Scale-Invariant Hidden Local Symmetric Model and its application to Dense Nuclear Matter by using $V_{\text{low k}}$

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Won-Gi Paeng, RISP/IBS

In collaboration with Thomas T. S. Kuo(Stony Brook), Hyun Kyu Lee(Hanyang Univ.) and Mannque Rho(CEA Saclay)

Model construction

- The dilatation current respect to the scale transformation by $x^{\mu} \to \lambda^{-1} x^{\mu}$ denoted as $\hat{d}[x] = -1$ is given by

$$D^{\mu} = x_{\nu}\theta^{\mu\nu},$$

and the scale symmetry is conserved when $\partial^{\mu} D_{\mu} = \theta^{\mu}{}_{\mu} = 0$, where $\theta^{\mu}{}_{\mu}$ is the trace of the energy-momentum tensor.

- But, the quantum effect in QCD breaks the scale symmetry as

$$\left(\theta^{\mu}{}_{\mu}\right)_{QCD} = \frac{\beta(\alpha_s)}{4\alpha_s} G^{a}{}_{\mu\nu} G^{a\,\mu\nu} + \sum_{q} m_q \bar{q} q \neq 0.$$

- The effective Lagrangian for the low energy region, which accounts for the explicit scale symmetry breaking of QCD, can be constructed by implementing a dilaton χ which transforms as $\chi \rightarrow \lambda\,\chi$ under the scale transformation.
- The terms for the explicit scale symmetry breaking are given by

$$-V(\chi) + \frac{{f_{0\pi}}^2}{4}\left(\frac{\chi}{f_{0\sigma}}\right)^3 \mathrm{tr}[MU^+ + h.c.]$$

in the effective Lagrangian, where $M = 2B^0\begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix}$. The second term in the above equation is given to have the same scale dimension as '$m_q\bar{q}q$'. If we take Coleman-Weinberg potential for $V(\chi)$ as $V(\chi) = \frac{B}{4}\chi^4 \mathrm{tr}[\ln\left(\frac{\chi}{f_{0\sigma}}\right)^4 - 1]$, we get

$$\left(\theta^{\mu}{}_{\mu}\right)_{eff} = B\chi^4 - \frac{{f_{0\pi}}^2}{4}\left(\frac{\chi}{f_{0\sigma}}\right)^3 \mathrm{tr}[MU^+ + h.c.].$$

- At the matching scale $-q^2=Q^2 \approx {\Lambda_M}^2$, the effective Lagrangian is matched with QCD by

$$\left\langle \left(\theta^{\mu}{}_{\mu}\right)_{QCD} \right\rangle = \left\langle \left(\theta^{\mu}{}_{\mu}\right)_{eff} \right\rangle,$$

so $\langle \chi \rangle$ carries the information of the density dependence by relating it to $\langle \bar{q}q \rangle$ and $\langle G^2 \rangle$. We call this density dependence Intrinsic Density Dependence(IDD).

- We would like to construct the scale-invariant effective model(χPT) with a scalar field(dilaton), where ρ and ω are given as a gauge boson of the hidden local symmetry and the baryon also included. We call this "bsHLS". We show that the parameters in the effective model are related to $\langle \bar{q}q \rangle$ and $\langle G^2 \rangle$ by $\langle \chi \rangle$.

We are interested in the terms,

$$\begin{aligned}\mathcal{L}_{bsHLS} &= \frac{1}{2}\left(\frac{\chi}{f_{0\sigma}}\right)^2 \partial_\mu \pi^a \partial^\mu \pi^a + \frac{1}{2}\partial^\mu \chi \partial_\mu \chi + \bar{N} i \gamma^\mu \partial_\mu N - \sum_{V=\rho,\omega} \frac{1}{2} \mathrm{tr}\left[V_{\mu\nu} V^{\mu\nu}\right] \\ &+ \sum_{V=\rho,\omega} \frac{1}{2} m_V^2 \left(\frac{\chi}{f_{0\sigma}}\right)^2 V^\mu V_\mu - \frac{f_{0\pi}^2}{2} m_\pi^2 \left(\frac{\chi}{f_{0\sigma}}\right)^3 \frac{\pi^a \pi_a}{f_{0\pi}^2} + V(\chi) - m_N \frac{\chi}{f_{0\sigma}} \bar{N} N \\ &- \sum_{V=\rho,\omega} g_V \left(g_{VN} - 1\right) \bar{N} \gamma_\mu V^\mu N + g_A \bar{N} \gamma^\mu \gamma_5 \frac{\partial_\mu \pi}{f_{0\pi}} N + \cdots ,\end{aligned}$$

where one boson exchange NN interactions are shown in the leading order of scale-chiral counting. Here, $V_\mu = \rho_\mu^a \frac{\tau^a}{2}$ or $\frac{\omega_\mu}{2}$, $\pi = \pi^a \frac{\tau^a}{2}$, $m_V^2 = f_{0\sigma_V}^2 g_V^2$ and

$$V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu - i g_V \left[V_\mu, V_\nu\right] .$$

$$(m_\pi)^2 = 2B^0 m, \; m_u = m_d = m$$

If we define σ as

$$\chi = f_{0\sigma} \exp\left(\frac{\sigma}{f_{0\sigma}}\right)$$

and take $\sigma \to \langle\sigma\rangle - \sigma'$, $\mathcal{L}_{bs\mathrm{HLS}}$ is rewritten as

$$\begin{aligned}\mathcal{L}_{bs\mathrm{HLS}} &= \frac{1}{2}\partial_\mu \pi^{*\,a}\partial^\mu \pi^{*\,a} + \frac{1}{2}\partial^\mu \sigma^* \partial_\mu \sigma^* + \bar{N} i\gamma^\mu \partial_\mu N - \sum_{V=\rho,\omega} \frac{1}{2}\mathrm{tr}\left[V_{\mu\nu}V^{\mu\nu}\right] \\ &+ \sum_{V=\rho,\omega} \frac{1}{2} m_V^{*\,2} V^\mu V_\mu - \frac{1}{2} m_\pi^{*\,2}\left(\pi^{*\,a}\right)^2 + \frac{1}{2} m_\sigma^{*\,2} \sigma^{*\,2} \\ &- m_N^* \bar{N} N + g_\sigma \sigma^* \bar{N} N - \sum_{V=\rho,\omega} g_{VNN} \bar{N}\gamma_\mu V^\mu N + g_A \bar{N}\gamma^\mu \gamma_5 \frac{\partial_\mu \pi^*}{f_\pi^*} N + \cdots ,\end{aligned}$$

where π^* and σ^* are defined as $\pi^* = \frac{\langle\chi\rangle}{f_{0\sigma}}\pi$ and $\sigma^* = \frac{\langle\chi\rangle}{f_{0\sigma}}\sigma'$, $\langle\chi\rangle = f_{0\sigma}\exp\left(\frac{\langle\sigma\rangle}{f_{0\sigma}}\right)$ and

$$m_V^* = g_V f_{0\sigma V} \frac{\langle\chi\rangle}{f_{0\sigma}},\ m_\pi^* = m_\pi \left(\frac{\langle\chi\rangle}{f_{0\sigma}}\right)^{1/2},\ m_N^* = m_N \frac{\langle\chi\rangle}{f_{0\sigma}},\ f_\pi^* = f_\pi \frac{\langle\chi\rangle}{f_{0\sigma}}$$

$$m_\sigma^{*\,2} = -\left.\frac{\partial^2}{\partial\sigma^{*\,2}} V\left(\langle\chi\rangle - \sigma^*\right)\right|_{\sigma^*=0} \approx m_\sigma^2 \left(\frac{\langle\chi\rangle}{f_{0\sigma}}\right)^2 ,$$

$$g_\sigma = \frac{m_N}{f_{0\sigma}},\ g_{\rho NN} = g_\rho\left(g_{\rho N} - 1\right) \text{ and } g_{\omega NN} = g_\omega\left(g_{\omega N} - 1\right) .$$

Application to Dense nuclear matter

- Now, we get the three free parameters in the model, which has a

density dependence. They are $\langle\chi\rangle$, g_ρ and g_ω. The all other parameters are related to them by

$$\frac{m^*{}_N}{m_N} \approx \frac{m^*{}_\sigma}{m_\sigma} \approx \frac{f^*{}_\pi}{f_{0\pi}} \approx \frac{\langle\chi\rangle}{f_{0\sigma}} \ \& \ \frac{m^*{}_V}{m_V} \propto g_V \frac{\langle\chi\rangle}{f_{0\sigma}}$$

$$g_{VNN} \propto g_V,$$

where $V = \rho$ and ω. Here, please note that g_ρ also has a density dependence by matching the current correlators of HLS with the current correlators of QCD at the matching scale Λ_M.

- $\langle\chi\rangle$ is defined as the classical solution for the equation of the motion of χ given by

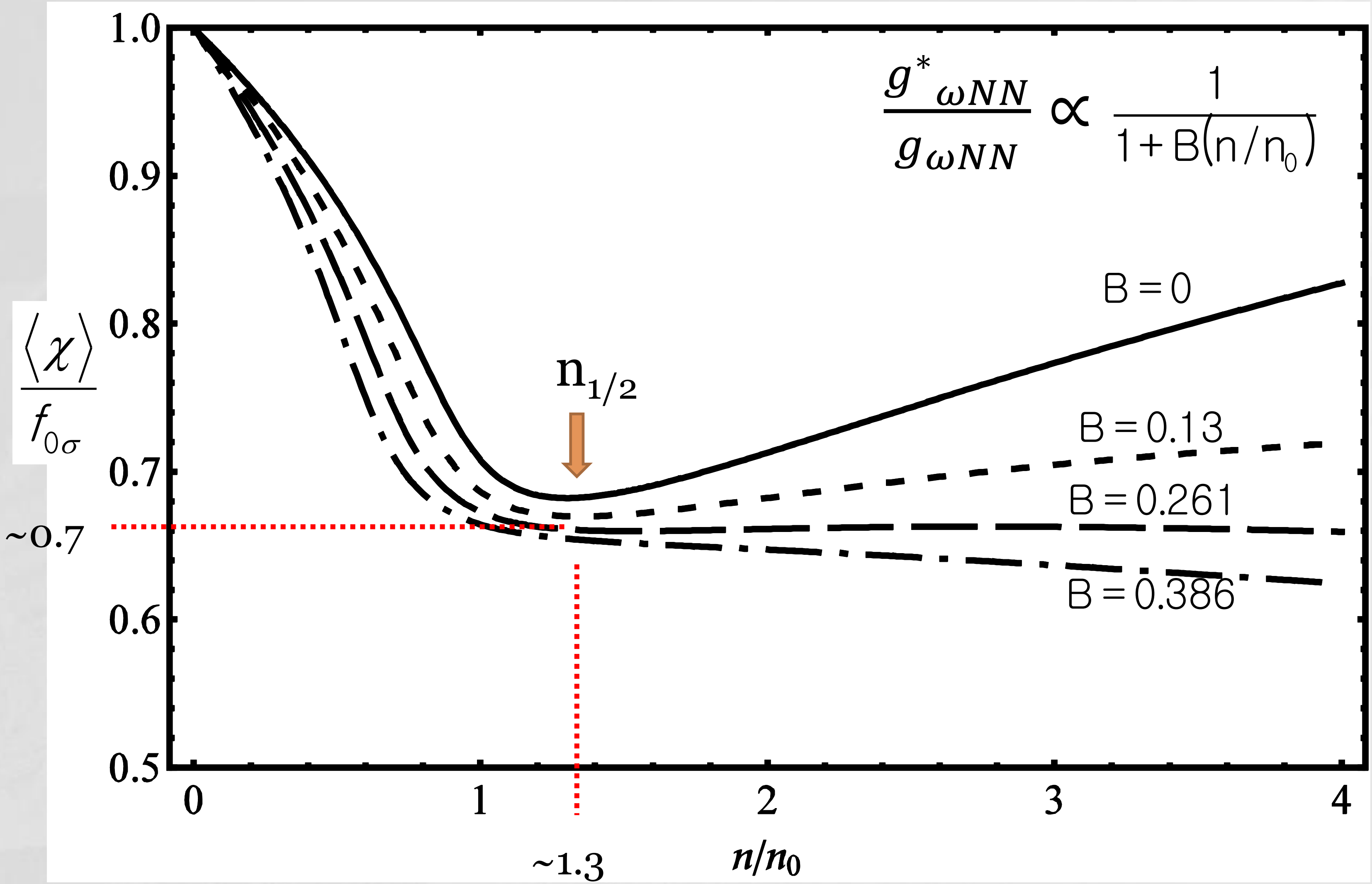
$$\frac{\partial}{\partial\chi}\mathcal{L}(\chi)|_{\chi=\langle\chi\rangle} = 0.$$

If we go into the nuclear matter, the value of $\langle\chi\rangle$ will be changed with depending on $\langle N^+N\rangle$ because χ is coupled with a nucleon by $\sim\chi\bar{N}N$.

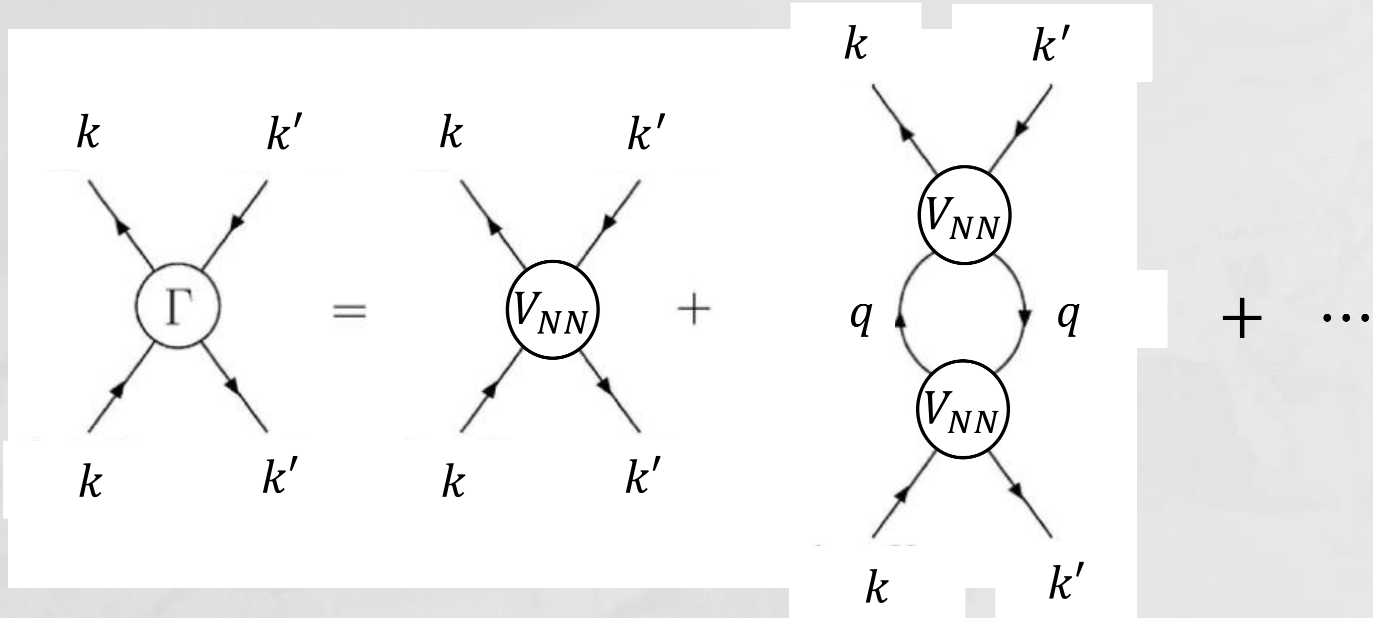
- We calculated the density dependence of $\langle\chi\rangle$ in bsHLS, but without

$$\frac{{f_{0\pi}}^2}{4}\left(\frac{\chi}{f_{0\sigma}}\right)^3 \mathrm{tr}[MU^+ + h.c.].$$

The calculation was done in the mean-field approximation.
[W.-G. Paeng, H. K. Lee, M. Rho and C. Sasaki, Phys. Rev. D 88, 105019 (2013)]



- We would like to check whether we can have the same scaling behavior of $\langle\chi\rangle$ in dense nuclear matter by using $\mathrm{V}_{low\ k}$.

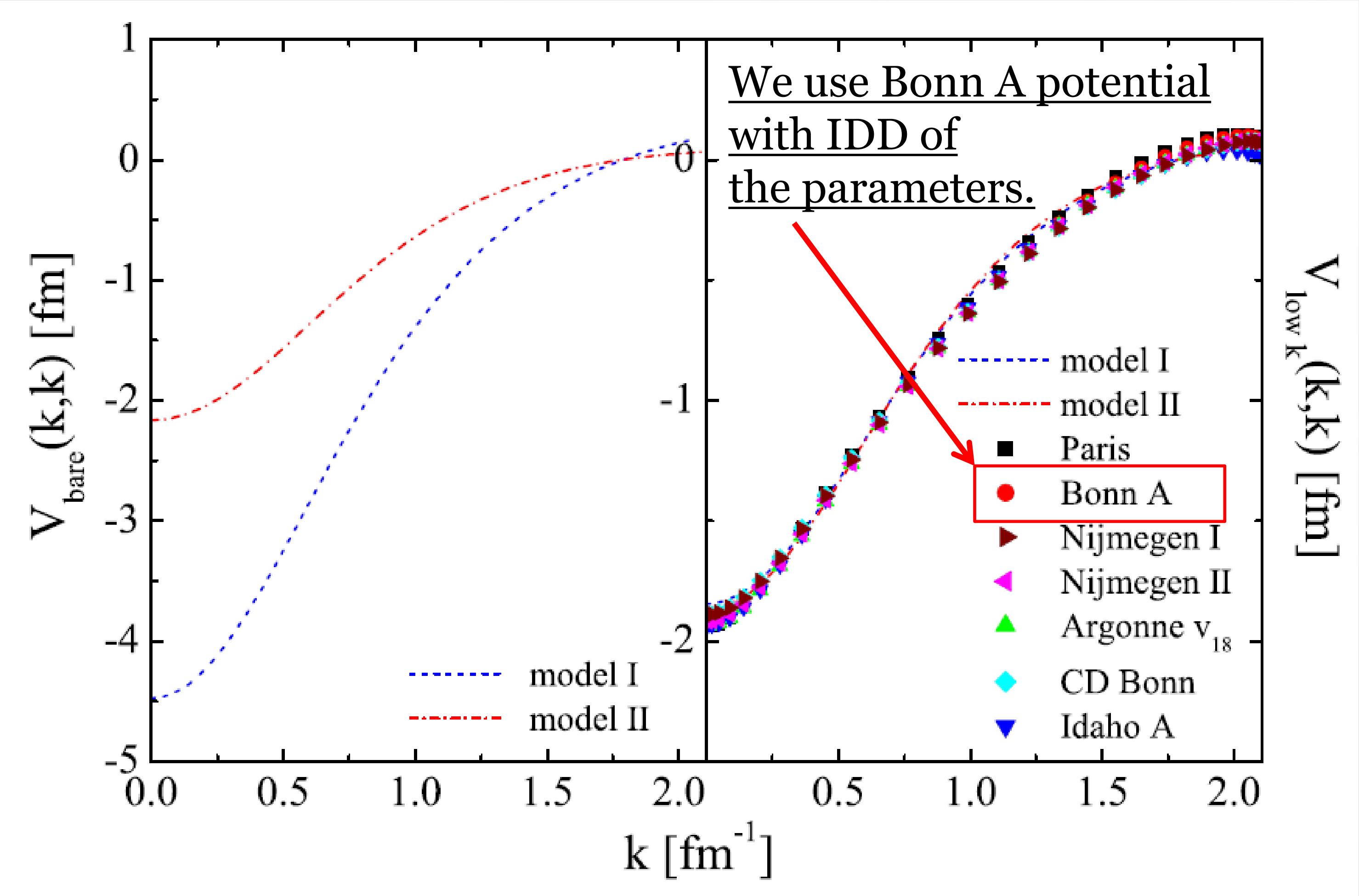


$$T(k',k,k^2) = V_{\mathrm{NN}}(k',k) + \frac{2}{\pi}\mathcal{P}\int_0^{\infty} \frac{V_{\mathrm{NN}}(k',q)T(q,k,k^2)}{k^2-q^2}q^2dq,$$

$$T_{\mathrm{lowk}}(k',k,k^2) = V_{\mathrm{lowk}}(k',k) + \frac{2}{\pi}\mathcal{P}\int_0^{\Lambda} \frac{V_{\mathrm{lowk}}(k',q)T_{\mathrm{lowk}}(q,k,k^2)}{k^2-q^2}q^2dq,$$

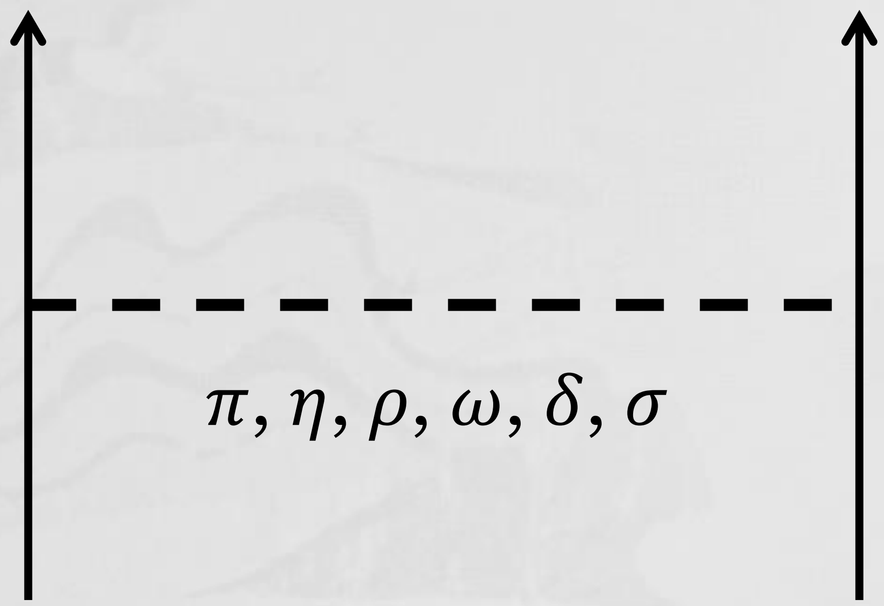
$$T(k',k,k^2) = T_{lowk}(k',k,k^2);\ (k',k) \leq \Lambda.$$

- The model-independent low momentum interaction, called V_{lowk}, is obtained, which reproduces the same phase shifts for the NN scattering data and deuteron pole which are the inputs.



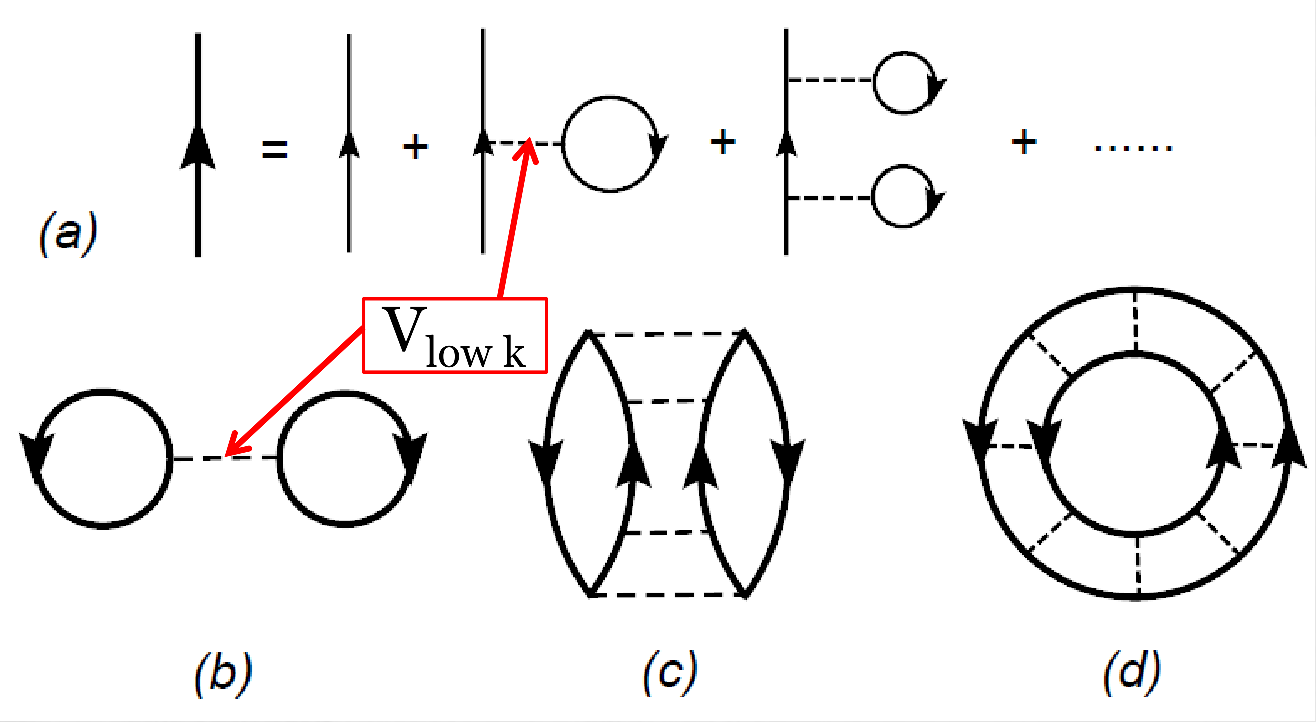
S. K. Bogner, T. T. S. Kuo and A. Schwenk, Phys. Rept. 386, 1 (2003)

- The Bonn A potential is a phenomenological potential given by one boson exchange expressed as



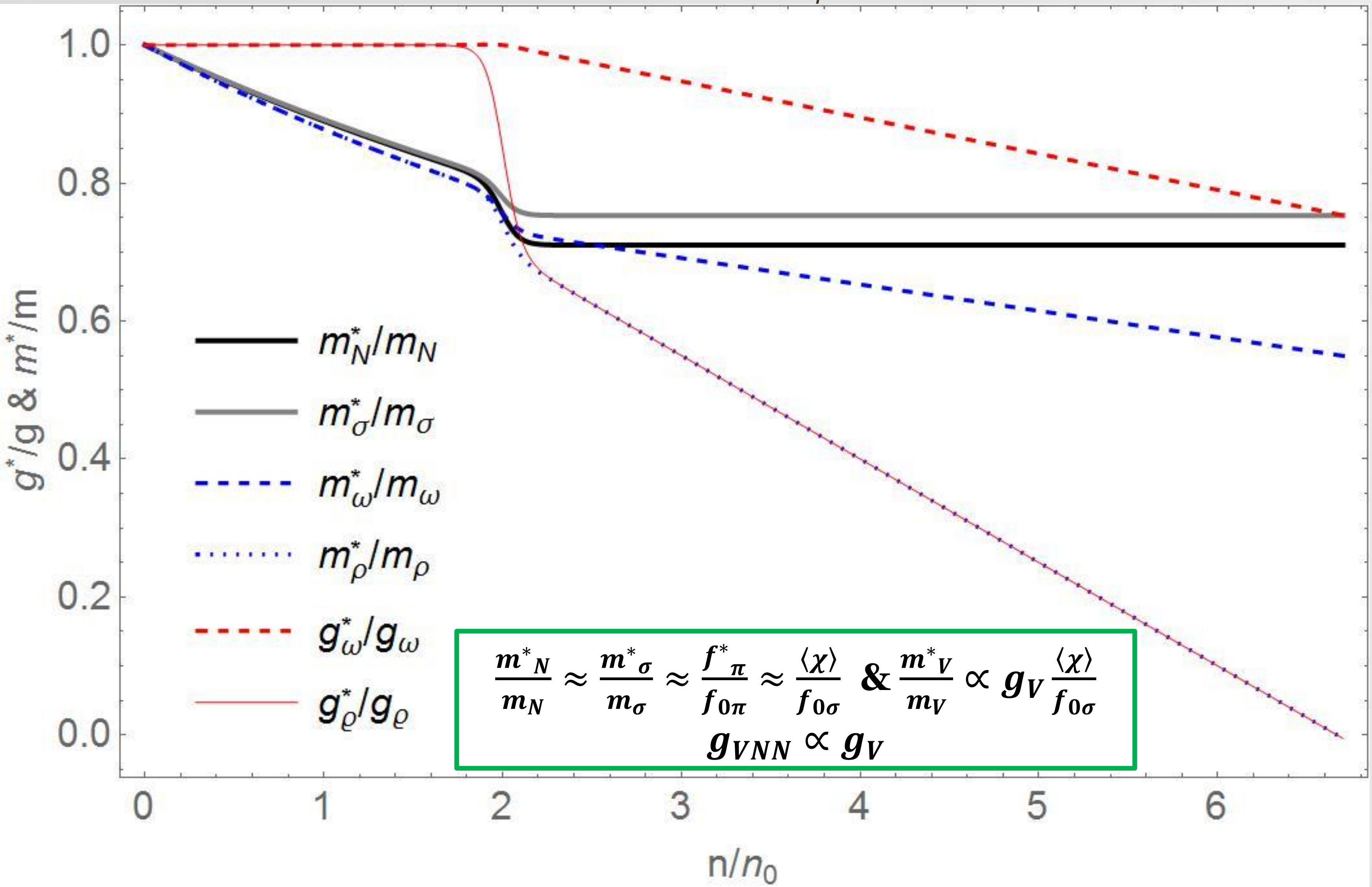
- When we go to the nuclear matter, the density dependence of the parameters obtained in bsHLS for the mass and coupling constants are used for π, ρ, ω, σ as $\frac{m^*}{m} = \frac{\langle\chi\rangle}{f_{0\sigma}}$ and $\frac{g^*}{g}$.

- We obtain the ground state energy of the symmetric nuclear matter and pure neutron matter by calculating the single particle energy for the diagram (a) and the pphh ring diagrams for the diagram (b), (c) and (d) summed to all orders within a model space of the cutoff Λ.



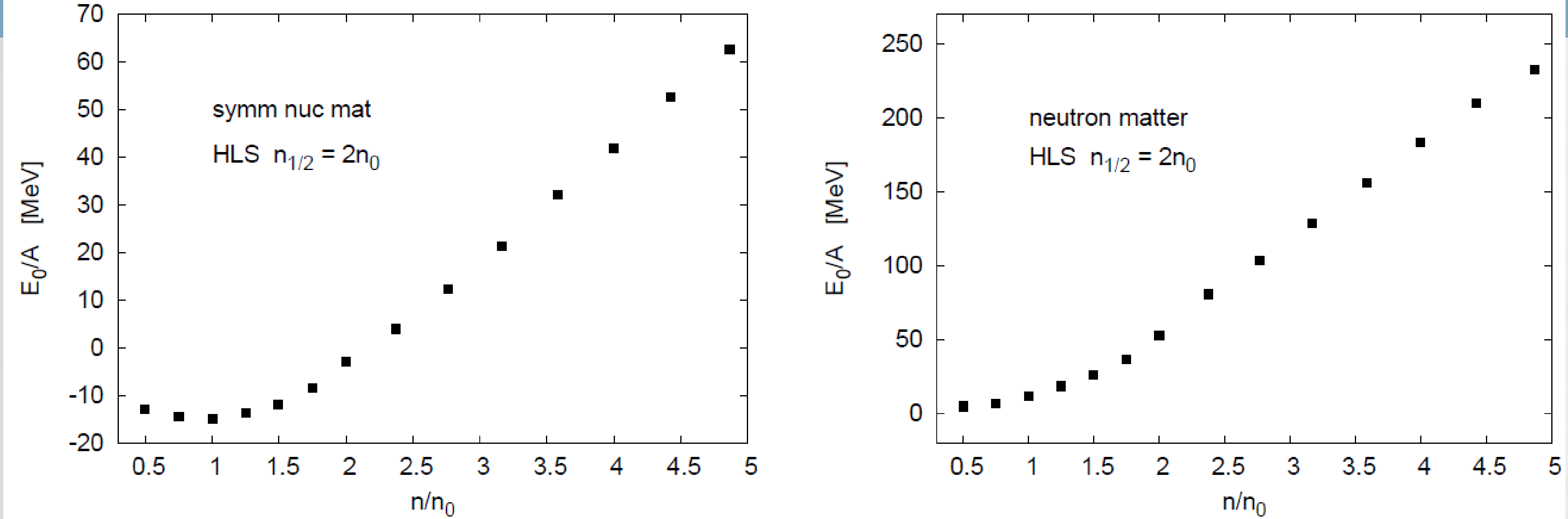
Result

- We obtain the best scaling of the parameters which give the reasonable EoS for the dense nuclear matter and satisfy VM property of $m^*_\rho \sim g^*_\rho \sim \langle \bar{q}q \rangle \rightarrow 0$ at the critical density.

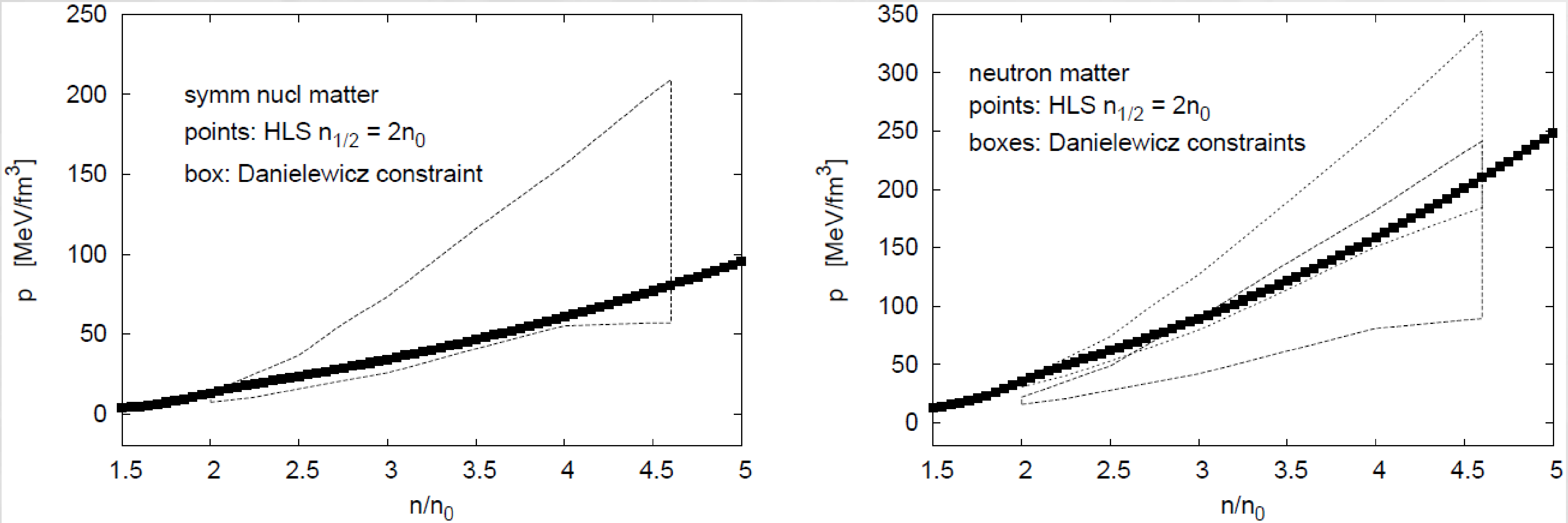


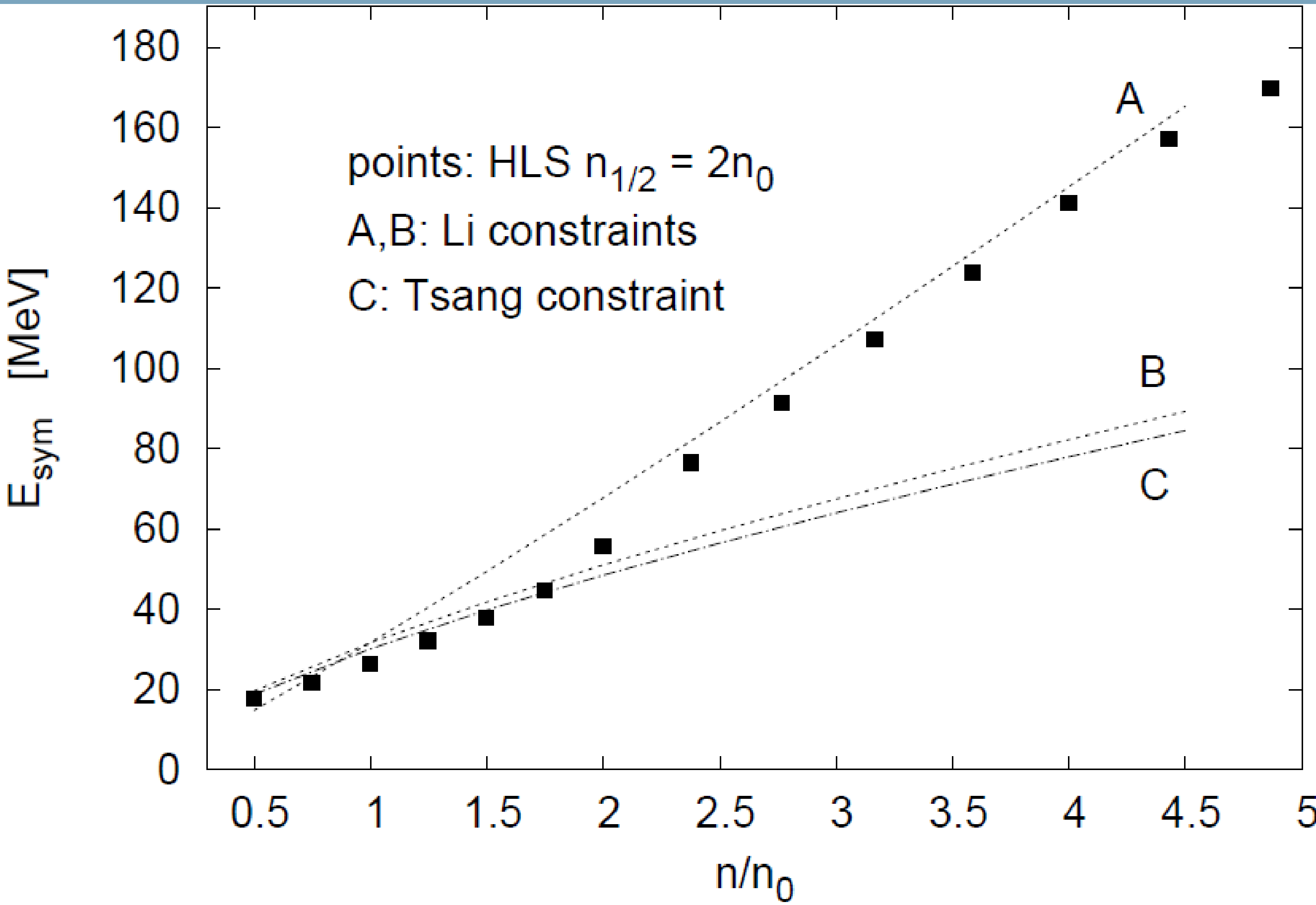
$$\frac{m^*_N}{m_N} \approx \frac{m^*_\sigma}{m_\sigma} \approx \frac{f^*_\pi}{f_{0\pi}} \approx \frac{\langle \chi \rangle}{f_{0\sigma}} \ \& \ \frac{m^*_V}{m_V} \propto g_V \frac{\langle \chi \rangle}{f_{0\sigma}}$$

$$g_{VNN} \propto g_V$$



The scaling gives, for symmetric nuclear matter, the saturation energy E_0/A=-15.1 MeV, saturation density $n_{sat} \approx 0.16$ fm^{-3} and compression modulus K = 183.2 MeV.





Li:

$E_{sym}(n) \approx 31.6(n/n_0)^{\gamma}$;
γ = 0.69(B) – 1.1(A)

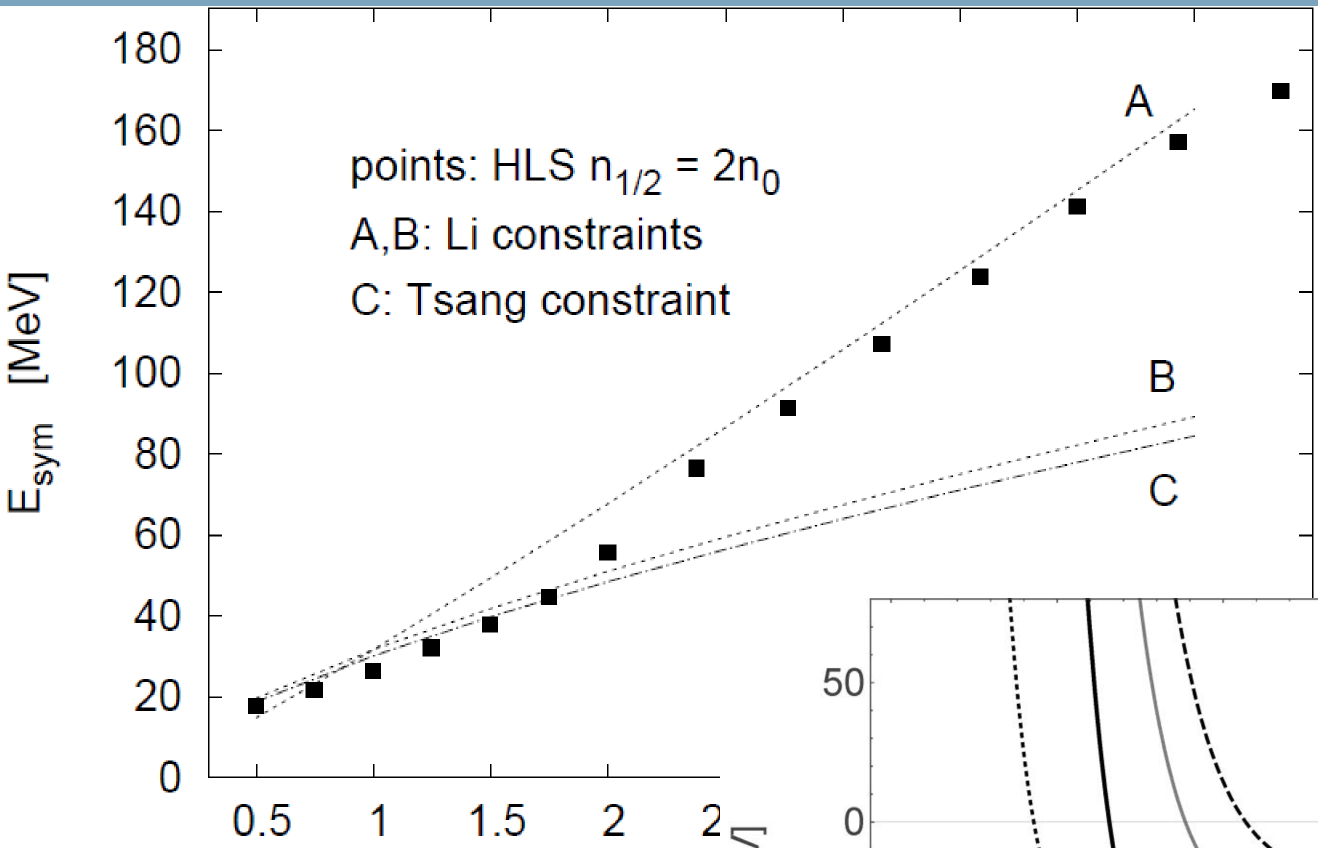
Tsang:

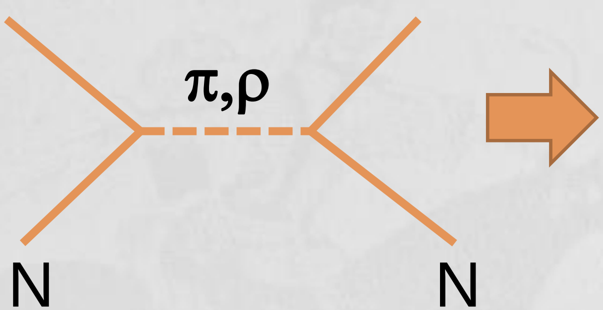
$$E_{sym}(n) = \frac{C_{s,k}}{2}\left(\frac{n}{n_0}\right)^{2/3} + \frac{C_{s,p}}{2}\left(\frac{n}{n_0}\right)^{\gamma_i};$$

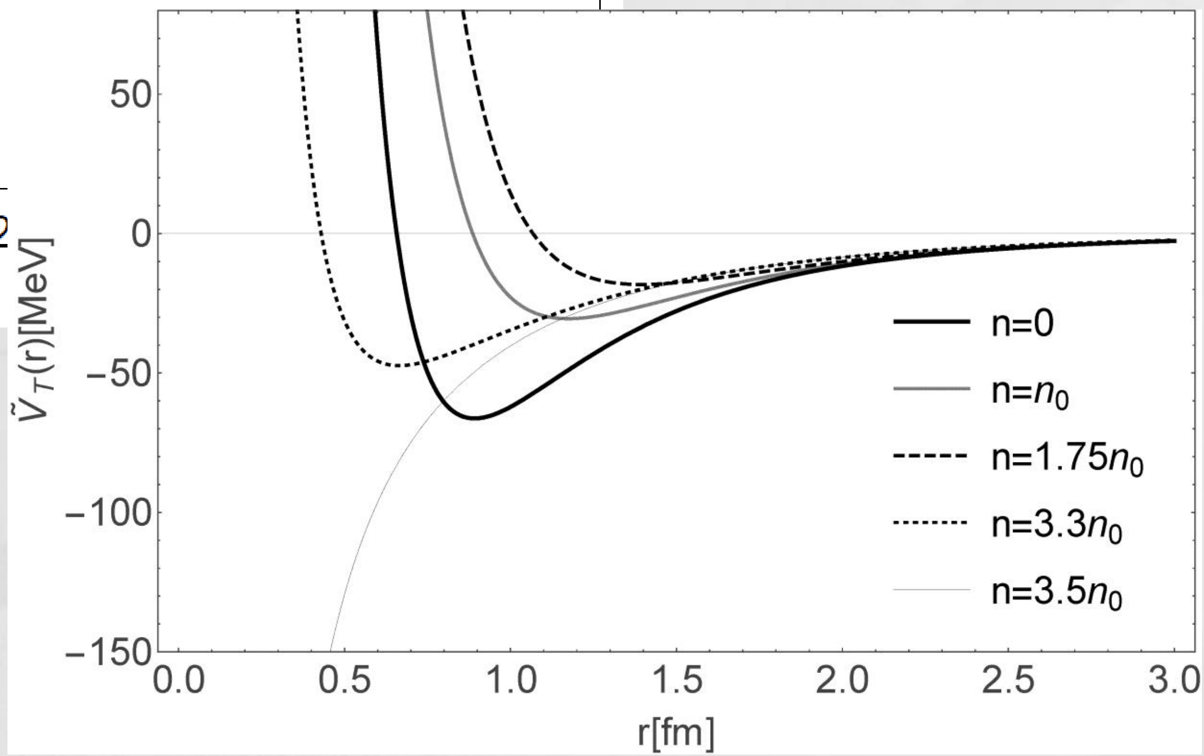
$C_{s,k}$ = 25 MeV, $C_{s,p}$ = 35.2 MeV and $\gamma_i \approx 0.7$(C).

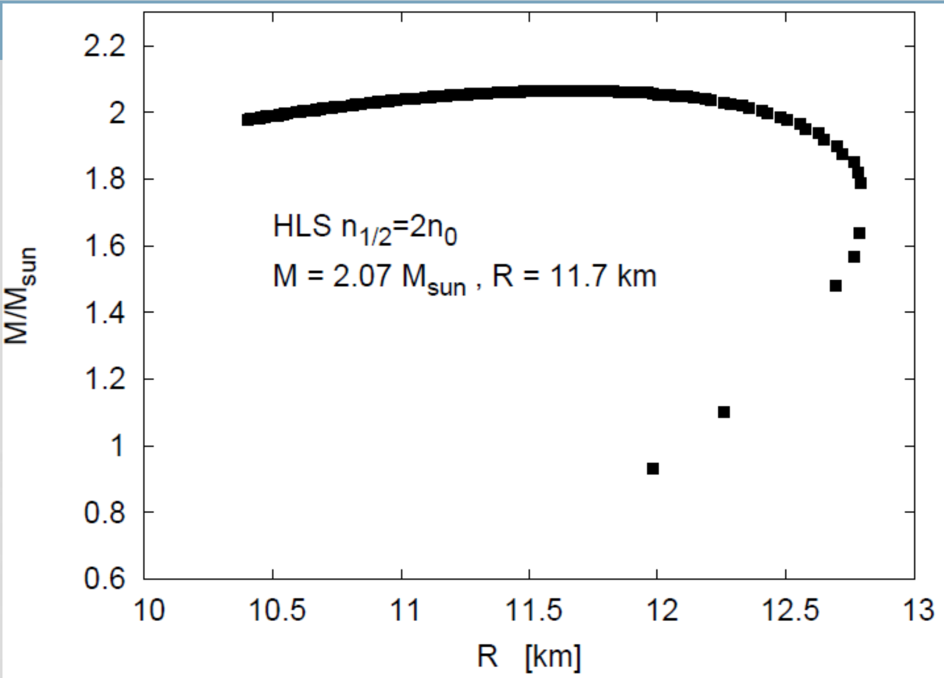
E_{sym}/MeV	L/MeV	
27	57.3	*bs*HLS
31.6	65.4-104.2	Li
30.1	62.0	Tsang
28-32	40-60	Lattimer

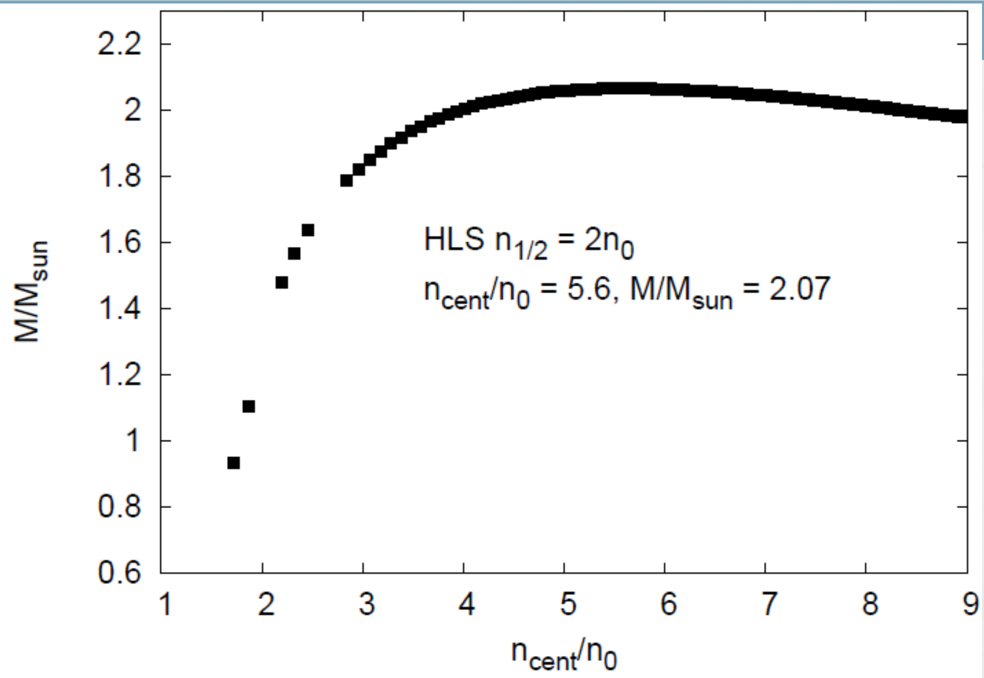
[Li]B. A. Li and L. W. Chen, Phys. Rev. C **72**, 064611 (2005).
[Tsang] M. B. Tsang, Y. Zhang, P. Danielewicz, M. Famiano, Z. Li, W. G. Lynch and A. W. Steiner, Phys. Rev. Lett. **102**, 122701 (2009) [Int. J. Mod. Phys. E **19**, 1631 (2010)] .
[Lattimer] J. M. Lattimer and Y. Lim, Interaction, Astrophys. J. **771**, 51 (2013).

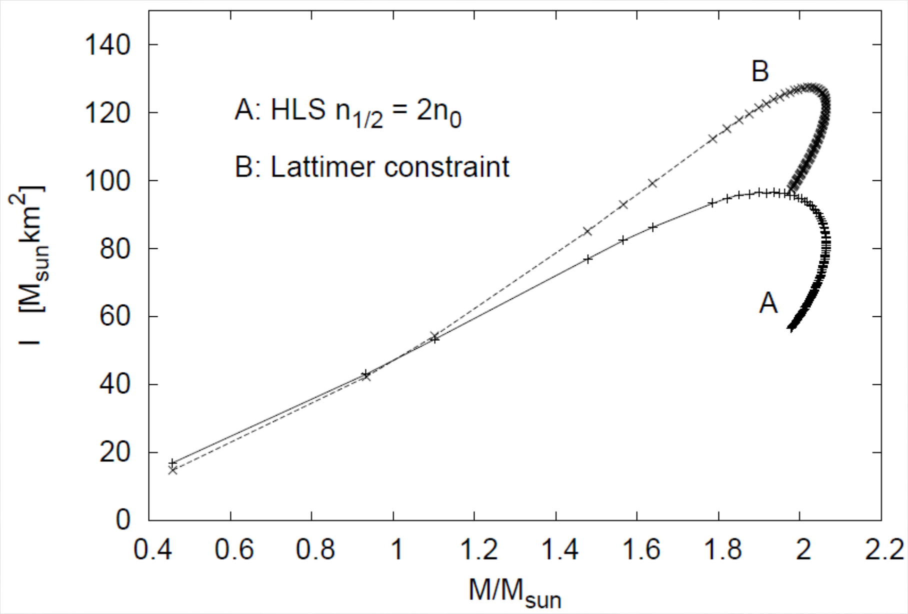

points: HLS $n_{1/2} = 2n_0$
A,B: Li constraints
C: Tsang constraint
A
B
C
E_{sym} [MeV]
180
160
140
120
100
80
60
40
20
0
0.5
1
1.5
2


π,ρ
N
N


$\tilde{V}_T(r)$[MeV]
r[fm]
n=0
n=n_0
n=1.75n_0
n=3.3n_0
n=3.5n_0
50
0
−50
−100
−150
0.0
0.5
1.0
1.5
2.0
2.5
3.0



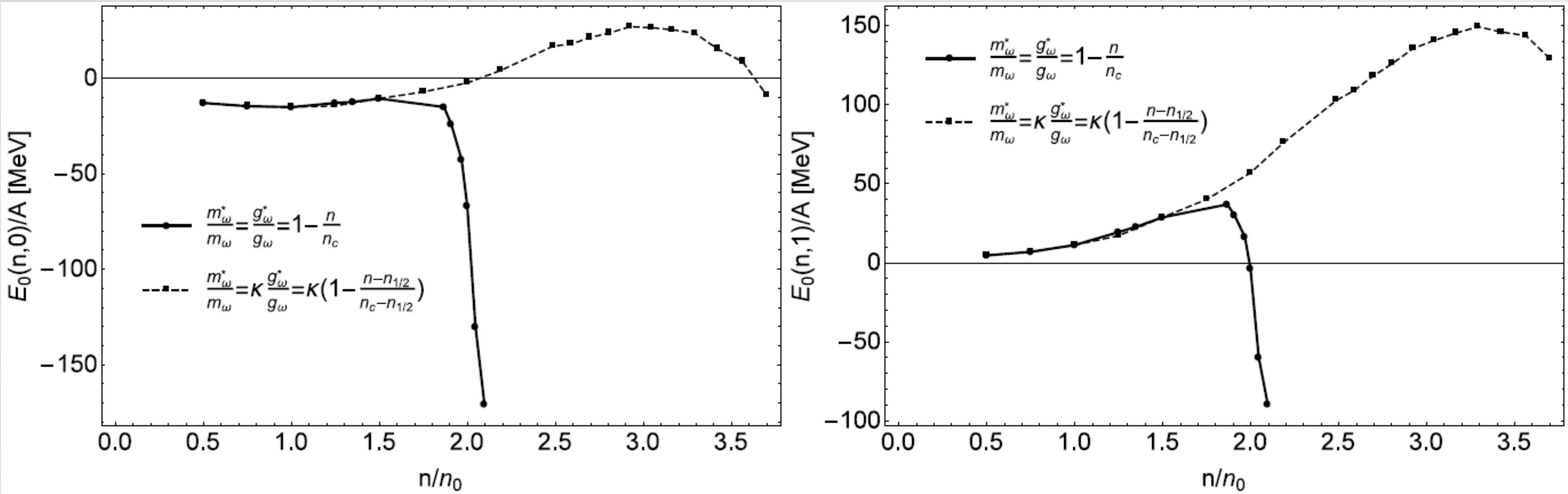




$$I \simeq (0.237 \pm 0.008) M R^2 [1 + 4.2 \frac{M}{M_\odot} \frac{km}{R} + 90 (\frac{M}{M_\odot} \frac{km}{R})^4]$$

J. M. Lattimer and B. F. Schutz, Astrophys. J. **629**, 979 (2005).

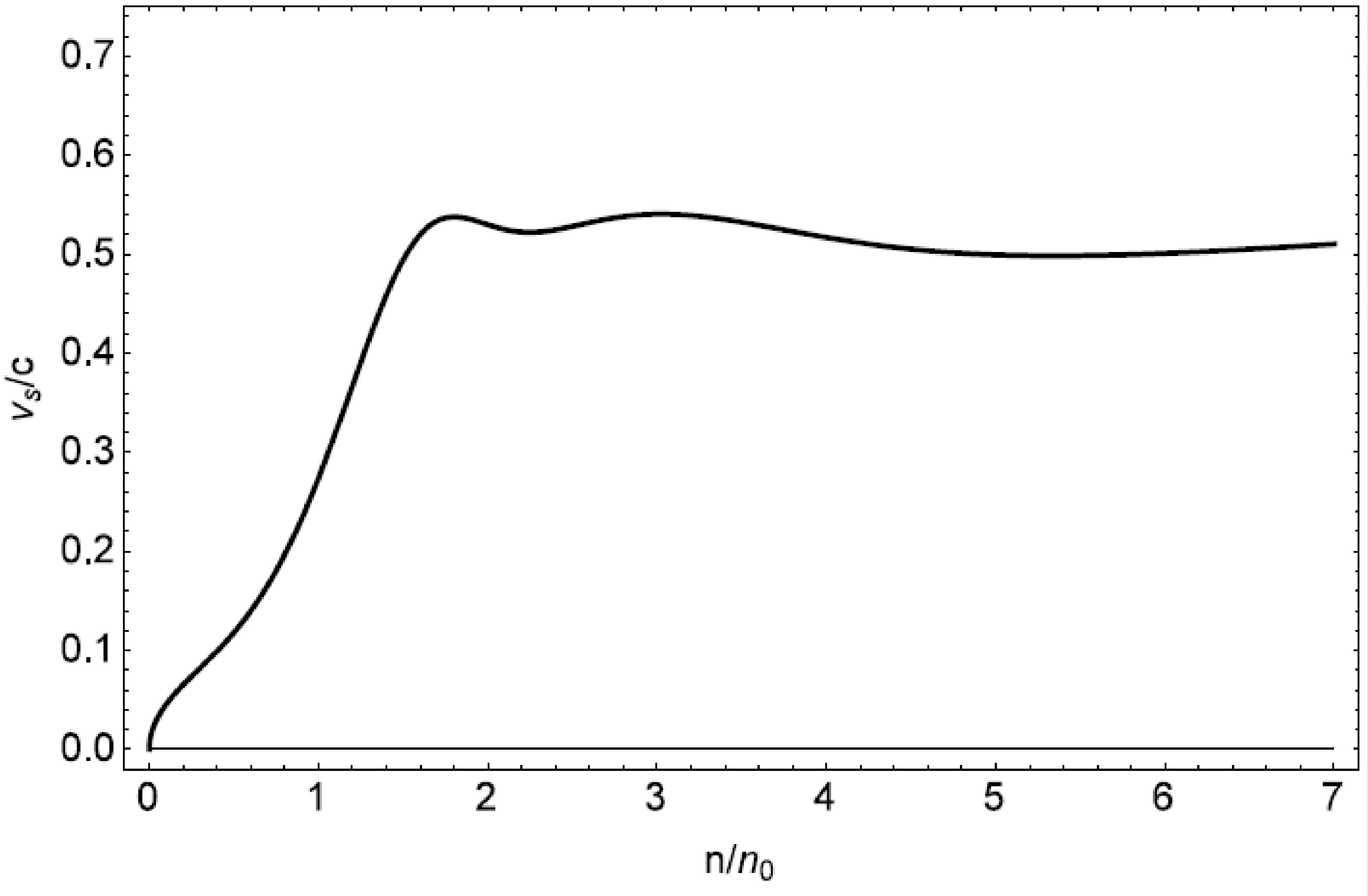
U(2) symmetry for ρ and ω is broken in dense nuclear matter



$$m^*{}_\omega/m_\omega \approx g^*{}_\omega/g_\omega \approx g^*{}_\rho/g_\rho \approx (1-n/n_c)$$

$$m^*{}_\omega/m_\omega \approx \frac{\langle\chi\rangle}{f_{0\sigma}} g^*{}_\omega/g_\omega \approx \frac{\langle\chi\rangle}{f_{0\sigma}} g^*{}_\rho/g_\rho \approx \frac{\langle\chi\rangle}{f_{0\sigma}}(1-n/n_c)$$

Sound velocity



Summary

- We studied the scale-invariant effective model implementing the scalar field(dilaton) with having the explicit scale symmetry breaking term analogous to the trace anomaly of QCD, which predicts the large amount of the nucleon and scalar meson mass stay more or less constant.
- What we found is that the scaling of the parameters predicts the soft EoS for the symmetric nuclear matter but the stiff EoS for the neutron matter. And, the symmetry energy is soft in low density region, but it is stiff in high density region.
- The model says that U(2) symmetry should be broken in dense nuclear matter to explain the two times solar mass neutron star.

Back up

Pion tensor force

$$V_M^T(r) = S_M \frac{f_{NM}^{*2}}{4\pi} \tau_1 \tau_2 S_{12} \mathcal{I}(m_M^* r)$$

$$\mathcal{I}(m_M^* r) \equiv m_M^* \left(\left[\frac{1}{(m_M^* r)^3} + \frac{1}{(m_M^* r)^2} + \frac{1}{3 m_M^* r} \right] e^{-m_M^* r} \right)$$

where $M = \pi, \rho$, $S_{\rho(\pi)} = +1(-1)$ and

$$S_{12} = 3 \frac{(\vec{\sigma}_1 \cdot \vec{r})(\vec{\sigma}_2 \cdot \vec{r})}{r^2} - \vec{\sigma}_1 \cdot \vec{\sigma}_2$$

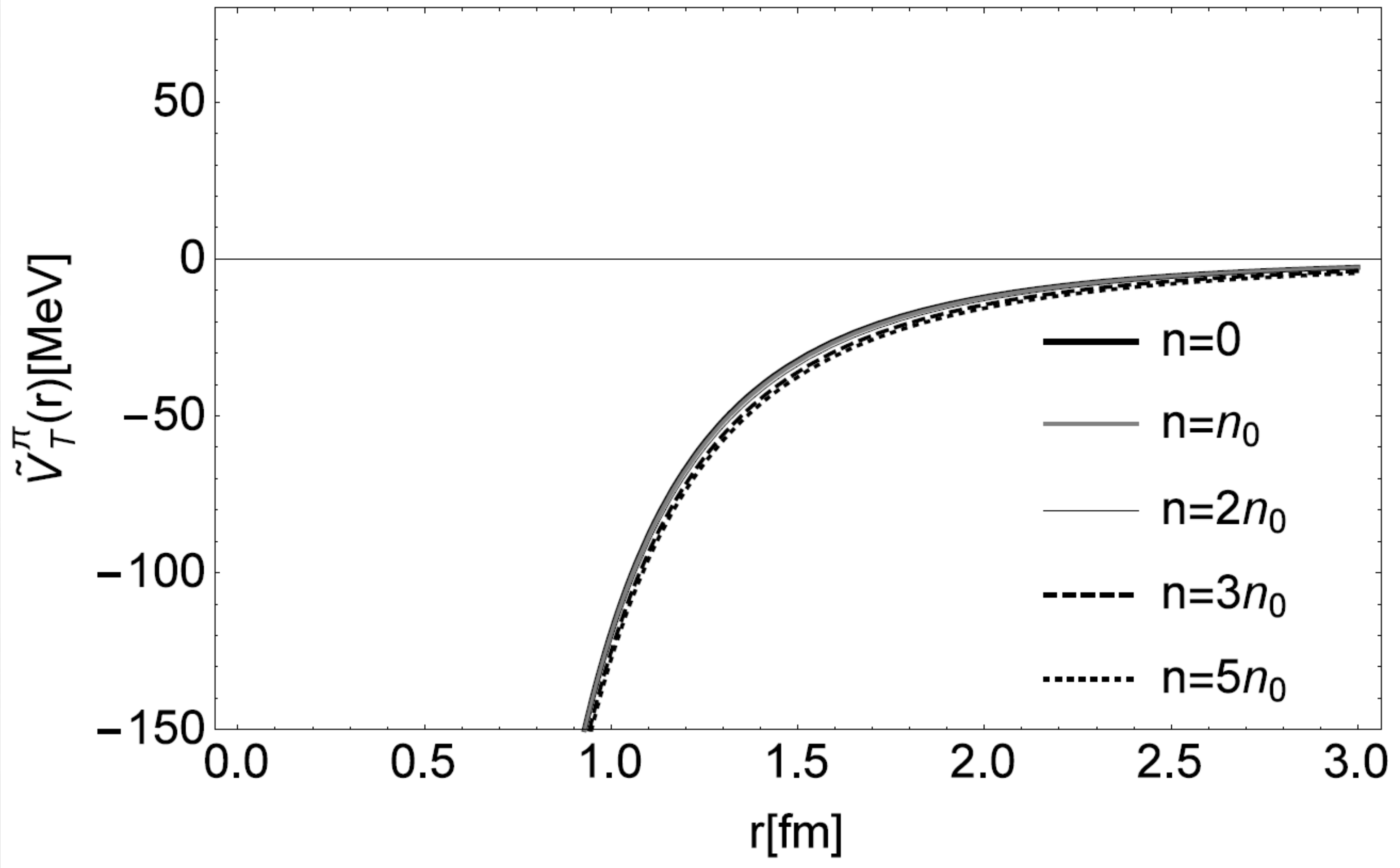
$$R_\pi \approx \frac{g_{\pi NN}^*}{g_{\pi NN}} \frac{m_N}{m_N^*} \frac{m_\pi^*}{m_\pi}$$

$$\approx \begin{cases} \Phi_I \times \Phi_I^{-1} \left(\frac{m_\pi^*}{m_\pi} \right) & \text{for R-I} \\ \kappa \times \kappa^{-1} \left(\frac{m_\pi^*}{m_\pi} \right) & \text{for R-II} \end{cases}$$

$$f_\pi^{*2} m_\pi^{*2} = m_q \langle \bar{q} q \rangle + \sum_{n>1} c_n \langle (\bar{q} q)^n \rangle$$

$$\Rightarrow \quad \kappa^2 m_\pi^{*2} = \sum_{n>1} c_n \langle (\bar{q} q)^n \rangle .$$

$$\frac{m_\pi^*}{m_\pi} = \left(\frac{1}{1+0.13*n/n_0}\right)^{1/2} \frac{1}{1+\exp\left(\frac{n-n_{1/2}}{0.05n_0}\right)} + \left(1-0.15*\frac{n}{n_0}\right)\frac{1}{1+\exp\left(-\frac{n-n_{1/2}}{0.05n_0}\right)}$$



Rho tensor force

$$V_M^T(r) = S_M \frac{f_{NM}^{*2}}{4\pi} \tau_1 \tau_2 S_{12} \mathcal{I}(m_M^* r)$$

$$\mathcal{I}(m_M^* r) \equiv m_M^* \left(\left[\frac{1}{(m_M^* r)^3} + \frac{1}{(m_M^* r)^2} + \frac{1}{3 m_M^* r} \right] e^{-m_M^* r} \right)$$

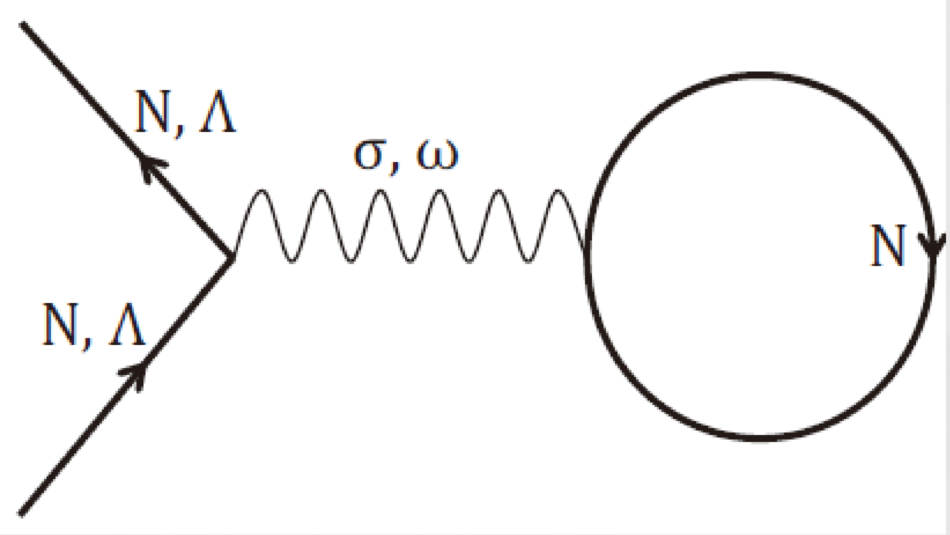
where $M = \pi,\ \rho,\ S_{\rho(\pi)} = +1(-1)$ and

$$S_{12} = 3\frac{(\vec{\sigma}_1 \cdot \vec{r})(\vec{\sigma}_2 \cdot \vec{r})}{r^2} - \vec{\sigma}_1 \cdot \vec{\sigma}_2$$

$$R_\rho \approx \frac{g_{\rho NN}^*}{g_{\rho NN}} \frac{m_N}{m_N^*} \frac{m_\rho^*}{m_\rho} \approx \left(\frac{g^*}{g}\right)^2 \approx \begin{cases} 1 & \text{for R-I} \\ \Phi_{II}^2 & \text{for R-II} \end{cases}$$

$$m_\rho^*/m_\rho \approx \left(\frac{g^*}{g}\right)\left(\frac{f_\pi^*}{f_\pi}\right) \approx \begin{cases} \Phi_I & \text{for R-I} \\ \Phi_{II} \times \kappa & \text{for R-II} \end{cases}$$

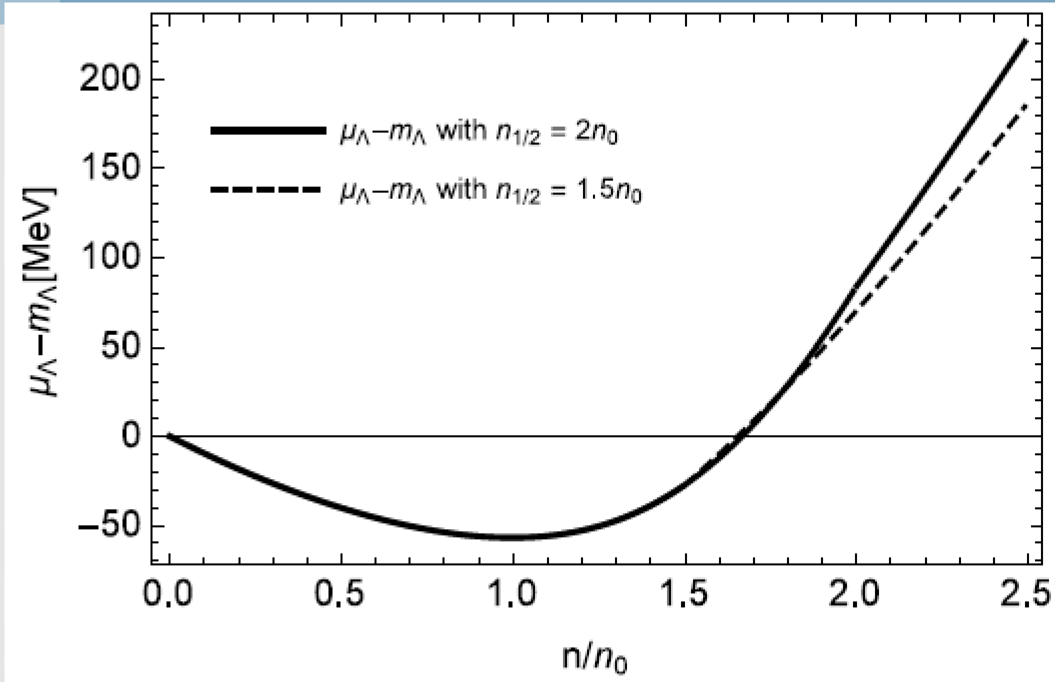
Hyperon in the nuclear matter



$$\mu_\Lambda = m_\Lambda^* - \frac{g_{\sigma\Lambda}^* g_{\sigma N}^*}{m_\sigma^{*\,2}} n_s + \frac{g_{\omega\Lambda}^* g_{\omega N}^*}{m_\omega^{*\,2}} n$$

$$= m_\Lambda^* + \frac{2}{3}\left(-\frac{g_{\sigma N}^{*\,2}}{m_\sigma^{*\,2}} n_s + \frac{g_{\omega N}^{*\,2}}{m_\omega^{*\,2}} n\right)$$

$$g_{\sigma\Lambda} \approx \frac{2}{3} g_{\sigma N} \text{ and } g_{\omega\Lambda} \approx \frac{2}{3} g_{\omega N}$$



	Parameters for Λ Mass Shift
R-I	$\frac{m_\Lambda^*}{m_\Lambda} = \frac{m_N^*}{m_N} = \frac{m_\sigma^*}{m_\sigma} = \frac{m_\omega^*}{m_\omega} = \frac{1}{1+c_I*\frac{n}{n_0}}$ $\frac{g_{\omega\Lambda}^*}{g_{\omega\Lambda}} = \frac{g_{\omega N}^*}{g_{\omega N}} = \frac{g_\omega^*}{g_\omega} = \frac{g_{\sigma\Lambda}^*}{g_{\sigma\Lambda}} = \frac{g_{\sigma N}^*}{g_{\sigma N}} = 1$
R-II	$\frac{m_\Lambda^*}{m_\Lambda} = \frac{m_N^*}{m_N} = \frac{m_\sigma^*}{m_\sigma} = \kappa = \frac{1}{1+c_I*\frac{n_{1/2}}{n_0}}$ $\frac{m_\omega^*}{m_\omega} = \kappa\frac{g_\omega^*}{g_\omega}$ & $\frac{g_{\omega\Lambda}^*}{g_{\omega\Lambda}} = \frac{g_{\omega N}^*}{g_{\omega N}} = \frac{g_\omega^*}{g_\omega}$ $\frac{g_{\sigma\Lambda}^*}{g_{\sigma\Lambda}} = \frac{g_{\sigma N}^*}{g_{\sigma N}} = 1$

Table 4: The "bare" parameter scaling for mean-field estimate of Λ mass shift in dense matter. The only scaling parameter is chosen to be $c_I = 0.13$ as in Section 5. The vacuum scalar (dilaton) mass is taken to be $m_\sigma = 720\,\text{MeV}$ so as to give $\sim$ 600 MeV at nuclear matter density appropriate for RMF approach. We have taken $\frac{3}{2}g_{\omega\Lambda} = g_{\omega N} = 12.5$ and $\frac{3}{2}g_{\sigma\Lambda} = g_{\sigma N} = m_N/f_\pi$. The empirical values $m_N = 939\,\text{MeV}$, $m_\Lambda = 1116\,\text{MeV}$ and $m_\omega = 783\,\text{MeV}$ are taken from the particle data booklet. The scaling $\frac{g_\omega^*}{g_\omega} \approx (1 - 0.053(n - n_{/2})/n_0)$ is taken as the "best fit" from the analysis in Section 5.

The density where $\mu_\Lambda - m_\Lambda$ becomes positive is in the range of BS constraint.[P. F. Bedaque and A. W. Steiner, "Hypernuclei and the hyperon problem in neutron stars," arXiv:1412.8686 [nucl-th].]

The notation of the particles

Coupling	Bosons (Strength of Coupling) $I = 0$ [1]	$I = 1$ $[\tau_1 \cdot \tau_2]$	Central [1]	Spin-Spin $[\sigma_1 \cdot \sigma_2]$	Tensor $[S_{12}]$	Spin-Orbit $[L \cdot S]$
ps	η (weak)	π (strong)	—	weak, coherent with *v*, *t*	strong	—
s	σ (strong)	δ (weak)	strong, attractive	—	—	coherent with *v*
v	ω (strong)	ρ (weak)	strong, repulsive	weak coherent with *ps*	opposite to *ps*	strong, coherent with *s*
t	ω (weak)	ρ (strong)	—	weak, coherent with *ps*	opposite to *ps*	—

I denotes the isospin of a boson. The characteristics quoted refer to $I = 0$ bosons (no isospin dependence). The isovector ($I = 1$) boson contributions, carrying a factor $\tau_1 \cdot \tau_2$, provide the isospin-dependent forces.

The parameters for Bonn A potential

	Potential A		
	m_α (MeV)	$g_\alpha^2/4\pi$	Λ_α(GeV)
π	138.03	14.7	1.3
η	548.8	4	1.5
ρ	769	0.86	1.95
ω	782.6	25	1.35
δ	983	1.3	2.0
σ [b]	550	8.8	2.0
	(710–720)	(17.194)	(2.0)

The Lagrangian in Bonn A potential

Lagrangians for meson-nucleon couplings are

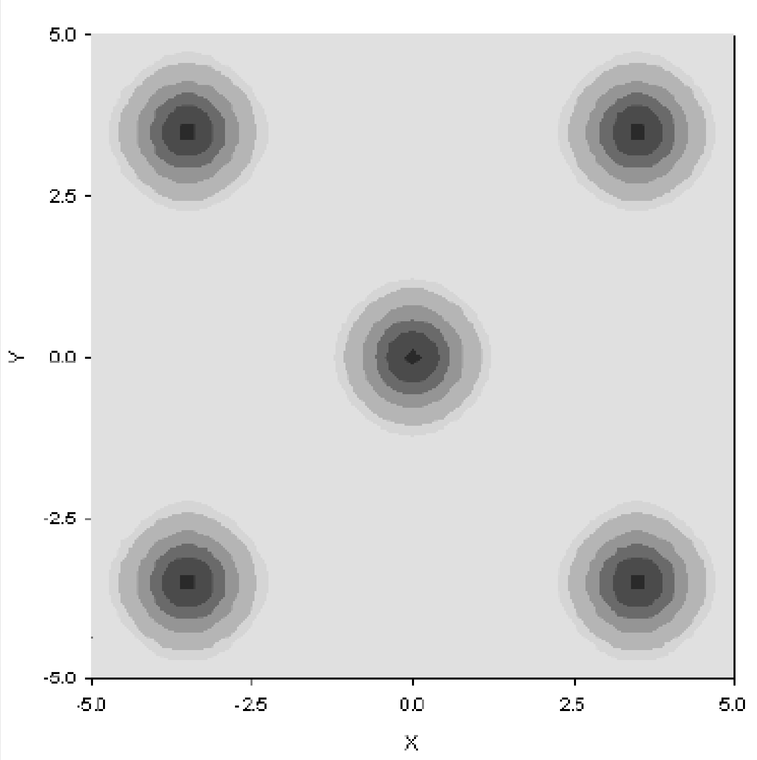
$$\mathcal{L}_{ps} = -g_{ps}\bar{\psi} i\gamma^5\psi\varphi^{(ps)}$$

$$\mathcal{L}_{pv} = -\frac{f_{ps}}{m_{ps}}\bar{\psi}\gamma^5\gamma^\mu\psi\partial_\mu\varphi^{(ps)}$$

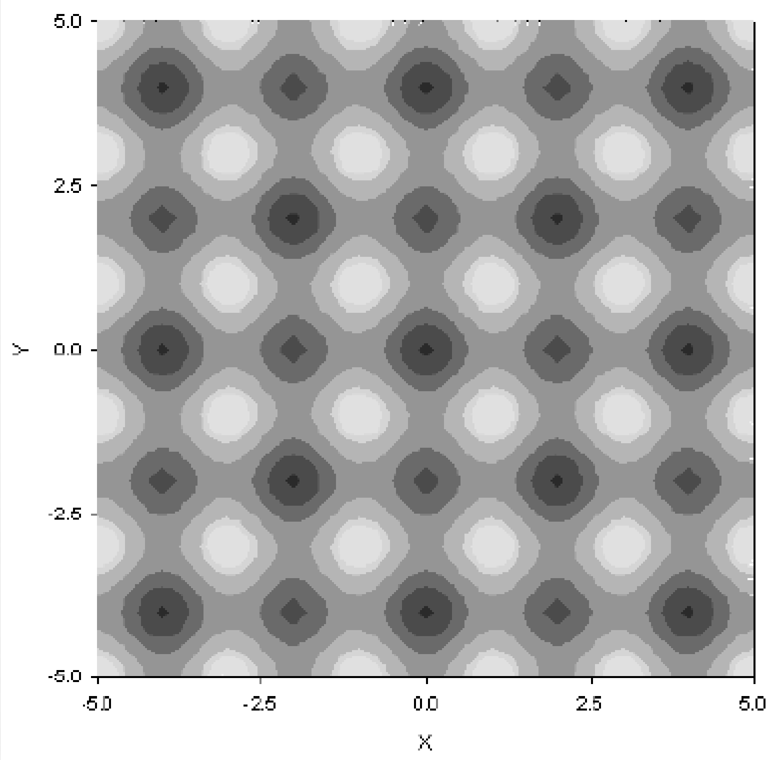
$$\mathcal{L}_s = +g_s\bar{\psi}\psi\varphi^{(s)}$$

$$\mathcal{L}_v = -g_v\bar{\psi}\gamma^\mu\psi\varphi_\mu^{(v)} - \frac{f_v}{4M}\bar{\psi}\sigma^{\mu\nu}\psi(\partial_\mu\varphi_\nu^{(v)} - \partial_\nu\varphi_\mu^{(v)})$$

H. K. Lee, B.-Y. Park and M. Rho, Phys. Rev. C **83**, 025206 (2011)
[Erratum-ibid. C 84, 059902 (2011)]



$$n = n_{1/2}$$



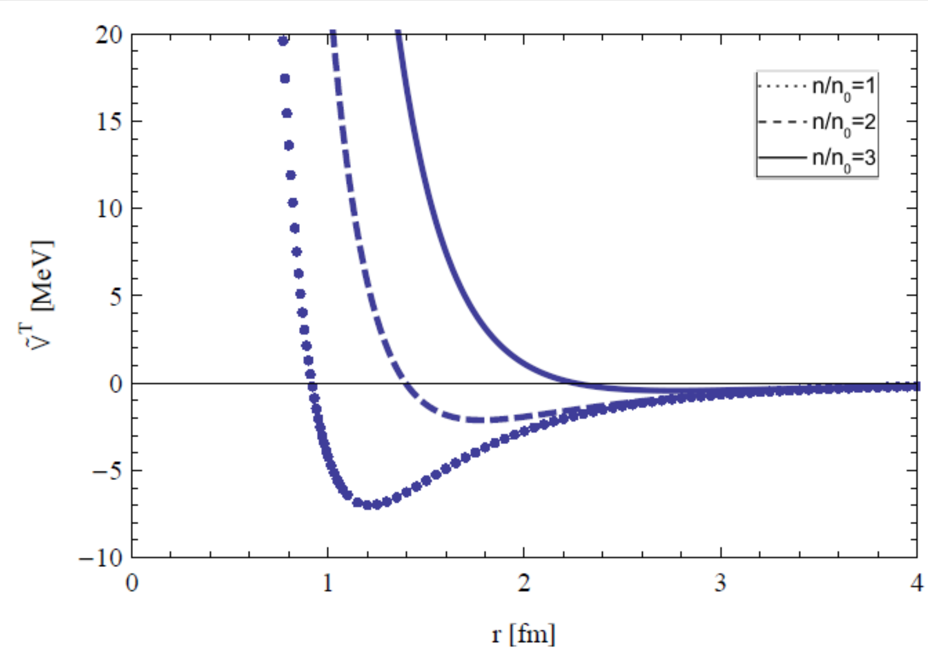
$$\frac{f_\pi^*}{f_\pi} \approx 1 - D_I\left(\frac{n}{n_0}\right)$$

$$n < n_{1/2}$$

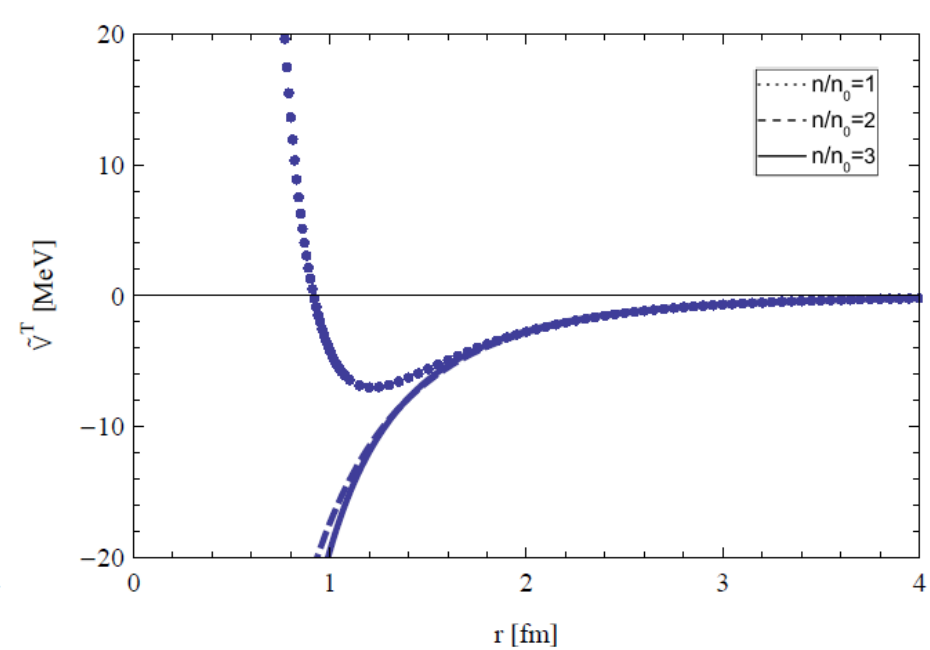
$$\frac{f_\pi^*}{f_\pi} \approx c \neq 0$$

$$n_{1/2} < n$$

Without topology change



With topology change



, where $D_I = D_{II} = 0.15$ was considered. So, we expect that the tensor force by exchanging ρ meson will be suppressed and only pion tensor force will remain.

$V^T(r) = V^T_\rho(r) + V^T_\pi(r)$

$$V^T_M(r) = S_M \frac{f^2_{NM}}{4\pi} m_M \tau_1 \cdot \tau_2 S_{12} \left(\left[\frac{1}{(m_M r)^3} + \frac{1}{(m_M r)^2} + \frac{1}{3 m_M r} \right] e^{-m_M r} \right)$$

where $M = \pi, \rho,\ S_{\rho(\pi)} = +1(-1)$.

$$R \equiv \frac{f^*_{N\rho}}{f_{N\rho}} \approx \frac{g^*_{\rho NN}}{g_{\rho NN}} \frac{m^*_\rho}{m_\rho} \frac{m_N}{m^*_N}.$$

Hatsuda and Kunihiro yielded the in-medium Gell-Mann-Oakes-Renner relation,

$$m^*_\pi(n)/m_\pi \approx (f^t_\pi(n)/f_\pi)^{-1} (\langle\bar{q}q\rangle^*(n)/\langle\bar{q}q\rangle)^{1/2}$$

Using the experimental information available at the nuclear matter density, $(f^t_\pi(n_0)/f_\pi)^2 \simeq 0.64$ and $\langle\bar{q}q\rangle(n_0)/\langle\bar{q}q\rangle \simeq 0.63$, we get $m_\pi{}^*/m_\pi \simeq 1$.

Tensor Force and Symmetry Energy

$$E(n,\delta) = E_0(n) + E_{sym}(n)\delta^2 + \cdots$$

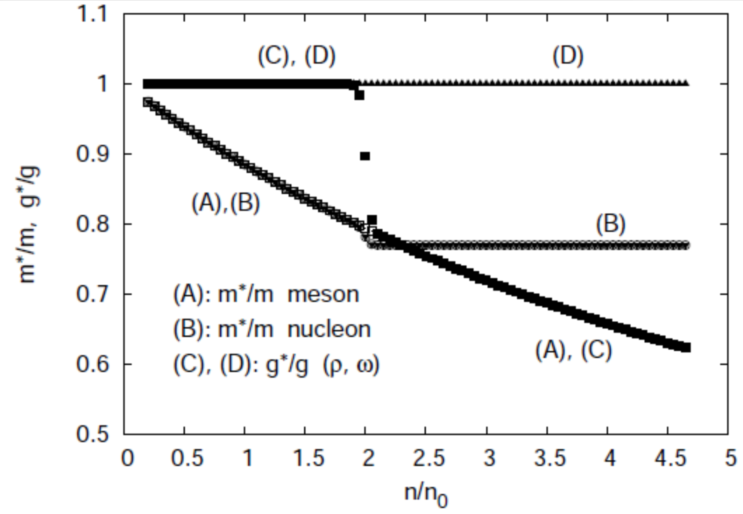
$$\delta = (N - Z)/(N + Z)$$

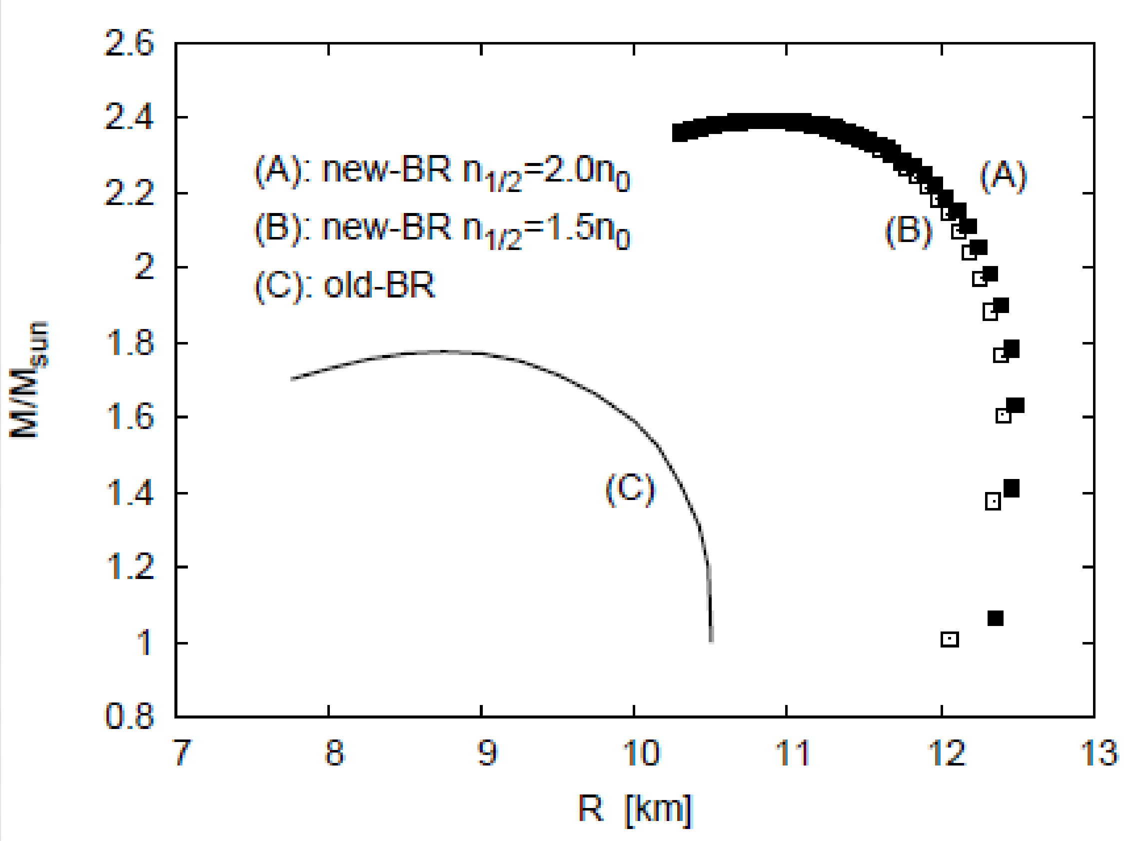
where E is the energy per baryon of the system,

$$E_{sym} \sim \langle V_{sym} \rangle \approx \frac{12}{\bar{E}} \langle V_T^2(r) \rangle$$

where $\bar{E} \approx 200$ MeV is the average energy typical of the tensor force excitation and V_T is the radial part of the net tensor force.

G. E. Brown and R. Machleidt, Phys. Rev. C 50, 1731 (1994)

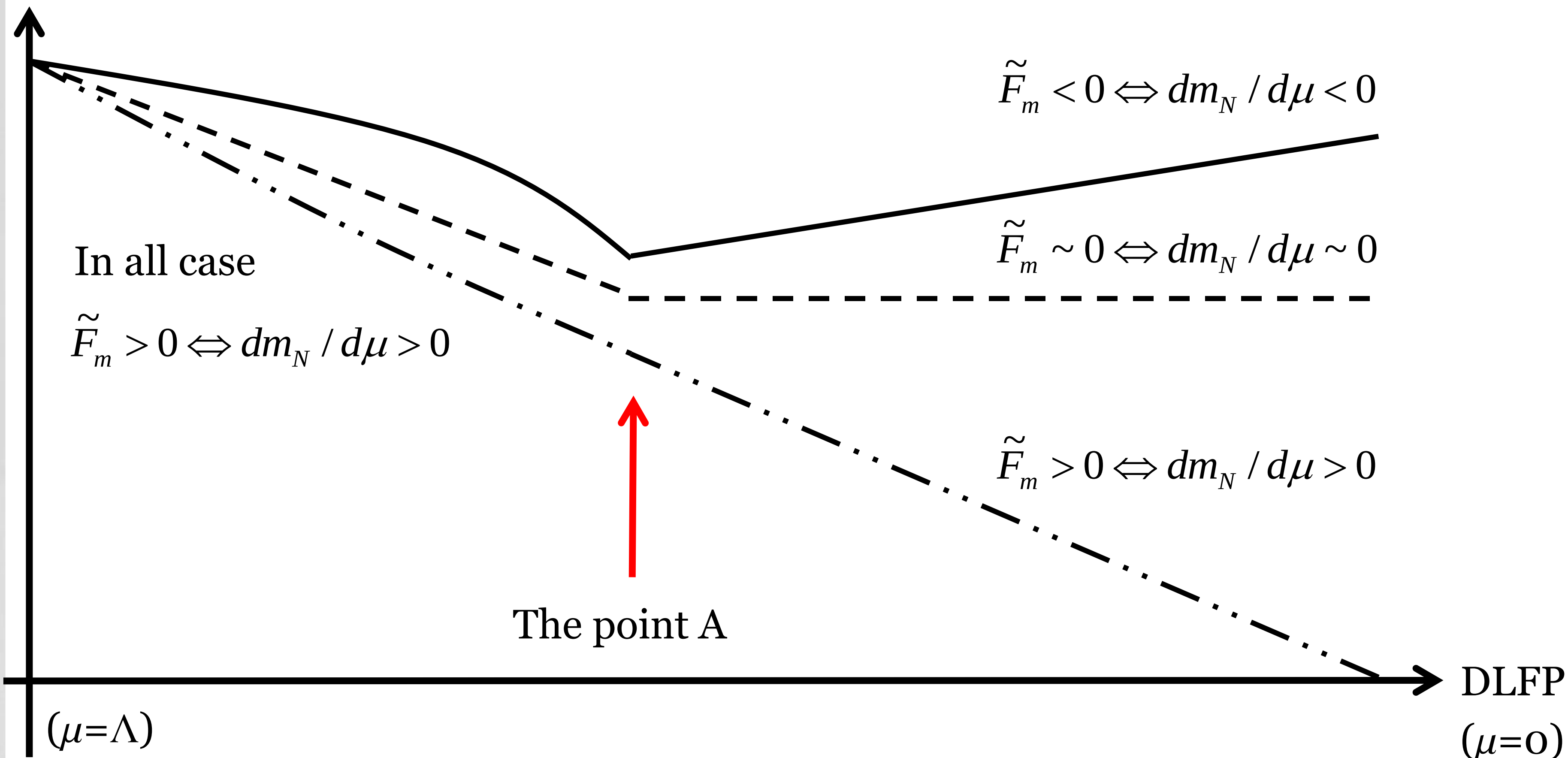




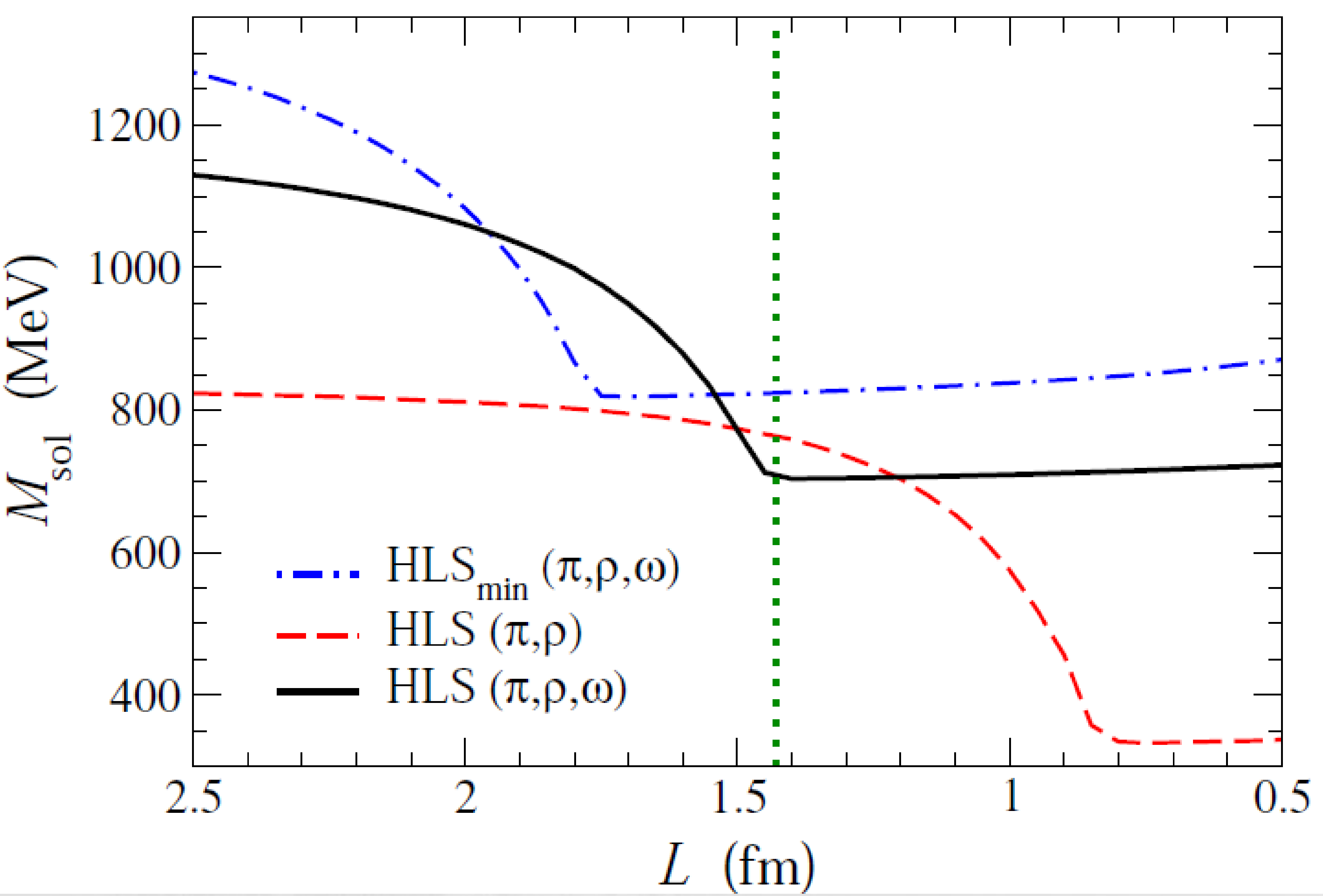
H. Dong, T. T. S. Kuo, H. K. Lee, R. Machleidt and M. Rho, Phys. Rev. C 87, no. 5, 054332 (2013)

It shows the interplay between the nucleon mass and the ω-NN coupling

$$\mu \frac{d m_N}{d\mu} = \frac{3 m_N}{16\pi^2} \tilde{\mathcal{F}}_m \,,$$



$$\tilde{\mathcal{F}}_m = \frac{g_A^2}{F_\pi^2}\left(\mu^2 - m_N^2\right) - \frac{3}{2}\left(1 - g_{V\rho}\right)^2 g_\rho^2 - \frac{1}{2}\left(g_{V\omega} - 1\right)^2 g_\omega^2$$



which is supported by the skyrmion calculation in a crystal
[Y.-L. Ma, M. Harada, H. K. Lee, Y. Oh, B.-Y. Park and M. Rho, Phys. Rev. D 88, 014016 (2013)]

$$
\begin{aligned}
\mathcal{L}_{bs\mathrm{HLS}}(\pi,\chi,V_\mu,N) &= (\frac{\chi}{f_{0\sigma}})^2\left(f_{0\pi}^2\mathrm{tr}\left[\hat{\alpha}_{\perp\mu}\hat{\alpha}_{\perp}^{\mu}\right] + f_{0\sigma_\rho}^2\mathrm{tr}\left[\hat{\alpha}_{\parallel\mu}\hat{\alpha}_{\parallel}^{\mu}\right] + f_0^2\mathrm{tr}\left[\hat{\alpha}_{\parallel\mu}\right]\mathrm{tr}\left[\hat{\alpha}_{\parallel}^{\mu}\right]\right) \\
&+ \frac{1}{2}\partial_\mu\chi\partial^\mu\chi + V(\chi) + \frac{f_{0\pi}^2}{4}(\frac{\chi}{f_{0\sigma}})^3\mathrm{Tr}(MU^\dagger + h.c.) + \bar{N}i\not{D}N - m_N\frac{\chi}{f_{0\sigma}}\bar{N}N \\
&+ g_A\bar{N}\gamma^\mu\gamma_5\hat{\alpha}_{\perp\mu}N + g_{\rho N}\bar{N}\gamma^\mu\hat{\alpha}_{\parallel\mu}N + g_{V0}\bar{N}\gamma^\mu\mathrm{tr}\left[\hat{\alpha}_{\parallel\mu}\right]N + \cdots
\end{aligned}
$$

with $D_\mu N = \left(\partial_\mu - ig_\rho\frac{\vec{\rho}_\mu\cdot\vec{\tau}}{2} - ig_\omega\frac{\omega_\mu}{2}\right)$, $g_{V0} = \frac{1}{2}(g_{\omega N} - g_{\rho N})$ and $f_0^2 = \frac{1}{2}\left(f_{0\sigma_\omega}^2 - f_{0\sigma_\rho}^2\right)$, where M is the spurion field with $\langle M\rangle = 2B^0\begin{pmatrix} m_u & 0 \\ 0 & m_d\end{pmatrix}$.

Under HLS transformation,

$$
\begin{aligned}
\alpha^\mu_{\perp,\parallel} &\to u(x)h(x)\alpha^\mu_{\perp,\parallel}u(x)^\dagger h(x)^\dagger\,, \\
N &\to u(x)h(x)N \quad \& \quad \chi\to\chi
\end{aligned}
$$

and

$$
M\to g_L M g_R^\dagger \quad \& \quad U\to g_L U g_R^\dagger\,,
$$

where $g_{L,R}\in[SU(2)_L\times SU(2)_R]_{global}$, $h(x)\in[SU(2)_V]_{local}$ and $u(x)\in[U(1)_V]_{local}$. Under scale transformation,

$$
\begin{aligned}
\partial,\,N,\,\alpha_{\perp,\parallel},\,M,\,\chi &\to \lambda\left(\partial,\,N,\,\alpha_{\perp,\parallel},\,M,\,\chi\right)\,, \\
U &\to U\,.
\end{aligned}
$$

C14 dating probes scaling

J.W. Holt et al, PRL **100**, 062501 (08)