

Nuclear Symmetry Energy in QCD degree of freedom

Phys. Rev. C87 (2013) 015204 ([arXiv:1209.0080](https://arxiv.org/abs/1209.0080))

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Some preliminary results

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Motivation and Outline

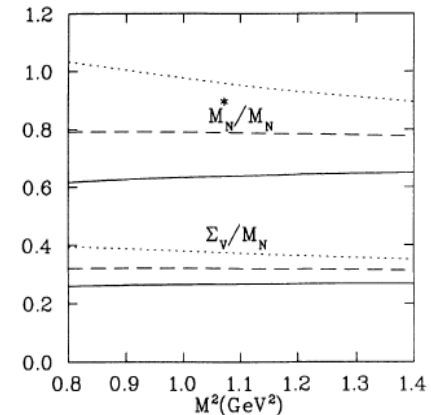
- Nuclear phenomenology – QCD sum rule

$$U \simeq S + V\gamma^0$$

$$G(q) = \frac{1}{\not{q} - M_n - \Sigma(q)} \rightarrow \lambda^2 \frac{\not{q} + \boxed{M^*} - \boxed{\psi\Sigma_v}}{(q_0 - E_q)(q_0 - \bar{E}_0)}$$

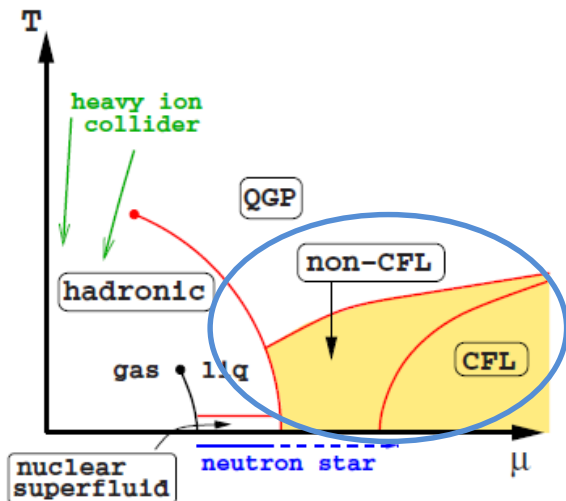
$M^* < M_n$

$\Sigma_v > 0$



Physical Review C49, 464 (1993)
(Thomas Cohen et al.)

- Extremely high density matter?
– QCD itself is main dynamics



Cold matter Symmetry Energy from

$$\mathcal{Z}_\Omega = \text{Tr} \exp \left[-\beta(\hat{H} - \vec{\mu} \cdot \vec{N}) \right]$$

$$= \int [D(\text{fields})] \exp \left[-\int_0^\beta d\tau \int x^3 \mathcal{L}_E(\text{fields}) \right]$$

Hard Dense Loop resummation
Color BCS pairing

Nuclear Symmetry Energy

- From equation of state

Bethe-Weisaker formula

$$m_{tot} = Nm_n + Zm_p - E_B/c^2$$

$$E_B = a_V A - a_S A^{2/3} - a_C (Z(Z-1))A^{-1/3} - a_A I^2 A + \delta(A, Z)$$

$$I = (N - Z)/A$$

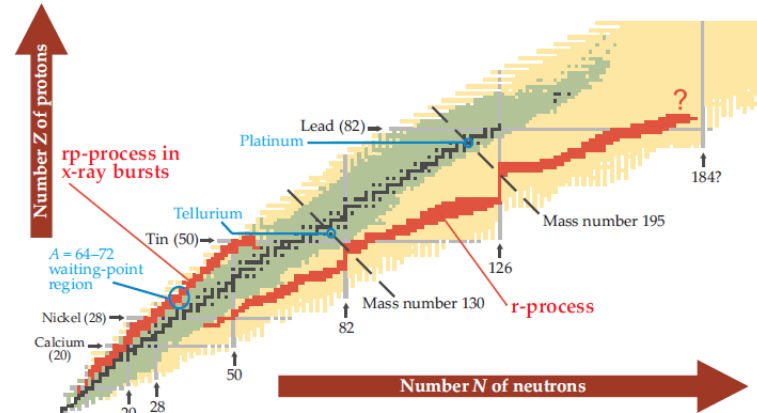
In continuous matter

$$\bar{E}(\rho_N, I) = \bar{E}(\rho_N) + \bar{E}_{sym}(\rho_N) I^2 + \dots$$

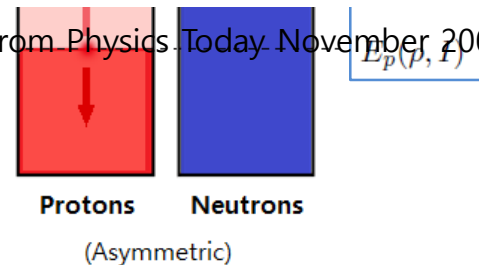
$$I = (\rho_n - \rho_p)/\rho$$

$$\bar{E} = \frac{1}{\int d^3k_n d^3k_p} \int d^3k_n d^3k_p E(\rho_n, \rho_p)$$

$$\Rightarrow E_{sym} = \frac{1}{2I} \cdot (\bar{E}_n - \bar{E}_p) \quad (\text{Up to linear density order})$$



(Quoted from Physics Today November 2008)



- RMFT propagator

$$G(q) = -i \int d^4x e^{iqx} \langle \Psi_0 | T[\psi(x) \bar{\psi}(0)] | \Psi_0 \rangle = \frac{1}{\not{q} - M_n - \Sigma(q)} \rightarrow \lambda^2 \frac{\not{q} + M^* - \not{q} \Sigma_v}{(q_0 - E_q)(q_0 - \bar{E}_0)}$$

QCD Sum Rule

- Correlation function

$$\begin{aligned}\Pi(q) &\equiv i \int d^4x e^{iqx} \langle \Psi_0 | T[\eta(x) \bar{\eta}(0)] | \Psi_0 \rangle \\ &= \Pi_s(q^2, q \cdot u) + \Pi_q(q^2, q \cdot u) \not{q} + \Pi_u(q^2, q \cdot u) \not{u}\end{aligned}$$

$\eta(x) = \epsilon_{abc} [u_a^T(x) C \gamma_\mu u_b(x)] \gamma_5 \gamma^\mu d_c(x)$
Ioffe's interpolating field for proton

- Energy dispersion relation and OPE

$$\Pi_i(q_0, |\vec{q}|) = \frac{1}{2\pi i} \int_{-\infty}^{\infty} d\omega \frac{2\text{Im}\Pi_i(\omega, |\vec{q}|)}{\omega - q_0} + \text{polynomials}$$

Contains **all possible hadronic resonance states** in **QCD degree of freedom**

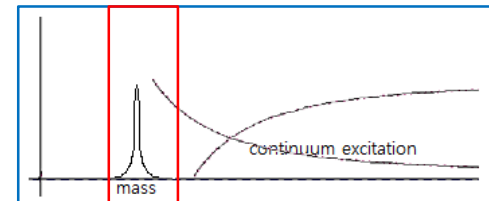
- Phenomenological ansatz in **hadronic degree of freedom**

$$\Pi(q_0, |\vec{q}|) \sim \frac{1}{(q^\mu - \tilde{\Sigma}_v^\mu) \gamma_\mu - M_N^*}$$

Equating both sides, hadronic quantum number can be expressed in **QCD degree of freedom**

- Weighting - Borel transformation

$$\mathcal{B}[\Pi_i(q_0, |\vec{q}|)] \equiv \lim_{\substack{-q_0^2, n \rightarrow \infty \\ -q_0^2/n = M^2}} \frac{(-q_0^2)^{n+1}}{n!} \left(\frac{\partial}{\partial q_0^2} \right)^n \Pi_i(q_0, |\vec{q}|)$$



QCD Sum Rule

- Operator Product Expansion

$$\Pi_i(q_0, |\vec{q}|) = \frac{1}{2\pi i} \int_{-\infty}^{\infty} d\omega \frac{2\text{Im}\Pi_i(\omega, |\vec{q}|)}{\omega - q_0} + \text{polynomials} = \sum_n C_n^i(q^2, q_0^2) \langle \hat{O}_n \rangle_{\rho, I}$$

Non-perturbative contribution

- In-medium condensate near normal nuclear density

$$\langle \hat{O}_{u,d} \rangle_{\rho, I} = \langle \hat{O}_{u,d} \rangle_{\text{vac}} + \langle \langle p | \hat{O}_0 | p \rangle \mp \langle p | \hat{O}_1 | p \rangle I \rangle \rho$$

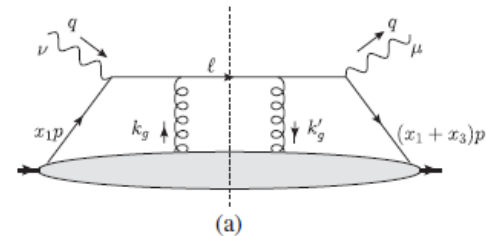
Medium property can be accounted by nucleon expectation value x density

- Multi-quark operators (Twist-4)

$$\langle (\bar{q}_1 \gamma^\alpha \gamma_5 t^A q_1) (\bar{q}_2 \gamma^\beta \gamma_5 t^A q_2) \rangle_p$$

$$\langle (\bar{q}_1 \gamma^\alpha t^A q_1) (\bar{q}_2 \gamma^\beta t^A q_2) \rangle_p$$

can be estimated from DIS experiments data

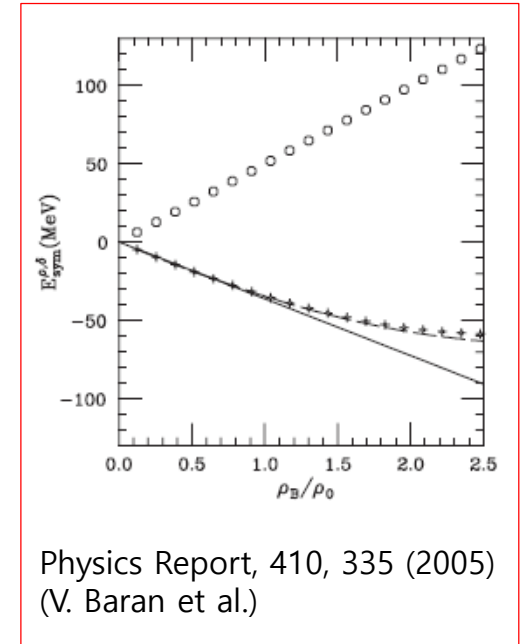
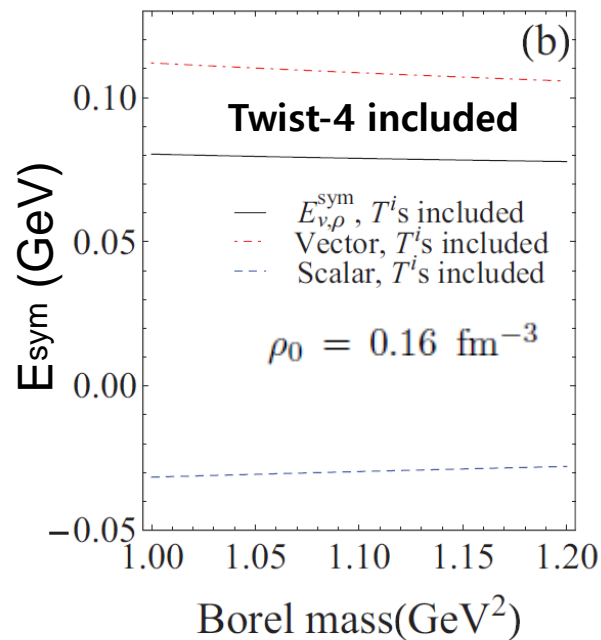
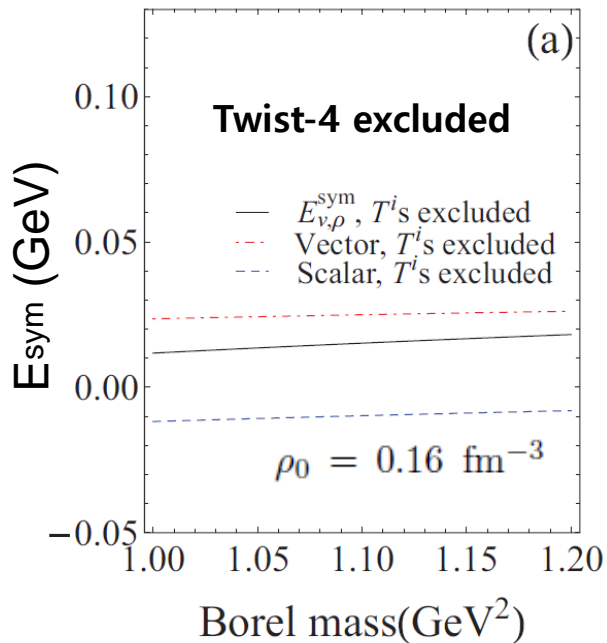


Nuclear Symmetry Energy from QCD SR

- **Iso-vector** scalar / **vector** decomposition

$$\langle \bar{d}d \rangle - \langle \bar{u}u \rangle \quad \langle d^\dagger d \rangle - \langle u^\dagger u \rangle$$

$$\langle (\bar{d}\Gamma^\mu d)(\bar{d}\Gamma^\nu d) \rangle - \langle (\bar{u}\Gamma^\mu u)(\bar{u}\Gamma^\nu u) \rangle$$



- Comparison with RMFT result

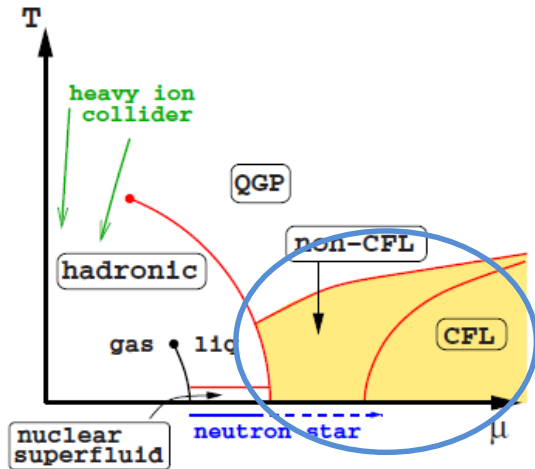
$$E_V^{\text{sym}} = \frac{1}{2} \left[f_\rho - f_\delta \left(\frac{m^*}{E_F^*} \right) \right] \rho_B$$

Iso-vector meson exchange ←

High density behavior -> high density dependence of condensates

At extremely high density?

- QCD phase transition



- In $1/\mu \ll 1/\Lambda_{\text{QCD}}$ region, **QCD** can be immediately applicable
- Statistical partition function for dense QCD

$$\mathcal{Z}_\Omega = \text{Tr} \exp \left[-\beta(\hat{H} - \vec{\mu} \cdot \vec{N}) \right]$$

$$= \int [D(\text{fields})] \exp \left[-\int_0^\beta d\tau \int x^3 \mathcal{L}_E(\text{fields}) \right]$$
- Normal QM phase - BCS paired phase

- Euclidean Lagrangian for dense QCD **at normal phase**

$$\mathcal{L}_E = \frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + \frac{1}{2\xi} (\partial_\mu A_\mu^a)^2 + \bar{\eta}^a (\partial^2 \delta_{ab} + g f_{abc} \partial_\mu A_\mu^c) \eta^b$$

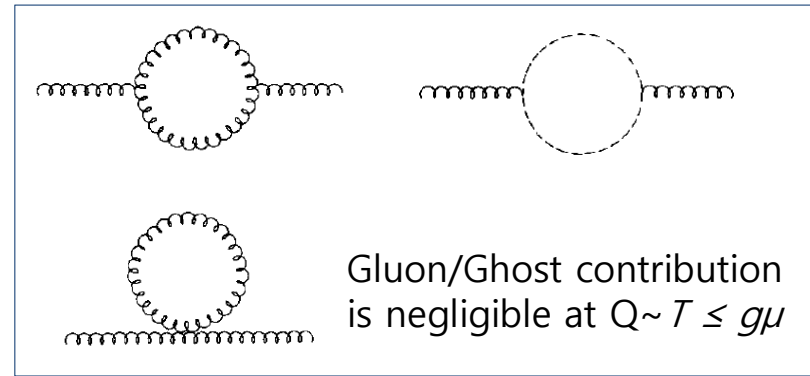
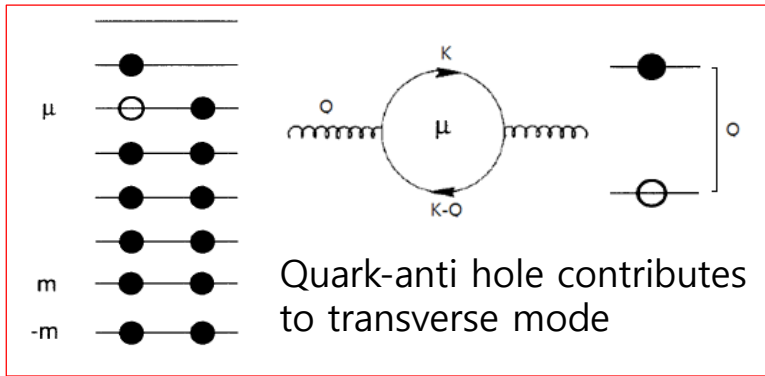
$$+ \sum_f^{n_f} \left[\psi_f^\dagger \partial_\tau \psi_f + \bar{\psi}_f (-i\gamma^i \partial_i + m_f) \psi_f - \mu_f \psi_f^\dagger \psi_f - g \bar{\psi}_f \mathbf{A} \psi_f \right]$$

$$\int \frac{d^4 Q}{(2\pi)^4} \equiv T \sum_n \int \frac{d^3 q}{(2\pi)^3}, \quad Q_\mu = (-\omega, \vec{q}) \quad \begin{array}{l} \omega_n = (2n+1)\pi/\beta \quad (\text{For fermion}) \\ \omega_n = 2n\pi/\beta \quad (\text{For boson}) \end{array}$$

Continuous energy integration -> Discrete sum over Matsubara frequency

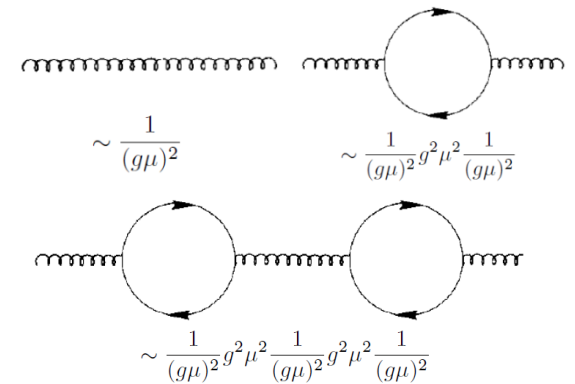
Hard Dense Loop resummation

- Quark-hole excitation is dominant ($Q \sim T \leq g\mu$)



- Gluon self energy in cold matter ($Q \sim T \leq g\mu$)

$$\begin{aligned} \Pi_{\mu\nu}(Q) &= g^2 \text{Tr} [\gamma_\mu S_F(K) \gamma_\nu S_F(K - Q)] \\ &= \frac{1}{2} g^2 \left(\sum_f \frac{\mu_f^2}{\pi^2} \right) \int \frac{d\Omega}{4\pi} \left(\delta_{\mu\nu} + \hat{K}_\mu \hat{K}_\nu \frac{i\omega}{Q \cdot \hat{K}} \right) \\ &\sim g^2 \mu^2 \text{ order} \end{aligned}$$



Phys. Rev. D.53.5866 (1996) C. Manuel

Phys. Rev. D.48.1390 (1993) J. P. Blaizot and J. Y. Ollitrault

All equivalent 1PI diagrams should be resummed!

Hard Dense Loop resummation

- Projection along polarization

Euclidean propagator

$$*D_{\mu\nu} = \frac{1}{Q^2 + \delta\Pi^L} P_{\mu\nu}^L + \frac{1}{Q^2 + \delta\Pi^T} P_{\mu\nu}^T + \frac{1}{f_e} \frac{Q_\mu Q_\nu}{Q^2}$$

$$P_{ij}^T = \delta_{ij} - \hat{q}_i \hat{q}_j, P_{44}^T = P_{4i}^T = 0$$

$$P_{\mu\nu}^L = \delta_{\mu\nu} - \frac{Q_\mu Q_\nu}{Q^2} - P_{\mu\nu}^T$$

Longitudinal and transverse part

$$\delta\Pi^L(Q) = 2 \sum_f \left(\frac{1}{2} g^2 \frac{\mu_f^2}{\pi^2} \right) \frac{Q^2}{q^2} \left(1 - \left(\frac{i\omega}{q} \right) Q_0 \left(\frac{i\omega}{q} \right) \right) \Rightarrow 2m_g^2 = g^2 \frac{\mu_f^2}{\pi^2} \quad (\text{In } \omega \rightarrow 0 \text{ limit})$$

$$\delta\Pi^T(Q) = \sum_f \left(\frac{1}{2} g^2 \frac{\mu_f^2}{\pi^2} \right) \left(\frac{i\omega}{q} \right) \left[\left(1 - \left(\frac{i\omega}{q} \right)^2 \right) Q_0 \left(\frac{i\omega}{q} \right) + \left(\frac{i\omega}{q} \right) \right] \Rightarrow 0$$

- Debye mass and effective Lagrangian

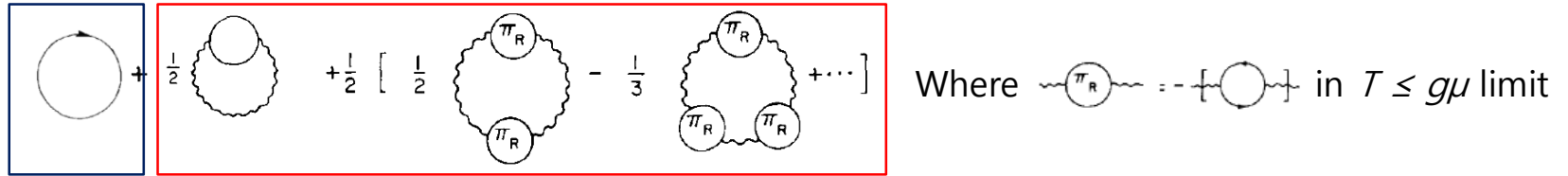
Effective Lagrangian for soft gluon in cold dense matter

$$\mathcal{L} = -\frac{1}{4} F^2 \rightarrow \frac{1}{2} A_\mu (-Q^2 g^{\mu\nu} + M_g^2 P_L^{\mu\nu} + O(\omega/q) P_T^{\mu\nu} + \dots) A_\nu$$

Debye mass from HDL

HDL resummed thermodynamic potential

- Relevant ring diagrams



$$\Omega(\mu) = - \left[\frac{1}{4} \sum_f \frac{\mu_f^4}{\pi^2} \right] + \alpha_s^2 \frac{2}{\pi} \left(\sum_f \frac{\mu_f^2}{\pi^2} \right)^2 \left[\Lambda_1 - \Lambda_2 - \alpha \ln 2 - \ln \left(\sum_f \frac{\mu_f^2}{\pi^2} \frac{1}{\bar{\mu}^2} \right) \Lambda_1 \right] - \alpha_s^2 \ln \alpha_s \frac{2}{\pi} \left(\sum_f \frac{\mu_f^2}{\pi^2} \right)^2 \Lambda_1$$

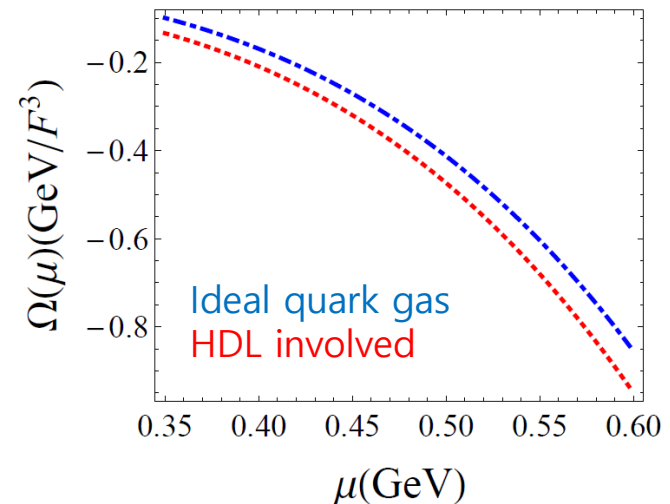
- Thermodynamic quantities can be obtained from $\Omega(\mu)$

$$\Omega(\mu) = \langle \hat{H} \rangle - \vec{\mu} \cdot \langle \vec{\hat{N}} \rangle = -\frac{1}{\beta V} \ln \mathcal{Z}_\Omega,$$

$$\rho_i(\mu) = \frac{\langle \hat{N}_i \rangle}{V} = \frac{1}{\beta V} \frac{\partial}{\partial \mu_i} \ln \mathcal{Z}_\Omega,$$

$$\epsilon(\mu) = \frac{\langle \hat{H} \rangle}{V} = -\frac{1}{V} \left(\frac{\partial}{\partial \beta} - \frac{1}{\beta} \vec{\mu} \cdot \frac{\partial}{\partial \vec{\mu}} \right) \ln \mathcal{Z}_\Omega,$$

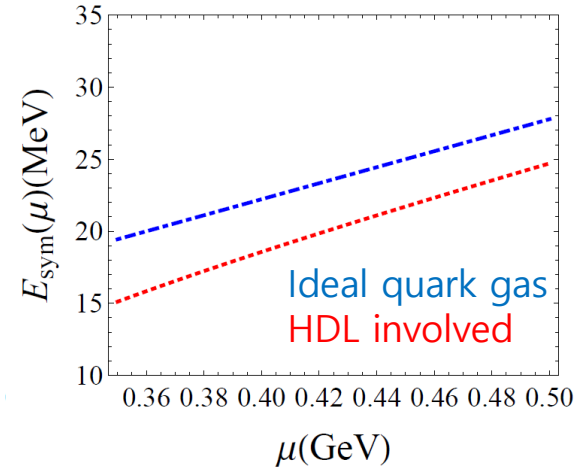
$$I_B = \frac{\rho_3}{\rho_B} = 3 \frac{\rho_u - \rho_d}{\rho_u + \rho_d} \quad \mu_d^u = \mu \left(1 \pm \frac{1}{3} I_B \right)^{\frac{1}{3}}$$



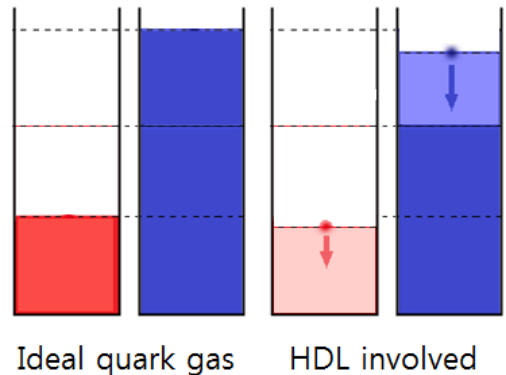
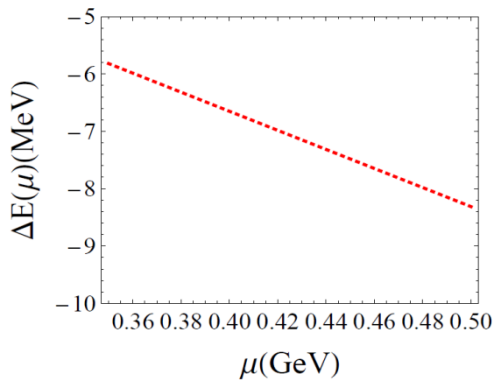
Symmetry Energy at normal phase

- Symmetry Energy**

$$\begin{aligned} \frac{\epsilon(\mu, I_B)}{\rho(\mu, I_B)} &= \frac{E(\mu, I_B)}{N_B} \\ &= \bar{E}(\mu, I_B) = \bar{E}(\mu) + \bar{E}_{sym}(\mu) I_B^2 + \dots \\ \bar{E}_{sym}(\mu) &= \frac{1}{2!} \frac{\partial^2}{\partial I_B^2} \bar{E}(\mu, I_B) \\ &= \boxed{\tilde{E}_{sym}^{q,0}(\mu)} + \boxed{\tilde{E}_{sym}^{g,HDL}(\mu)} \end{aligned}$$



- HDL correction suppress Quasi-Fermi sea



As density becomes higher,
suppression becomes stronger

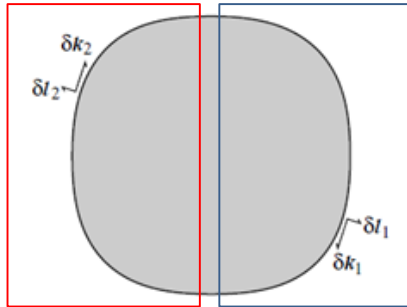
The difference between quasi-Fermi seas becomes smaller

-> Costs less energy than ideal gas

-> **Reduced symmetry energy**

Color Superconductivity

- **BCS** Pairing near **Fermi sea**



- In terms of effective interaction near Fermi sea

$$\mathcal{S}_{4q} \sim (\psi^\dagger(-p_f)\psi(-p_i)) (\psi^\dagger(p_f)\psi(p_i))$$
 is marginal along to Fermi velocity
- Fermion – conjugated fermion interaction
- When $V < 0$ two states form a **condensate (gap)**

- **Wilsonian HDET** and **Nambu-Gorkov** formalism

- BCS action as Fermion – conjugated Fermion

$$\mathcal{S}_{BCS} \sim \frac{1}{2} \left[\psi(-p)^T C \Delta(p) \psi(p) + \psi(p) \tilde{\Delta}(p) C \bar{\psi}^T(-p) \right]$$

- HDET Lagrangian - Irrelevant high energy excitation has been integrated out

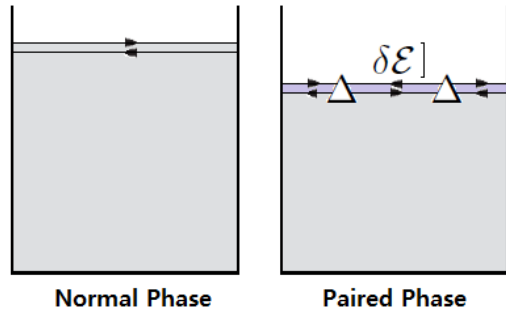
$$\mathcal{L}_D = \sum_{\vec{v}} \left[\psi^\dagger i\vec{V} \cdot D \psi - \psi^\dagger \frac{1}{2\mu + i\tilde{V} \cdot D} D_\perp^2 \psi \right]$$

Diagrammatically described gapped quasi-state

$$\rightarrow \Delta \leftarrow \bar{\Delta} \rightarrow \quad \rightarrow \quad S_\Delta(l) = \frac{l_0 + l_v}{l_0^2 - l_v^2 - \Delta^2} \gamma_0$$

Color BCS paired state

- **BCS** Pairing locks the gapped quasi-states

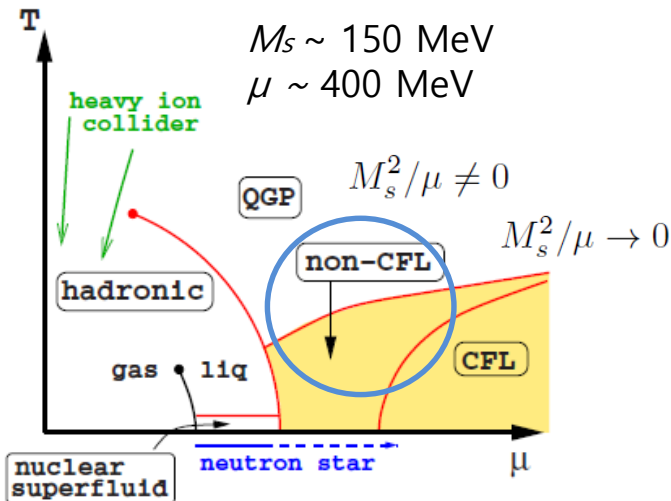


- In QCD, color anti-triplet gluon exchange interaction is attractive ($V < 0$)

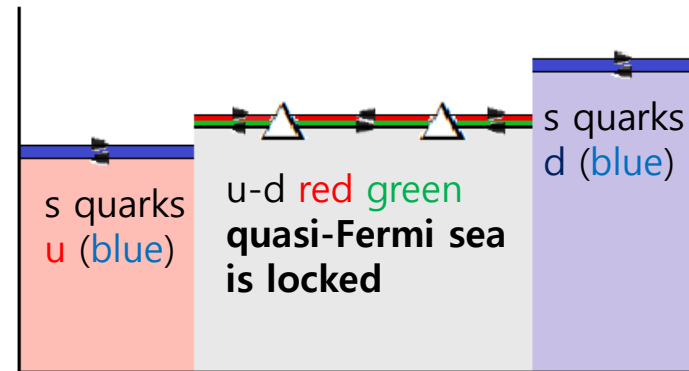
$$\langle \psi_a^\alpha C \gamma_5 \psi_b^\beta \rangle \sim \Delta_1 \epsilon^{\alpha\beta 1} \epsilon_{ab1} + \Delta_2 \epsilon^{\alpha\beta 2} \epsilon_{ab2} + \Delta_3 \epsilon^{\alpha\beta 3} \epsilon_{ab3}$$

- In non negligible M_s^2/μ **2SC** state is favored

- **2 color superconductivity**



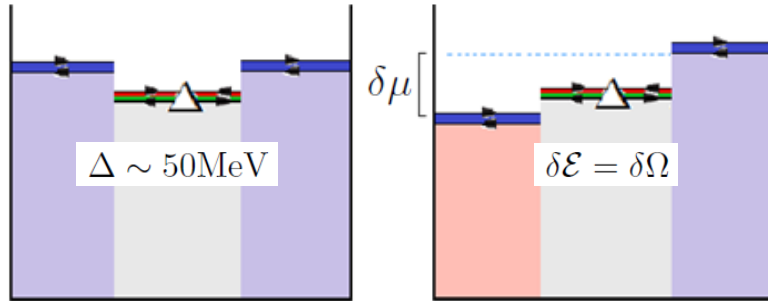
In 2SC phase, u-d red-green states are gapped



Only s quarks and u-d blue quarks are liberal

Asymmetrization in 2SC phase

- Only **Blue state (1/3)** can affect iso-spin asymmetry



- BCS phase remains in $\delta\mu < (1/\sqrt{2})\Delta \sim \Lambda$ (Phys. Rev. Lett. 9, 266 (1962) A. M. Clogston)
- Only **u-d blue** states can be asymmetrized
- The other 4 gapped quasi-states are **locked**

- In HDET formalism $\delta\mathcal{E} = \delta\Omega = - \sum_{ud,rg} \frac{\mu_f^2}{\pi^2} \left[\Lambda \sqrt{\Lambda^2 + \Delta^2} + \Delta^2 \ln((\Lambda + \sqrt{\Lambda^2 + \Delta^2})/\Delta) \right]$

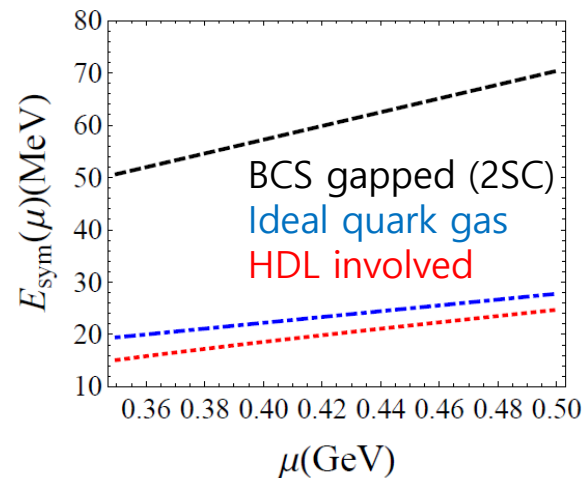
- **Symmetry energy**

$$\frac{\epsilon(\mu, \Delta, I_{B\Delta})}{\rho(\mu, \Delta, I_{B\Delta})} = \frac{E(\mu, \Delta, I_{B\Delta})}{N_{B\Delta}}$$

$$= \bar{E}(\mu, \Delta) + \bar{E}_{sym}(\mu, \Delta) I_{B\Delta}^2 + \dots$$

$$\bar{E}_{sym}(\mu, \Delta) = \frac{1}{2} \frac{\partial^2}{\partial I_{B\Delta}^2} \bar{E}_{\Delta}(\mu, I_{B\Delta})$$

$$I_{B\Delta} = 3 \frac{\rho_d - \rho_u}{\rho_d + \rho_u} \times \frac{1}{3}$$



Quasi-fermion state in 2SC phase

- Meissner mass effect?

Quasi-fermion in gapped state



$$S_{\Delta}(l) = \frac{l_0 + l_v}{l_0^2 - l_v^2 - \Delta^2} \gamma_0$$

Fermion in ungapped state



$$S(l) = \frac{l_0 + l_v}{l_0^2 - l_v^2} \gamma_0$$

- Gapped states and dense loop

1) If gap size is quite large $\Delta \sim g\mu$

-> matter loop do not have hard contribution

-> do not need resummation -> **reduction vanish**



2) But if gap size is quite small $\Delta < g\mu$

-> **needs resummation** -> **reduction remains**



3) For ungapped quark loop -> **reduction remains**



High density effective formalism

- **2SC** description as linear combination of Gellman matrices

Gapped and un-gapped quasi-state

$$\psi_{+, \alpha i} = \sum_{A=0}^5 \frac{(\tilde{\lambda}_A)_{\alpha i}}{\sqrt{2}} \psi_+^A \quad \tilde{\lambda}_0 = \frac{1}{\sqrt{3}} \lambda_8 + \sqrt{\frac{2}{3}} \lambda_0; \quad \tilde{\lambda}_A = \lambda_A \quad (A = 1, 2, 3); \quad \tilde{\lambda}_4 = \frac{\lambda_{4-i5}}{\sqrt{2}}; \quad \tilde{\lambda}_5 = \frac{\lambda_{6-i7}}{\sqrt{2}}$$

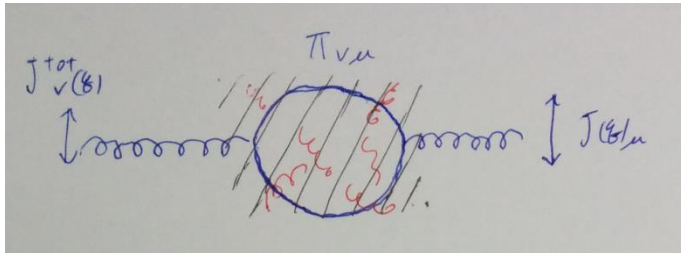
Effective Lagrangian

$$\mathcal{L}_D = \sum_{\vec{v}} \sum_{A,B=0}^5 \chi^{A\dagger} \left(\frac{i \text{Tr}[\tilde{T}_A V \cdot D \tilde{T}_B]}{\Delta_{AB}} \frac{\Delta_{AB}}{i \text{Tr}[\tilde{T}_A^* \tilde{V} \cdot D^* \tilde{T}_B^*]} \right) \chi^B + (L \rightarrow R) \cdot \quad \tilde{T}_A = \frac{\tilde{\lambda}_A}{\sqrt{2}} \quad (A = 0, \dots, 5)$$

$$\chi = \begin{pmatrix} \psi_+ \\ C\psi_-^* \end{pmatrix} \quad + \text{ and } - \text{ represents Fermi velocity}$$

Determines gluon-quasi Fermion coupling

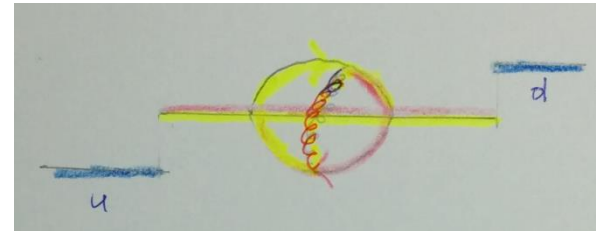
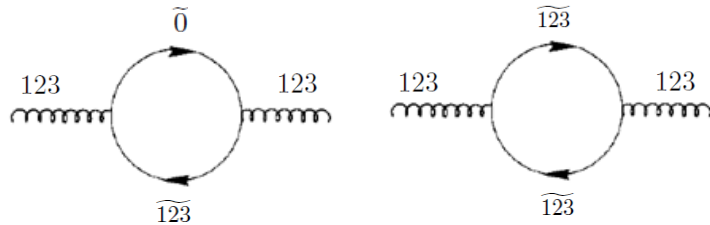
- Gluon self energy as kernel for linear response



$$\begin{aligned} J_{\mu}^{A, \text{ind}}(P) &= J_{\mu}^{A, \text{tot}}(P) - \mathbf{J}_{\mu}^A(P) \\ &= i[(D^{-1})_{\mu\nu}^{AB}(P) - (\mathcal{D}^{-1})_{\mu\nu}^{AB}(P)] \langle A^{B, \nu}(P) \rangle \\ &\equiv \Pi_{\mu\nu}^{AB}(P) \langle A^{B, \nu}(P) \rangle, \\ \langle A_{\mu}^A(P) \rangle &= -i \mathcal{D}_{\mu\nu}^{AB}(P) \mathbf{J}^{\nu, B}(P) \end{aligned}$$

Gluon self energy in 2SC phase

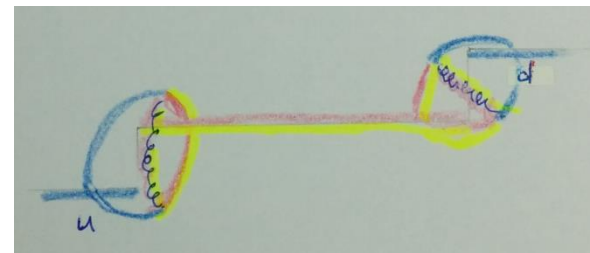
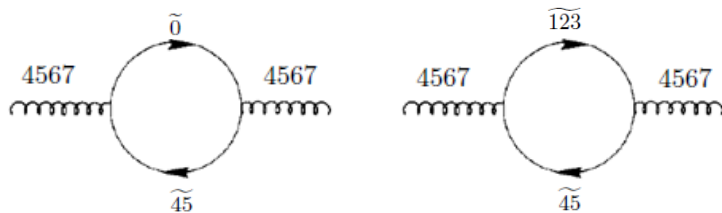
- Adjoint color 1,2,3 only couple with gapped states
 - In $p \rightarrow 0$ limit, $\Pi_{ab}^{\mu\nu}(0) = 0$
 - Gluons are trapped in gapped state
 - Symmetry energy do not reduced by **HDL**



Adjoint 1,2,3 trapped in BCS gap state

- Adjoint color 4,5,6,7 partially couple with gapped state

- In $p \rightarrow 0$ $\Pi_{ab}^{00}(0) = \frac{3}{2}m_g^2$ $-\Pi_{ab}^{ij}(0) = \frac{1}{2}m_g^2$
- Transition from gapped-ungapped state
- **HDL** through asymmetric Fermi sea can contribute reduction of symmetry energy

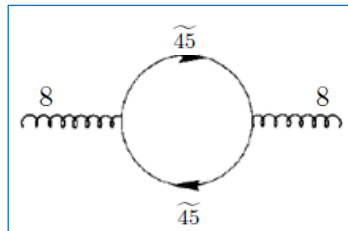
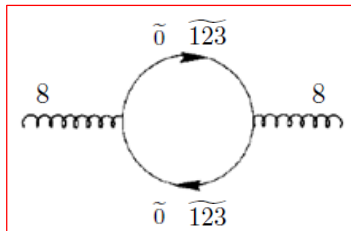
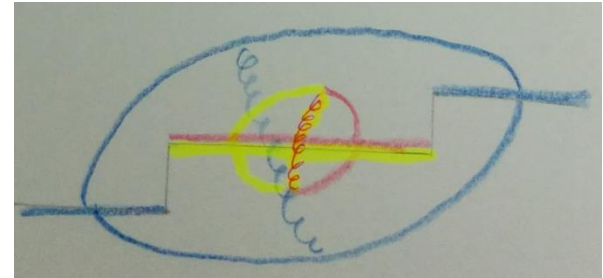


Adjoint 4,5,6,7 mediate transition between BCS gapped – ungapped state

Gluon self energy in 2SC phase

- Adjoint color 8

- In $p \rightarrow 0$ limit, $\Pi_{ab}^{00}(0) = 3m_g^2$ $-\Pi_{ab}^{ij}(0) = \frac{1}{3}m_g^2$
- HDL** from **gapped loop** \rightarrow locked in symmetric gapped sea
- HDL** from **ungapped loop** \rightarrow can be asymmetrized as in normal phase

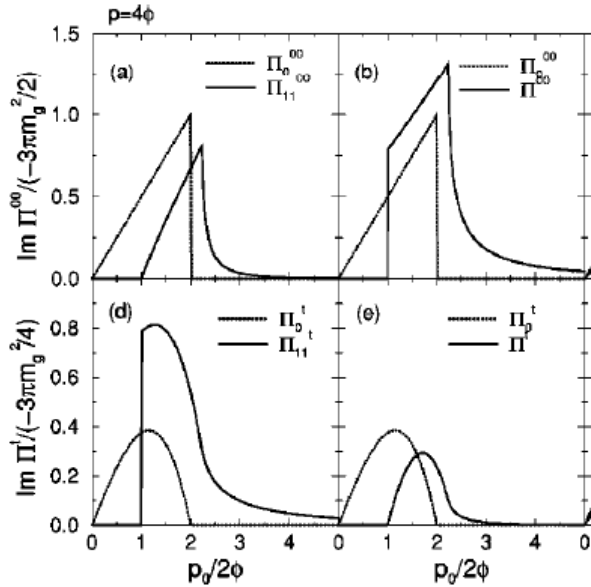


Adjoint 8 has contributions from **gapped loop** and **ungapped loop**

- All gapped state can be liberated at $T, \delta\mu > (1/\sqrt{2})\Delta$
- Reduction from HDL significantly dropped**

Recent works

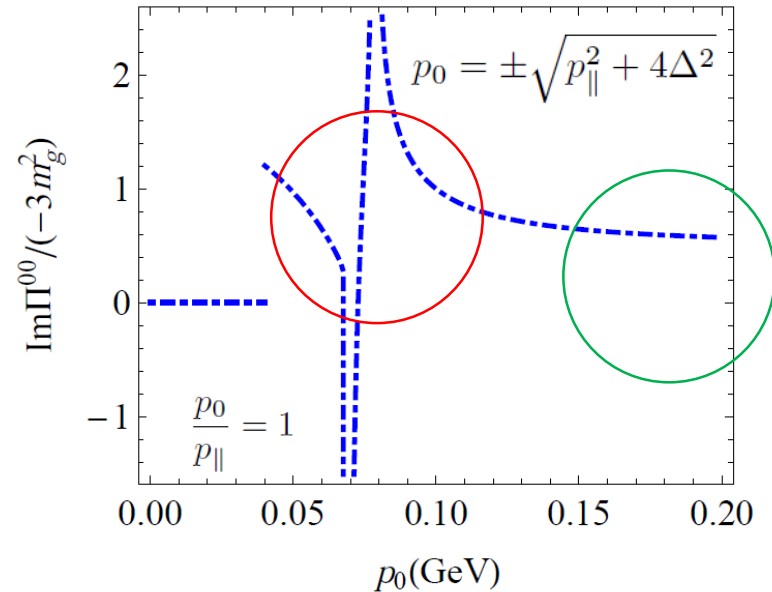
- Calculation in TFT



(PRD 64, 094003 (2001) Dirk H. Rishke)

- Imaginary part of gluon self energy
- In 2SC, no gluon mass in static limit (for adjoint 1,2,3)

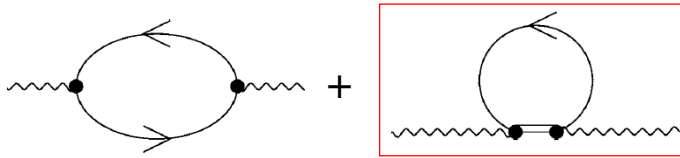
- Calculation in HDET



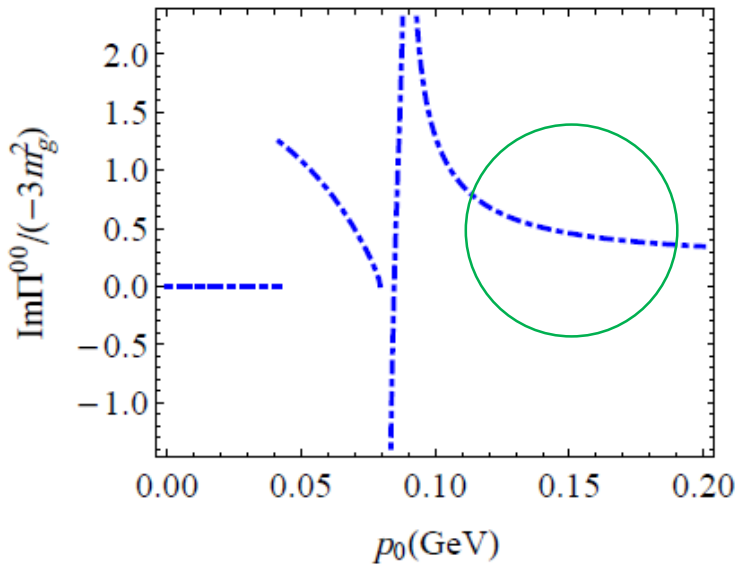
- Imaginary part of gluon self energy
- Two singularities
- Irrelevant loop correction needs

Recent works

- Irrelevant loop correction

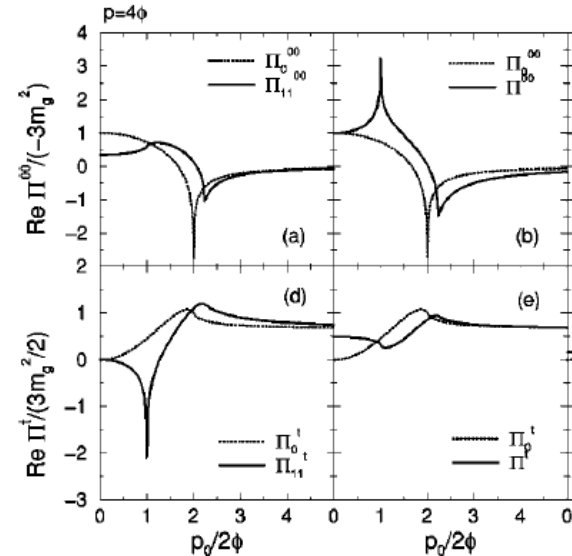


$$\mathcal{L}_D = \sum_{\vec{v}} \left[\psi^\dagger i\vec{V} \cdot D \psi - \psi^\dagger \frac{1}{2\mu + i\vec{V} \cdot D} D_\perp^2 \psi \right]$$



- Real part can be obtained by dispersion relation

$$\text{Re}\Pi(p_0, p) \equiv \frac{1}{\pi} \mathcal{P} \int_{-\infty}^{\infty} d\omega \frac{\text{Im}\Pi(p_0, p)}{\omega - p_0}$$

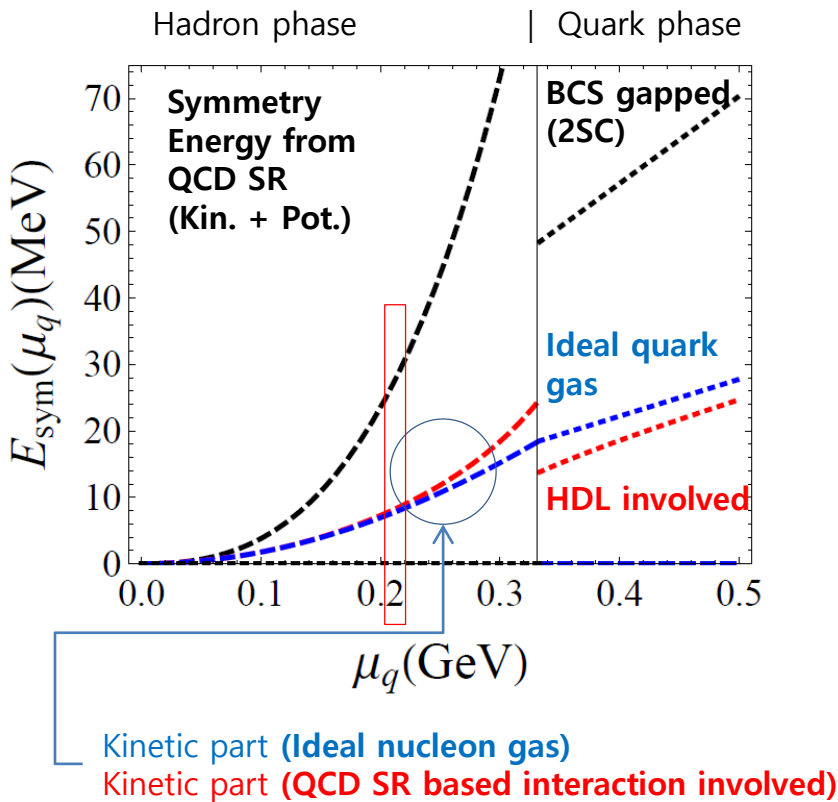


(PRD 64, 094003 (2001) Dirk H. Rishke)

- Real part of gluon self energy
- Scale comparison

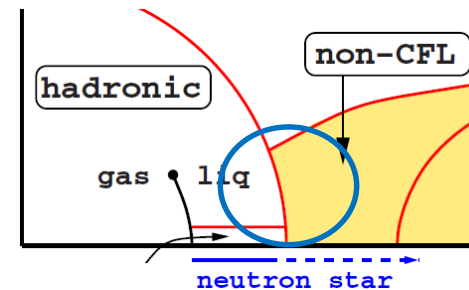
Summary and Future goals

- **Nuclear Symmetry Energy** in hadron and quark phase
- Evaluating gluon self energy by analytic calculation



Cold matter symmetry energy from **correct statistics** can be obtained

- Quark-hadron continuity?



Important quantum numbers?
(e.g. strangeness)
-> High density behavior at hadron phase