

# **Experimental data on collective flow and correlations**

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**Heavy Ion Meeting 2014-06  
20 June 2014**

# ALICE Results

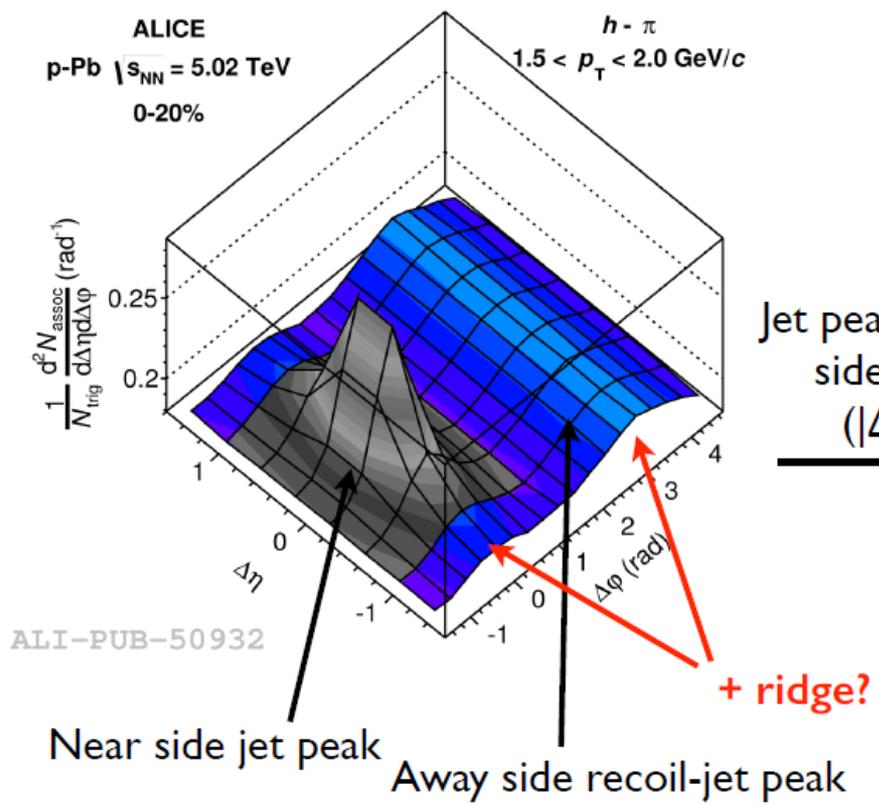
Extracted from the following talks.

1. “Long-range angular correlations at the LHC with ALICE” by [Leonardo Milano](#)
2. “Elliptic flow of identified particles in Pb-Pb collisions at the LHC” by [A. Dobrin](#)
3. “Searchs for azimuthal flow in pp, p-Pb and Pb-Pb collisions” by [Anthony Timmins](#)

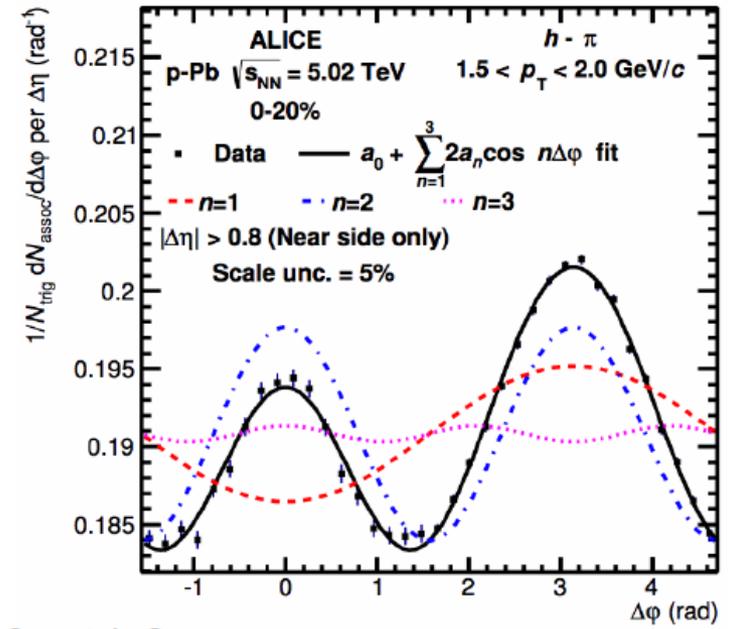
# Associated yield per trigger particle

$$\frac{1}{N_{\text{trig}}} \frac{d^2 N_{\text{assoc}}}{d\Delta\eta d\Delta\phi} = \frac{S(\Delta\eta, \Delta\phi)}{B(\Delta\eta, \Delta\phi)} \quad S(\Delta\eta, \Delta\phi) = 1/N_{\text{trig}} d^2 N_{\text{same}}/d\Delta\eta d\Delta\phi$$

$$B(\Delta\eta, \Delta\phi) = \alpha d^2 N_{\text{mixed}}/d\Delta\eta d\Delta\phi$$



Jet peak in the near side excluded ( $|\Delta\eta| < 0.8$ )



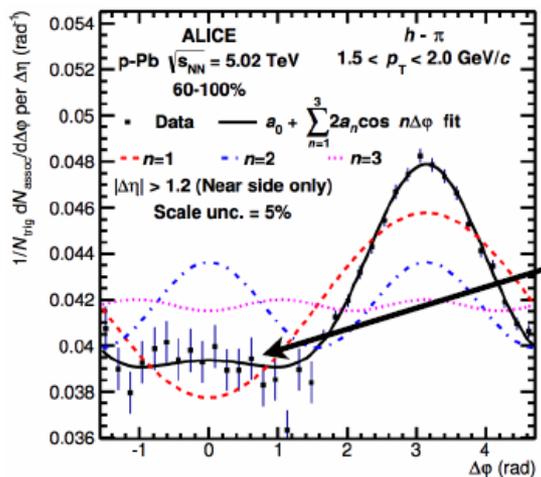
- good fit with 3 components:
- first component large due to recoil jet
  - $a_2$  given by jet+ridge
  - $a_3$  much smaller than the other components

## How to get rid of the jet contribution?

# The subtraction procedure

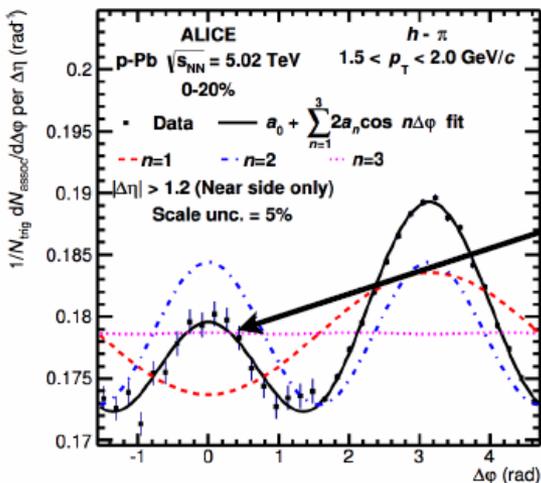
jet contribution reduced assuming:

- Mostly jet contribution (i.e. no significant ridge) in low multiplicity p-Pb events
- No significant medium effect in the energy loss / jet fragmentation



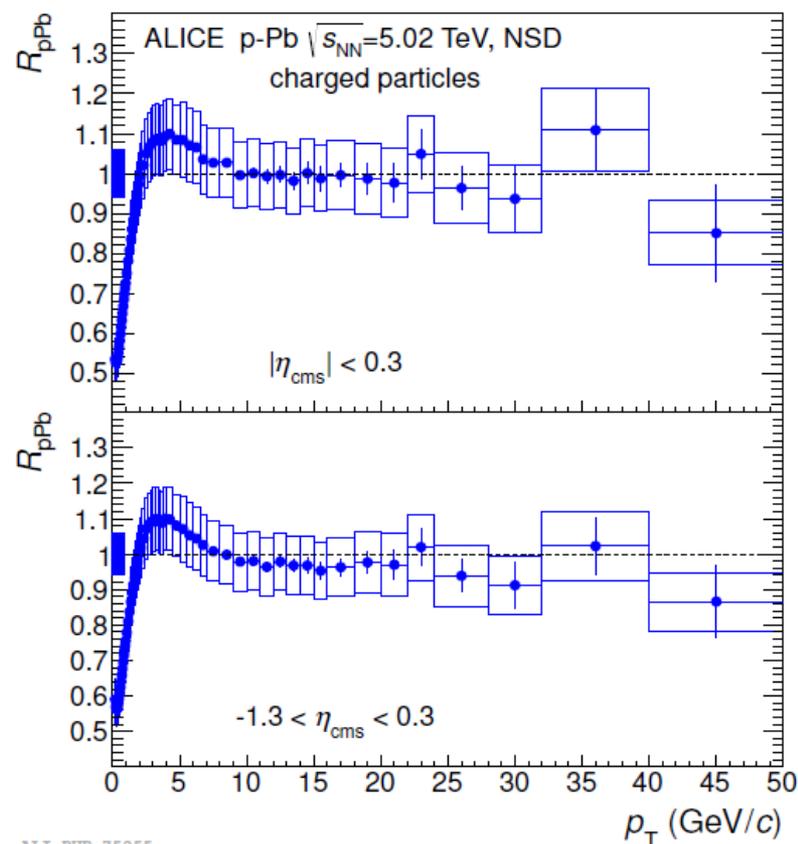
**low mult.**

No significant ridge in  
60-100%



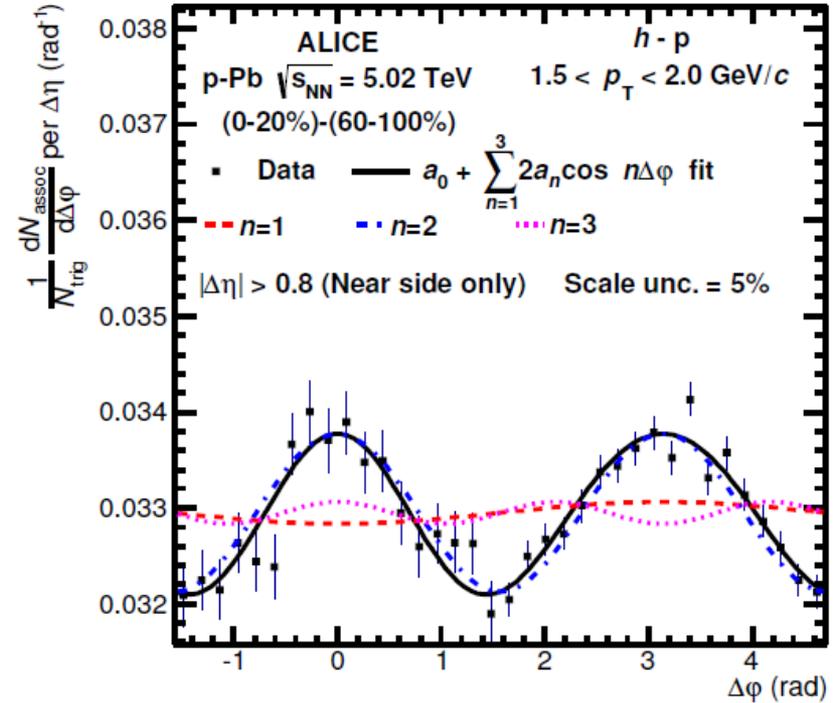
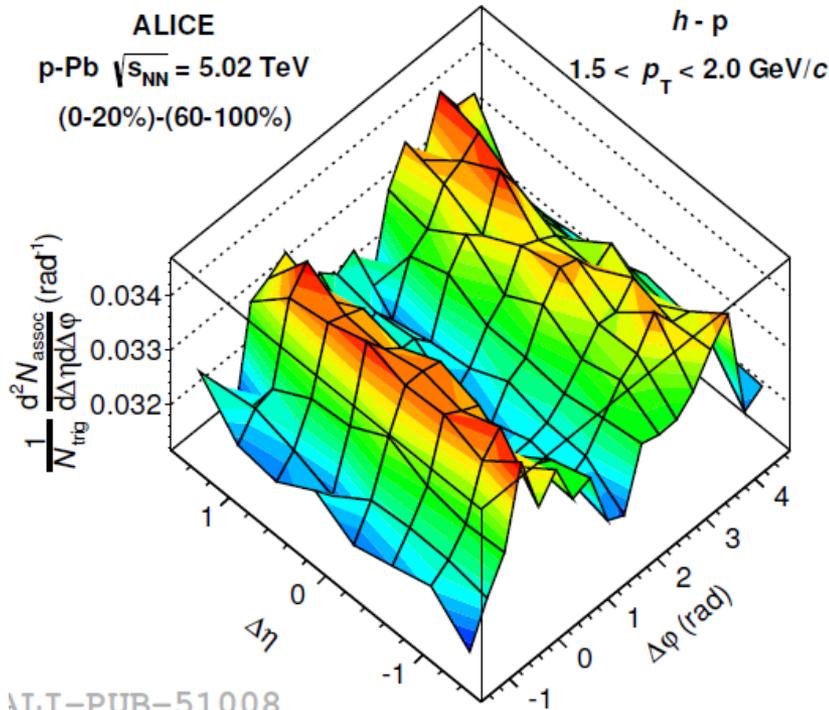
**high mult.**

Not the case for 0-20%



ALI-PUB-75255

# The subtraction procedure



$$V_{n\Delta}\{2\text{PC, sub}\} = a_n / (a_0 + b)$$

$$v_n^h\{2\text{PC}\} = \sqrt{V_{n\Delta}^{h-h}}$$

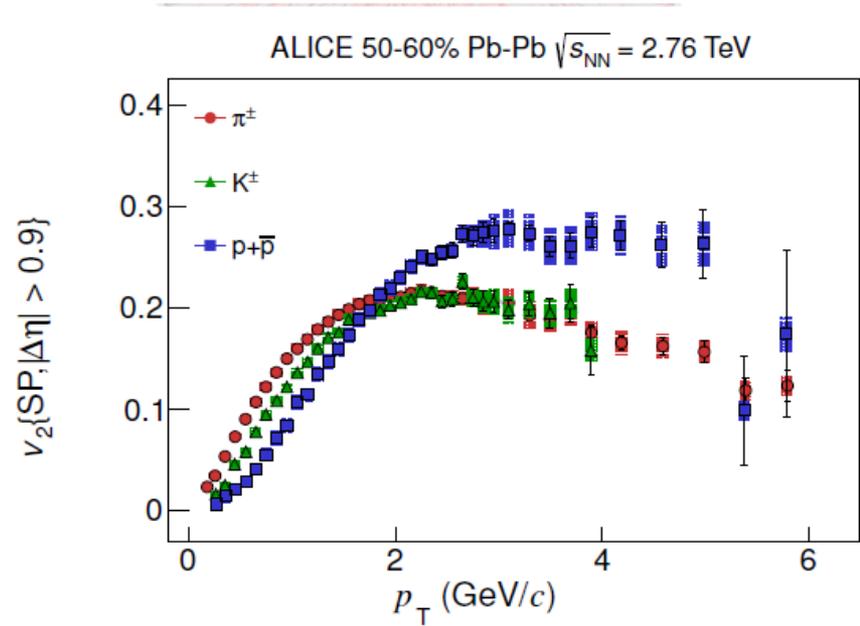
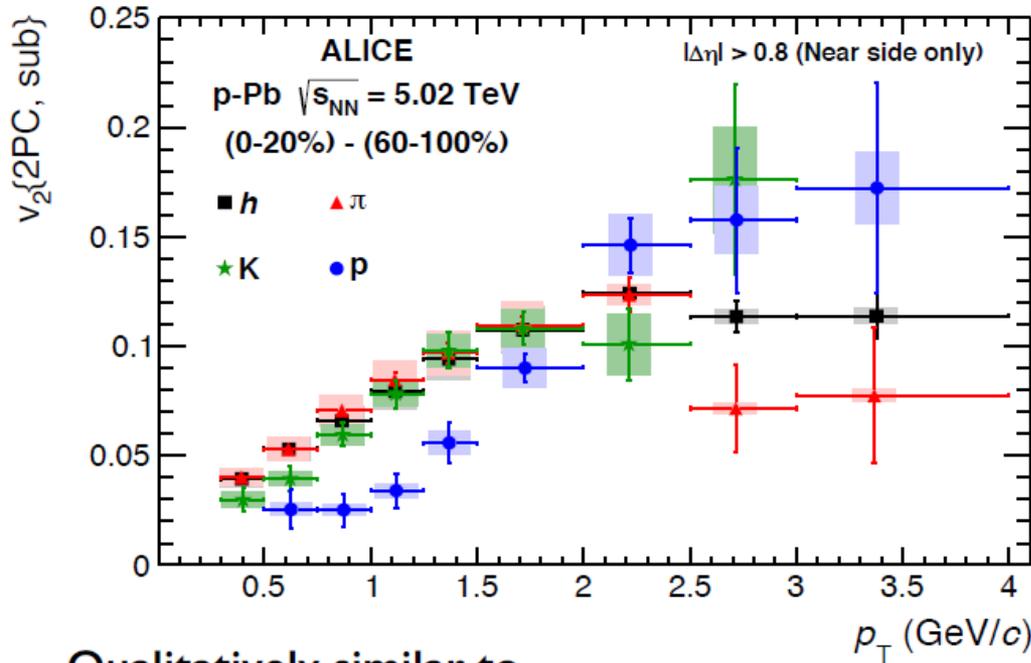
$$v_n^i\{2\text{PC}\} = V_{n\Delta}^{h-i} / \sqrt{V_{n\Delta}^{h-h}}$$

(When each of the particles is correlated with a common plane)

$$v_n\{2\} \equiv \sqrt{c_n\{2\}}$$

\*b calculated in 60-100% class

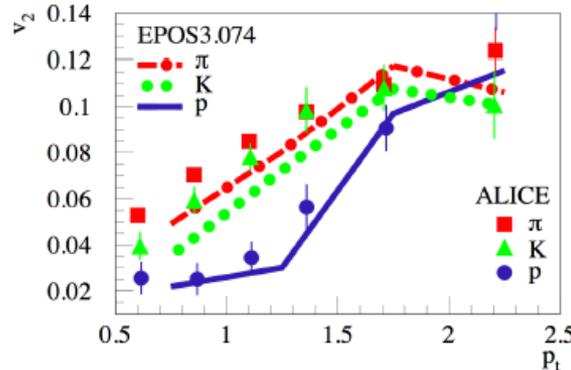
# $v_2$ of $\pi$ , $K$ , $p$ in high-multiplicity p-Pb



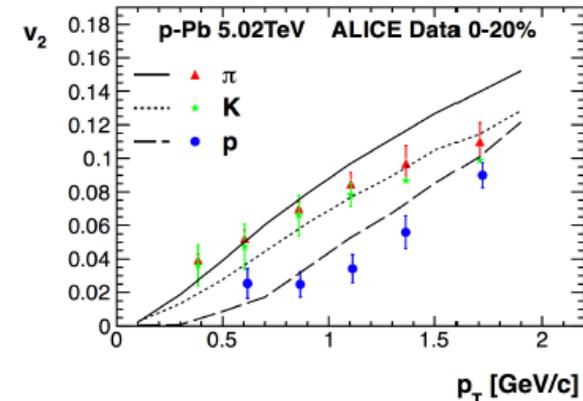
Qualitatively similar to  
Pb-Pb collisions  
... and consistent with  
hydro predictions

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arXiv:1405.4632 [nucl-ex]



K. Werner, M. Bleicher, B. Guiot, I. Karpenko, T. Pierog, arXiv:1307.4379 [nucl-th]

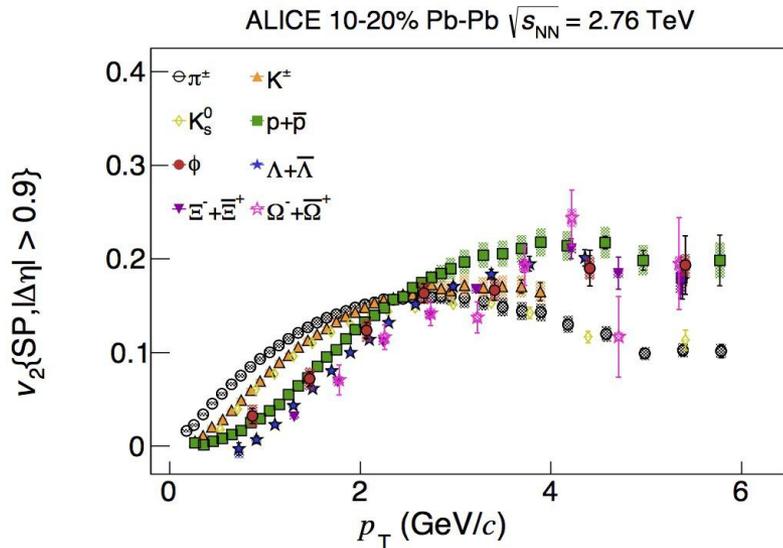


P. Bozek, W. Broniowski, G. Torrieri, Phys. Rev. Lett. 111, 172303 (2013)

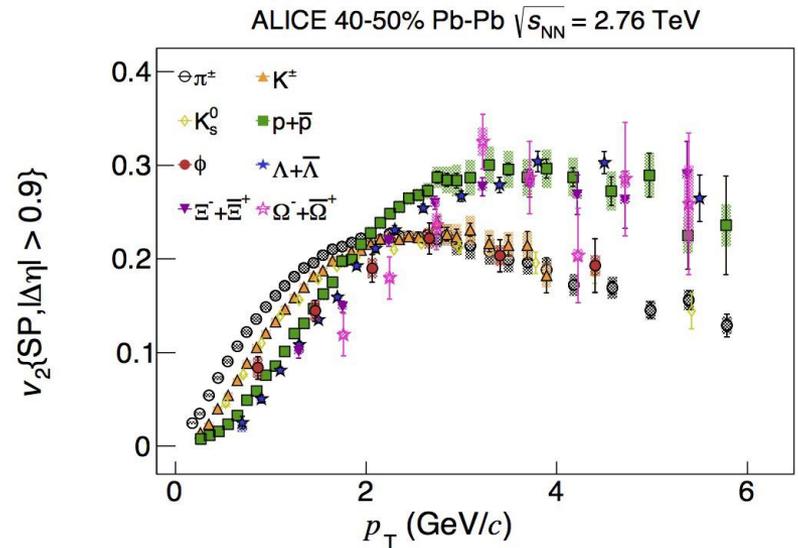
# Conclusions

- experimental observations are highly **suggestive** of collective effects in high-multiplicity p-Pb collisions
- **mass ordering** of the second order coefficients of the double ridge

# Identified particle $v_2$



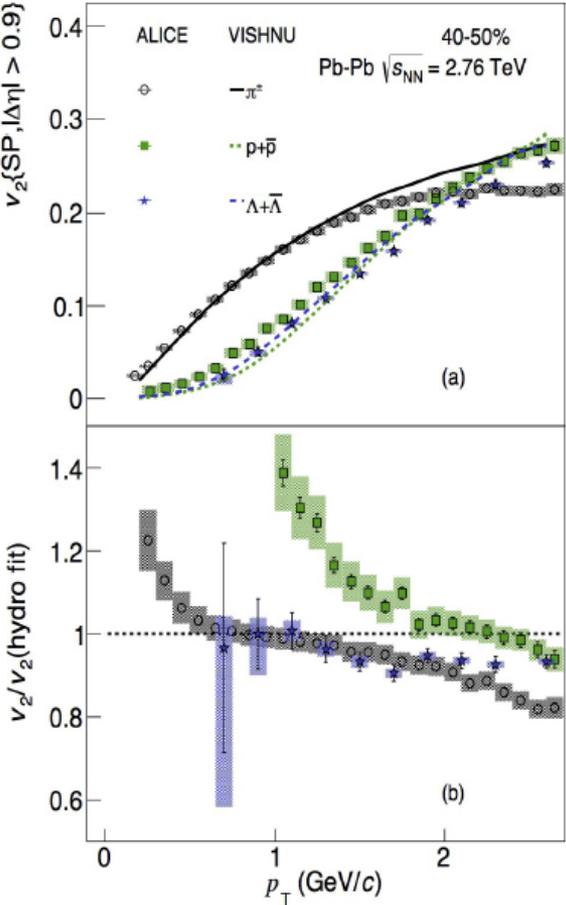
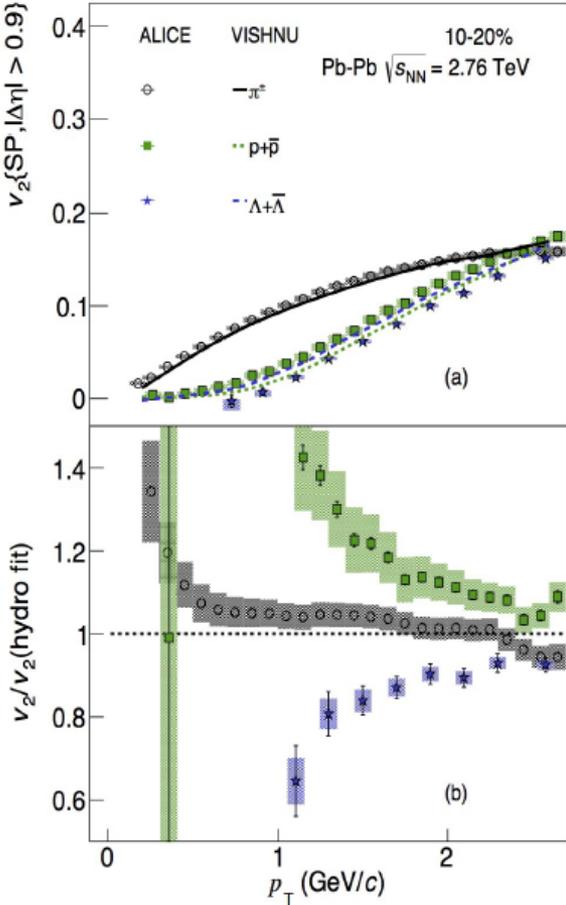
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ALICE-PUB-82660

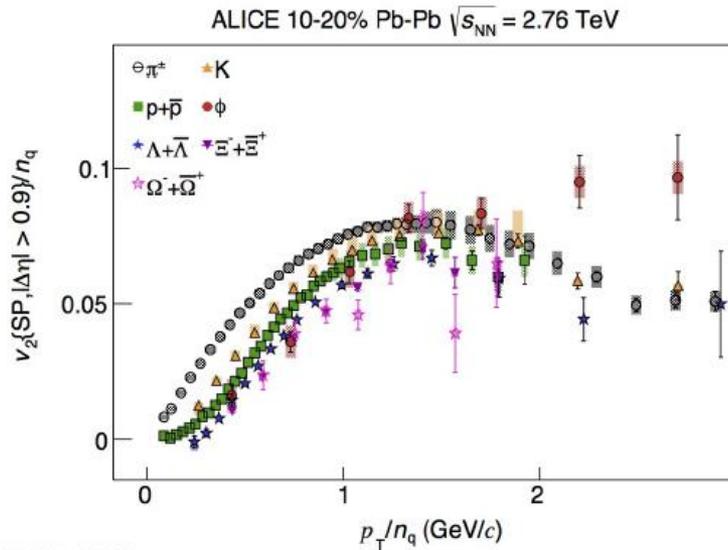
- Small difference between  $v_2$  for  $K^\pm$  and  $K_s^0$ 
  - Physics mechanism/detector effect responsible not understood yet
  - $v_2$  for  $K^\pm$  and  $K_s^0$  averaged for  $p_T < 4.0$  GeV/c in the following slides
- For  $p_T < 2$  GeV/c: observe mass ordering indicative of radial flow
- For  $p_T \sim 2-3.5$  GeV/c: crossing between  $v_2$  of p and  $\pi^\pm$
- For  $p_T > 3$  GeV/c: particles tend to group into mesons and baryons
  - $v_2$  of  $\phi$  follows baryons for central collisions and shift progressively to mesons for peripheral collisions

# Comparison with hydrodynamical calculations ( $\pi^\pm$ , p, $\Lambda$ )

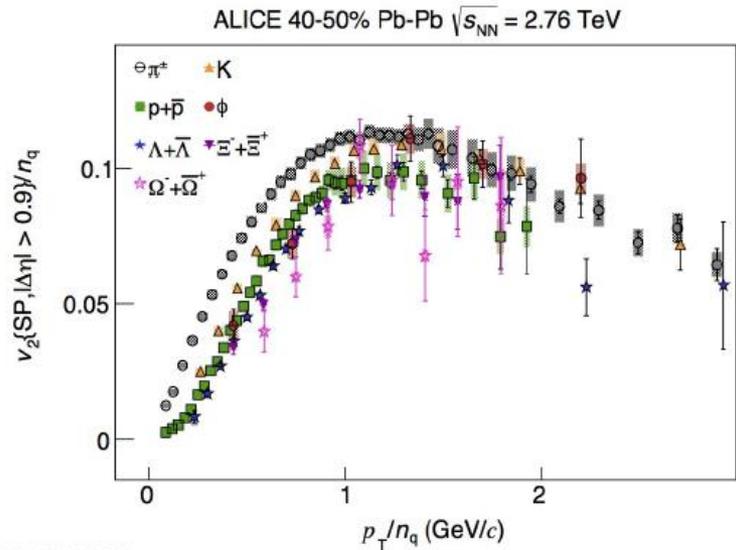


- Hydrodynamical calculations (MC-KLN,  $\eta/s=0.16$ ) coupled to a hadronic cascade model (VISHNU) reproduce the main features of  $v_2$  for  $p_T < 2$  GeV/c
  - Underestimates the  $v_2$  for  $\pi^\pm$
  - Underpredicts the  $v_2$  for p
  - Overestimates the  $v_2$  for  $\Lambda$ 
    - Mass ordering is broken in the model

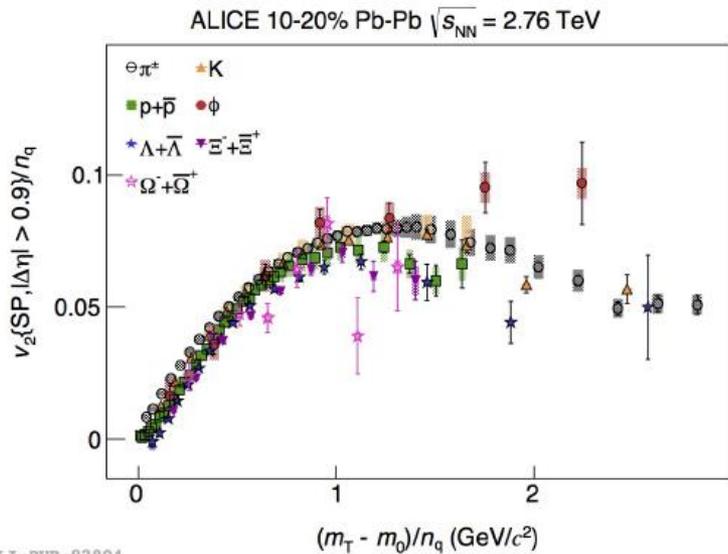
# $p_T/n_q$ or $KE_T/n_q$ scaling?



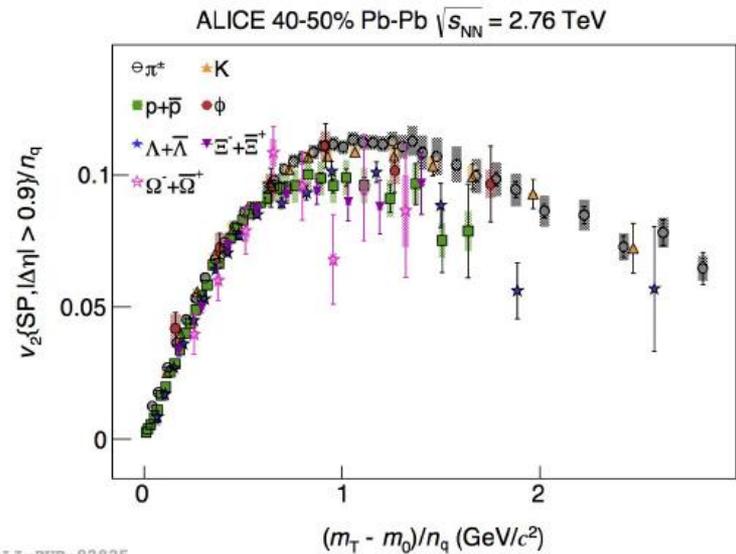
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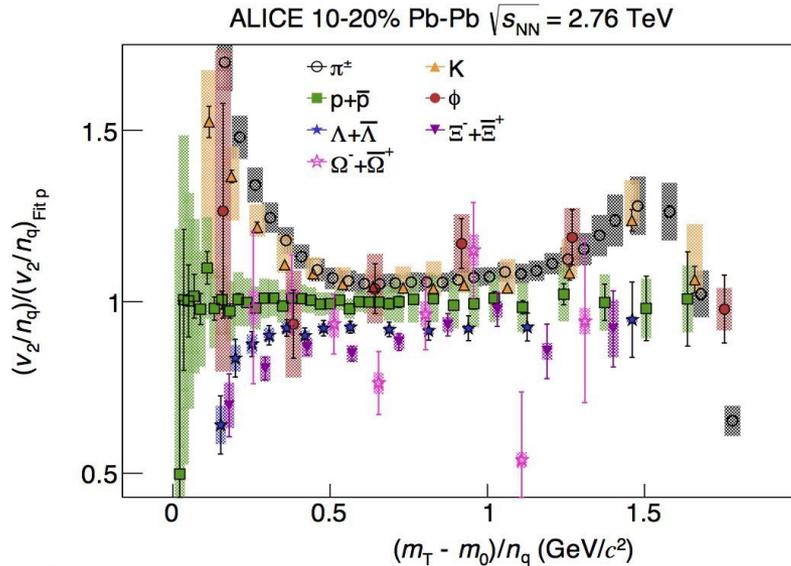


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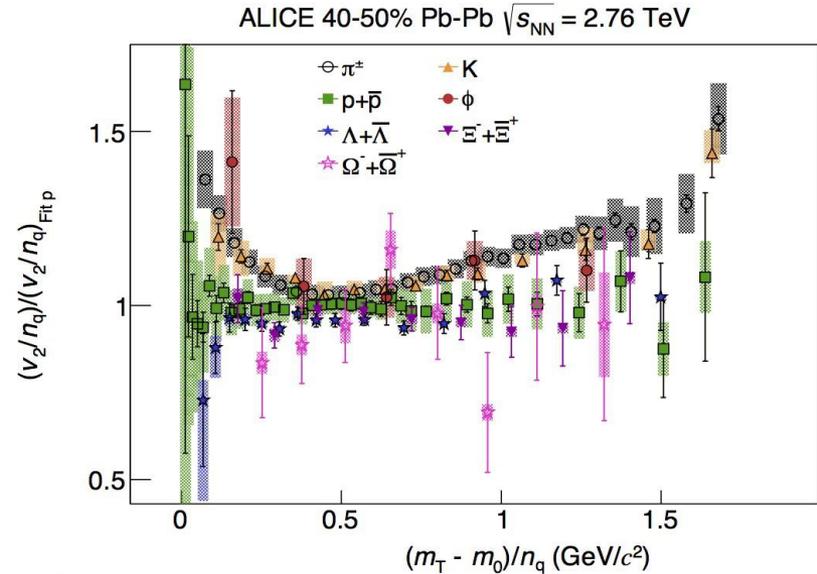


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# $KE_T/n_q$ scaling?



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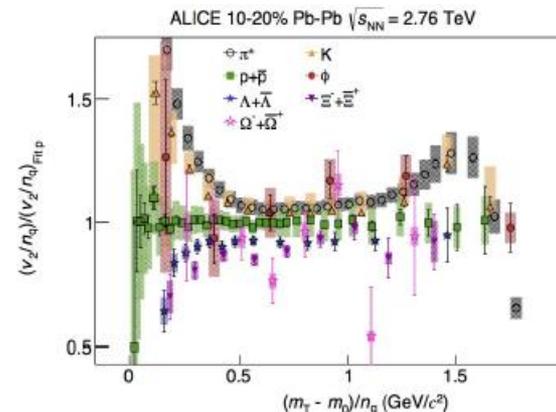
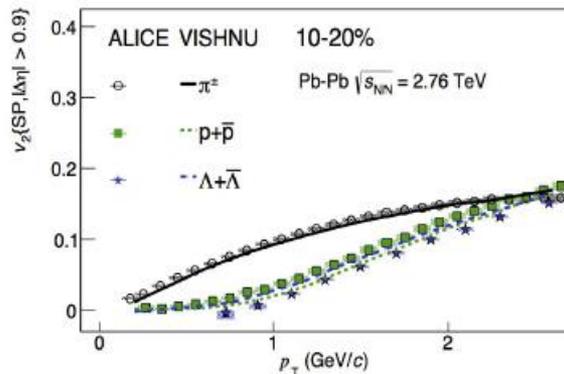
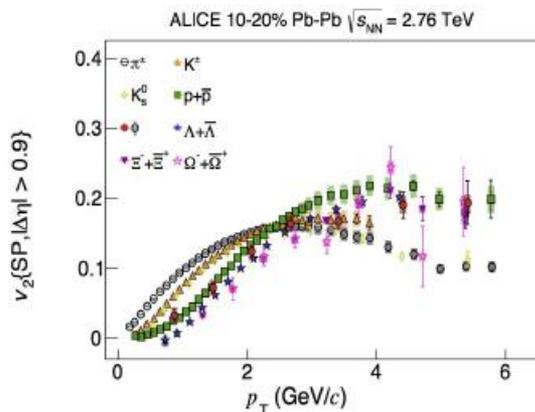
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$$KE_T = m_T - m_0 \quad m_T = \sqrt{p_T^2 + m_0^2}$$

- For  $KE_T/n_q < 0.6-0.8$  GeV/c<sup>2</sup>: NCQ scaling is broken at the LHC
- For  $KE_T/n_q > 0.8$  GeV/c<sup>2</sup>: NCQ scaling deviations at the level of  $\pm 20\%$ 
  - Similar magnitude for all centrality classes

# Summary

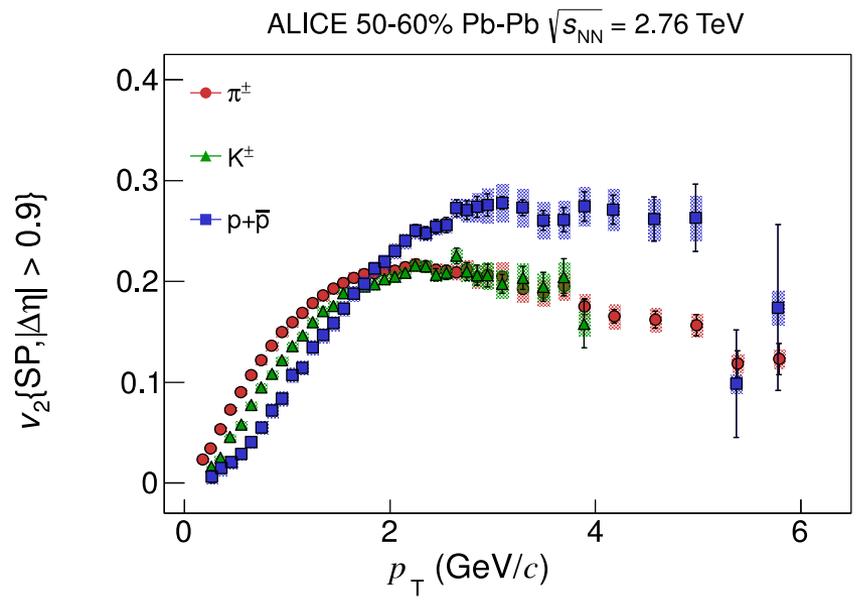
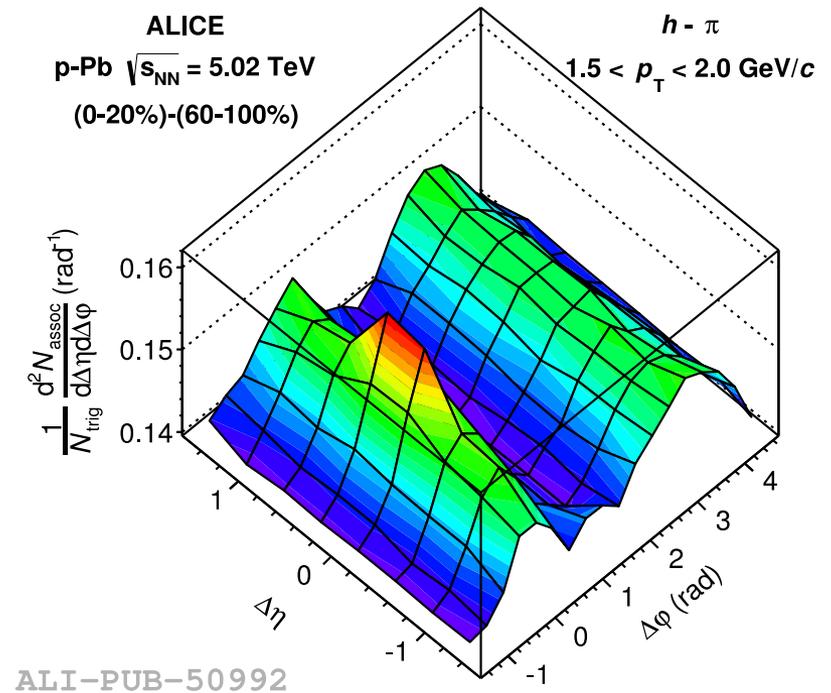
- $v_2$  for  $\pi^\pm$ ,  $K^\pm$ ,  $p$ ,  $K_s^0$ ,  $\Lambda$ ,  $\phi$ ,  $\Xi$ ,  $\Omega$  is measured in Pb-Pb collisions using the ALICE detector
  - Observe mass ordering for  $p_T < 2$  GeV/c
  - Crossing between  $v_2$  of  $p$  and  $\pi$  for  $p_T \sim 2-3.5$  GeV/c
  - Particles tend to group into mesons and baryons for  $p_T > 3$  GeV/c
    - $v_2$  of  $\phi$  follows baryons for central collisions and shift to mesons for peripheral collisions
  - Hydrodynamical calculations (MC-KLN,  $\eta/s=0.16$ ) coupled to a hadronic cascade model describe qualitatively the measurements
  - Observe deviations from NCQ scaling at the level of  $\pm 20\%$



# Motivation

- ❑ Double ridge observed in p-Pb collisions
  - ✓ Few or many particle correlations?
  
- ❑ Flow cumulants sensitive to multi-particle correlations
  - ✓ How do they compare to Pb-Pb at same multiplicity?
  
- ❑ Mass dependence of  $v_2$  observed in Pb-Pb collisions
  - ✓ Interplay of radial and elliptic flow
  - ✓ What happens in pp and p-Pb collisions?

*Phys. Lett. B726 (2014) 164*  
 ALICE PID flow paper just submitted to arivx  
 p-Pb and Pb-Pb results to be submitted this week



# Flow cumulants and coefficients

- Cumulants formed from  $v_n$  moments. Moments from multi-particle correlations ( $n$ =flow harmonic,  $\langle v_n \rangle^m = \langle m \rangle$ ).

$$c_n \{2\} = \langle\langle 2 \rangle\rangle$$

$$c_n \{4\} = \langle\langle 4 \rangle\rangle - 2\langle\langle 2 \rangle\rangle^2$$

$$c_n \{6\} = \langle\langle 6 \rangle\rangle - 9\langle\langle 4 \rangle\rangle\langle\langle 2 \rangle\rangle + 12\langle\langle 2 \rangle\rangle^3$$

→

- Flow coefficients formed from cumulants

$$v_n \{2\} = \sqrt{c_n \{2\}}$$

$$v_n \{4\} = \sqrt[4]{-c_n \{4\}}$$

$$v_n \{6\} = \sqrt[6]{\frac{1}{4}c_n \{6\}}$$

- Methods have different sensitivity to flow fluctuations and non-flow

$$\begin{aligned} v_n \{2\} &\cong v_n^2 + \sigma_n^2 + \delta \\ v_n \{4\} &\cong v_n^2 - \sigma_n^2 \end{aligned}$$

$v_2 \{2\}$  and  $v_2 \{4\}$  have different sensitivity to **flow fluctuations** ( $\sigma_n$ ) and non-flow ( $\delta$ )

## • Cumulants:

- 2- and 4-particle azimuthal correlations for an event:

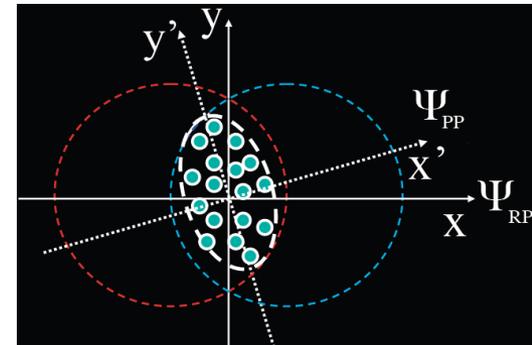
$$\langle 2 \rangle \equiv \langle \cos(n(\varphi_i - \varphi_j)) \rangle, \varphi_i \neq \varphi_j$$

$$\langle 4 \rangle \equiv \langle \cos(n(\varphi_i + \varphi_j - \varphi_k - \varphi_l)) \rangle, \varphi_i \neq \varphi_j \neq \varphi_k \neq \varphi_l$$

- Averaging over all events, the 2<sup>nd</sup> and 4<sup>th</sup> order cumulants are given:

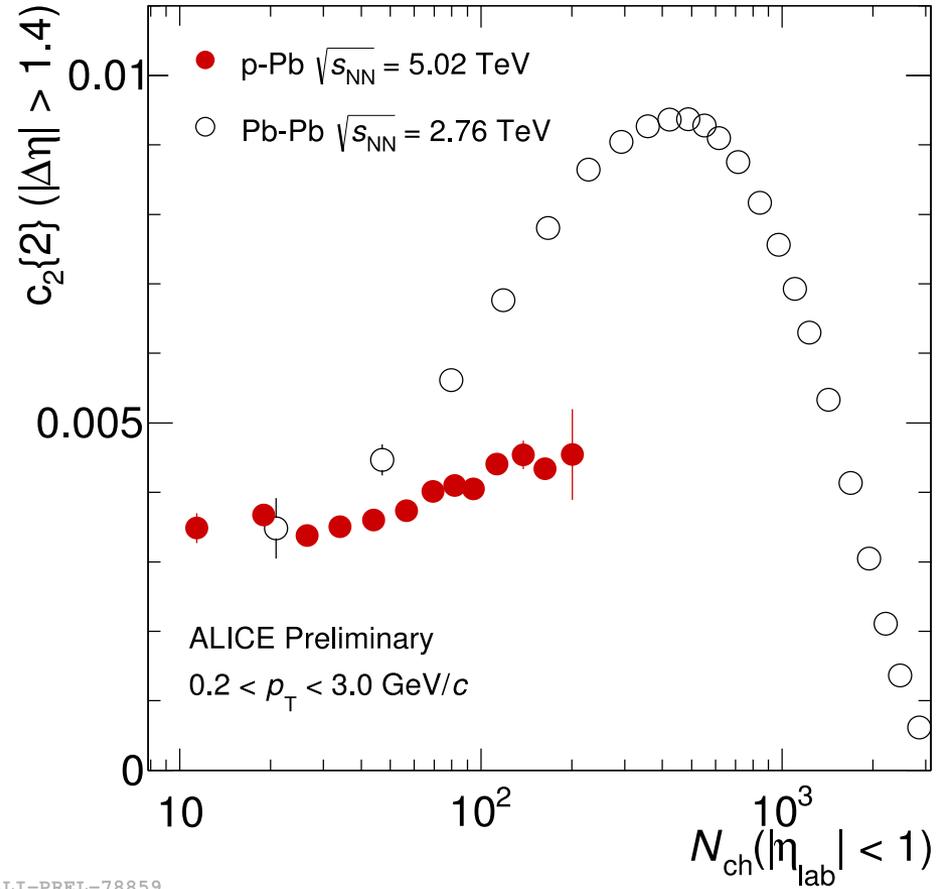
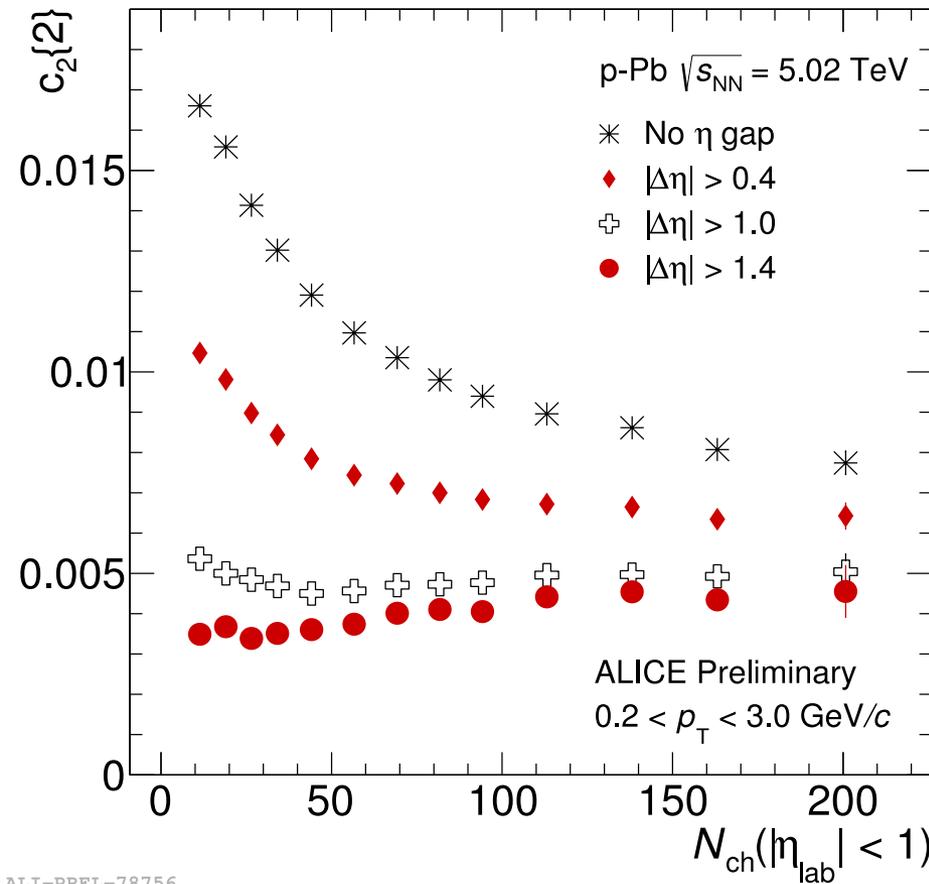
$$\begin{aligned} c_2 \{n\} &= \langle\langle 2 \rangle\rangle = v_n^2 + \delta_n \\ c_4 \{n\} &= \langle\langle 4 \rangle\rangle - 2\langle\langle 2 \rangle\rangle^2 = -v_n^4 \end{aligned}$$

Plane of symmetry ( $\Psi_{PP}$ ) fluctuate event-by-event around reaction plane ( $\Psi_{RP}$ ) => **flow fluctuation** ( $\sigma_n$ )



# $c_2\{2\}$ in p-Pb and Pb-Pb

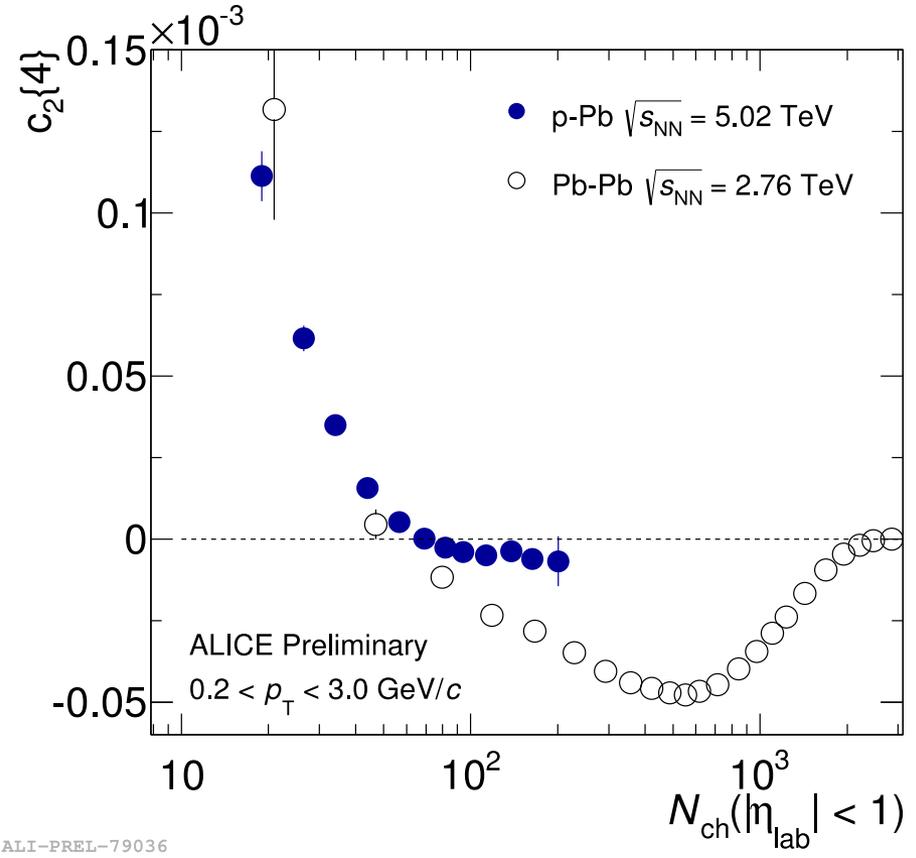
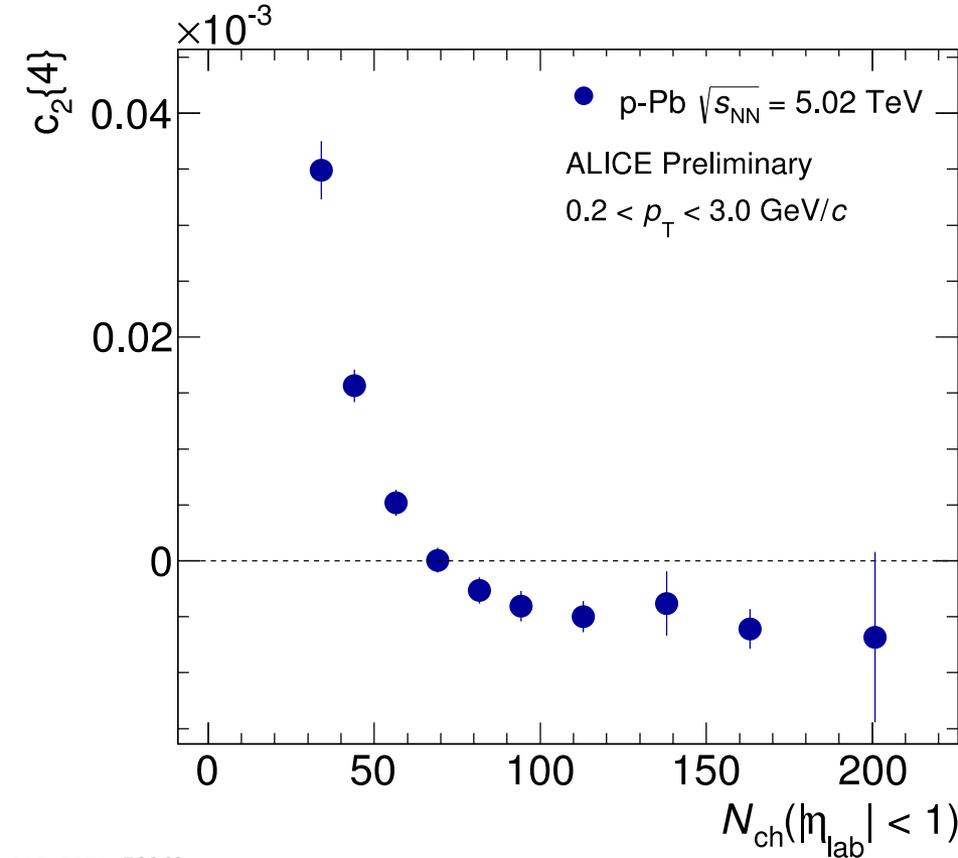
$$v_2\{2\} = \sqrt{c_2\{2\}}$$



- p-Pb  $c_2\{2\}$  rises for large  $\Delta\eta$  gap. Inconsistent with naïve expectations of non-flow
- Pb-Pb  $c_2\{2\}$  values bigger at same  $N_{\text{ch}}$ .
  - ✓  $\epsilon_2$  (Pb-Pb)<sub>RMS</sub> driven by geometry & fluctuations.
  - ✓  $\epsilon_2$  (p-Pb)<sub>RMS</sub> by just fluctuations?

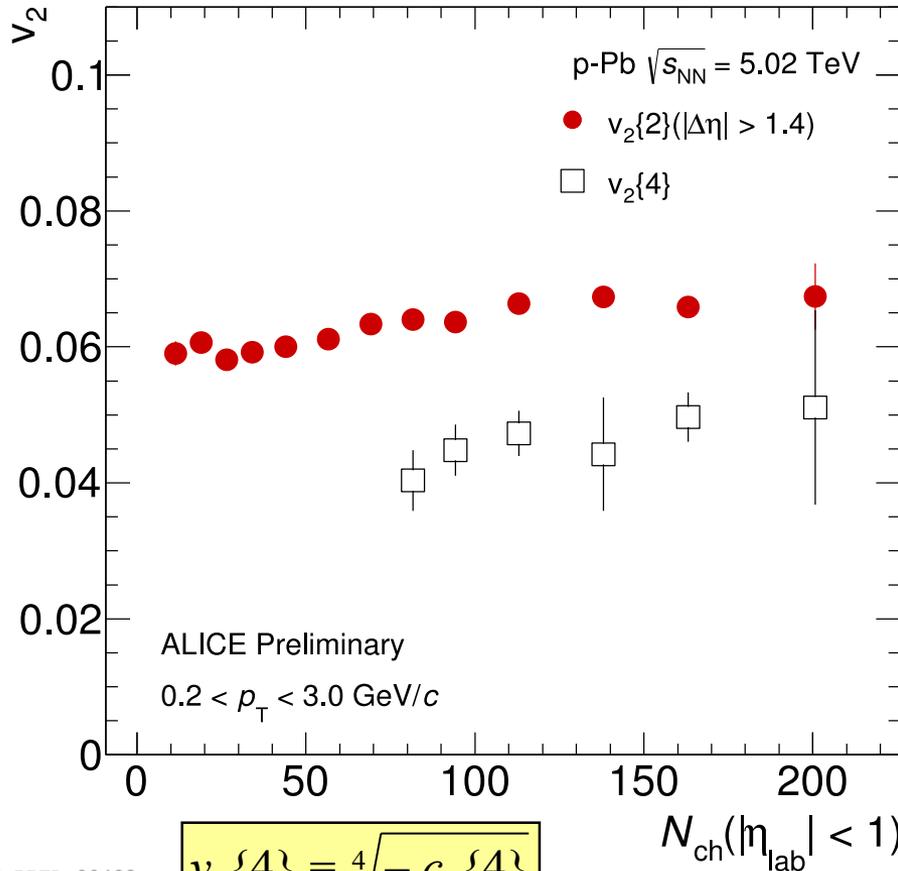
# $c_2\{4\}$ in p-Pb and Pb-Pb

$$v_2\{4\} = \sqrt[4]{-c_2\{4\}}$$

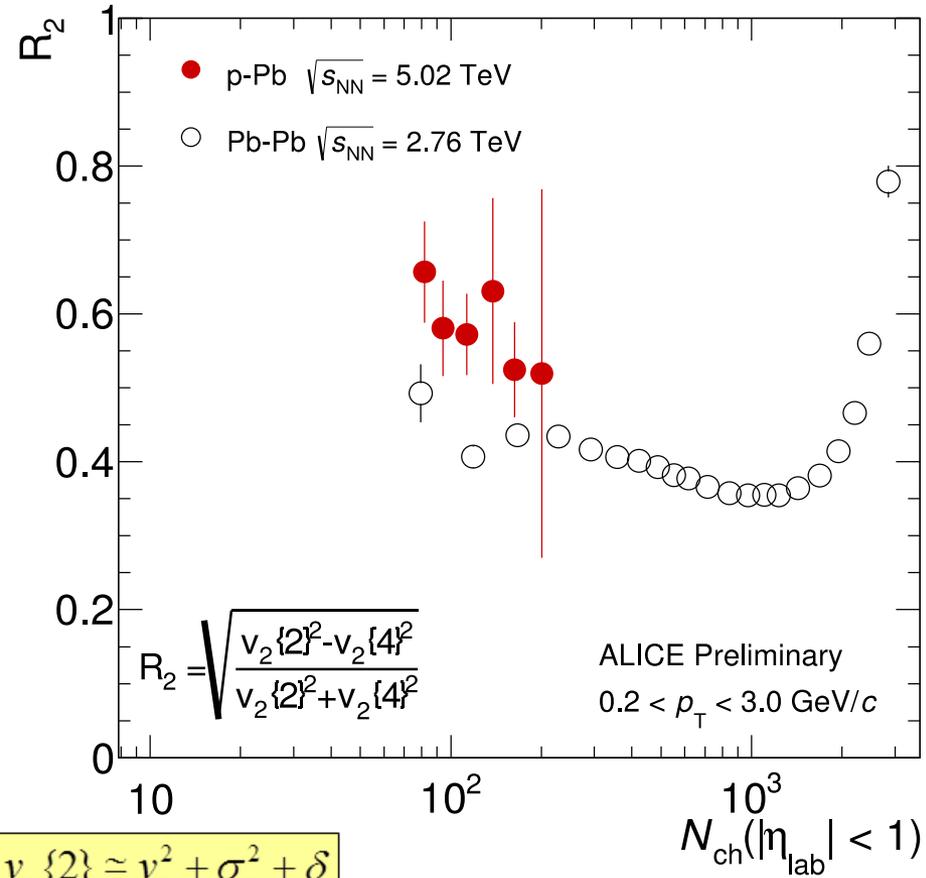


- p-Pb  $c_2\{4\}$  switches from pos. to neg. at high  $N_{ch}$ . ( $v_2\{4\}$  becomes real).
- Pb-Pb  $c_2\{4\}$  values more neg. at same  $N_{ch}$  after  $N_{ch} > 100$

# $v_2\{2\}$ and $v_2\{4\}$ in p-Pb



$$v_n\{4\} \equiv \sqrt[4]{-c_n\{4\}}$$

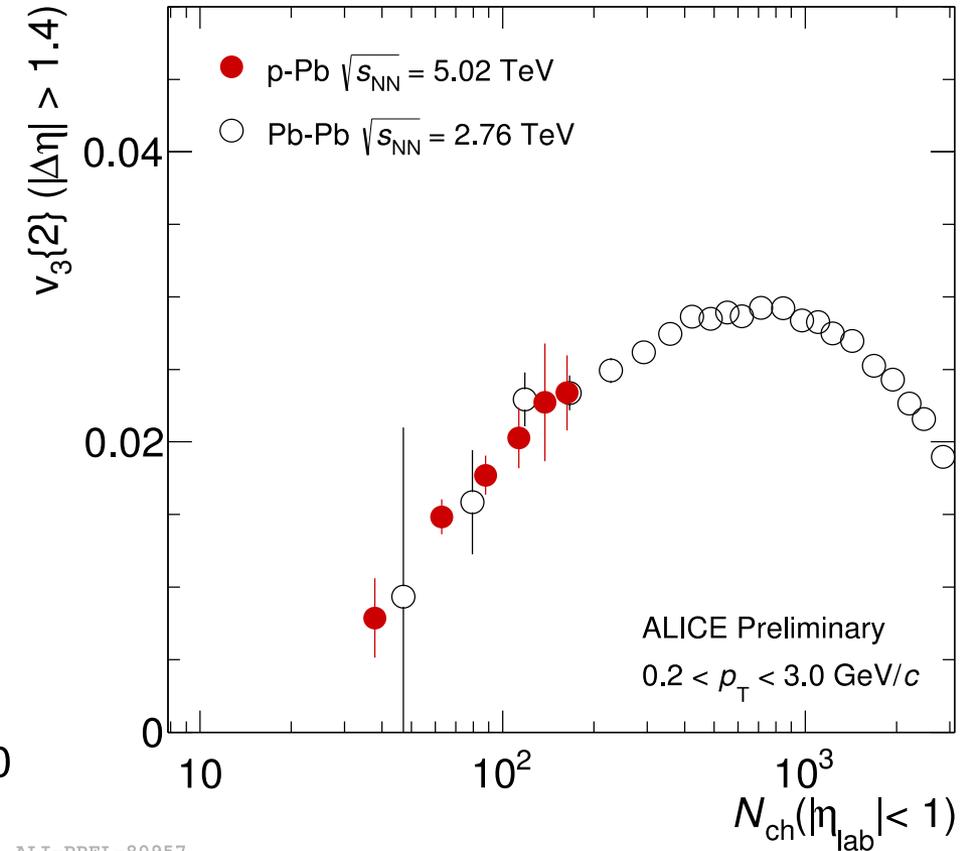
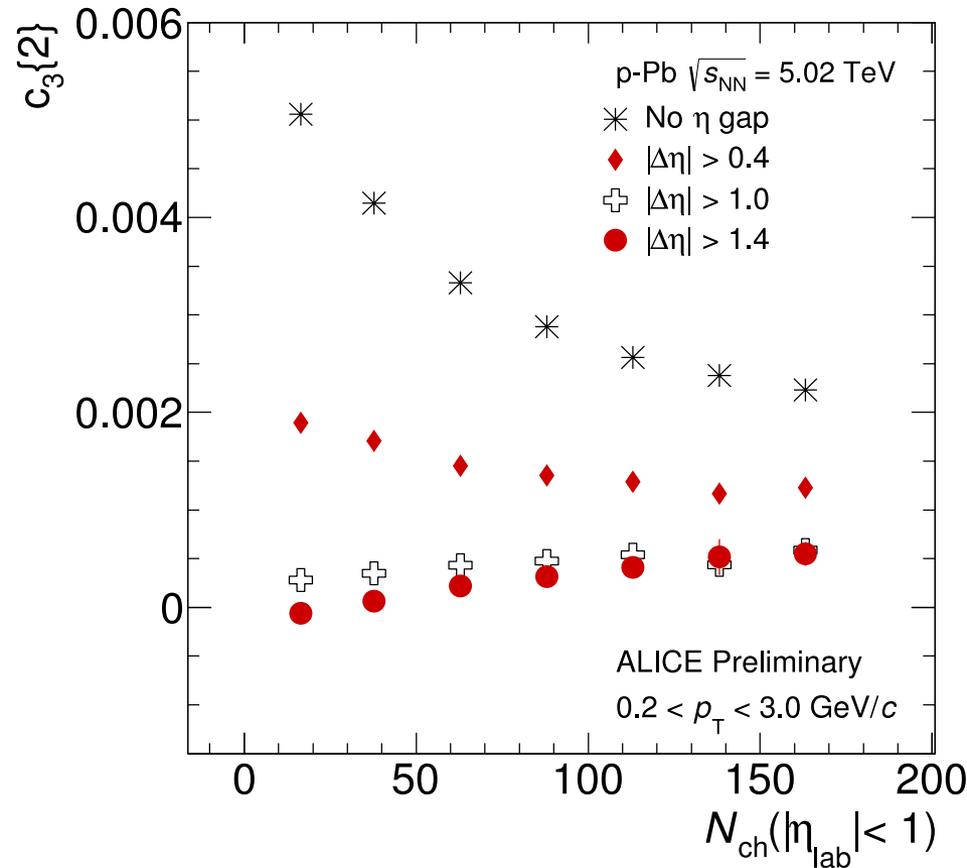


$$v_n\{2\} \cong v_n^2 + \sigma_n^2 + \delta$$

$$v_n\{4\} \cong v_n^2 - \sigma_n^2$$

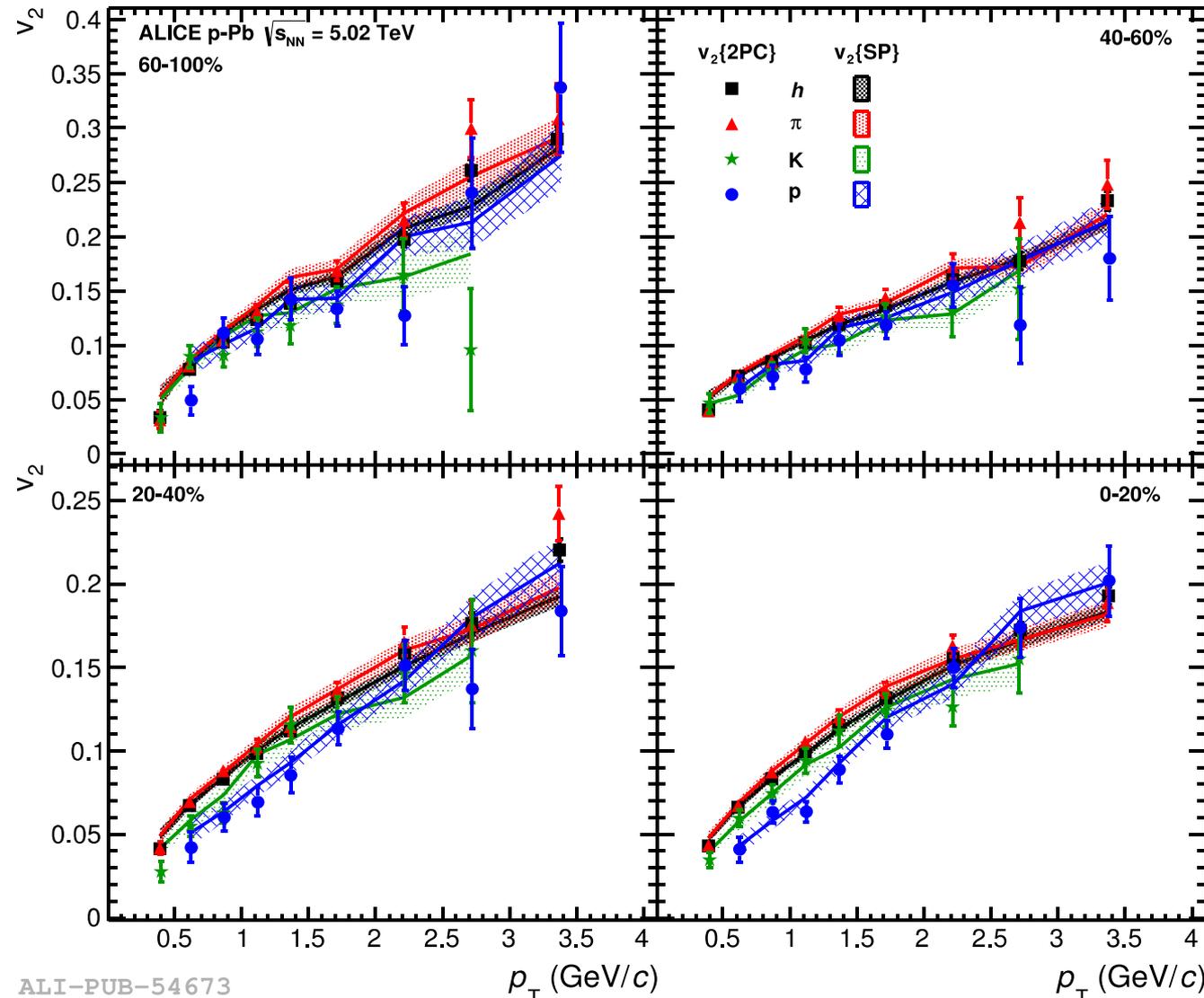
- $v_2\{2\} > v_2\{4\}$  in p-Pb -> Indicative of flow fluctuations? Contributions from non-flow?
- $R_2$  approximates  $\sigma_{v_2} / \langle v_2 \rangle$ . Fluctuations larger in p-Pb compared to Pb-Pb.

# Third harmonic in p-Pb and Pb-Pb collisions



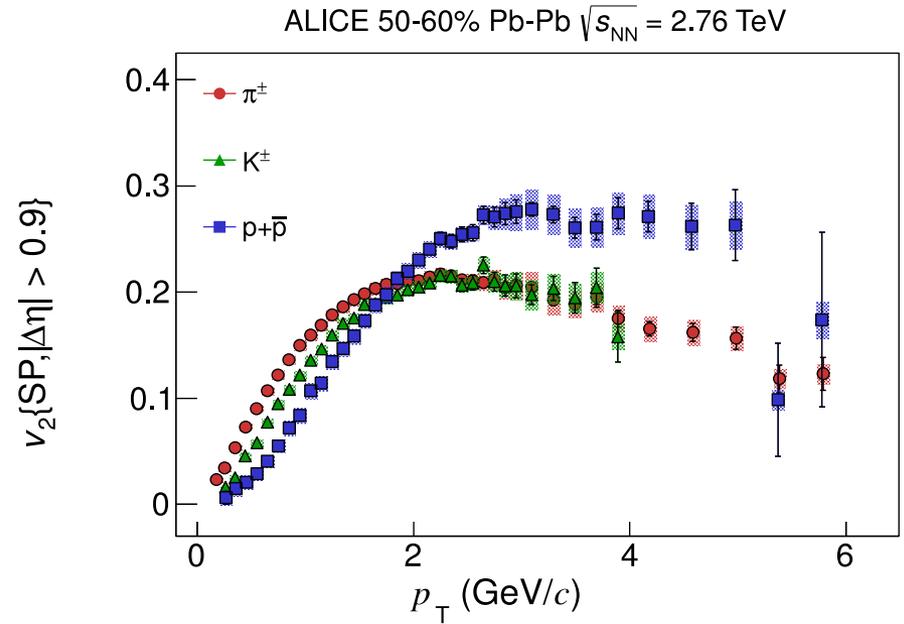
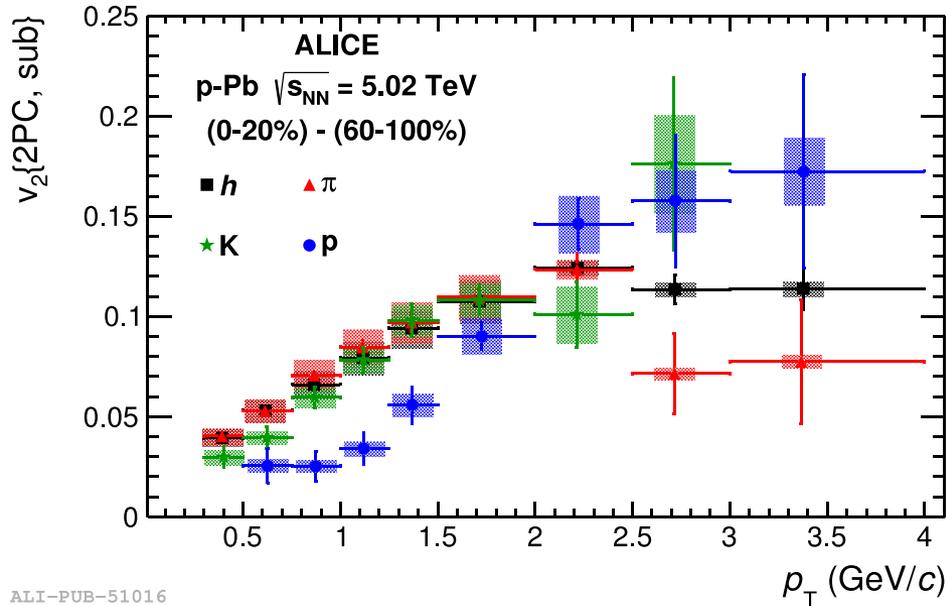
- Large dependence on  $\Delta\eta$  gap for  $c_3\{2\}$ . Increases with  $N_{ch}$  for large  $\Delta\eta$
- $v_3\{2\}$  consistent with Pb-Pb at same  $N_{ch}$   
 ✓  $\epsilon_3(\text{p-Pb})_{\text{RMS}} \sim \epsilon_3(\text{Pb-Pb})_{\text{RMS}}$  and driven by fluctuations?

# $v_2\{SP\}$ and $v_2\{2PC\}$ in p-Pb



- Both  $v_2\{SP\}$  and  $v_2\{2PC\}$  equivalent for current  $p_T$  selections
- “Centrality” characterized via multiplicity in V0 (Pb side)
- Mass ordering at high multiplicity
  - ✓ Different to pp
  - ✓  $v_2(p) < v_2(K)$
  - ✓  $v_2(\pi) \sim v_2(K)$
  - ✓ Hint of cross over in high mult. classes

# $v_2\{2PC, \text{sub}\}$ in p-Pb



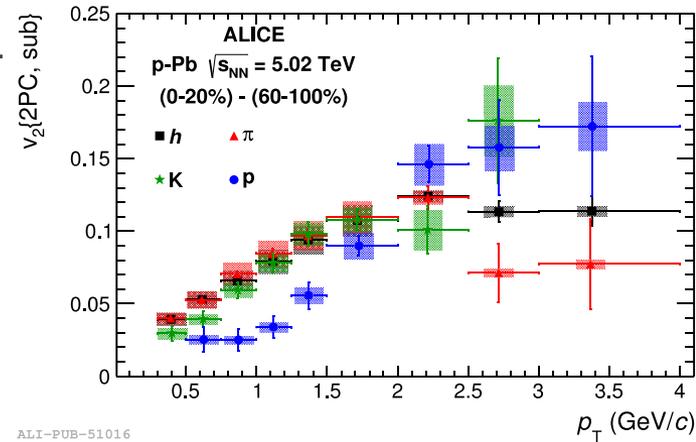
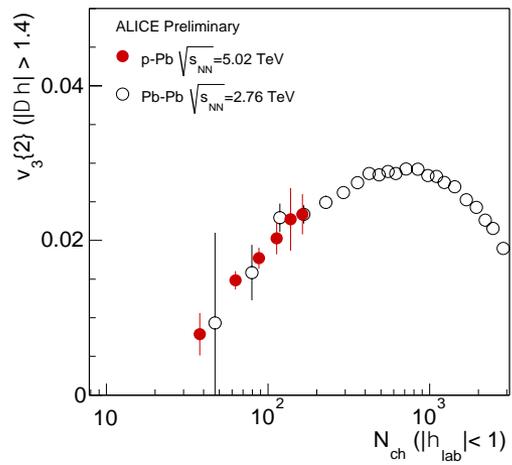
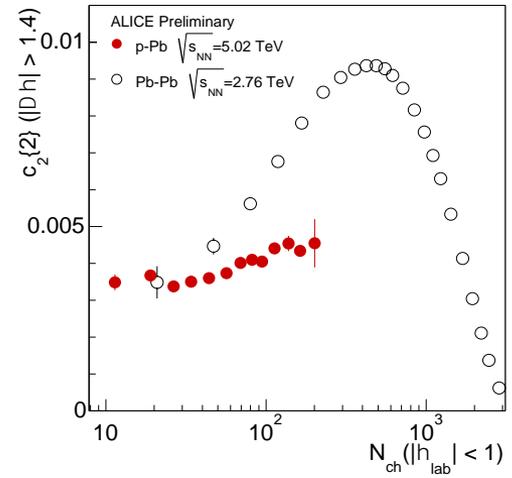
- $v_2\{2PC, \text{sub}\}$ : Obtained via **central yields – peripheral associated yields**
  - ✓ Aims to subtract non-flow
  - ✓ **Mass dependence more pronounced.**
  - ✓ Cross over of  $v_2(\pi)$  &  $v_2(p)$
- Qualitatively more similar to Pb-Pb.

# Summary

Experimental observations **highly suggestive of collective effects in high mult. p-Pb collisions**

- Integrated  $h^\pm v_n$  measurements
  - ✓  $c_2\{2\}$  rises in p-Pb with  $N_{ch}$  for large  $|\Delta\eta|$ . Naively inconsistent with non-flow.
  - ✓  $c_2\{4\}$  in p-Pb transitions from pos. to neg. values.  $v_2\{4\}$  becomes real.
  - ✓  $c_2\{m\}$  higher in Pb-Pb compared to p-Pb at same  $N_{ch}$ .
  - ✓  $v_3\{2\}$  in p-Pb and Pb-Pb similar at same  $N_{ch}$ .

- Differential PID  $v_2$  measurements
  - ✓ Different mass ordering in minbias pp and high mult. p-Pb.
  - ✓ Mass ordering in high mult. p-Pb more pronounced after non-flow subtraction.
  - ✓ Qualitative similar features to Pb-Pb collisions.

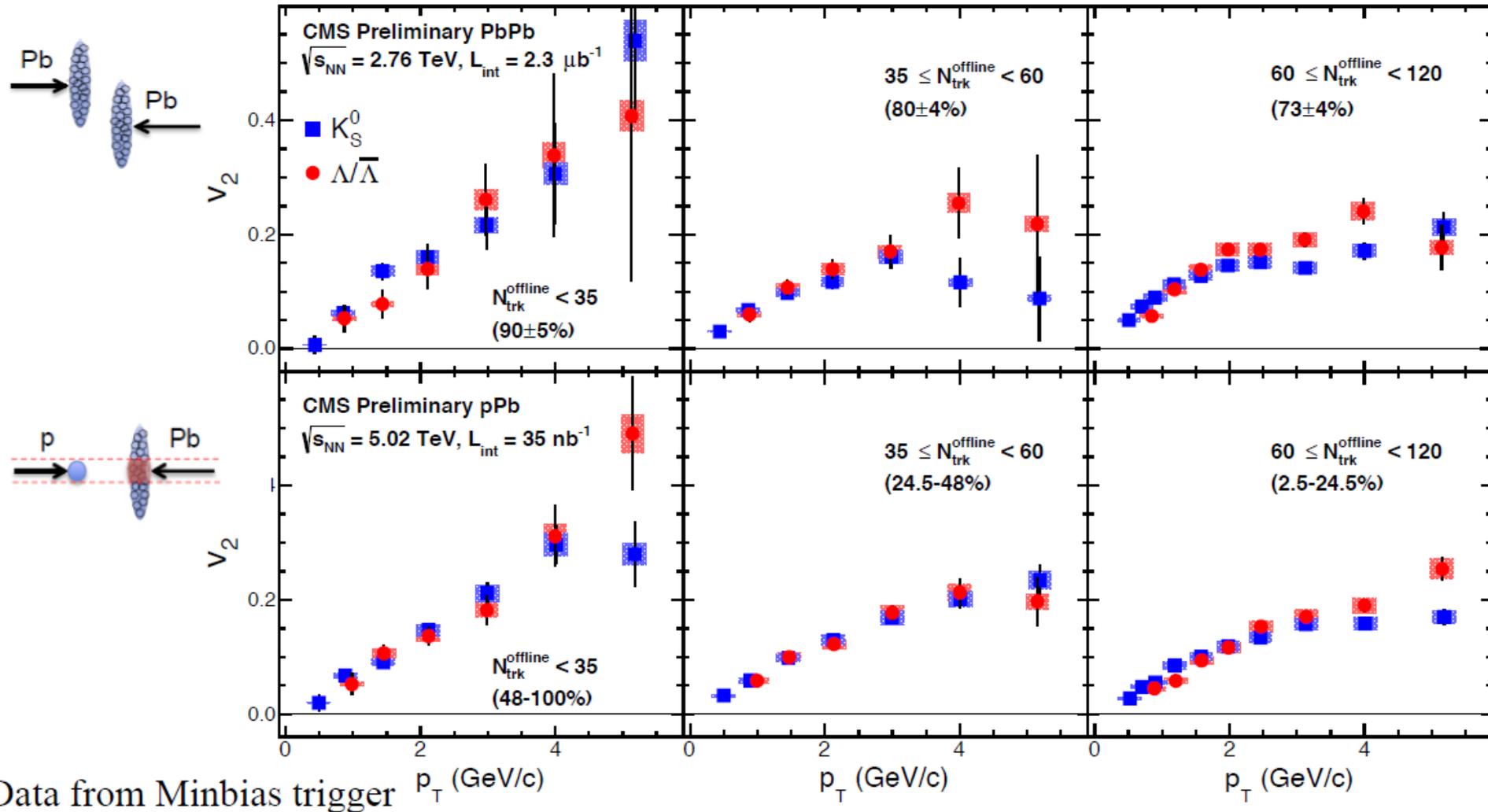


# CMS Results

Extracted from the following talks.

1. “Long range two-particle correlations with  $K_0^S$  and  $\Lambda$  in pPb and PbPb collisions” by [Monika Sharma](#)
2. “Pseudorapidity dependence of long-range two-particle correlations in pPb collision at CMS” by [Lingshan Xu](#)
3. “Azimuthal Anisotropy of Charged Particles from Multiparticle Correlations in pPb and PbPb Collisions” by [Quan Wang](#)

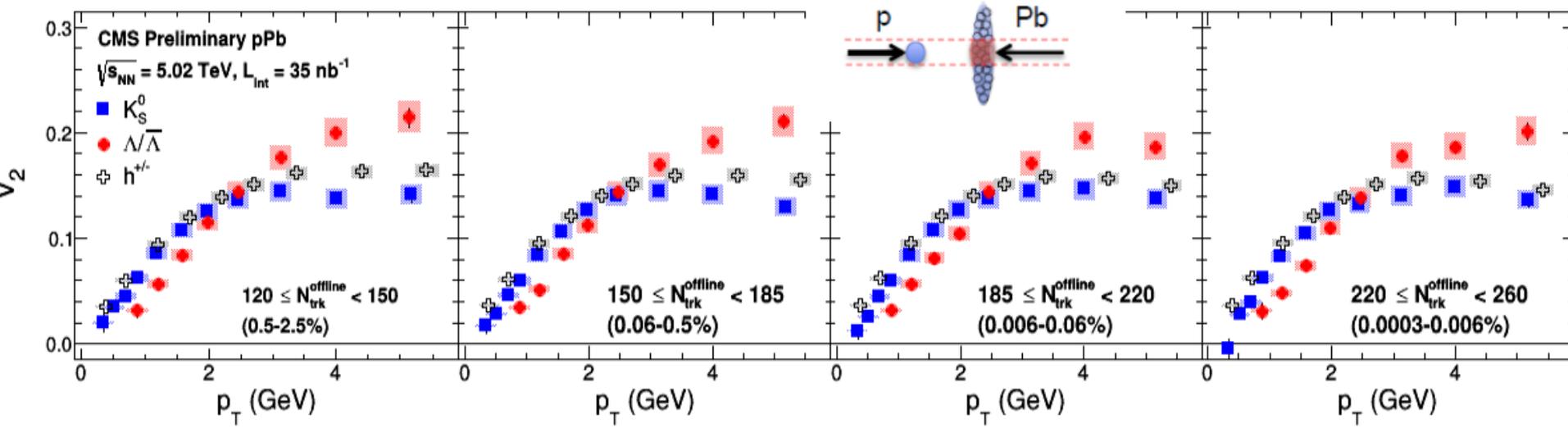
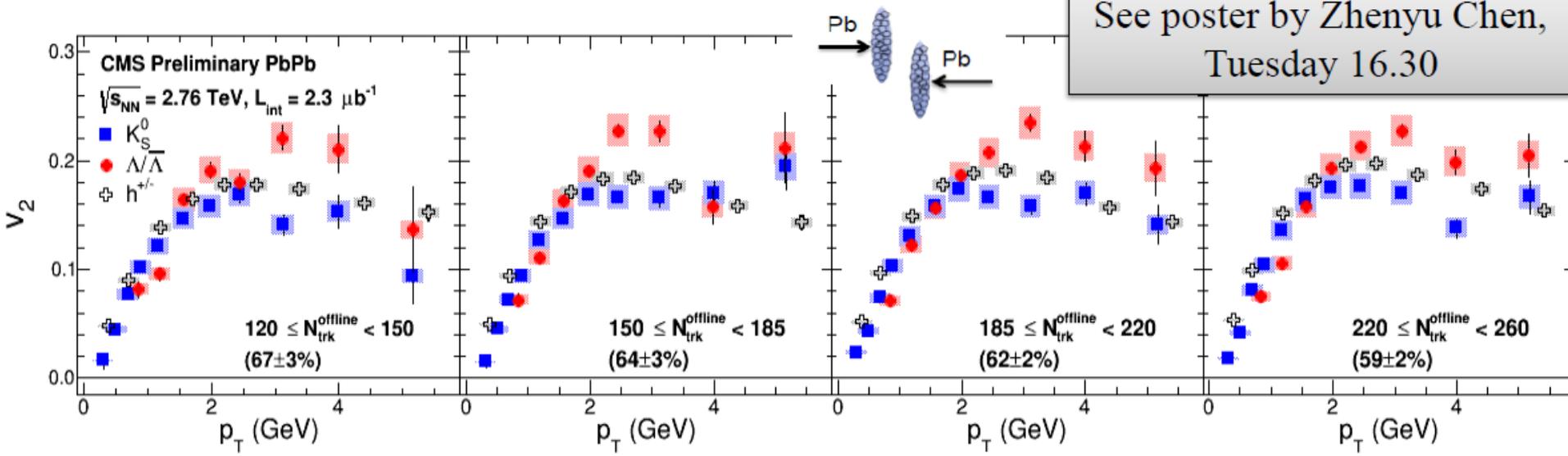
# Low multiplicity: pPb and PbPb collisions



- $v_2$  patterns almost the same for  $K_S^0$  and  $\Lambda$  at low multiplicity in both collision systems
- Crossing over observed for  $p_T$  around 2 GeV/c for 60 – 120 multiplicity range

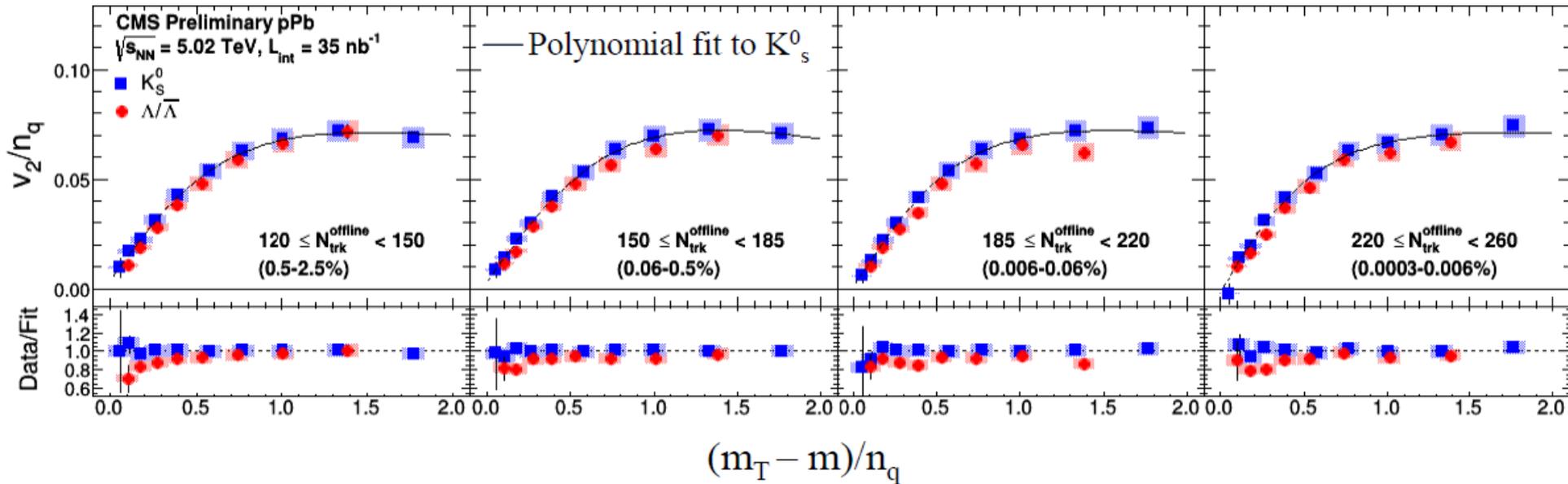
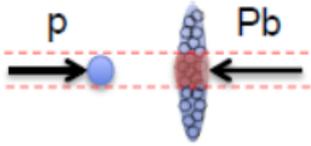
# Higher multiplicity class: pPb and PbPb collisions

See poster by Zhenyu Chen,  
Tuesday 16.30



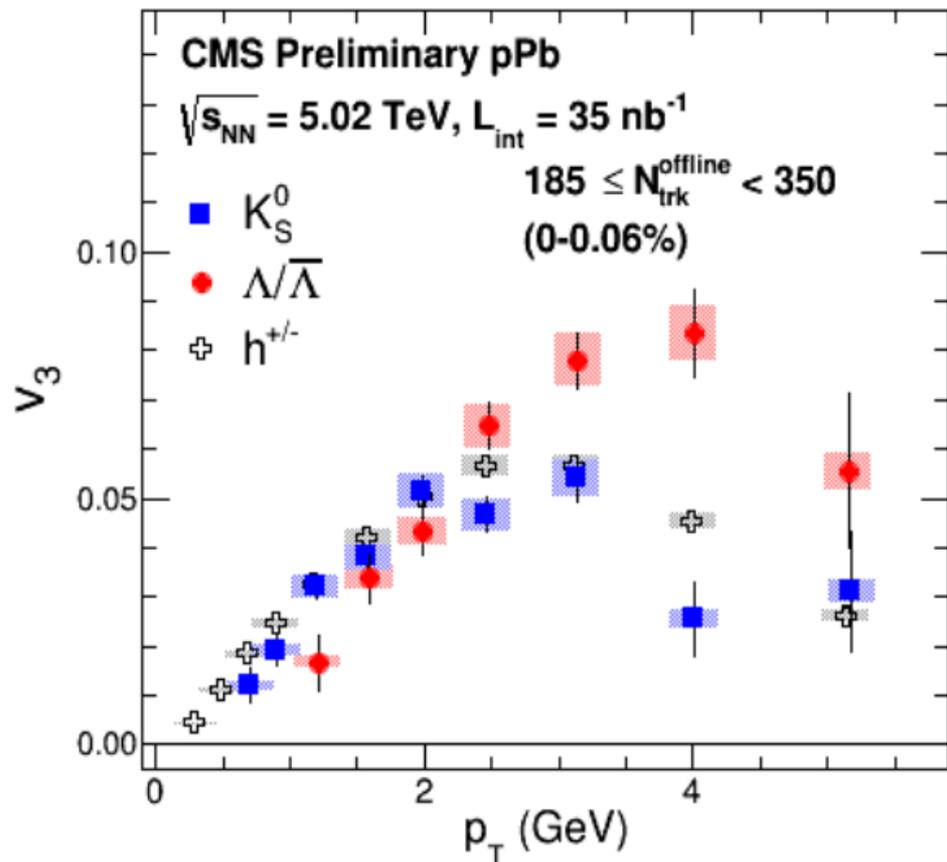
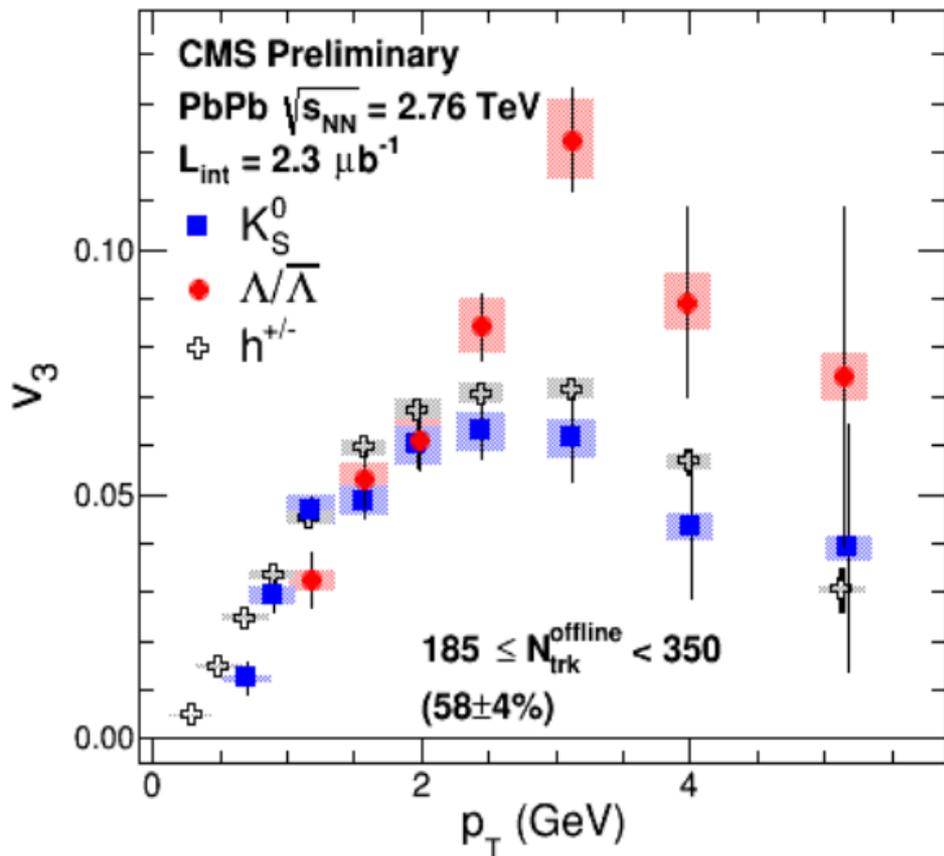
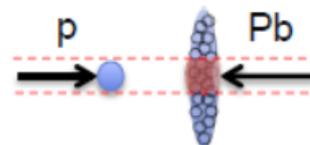
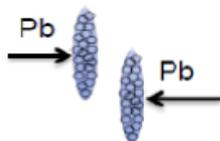
Mass ordering (below  $\sim 2 \text{ GeV}/c$ ) and cross-over (above  $\sim 2 \text{ GeV}/c$ ) observed

# NCQ scaling in high multiplicity pPb collisions



Azimuthal anisotropy develops at the partonic level in pPb collisions?

# $v_3$ in higher multiplicity



Cross-over (above  $\sim 2$  GeV/c) observed

# Conclusions

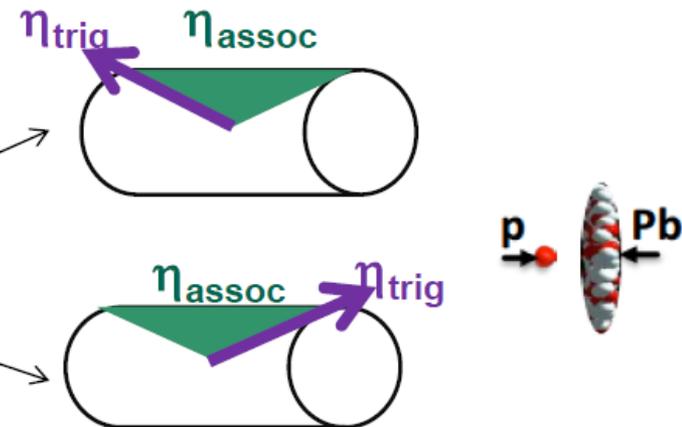
- Presented the second-order ( $v_2$ ) and third-order ( $v_3$ ) anisotropy harmonics of  $K_S^0$  and  $\Lambda$  for high-multiplicity in pPb collisions
  - Results were compared to similar multiplicities in PbPb collisions
- Low multiplicity in pPb and PbPb collisions:
  - $v_2$  patterns are almost the same for  $K_S^0$  and  $\Lambda$
- Higher multiplicity in pPb and PbPb collisions:
  - Mass ordering prominently observed in pPb collisions compared to PbPb collisions
  - Cross-over is seen at  $p_T$  around 2 GeV/c
- NCQ scaling observed for  $v_2$  in pPb collisions

# Analysis procedure

- Previous analyses integrated over trigger and associate  $\eta$ . Possible  $\Delta\eta$  dependence is averaged out.
- Use **fixed narrow trigger  $\eta$**  range:

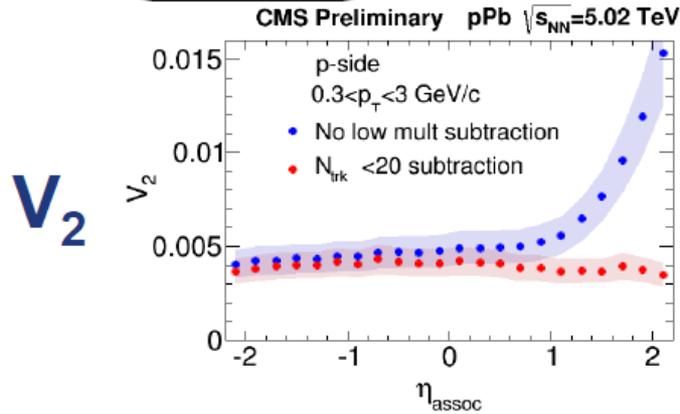
☐  $-2.4 < \eta_{trig} < -2.0$  (Pb-going side)

☐  $2.0 < \eta_{trig} < 2.4$  (p-going side)



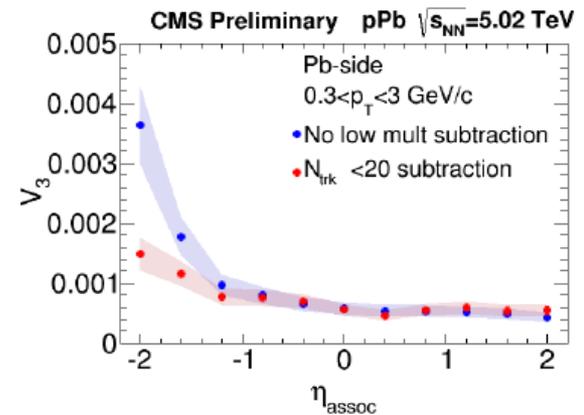
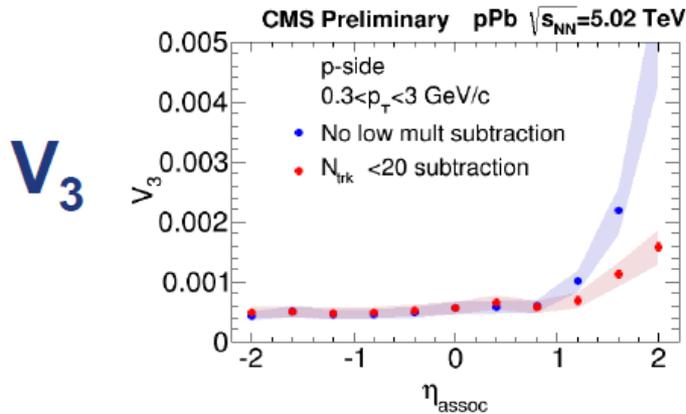
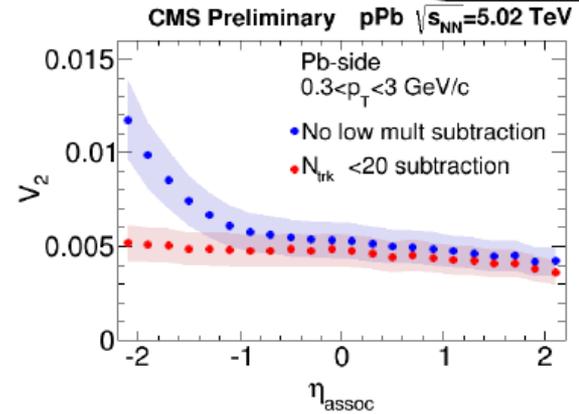
- Two-particle acceptance is 100%; no need to divide by mixed-events.
- Efficiency corrected for associated particles.
- Correlation normalized per trigger particle.
- $p_T^{trig} = 0.3-3$  GeV/c,  $p_T^{assoc} = 0.3-3$  GeV/c
- Low-multiplicity:  $2 \leq N_{trk}^{offline} < 20$ . High-multiplicity:  $220 \leq N_{trk}^{offline} < 260$

# Fourier coefficients $V_n$ from dihadron correlation



Blue: no jet subtraction

Red: with jet subtraction

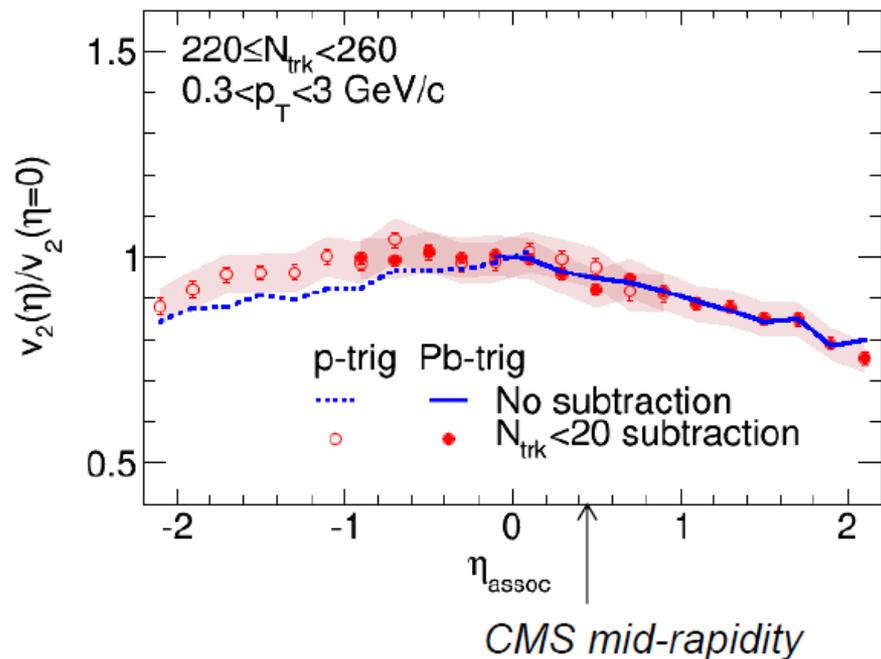


- Jet contribution mostly removed at short range.
- Small difference at long range: away jet contribution is small

# Extract $v_n(\eta)/v_n(0)$ from Fourier coefficient

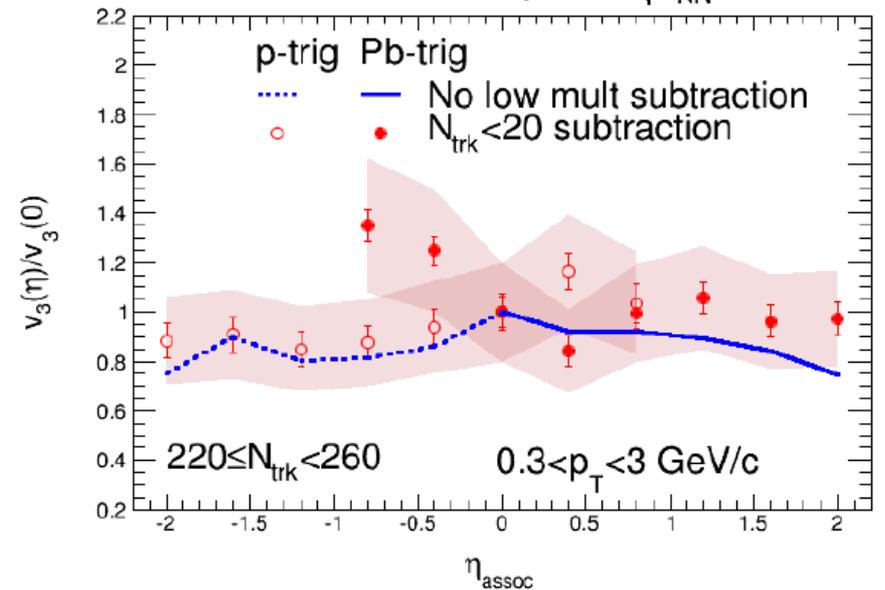
$v_2(\eta)/v_2(0)$ :

CMS Preliminary pPb  $\sqrt{s_{NN}}=5.02$  TeV



$v_3(\eta)/v_3(0)$ :

CMS Preliminary pPb  $\sqrt{s_{NN}}=5.02$  TeV

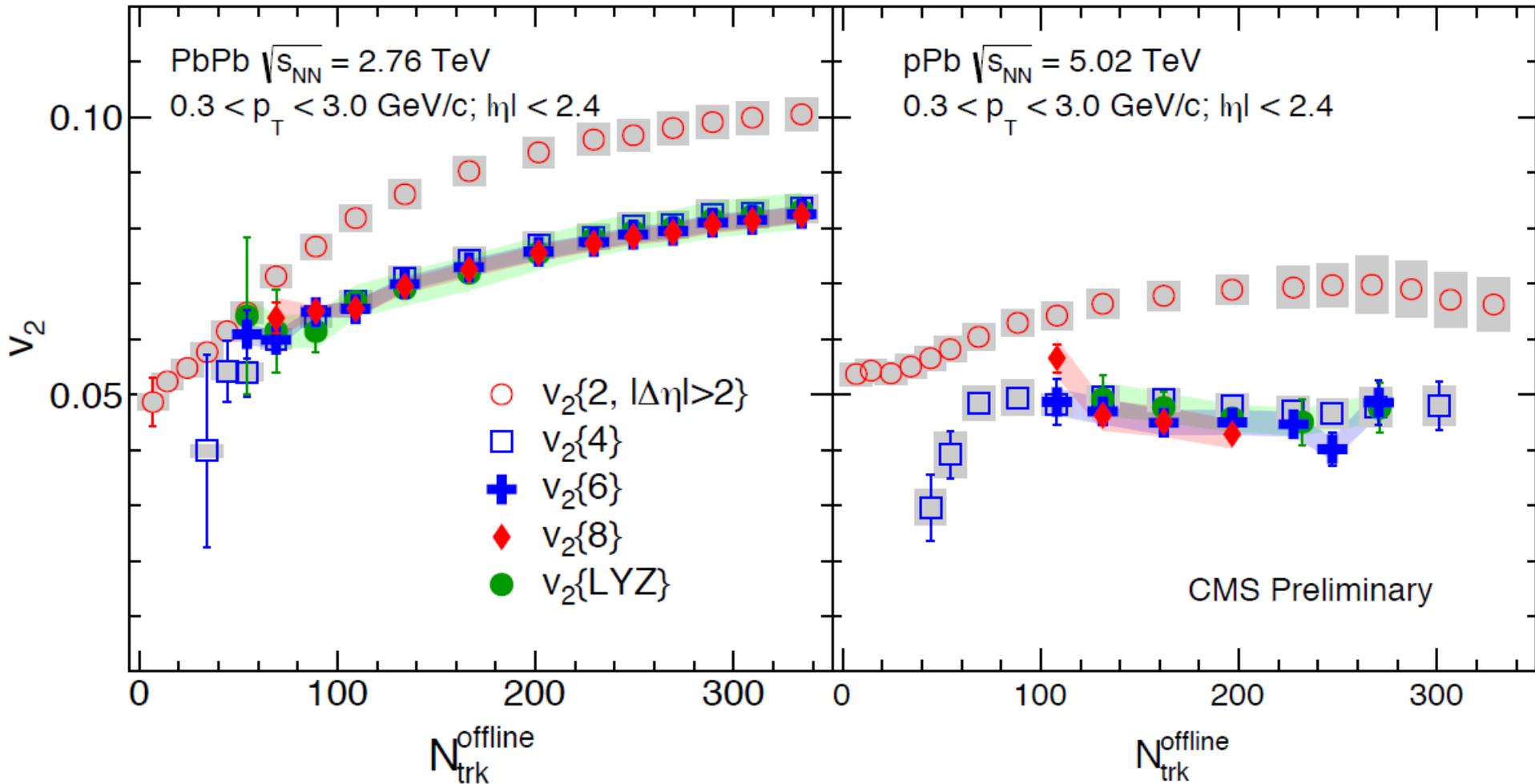


- $v_2$  shape is  $\eta$  dependent !
- $v_2$  from low-mult. subtraction: asymmetric about mid-rapidity
- With large errors, cannot draw conclusion for  $v_3$

# Conclusions

- Two-particle correlations studied in pPb, with trigger particles restricted to fixed, narrow windows, for Pb-going side ( $-2.4 < \eta_{trig} < -2.0$ ) and p-going side ( $2.0 < \eta_{trig} < 2.4$ )
- Near-side jet and ridge decomposed:  
Ridge yield depends on  $\eta$ , and different for Pb-going and p-going triggers.
- Fourier coefficients and self-normalized single-particle harmonics extracted:  
Significant  $\eta$  dependence observed for  $v_2$ .

# Results – $v_2$



*“Lee-Yang Zeros”*  
all-particle correlation

➤  $v_2\{4\}$ ,  $v_2\{6\}$ ,  $v_2\{8\}$  and  $v_2\{\text{LYZ}\}$  are in good agreement  $\pm 10\%$

# Result – Cumulant $v_2$ ratios

## ➤ In hydrodynamic picture

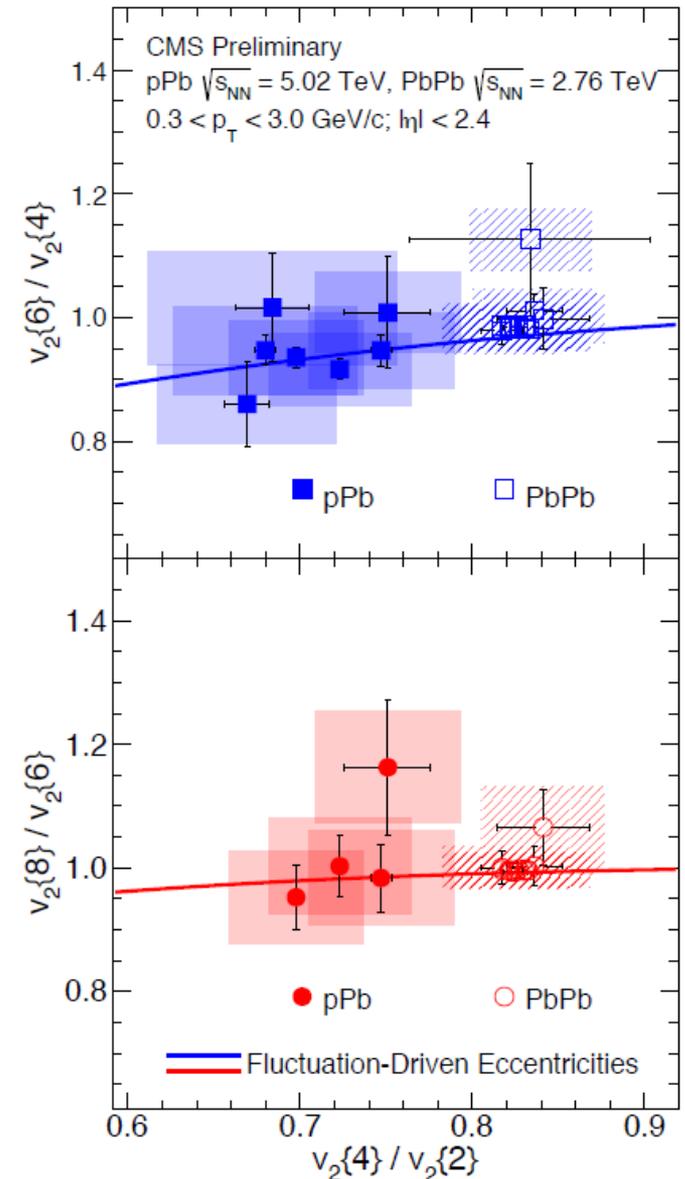
- arXiv:1311.7325  
(Bzdak, Bozek & McLerran)
- PRL 112 (2014) 082301  
(Yan & Ollitrault)

$$\varepsilon_2\{4\} \cong \varepsilon_2\{6\} \cong \varepsilon_2\{8\}$$

$$v_2\{4\} \cong v_2\{6\} \cong v_2\{8\}$$

## ➤ Fluctuation-driven initial-state eccentricities

[PRL 112 (2014) 082301 (Yan & Ollitrault)]



# Summary

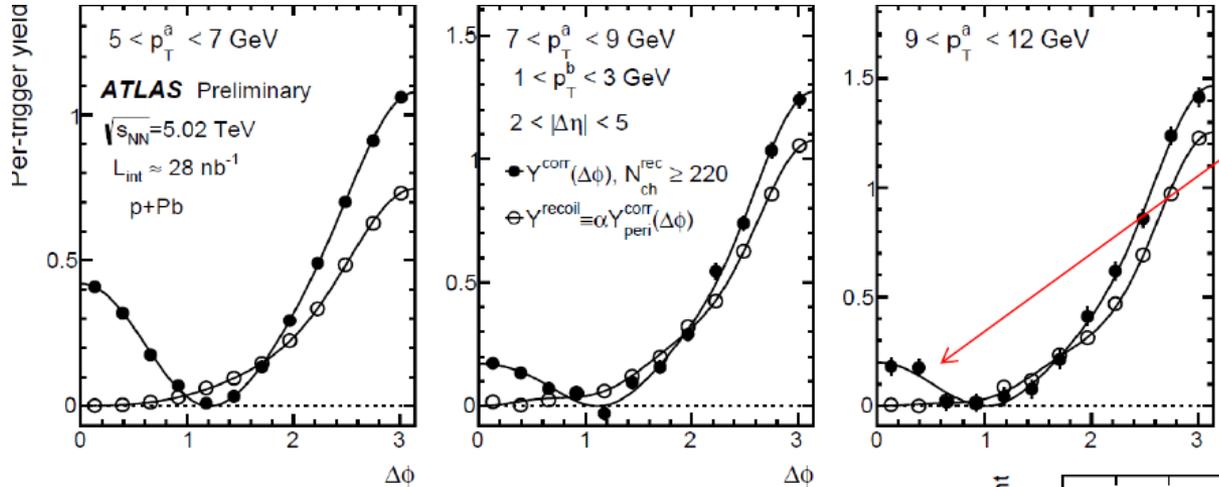
- 6-, 8- and all-particle correlations are measured for the first time in pPb collisions at 5.02 TeV
- A direct comparison is made between pPb and PbPb as a function of multiplicity
- $v_2\{4\}$ ,  $v_2\{6\}$ ,  $v_2\{8\}$  and  $v_2\{LYZ\}$  are consistent within 10% in pPb and PbPb, respectively
- Relative ratios of  $v_2$  from cumulant methods are consistent with hydrodynamic predictions within current statistical precision

# ATLAS Results

Extracted from the following talks.

1. “Measurement of the long-range pseudorapidity correlations and azimuthal harmonics in  $\sqrt{s}=5.02$  TeV proton-lead collisions with the ATLAS detector” by [Sooraj Radhakrishnan](#)
2. “Flow harmonics in Pb+Pb collisions at energy of  $\sqrt{s_{NN}} = 2.76$  TeV with the ATLAS detector” by [Dominik Derendarz](#)

# Ridge at higher $p_T$

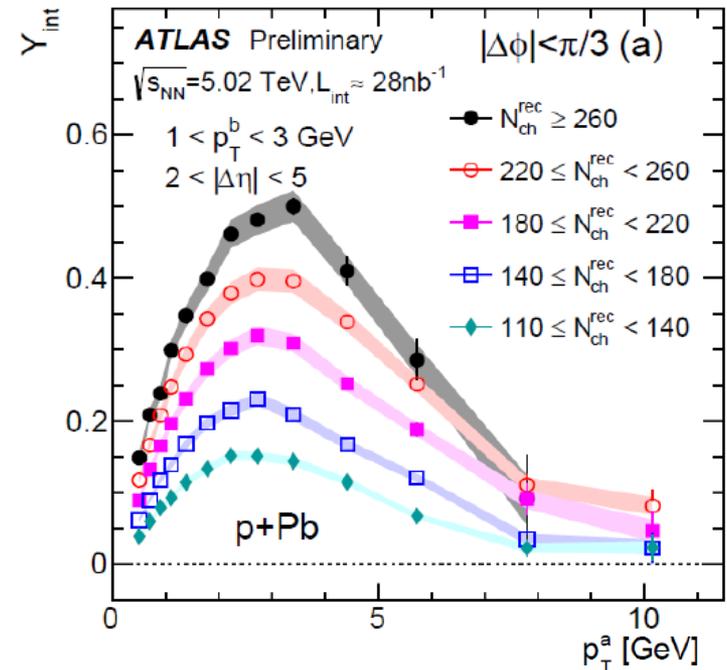


■ Near-side ridge visible through the entire  $p_T$  range studied.

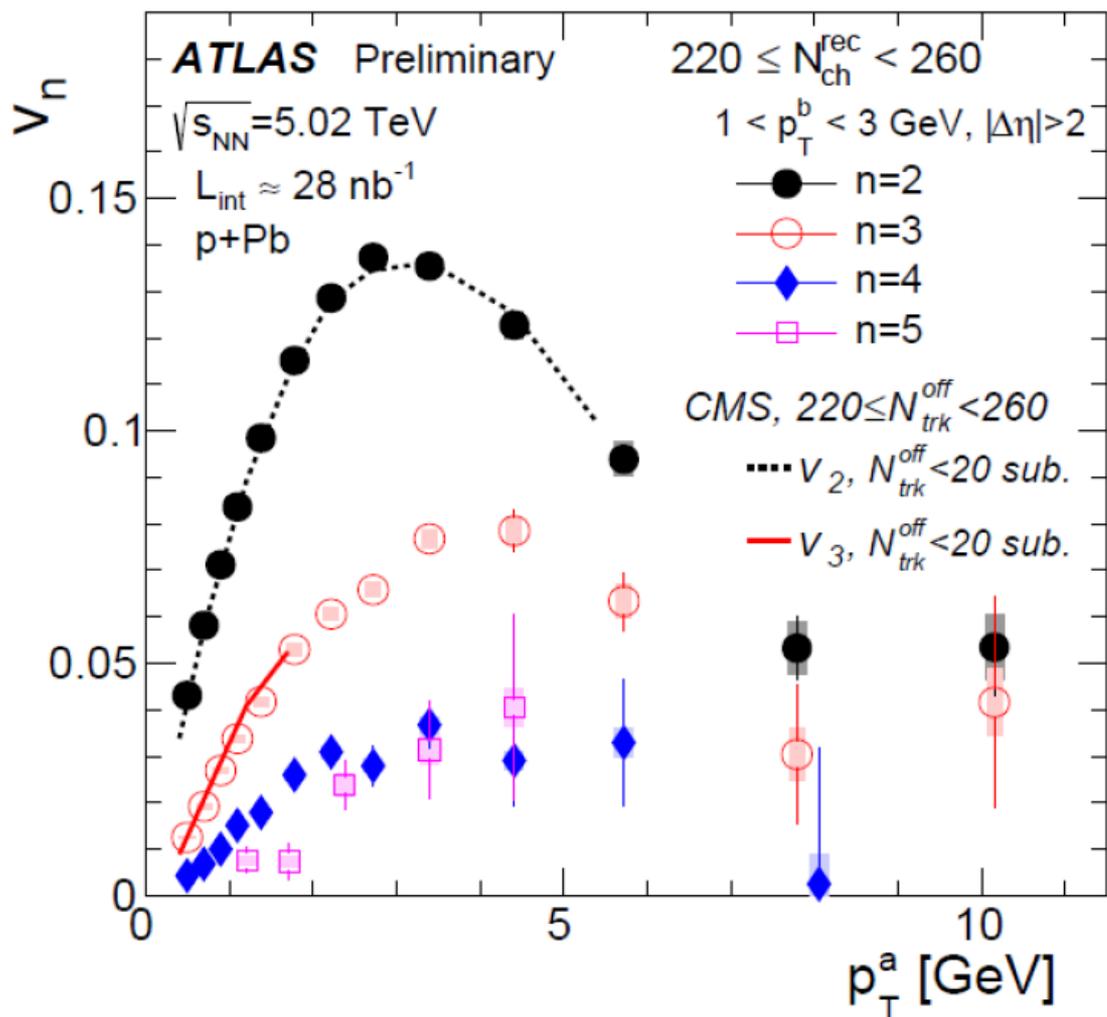
$$Y^{corr}(\Delta\phi) = \frac{\int B(\Delta\phi)d\Delta\phi}{\pi} \left( \frac{S(\Delta\phi)}{B(\Delta\phi)} - b_{ZYAm} \right)$$

- Integrated yield on near-side:
  - increase with  $p_T$
  - reaches maximum  $\sim 3$  --  $4$  GeV and then decreases.

$$Y_{int} = \int_a^b Y^{corr}(\Delta\phi) d\Delta\phi$$

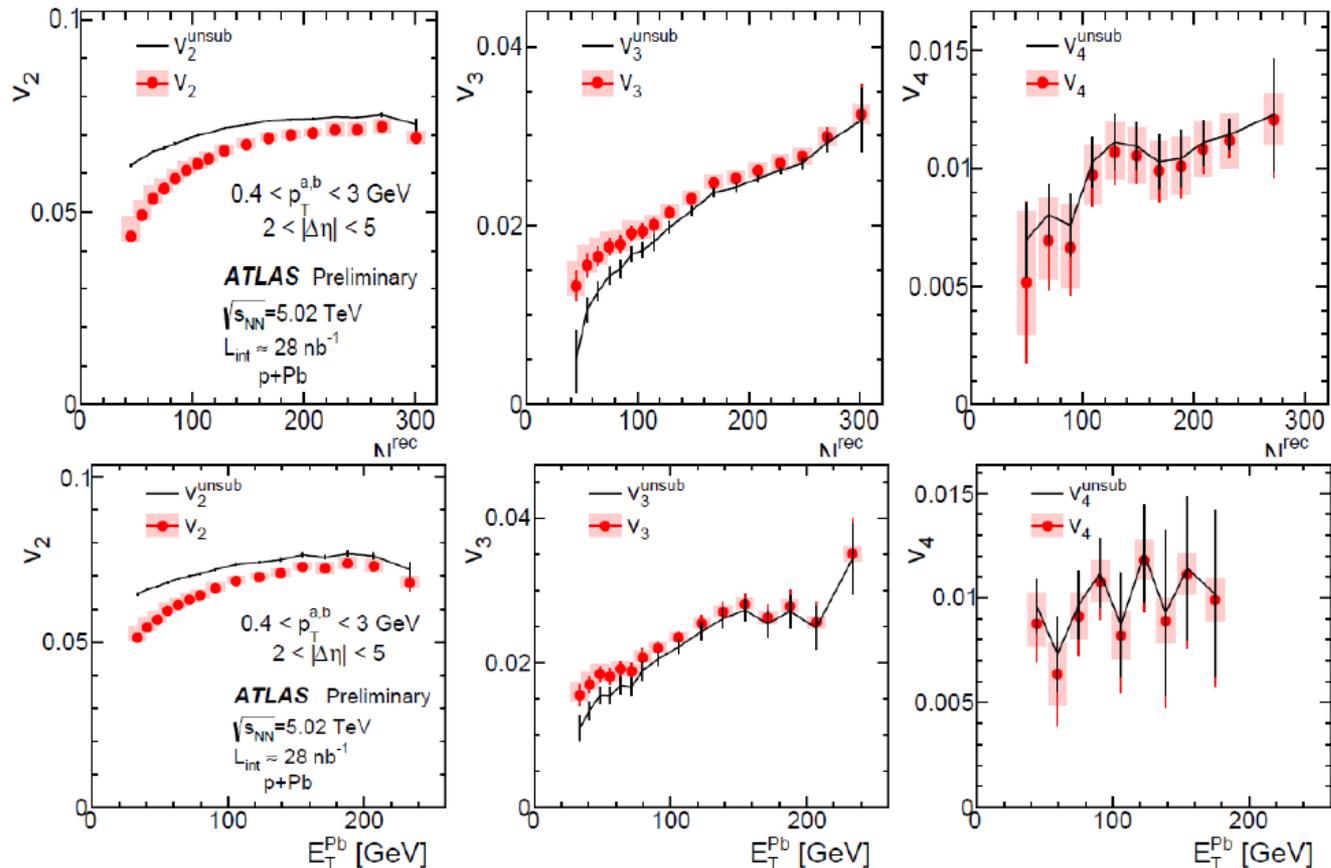


# $v_n$ vs $p_T$ for different $n$



- $v_n$  decrease with increasing  $n$ .
- Rise with  $p_T$  at low  $p_T$  and then decrease.
- Non-zero  $v_5$  in high multiplicity event classes.

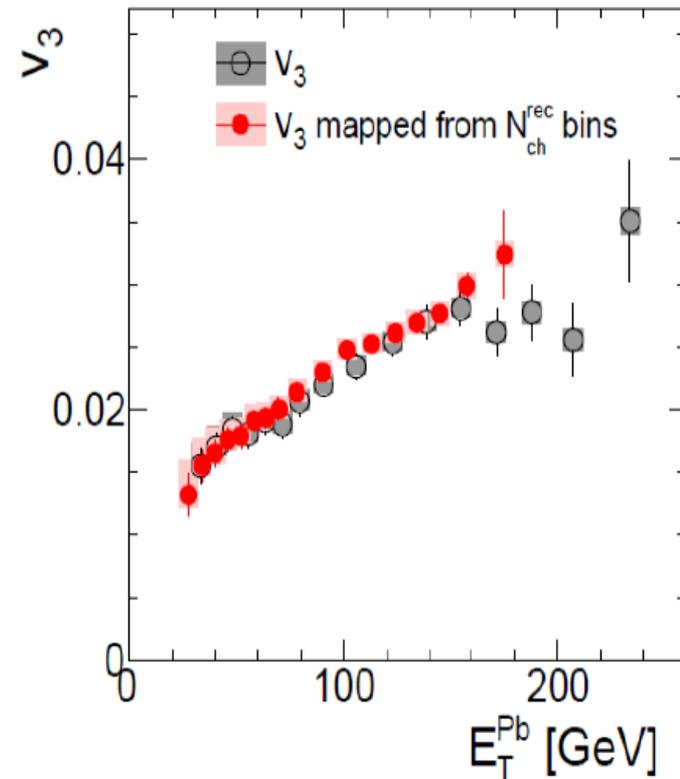
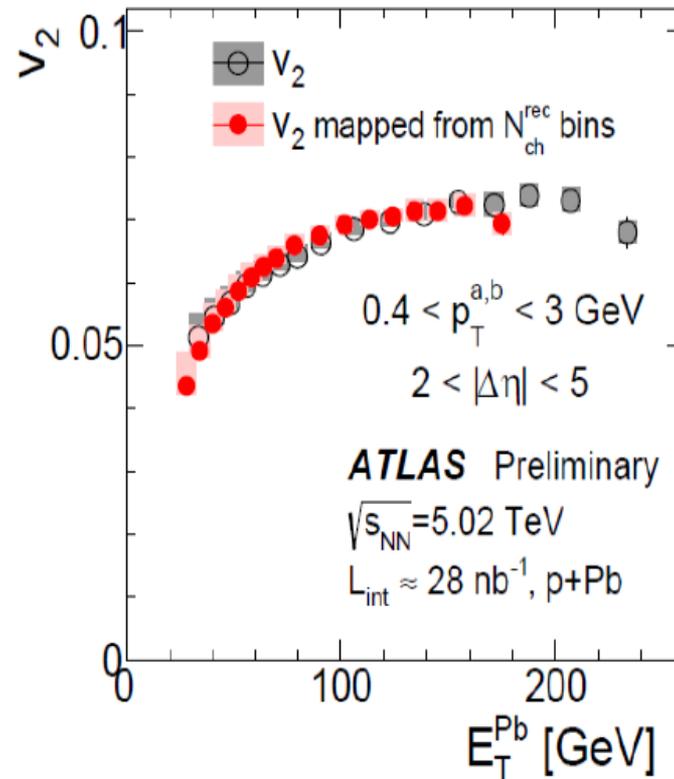
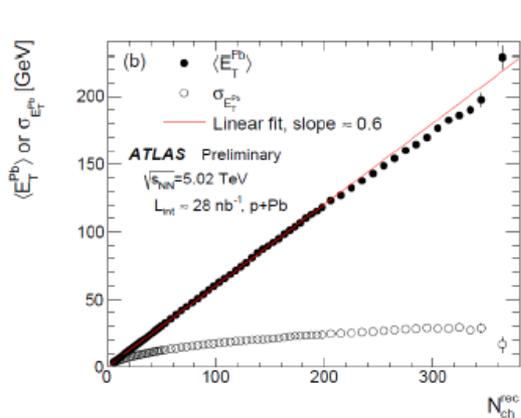
# $v_n$ : Event activity dependence



Similar behavior seen for  $E_T^{Pb}$  dependence

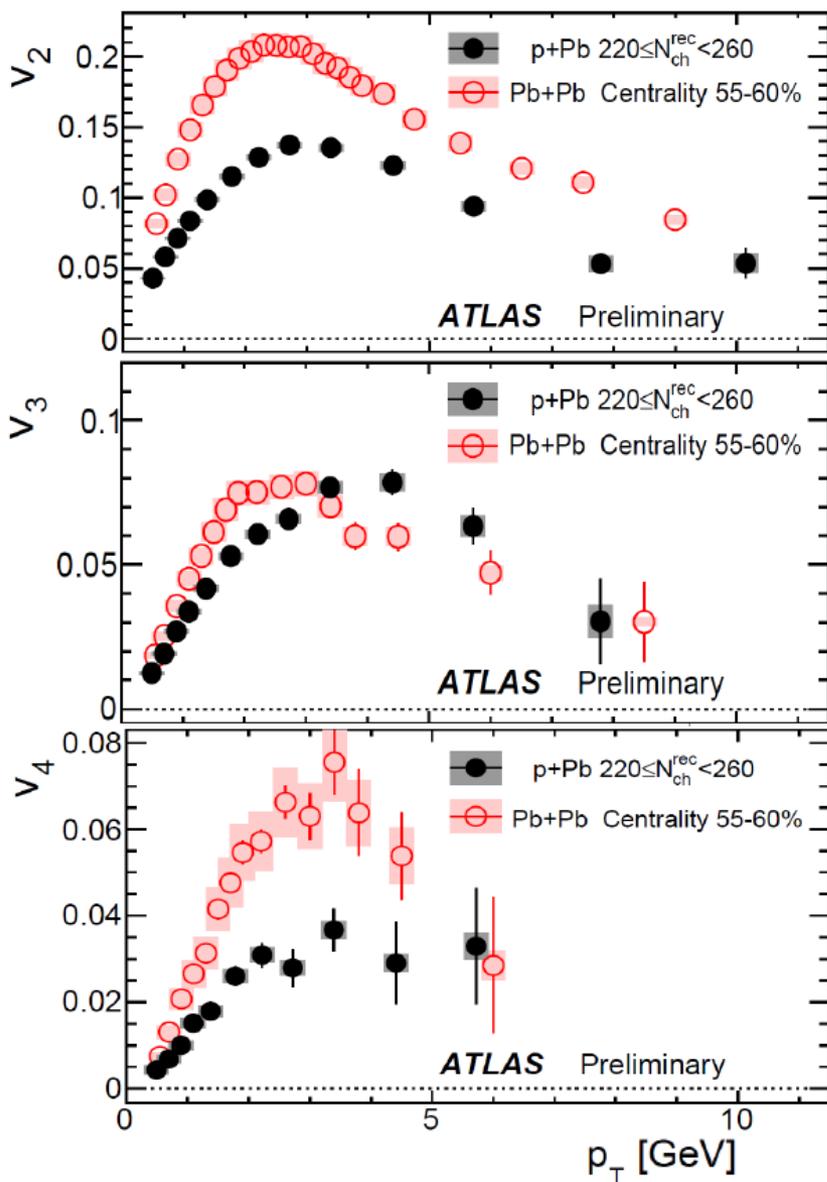
- $v_2$  show less variation for  $N_{ch}^{rec} > 150$ , while  $v_3$  continue to increase
- Recoil contribution does not affect the  $v_n$  for large  $N_{ch}^{rec}$ , but significant deviations see for smaller  $N_{ch}^{rec}$

# Mapping $N_{\text{ch}}^{\text{rec}}$ - dependence to $E_T^{\text{Pb}}$ dependence



- The  $v_n$  values for  $N_{\text{ch}}^{\text{rec}}$  is plotted at corresponding  $\langle E_T^{\text{Pb}} \rangle$  value, using the  $N_{\text{ch}}^{\text{rec}}$  vs  $E_T^{\text{Pb}}$  correlation data.
- Good consistency suggest that two event-activity definition captures the same azimuthal anisotropy of the long-range correlation.

# Comparison of $v_n$ in p+Pb and peripheral Pb+Pb



- Significantly larger  $v_2$  and  $v_4$  in Pb+Pb, but comparable magnitudes for  $v_3$ !

- Large elliptic geometry from overlap in PbPb

- $v_4$  and  $v_2$  are coupled

$$v_4 = \sqrt{c_0^2 + c_1^2 v_2^4} \quad (\text{see talk by Soumya})$$

- Compare  $v_n(p_T)_{p+Pb}$  with  $v_n(p_T/K)_{Pb+Pb}$ , (Teaney et al arXiv:1312.6770 [nucl-th].)

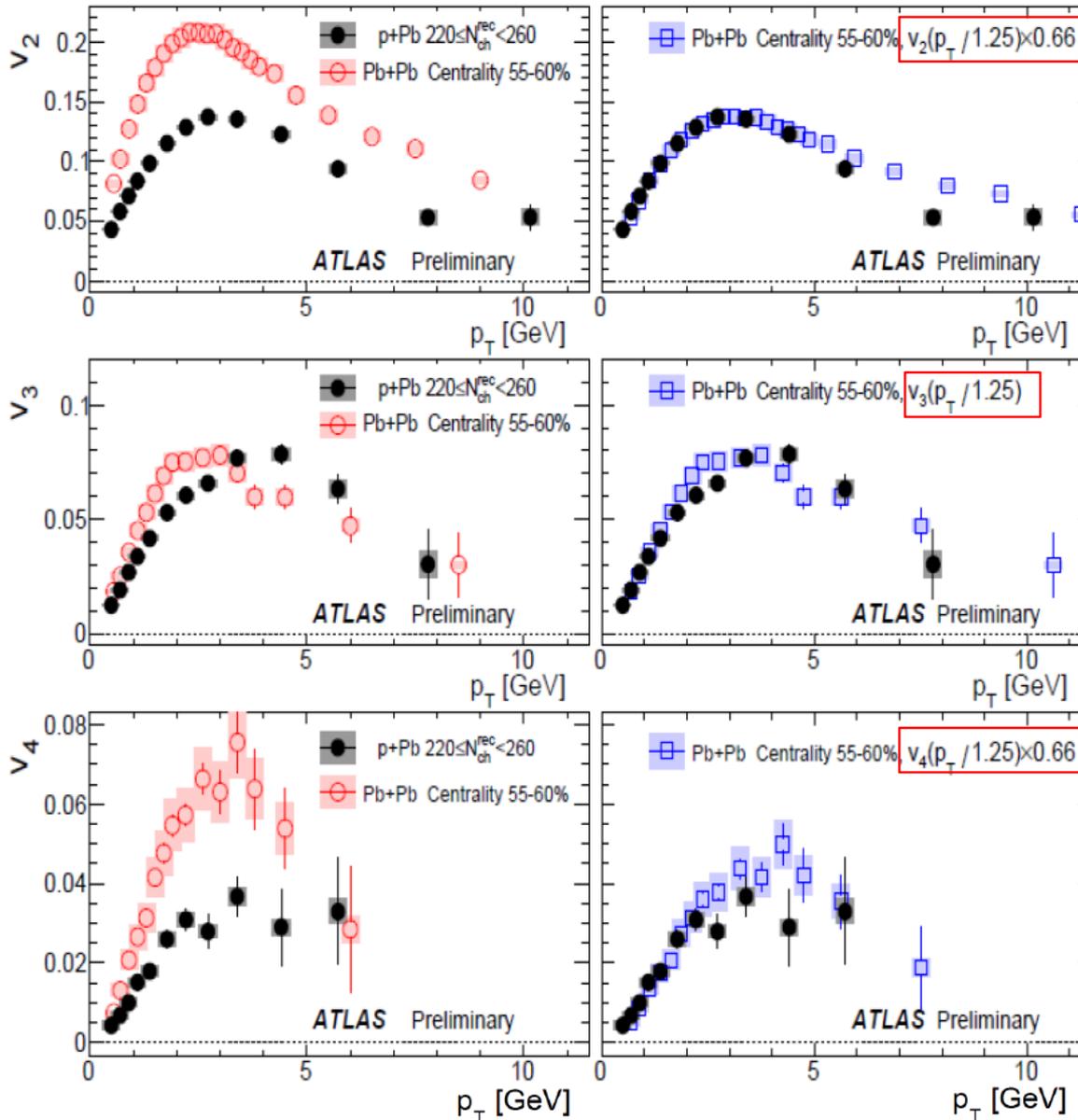
- $K=1.25$ , ratio of  $\langle p_T \rangle$ .

- p+Pb:  $\langle N_{ch} \rangle \pm \sigma = 259 \pm 13$
- Pb+Pb:  $\langle N_{ch} \rangle \pm \sigma = 241 \pm 43$

They both emerge from a collective response to the geometry dictated by  $\frac{l_{mfp}}{L} = f(dN/dy)$

$L$  is the transverse size of the high multiplicity events

# $v_n$ scaling between the p+Pb and Pb+Pb systems



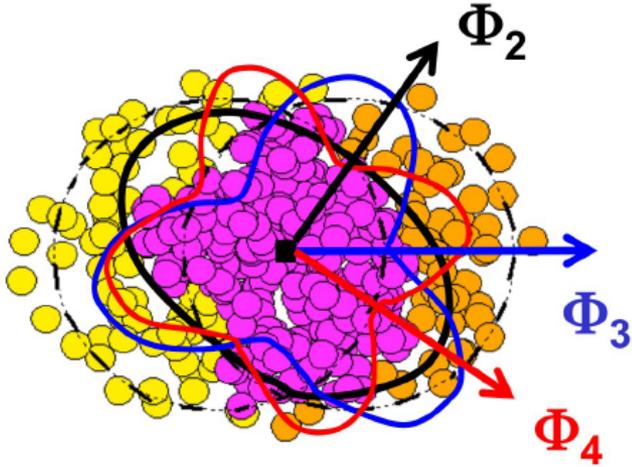
- $v_2$  values, after scaling the  $p_T$  axis, differ only by a scale factor between the two systems.
- Suggests a similar origin for  $v_2$  in the two systems and similar medium response to initial geometry?

# Summary and Conclusions

- The long-range correlation in high multiplicity events persists to  $p_T \sim 12$  GeV.
- $v_n$  vs  $p_T$  and event-activity.
  - First 5 Fourier harmonics measured
  - The magnitude of  $v_n$  decrease with increasing n.
  - $v_n$  found to increase with  $N_{ch}^{rec}$  and  $E_T^{Pb}$ , but  $v_2$  shows a saturation at higher event activity values.
- Comparison with peripheral Pb+Pb
  - $v_n(p_T)$ ,  $n = 1, 2, 3$ , are compared between p+Pb and Pb+Pb collisions with similar multiplicity.
  - Similar shape in  $p_T$  observed, once a scaling is applied to account for the difference in mean  $p_T$  between the two systems.

“Based on an independent cluster model and a simple conformal scaling argument, where the ratio of the mean free path to the system size stays constant at fixed multiplicity, we argue that flow in p+A emerges as a collective response to the fluctuations in the position of clusters, just like in A + A collisions.”  
(arXiv:1312.6770)

# Cumulant method



- Higher order harmonics arise due to event-by-event fluctuations in the initial geometry
- Cumulants technique allows for measurement of the  $v_n$  fluctuations

- Multi-particle ( $2k$ ) cumulants are insensitive to lower order correlations ( $< 2k$ ) – non flow eliminated by construction
- Generating function is used to obtain  $2k$ -particle correlation and cumulants  
(N. Borghini, P. M. Dinh, J. -Y. Ollitrault, arXiv:nucl -ex/0110016)
- Cumulants give estimate of relative fluctuations:

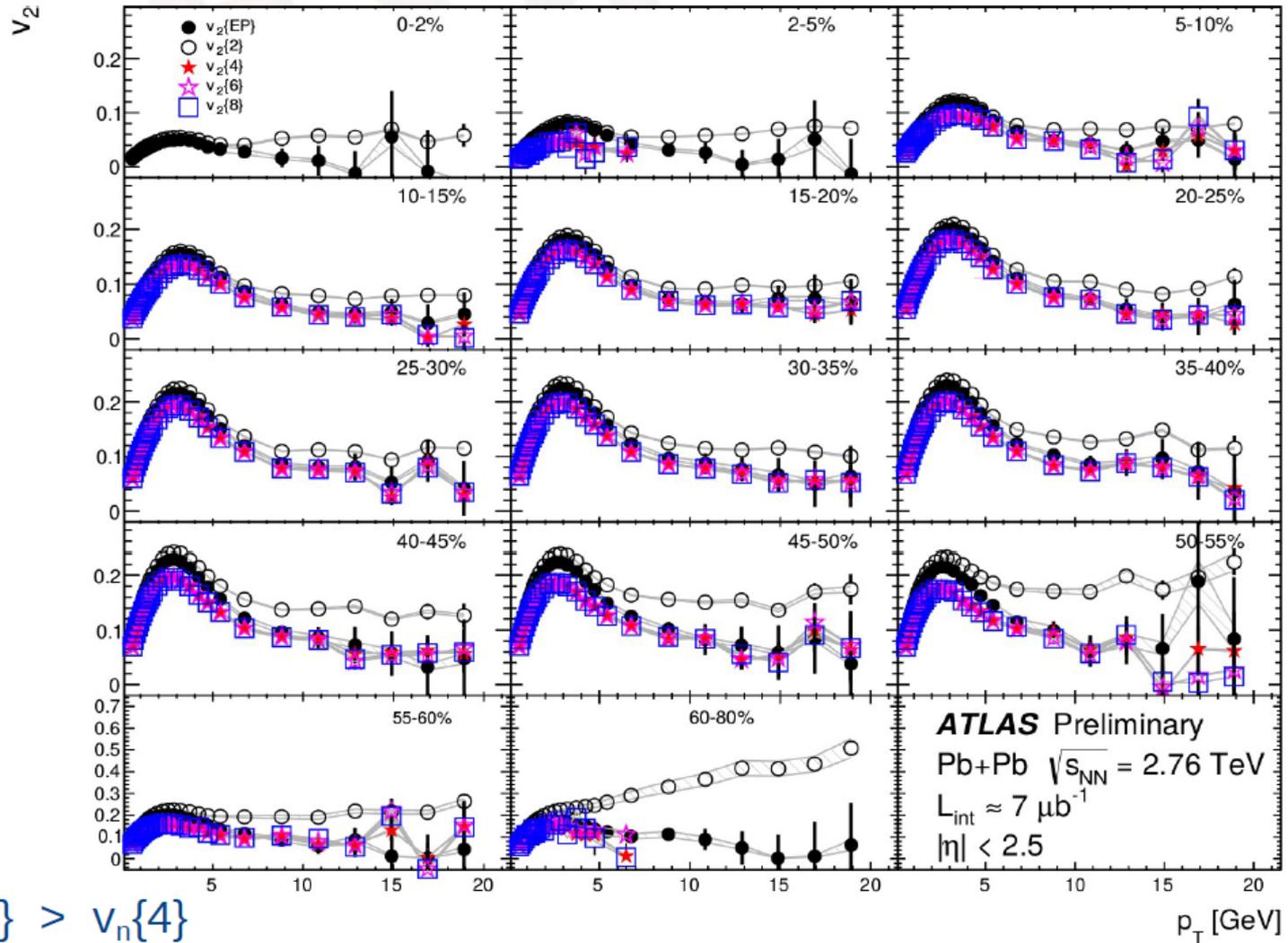
$$F(v_n) = \sqrt{\frac{v_n\{2\}^2 - v_n\{4\}^2}{v_n\{2\}^2 + v_n\{4\}^2}}$$

$$\begin{aligned} v_n\{2\} &\cong v_n^2 + \sigma_n^2 + \delta \\ v_n\{4\} &\cong v_n^2 - \sigma_n^2 \end{aligned}$$

- To lower the influence of non-flow effects in estimation of relative fluctuations  $v_n\{EP\}$  is used instead of  $v_n\{2\}$

# Transverse momentum dependence of $v_2$

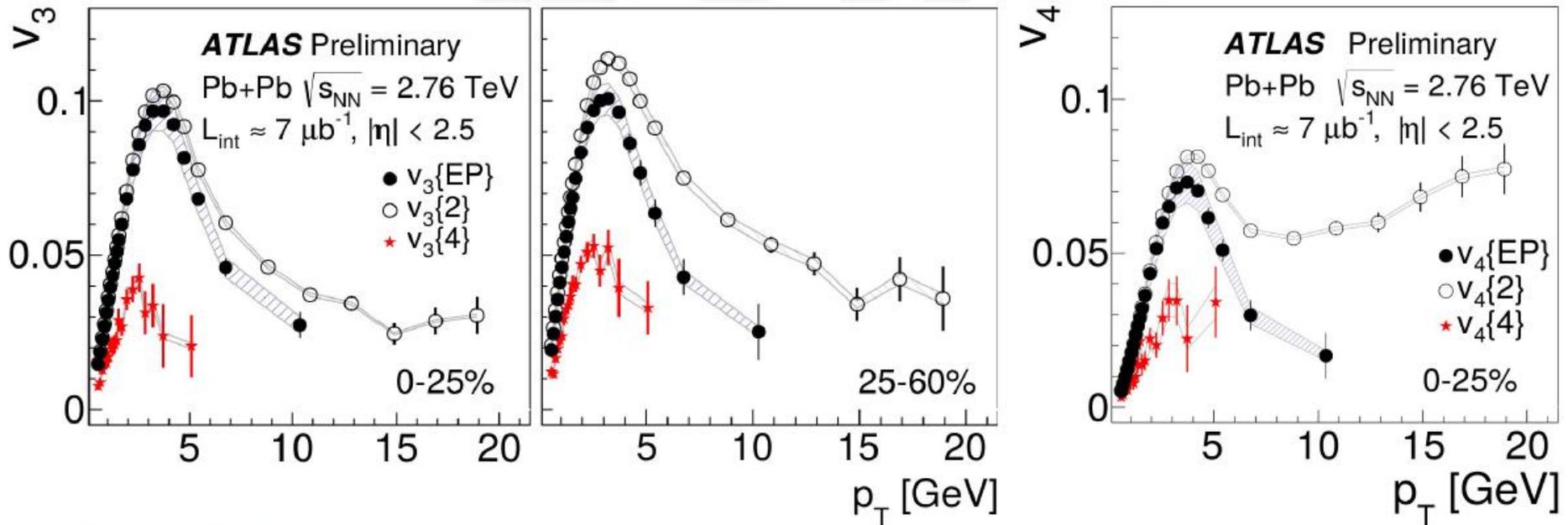
- $v_2\{2\}$  shows a strong flow signal at high  $p_T$  (jet-like correlations)
- Strong reduction of  $v_2$  by using more than 2 particle correlations



$$v_n\{2\} > v_n\{EP\} > v_n\{4\}$$

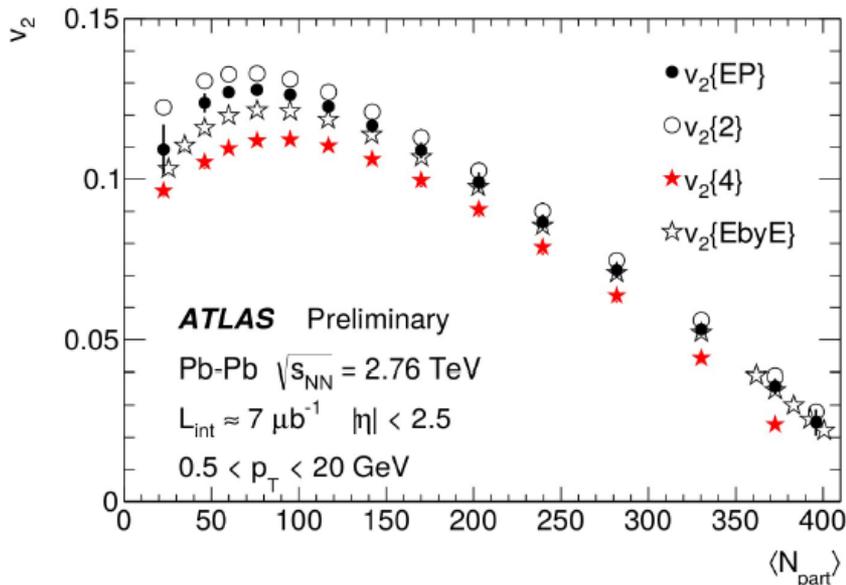
$$v_n\{4\} \approx v_n\{6\} \approx v_n\{8\}$$

# Transverse momentum dependence of $v_3$ and $v_4$

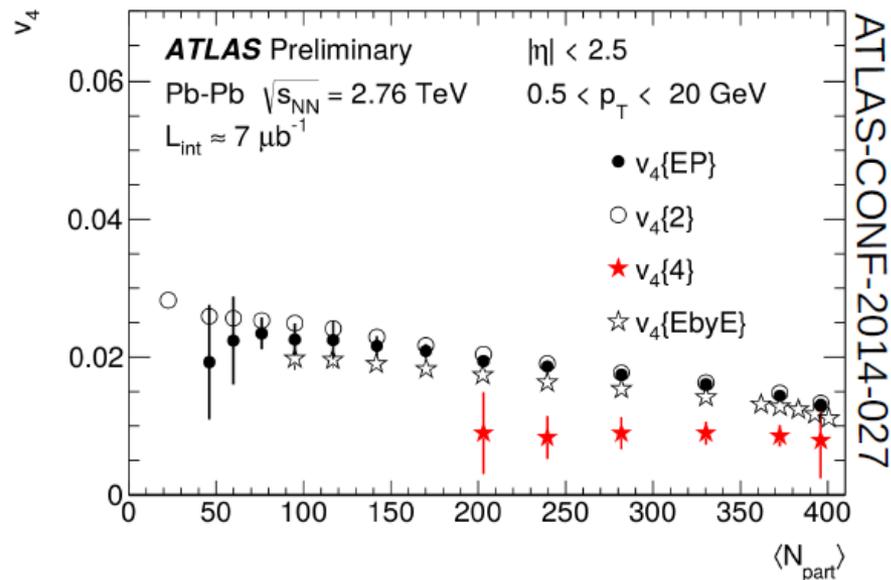
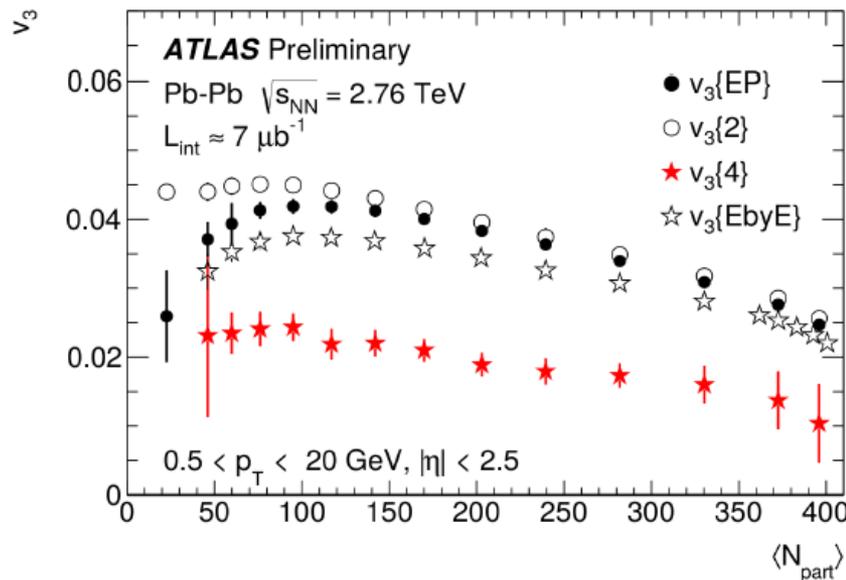


- Significant values of  $v_3\{4\}$  and  $v_4\{4\}$  calculated
- $v_{3,4}\{4\} < v_{3,4}\{2\}$  - expected from fluctuations and suppression of non-flow effects

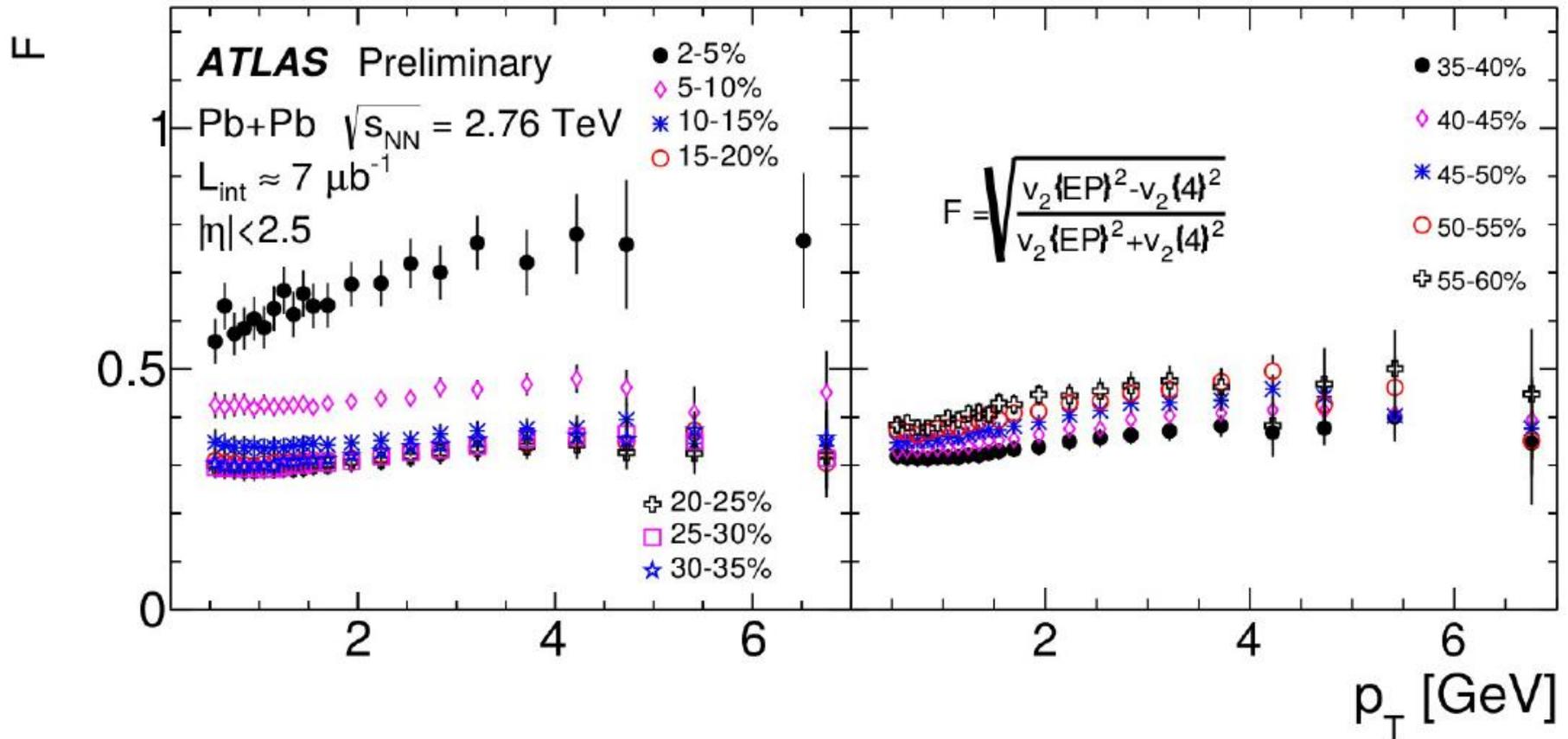
# Centrality dependence of $v_n$ harmonics



- Comparison of  $v_n\{2,4\}$  with  $v_n\{EP\}$  and  $v_n\{EbyE\}$  (mean of  $p(v_n)$  distribution)
- $v_n\{2\} > v_n\{EP\} > v_n\{EbyE\} > v_n\{4\}$
- Difference between  $v_n\{2\}$  and  $v_n\{4\}$  more pronounced for  $n = 3, 4$

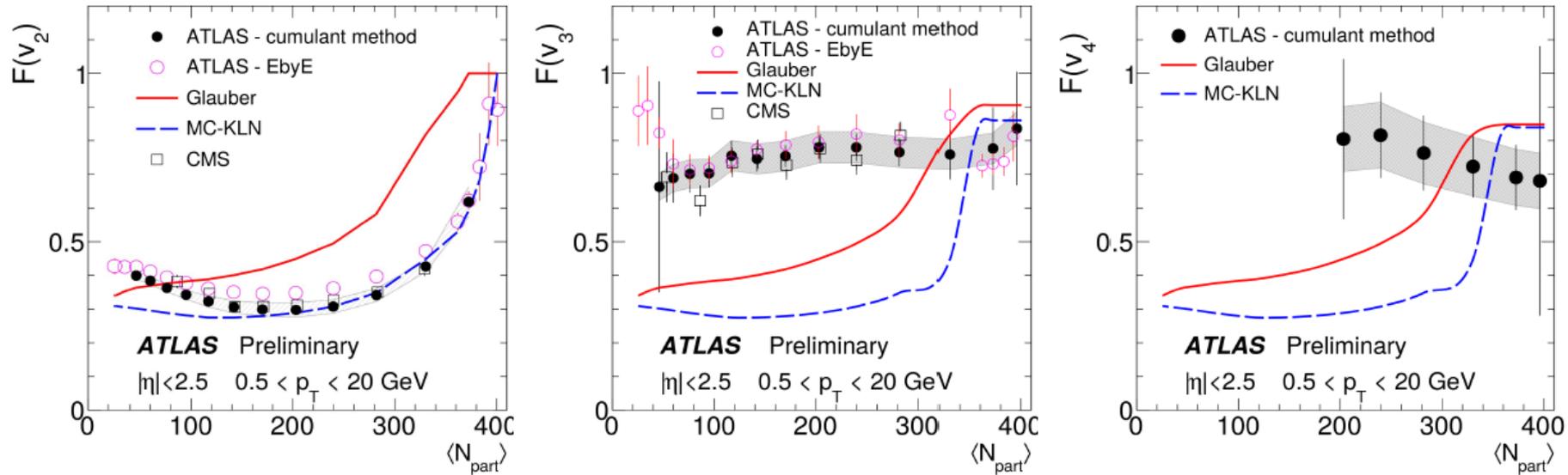


# Elliptic flow fluctuations $p_T$ dependence



- Event plane  $v_2\{EP\}$  method used instead of  $v_2\{2\}$  to lower the contribution of non-flow effects
- Only 2-5% bin shows significant  $p_T$  dependence

# Fluctuations of $v_n$ $\langle N_{part} \rangle$ dependence



- Strong dependence of  $F(v_2)$  on centrality with a minimum at  $\langle N_{part} \rangle \approx 200$
- Large triangular and quadrangular harmonic fluctuations are measured
- Weak dependence of  $F(v_3)$ ,  $F(v_4)$  on centrality
- Good agreement cumulant results with EbyE calculations
- ATLAS results are consistent with the CMS estimate of  $v_3$  relative fluctuations
- Both models fail to reproduce the relative fluctuations

# Conclusions

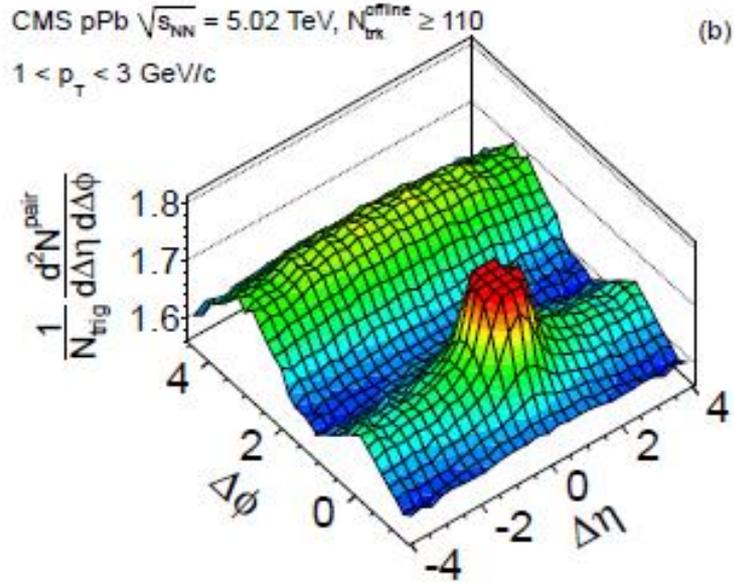
- Reduction of harmonics observed when calculated with higher order cumulants
  - $v_n\{2\} > v_n\{EP\} > v_2\{EbyE\} > v_2\{4\} \approx v_2\{6\} \approx v_2\{8\}$
  - $v_3\{2\} \gg v_3\{4\}$
  - $v_4\{2\} \gg v_4\{4\}$
- Relative fluctuations of  $v_n$  (for  $n=2,3,4$ )
  - Strong centrality dependence for elliptic flow
  - Large values and weak  $N_{part}$  dependence of fluctuations for  $n=3,4$  indicate large fluctuation in the shape of the initial geometry
  - Both Glauber and KLN-MC models did not reproduce these fluctuations in the full centrality range

# PHENIX Results

Extracted from the following talks.

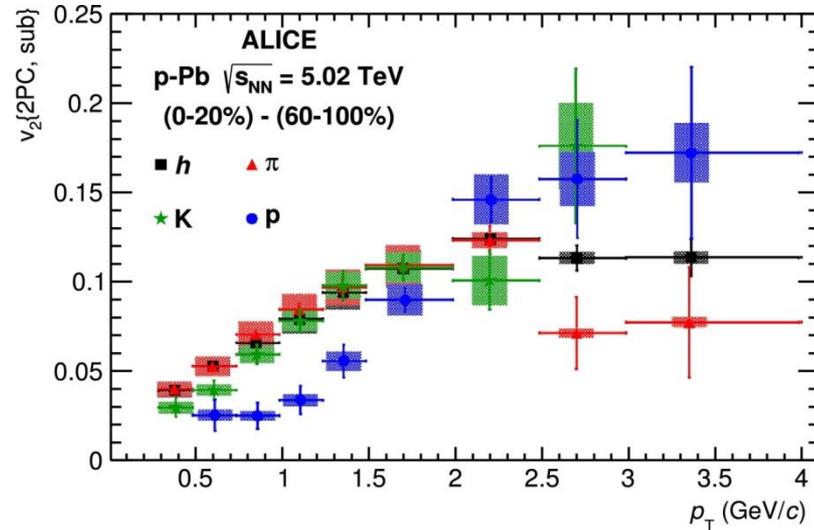
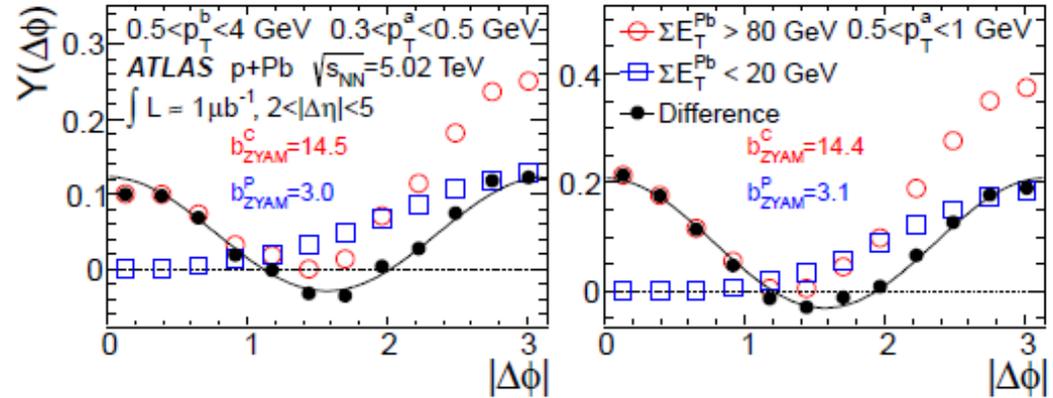
1. “Measurements of long-range angular correlation and identified particle  $v_2$  in 200 GeV d+Au collisions from PHENIX” by Shengli Huang

# “Ridge” and “ $v_2$ ” in p+Pb@5.02 TeV



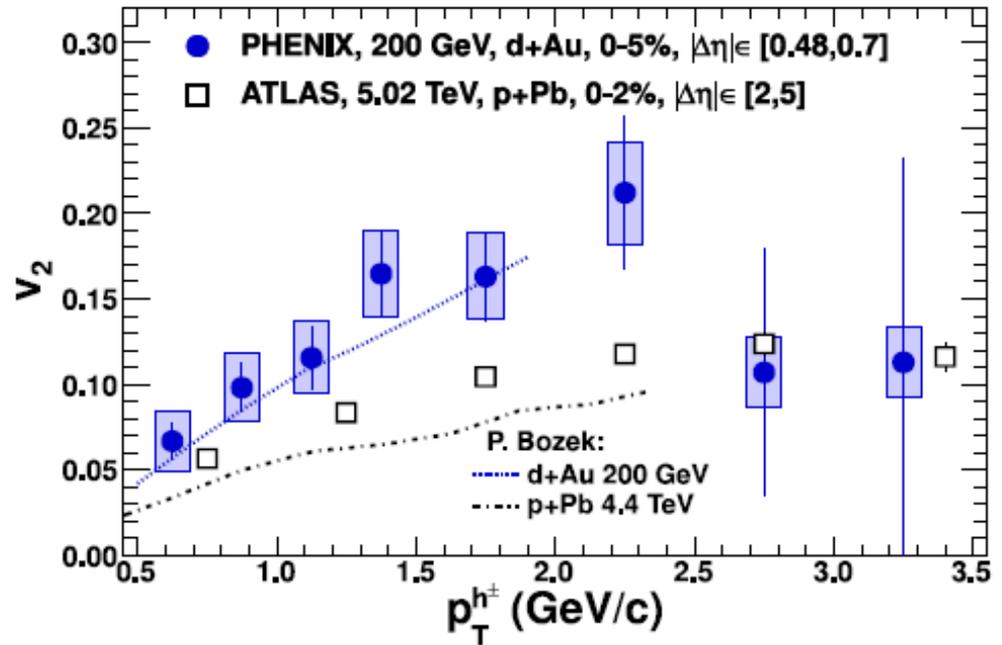
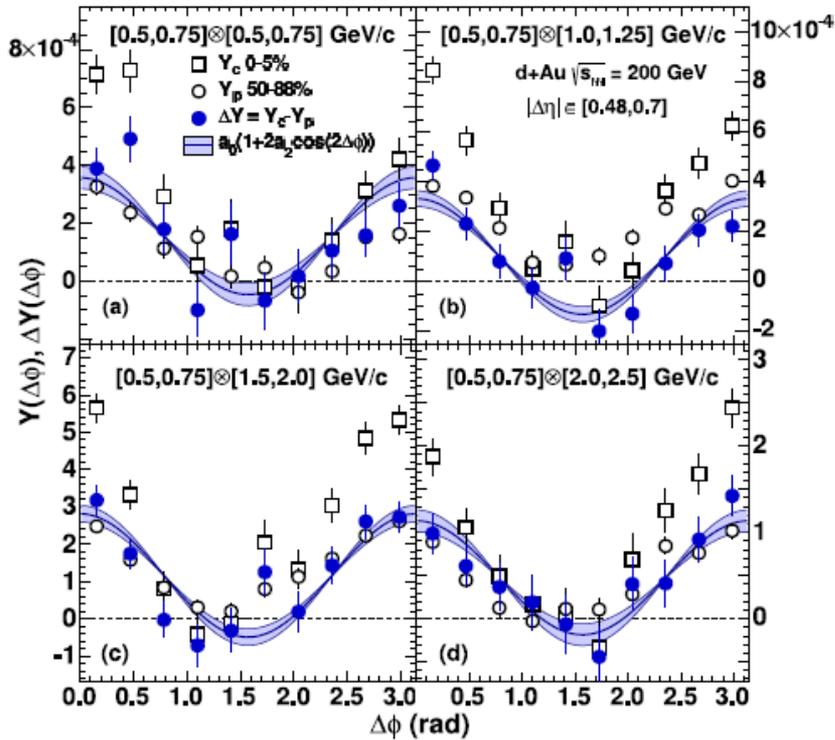
ALICE: Physics Letters B 726 (2013)  
 ATLAS: Phys. Rev. Lett. 110(2013)  
 CMS: Phys. Lett. B 7198(2013)

(b)



- ❑ A “ridge” is observed in the central p + Pb@5.02 TeV
- ❑ The  $\Delta\phi$  distribution shows a  $\cos(2\Delta\phi)$  structure
- ❑ The identified particle  $v_2$  shows a mass ordering

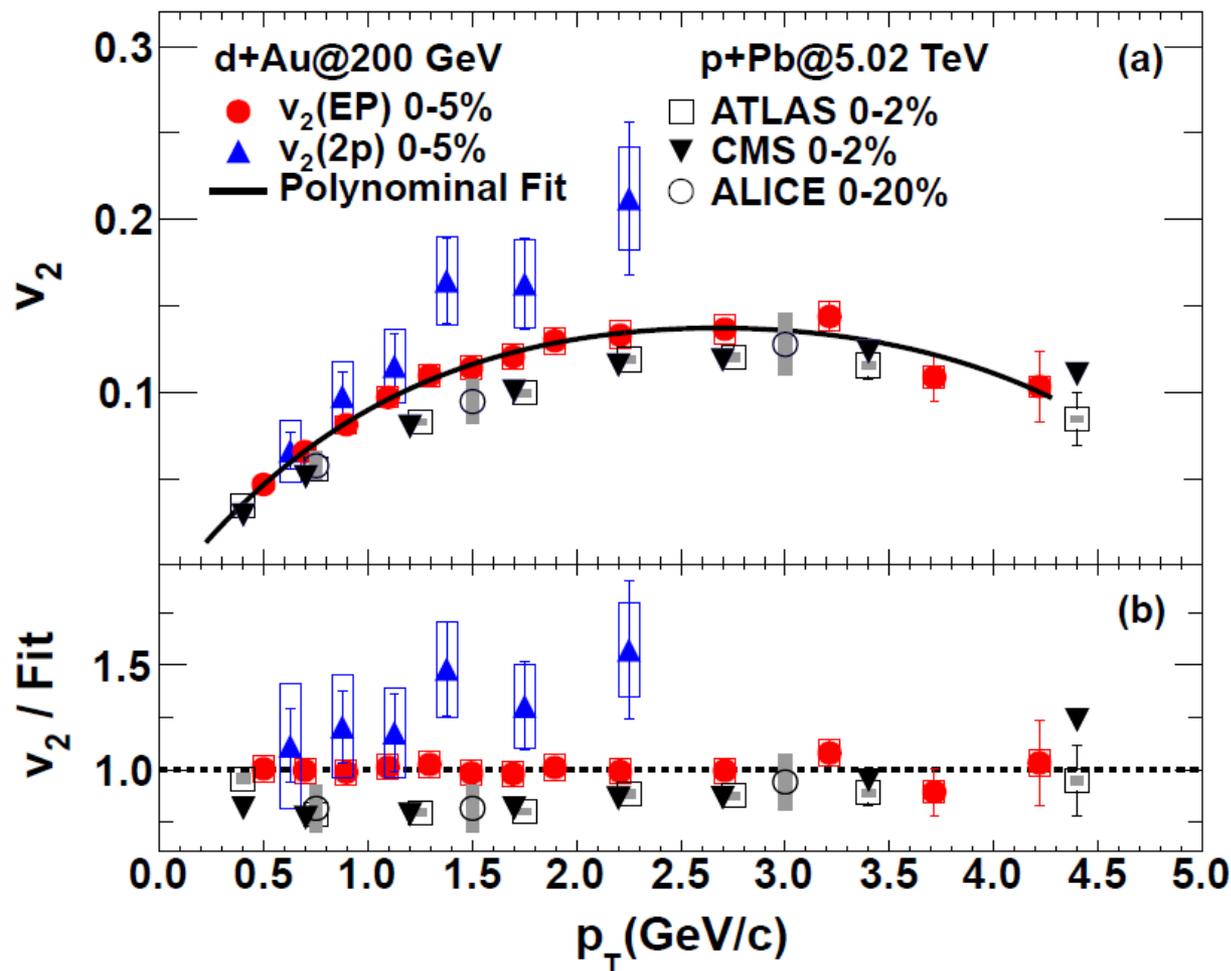
# $V_2$ in d+Au@ 200 GeV



PHENIX: [Phys. Rev. Lett. 111, 212301 \(2013\)](#)

- The  $\cos(2\Delta\phi)$  structure is also seen in 0-5% d+Au. The cut of  $|\Delta\eta| > 0.48$  is the limit of our central arm acceptance
- The  $v_2$  in 0-5% d+Au is higher than that in 0-2% p+Pb collisions, which is consistent with hydro calculation
- The measurement with large  $|\Delta\eta|$  is required!

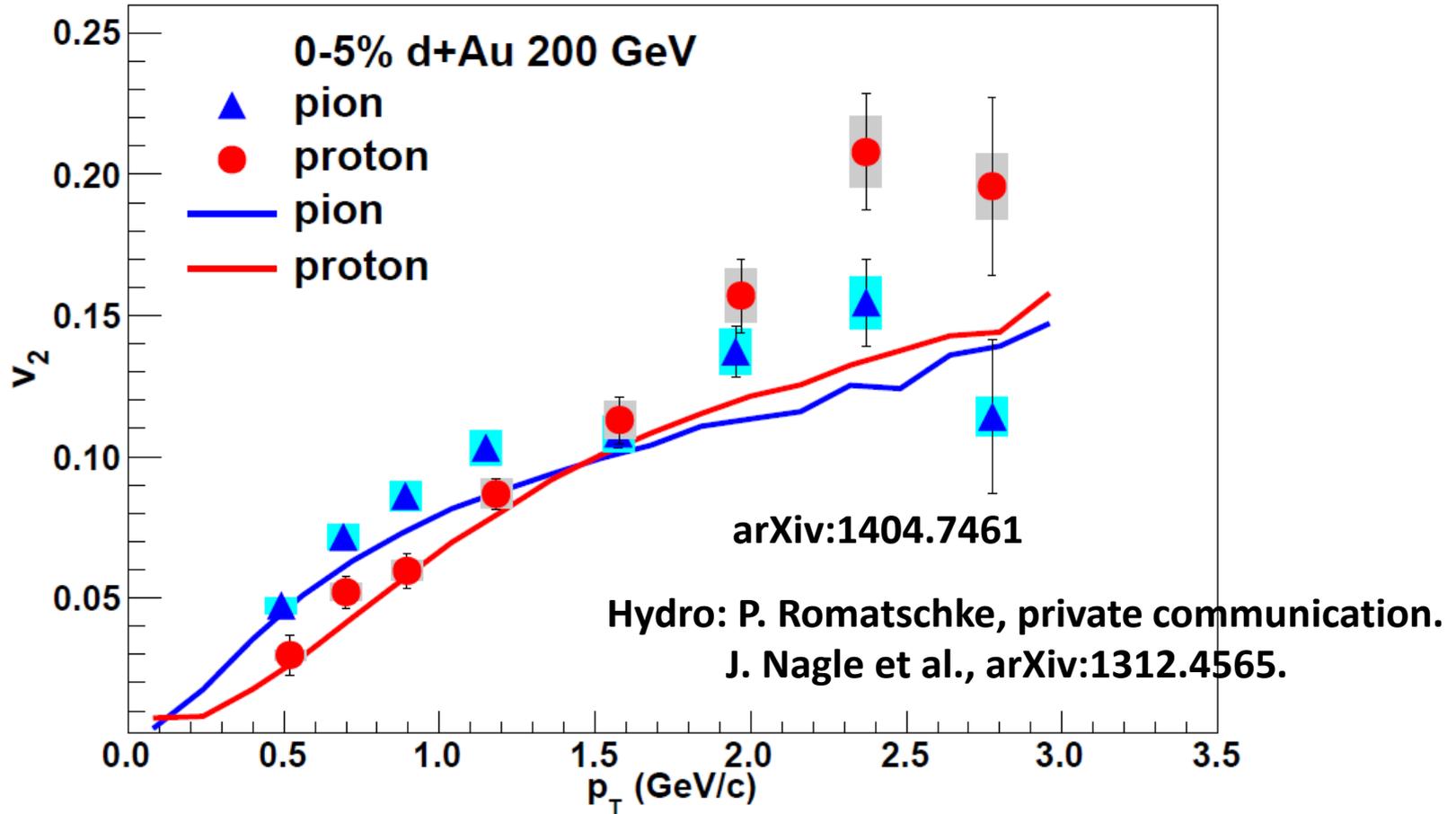
# $V_2$ (EP) of charged hadron in 0-5% d+Au



arXiv:1404.7461

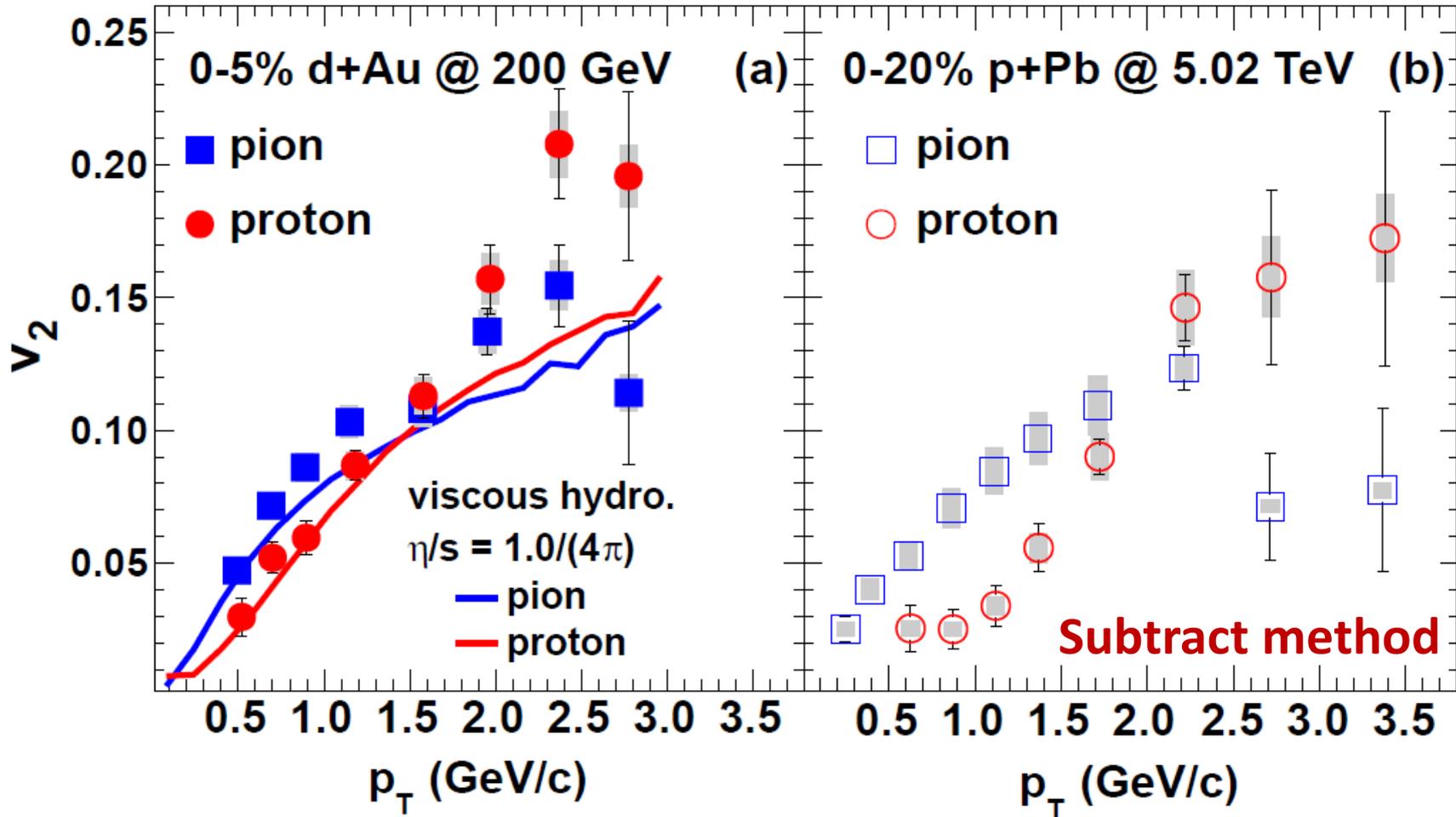
- The charged hadron  $v_2$  measured by the event plane method in central dAu is similar to that in central pPb

# Identified particles' $v_2$ from EP methods



- Mass ordering is observed in 0-5% d+Au
- This ordering can be reproduced in hydro calculation from P. Romatschke et al.

# Weaker radial flow in dAu?



- ❑ The magnitude of mass ordering in p+Pb is larger than that in d+Au
- ❑ Weaker radial flow in d+Au?

# Summary

- Is there “ridge” in dAu collisions?

**There is “Ridge” in dAu even with  $|\Delta\eta| > 6.0$**

- How about the difference between the  $v_2$  in dAu and pPb?

**$v_2$  in central dAu is similar to that in central pPb, while hydro calculation show a significant difference**

- Is there mass ordering for identified particle  $v_2$  in dAu?

**The mass ordering is observed in central dAu , while it is smaller comparing with central pPb, it may be due to a weaker radial flow in dAu**

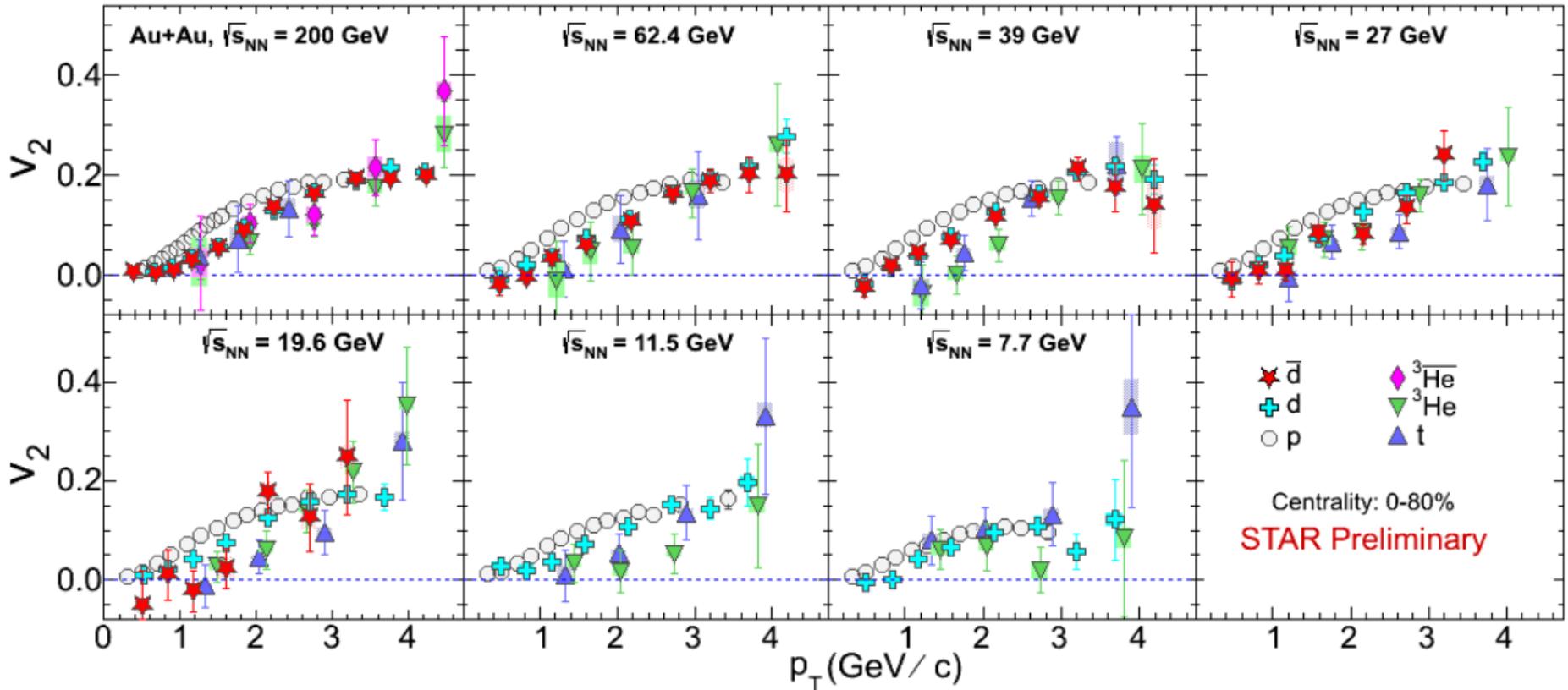
**The input from CGC model calculation is expected for the further understanding the physics of “ridge” and “ $v_2$ ” in small collision system**

# STAR Results

Extracted from the following talks.

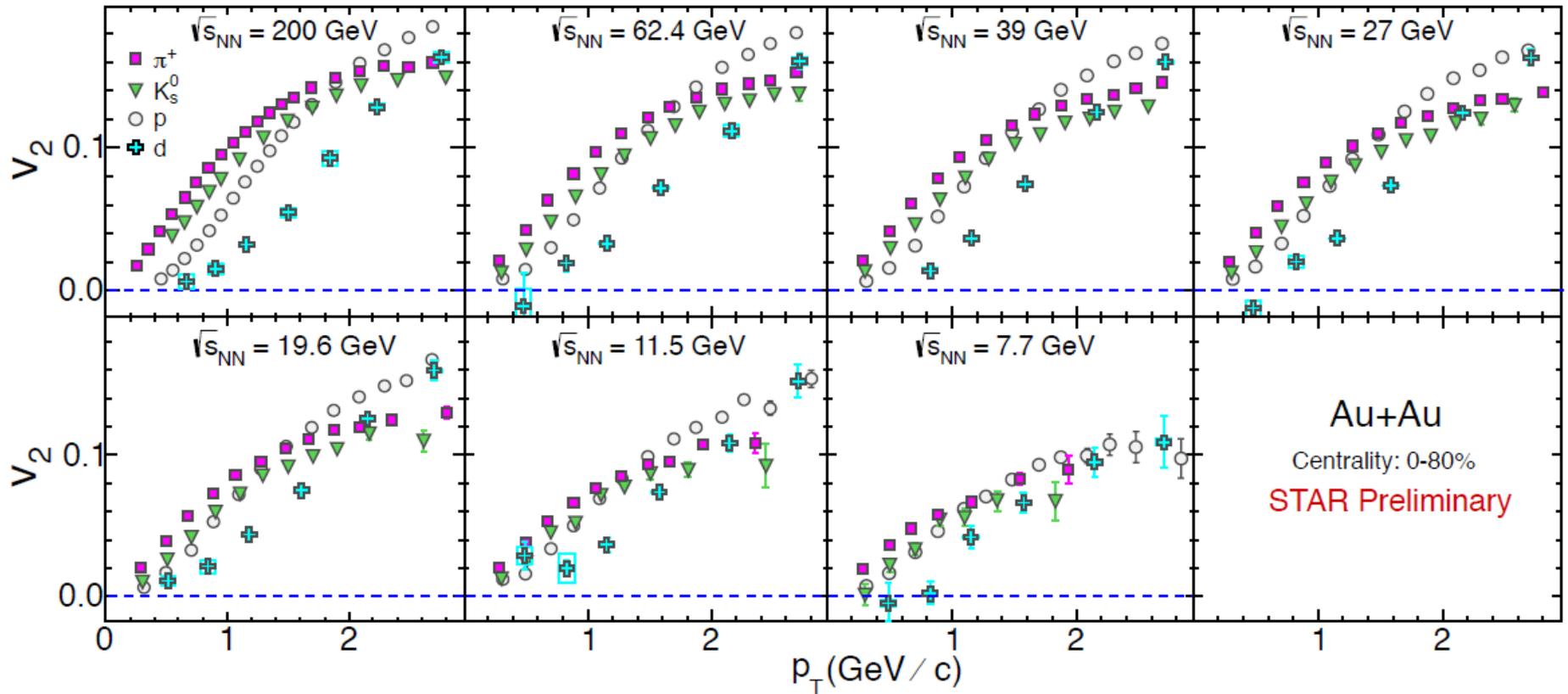
1. “The centrality and energy dependence of the elliptic flow of light nuclei and hadrons in STAR”  
by [Rihan Haque](#)

# Measurement of nuclei $v_2$



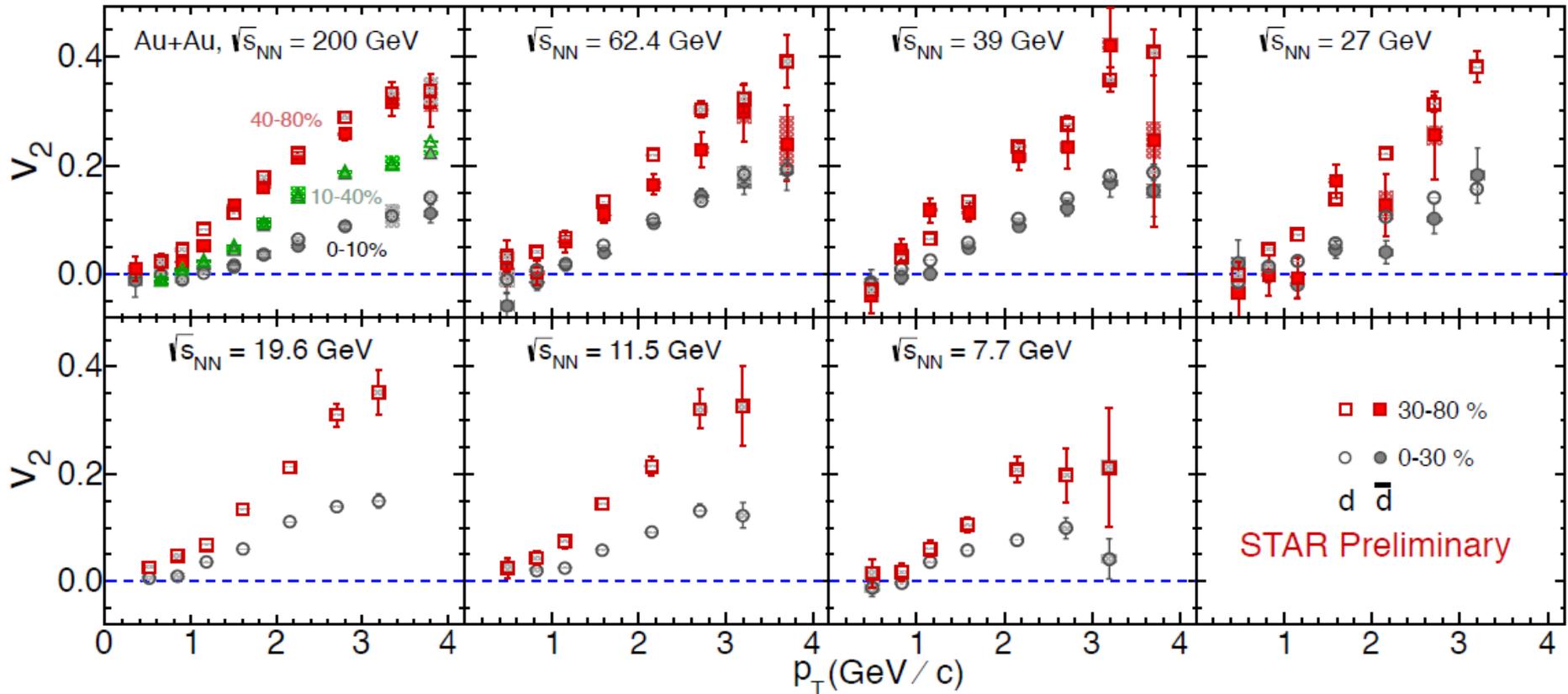
- ✓ Elliptic flow of  $d$ ,  $\bar{d}$ ,  $t$ ,  ${}^3\text{He}$ ,  ${}^3\bar{\text{He}}$  measured at mid-rapidity.
- ✓  $\eta$  sub-eventplane method was used with  $\eta$ -gap = 0.1

# Mass ordering of $v_2$



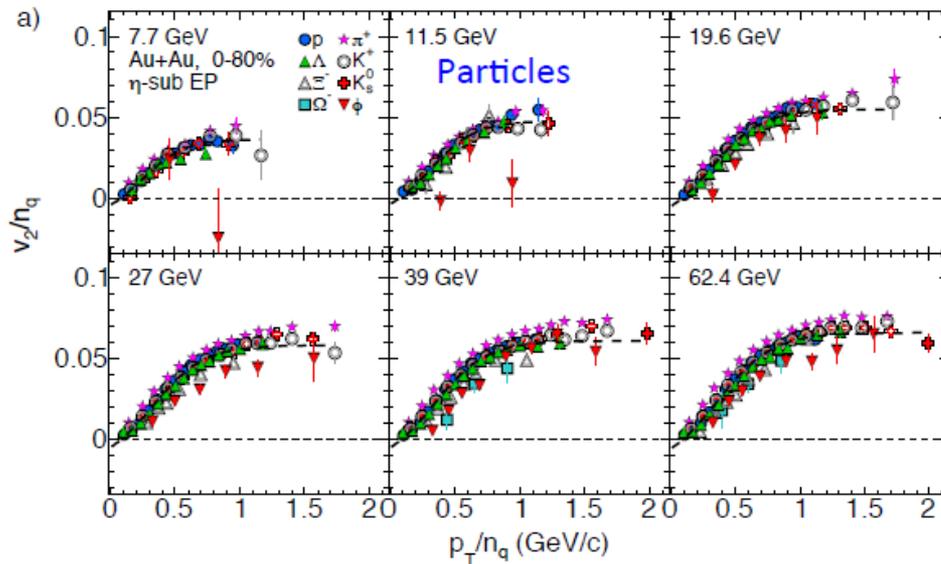
→ Nuclei  $v_2$  shows mass ordering at low  $p_T$  similar to hadrons

# Centrality dependence of nuclei $v_2$



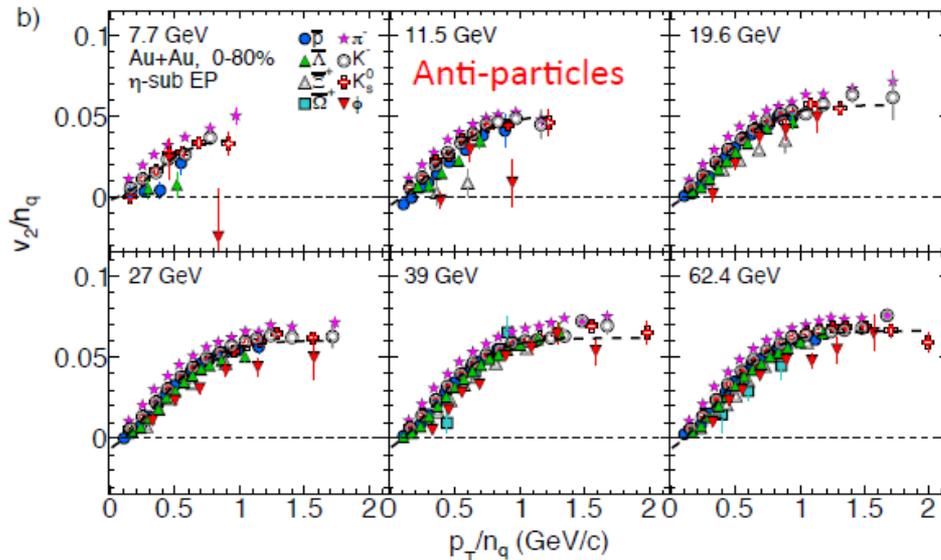
→ Nuclei  $v_2$  shows centrality dependence for all energies

# NCQ scaling of hadron $v_2$



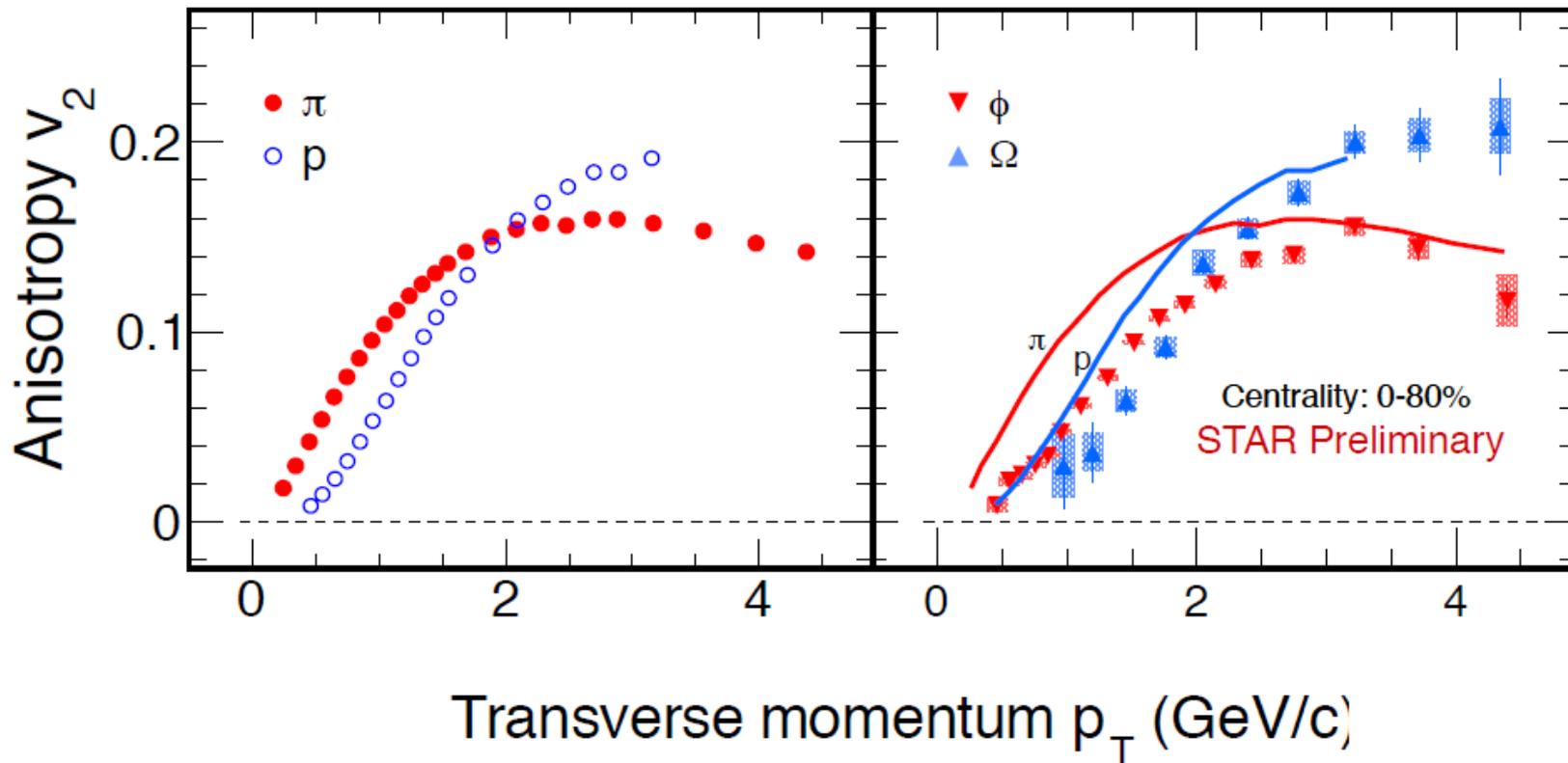
✓ NCQ scaling observed for particle and anti-particle groups separately for beam energy  $\geq 19.6$  GeV

✓ Scaling holds for  $1.5 < p_T < 5.0$  GeV/c



✓ More statistics is needed for 7.7 and 11.5 GeV/c

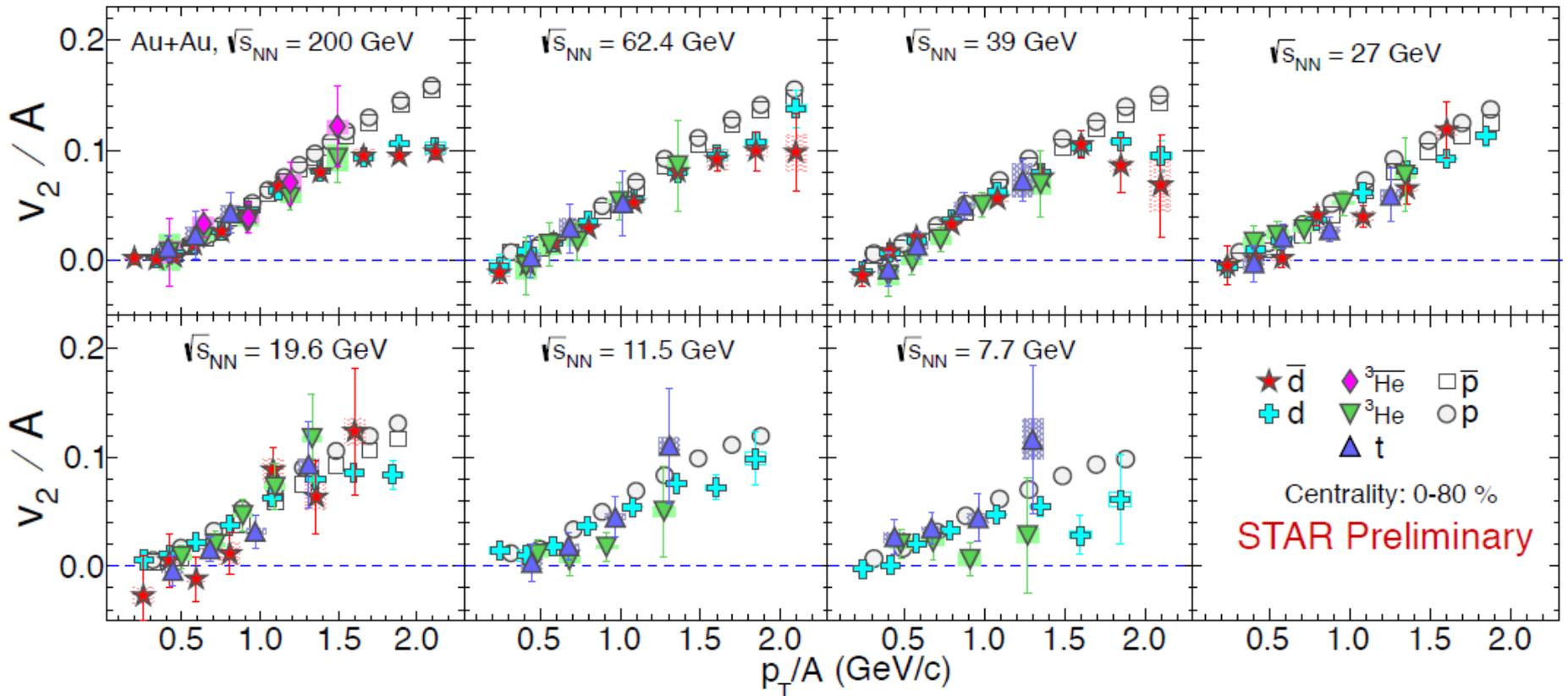
# Precision measurement of $v_2$ of $\phi$ and $\Omega$



- ✓ Mass ordering observed for  $p_T < 2.0$  GeV/c
- ✓ Baryon – meson splitting for  $2.0 < p_T < 5.0$  GeV/c

→ High precision measurement of  $\phi$  and  $\Omega$   $v_2$  agree with the previous physics conclusion of partonic collectivity at 200 GeV

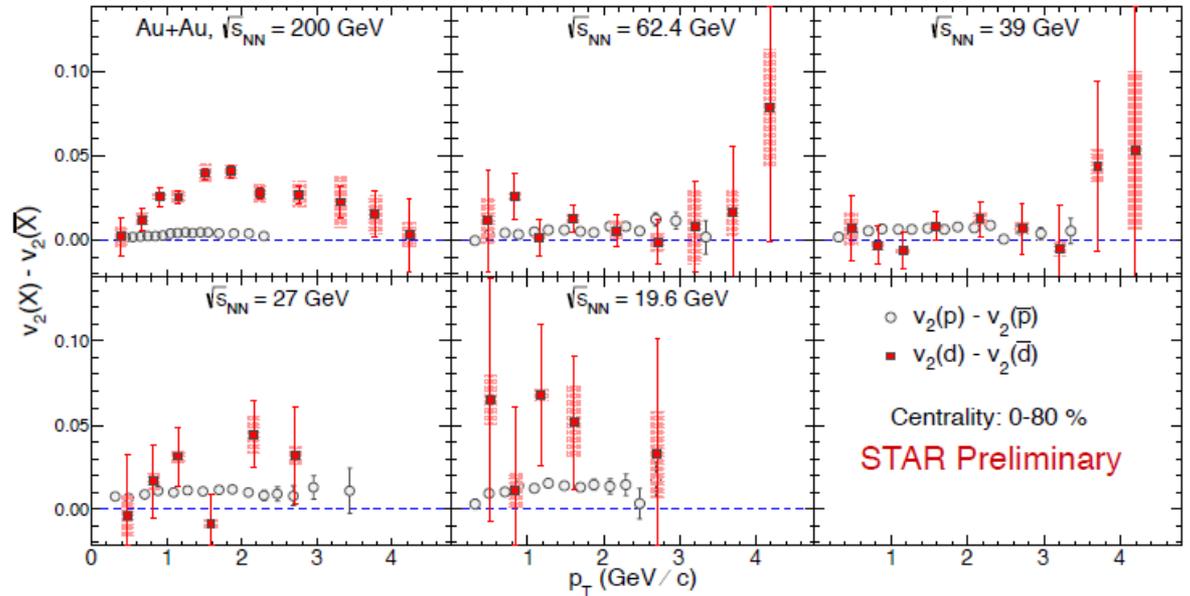
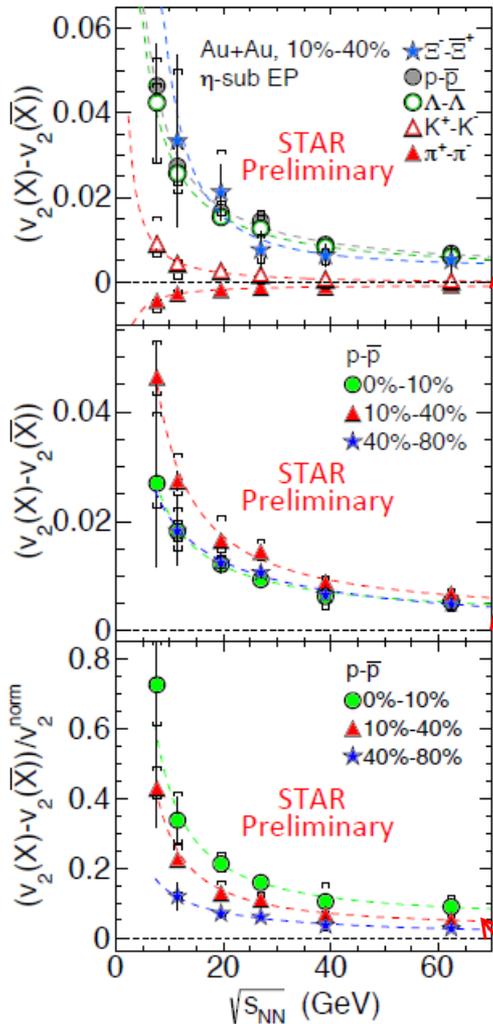
# Mass number scaling of $v_2$



Nuclei  $v_2$  show mass number scaling for  $p_T/A \sim 1.5$  GeV/c for all beam energies

$\rightarrow$  Support the general idea that nuclei are formed by coalescence of nucleons

# $v_2$ of particles and anti-particles



- Nuclei and anti-nuclei  $v_2$  shows a difference at 200 GeV
- Statistical uncertainties large at lower beam energies to make definite conclusions.
- $\Delta v_2$  for 10-40% centrality is similar to minimum bias result
- Centrality dependence not observed in  $\Delta v_2$
- $\Delta v_2$  relative to proton  $v_2$  shows a centrality dependence

$v_2^{norm} = v_2$  of proton

# Summary

1. Nuclei  $v_2$  versus  $p_T$  shows a clear centrality dependence and mass ordering when compared to identified hadrons at all beam energies studied  
→ *Mass ordering of  $v_2$  occurs naturally in a hydrodynamic model.*
2. Nuclei  $v_2$  versus  $p_T$  shows mass number scaling upto  $p_T/A = 1.5$  GeV/c and the magnitude of nuclei  $v_2$  versus  $p_T$  are reproduced by a Coalescence model.  
→ *Both these support the physics picture of coalescence of nucleons as the dominant mechanism of nuclei production.*
3. The difference in  $v_2$  of proton and anti-proton is observed to be similar at all collision centralities studied for the BES energies. A centrality dependence appears when this difference is normalized to proton  $v_2$  at the respective beam energies  
→ *The results implies hadronic interactions play an important role at lower beam energies.*

“with large  $\Delta v_2$ ”

# Backups

Fourier coefficients can be extracted from the  $\Delta\varphi$  projection of the per-trigger yield by a fit with:

$$\frac{1}{N_{\text{trig}}} \frac{dN_{\text{assoc}}}{d\Delta\varphi} = a_0 + 2a_1 \cos \Delta\varphi + 2a_2 \cos 2\Delta\varphi + 2a_3 \cos 3\Delta\varphi. \quad (2)$$

From the relative modulations  $V_{n\Delta}^{h-i}\{2\text{PC}\} = a_n^{h-i}/a_0^{h-i}$ , where  $a_n^{h-i}$  is the  $a_n$  extracted from  $h-i$  correlations, the  $v_n^i\{2\text{PC}\}$  coefficient of order  $n$  for a particle species  $i$  (out of  $h, \pi, K, p$ ) are then defined as:

$$v_n^h\{2\text{PC}\} = \sqrt{V_{n\Delta}^{h-h}} \quad v_n^i\{2\text{PC}\} = V_{n\Delta}^{h-i} / \sqrt{V_{n\Delta}^{h-h}}. \quad (3)$$

In the case that each of the particles is correlated with a common plane, the  $v_n^i\{2\text{PC}\}$  are the Fourier coefficients of the corresponding single-particle angular distributions.

$$\frac{2\pi}{N} \frac{dN}{d\phi} = 1 + 2 \sum_{n=1}^{\infty} v_n \cos n(\phi - \Phi_n)$$

The flow vector

$$\begin{aligned} \vec{Q}_n &\equiv (|Q_n| \cos(n\Psi_n), |Q_n| \sin(n\Psi_n)) & |Q_n| \cos(n\Psi_n) &= \frac{1}{N} \sum_j \cos(n\phi_j) \\ & & |Q_n| \sin(n\Psi_n) &= \frac{1}{N} \sum_j \sin(n\phi_j), \\ Q_n &= |Q_n| e^{in\Psi_n} \equiv \frac{1}{N} \sum_j e^{in\phi_j} \end{aligned}$$

The original idea behind the event-plane method is that the direction  $\Psi_n$  of the flow vector in a reference detector provides an estimate of the corresponding angle  $\Phi_n$  in the underlying probability distribution. Because a finite sample of particles is used, statistical fluctuations cause  $\Psi_n$  to differ from  $\Phi_n$ .

The nonlinear dependence of the resolution on the underlying flow is the origin of the difficulties of the event plane method, which arise when flow fluctuations are considered. A simpler quantity is the projection of the flow vector onto the underlying direction  $\Phi_n$ , which directly gives the underlying flow:

$$\langle Q_n e^{-in\Phi_n} \rangle_{|v_n} = v_n$$

In SP method, use flow vector  $Q_n$  to calculate flow  $v_n$

$$v_n = \frac{\langle |Q_n| \cos n(\phi - \psi_n^{measured}) \rangle}{\bar{Q}_n}$$

where  $\bar{Q}_n$  is define as

- $\sqrt{\langle Q_{nA} \cdot Q_{nB} \rangle}$  for 2 sub event
- $\frac{\sqrt{\langle Q_{nA} \cdot Q_{nB} \rangle \langle Q_{nA} \cdot Q_{nC} \rangle}}{\sqrt{\langle Q_{nB} \cdot Q_{nC} \rangle \text{ for 3 subevent}}}$



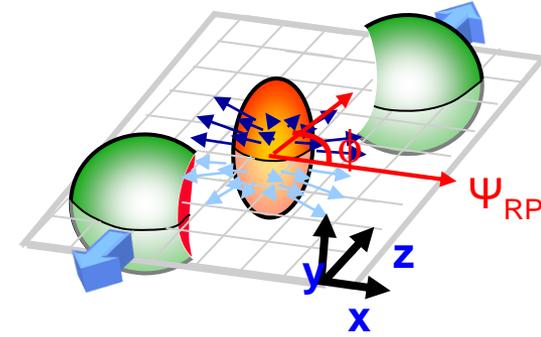
# Experimental methods



- Event plane (EP) method:

$$E \frac{d^3 N}{d^3 p} = \frac{1}{2\pi} \frac{d^2 N}{p_t dp_t dy} \left( 1 + \sum_{n=1}^{\infty} 2 v_n \cos(n(\varphi - \Psi_{RP})) \right)$$

$$v_n = \langle \cos(n(\varphi_i - \Psi_{RP})) \rangle$$



- Cumulants:

- 2- and 4-particle azimuthal correlations for an event:

$$\langle 2 \rangle \equiv \langle \cos(n(\varphi_i - \varphi_j)) \rangle, \varphi_i \neq \varphi_j$$

$$\langle 4 \rangle \equiv \langle \cos(n(\varphi_i + \varphi_j - \varphi_k - \varphi_l)) \rangle, \varphi_i \neq \varphi_j \neq \varphi_k \neq \varphi_l$$

- Averaging over all events, the 2<sup>nd</sup> and 4<sup>th</sup> order cumulants are given:

$$c_2\{n\} = \langle \langle 2 \rangle \rangle = v_n^2 + \delta_n$$

$$c_4\{n\} = \langle \langle 4 \rangle \rangle - 2 \langle \langle 2 \rangle \rangle^2 = -v_n^4$$

$v_n$  : reference\_flow  
 $\langle \rangle$  : average\_particles  
 $\langle \langle \rangle \rangle$  : average\_events

$$v_n\{2\} \equiv \sqrt{c_n\{2\}}$$

$$v_n\{4\} \equiv \sqrt[4]{-c_n\{4\}}$$

$$v_n\{2\} \cong v_n^2 + \sigma_n^2 + \delta$$

$$v_n\{4\} \cong v_n^2 - \sigma_n^2$$

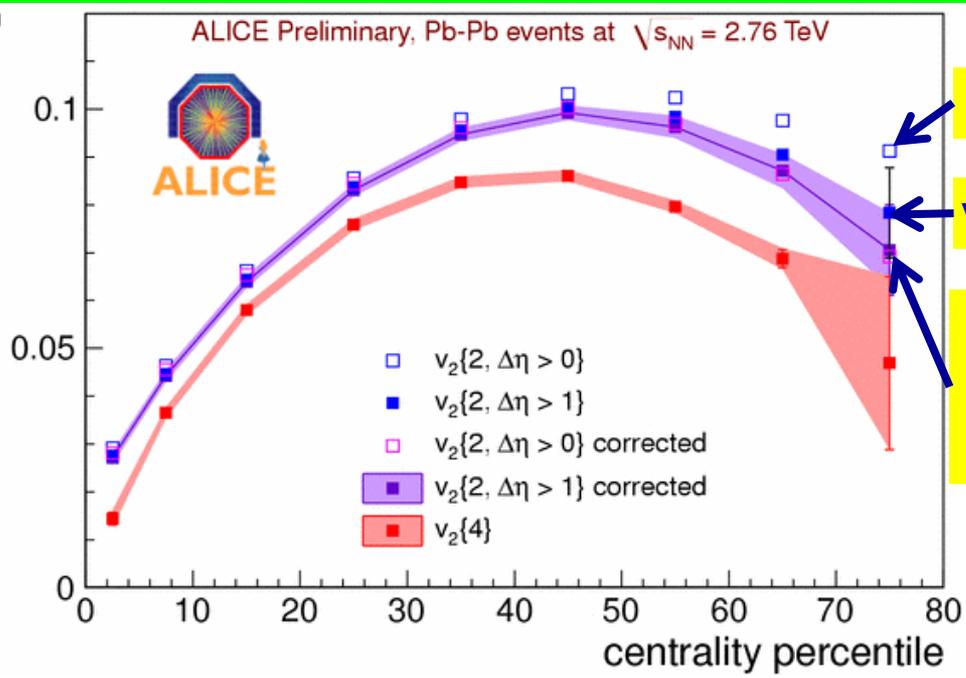
$v_2\{2\}$  and  $v_2\{4\}$  have different sensitivity to flow fluctuations ( $\sigma_n$ ) and non-flow ( $\delta$ )



# Elliptic Flow $v_2$



## Non-Flow corrections



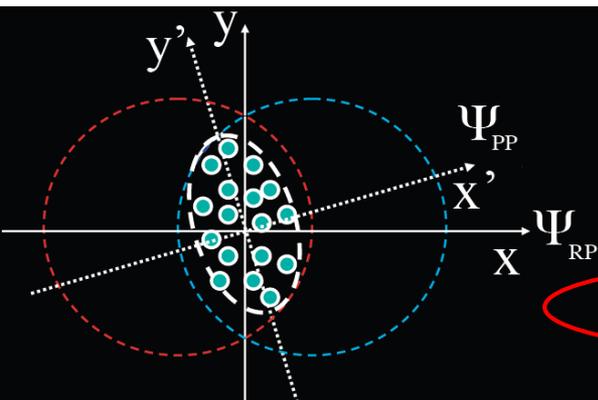
$v_2$  no eta gap between particles

$v_2$   $|\eta| > 1$  to reduce non-flow such as jets

both  $v_2$  corrected for remaining non-flow using Hijing or scaled pp

With this, we can remove most of non-flow ( $\delta$ )

Plane of symmetry ( $\Psi_{PP}$ ) fluctuate event-by-event around reaction plane ( $\Psi_{RP}$ ) => flow fluctuation ( $\sigma_n$ )



$$v_n^2\{2\} = \bar{v}_n^2 + \sigma_v^2$$

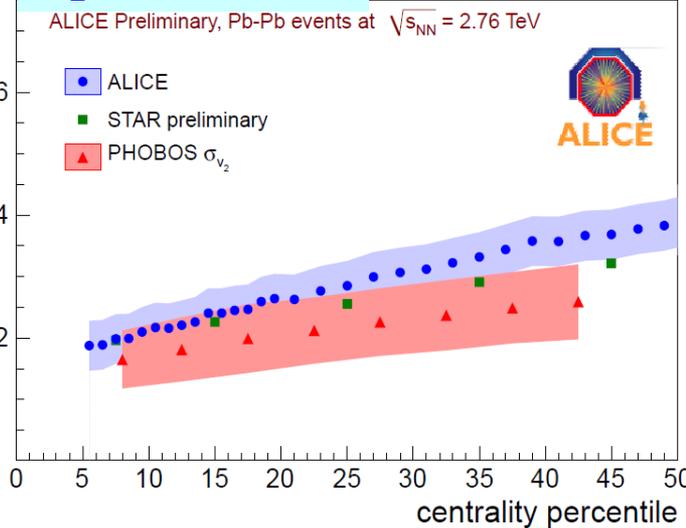
$$v_n^2\{4\} = \bar{v}_n^2 - \sigma_v^2$$

$$v_n^2\{2\} + v_n^2\{4\} = 2\bar{v}_n^2$$

$$v_n^2\{2\} - v_n^2\{4\} = 2\sigma_v^2$$

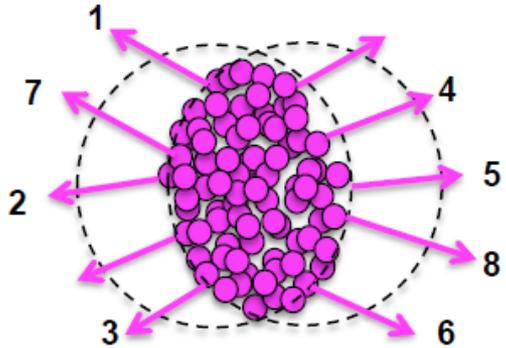
$$\left( (v_2\{2\}^2 - v_2\{4\}^2) / 2 \right)^{1/2}$$

## $v_2$ Fluctuations



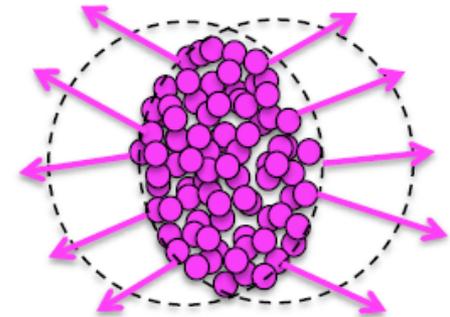
# Analysis Techniques

## 6- and 8-particle cumulant



- **Genuine 6- and 8-particle correlations**
- **Insensitive to non-flow contributions from  $< 6$  and 8 particles**

## Lee-Yang Zeros



- **Genuine all-particle correlation**
- **Built-in correction for non-uniform distribution**

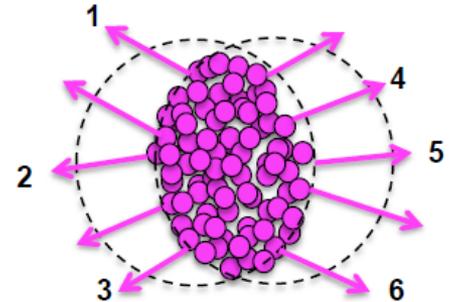
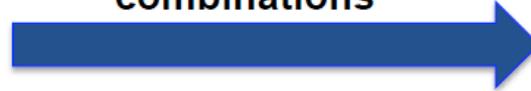
# Multiparticle Cumulant

- 6-particle correlator, per event

$$\langle\langle 6 \rangle\rangle \equiv \left\langle e^{in(\phi_1+\phi_2+\phi_3-\phi_4-\phi_5-\phi_6)} \right\rangle$$

$$\equiv \frac{1}{P_{M,6}} \sum_{\substack{i \neq j \neq k \\ \neq l \neq m \neq n}}^M e^{in(\phi_i+\phi_j+\phi_k-\phi_l-\phi_m-\phi_n)}$$

Distinctive 6-particle combinations



- 6-particle cumulant, all events

$$c_n\{6\} = \langle\langle 6 \rangle\rangle - 9 \cdot \langle\langle 4 \rangle\rangle \langle\langle 2 \rangle\rangle + 12 \cdot \langle\langle 2 \rangle\rangle^3$$

- Q-Cumulant: decompose  $\rightarrow$  flow vector  $Q_n \equiv \sum_{i=1}^M w_i e^{in\varphi_i}$

- Cumulant  $v_n \rightarrow$

$$v_n\{4\} = \sqrt[4]{-c_n\{4\}}, v_n\{6\} = \sqrt[6]{\frac{1}{4}c_n\{6\}}, v_n\{8\} = \sqrt[8]{-\frac{1}{33}c_n\{8\}}$$

# Lee-Yang Zeros Method

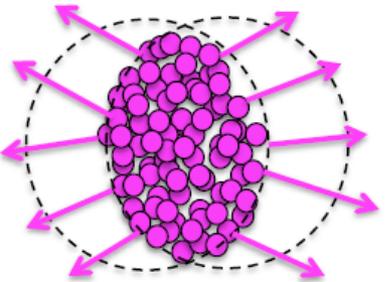
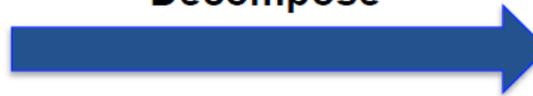
- All-particle correlation, per event

$$g(ir) \equiv \prod_{j=1}^M \left[ 1 + i \cdot r \cdot w_j \cos(n(\phi_j - \theta)) \right]$$

- Generating function, all events

$$G(ir) = \langle g(ir) \rangle = \frac{1}{N_{\text{evt events}}} \sum g(ir)$$

Decompose



**Flow vector:**

$$Q_n = (Q_{nx}, Q_{ny})$$

$$Q_{nx} = \sum_{j=1}^M w_j \cos(n\phi_j)$$

$$Q_{ny} = \sum_{j=1}^M w_j \sin(n\phi_j)$$

- Integrated  $v_n$  {LYZ}

$$V_2 \{LYZ\} = \frac{j_{01}}{r_0}$$

$j_{01} = 2.40483$   
 $r_0$  is the first zero of  $|G(ir)|$

