Nuclear Symmetry Energy in QCD degree of freedom

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Some preliminary results

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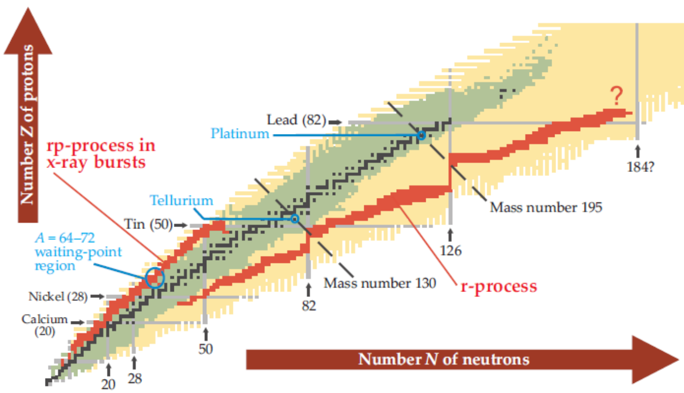
Outline – 2 phases

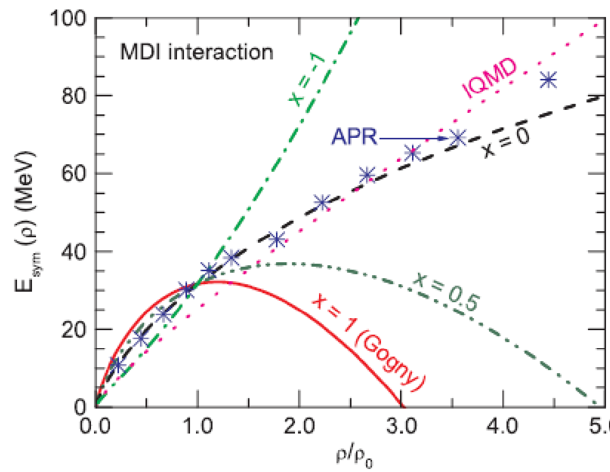
- Nuclear symmetry energy in hadronic phase

I. Using in-medium QCD Sum Rules

(PRL 102, 062502 Z. Xiao et al.)



(Physics Today November 2008)



II. Iso-spin effect on hyperon

- Symmetry energy in cold dense matter

I. Using thermal QCD and resummation

II. Considering color BCS pairing with High density effective field theory

Asymmetric nuclear matter

- From equation of state

Bethe-Weisaker formula

$$m_{tot} = Nm_n + Zm_p - E_B/c^2$$
$$E_B = a_V A - a_S A^{\frac{2}{3}} - a_C(Z(Z-1))A^{-\frac{1}{3}} - a_A I^2 A + \delta(A,Z)$$
$$I = (N-Z)/A$$

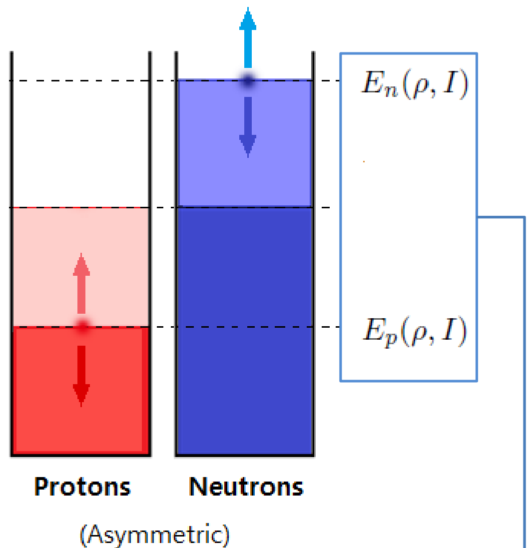
In continuous matter

$$\bar{E}(\rho_N, I) = \bar{E}(\rho_N) + \bar{E}_{sym}(\rho_N) I^2 + \cdots$$
$$I = (\rho_n - \rho_p)/\rho$$

$$\overline{E} = \frac{1}{\int d^3k_n d^3k_p} \int d^3k_n d^3k_p E(\rho_n, \rho_p)$$

$$\Rightarrow E_{sym} = \frac{1}{2I} \cdot (\bar{E}_n - \bar{E}_p)$$ (Up to linear density order)

- Quasi-nucleon on the asymmetric Fermi sea



- RMFT propagator

$$G(q) = -i \int d^4x e^{iqx} \langle \Psi_0 | \mathrm{T}[\psi(x)\bar{\psi}(0)] | \Psi_0 \rangle = \frac{1}{\not{q} - M_n - \Sigma(q)} \to \lambda^2 \frac{\not{q} + M^* - \not{u}\Sigma_v}{(q_0 - E_q)(q_0 - \bar{E}_0)}$$

QCD Sum Rule

- Correlation function

$$\Pi(q) \equiv i \int d^4x\, e^{iqx} \langle \Psi_0 | \mathrm{T}[\eta(x)\bar{\eta}(0)] | \Psi_0 \rangle$$
$$= \Pi_s(q^2, q\cdot u) + \Pi_q(q^2, q\cdot u)\not{q} + \Pi_u(q^2, q\cdot u)\not{u}$$

$$\eta(x) = \epsilon_{abc}[u_a^T(x) C \gamma_\mu u_b(x)] \gamma_5 \gamma^\mu d_c(x)$$

Ioffe's interpolating field for proton

- Energy dispersion relation and OPE

$$\Pi_i(q_0, |\vec{q}|) = \frac{1}{2\pi i} \int_{-\infty}^{\infty} d\omega \frac{2\mathrm{Im}\Pi_i(\omega, |\vec{q}|)}{\omega - q_0} + \text{polynomials}$$

Contains **all possible hadronic resonance states** in **QCD degree of freedom**

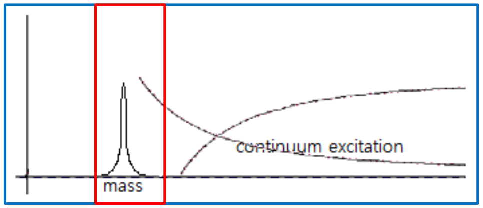
- Phenomenological ansatz in **hadronic degree of freedom**

$$\Pi(q_0, |\vec{q}|) \sim \frac{1}{(q^\mu - \tilde{\Sigma}_v^\mu)\gamma_\mu - M_N^*}$$

Equating both sides, hadronic quantum number can be expressed in QCD degree of freedom

- Weighting - Borel transformation

$$\mathcal{B}[\Pi_i(q_0, |\vec{q}|)] \equiv \lim_{\substack{-q_0^2, n \to \infty \\ -q_0^2/n = M^2}} \frac{(-q_0^2)^{n+1}}{n!} \left(\frac{\partial}{\partial q_0^2} \right)^n \Pi_i(q_0, |\vec{q}|)$$



OPE and Condensates

- OPE and self energies

$$\Pi = \Pi_s + \Pi_q \not{q} + \Pi_u \not{u}$$
$$E_q = \Sigma_v + \sqrt{\vec{q}^2 + M^{*2}}$$

where $M^* = \dfrac{\mathcal{B}(\Pi_s)}{\mathcal{B}(\Pi_q)}$ $\quad \Sigma_v = \dfrac{\mathcal{B}(\Pi_u)}{\mathcal{B}(\Pi_q)}$

- For Nucleons (proton OPE)

$$\mathcal{B}(\Pi_q) \sim \frac{1}{32\pi^4}(M^2)^3$$
$$\mathcal{B}(\Pi_s) \sim \frac{1}{4\pi^2}(M^2)^2\langle \bar{d}d\rangle_{\rho,I} - \frac{4}{3}\bar{E}_q\langle\bar{q}q\rangle_{vac}\langle u^\dagger u\rangle_{\rho,I}$$
$$\mathcal{B}(\Pi_u) \sim \frac{1}{12\pi^2}(M^2)^2(7\langle u^\dagger u\rangle_{\rho,I} + \langle d^\dagger d\rangle_{\rho,I})$$
$$-\frac{\bar{E}_q}{9\pi^2}M^2\left(\boxed{4m_u\langle\bar{u}u\rangle_{\rho,I}} - 16\langle u^\dagger iD_0 u\rangle_{\rho,I} + \boxed{m_d\langle\bar{d}d\rangle_\rho} - 4\langle d^\dagger iD_0 d\rangle_{\rho,I}\right)$$

1. Leading contribution comes from chiral condensate and density operator
2. Non-negligible correction comes from spin-1 4quark operator and $\langle\bar{q}\gamma_\mu iD_\nu q\rangle_\rho$

Trace part of $\langle\bar{q}\gamma_\mu iD_\nu q\rangle_\rho$ vanishes in m_q->0 limit

Only twist-2 part contributes

OPE and Condensates

- For Sigma hyperon (Σ^+ OPE)

$$\mathcal{B}(\Pi_q) \sim \frac{1}{32\pi^4}(M^2)^3$$

$$\mathcal{B}(\Pi_s) \sim \frac{m_s}{16\pi^4}(M^2)^3 - \frac{1}{4\pi^2}(M^2)^2\langle\bar{s}s\rangle_{\rho,I} - \frac{4}{3}E_q\langle\bar{s}s\rangle_{vac}\langle u^\dagger u\rangle_{\rho,I}$$

$$\mathcal{B}(\Pi_u) \sim \frac{1}{12\pi^2}(M^2)^2(7\langle u^\dagger u\rangle_{\rho,I} + \langle s^\dagger s\rangle_{\rho,I})$$

$$- \frac{E_q}{9\pi^2}M^2(4m_u\langle\bar{u}u\rangle_{\rho,I} - 16\langle u^\dagger iD_0 u\rangle_{\rho,I} + m_s\langle\bar{s}s\rangle_{\rho,I} - 4\langle s^\dagger iD_0 s\rangle_{\rho,I})$$

Trace part of $\langle\bar{q}\gamma_\mu iD_\nu q\rangle_\rho$
vanishes in md->0 limit
$\langle u^\dagger iD_0 u\rangle_{\rho,I} = (1-(0.35)I)(258\text{MeV})\rho_N$
determines large correction

Trace part of $\langle\bar{s}\gamma_\mu iD_\nu s\rangle_\rho$
Do not vanish -> minimal contribution
$\langle s^\dagger iD_0 s\rangle_{\rho,I} = \frac{1}{4}m_s\langle\bar{s}s\rangle_{\rho,I} + (18\text{MeV})\rho_N$

- For Sigma hyperon, vector potential determines iso-spin dependence

Iso-spin asymmetric condensates

- Linear density approximation

$$\langle \hat{O} \rangle_{\rho,I} = \langle \hat{O} \rangle_{\text{vac}} + \langle n|\hat{O}|n\rangle \rho_n + \langle p|\hat{O}|p\rangle \rho_p$$
$$= \langle \hat{O} \rangle_{\text{vac}} + \frac{1}{2}(\langle n|\hat{O}|n\rangle + \langle p|\hat{O}|p\rangle)\rho$$
$$+ \frac{1}{2}(\langle n|\hat{O}|n\rangle - \langle p|\hat{O}|p\rangle) I \rho .$$

Vacuum condensate +
nucleon expectation value x density

Iso-spin symmetric and asymmetric part

- Iso-spin symmetric part

$$\langle \bar{q}q \rangle_\rho = \langle \bar{q}q \rangle_{vac} + \frac{\sigma_n}{2m_q}\rho_n$$
$$\langle \bar{s}s \rangle_\rho = (0.8)\langle \bar{q}q \rangle_{vac} + y\frac{\sigma_n}{2m_q}\rho_n$$

Nucleon expectation value can be determined from nucleon sigma term

y parameter determines hyperon sigma term

- Iso-spin asymmetric part

$$\langle [\bar{q}q]_1 \rangle_p = \frac{1}{2}(\langle p|\bar{u}u|p\rangle - \langle p|\bar{d}d|p\rangle)$$
$$= \frac{1}{2}\left[\frac{(m_{\Xi^0} + m_{\Xi^-}) - (m_{\Sigma^+} + m_{\Sigma^-})}{2m_s - 2m_q}\right]$$

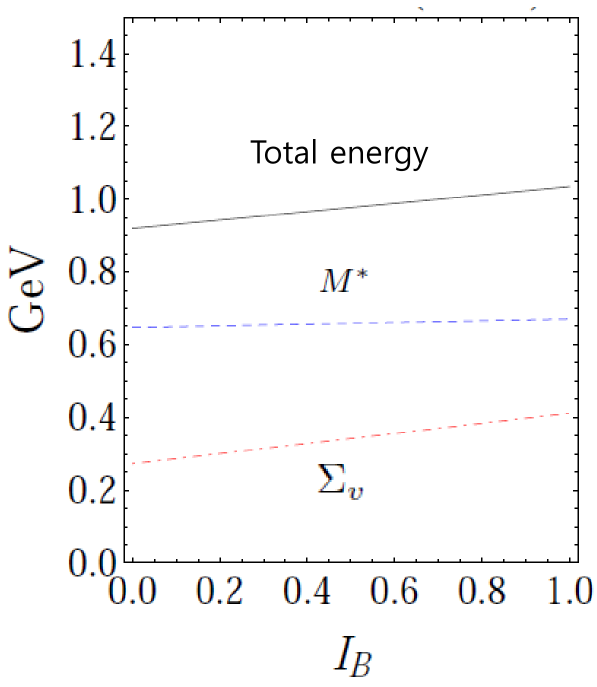
Can be determined from baryon octet relation

$$m_p = A + m_u B_u + m_d B_d + m_s B_s$$
$$m_n = A + m_u B_d + m_d B_u + m_s B_s$$
$$m_{\Sigma^+} = A + m_u B_u + m_d B_s + m_s B_d$$
$$m_{\Sigma^-} = A + m_u B_s + m_d B_u + m_s B_d$$
$$m_{\Xi^0} = A + m_u B_d + m_d B_s + m_s B_u$$
$$m_{\Xi^-} = A + m_u B_s + m_d B_d + m_s B_u$$
$$A \equiv \langle (\bar{\beta}/4\alpha_s) G^2 \rangle_p, \quad B_u \equiv \langle \bar{u}u \rangle_p, \quad B_d \equiv \langle \bar{d}d \rangle_p$$

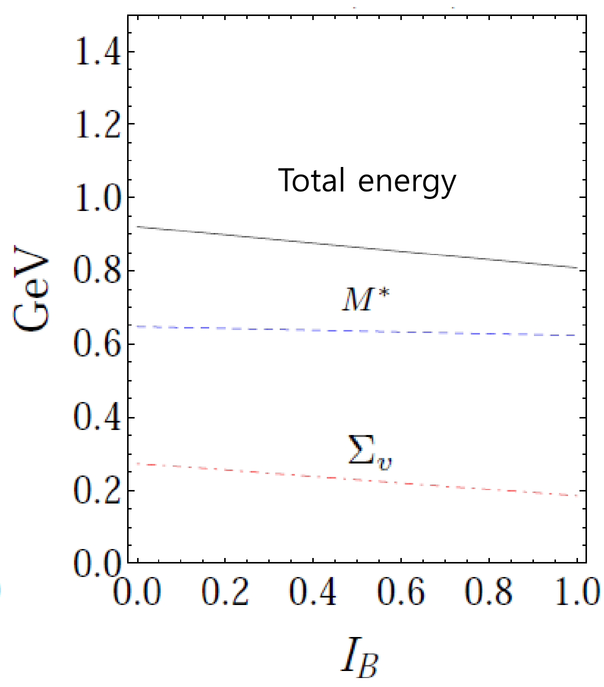
Sum rule result I

- Nucleon self energies (at normal density)

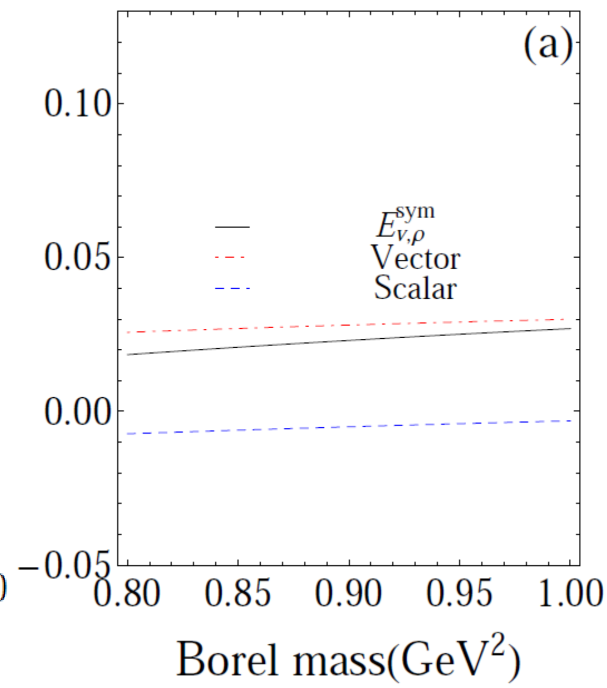
Neutron



Proton



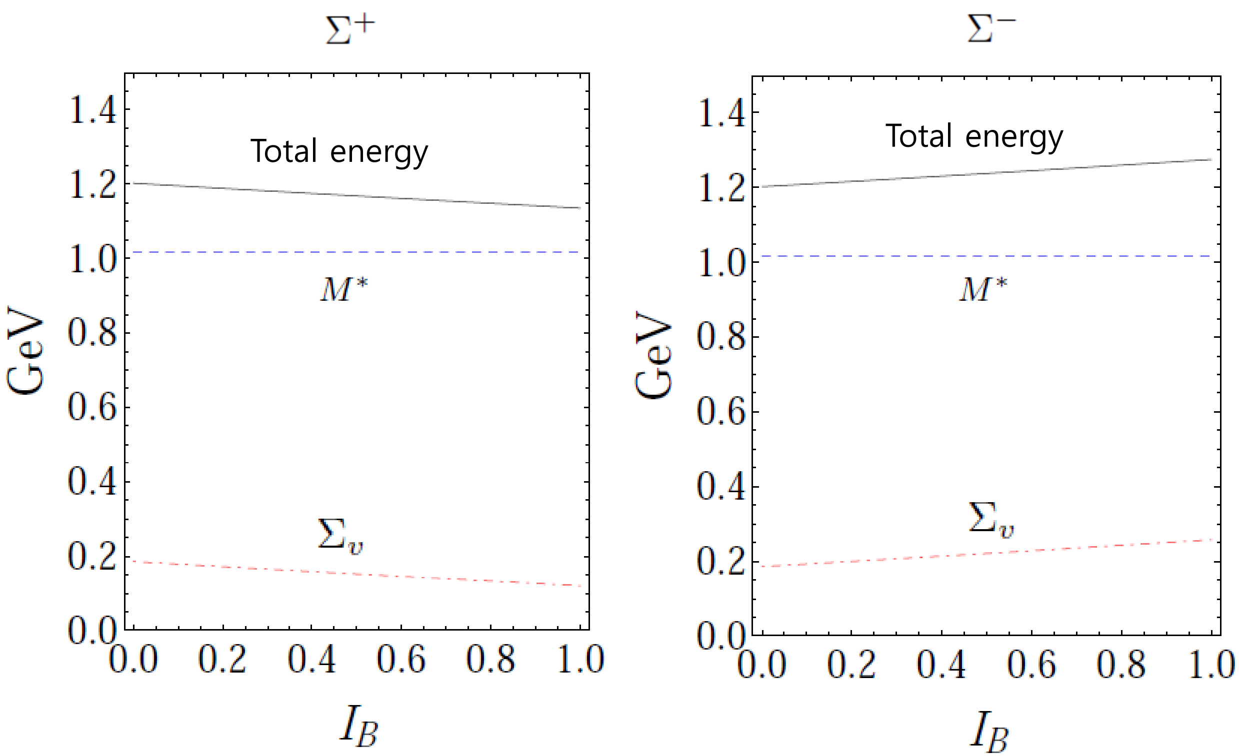
Symmetry energy
(up to linear density order)



1) Total energy (black line), Effective mass (blue line), Vector self energy (red line)
2) Quite strong iso-spin dependence (Vector self energy)

Sum rule result II

- Sigma self energies (at normal density)



1) Effective mass do not strongly depend on I_B

2) Total energy is quite repulsive

1) Total energy (black line), Effective mass (blue line), Vector self energy (red line)
2) Weak iso-spin dependence

Sum rule result III

- Potential

	Neutron	Σ^{-}
(I=0)	S = -300 MeV	S = -107 MeV
	V= +263 MeV	V= +178 MeV
(I=1)	S = -270 MeV	S = -107 MeV
	V= +357 MeV	V= +245 MeV
(I=1) - (I=0)	ΔS = +30 MeV	ΔS = 0 MeV
	ΔV = +94 MeV	ΔV = +67 MeV

For neutron, S=-300+30 I_B (MeV), V=263+94 I_B (MeV)
For sigma- , S=-107+0 I_B (MeV), V=178+67 I_B (MeV)

- The ratios

If one consider phenomenological meson exchange channel, as a optical potential,

$$M^* = M - g_\sigma \sigma - g_\delta \vec{\tau} \cdot \vec{\delta}$$ → Very weak!

$$V^\mu = g_\omega \omega^\mu + g_\rho \vec{\tau} \cdot \vec{\rho}$$

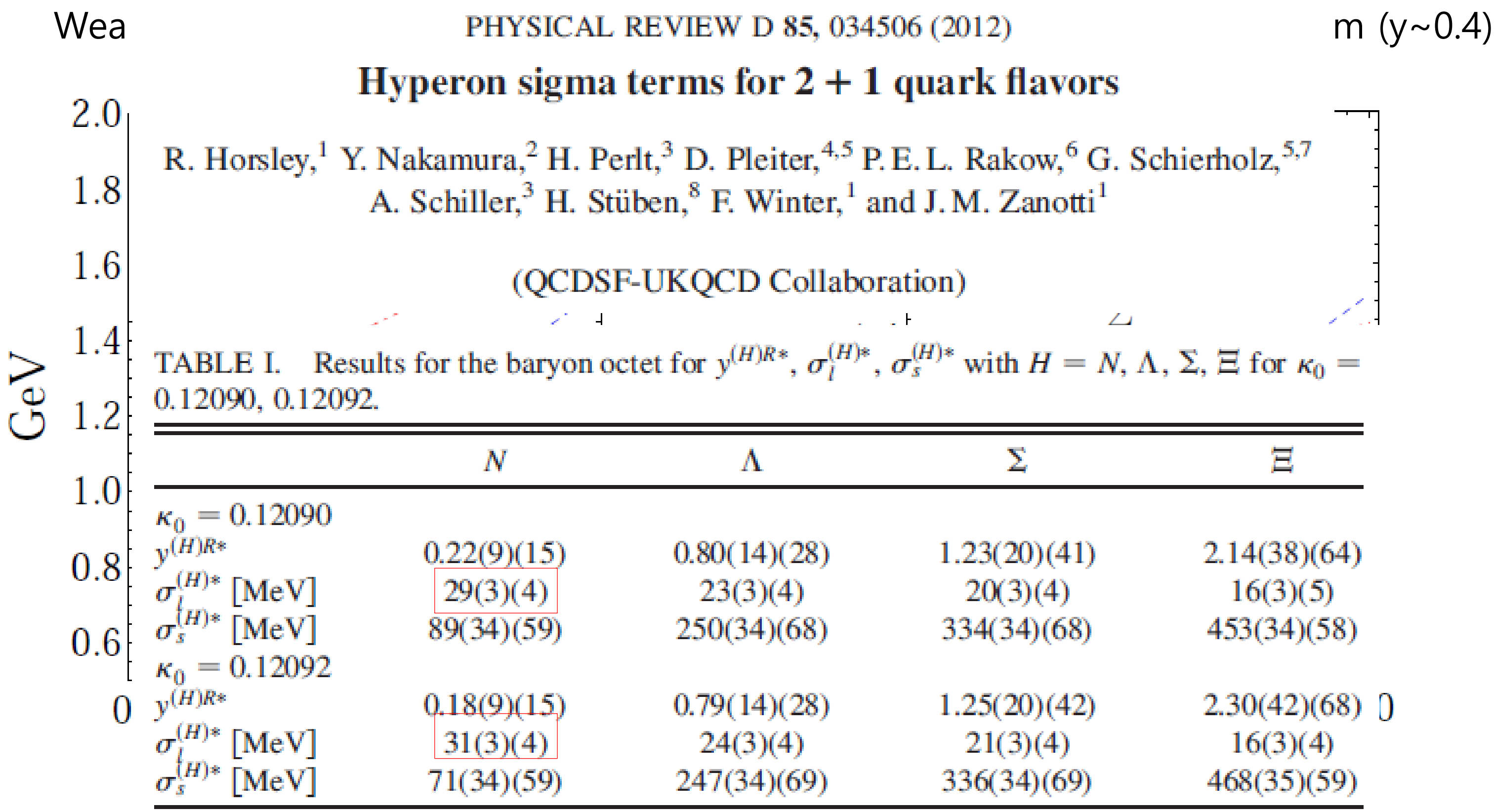
Ratio for coupling can be suggested as

$$g_{\rho\Sigma}/g_{\rho N} \sim 0.7$$
$$g_{\omega\Sigma}/g_{\omega N} \sim 0.3$$
$$g_{\sigma\Sigma}/g_{\sigma N} \sim 0.7$$

Sum rule result IV

- Density behavior in pure neutron matter $\langle \bar{s}s \rangle_\rho = (0.8)\langle \bar{q}q \rangle_{vac} + y\frac{\sigma_n}{2m_q}\rho_n$

Wea … m (y~0.4)



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Hyperon sigma terms for 2 + 1 quark flavors

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(QCDSF-UKQCD Collaboration)

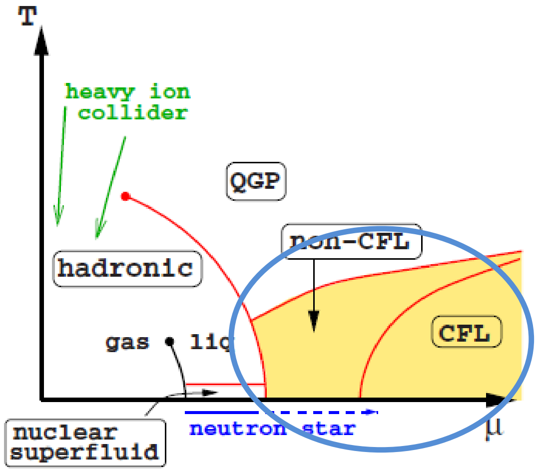
TABLE I. Results for the baryon octet for $y^{(H)R*}$, $\sigma_l^{(H)*}$, $\sigma_s^{(H)*}$ with $H = N, \Lambda, \Sigma, \Xi$ for $\kappa_0 =$ 0.12090, 0.12092.

	N	Λ	Σ	Ξ
$\kappa_0 = 0.12090$				
$y^{(H)R*}$	0.22(9)(15)	0.80(14)(28)	1.23(20)(41)	2.14(38)(64)
$\sigma_l^{(H)*}$ [MeV]	29(3)(4)	23(3)(4)	20(3)(4)	16(3)(5)
$\sigma_s^{(H)*}$ [MeV]	89(34)(59)	250(34)(68)	334(34)(68)	453(34)(58)
$\kappa_0 = 0.12092$				
$y^{(H)R*}$	0.18(9)(15)	0.79(14)(28)	1.25(20)(42)	2.30(42)(68)
$\sigma_l^{(H)*}$ [MeV]	31(3)(4)	24(3)(4)	21(3)(4)	16(3)(4)
$\sigma_s^{(H)*}$ [MeV]	71(34)(59)	247(34)(69)	336(34)(69)	468(35)(59)

With small medium dependence of $\langle \bar{s}s \rangle_\rho$, Σ^-always **heavier than neutron**

At extremely high density?

- QCD phase transition



- In $1/\mu \ll 1/\Lambda_{\rm QCD}$ region, **QCD** can be immediately applicable
- Statistical partition function for dense QCD

$$\mathcal{Z}_\Omega = \mathrm{Tr}\exp\left[-\beta(\hat{H} - \vec{\mu}\cdot\hat{\vec{N}})\right]$$
$$= \int [D(\text{fields})] \exp\left[-\int_0^\beta d\tau \int x^3 \mathcal{L}_E(\text{fields})\right]$$

- Normal QM phase (**HDL**) - BCS paired phase (**HDET**)

- Euclidean Lagrangian for dense QCD **at normal phase**

$$\mathcal{L}_E = \frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + \frac{1}{2\xi}(\partial_\mu A^a_\mu)^2 + \bar{\eta}^a(\partial^2\delta_{ab} + gf_{abc}\partial_\mu A^c_\mu)\eta^b$$
$$+ \sum_f^{n_f}\left[\psi_f^\dagger \partial_\tau \psi_f + \bar{\psi}_f(-i\gamma^i\partial_i + m_f)\psi_f - \mu_f\psi_f^\dagger\psi_f - g\bar{\psi}_f \not{A}\psi_f\right]$$

$$\int \frac{d^4Q}{(2\pi)^4} \equiv T\sum_n \int \frac{d^3q}{(2\pi)^3}, \quad Q_\mu = (-\omega, \vec{q})$$

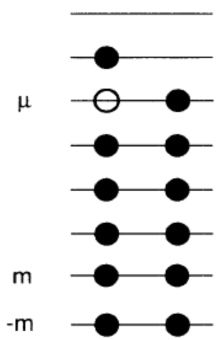
$\omega_n = (2n+1)\pi/\beta$ (For fermion)

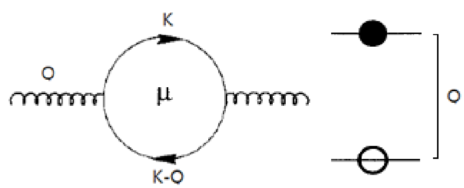
$\omega_n = 2n\pi/\beta$ (For boson)

Continuous energy integration -> Discrete sum over Matsubara frequency

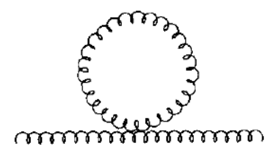
Hard Dense Loop

- Quark-hole excitation is dominant ($Q \sim T \leq g\mu$)





Quark-antiquark loop contributes to transverse mode



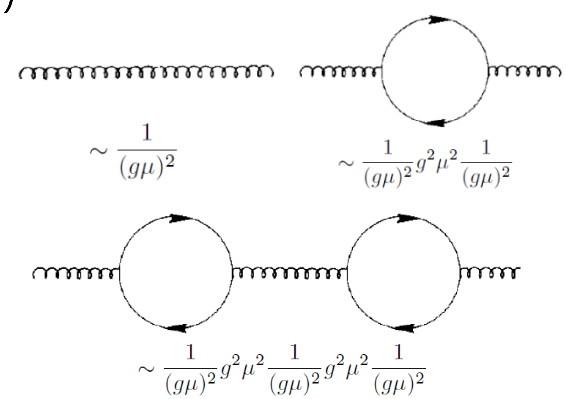
Gluon/Ghost contribution is negligible at $Q \sim T \leq g\mu$

- Gluon self energy in cold matter ($Q \sim T \leq g\mu$)

$$\Pi^{ab}_{\mu\nu}(Q) = g^2\delta^{ab}\int \frac{d^4K}{(2\pi)^4}\mathrm{Tr}\left[\gamma_\mu S_F(K)\gamma_\nu S_F(K-Q)\right]$$

$$= m^2\delta^{ab}\int \frac{d\Omega}{4\pi}\left(\delta_{\mu4}\delta_{\nu4} + \hat{K}_\mu\hat{K}_\nu\frac{i\omega}{Q\cdot\hat{K}}\right),$$

$$m^2 = \frac{1}{3}g^2T^2\left(C_A + \frac{1}{2}n_f\right) + \boxed{\frac{1}{2}g^2\sum_f\frac{\mu_f^2}{\pi^2}}$$



Phys. Rev. D.53.5866 (1996) C. Manuel
Phys. Rev. D.48.1390 (1993) J. P. Blaizot and J. Y. Ollitrault

All equivalent 1PI diagrams should be resumed!

Hard Dense Loop resumation

- Projection along polarization

Euclidean propagator

$$*D_{\mu\nu} = \frac{1}{Q^2+\delta\Pi^L} P^L_{\mu\nu} + \frac{1}{Q^2+\delta\Pi^T} P^T_{\mu\nu} + \frac{1}{f_e}\frac{Q_\mu Q_\nu}{Q^2}$$

$$P^T_{ij} = \delta_{ij} - \hat{q}_i \hat{q}_j, P^T_{44} = P^T_{4i} = 0$$

$$P^L_{\mu\nu} = \delta_{\mu\nu} - \frac{Q_\mu Q_\nu}{Q^2} - P^T_{\mu\nu}$$

Longitudinal and transverse part

$$\delta\Pi^L(Q) = 2\sum_f \left(\frac{1}{2} g^2 \frac{\mu_f^2}{\pi^2}\right) \frac{Q^2}{q^2} \left(1 - \left(\frac{i\omega}{q}\right) Q_0 \left(\frac{i\omega}{q}\right)\right)$$

In w->0 limit $\Rightarrow 2m_g^2 = g^2 \frac{\mu_f^2}{\pi^2}$

$$\delta\Pi^T(Q) = \sum_f \left(\frac{1}{2} g^2 \frac{\mu_f^2}{\pi^2}\right) \left(\frac{i\omega}{q}\right) \left[\left(1 - \left(\frac{i\omega}{q}\right)^2\right) Q_0 \left(\frac{i\omega}{q}\right) + \left(\frac{i\omega}{q}\right)\right]$$

$\Rightarrow 0$

$$Q_0(x) = \tfrac{1}{2} \ln\left[(x+1)/(x-1)\right]$$

- Debye mass and effective Lagrangian

Effective Lagrangian for soft gluon in cold dense matter

$$\mathcal{L} = -\frac{1}{4} F^2 \rightarrow \frac{1}{2} A_\mu (-Q^2 g^{\mu\nu} + \boxed{2m_g^2 P_L^{\mu\nu}} + O(\omega/q) P_T^{\mu\nu} + \cdots) A_\nu$$

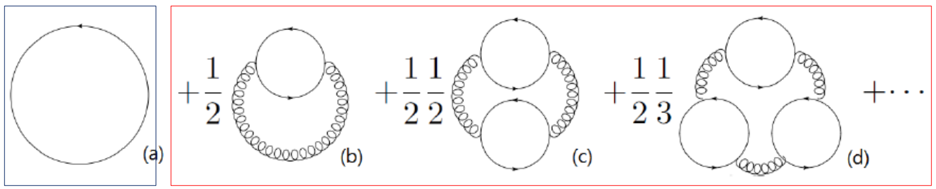
Debye mass from hard(dense) quark loop -> Iso-spin dependence

HDL resumed thermodynamic potential

- Free energy from partition function

$$\Omega(\mu) = \langle \hat{H} \rangle - \vec{\mu} \cdot \langle \hat{\vec{N}} \rangle = -\frac{1}{\beta} \ln \mathcal{Z}_\Omega \qquad \mathcal{Z}_\Omega \sim \mathrm{Exp}(\text{Connected diagrams})$$

- Relevant ring diagrams in HDL resummation



(T~Q~$g\mu$, only gluon line is soft)

$$\ln \mathcal{Z}_{\Omega_{q,0}} \simeq \beta V \left(\frac{N_c}{12} \sum_{f=u,d} \frac{\mu_f^4}{\pi^2} \right)$$

Ideal quark gas

$$\begin{aligned} \ln \mathcal{Z}_{\Omega_{g,\mathrm{HDL}}} &= -\frac{(N_c^2-1)}{2} \beta V \int \frac{d^4Q}{(2\pi)^4} \ln\left[1 + \Pi_{\mu\nu}(Q) D_F^{\nu\mu}(Q)\right] \\ &= -\frac{(N_c^2-1)}{2} \beta V \int \frac{d^4Q}{(2\pi)^4} \left(\ln\left[1 + \delta\Pi^L(Q)\frac{1}{Q^2}\right] + 2\ln\left[1 + \delta\Pi^T(Q)\frac{1}{Q^2}\right] \right) \end{aligned}$$

$$\mathcal{L} \equiv -\frac{(N_c^2-1)}{2} \beta V \int \frac{d^4Q}{(2\pi)^4} \ln\left[1 + m^2\left(1 - \frac{i\omega}{2q} \ln\frac{i\omega + q}{i\omega - q}\right)\frac{1}{q^2}\right],$$

$$\mathcal{T} \equiv -\frac{(N_c^2-1)}{2} \beta V \int \frac{d^4Q}{(2\pi)^4} \ln\left[1 + \frac{m^2}{2}\left(\frac{i\omega}{q}\right)\left[\left(1 - \left(\frac{i\omega}{q}\right)^2\right) Q_0\left(\frac{i\omega}{q}\right) + \left(\frac{i\omega}{q}\right)\right]\frac{1}{Q^2}\right]$$

HDL resumed thermodynamic potential

- After regularization

Longitudinal mode is important

$$\mathcal{L} = (N_c^2-1)\beta V \frac{1}{(2\pi)}\frac{d\Omega_3}{(2\pi)^3}\frac{(m^2)^2}{4}\left[\left(1-\ln\frac{m^2}{\pi\mu_4^2}\right)\alpha-\beta+\frac{1}{\epsilon}\alpha\right]$$

$$\Rightarrow \beta V\left[\alpha_s^2\frac{2}{\pi}\left(\sum_{f=u,d}\frac{\mu_f^2}{\pi^2}\right)^2\left[\left(1-\ln 2-\ln\left(\sum_{f=u,d}\frac{\mu_f^2}{\pi^2}\frac{1}{\mu_4^2}\right)-\ln\alpha_s\right)\alpha-\beta\right]\right]_{\text{finite}}$$

$\alpha = 0.321336$

$\beta = -0.176945$

$$\mathcal{T} = (N_c^2-1)\beta V \frac{1}{(2\pi)}\frac{d\Omega_3}{(2\pi)^3}\frac{(m^2)^2}{8}\left[\left(1-\ln\frac{m^2}{2\pi\mu_4^2}\right)\frac{1}{2}\bar{\alpha}-\frac{1}{2}\bar{\beta}+\frac{1}{2\epsilon}\bar{\alpha}\right]$$

$$\Rightarrow \beta V\left[\alpha_s^2\frac{1}{\pi}\left(\sum_{f=u,d}\frac{\mu_f^2}{\pi^2}\right)^2\left[\left(1-\ln\left(\sum_{f=u,d}\frac{\mu_f^2}{\pi^2}\frac{1}{\mu_4^2}\right)-\ln\alpha_s\right)\frac{1}{2}\bar{\alpha}-\frac{1}{2}\bar{\beta}\right]\right]_{\text{finite}}$$

$\bar{\alpha} = 0.142727$

$\bar{\beta} = -0.200869$

- Total logarithm

$$\ln\mathcal{Z}_\Omega = \beta V\left(\frac{1}{4}\sum_{f=u,d}\frac{\mu_f^4}{\pi^2}\left[1-4\left(\frac{\alpha_s}{\pi}\right)+\left(\frac{8}{3}-\frac{4}{9}\pi^2\right)\left(\frac{\alpha_s}{\pi}\right)^2\right]\right.$$

$$\left.+\alpha_s^2\frac{2}{\pi}\left(\sum_{f=u,d}\frac{\mu_f^2}{\pi^2}\right)^2\left[\left(1-\ln\alpha_s-\ln\left(\sum_{f=u,d}\frac{\mu_f^2}{\pi^2}\frac{1}{\mu_4^2}\right)\right)\Lambda_1-\Lambda_2-\alpha\ln 2\right]\right)$$

→ Quark resummation (optionally considered)

$\Lambda_1 \equiv \alpha + \frac{1}{2}\bar{\alpha}$

$\Lambda_2 \equiv \beta + \frac{1}{2}\bar{\beta}$

HDL resumed symmetry energy

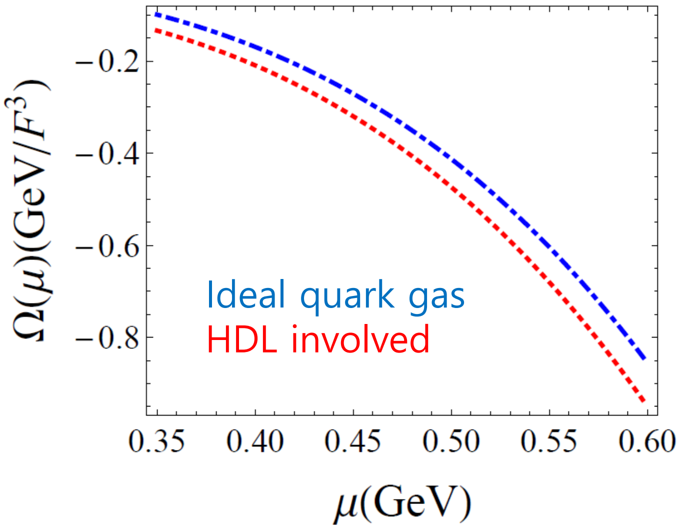
- Thermodynamic quantities can be obtained from $\Omega(\mu)$

$$\Omega(\mu) = \langle \hat{H} \rangle - \vec{\mu} \cdot \langle \hat{\vec{N}} \rangle = -\frac{1}{\beta V} \ln \mathcal{Z}_\Omega,$$

$$\rho_i(\mu) = \frac{\langle \hat{N}_i \rangle}{V} = \frac{1}{\beta V} \frac{\partial}{\partial \mu_i} \ln \mathcal{Z}_\Omega,$$

$$\epsilon(\mu) = \frac{\langle \hat{H} \rangle}{V} = -\frac{1}{V} \left(\frac{\partial}{\partial \beta} - \frac{1}{\beta} \vec{\mu} \cdot \frac{\partial}{\partial \vec{\mu}} \right) \ln \mathcal{Z}_\Omega,$$

$$I_B = \frac{\rho_3}{\rho_B} = 3 \frac{\rho_u - \rho_d}{\rho_u + \rho_d} \qquad \mu^u_d = \mu \left(1 \pm \frac{1}{3} I_B \right)^{\frac{1}{3}}$$



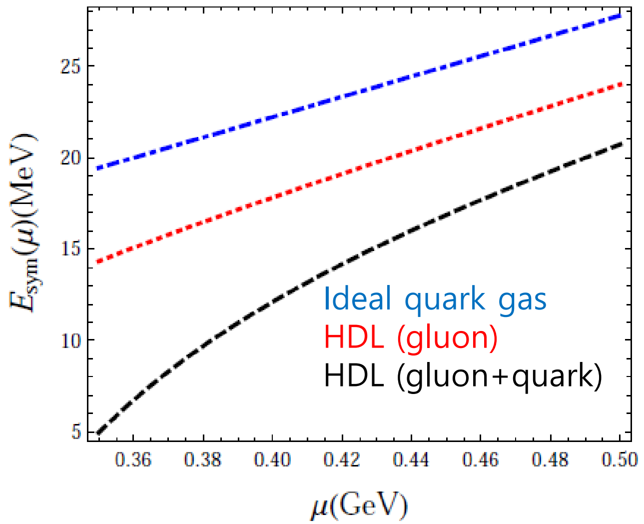
- **Quark matter symmetry energy**

$$\frac{\epsilon(\mu, I_B)}{\rho_B(\mu, I_B)} = \bar{E}(\mu, I_B) = E_0(\mu, I_B) + \bar{E}_{sym}(\mu) I_B^2 + O(I_B^4) + \cdots,$$

$$E_{sym}(\mu) = \frac{1}{2!} \frac{\partial^2}{\partial I_B^2} E(\mu, I_B),$$

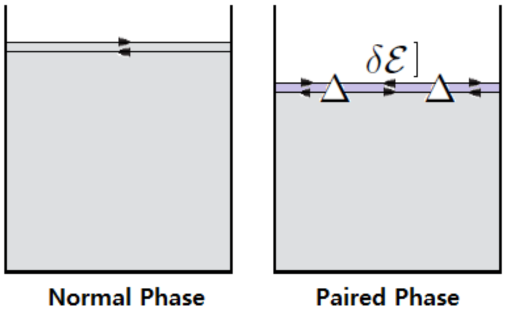
$$= \widetilde{E}^{q,0}_{sym}(\mu) - \widetilde{E}^{g,HDL}_{sym}(\mu)$$

With gauge interaction, the symmetry energy becomes **even smaller**



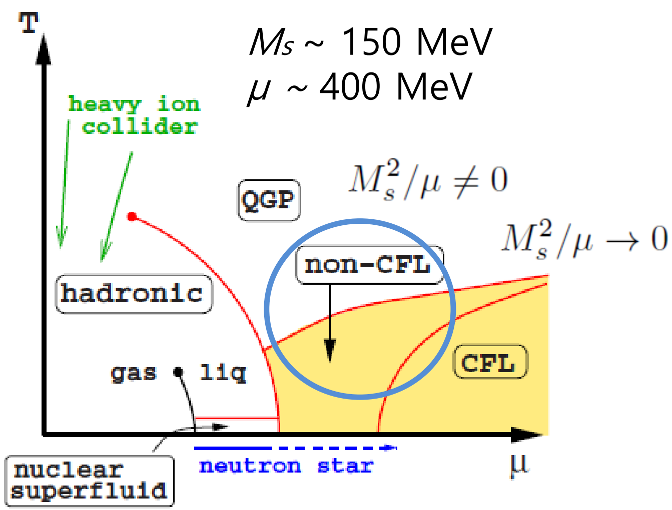
Color BCS paired states

- **BCS** Pairing locks the gapped quasi-states

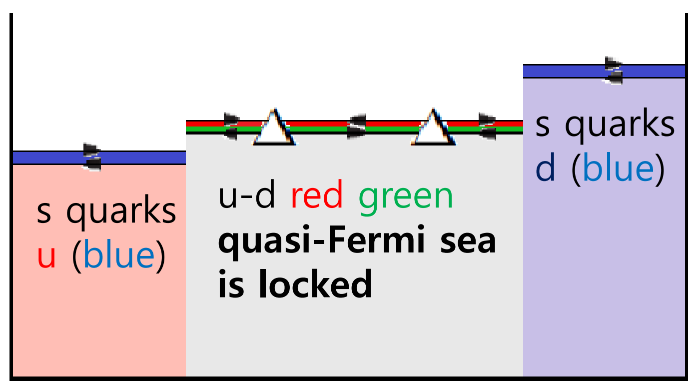


- 4-Fermion interaction with opposite momenta becomes important
- In QCD, color anti-triplet gluon exchange interaction is attractive (V<0)
- $\langle\psi_a^\alpha C\gamma_5\psi_b^\beta\rangle \sim \Delta_1\epsilon^{\alpha\beta 1}\epsilon_{ab1} + \Delta_2\epsilon^{\alpha\beta 2}\epsilon_{ab2} + \Delta_3\epsilon^{\alpha\beta 3}\epsilon_{ab3}$
- In non negligible M_s^2/μ **2SC** state is favored

- **2 color superconductivity**



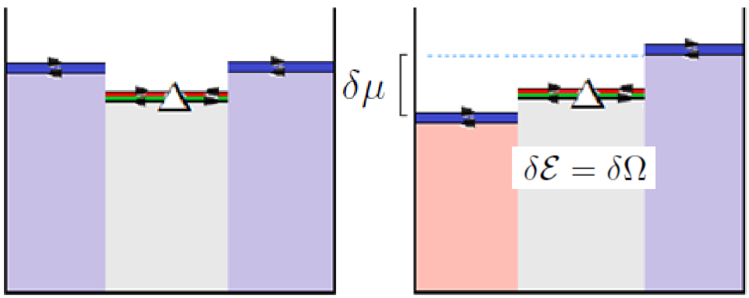
In 2SC phase, u-d red-green states are gapped



Only s quarks and u-d blue quarks are liberal

Asymmetrizing in 2SC phase

- **Only Blue state (1/3)** can affect iso-spin asymmetry



- BCS phase remains in $\delta\mu < (1/\sqrt{2})\Delta \sim \Lambda$
  (Phys. Rev. Lett. 9, 266 (1962) A. M. Clogston)
- Only u-d blue states can be asymmetrized
- The other 4 gapped quasi-states are **locked**

- **Thermodynamic potential and Symmetry energy**

$$\Omega_\Delta(\mu) \simeq -\frac{1}{12}\sum^{N_c}\sum_{f=u,d}\frac{\mu_f^4}{\pi^2} - \sum_i^{2SC}\frac{\mu_i^2\Delta^2}{4\pi^2},$$

$$\rho_i(\mu) = \frac{1}{3}\frac{\mu_i^3}{\pi^2}, \quad \rho_{\Delta i}(\mu) = \frac{1}{3}\frac{\mu_i^3}{\pi^2} + \frac{\mu_i\Delta^2}{2\pi^2},$$

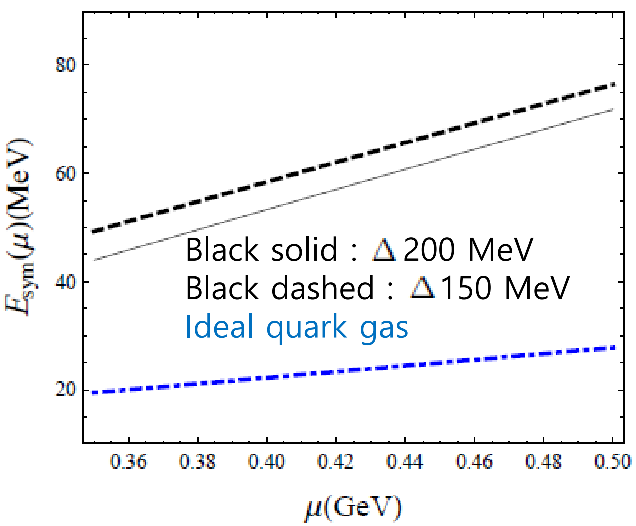
$$\epsilon_\Delta(\mu) = \epsilon_{\text{unpaired}}(\mu) + \epsilon_{\text{paired}}(\mu)$$

$$= \frac{1}{4}\sum_i^{\text{unpaired}}\frac{\mu_i^4}{\pi^2} + \frac{1}{4}\sum_i^{2SC}\left[\frac{\mu_i^4}{\pi^2} + \frac{\mu_i^2\Delta^2}{\pi^2}\right]$$

$$\frac{\epsilon(\mu, I_{\tilde{B}})}{\rho_{\tilde{B}}(\mu, I_{\tilde{B}})} = \bar{E}(\mu, I_{\tilde{B}})$$

$$I_{\tilde{B}} = I_B/3$$

$$E_{sym}^{2SC}(\mu) = \frac{1}{2!}\frac{\partial^2}{\partial I_{\tilde{B}}^2}\bar{E}(\mu, I_{\tilde{B}}),$$



Gluon rest masses from HDET (2SC)

- **2SC** description in linear combination of Gellman matrices

Gapped and un-gapped quasi-state

$$\psi_{+,\alpha i} = \sum_{A=0}^{5} \frac{(\tilde{\lambda}_A)_{\alpha i}}{\sqrt{2}} \psi_+^A \qquad \chi = \begin{pmatrix} \psi_+ \\ C\psi_-^* \end{pmatrix}$$

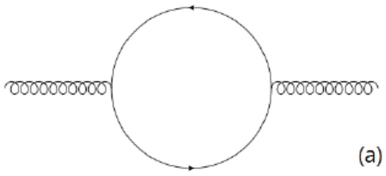
+ and – represents Fermi velocity

$$\tilde{\lambda}_0 = \frac{1}{\sqrt{3}}\lambda_8 + \frac{2}{3}I;\ \tilde{\lambda}_A = \lambda_A\ (A=1,2,3);\ \tilde{\lambda}_4 = \frac{1}{\sqrt{2}}(\lambda_4 - i\lambda_5);\ \tilde{\lambda}_5 = \frac{1}{\sqrt{2}}(\lambda_6 - i\lambda_7),$$

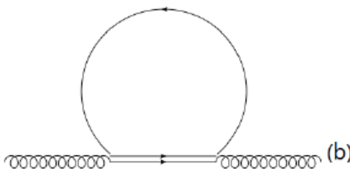
High Density Effective Lagrangian in Nambu-Gorkov form

$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \sum_{\vec{v}_f}^{\text{half}} \sum_{A,B=0}^{5} \left[\chi^{A\dagger} \begin{pmatrix} iV\cdot\partial\delta_{AB} & \Delta_{AB} \\ \Delta_{AB} & i\bar{V}\cdot\partial\delta_{AB} \end{pmatrix} \chi^B + igA^a_\mu \chi^{A\dagger} \begin{pmatrix} iV^\mu\kappa_{AaB} & 0 \\ 0 & -i\bar{V}^\mu\kappa^*_{AaB} \end{pmatrix} \chi^B \right.$$

$$\left. + g^2 A^c_\mu A^d_\nu \chi^{A\dagger} \begin{pmatrix} \frac{1}{2\mu_f + i\bar{V}\cdot D}\xi^{cd}_{AB} & 0 \\ 0 & \frac{1}{2\mu_f + iV\cdot D^*}\xi^{cd*}_{AB} \end{pmatrix} P^{\mu\nu}\chi^B \right] + (L\to R), \qquad P^{\mu\nu} = g^{\mu\nu} - \tfrac{1}{2}(V^\mu\bar{V}^\nu + V^\nu\bar{V}^\mu)$$



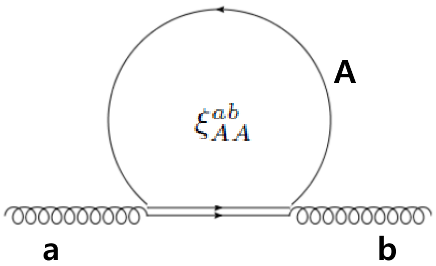
$$\Pi^{ab}_{\mu\nu}(q) = g^2 \int d^4x e^{-iq\cdot x} \sum_{\vec{v}_f}^{\text{half}} \langle \Omega | T[J^a_\mu(x,\vec{v}_f) J^b_\nu(0,\vec{v}_f)] | \Omega \rangle$$



Only **(b)** shows explicit iso-spin dependence

Gluon rest masses from HDET (2SC)

- Relevant diagram



- Transverse mode
- Double line denotes propagating in Dirac sea mode
- Tadpole can be obtained by calculating ξ_{AA}^{ab}
- Total counter term $\delta\Pi_{ij}^{ab}(0) = -\frac{1}{3}m^2\delta_{ij}\delta^{ab}$

- ξ_{AA}^{ab} and rest masses

	$a,b=1,2,3$	$a,b=4,5,6,7$	$a,b=8$
Paired (A=0,1,2,3)	$\xi_{AA}^{ab}=\frac{1}{2}\delta^{ab}$	$\xi_{AA}^{ab}=\frac{1}{4}\delta^{ab}$	$\xi_{AA}^{ab}=\frac{1}{6}\delta^{ab}$
Unpaired (A=4,5)	$\xi_{AA}^{ab}=0$	$\xi_{AA}^{ab}=\frac{1}{4}\delta^{ab}$	$\xi_{AA}^{ab}=\frac{1}{3}\delta^{ab}$

	$a,b=1,2,3$	$a,b=4,5,6,7$	$a,b=8$
$\Pi_{00}^{ab}(0)$	0	$\frac{1}{2}m^2\delta^{ab}$	$m^2\delta^{ab}$
$-\Pi_{ij}^{ab}(0)$	0	$\frac{1}{12}g^2\sum_f^{4,5}(\mu_f^2/\pi^2)\delta_{ij}\delta^{ab}$	$\frac{1}{9}g^2\sum_f^{4,5}(\mu_f^2/\pi^2)\delta_{ij}\delta^{ab}$

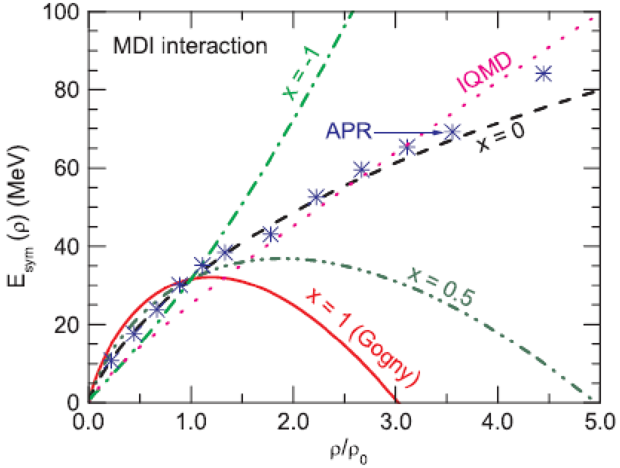
$m^2 = (g^2\mu^2/\pi^2)$

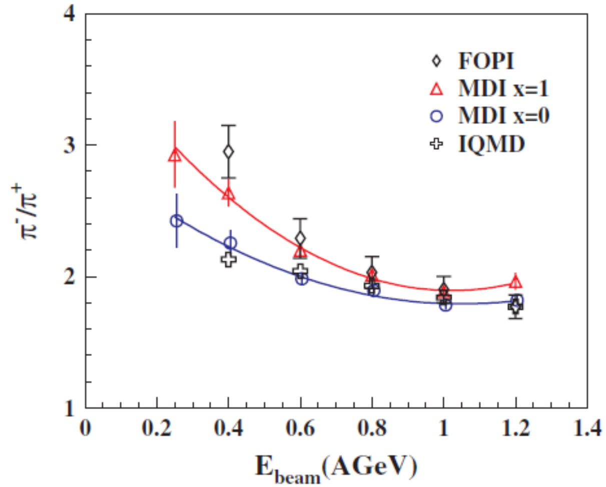
- Only Meissner mass has iso-spin dependence
- The portion is very small : minimal contribution for static quantities
- For super-soft gluons, transverse mode can be important
- Dynamical process may strongly depends on iso-spin asymmetry

Stiff or Soft?

- Density behavior of symmetry energy

(PRL 102, 062502 Z. Xiao et al.)



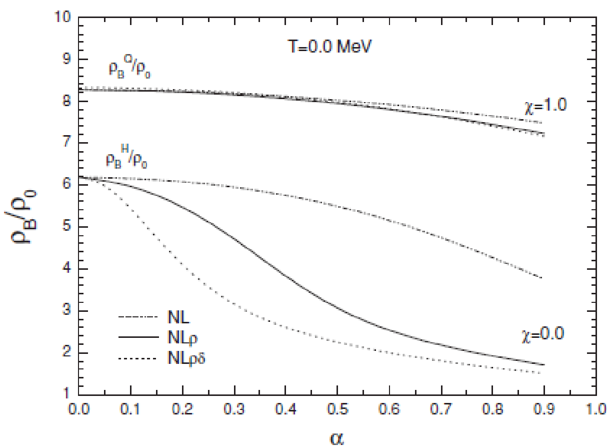


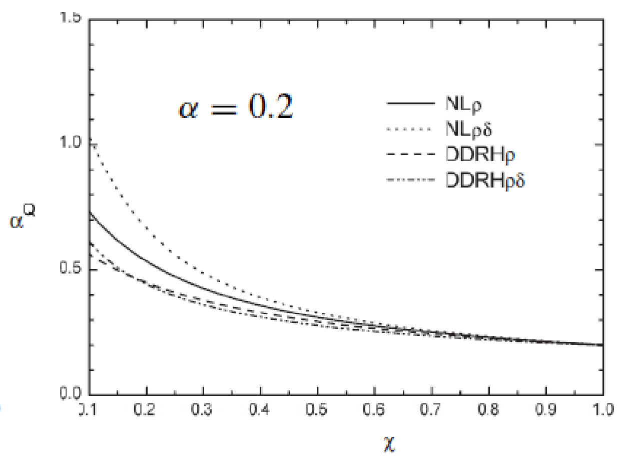
X=1 corresponds to soft symmetry energy

To confirm this, direct n/p ratio should be measured

Soft symmetry energy is needed to keep high iso-spin density

(PRC 83, 014911 M.Ditoro et al.)





$$\alpha \equiv -\frac{\rho_3}{\rho_B} = \frac{(1-\chi)\alpha^H}{(1-\chi) + \chi\frac{\rho_B^Q}{\rho_B^H}} + \frac{\chi\alpha^Q}{(1-\chi)\frac{\rho_B^H}{\rho_B^Q} + \chi}$$

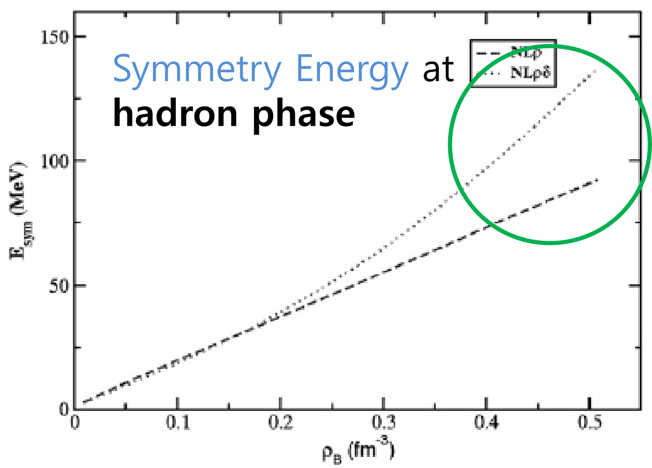
Iso-spin density can remain in high value with large iso-spin asymmetry of quark phase

Persisting high iso-spin density can cause large π^-/π^+

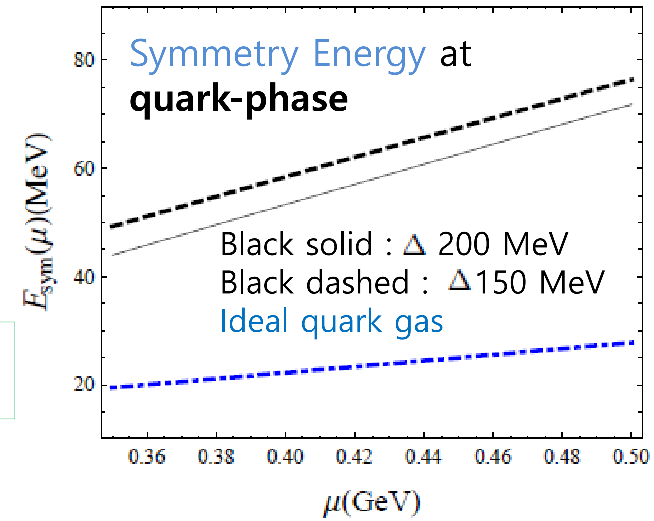
Hadron–Quark mixed phase

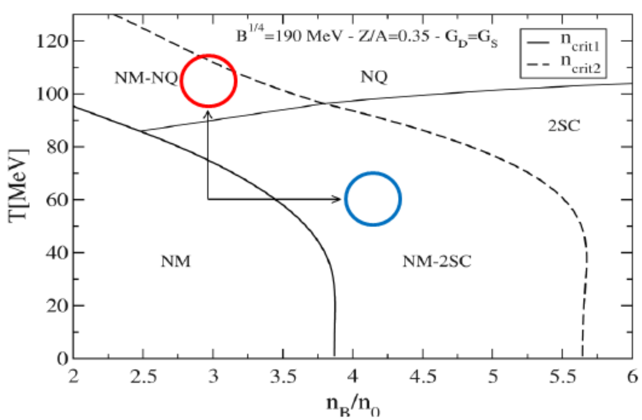
- Iso-spin distillation and π^-/π^+ ratio (in agreement with Phys. Rev. D **81**, 094024 (2010))

Physics Report, 410, 335 (2005) (V. Baran et al.)



Pushing neutron
-> Evaporation

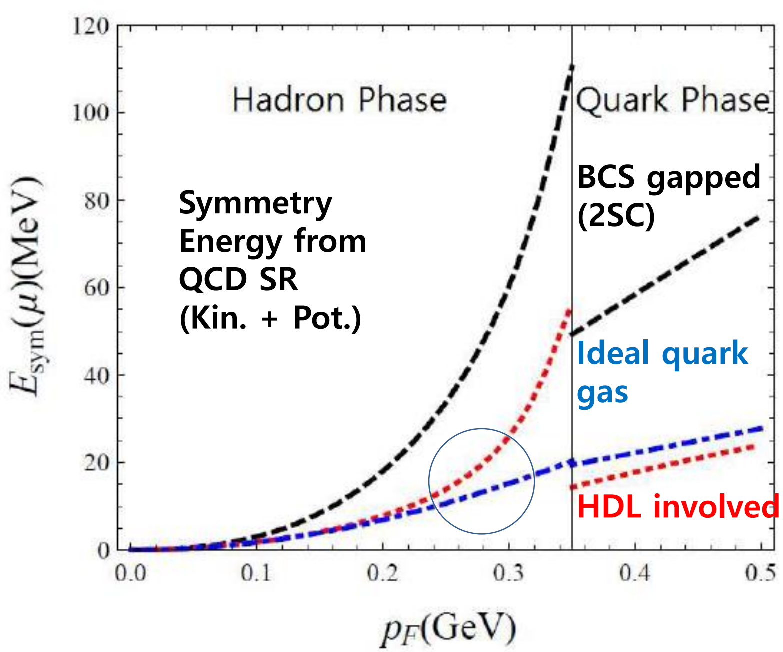




- Large symmetry energy can cause iso-spin evaporation
- As nuclear symmetry energy is **larger** than quark matter symmetry energy, iso-spin distillation can occur at mixed phase
- At 2SC, the distillation will be **reduced**
- Eventually, π^-/π^+ ratio will be **reduced**

Summary and Future goals

- **Nuclear Symmetry Energy** in hadron and quark phase

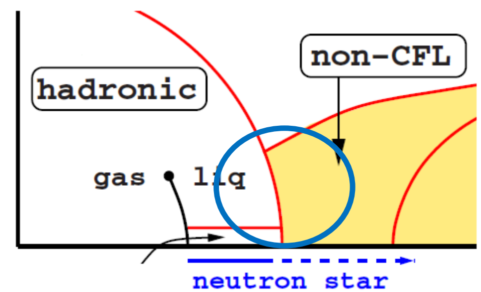


Kinetic part **(Ideal nucleon gas)**
Kinetic part **(QCD SR based interaction involved)**

- Transverse gluonic mode at super-soft condition

Iso-spin dependent dynamical process can be important probe for quark matter symmetry energy

- Strangeness in high density



Important quantum numbers?
-> High density behavior at hadron phase
-> Kaon production may give answer