

Systematic studies of correlations between different order flow harmonics in Pb–Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV

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HIM 2016 05

based on arXiv:1604.07663

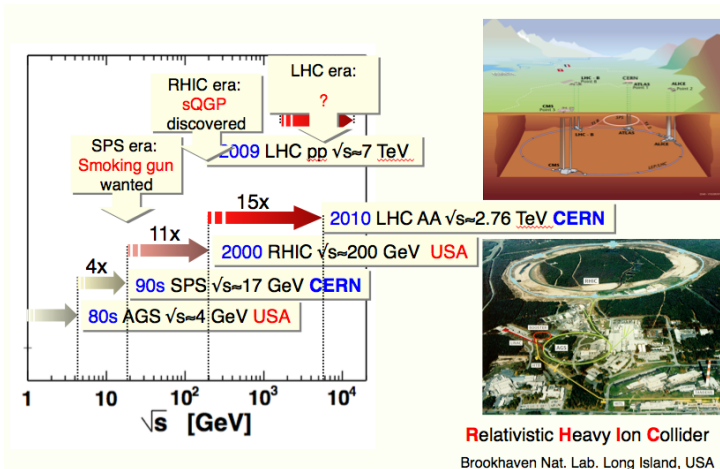


ALICE @ CERN

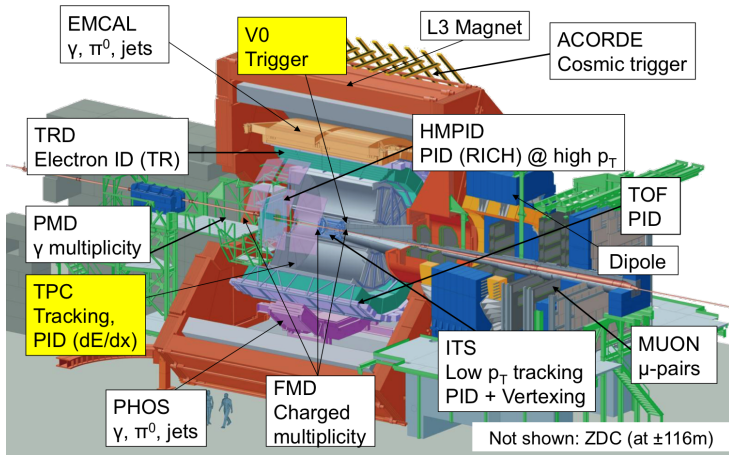


- LHC(Large Hadron Collider), SPS, PS
- 4 major experiments
- p+p collisions(as reference), Pb+Pb collisions

Compare with other experiments

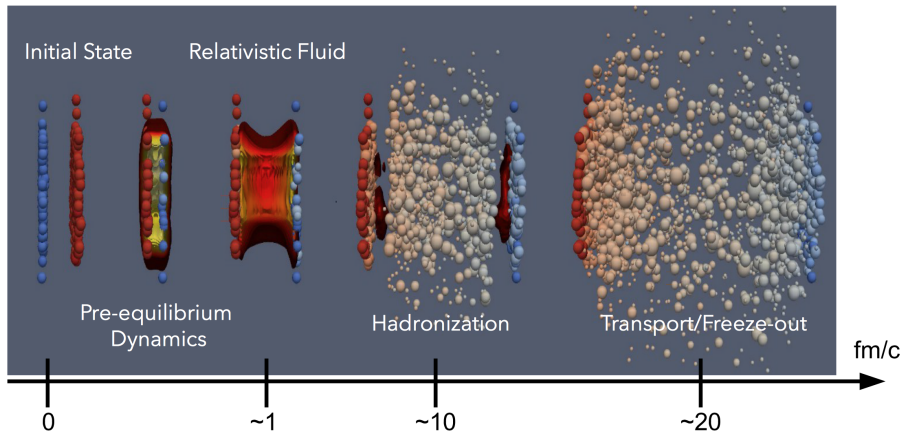


ALICE Detectors



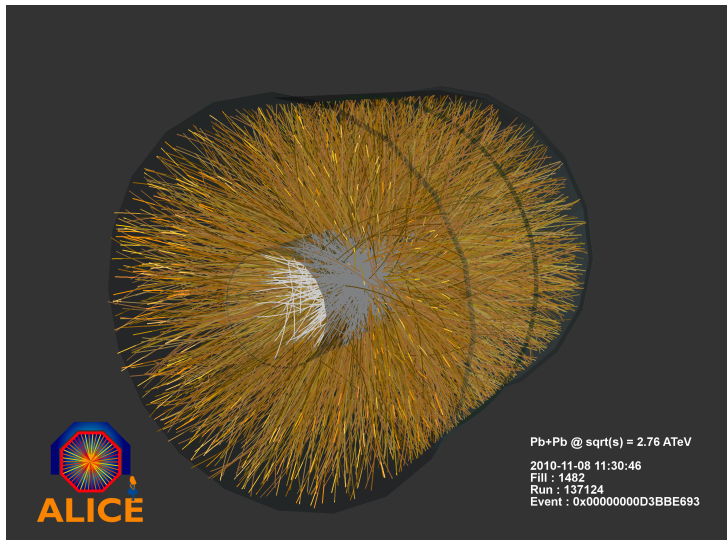
- Centrality determination by V0(N_{ch} with scintillators in $2.8 < \eta < 5.1$ and $-3.7 < \eta < -1.7$) in **Pb–Pb collisions** at $\sqrt{s_{NN}} = 2.76 \text{ TeV} \approx 20M$ events.
- Tracking - TPC tracks constrained to the primary vertex and **full azimuthal acceptance** (Unidentified charged particles $|\eta| < 0.8$, $0.2 < p_T < 5.0 \text{ GeV}/c$)

Evolution of heavy ion collision



Visualization: madai.us

And it's results



First event from ALICE experiments in 2010

Flow analysis

As $\frac{dN}{d\phi}$ is a periodic function ($0 \sim 2\pi$), it can be expressed with Fourier transformation.

$$\frac{dN}{d\phi} = \frac{x_0}{2\pi} + \frac{1}{\pi} \sum_{n=1} (A_n \cos n\phi + B_n \sin n\phi) \quad (1)$$

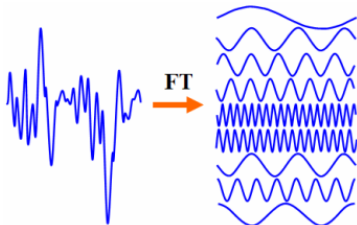
If we define v_n and ψ_n such as,

$$v_n^2 = A_n^2 + B_n^2, \quad 0 \leq \psi_n \leq \frac{2\pi}{n} \quad (2)$$

Then we can express A_n and B_n with v_n and ψ_n . if we put back these into original equation (1) then

$$\frac{dN}{d\phi} = \frac{x_0}{2\pi} + \frac{1}{2\pi} \sum_{n=1} (2v_n \cos n(\phi - \psi_n)) \quad (3)$$

And, we called v_n as flow constant, and ψ_n as event plane angle.



How to measure flow?

(P. Danielewicz, G. Odyniec, Phys. Lett. 157B, 146 (1985))

- Fourier decomposition is used to quantify the anisotropic distribution of produced particles

$$\frac{dN}{d\phi} = \frac{v_0}{2\pi} + \frac{1}{2\pi} \sum_{n=1} (2v_n \cos n(\varphi - \psi_n))$$

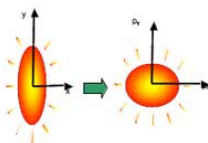
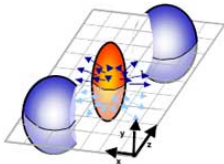
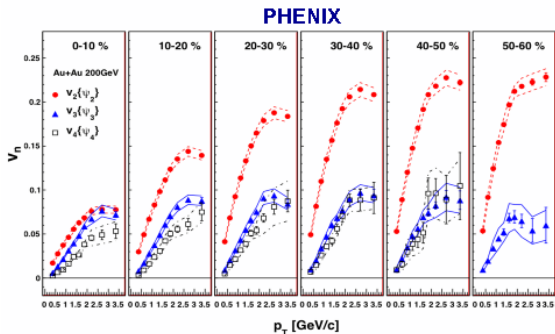
- Flow magnitude v_n can be estimated with Event Plane method

$$v_n\{EP\} = \langle \cos n(\varphi - \psi_n) \rangle$$

- or by measuring multi-particle correlation (Cumulant method)

$$v_n\{2\} = \sqrt{\langle \cos n(\varphi_1 - \varphi_2) \rangle}$$

First results from PHENIX



- The second coefficient of Fourier's harmonics (v_2) is significantly larger than any other harmonics
- This v_2 values are grow as function of p_T and Centrality
- This phenomenon was unique in heavy-ion collisions

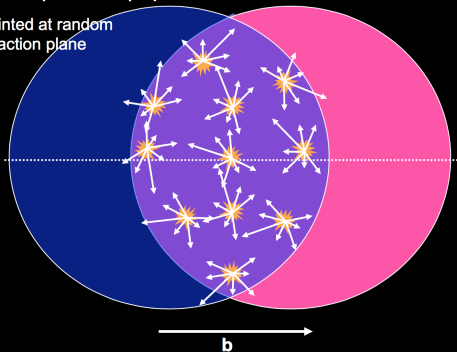
Elliptic Flow

Ollitrault 1992

1) superposition of independent p+p:

momenta pointed at random
relative to reaction plane

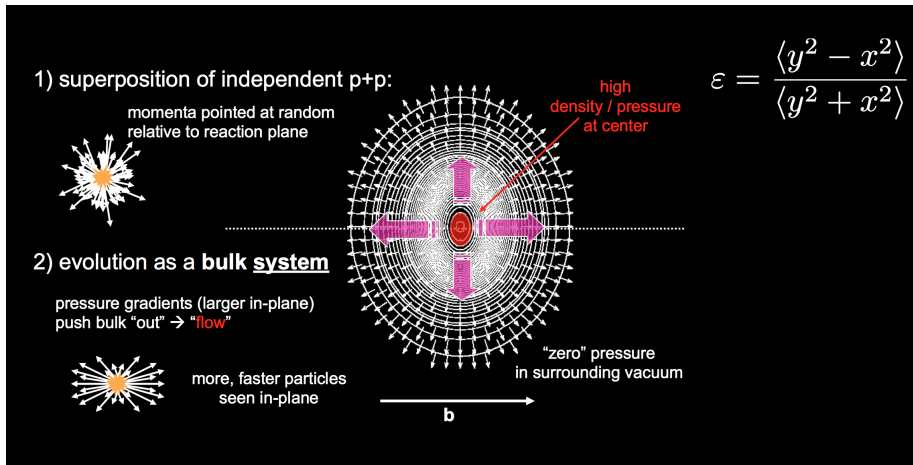
Animation: Mike Lisa



$$\varepsilon = \frac{\langle y^2 - x^2 \rangle}{\langle y^2 + x^2 \rangle}$$

courtesy of Raimond Snellings (New J.Phys. 13 (2011) 055008)

Elliptic Flow

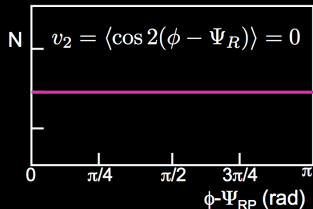


courtesy of Raimond Snellings (New J.Phys. 13 (2011) 055008)

Elliptic Flow

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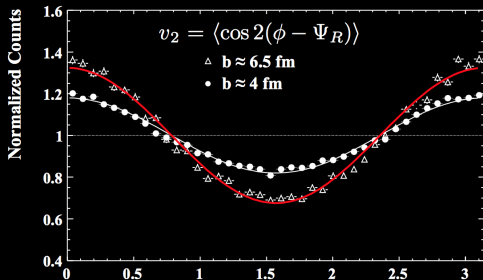


2) evolution as a **bulk system**

pressure gradients (larger in-plane)
push bulk "out" \rightarrow "flow"



more, faster particles
seen in-plane

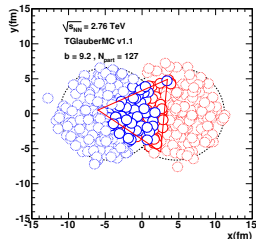
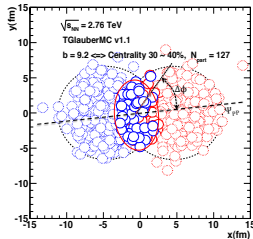
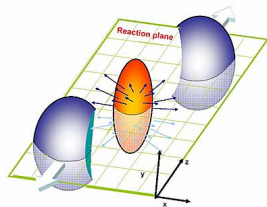


courtesy of Raimond Snellings (New J.Phys. 13 (2011) 055008)

Note

For better understanding, I'd like to make it sure that

- Reaction plane (RP) : Plane which is defined by IP and z-axis (beam direction)
- Participant plane (PP) : Effective RP affected by non-perfect isotropic shape
- (n-th order) Event plane (EP) : Mathmatically defined by above equation



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Note!

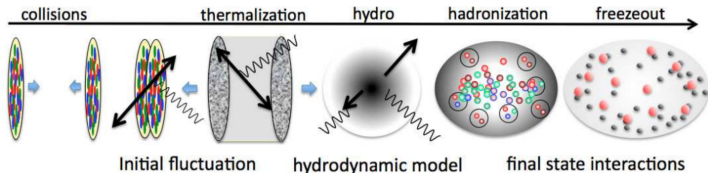
$$\Psi_{RP} \neq \Psi_{PP} \neq \Psi_{EP}$$

Also,

$$\frac{dN}{d\phi} = \frac{v_0}{2\pi} + \frac{1}{2\pi} \sum_{n=1} (2v_n \cos n(\varphi - \psi_{RP}))$$

is not hold when we consider non-flow effects and non-ideal case

Schematics of Heavy ion collision



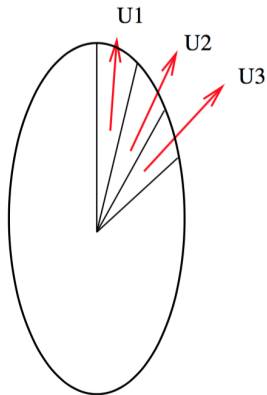
Schematic sketch of relativistic heavy ion collisions arXiv:1204.4795

A heavy ion collision can be divided into several stages,

- Pre-equilibrium : Immediately after collision
- Deconfined state : QGP is formed and starts expanding
- Hadron gas : Quarks and gluons are bound into hadrons when temperature is sufficiently low
- Free streaming : Hadrons stop interacting and fly to the detector

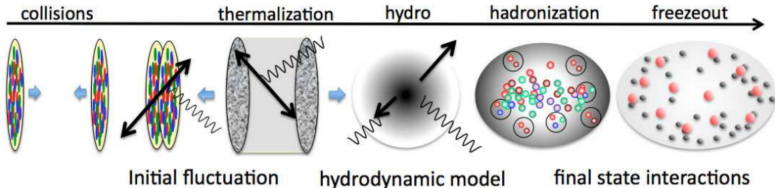
Flow in Heavy-ion collisions

Correlation between "Flow" and "System properties"

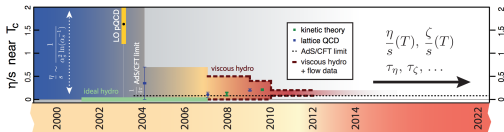
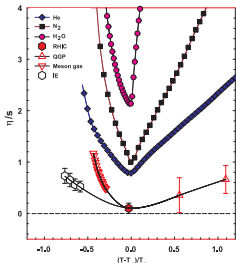


- Weakly coupled \rightarrow Long distance until next collision \rightarrow easy mixing
- Strongly coupled \rightarrow Short distance until next collision \rightarrow mixing take long time
- $\eta \propto l_{mfp} \propto 1/n_{\sigma}$
- The larger the cross-section, the smaller η , large v_2
- Stronger interaction \rightarrow Less viscous fluid
- Shear viscosity smears out flow differences (it's a diffusion)
- Shear viscosity **reduces** non-sphericity

Result : Large v_2 means, low η/s



Initial geometry and its fluctuations \rightarrow Transport properties ($\eta/s(T)$) \rightarrow final-state particles

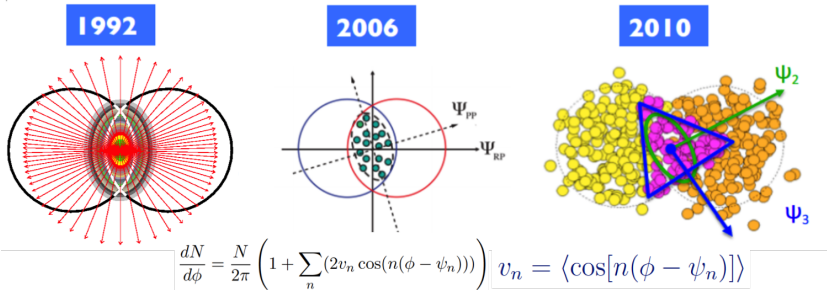


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R. A. Lacey et al., Phys. Rev. Lett. 98, 092301 (2007), "It is argued that such a low value is indicative of thermodynamic trajectories for the decaying matter which lie close to the QCD critical end point."

courtesy of Bjorn Schenke, "String theory (AdS/CFT correspondence) finds η/s is $1/4\pi$ a strongly coupled conformal theory \rightarrow hints at a lower bound of that order."

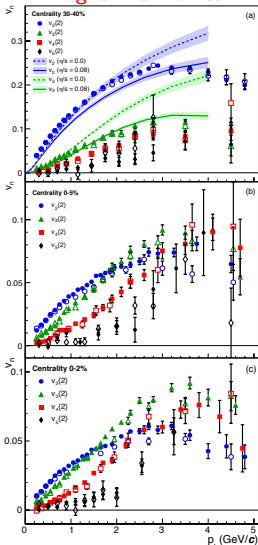
Flow measurements in Heavy-Ion Collisions



- The magnitudes of Flow-vector, anisotropic flow harmonics v_n , have been measured in great details (centrality, p_T , η , PID)
 - Large elliptic flow has indicated **fluid behavior of matter created at RHIC** in early 2000's (BNL announces perfect liquid in 2005 press release)
 - The importance of fluctuations was realized later and **analysis of odd flow harmonics** began in 2010 (since B. Alver, G. Roland, Phys.Rev. C81, 054905)
- The fluctuations of each individual flow harmonic have been investigated in great details in recent years

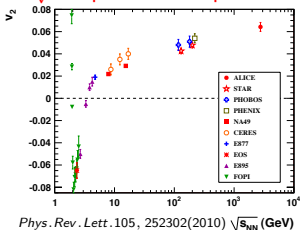
Selected flow measurements at LHC in one slide

Higher Harmonics

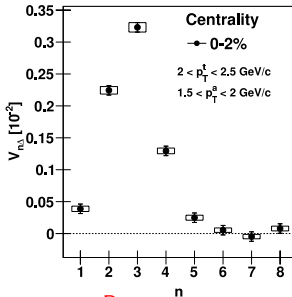


ALI-PUB-50346

ALICE, Phys. Rev. Lett. 107, 032301(2011)

 \sqrt{s} dependence elliptic flowPhys. Rev. Lett. 105, 252302(2010) $\sqrt{s_{NN}}$ (GeV)

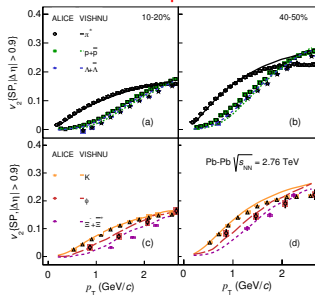
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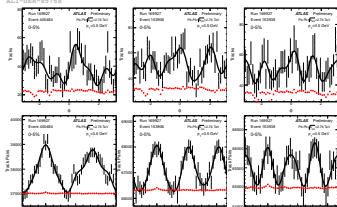
ALI-PUB-44119

Power spectra
ALICE, Phys. Lett. B708(2012)249 - 264

PID elliptic flow

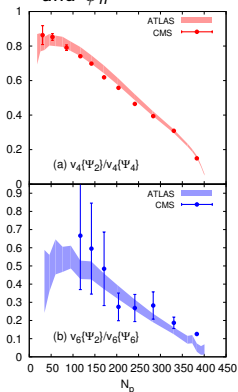


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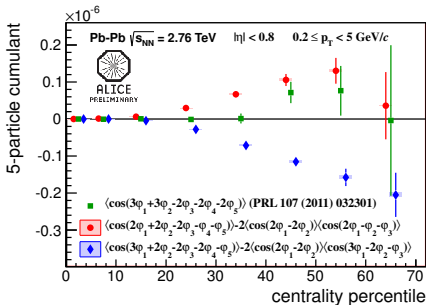
Event-by-event flow harmonics
ATLAS, JHEP1311(2013)183

Correlation between flow-vectors

- Flow direction correlations: ψ_n and ψ_m correlations
- Flow magnitude correlations: v_m and v_n correlations
 - Are v_n and v_m correlated? anti-correlated? or not correlated?
 - How can we investigate the relationship between v_n and v_m without contribution of ψ_m and ψ_n



$\langle \cos 4(\Phi_2 - \Phi_4) \rangle_w \equiv \frac{v_4\{\Psi_2\}}{v_4\{\Psi_4\}}$, which includes not only event plane angle correlations but also its magnitude (J.Y.Ollitrault et. al., Phys.Lett. B744 (2015) 82-87)



ALI-PREL-29328

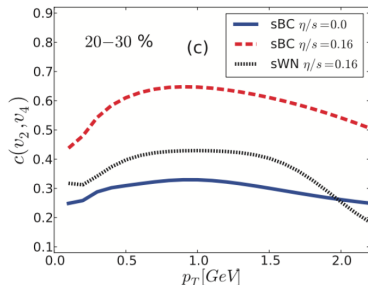
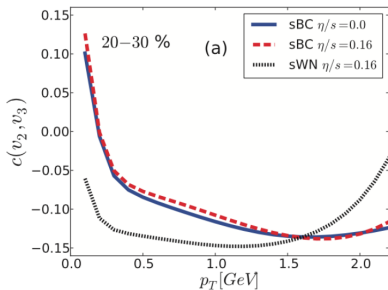
multi-particle cumulant from ALICE (PRL 107 (2011) 032301)

$\frac{v_4\{\Psi_2\}}{v_4\{\Psi_4\}}$, $\frac{v_6\{\Psi_2\}}{v_6\{\Psi_6\}}$ from ATLAS(arXiv:1403.0489), CMS (arXiv:1310.8651)

Correlations of v_m and v_n

A linear correlation coefficient $c(v_n, v_m)$ was proposed (H. Niemi et al., Phys. Rev. C 87, 054901 (2013)) to study the correlations between v_n and v_m

$$c(v_m, v_n) = \left\langle \frac{(v_m - \langle v_m \rangle_{ev})(v_n - \langle v_n \rangle_{ev})}{\sigma_{v_n} \sigma_{v_m}} \right\rangle_{ev}$$



- $c(v_2, v_3)$ is sensitive to initial conditions and insensitive to η/s , $c(v_2, v_4)$ is sensitive to both
- However, this observable is not easily accessible in flow measurements which are relying on two- and multi-particle correlations.

Symmetric 2-harmonic 4-particle Cumulants

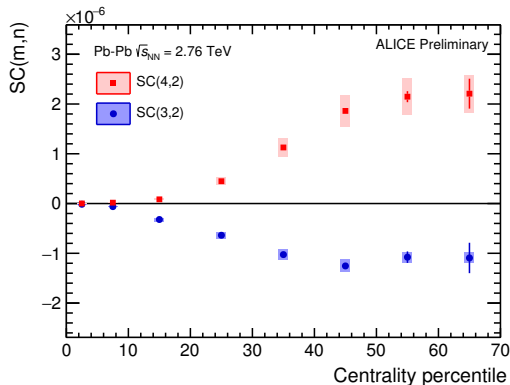
New Observable : Symmetric 2-harmonic 4-particle Cumulants (SC) ¹

$$\begin{aligned} \langle\langle \cos(m\varphi_1 + n\varphi_2 - m\varphi_3 - n\varphi_4) \rangle\rangle_c &= \langle\langle \cos(m\varphi_1 + n\varphi_2 - m\varphi_3 - n\varphi_4) \rangle\rangle \\ &\quad - \langle\langle \cos[m(\varphi_1 - \varphi_2)] \rangle\rangle \langle\langle \cos[n(\varphi_1 - \varphi_2)] \rangle\rangle \\ &= \langle v_m^2 v_n^2 \rangle - \langle v_m^2 \rangle \langle v_n^2 \rangle \end{aligned}$$

- By construction not sensitive to
 - non flow effects
 - inter-correlations of various symmetry planes
- It is non-zero if the event-by-event amplitude fluctuations of v_n and v_m are (anti-)correlated.

¹Ante Bilandzic et al., Phys. Rev. C 89, 064904 (2014)

SC(m, n) results

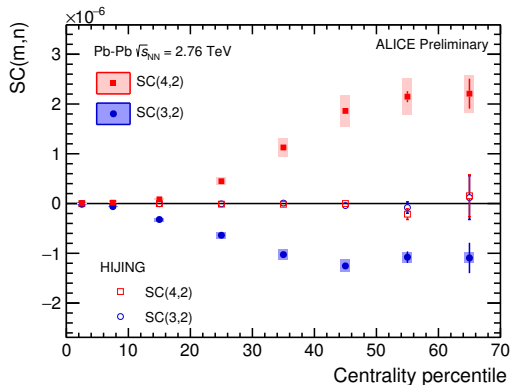


$$\begin{aligned} \langle \langle \cos(m\varphi_1 + n\varphi_2 - m\varphi_3 - n\varphi_4) \rangle \rangle_c \\ = \langle v_m^2 v_n^2 \rangle - \langle v_m^2 \rangle \langle v_n^2 \rangle \end{aligned}$$

ALI-PREL-96651

- The positive values of SC(4,2) and negative SC(3,2) are observed for all centralities.
 - suggests a correlation between v_2 and v_4 , and an anti-correlations between v_2 and v_3 .
 - indicates finding $v_2 > \langle v_2 \rangle$ in an event enhances the probability of finding $v_4 > \langle v_4 \rangle$ and finding $v_3 < \langle v_3 \rangle$ in that event.

SC(m, n) results with HIJING: is Non-flow contribution?



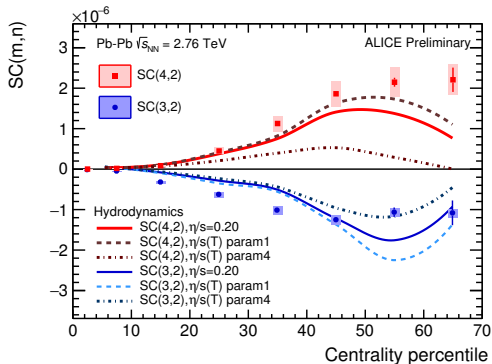
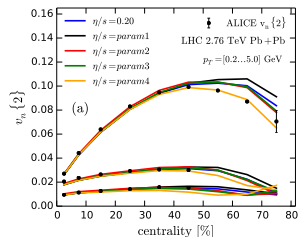
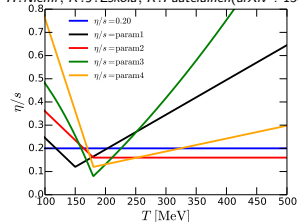
ALI-PREL-96655

- It is found that both $\langle v_m^2 v_n^2 \rangle$ and $\langle v_m^2 \rangle \langle v_n^2 \rangle$ are non-zero in HIJING, but calculation of SC(m,n) from HIJING are compatible with zero
 - suggests SC measurements are nearly insensitive to non-flow correlations
- non-zero values of SC measurements cannot be explained by non-flow effects, thus confirms the existence of (anti-)correlations between v_n and v_m harmonics.

$$\begin{aligned} & \langle \langle \cos(m\varphi_1 + n\varphi_2 - m\varphi_3 - n\varphi_4) \rangle \rangle_c \\ & = \langle v_m^2 v_n^2 \rangle - \langle v_m^2 \rangle \langle v_n^2 \rangle \end{aligned}$$

SC(m, n) results : Comparisons to hydrodynamics

H. Niemi, K.J. Eskola, R. Paatelainen (arXiv : 1505.02677)



ALI-PREL-96671

$$\langle\langle \cos(m\varphi_1 + n\varphi_2 - m\varphi_3 - n\varphi_4) \rangle\rangle_c = \langle v_m^2 v_n^2 \rangle - \langle v_m^2 \rangle \langle v_n^2 \rangle$$

- Although hydro describes the v_n fairly well, there is not a single centrality bin for which a given η/s parameterization describes simultaneously SC(4,2) and SC(3,2)
- SC measurements provide stronger constrains on the η/s in hydro in combination with standard v_n measurements

Working on progress : Symmetric 2-harmonic 4-particle Cumulants

New Observable : Symmetric 2-harmonic 4-particle Cumulants (SC) ²

$$\begin{aligned} \langle\langle \cos(m\varphi_1 + n\varphi_2 - m\varphi_3 - n\varphi_4) \rangle\rangle_c &= \langle\langle \cos(m\varphi_1 + n\varphi_2 - m\varphi_3 - n\varphi_4) \rangle\rangle \\ &\quad - \langle\langle \cos[m(\varphi_1 - \varphi_2)] \rangle\rangle \langle\langle \cos[n(\varphi_1 - \varphi_2)] \rangle\rangle \\ &= \langle v_m^2 v_n^2 \rangle - \langle v_m^2 \rangle \langle v_n^2 \rangle \end{aligned}$$

- It is non-zero if the event-by-event amplitude fluctuations of v_n and v_m are (anti-)correlated.

Also SC(m,n) can be normalizable with $\langle v_m^2 \rangle \langle v_n^2 \rangle$

$$SC(m, n)_{norm} = SC(m, n) / \langle v_m^2 \rangle \langle v_n^2 \rangle$$

- Normalized SC(m,n) reflects the degree of the correlation.
- While SC(m,n) contains both the degree of the correlation and individual v_n .

²Ante Bilandzic et al., Phys. Rev. C 89, 064904 (2014)

Measuring correlation with moments

This SC(m,n) can be calculated with multi-particle cumulants(QC) but also can be calculated with Scalar Product method(SP) by using Moments³

$$\mathcal{M} \equiv \left\langle \prod_n (V_n)^{k_n} (V_n^*)^{l_n} \right\rangle = \left\langle \prod_n (Q_{nA})^{k_n} (Q_{nB}^*)^{l_n} \right\rangle \quad (4)$$

Then SC(m, n) can be expressed as

- $\langle (Q_{An} Q_{Bn}^* Q_{Am} Q_{Bm}^*) \rangle - \langle (Q_{An} Q_{Bn}^*) \rangle \langle (Q_{Am} Q_{Bm}^*) \rangle$

where Q_n is normalized flow Q-vector ($\frac{1}{M} \sum_{i=1}^M e^{in\phi_i}$), and A, B denotes sub event groups which are divided with η gap

Auto(self) correlation term in red part with 4p correlation between $Q_{An} - Q_{Am}$ and $Q_{Bn} - Q_{Bm}$, these could be corrected by correction term
(credit : Ante and Sergei)

$$\frac{1}{M_B} \text{Re}(Q_{Bm+n}^* Q_{Am} Q_{An}) - \frac{1}{M_A} \text{Re}(Q_{Am+n} Q_{Bn}^* Q_{Bm}^*) + \frac{1}{M_A M_B} \text{Re}(Q_{Am+n} Q_{Bm+n}^*)$$

³Rajeev S. Bhalerao et al, <http://doi.org/10.1016/j.physletb.2015.01.019>

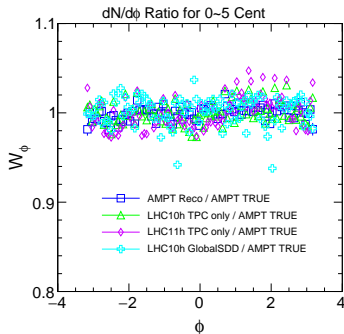
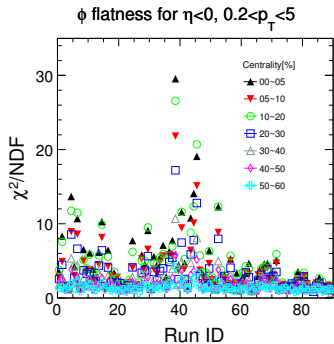
Summary

- Moments of the distribution of V_n provide a complete set of multiparticle correlation, which can be used to probe the physics of flow fluctuations.
- Flow fluctuations have been measured as SC and Normalized SC
 - SC(m,n) results with Q-Cumulants and Scalar Product are consistent within errors up to 40% centrality bins
 - SC results($v_n^2 - v_m^2$ correlation) and normalized SC(scaled with $\langle v_n^2 \rangle \langle v_m^2 \rangle$) results shows similar trends with Hydrodynamics and AMPT simulation
 - Higher order SC correlations(SC(5,2), SC(5,3), SC(4,3)) are smaller than lower order SC correlation(SC(3,2), SC(4,2))
 - But in normalized results, correlation between higher order flow harmonics are stronger than lower order flow correlations
- p_T dependence of SC(m,n)
 - p_T dependence of SC(3,2) and SC(4,2) are checked both in Data and AMPT simulation but, no p_T dependence for normalized SC(m,n) results up to 1.0GeV/c
 - we go to more higher p_T cuts $\gtrsim 1\text{GeV}/c$, we start to see a clear p_T dependence of normalized SC, which might indicate the p_T dependent flow angle fluctuations.

Backup Slides

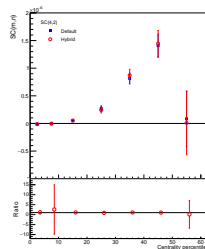
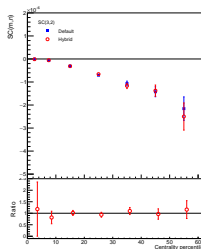
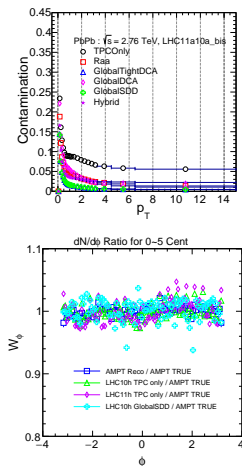
Backup

How to estimate the systematics from the non-uniform ϕ efficiency ?



- 1 Check the deviations of the observables with 3 different group of runs based on χ^2/NDF cuts.
- 2 Check the deviations between track selection cuts (TPCOnly:FilterBit128, GlobalSDD:96..).
- 3 MC method using the large statistics AMPT sets (LHC13f3c,b,a)
 - Physical Primary particle only + imposing non-uniform ϕ distribution
 - ϕ distribution taken from the data

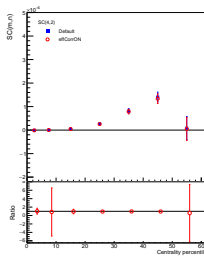
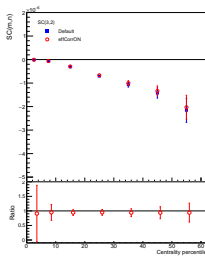
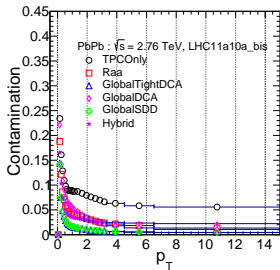
SC(m,n) results with different TrackFilter bit



cut	filter bit	comments
TPCOnly	128 (7)	GetStandardTPCOnlyTrackCuts() + SetMinNClustersTPC(70)
GlobalSDD	96 (5 6)	GetStandardTSTPCTrackCuts2010() with requiring the first SDD cluster instead of an SPD cluster

- more fake and secondary tracks for TPCOnly track cut
- two track cuts give relatively good uniform ϕ distribution

Systematics of SC(m,n) with Efficiency correction



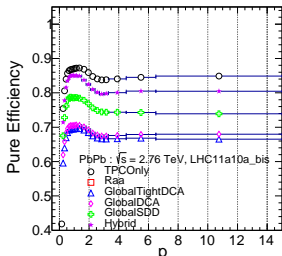
correction to p_T dependent efficiency

In following equations G_{trigvtx} stands for the number of true charged physical primaries emitted to $|\eta| < 0.8$ in triggered events where an event vertex was reconstructed. $C(p_T)$,

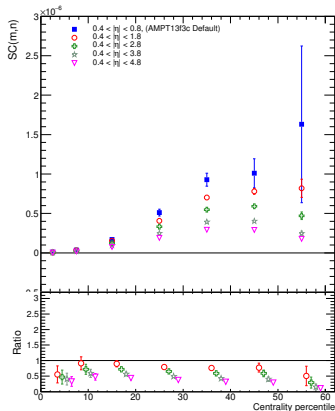
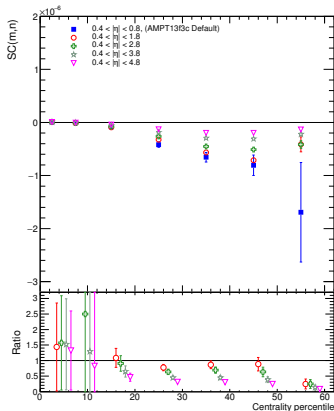
$$C^{-1}(p_T) = \frac{M_{\text{trigvtx}}(p_T) + B(p_T)}{G_{\text{trigvtx}}(p_T)}, \quad (5)$$

$$\text{true efficiency} = M_{\text{trigvtx}}(p_T) / G_{\text{trigvtx}}(p_T), \quad (6)$$

$$\text{contamination} = B(p_T) / [M_{\text{trigvtx}}(p_T) + B(p_T)] \quad (7)$$



SC(m,n) AMPT results with large η

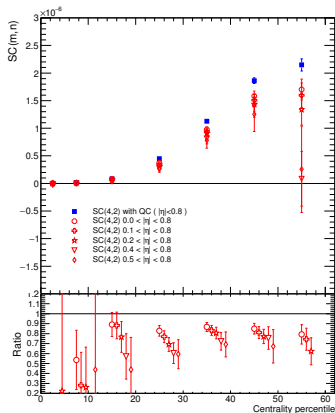
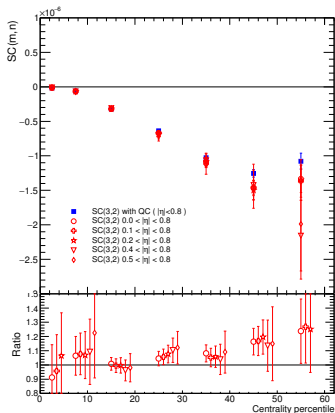


- When η region extend to large(forward) region, $SC(m,n)$ values getting smaller
- $SC(m,n)$ with $0.4 < |\eta| < 0.8$ have 5 times larger then $SC(m,n)$ with $0.4 < |\eta| < 4.8$

Deviation of two different method comes from Non-flow effects?

Applying different $\Delta\eta$ for SP method. Generally we can easily expect that

- Small η gap between subevent groups \rightarrow big non-flow effect
- Large η gap between subevent groups \rightarrow small non-flow effect



But, SP method with smaller $\Delta\eta$ results are more closes to QC method results.

How about Normalized SC(m,n)?

