A Pathway to a Transport Model for RAON

Kyungil Kim

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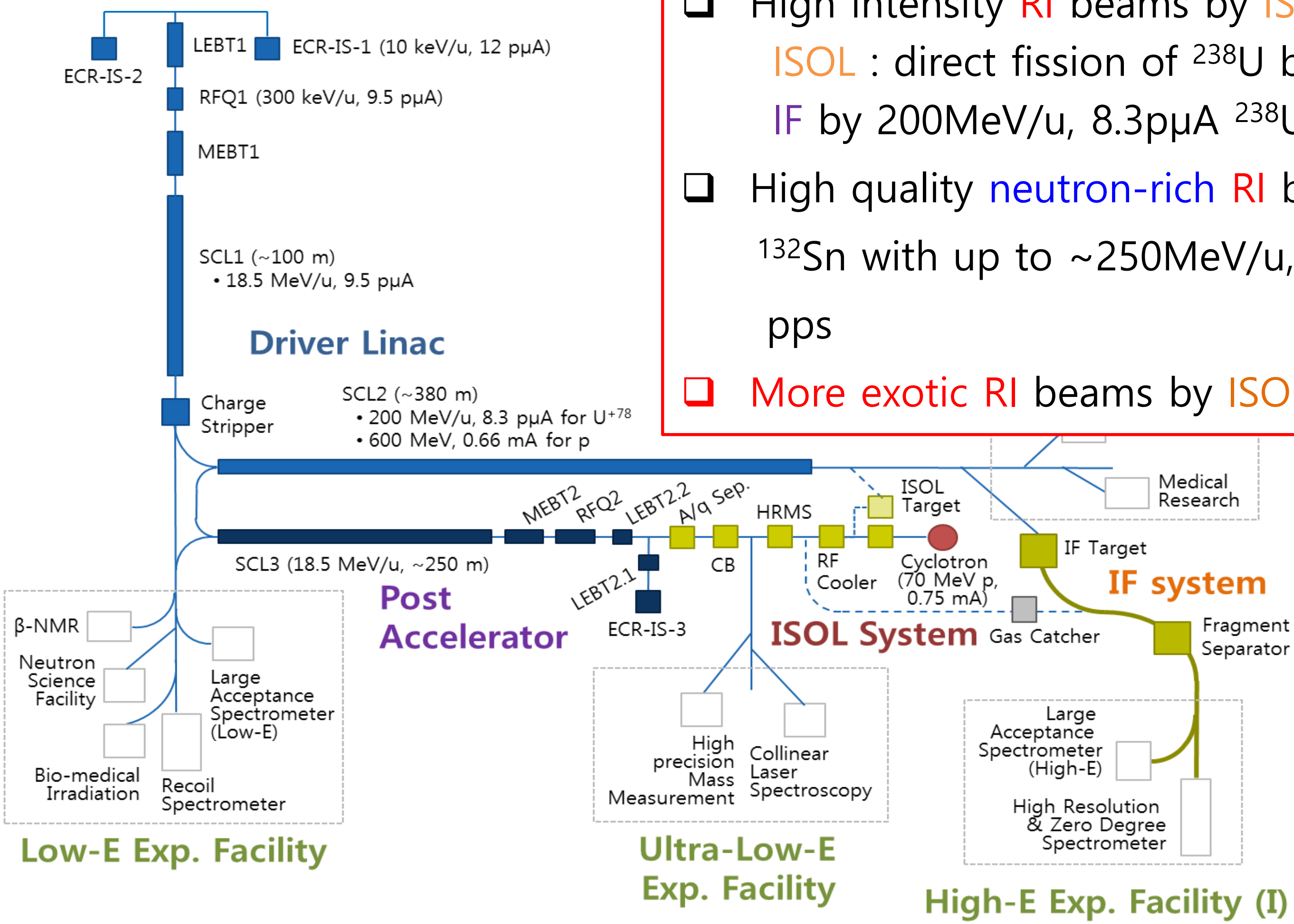
Contents

RAON and Nuclear Reactions

- High intensity RI beams by ISOL & IF
  - ISOL : direct fission of ^{238}U by p 70MeV
  - IF by 200MeV/u, 8.3pμA ^{238}U
- High quality neutron-rich RI beams
  - ^{132}Sn with up to ~250MeV/u, up to 10^7 pps
- More exotic RI beams by ISOL+IF



Slide from Y.K.Kim's presentation

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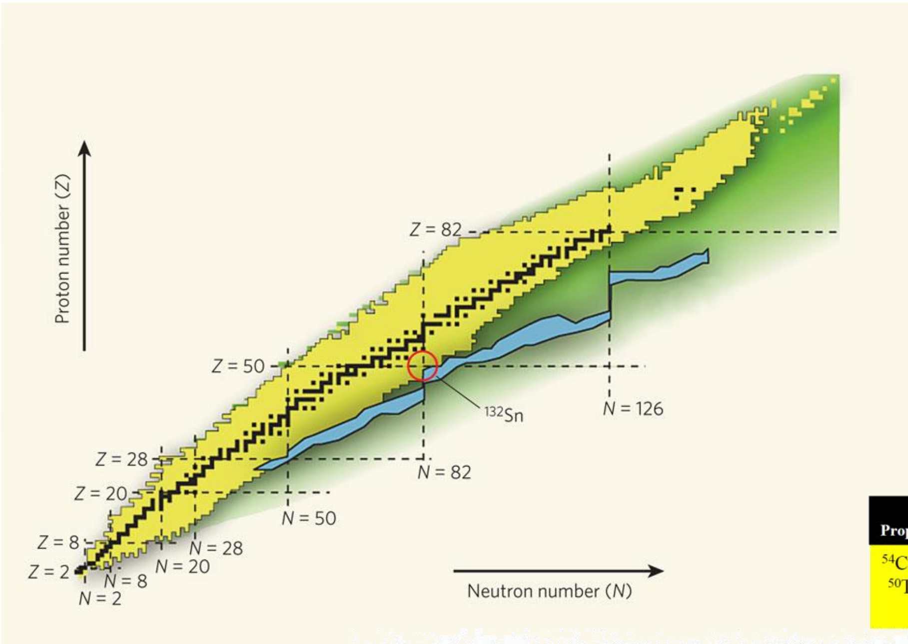
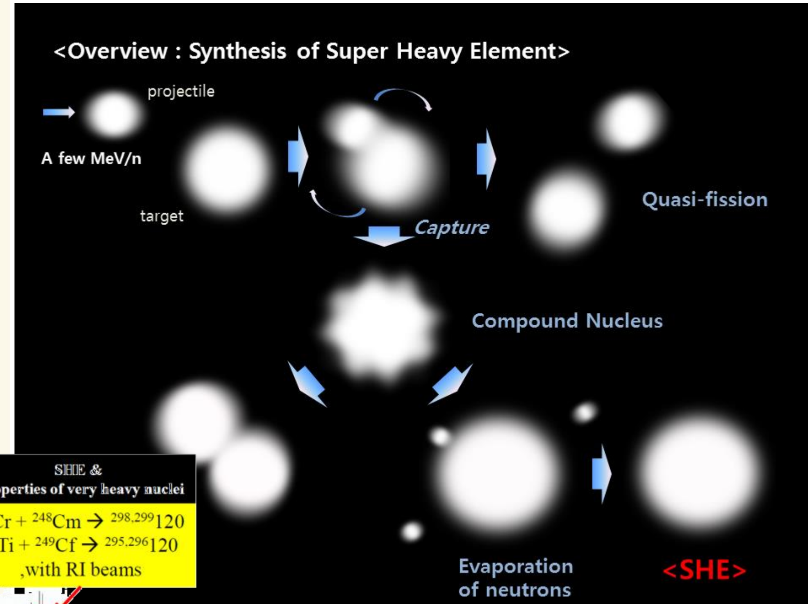
RAON and Nuclear Reactions





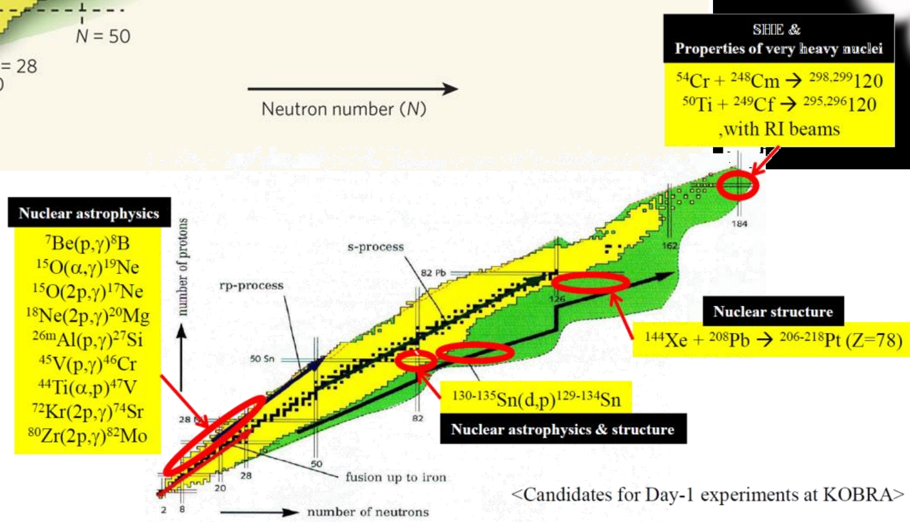


figure from Y.K.Kim's presentation

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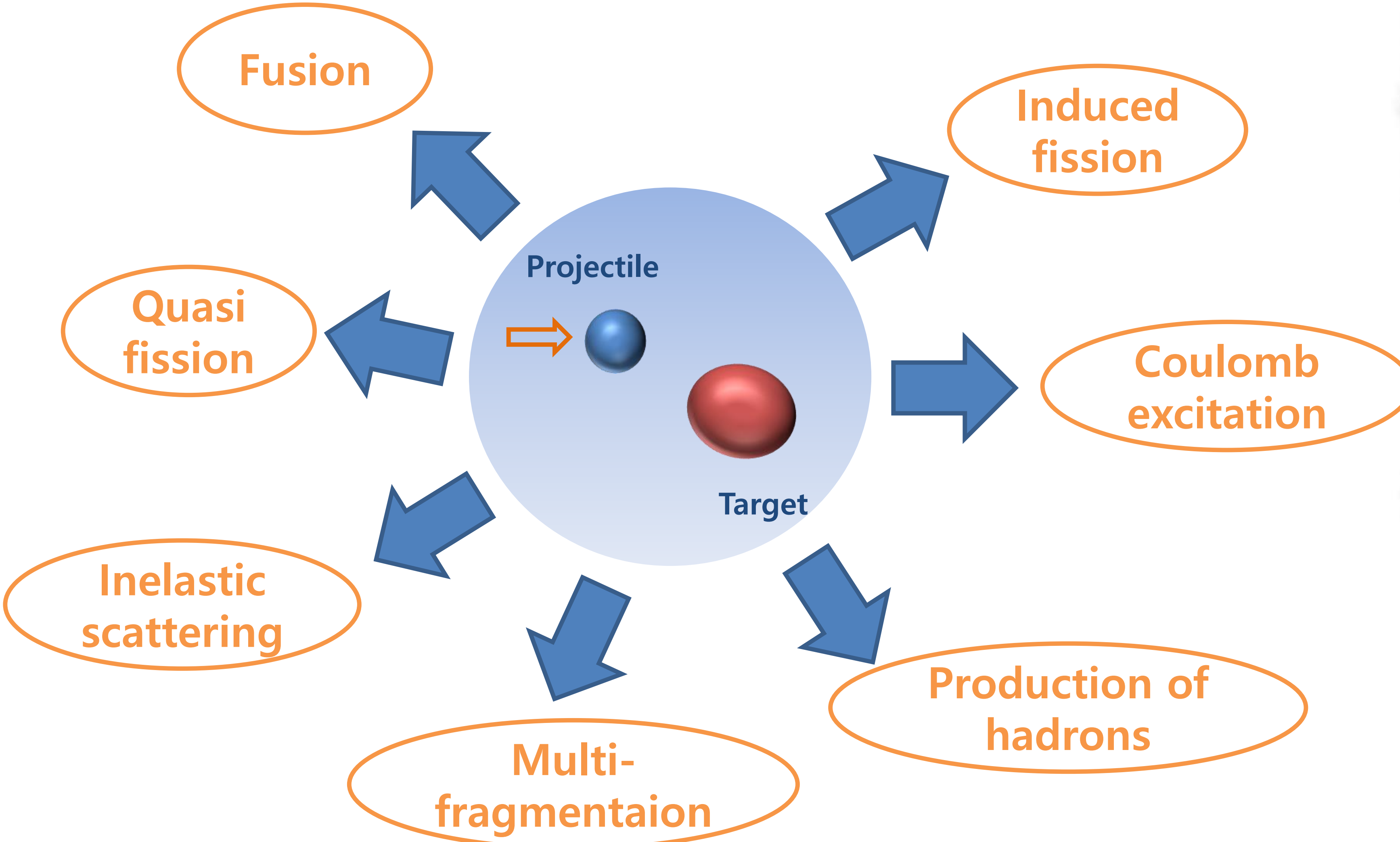
RAON and Nuclear Reactions



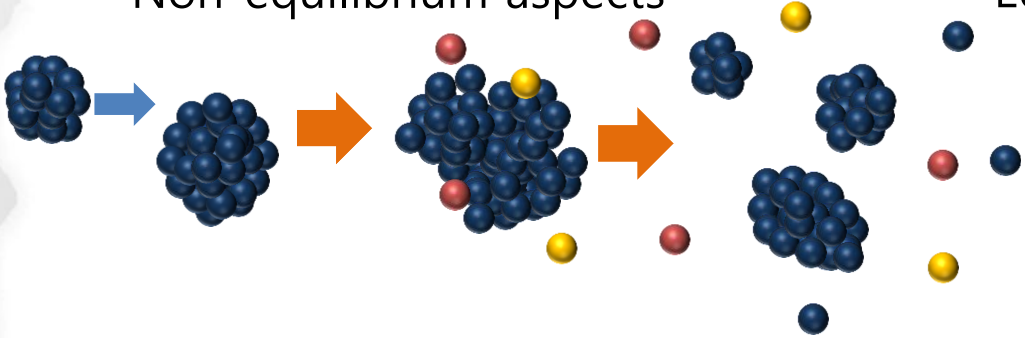
RAON and Nuclear Reactions

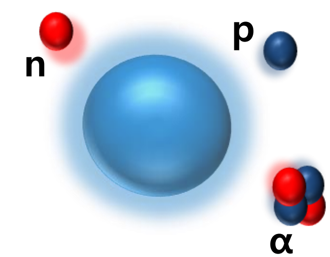
Direct Reactions ***Compound Nuclear Reactions***

~10^{-20} sec — Time — ~10^{-16} sec

Non-equilibrium aspects

Equilibrated (decay statistically)





Low and intermediate energy regime : *~ 100 MeV/n*

- fusion, quasi-fission (multinucleon transfer)
, fragmentation

Relativistic regime : *100 MeV/n ~ a few GeV/n*

- production of hadrons, collective flow phenomena

Kinetic Energy	λ=h/p
1 MeV	3 x 10^{-14} m
10 MeV	9 x 10^{-15} m
100 MeV	3 x 10^{-15} m
1 GeV	7 x 10^{-16} m

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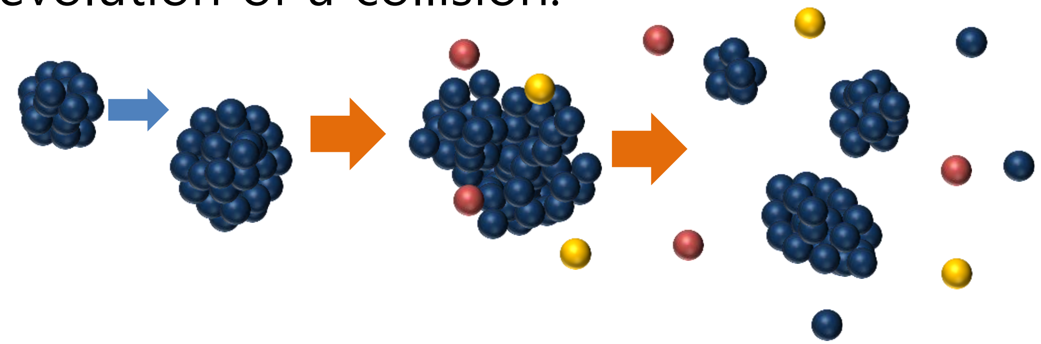
Transport Model?

Transport model : Model to treat non-equilibrium aspects of the temporal evolution of a collision.



- ✓ Many-body problem with nucleons
- ✓ Numerical simulation
- ✓ Direct reaction regime (compound nuclear reaction -> statistical model)
- ✓ Different methods with different energies

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Various Codes

Transport Model

BUU Type

- Boltzmann : Collision term
- Uehling & Uhlenbeck, Nordheim : Pauli blocking
- Vlasov : Mean field w/o collision
- Landau : Averaged collisions

Many names,

- Boltzmann-Uehling-Uhlenbeck : BUU
- BNL , VUU, LV, …
- Isospin dependent BUU : IBUU
- Relativistic BUU : RBUU
- …

QMD Type

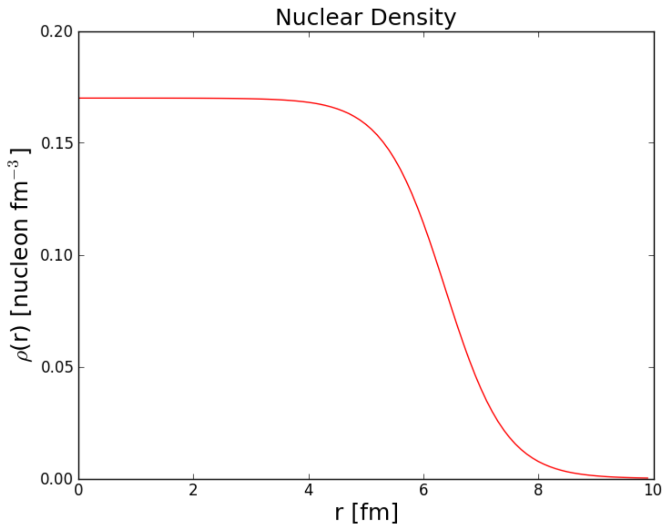
- Classical Molecular Dynamics : CMD
- Quantum Molecular Dynamics : QMD
- Antisymmetrized MD : AMD
- Fermionic MD : FMD
- Constrained MD : CoMD
- Improved QMD : ImQMD
- Ultra-relativistic QMD : UrQMD
- …

Introduction to Transport Model

1. Initialization

$$\rho = \rho_0 \left[1 + \exp\left\{\frac{r-R}{a}\right\}\right]^{-1}$$

$$R = 1.12A^{1/3} \quad a = 0.53 \text{ [fm]}$$



<Phase Space Density and Wigner Transforamtion>

- Both a position and a momentum of nucleons are needed.
  - ➢ **Phase space density!!**
- **Wigner transformation**

$$f(r,p;t) = \int d^4\zeta \exp(ip_\mu\zeta^\mu)\tilde{f}\left(r+\frac{\zeta}{2}, r-\frac{\zeta}{2}\right), \qquad \zeta = r_1 - r_2, \quad r = \frac{r_1+r_2}{2}$$

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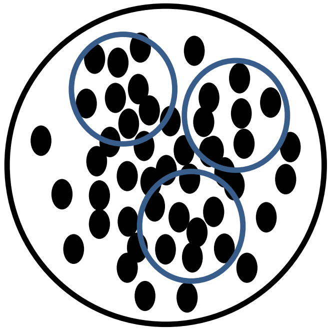
Introduction to Transport Model

➢ BUU Model

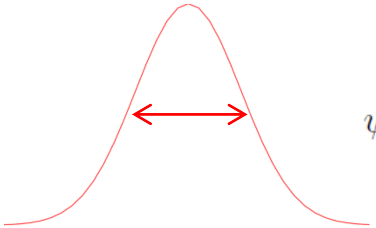


- Semi-classical approach : point particles
- Test particle method : 1~500 test particles per nucleon

$$f(\mathbf{r},\mathbf{p};t) = \frac{1}{N_{TP}} \sum_{i=1}^{AN_{TP}} \delta(\mathbf{r}-\mathbf{r}_i(t))\delta(\mathbf{p}-\mathbf{p}_i(t))$$

Randomly distributed !

➢ QMD Model



L=1.08 fm^2
-> r_N=1.8 fm

- Gaussian wave packets

$$\psi_i(\mathbf{r},\mathbf{p}_{i0},\mathbf{r}_{i0},t) = \frac{\exp\{i[\mathbf{p}_{i0}\cdot(\mathbf{r}-\mathbf{r}_{i0}) - p_{i0}^2 t/2m]\}}{[\sqrt{\pi/2L}2L(t)]^{2/3}} \exp\{-[\mathbf{r}-\mathbf{r}_{i0}-\mathbf{p}_{i0}t/m]^2/4L(t)\}$$

$$f(\mathbf{r},\mathbf{p},t) = \frac{1}{(2\pi)^3} \int e^{-i\mathbf{p}\cdot\mathbf{r}_{12}} \psi_i(\mathbf{r}+\mathbf{r}_{12}/2,t)\psi_i^*(\mathbf{r}-\mathbf{r}_{12}/2,t) d^3 r_{12}$$
$$= \frac{1}{\pi^3} \exp[-(\mathbf{r}-\mathbf{r}_{i0}-\mathbf{p}_{i0}t/m)^2/sL - (\mathbf{p}-\mathbf{p}_{i0})^2 \cdot 2L]$$

Introduction to Transport Model

2. Propagation

Boltzmann-Uehling-Uhlenbeck equation

$$\frac{df}{dt} = \underbrace{\frac{\partial f}{\partial t} + \frac{\vec{p}}{m}\vec{\nabla}^{(r)} f}_{\text{drift term}} - \underbrace{\vec{\nabla}^{(r)} U \vec{\nabla}^{(p)}}_{\text{accelerating term}} f = \underbrace{I_{coll}}_{\text{collision term}}$$

cf) RBUU eq. : $[p_\mu^* \partial_x^\mu + (p_\nu^* F^{\mu\nu} + m^*(\partial_x^\mu m^*))\partial_\mu^{p^*}] f(x, p^*) = \mathcal{I}_{coll}$

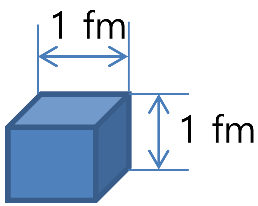
- **BUU Model**

  Equation of motion

  $$\frac{\partial r_i}{\partial t} = \frac{p_i}{m}, \qquad \frac{\partial p_i}{\partial t} = -\nabla U|_{r_i}$$

  Density dependent mean field

  $$U(\rho) = \alpha(\rho/\rho_0) + \beta(\rho/\rho_0)^\gamma$$



(Unit box for density estimation)

Introduction to Transport Model

➢ QMD Model

Equation of motion

$$\frac{d}{dt}\mathbf{r}_i = \{\mathbf{r}_i, \mathcal{H}\}, \quad \frac{d}{dt}\mathbf{p}_i = \{\mathbf{p}_i, \mathcal{H}\} \qquad \mathcal{H}\{\mathbf{r}_n, \mathbf{p}_n\} = \sum_{i=1}^{A} \frac{\mathbf{p}_i^2}{2m_i} + \sum_{i<j} V(|\mathbf{r}_i - \mathbf{r}_j|)$$

Nucleon-Nucleon interaction

: Skyrme , Volkov , Gogny , …

ex) Skyrme force

$$V^{\mathrm{loc}} = t_1\delta(\boldsymbol{r}_1 - \boldsymbol{r}_2) + t_2\delta(\boldsymbol{r}_1 - \boldsymbol{r}_2)\delta(\boldsymbol{r}_1 - \boldsymbol{r}_3)$$

$$\Rightarrow \quad U^{\mathrm{loc}} = \alpha(\rho/\rho_0) + \beta(\rho/\rho_0)^{\gamma}$$

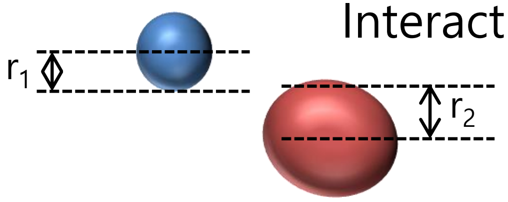
ex) Gogny force

$$v_{ij} = \sum_{k=1}^{2} v_{0k}(W_k + B_k P_\sigma - H_k P_\tau - M_k P_\sigma P\tau)\exp[-(\mathbf{r}_i - \mathbf{r}_j)^2/a_k^2]$$

$$+ \frac{t_\rho}{6}(W_\rho + B_\rho P_\sigma - H_\rho P_\tau - M_\rho P_\sigma P_\tau)\rho(\mathbf{r}_i)^\sigma \delta(\mathbf{r}_i - \mathbf{r}_j),$$

Introduction to Transport Model

3. Collision



Interaction radius = $\pi(r_1 + r_2)^2$

$$b < \frac{1}{\pi}\sqrt{\sigma^{tot}(\sqrt{s})}$$ Scatter with probability 1 !

Pauli blocking factor

$$I_{coll} = \int d\mathbf{v}_2 d\mathbf{v}_{1'} d\mathbf{v}_{2'} |\mathbf{v}_2 - \mathbf{v}_1| \sigma(\Omega)(2\pi)^3 \delta(\mathbf{p}_1 + \mathbf{p}_2 - \mathbf{p}_{1'} - \mathbf{p}_{2'})$$

$$[f_{1'} f_{2'} (1 - f_1)(1 - f_2) - f_1 f_2 (1 - f_{1'})(1 - f_{2'})]$$

In-medium cross-section

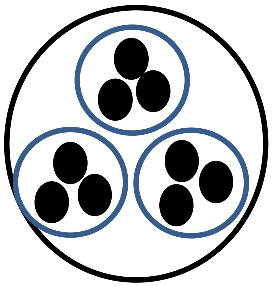
<Elastic and In-elastic scattering>

NN -> NN , NN -> NΛK , NN -> NΔ ,
NΔ -> NΔ , Δ -> Nπ , …

Introduction to Transport Model

- **BUU Model**

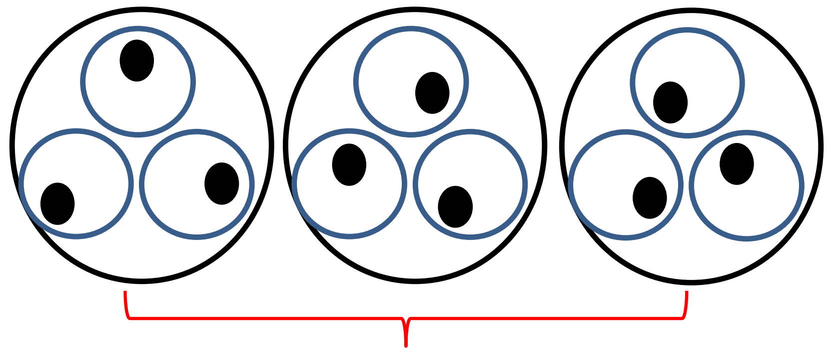
<Full ensemble method>



$\sigma_{TP} = \sigma_{NN}/N_{TP}$

$\sim (AN_{TP})^2$

<Parallel ensemble method>

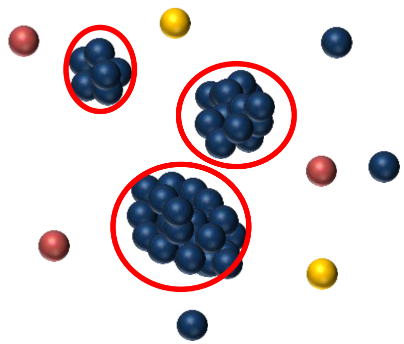


N_{TP} ensembles

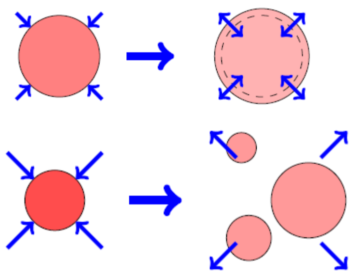
$\sim N_{TP}A^2$

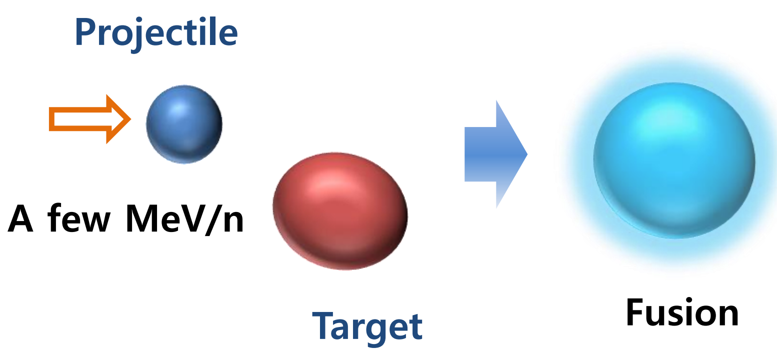
Introduction to Transport Model

4. Clustering

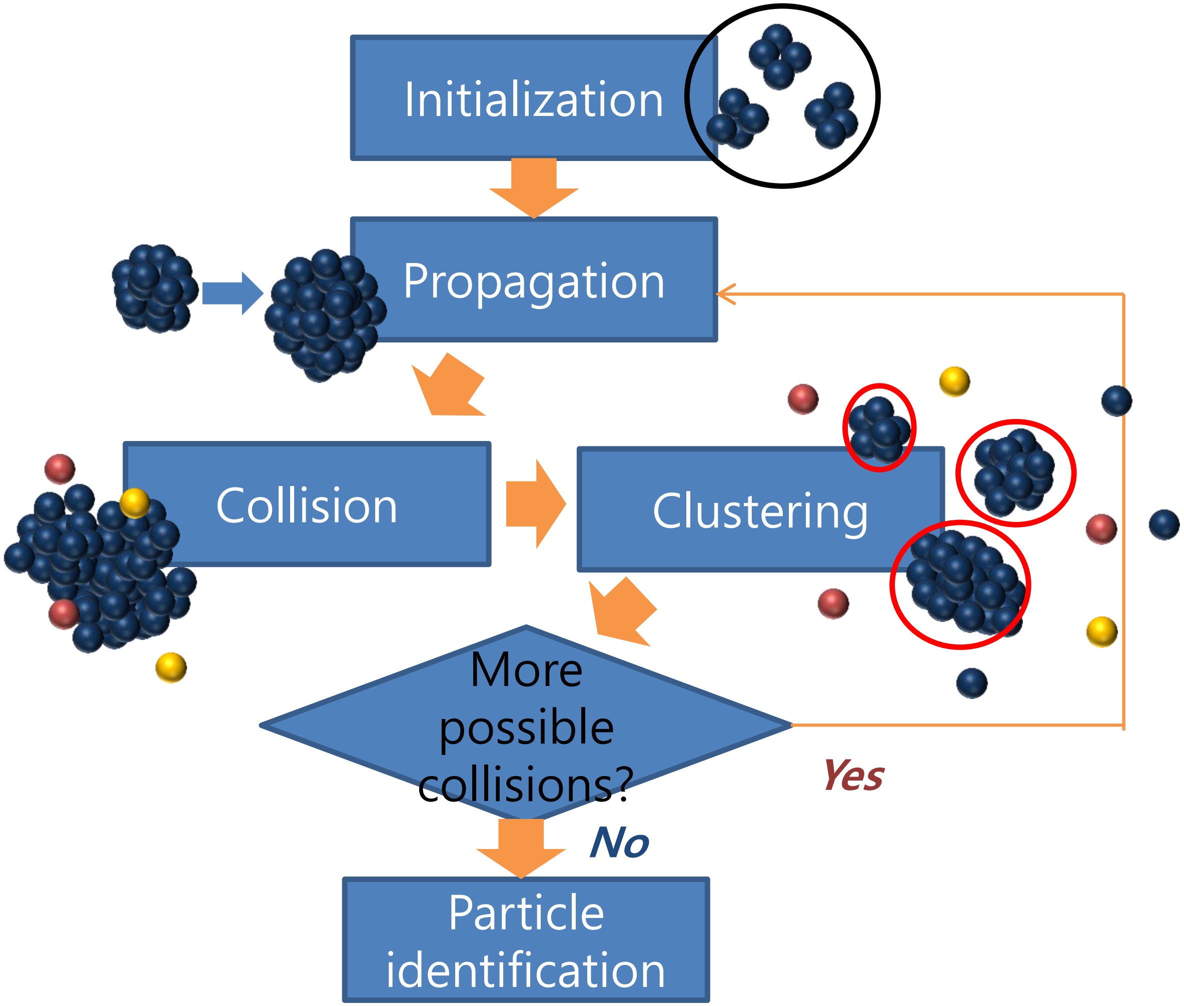


Identifications of collision fragments are performed by clustering nearby nucleons.





Introduction to Transport Model

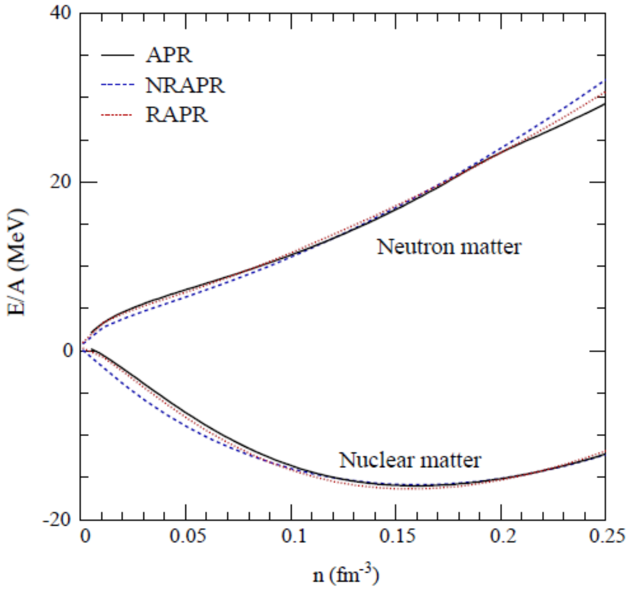


EOS and Symmetry Energy

Equation of State

$$E(\rho,\delta) = E(\rho,\delta=0) + E_{sym}(\rho)\delta^2 + \mathcal{O}(\delta^4)$$

where, $\delta = \dfrac{\rho_n - \rho_p}{\rho_n + \rho_p}$



Ref.) A.Steiner et al. P.Rept 411 (2005) 325

- ➤ Astrophysics (super novae, neutron star)
- ➤ Giant monopole resonance
- ➤ Heavy-ion collisions

Incompressibility of symmetry nuclear matter at its saturation density $\rho \approx 0.16\ \mathrm{fm}^{-3}$

$K_0 = 231 \pm 5\ [\mathrm{MeV}]$ ➡ from Giant Monopole Resonance

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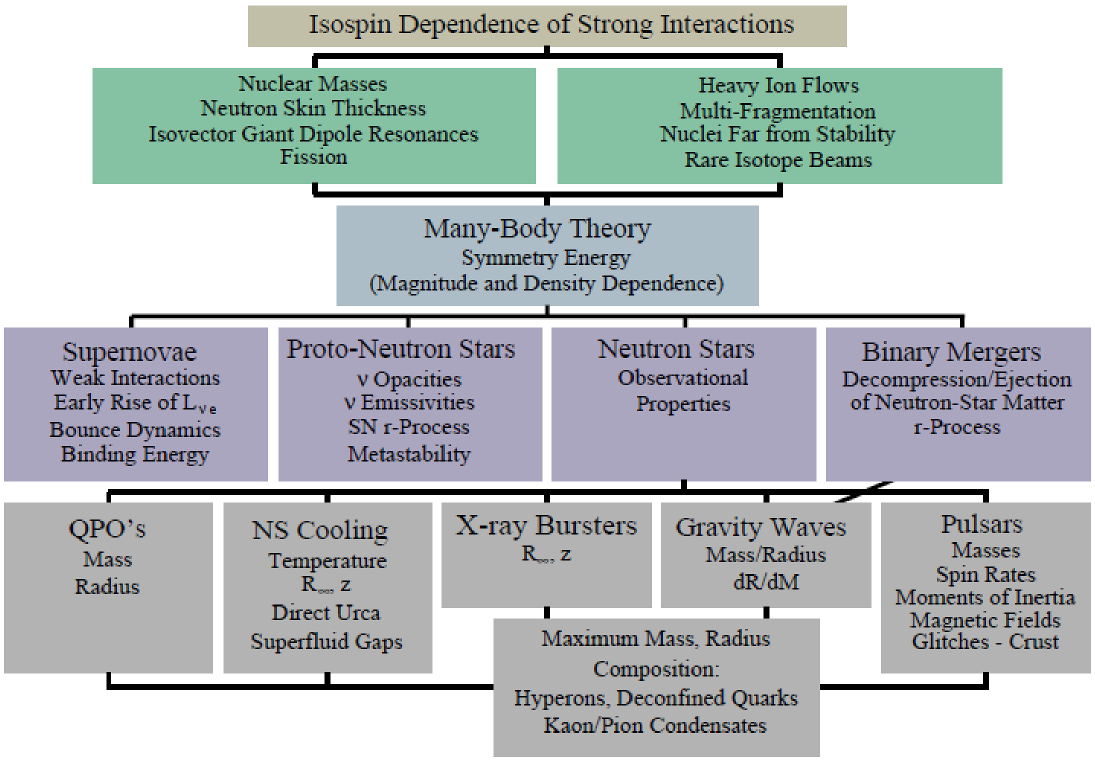
EOS and Symmetry Energy

<The multifaceted influence of the nuclear symmetry energy>



Ref.) A.Steiner et al. P.Rept 411 (2005) 325

To explore the EOS of isospin asymmetric matter from heavy-ion reactions induced by neutron-rich beams, we need appropriate theoretical tools.

-> Transport Model !!

CoMD and ImQMD

Typical QMD + constraints -> less CPU time

Constrained Molecular Dynamics (CoMD)

$$\overline{f_i} \leq 1 \quad (\text{for all } i),$$

$$\overline{f_i} \equiv \sum_j \delta_{\tau_i,\tau_j} \delta_{s_i,s_j} \int_{h^3} f_j(\mathbf{r},\mathbf{p}) d^3r d^3p.$$

Restriction on phase space density

Improved Quantum Molecular Dynamics (ImQMD)

$$U_{loc} = \frac{\alpha}{2} \sum_i \left\langle \frac{\rho}{\rho_0} \right\rangle_i + \frac{\beta}{3} \sum_i \left\langle \frac{\rho^2}{\rho_0^2} \right\rangle_i + \frac{C_s}{2} \int \frac{(\rho_p - \rho_n)^2}{\rho_0} d^3\mathbf{r} + \int \frac{g_1}{2} (\nabla \rho)^2 d^3\mathbf{r}.$$

$$\sum_i \left\langle \frac{\rho^2}{\rho_0^2} \right\rangle_i \approx \sum_i \left\langle \frac{\rho}{\rho_0} \right\rangle_i^2 + \int \frac{g_2}{2} (\nabla \rho)^2 d^3\mathbf{r},$$

Consideration of surface energy

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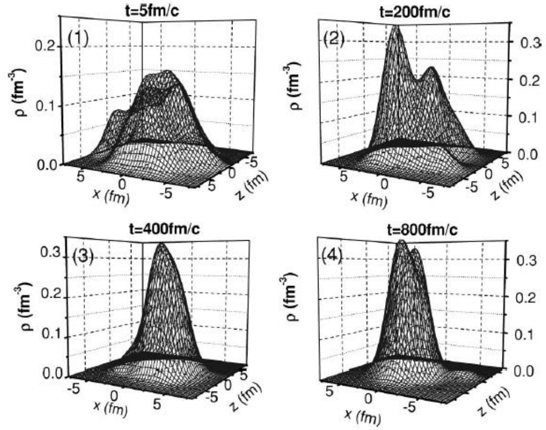
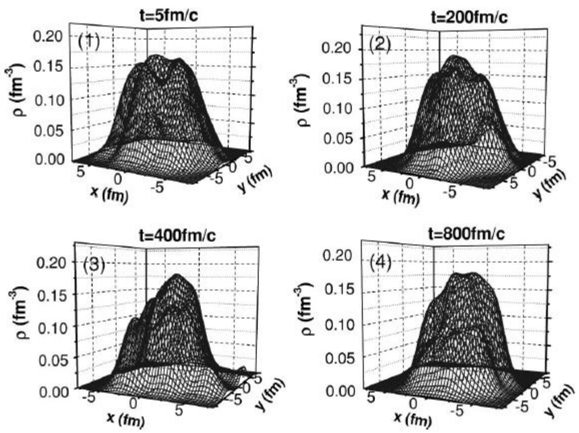
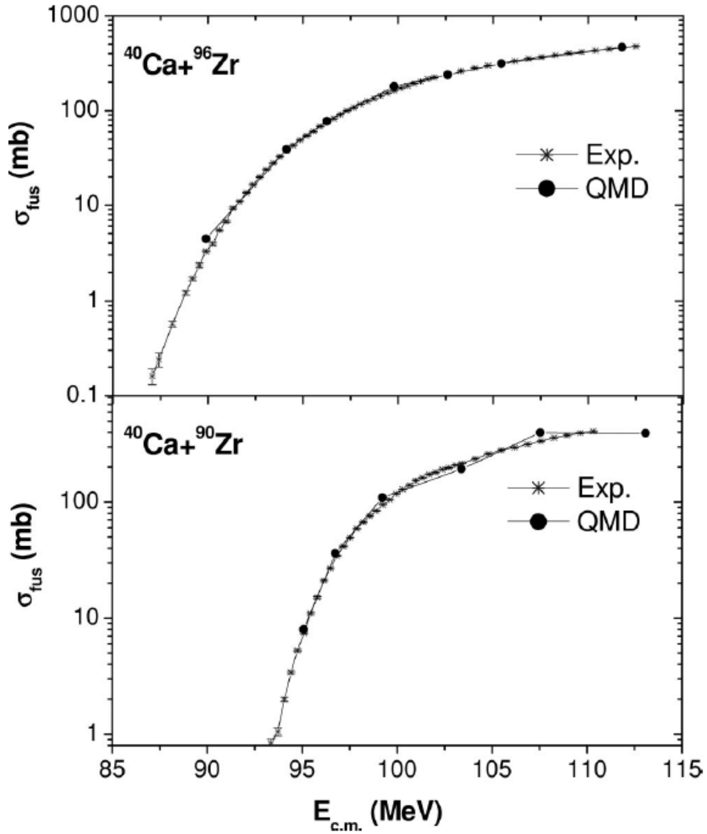
CoMD and ImQMD

Testing stability of nuclei





Fusion cross-sections



N.Wang – Phys.Rev.C 65 (2002) 064608

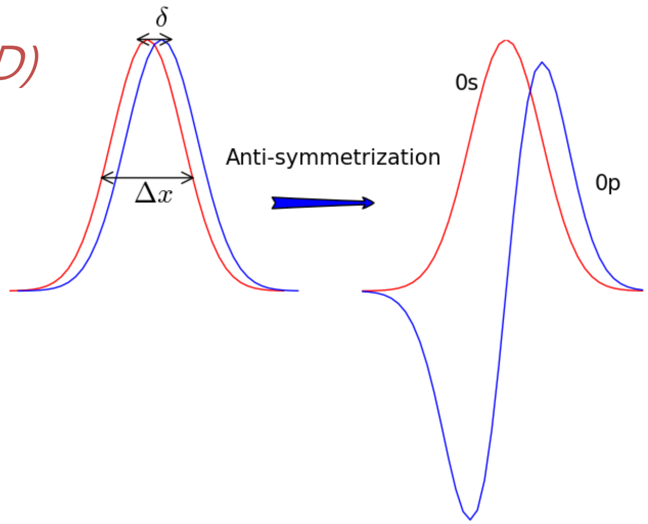
FMD and AMD

Fermionic Molecular Dynamics (FMD)
Antisymmetrized Molecular Dynamics (AMD)

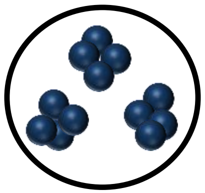
AMD wave function

$$|\Phi(Z)>= det_{ij}\left[\exp\left\{-\nu\left(\mathbf{r}-\frac{\mathbf{Z}}{\sqrt{\nu}}\right)^2\right\}\chi_{\alpha_i}(j)\right]$$

Slater Determinant



Ex.) ^{12}C in AMD



3α structure

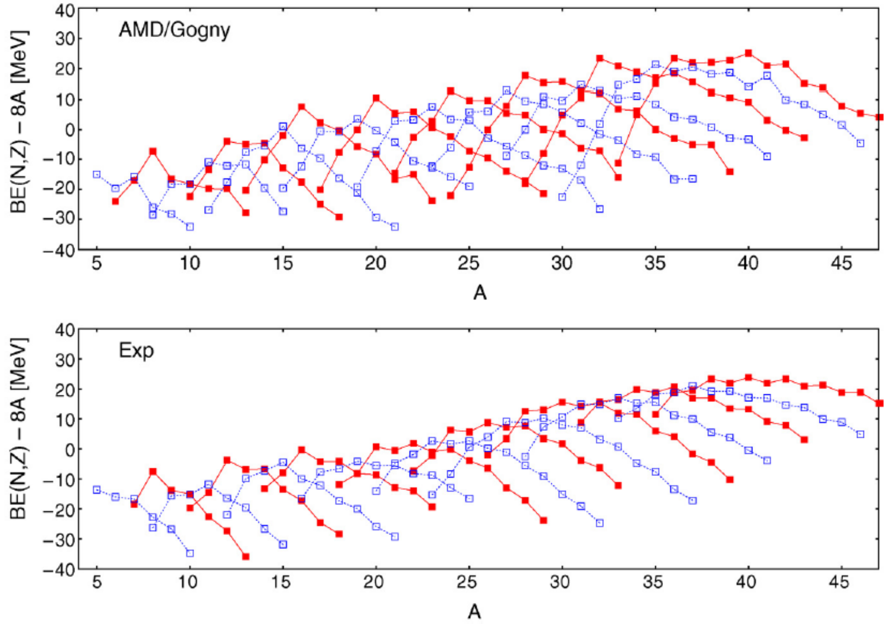
Equation of motion for the wave packet centroids *Z*

$$\frac{d}{dt}Z_i = \{Z_i, \mathcal{H}\}_{PB} \quad \text{and} \quad i\hbar\sum_{j=1}^{A}\sum_{\tau=x,y,z} C_{i\sigma,j\tau}\frac{dZ_{j\tau}}{dt} = \frac{\partial\mathcal{H}}{\partial Z_{i\sigma}}$$

$$(\text{c.f. } C_{i\sigma,j\tau} = \delta_{ij}\delta_{\sigma\tau} \text{ in QMD})$$

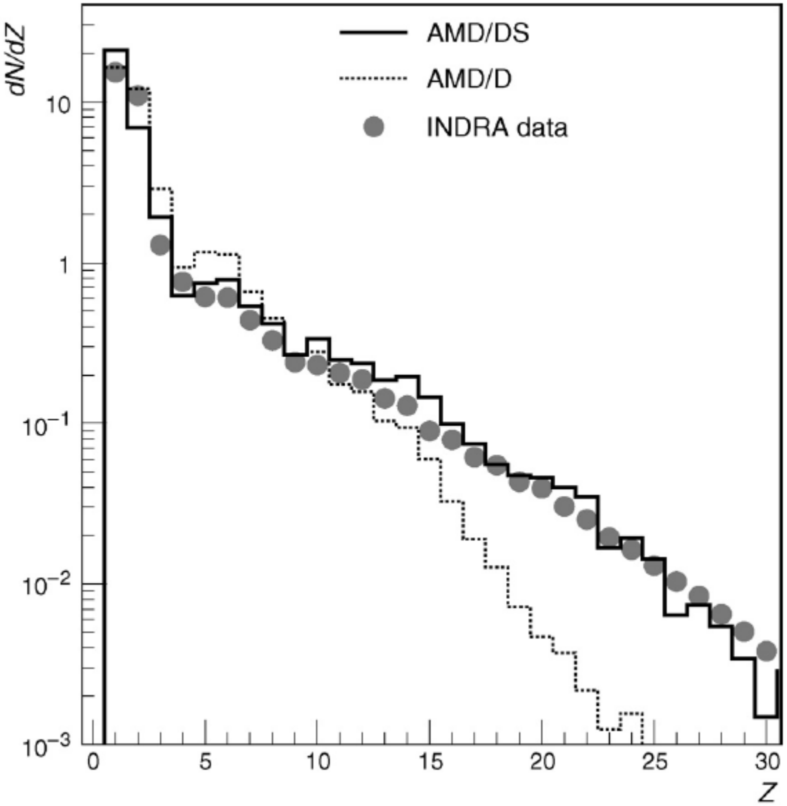
Better fermionic nature -> more CPU time!

FMD and AMD



<Binding energies of nuclei>

The charge distribution of the produced clusters in ^{129}Xe+Sn at 50 MeV/n

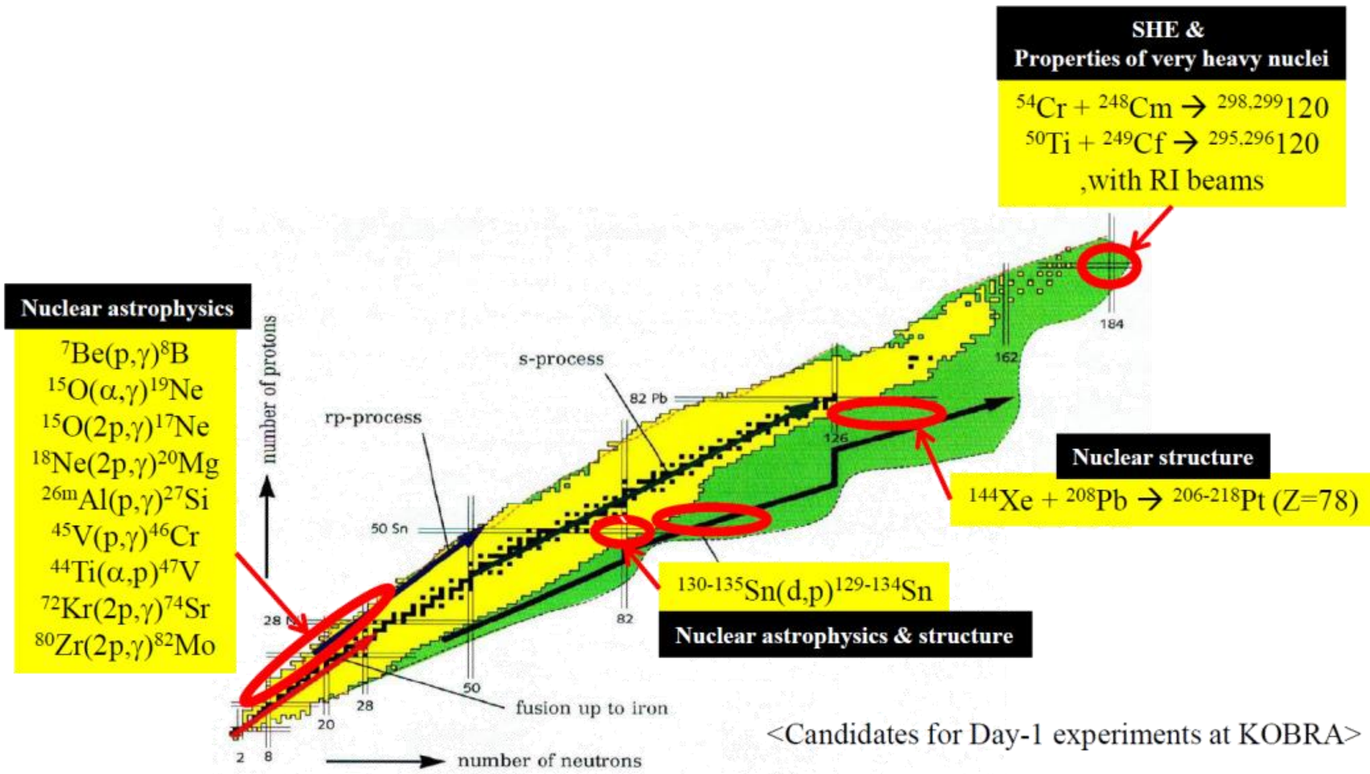


A.Ono – Prog.Part.Nucl.Phys 53 (2004) 501

Transport Model for RAON

Unstable nuclei ➡ $r \neq 1.12A^{1/3}$ ***e.g. ^{11}Li is bigger than ^{208}Pb***

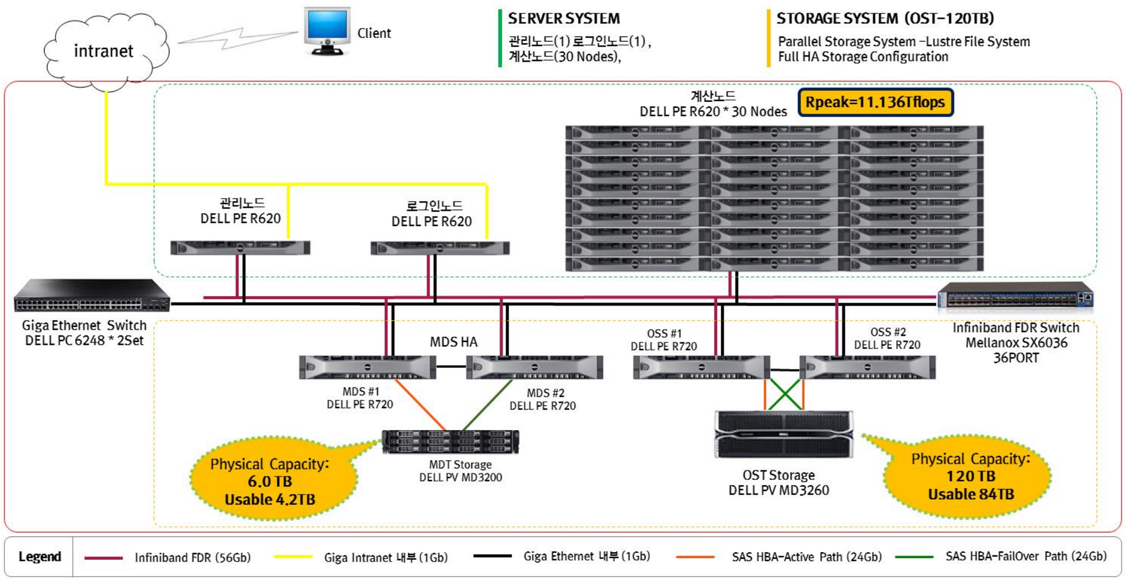
Nuclear Structure !!



<Candidates for Day-1 experiments at KOBRA>

Transport Codes and Super Computer

Cluster at RISP : 480 CPU cores



Tachyon II at KISTI : 25408 CPU cores

Summary and Outlook

- ❖ RAON facility will provide opportunities to study isospin asymmetric matters and exotic nuclei by heavy-ion collisions induced by neutron-rich beams.
- ❖ As a reliable theoretical tools, we need a transport model.
- ❖ Transport model is a model to treat non-equilibrium aspects of the temporal evolution of a collision.
- ❖ Many transport model codes are developed for different energy regions and observables.
- ❖ To simulate HIC at RAON, we need a transport model which describes well low energy reactions and nuclear structures.

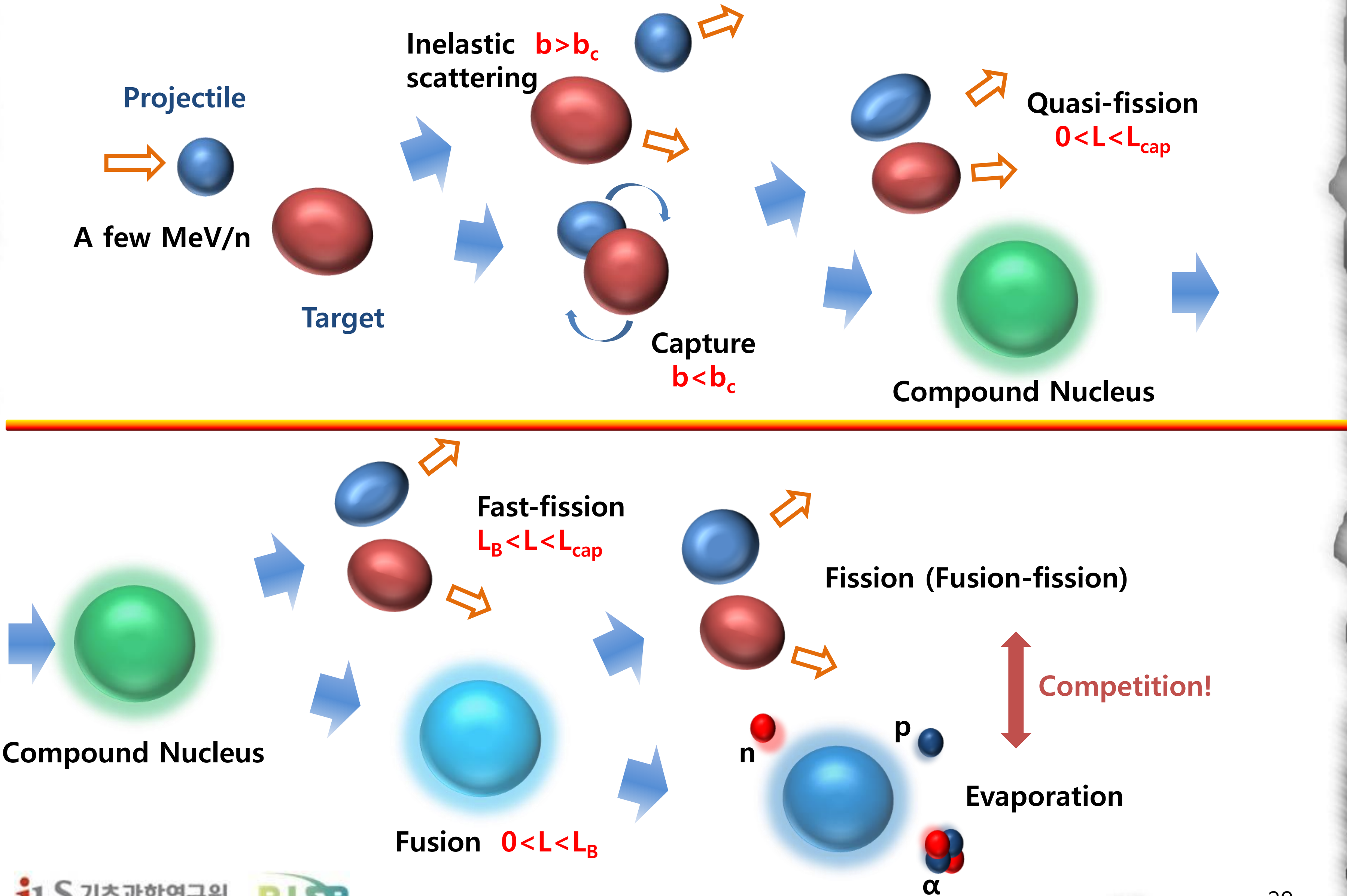
References

- BUU
  - G.Bertsch – Phys.Rept 160 (1988) 189
  - P.Danielewicz – Ann.Phys 152 (1984) 239, 305
- RBUU
  - O.Buss – Phys.Rept 512 (2012) 1
- QMD
  - J.Aichelin – Phys.Rept 202 (1991) 233
- AMD
  - A.Ono – Prog.Part.Nucl.Phys 53 (2004) 501
- FMD
  - H.Feldmeier – Nucl.Phys A515 (1990) 147
- ImQMD
  - N.Wang – Phys.Rev.C 65 (2002) 064608
- CoMD
  - M.Papa – Phys.Rev.C 64 (2001) 024612

Thank you for your attention!!

Backup Slides

<Schematic View of the Fusion Process>



Super Heavy Elements at RAON