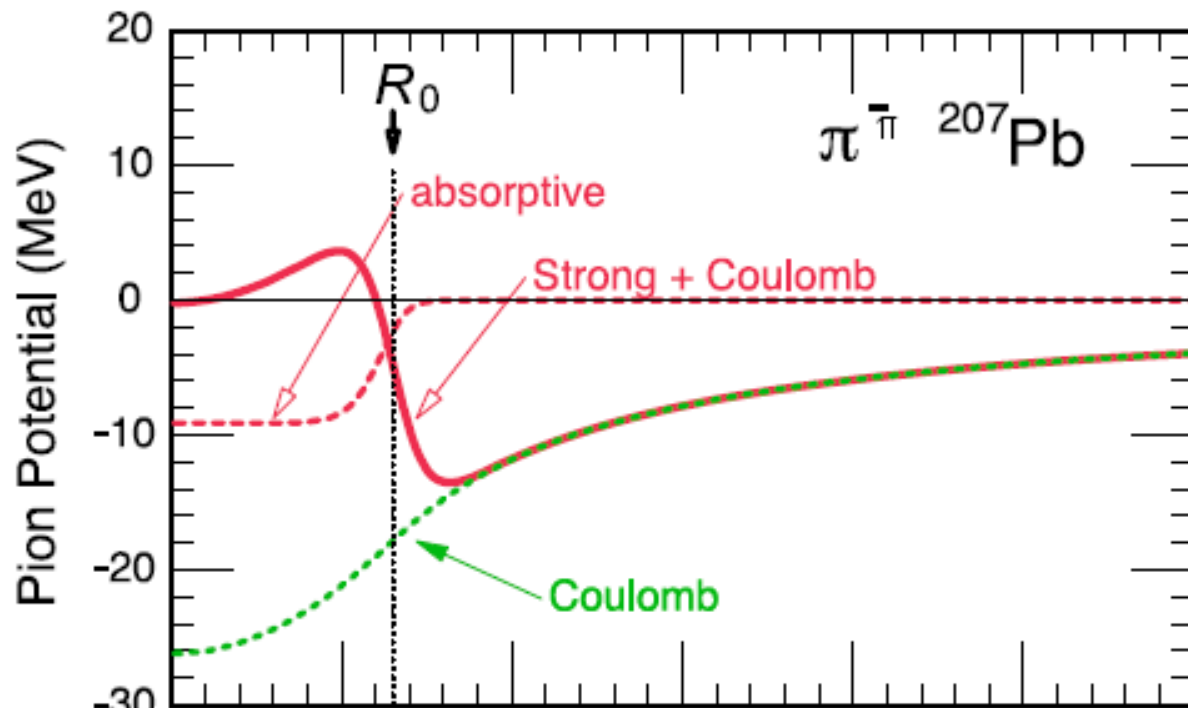


Discovery of Deeply bound pionic atom(1996); T.Yamazaki et al, ZPA (1996),  
 As predicted by Hirenzaki, Toki and Yamazaki, PRC (1991).



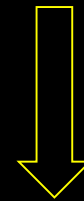
$(d, {}^3\text{He})$  @GSI





Attractive Coulomb  
+  
Repulsive pi-N int.

“Coulomb-assisted pionic Nuclei”

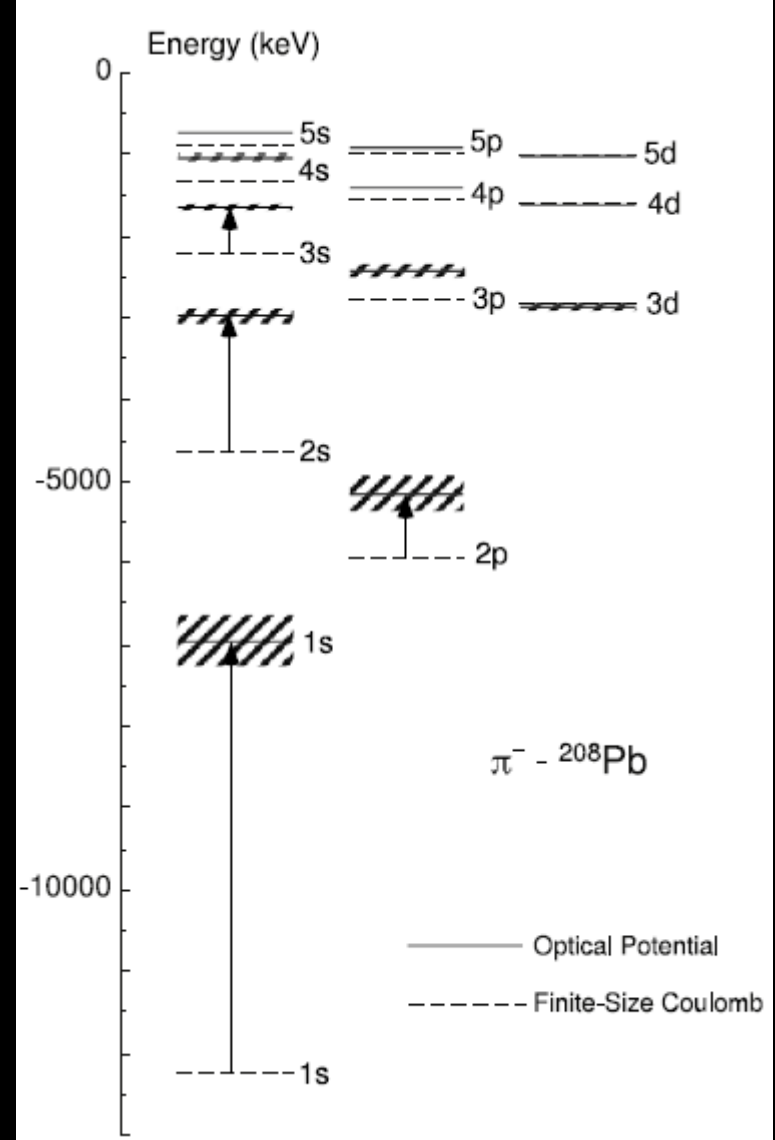
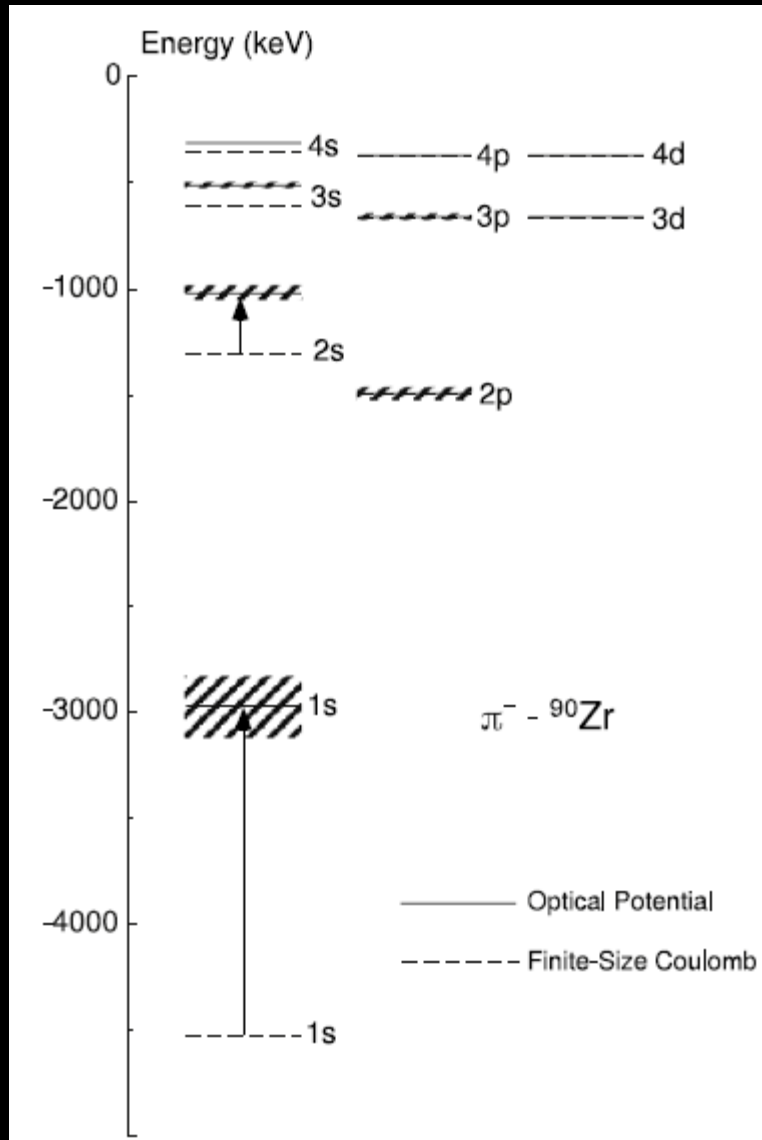


Localized around the surface of the nucleus, i.e.,

“halo-type bound states”

T. Yamazaki, S. Hirenzaki,  
R. S. Hayano and H. Toki,  
Phys. Rep. 514 (2012), 1

# Enhanced repulsion of pi-N interaction in s-wave



# Deeply bound pionic atom/nuclei and in-medium chiral condensate

- An enhanced repulsion due to Tomozawa-Weinberg term  $b_1^*$  as characterized by the reduced in-medium pion decay constant  $f_\pi^*$  :

$$T^{(+)} = \frac{1}{2}(T_{\pi-p} + T_{\pi-n}) \equiv 4\pi \varepsilon_1 b_0 = 0$$

$$T^{(-)} = \frac{1}{2}(T_{\pi-p} - T_{\pi-n}) \equiv -4\pi \varepsilon_1 b_1 = \frac{\omega}{2f_\pi^2}$$

**s-wave optical potential for  $\pi^-$  reads**

$$2m_\pi U_s = -4\pi \left[ 1 + \frac{m_\pi}{m_N} \right] (b_0^*(\rho)\rho - b_1^*(\rho)\delta\rho)$$

$$= -T^{(+)*}(\omega = m_\pi; m_\pi)\rho - T^{(-)*}(\omega = m_\pi; m_\pi)\delta\rho$$



$$\frac{b_1}{b_1^*} = \left( \frac{F_\pi^t}{F_\pi} \right)^2$$

$$T^{(-)*}(\omega; 0) \simeq \frac{\omega}{2(F_\pi^t)^2}$$

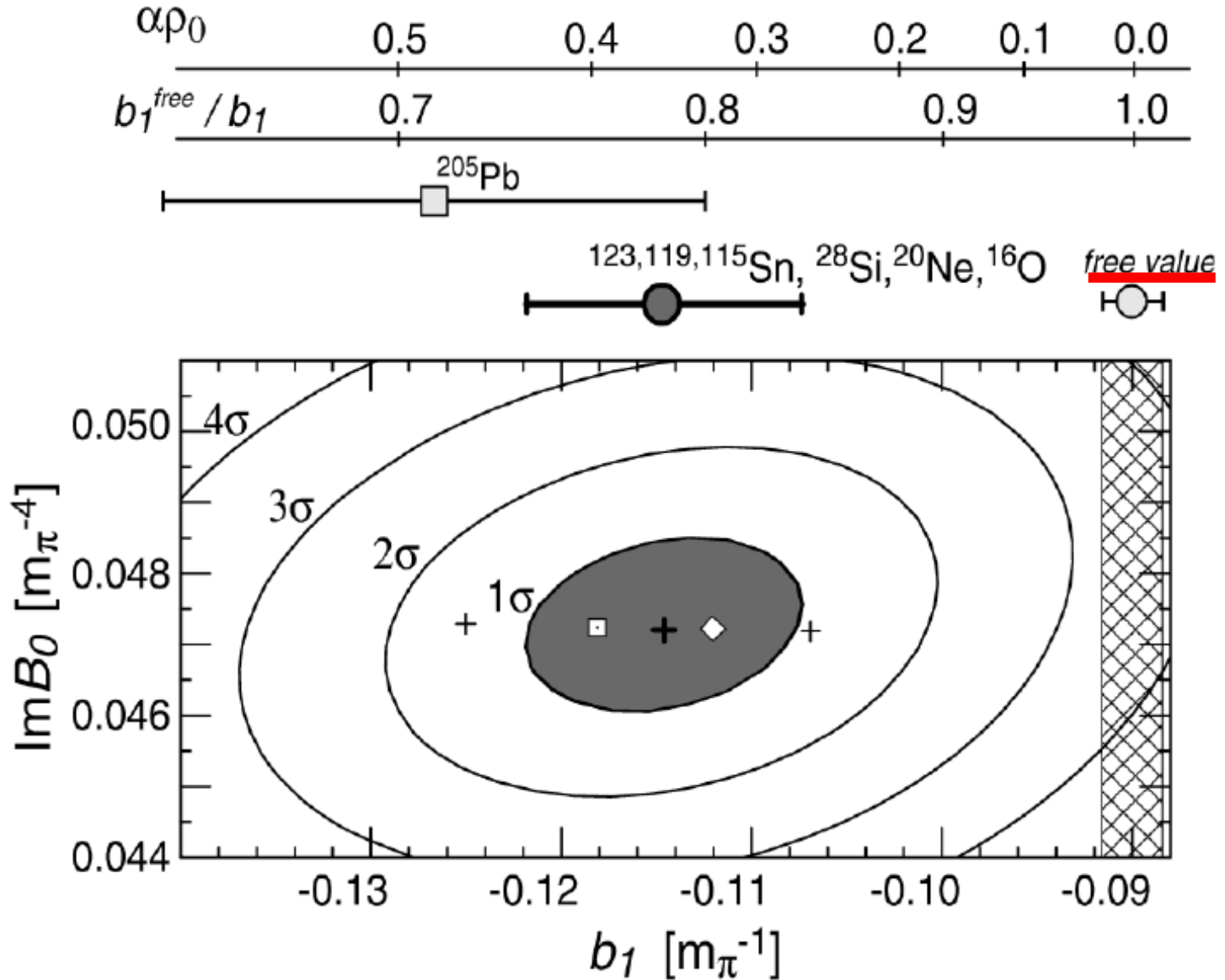
Kolomeitsev-Kaiser-Weise,  
PRL90 (2003)

which can be related to that of the chiral condensate directly as

$$\frac{\langle \bar{q}q \rangle^*}{\langle \bar{q}q \rangle} \approx \left( \frac{b_1}{b_1^*} \right)^{1/2} \left( 1 - \gamma \frac{\rho}{\rho_0} \right)$$

**Jido, Hatsuda and TK, PLB670(2008)**

QCD vacuum may be effectively changed and chiral symmetry is partially restored even at density lower than the normal nuclear density in finite nuclei!



The absolute value to the quark condensate is expected to decrease in average in hot and/or dense matter.

## Finite T

$$\delta\langle\langle\bar{u}u\rangle\rangle \simeq \frac{\partial F_{\pi\text{-gas}}}{\partial m_u} = 3 \sum_{\mathbf{k}} \frac{\partial E_{\pi}(\mathbf{k})}{\partial m_u} n_{\pi}(\mathbf{k}) \quad \frac{\partial E_{\pi}(\mathbf{k})}{\partial m_u} = \frac{m_{\pi}}{E_{\pi}(\mathbf{k})} \frac{\partial m_{\pi}}{\partial m_u} = \langle\pi(\mathbf{k})|\bar{u}u|\pi(\mathbf{k})\rangle$$

$$\langle\pi(0)|\bar{u}u|\pi(0)\rangle \simeq \langle\pi(0)|\bar{d}d|\pi(0)\rangle \simeq \frac{\partial m_{\pi}}{\partial \hat{m}} \simeq \frac{m_{\pi}}{m_u + m_d} = 7 \sim 10 > 0.$$

q-bar q probes either the vacuum or pions that are thermally excited. The latter gives a positive number, and thus the absolute value of the averaged condensate decreases.

## Finite density

$$\langle\Psi|\bar{q}q|\Psi\rangle = \frac{\partial\langle\Psi|\mathcal{H}_{\text{QCD}}|\Psi\rangle}{\partial m_q}$$

$$\langle\text{nm}|\mathcal{H}_{\text{QCD}}|\text{nm}\rangle = E_{\text{vac}} + n_B[M_N + E_b.]$$

$$\frac{\partial M_N}{\partial m_q} = \langle N|:\bar{q}q:|N\rangle$$

$$\Sigma_{\pi N} = \hat{m}\langle N|:\bar{u}u + \bar{d}d:|N\rangle$$

$$\frac{f_{\pi}^* m_{\pi}^{*2}}{f_{\pi} m_{\pi}^2} = \frac{\langle\text{nm}|\bar{u}u + \bar{d}d|\text{nm}\rangle}{\langle\bar{u}u + \bar{d}d\rangle_0} \simeq 1 - \frac{n_B}{f_{\pi}^2 m_{\pi}^2} \Sigma_{\pi N}$$

$$\langle\bar{s}s\rangle_P = .53,$$

In the nuclear medium, nucleons play the same role as the pions do in hot matter.

# Phenomena expected when chiral symm. is (partially) restored.

Chiral restoration implies that correlators in the positive/negative parity get degenerate.

$$\langle S(\mathbf{x})S(\mathbf{y}) \rangle \rightarrow \langle P^a(\mathbf{x})P^a(\mathbf{y}) \rangle, \quad \langle A_\mu^a(\mathbf{x})A_\nu^b(\mathbf{y}) \rangle \rightarrow \langle V_\mu^a(\mathbf{x})V_\nu^b(\mathbf{y}) \rangle$$

Scalar-Pseudoscalar

Axial vector-Vector

**Chiral symmetry in Baryon sector;**

parity doubling? What is the nature of  $N^*(1535)$ ?

Ref. C. DeTar and T.K. Phys.Rev.D39,2805(1989)

Axial anomaly:  $\eta'$  in hot and dense matter



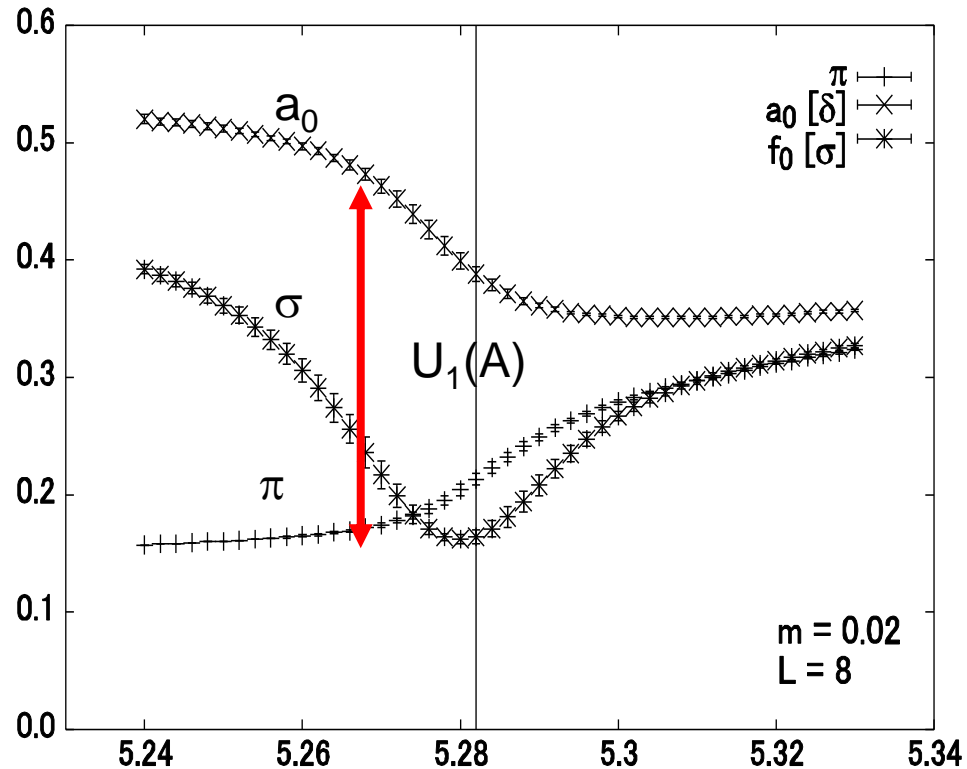
Cf. Lattice Calculation of the *generalized masses* : Screening masses

F. Karsch, Lect. Note Phys. **583** (2002), 209.  $N_f = 2$ ,  $8^3 \times 4$ ; Staggered fermion

$$m_\sigma^2 = \chi_\sigma^{-1}$$

$$\chi_\sigma = \langle (\bar{q}q)^2 \rangle$$

the **softening** of the  $\sigma$  with increasing  $T$



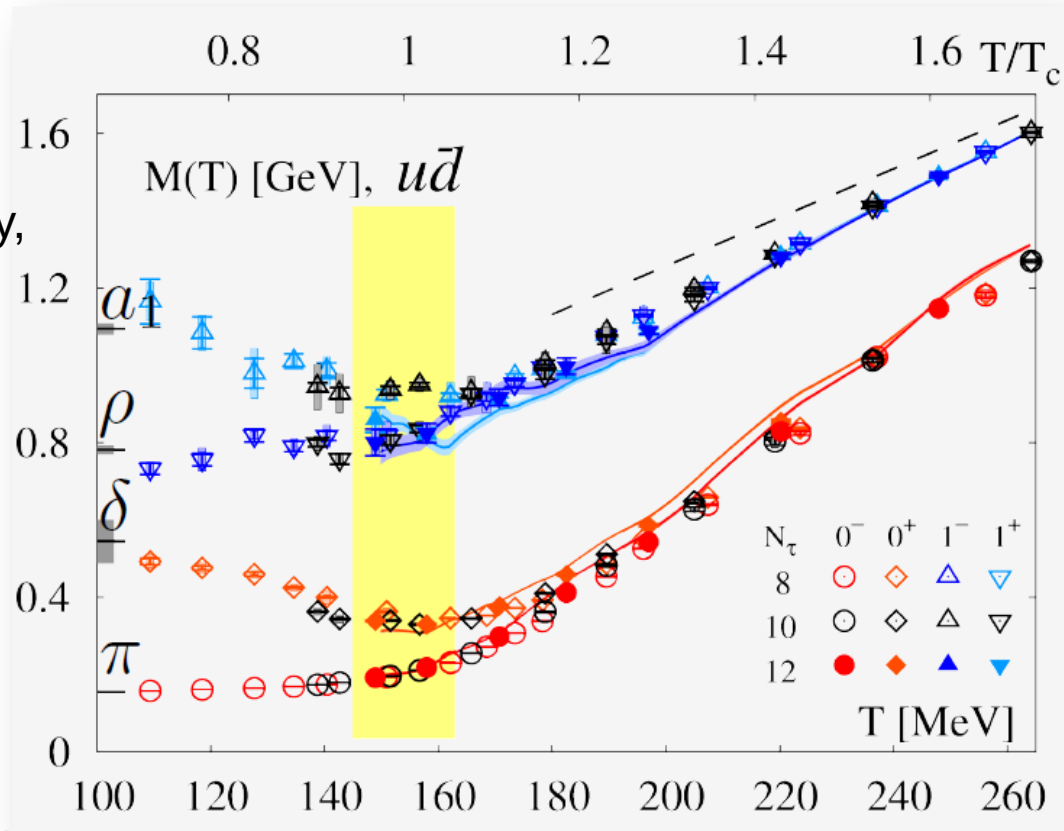
and

a degeneracy of the  $\sigma$  and  $\pi$  at high  $T$

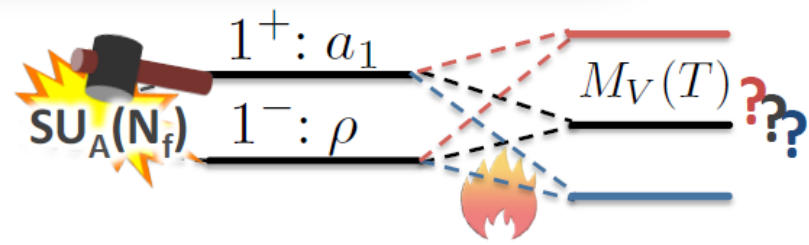
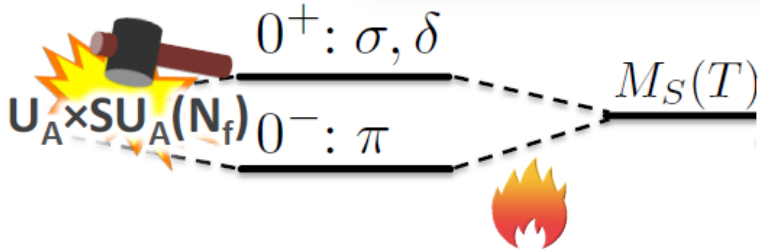
# Screening mass in light mesons

Karsch, Maezawa,  
Mukhrjee, Petereczky,  
In preparation

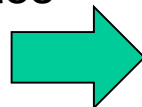
Open:  $N_\tau = 8$   
Black:  $N_\tau = 10$   
Filled:  $N_\tau = 12$



Restoration pattern



Positive parity and negative parity states  
Tend to come close to each other.



Yu Maezawa @RCNP seminar Feb. 2016

Dynamical masses? And/or @ finite  $\rho$

# Effective Lagrangian with axial anomaly

$$\begin{aligned}\text{Def. } \Phi_{ij} &\equiv \bar{q}_j(1 - \gamma_5)q_i = 2\bar{q}_{jR}q_{iL} = \bar{q}_j q_i + i\bar{q}_j i\gamma_5 q_i \\ (\Phi^\dagger)_{ij} &\equiv \bar{q}_j(1 + \gamma_5)q_i = 2\bar{q}_{jL}q_{iR}\end{aligned}$$

Transformation properties:

$$SU_L(3) \otimes SU_R(3) \quad \Phi_{ij} \rightarrow L_{ik}\Phi_{kl}R_{lj}^\dagger, \quad (\Phi^\dagger)_{ij} \rightarrow R_{ik}(\Phi^\dagger)_{kl}L_{lj}^\dagger$$

$$U_L(1) \otimes U_R(1) \simeq U_V(1) \otimes U_A(1)$$

$$\Phi_{ij} \rightarrow e^{i(\theta_{0L}-\theta_{0R})/2}\Phi_{ij} \quad (\Phi^\dagger)_{ij} \rightarrow e^{-i(\theta_{0L}-\theta_{0R})/2}(\Phi^\dagger)_{ij}$$

$$I_n = \text{tr}(\Phi\Phi^\dagger)^n, \quad (n = 1, 2, 3, \dots) \quad U_L(3) \otimes U_R(3)\text{-invariant}$$

$$\Phi = \sum_{a=0}^8 \Phi_a \lambda_a / \sqrt{2} \quad (\because \text{tr} \lambda_a \lambda_b = 2\delta_{ab})$$

$$\Phi_a = \text{tr} \Phi \lambda_a / \sqrt{2} = \bar{q}(1 - \gamma_5) \lambda_a q / \sqrt{2}$$

$$= \hat{\sigma}_a + i\hat{p}_a, \quad \text{with } \hat{\sigma}_a = \bar{q} \lambda_a q / \sqrt{2} \quad \hat{p}_a = \bar{q} i \gamma_5 \lambda_a q / \sqrt{2}$$

$$I_1 = \sum_{a,b=0}^8 \Phi_a \Phi_b^\dagger \text{tr} \lambda^a \lambda^b / 2 = \sum_{a=0}^8 \Phi_a \Phi_a^\dagger,$$

$$= \sum_{a=0}^8 [\hat{\sigma}_a^2 + \hat{p}_a^2]$$

$\exists U_A(1)$  Anomaly:  $\bar{q}_i q_j$ ; particle # is always conserved.

$\det \Phi, \det \Phi^\dagger: U_V(1) \otimes SU_L(3) \otimes SU_R(3)$  - invariant

$$\det \Phi \rightarrow \det (L \Phi R^\dagger) = \det L \det \Phi \det R^\dagger = \det \Phi$$

but  ~~$U_A(1)$~~

$$\Phi_{ij} \rightarrow e^{i(\theta_{0L} - \theta_{0R})/2} \Phi_{ij} \rightarrow \det \Phi \rightarrow e^{i3(\theta_{0L} - \theta_{0R})} \det \Phi \neq \det \Phi$$

$$I_D = \det \Phi + \det \Phi^\dagger ; \text{Hermite}$$

# Effective Model; $SU_L(3) \otimes SU_R(3)$ - $\sigma$ model

$$\mathcal{L}_\sigma^{(0)} = 1/2 \cdot (\text{tr} \partial_\mu \Phi \partial^\mu \Phi) - 1/2 \cdot \mu^2 I_1 - \lambda I_1^2 - \gamma I_2 + \tau I_D$$

In the chiral limit.

## I. Vacuum:

$$\left\langle \frac{\partial \mathcal{L}_0}{\partial \Phi^\dagger} \right\rangle = 0$$



Ansatz:

$$\langle \Phi \rangle = \varphi_0 \mathbf{1}, \quad \mathbf{1} = \text{diag}(1, 1, 1)$$

$$\langle \Phi \rangle \rightarrow L(\vartheta_L) \langle \Phi \rangle R^\dagger(\vartheta_R)$$

If  $\vartheta_L = \vartheta_R$ , i.e.,  $SU_V(3)$ ,

$\langle \Phi \rangle$  is invariant, but otherwise not..

$$2(3\lambda + \gamma)\varphi_0^2 - \tau\varphi_0 + \mu^2/2 = 0$$

$$\therefore \varphi_0 = \frac{\tau + \sqrt{\tau^2 - 4\mu^2(3\lambda + \gamma)}}{4(3\lambda + \gamma)} \quad \text{for } \mu^2 < 0$$

## 2. Meson spectra:

$$\Phi = \varphi_0 \mathbf{1} + \Phi', \quad \Phi' = \frac{1}{\sqrt{2}}(S + iP)$$

$$S = \sum_{a=0}^8 S_a \lambda_a \quad P = \sum_{a=0}^8 P_a \lambda_a$$

Meson masses;

(1) ps-mesons

from the coefficients of  $-1/2 \cdot P_a^2$

8重項

$\pi, K, \eta_8$

$$m_{\text{ps}}^{(8)2} = \mu^2 + 4\varphi_0^2(3\lambda + \gamma) - 2\varphi_0\tau = 0$$

NG-boson

一重項

$\eta_1$

$$m_{\text{ps}}^{(0)2} = 6\tau\varphi_0 \neq 0$$

$$\left\langle \frac{\partial \mathcal{L}_0}{\partial \Phi^\dagger} \right\rangle = 0$$

Anomaly term,  
the strength of which hardly  
changes.

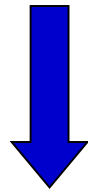
Chiral condensate, which is reduced in  
Hot and/or dense matter rather significantly.

Experiment to explore a possible change of the eta' in **finite nuclei** is being made in GSI at Germany.

# A dynamical Chiral Lagrangian with Axial Anomaly

M. Kobayashi and T. Maskawa ('70),  
G. 't Hooft ('76)

$$\mathcal{L} = \bar{q}i\gamma \cdot \partial q + \sum_{a=0}^8 \frac{g_S}{2} [(\bar{q}\lambda_a q)^2 + (\bar{q}i\lambda_a \gamma_5 q)^2] - \bar{q}mq + g_D [\det \bar{q}_i(1 - \gamma_5)q_j + \text{h.c.}]$$



T.K. Soryushiron Kenkyu (1988),  
T.K. and T. Hatsuda, Phys. Lett. B (1988);  
Phys. Rep. 247 (1994)

A presentation of Chiral Anomaly:

$$\partial_\mu A_5^\mu = 2iN_f g_D (\det \Phi - \text{h.c.}) + 2i\bar{q}m\gamma_5 q \quad \Phi_{ij} = \bar{q}_j(1 - \gamma_5)q_i$$



$$\partial_\mu A_5^\mu = 2N_f \frac{g^2}{32\pi^2} F_{\mu\nu}^a \tilde{F}_a^{\mu\nu} + 2i\bar{q}m\gamma_5 q \quad \text{Anomaly eq. of QCD}$$

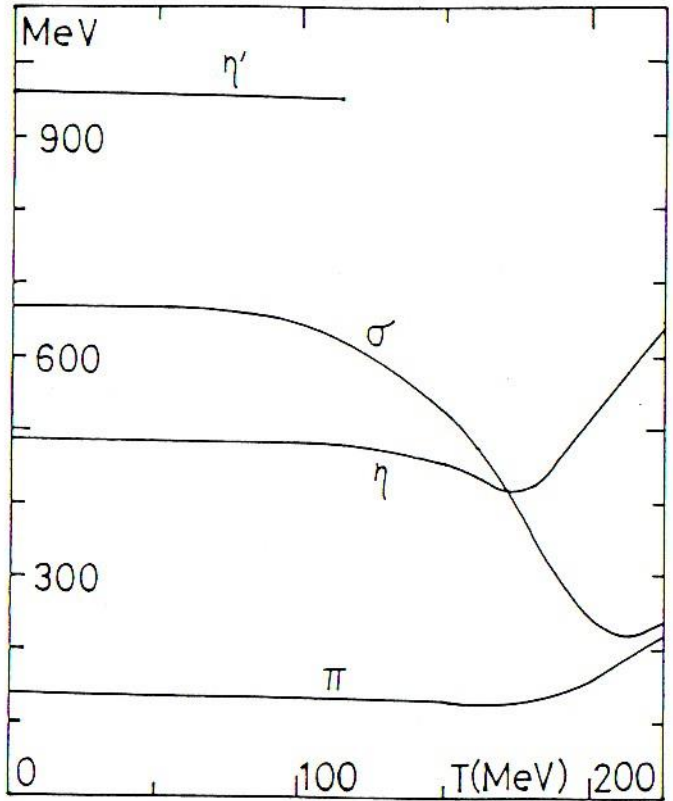
*Note:*  $g_D < 0$  consistent with the instanton-induced interaction

# Effective restoration of axial symmetry at finite temperature

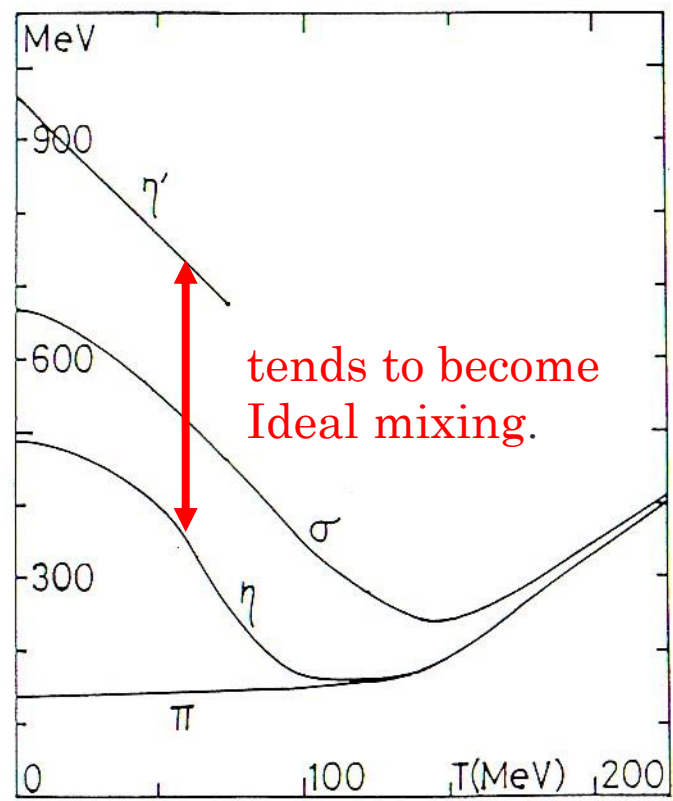
R. Pisarski and F. Wilczek(1984)

$\eta - \eta'$

T. K. Phys. Lett. B (1989)



T-independent  $g_D$



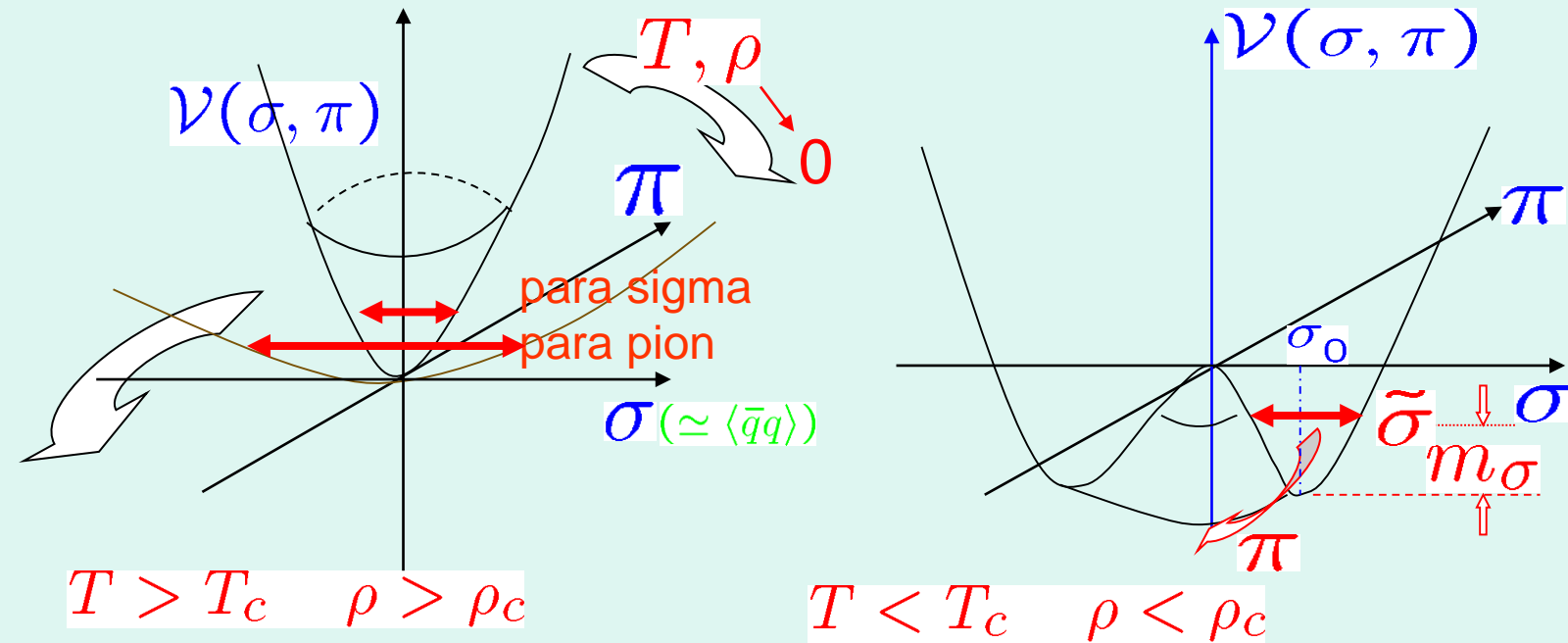
$$g_D(T) = g_D(T = 0) \exp[-(T/T_0)^2].$$

$$T_0 = 100 \text{ MeV}$$

NJL model with Kobayashi-Maskawa-'t Hooft term; T.K. and T.Hatsuda (1988)



# The sigma meson as a Higgs particle of chiral symmetry breaking in QCD



The low mass sigma in vacuum is now established:  
 pi-pi scattering; Colangelo, Gasser, Leutwyler('06) and many others  
 Full lattice QCD ; SCALAR collaboration ('03)

**q-qbar, tetra quark, glue balls, or their mixed st's?**

M.Wakayama et al(SCALAR Collab.), PRD91(2015)

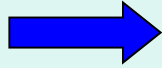
c.f. The sigma as the Higgs particle in QCD  $\sigma = \sigma_0 + \tilde{\sigma}$ ; a composite particle  
 $\phi$ ; Higgs field  $\longrightarrow \phi = \langle \phi \rangle + \tilde{\phi}$

Higgs particle (discovered @2012) with mass=125 GeV

Is the Higgs a structureless elementary particle? Recent anomalous events in LHC?

# The sigma as the Higgs particle in QCD

$$\sigma = \sigma_0 + \tilde{\sigma} \quad ; \text{a composite particle}$$

$\phi$  ; Higgs field   $\phi = \langle \phi \rangle + \tilde{\phi}$

Higgs particle (discovered @2012) with mass=125 GeV:

The corresponding NG bosons are absorbed into W and Z to make Them massive.

Is the Higgs a structureless elementary particle?

What are the meaning of the recent anomalous events in LHC?  
Diphoton excess @ 750 GeV with  $3\sigma$  by ATLAS and CMS

Techni-pion (pseudo scalar)

Diboson excess @ 2 TeV with  $3\sigma$  by ATLAS; arXiv:1506.00962

Techni-rho(vector)

An interpretation based on one-familt Technicolor model:

S.Matsuzaki and K.Yamawaki, arXiv:1512.05564

Understanding of the  $\sigma$  may help the underlying physics of the Higgs and its possible brothers/sisters.

T.Hyodo, D. Jido and TK, NPA848(2010)

## Linear sigma model with parity doubling

Carleton DeTar\*

*Research Institute for Fundamental Physics, Kyoto University, Kyoto 600, Japan*

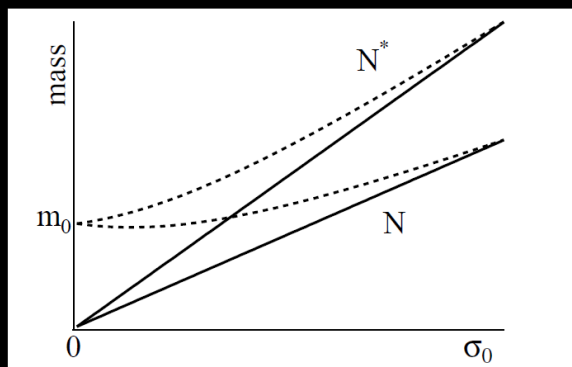
Teiji Kunihiro†

*Department of Natural Sciences, Ryukoku University, Fushimi-ku, Kyoto 612, Japan*

(Received 27 May 1988)

Recent lattice-gauge-theory simulations at finite temperatures have suggested that chiral-symmetry restoration at finite temperatures entails parity doubling of the baryon spectrum. We show that a natural extension of the Gell-Mann–Lévy model incorporates this effect. Predictions of this candidate effective model for the hadronic component of high-density and high-temperature nuclear matter are discussed. The model suggests a parametrization of the dependence of the baryon-doublet masses on the quark mass. This parametrization is compared with the recent lattice results.

$$\mathcal{L} = \bar{\Psi} i \gamma \cdot \partial \Psi - g_1 \bar{\Psi} (\sigma + i \pi \cdot \pi \rho_3 \gamma_5) \Psi + g_2 \bar{\Psi} (\rho_3 \sigma + i \pi \cdot \pi \gamma_5) \Psi - i m_0 \bar{\Psi} \rho_2 \gamma_5 \Psi + \mathcal{L}_M(\sigma, \pi)$$

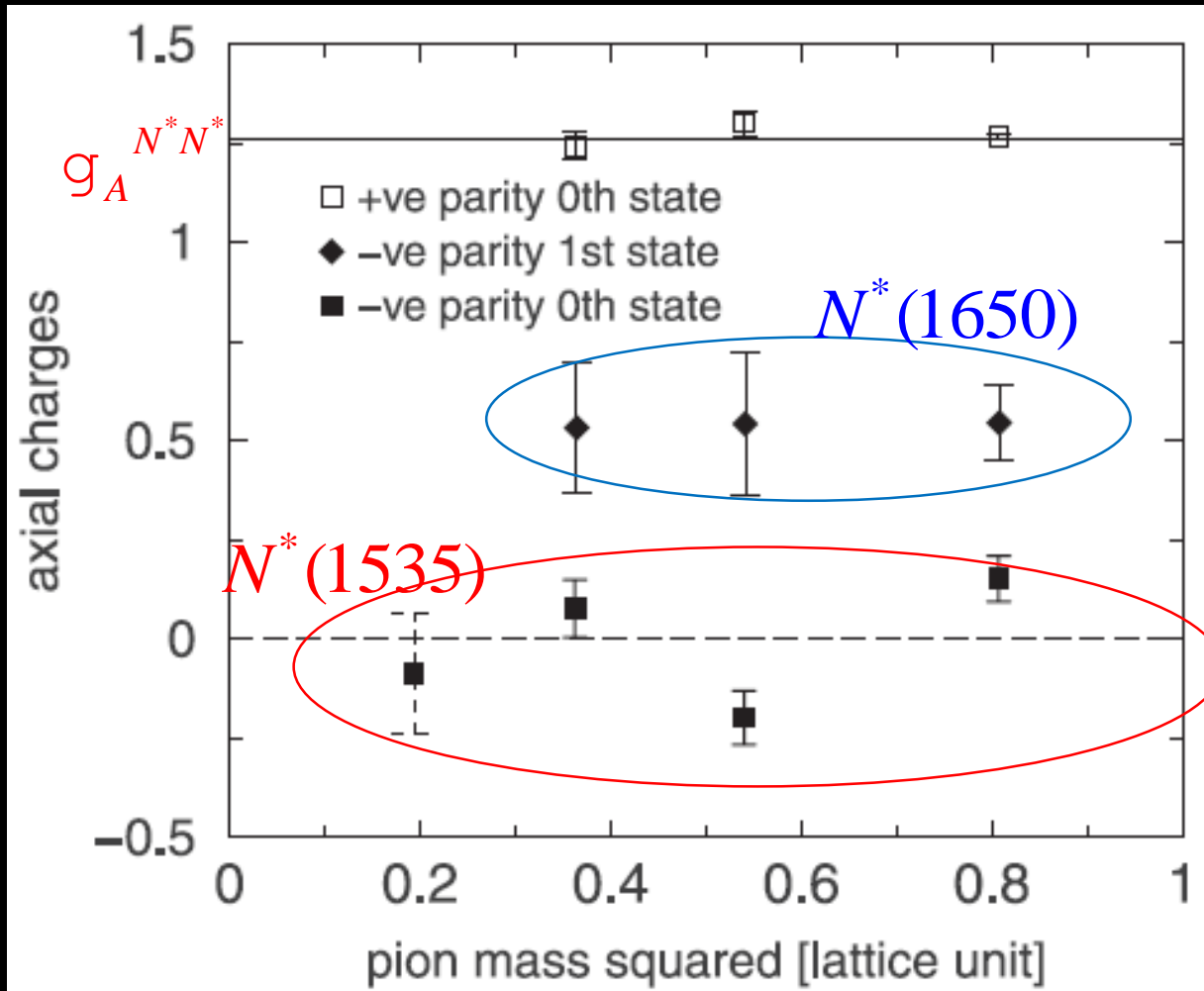


$$g_A = \begin{bmatrix} \tanh \delta & -1/\cosh \delta \\ -1/\cosh \delta & -\tanh \delta \end{bmatrix}$$

The axial charge of  $N^*(1535)$  could be negative!

# Full QCD lattice calculation of the axial charges of $N^*(1535)/N^*(1650)$

T.T.Takahashi and T.Kunihiro, PRD78 (2008)



Surprisingly, The lattice simulation with full QCD tells as that  
**the axial charge of  $N^*(1535)$  is vanishingly small!**

# In-medium $\eta \rightarrow 3\pi$ decay

S.Sakai and T.K., PTEP(2015), (2016)

Possible effects of isospin asymmetry on hadron decay in nuclear medium

## ○ $\eta \rightarrow 3\pi$ ( $\pi^+\pi^-\pi^0, 3\pi^0$ ) decay (in free space)

- ✓ Isospin-symmetry breaking in QCD ( $u-d$  quark mass difference)
  - G parity violating process ( $\eta$ :even, $\pi$ :odd)

✳ Small QED corrections (Sutherland(1966), Baur et al.(1996), Ditsche et al.(2009))



Small decay width ( $\sim 70$  eV from current algebra)

Osborn and Wallace (1970)

- ✓ Final-State Interaction among  $\pi \leftarrow$  Significance of  $\sigma$  (s-wave  $2\pi$ ) channel

### - Perturbative approach

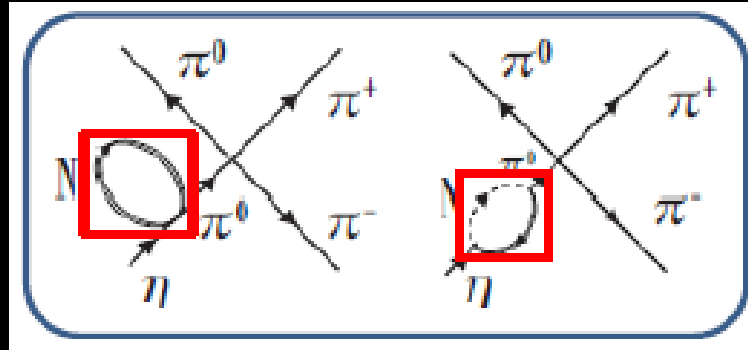
Chiral perturbation theory: Gasser and Leutwyler(1985), Bijnens and Ghorbani(2007)



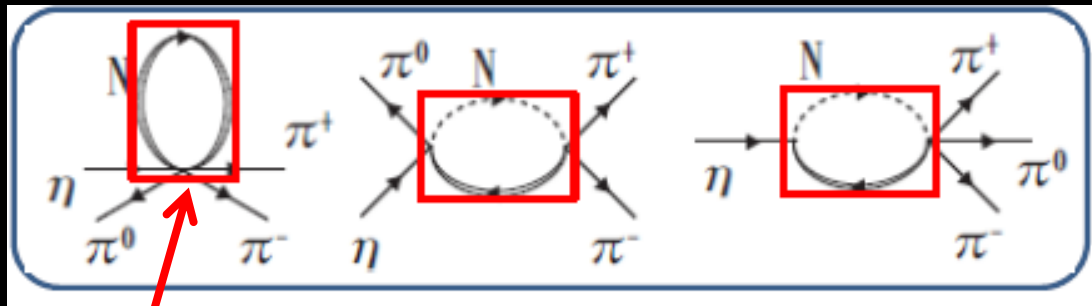
### - Non-perturbative approach

- Chiral Unitary approach (resummation scheme): Borasoy and Nissler(2005)
- Dispersive approach (Roiesnel and Truong(1981), Kambor et al.(1996), Anisovich and Leutwyler(1996),...)

# Modification of the mixing angle in the asymmetric nuclear medium

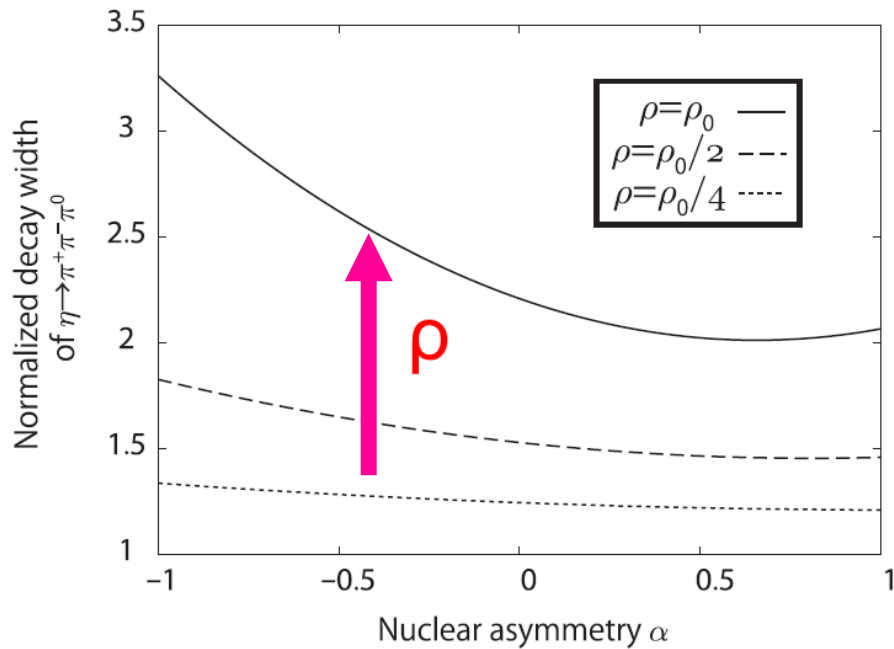


## Vertex corrections in the (asymmetric) nuclear medium

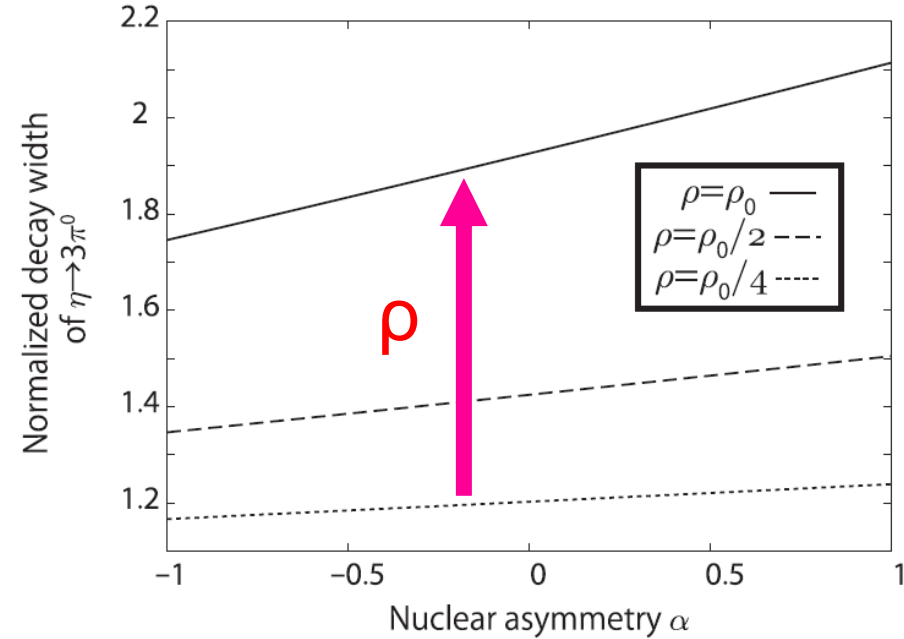


$\rho$  -dependent

## Charged decay



## Neutral decay



$$\alpha \equiv \delta\rho/\rho$$

$$\delta\rho = \rho_n - \rho_p$$

Isospin asymmetry of the nuclear medium does affect the  $\eta \rightarrow 3\pi$  decay, but the total density dependence overwhelms it, which is caused by the Enhancement in the sigma channel and can reflect the partial restoration of the chiral symmetry.



## Brief summary and concluding remarks

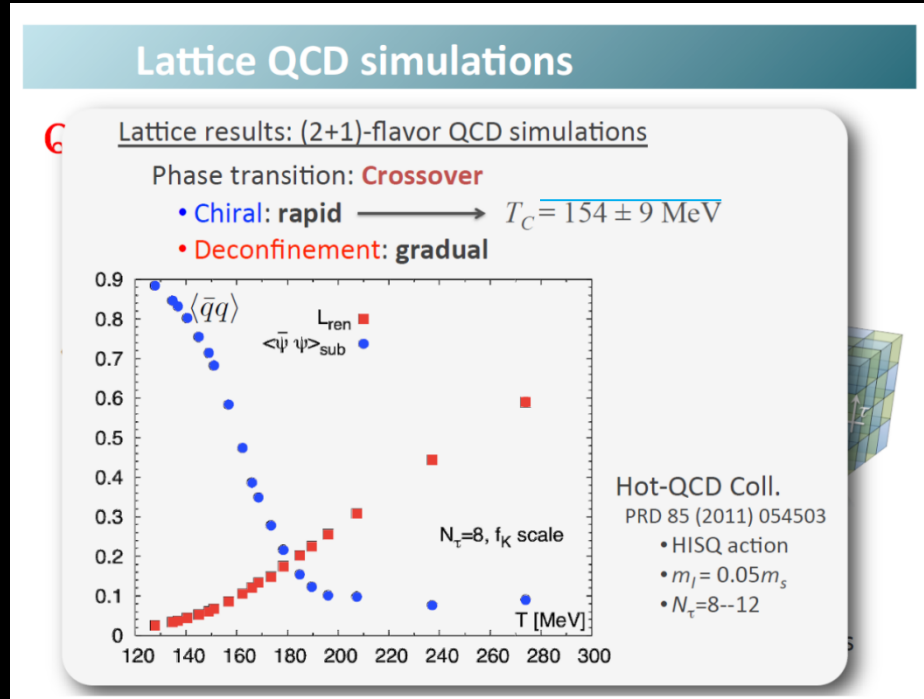
1. Hadrons are elementary excitations on top of the nonperturbative vacuum of the QCD vacuum.
2. Chiral symmetry is spontaneously broken in the QCD vacuum, but the symmetry may get (partially) restored
3. In various environment characterized by temperature, density, magnetic field and so on.
4. The deeply bound pionic atoms/nuclei may show that the chiral symmetry is partially restored even in finite nuclei existing in Nature.
5. Various exotic phenomena can be expected to occur along with the (partial) restoration of chiral symmetry.

### Physics of pionic atoms/nuclei involves

- 1) Interplay of EM with the Strong int.
- 2) Effects of large isospin asymmetry

• Magnetic field  $B$  induces isospin asymmetry due the different charges of  $u$  and  $d$ . EM field v.s. Chiral symm. Breaking ;eg. S. Klevansky,RMP (1992) and many many others.

• Hadron-`QGP' transition at finite  $T$  is crossover!



Yu Maezawa @RCNP seminar Feb. 2016

What is the physical picture of `hadrons' around the crossover region?  
**Swelled?** Quarks/gluons **are percolated?**  
**Super-multi quark hadrons?**  
**Tetraquarks** or **diquarks** play significant roles?



**Implications to finite density systems?**

**H-dibaryon** matter in the intermediate stage? R. Tamagaki, PTP85 (1991)

Hadron-quark **transition at finite  $\mu$  also crossover?**

Masuda, Hatsuda and Takatsuka, ApJ764(2013)

The role of the **vector interaction  $g_v$**  for the crossover important? TK, PLB271 (1991), As well as the axial anomaly; Kitazawa et al,PTP108(2002); Hatsuda et a; PRL97 (2006)