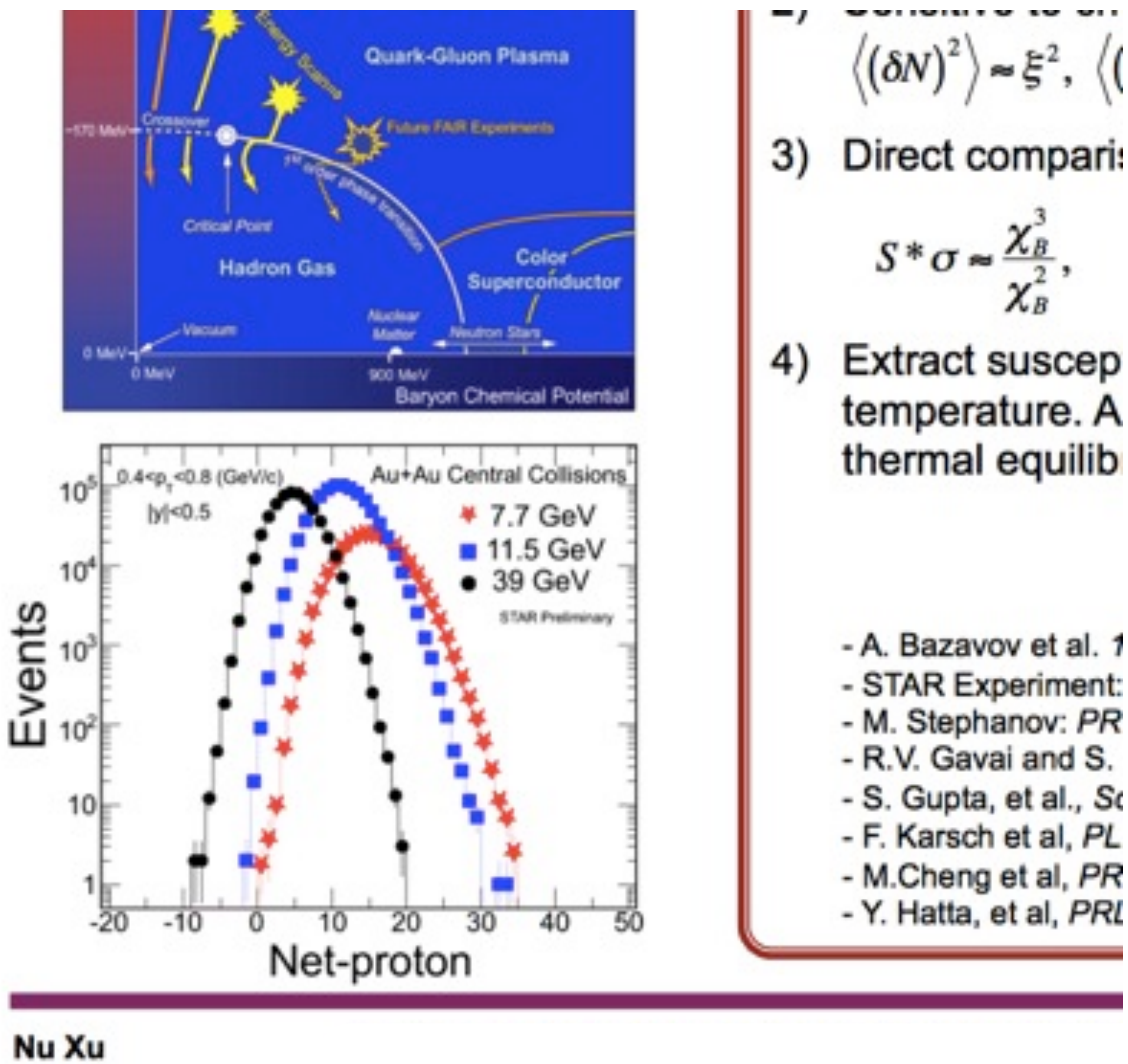
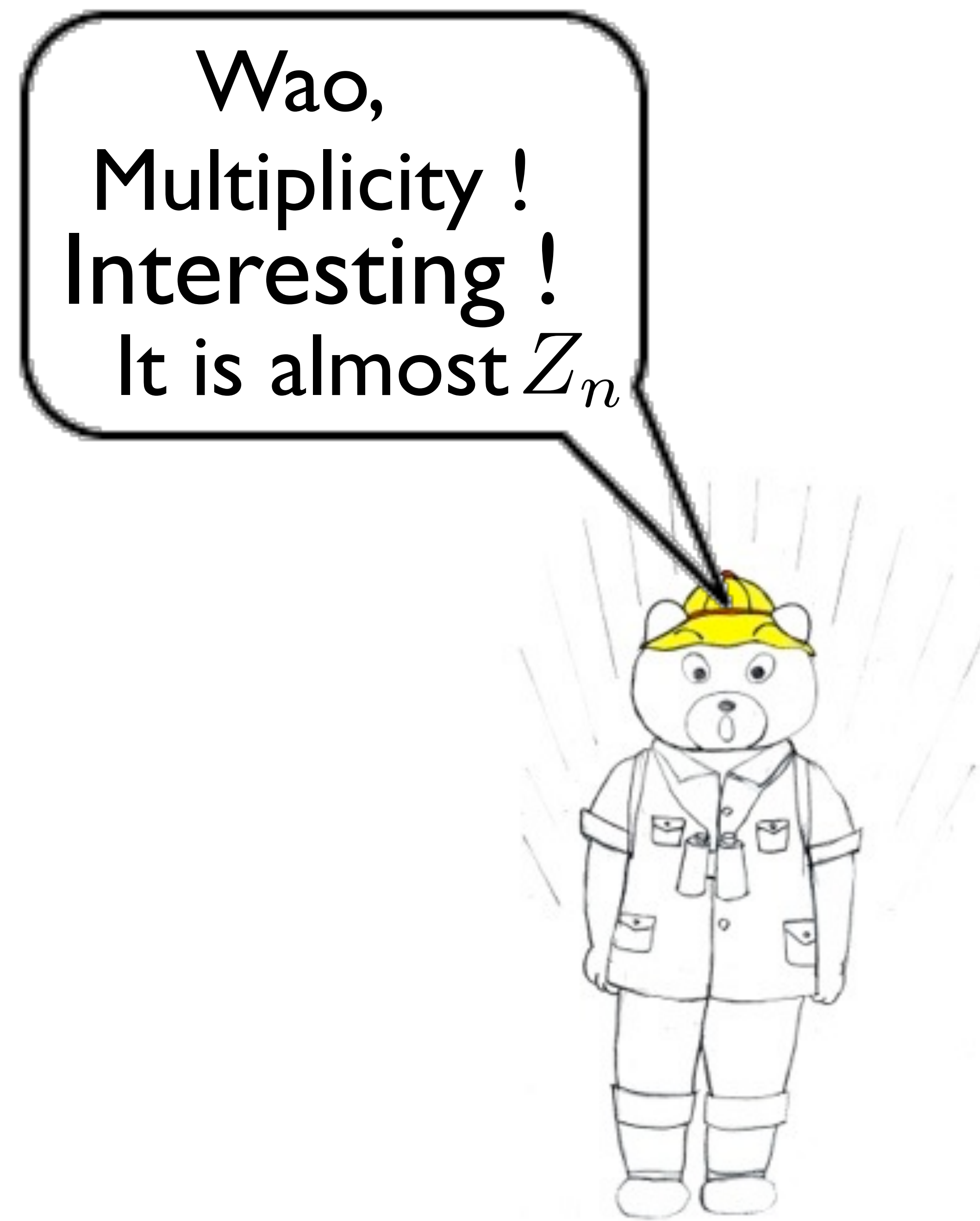
Probing QCD phase Structure by Baryon Multiplicity Distributionole

Atsushi Nakamura
in Collaboration with K.Nagata
HIM(Heavy Ion Meeting) 2013-10
Incheon, 2013 Nov.

Last Year, I saw Multiplicity Distribution of Heavy Ion Collisions (RHIC)





Message of This Talk

From Experiments
we can get Zn

Zn are Good
Quntities to study
QCD Phase

Canonical Partition
Functions

Plan of the Talk

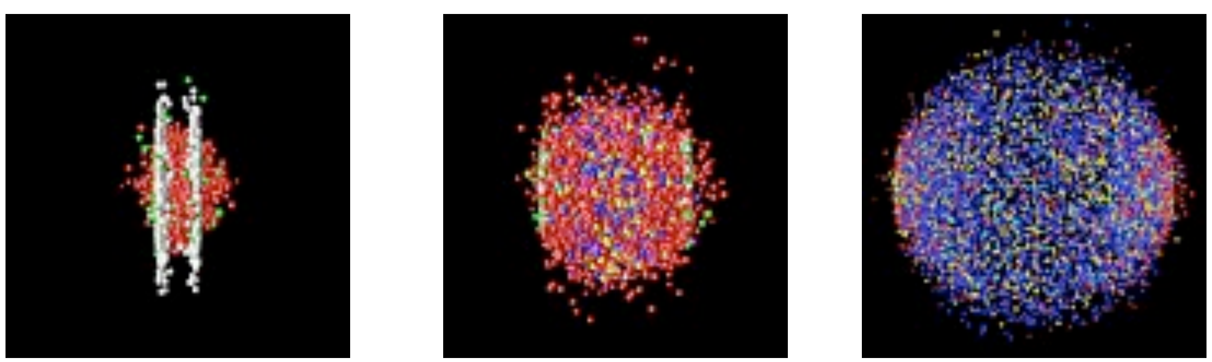
1. Getting Zn
2. Z as a function of μ
3. Lee-Yang zero

1.Getting Zn

We assume

the Fireballs created in High Energy Nuclear Collisons are described as a Statistical System.

with μ (chemical Potential)
and T (Temperature)



$$Z(\mu, T)$$

Grand Canonical

$$Z(\mu, T) \Leftrightarrow Z_n(T)$$

$$Z(\mu, T) = \mathrm{Tr}\, e^{-(H-\mu\hat{N})/T}$$

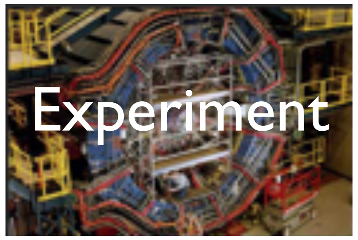
$$= \sum_n \langle n | e^{-(H-\mu\hat{N})/T} | n \rangle$$

If $[H, \hat{N}] = 0$

$$= \sum_n \langle n | e^{-H/T} | n \rangle \, e^{\mu n/T}$$

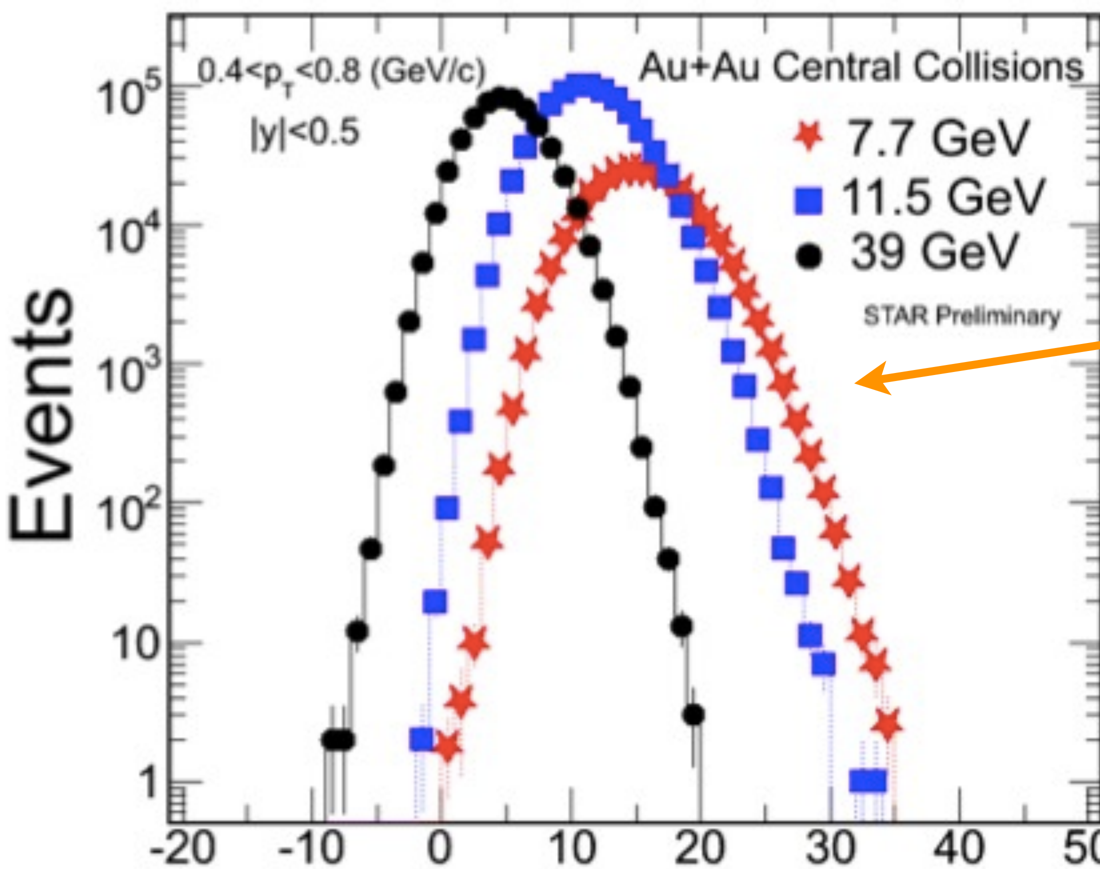
$$= \sum_n Z_n(T) \xi^n \qquad \left(\xi \equiv e^{\mu/T} \right)$$

Fugacity

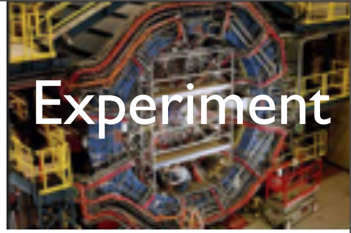


$$Z(\xi, T) = \sum_n Z_n(T)\, \xi^n$$

Partition Function is
Sum of the Probabilities
with n ...
If I know ξ, then I have Zn.



Net-proton number



How can we extract Z_n from $P_n = Z_n \xi^n$?

ξ unknown

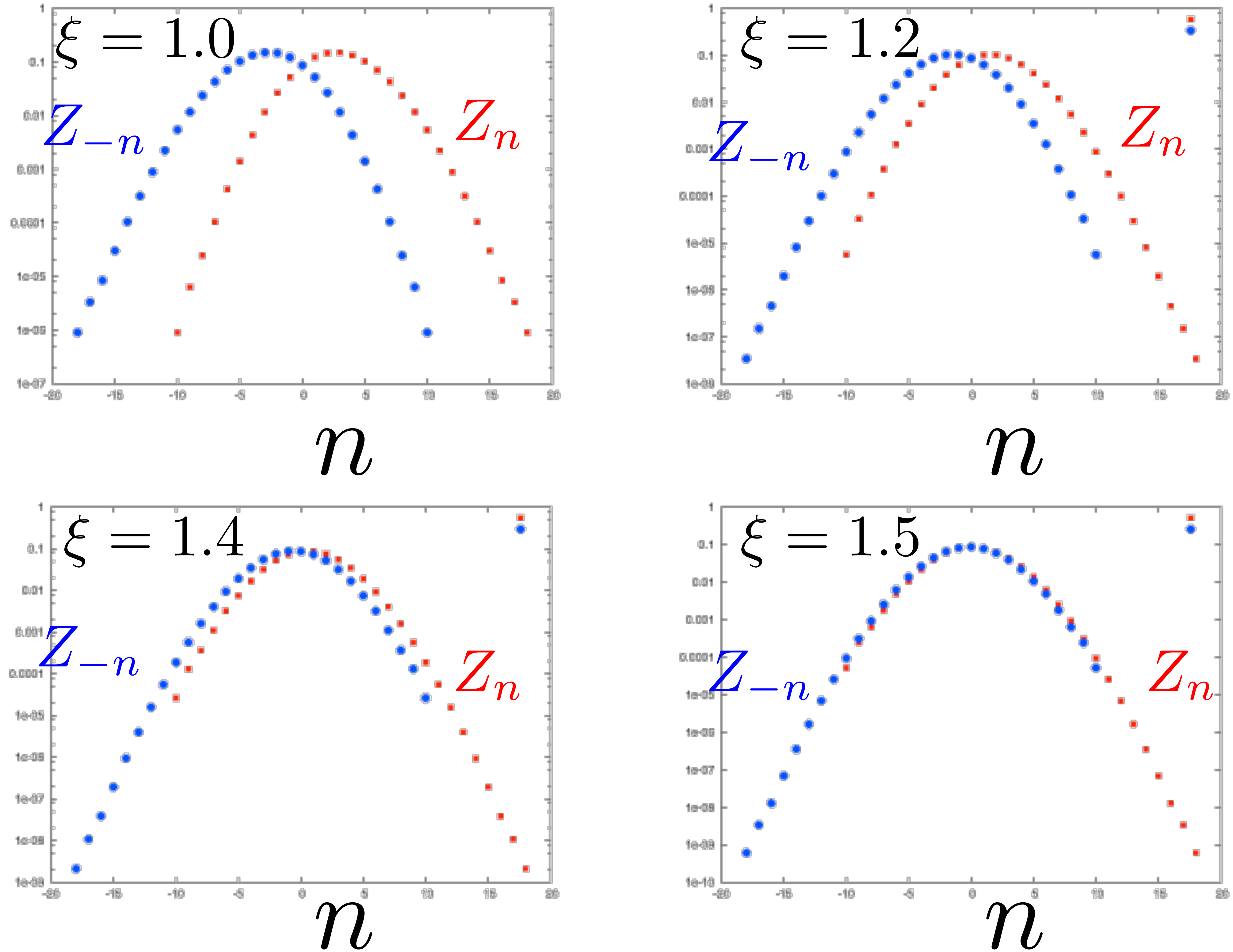
$$Z_n = P_n / \xi^n$$

We require (Particle-AntiParticle Symmetry)

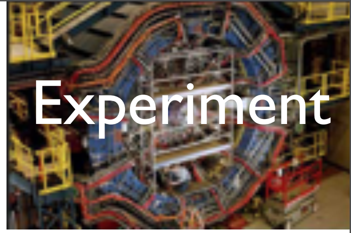
$$Z_{+n} = Z_{-n}$$

$$Z_n = P_n / \xi^n$$

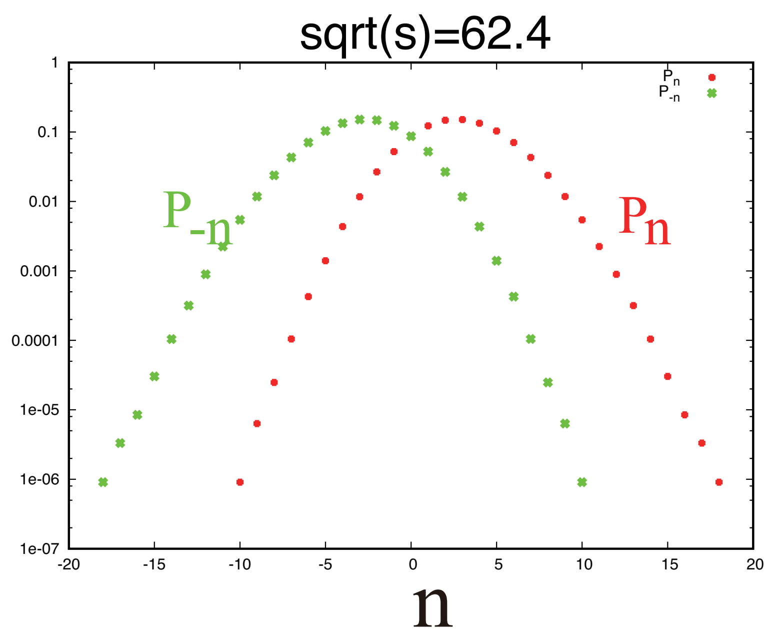
From Experiment



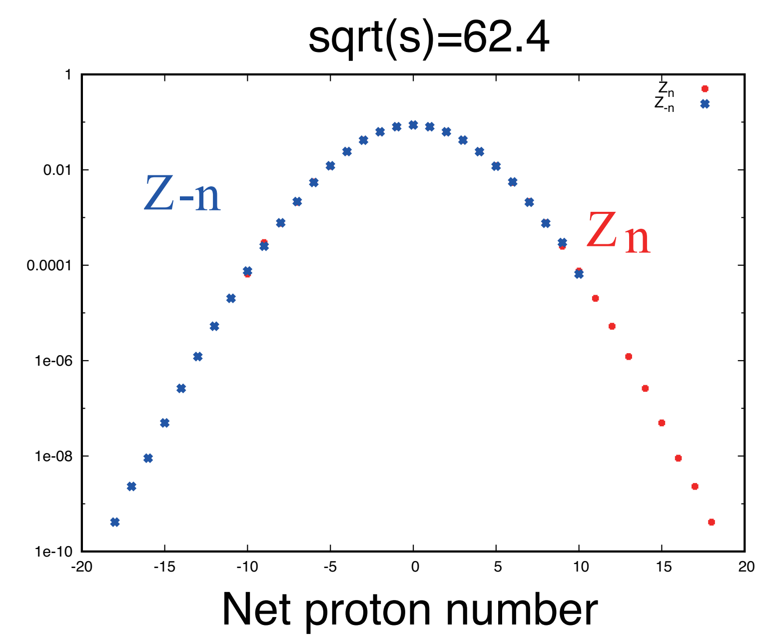
Final Value $\xi = 1.534$



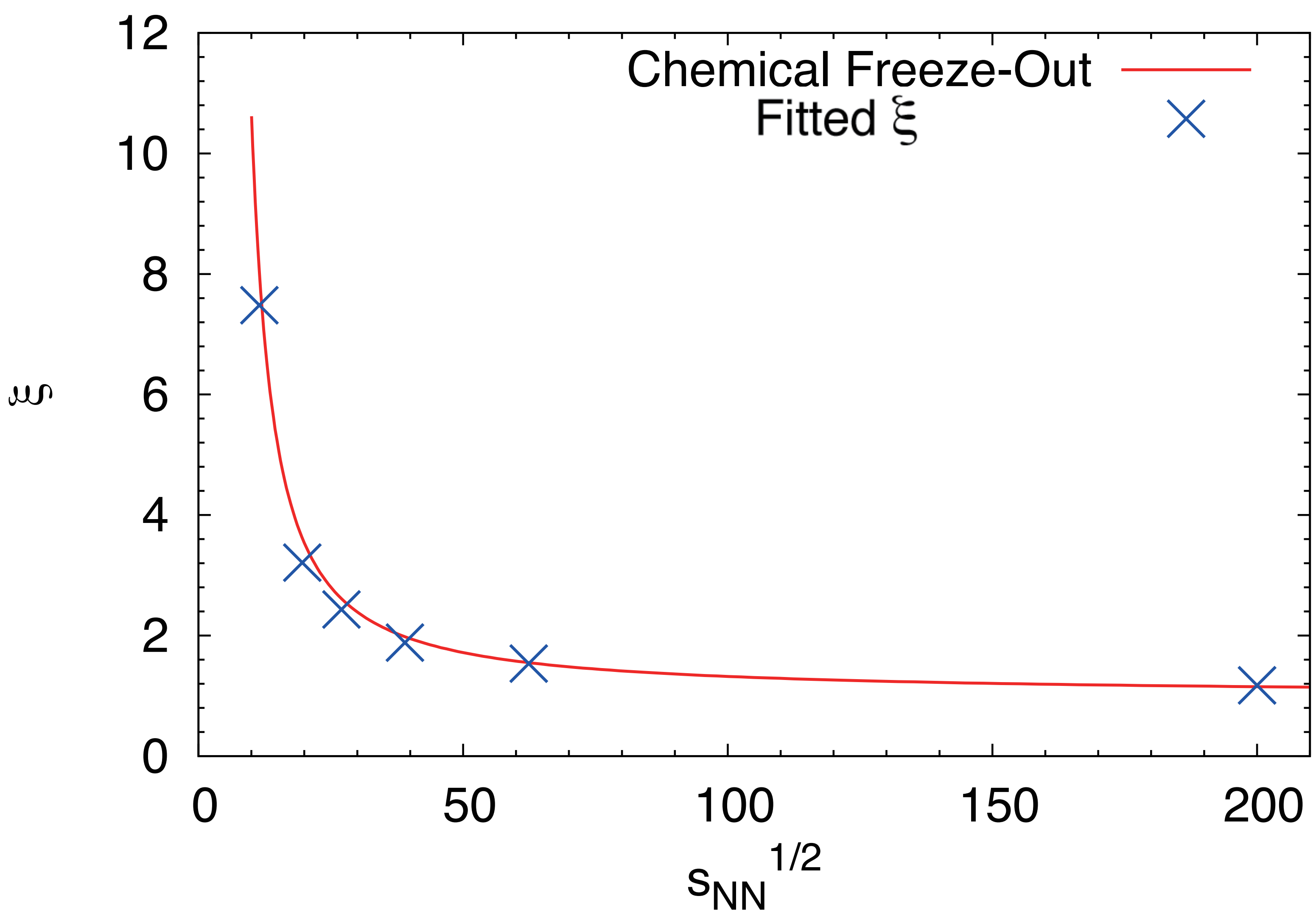
Demand $\boxed{Z_{+n} = Z_{-n}}$



Fit ξ



Fitted ξ are very consistent with those by Freeze-out Analysis.

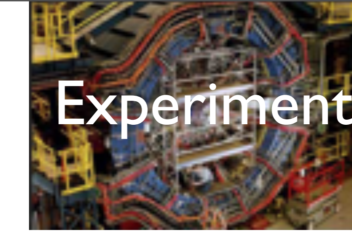


x This work

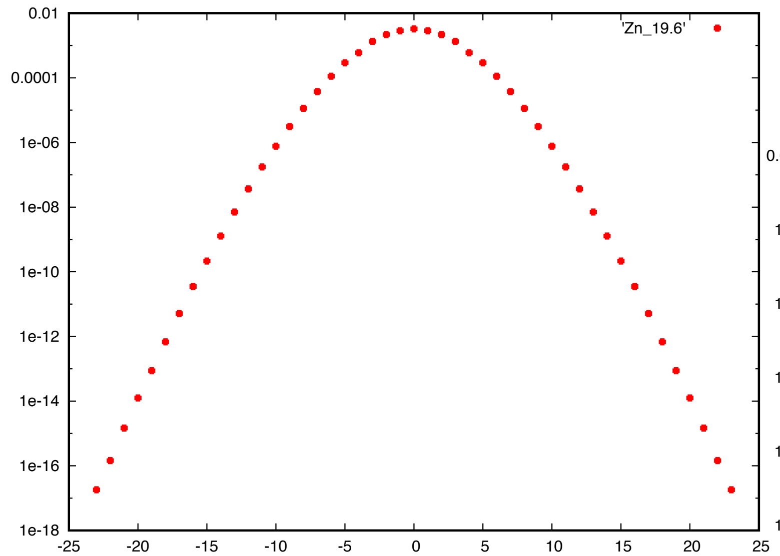
— Freeze-out

J.Cleymans, H.Oeschler, K.Redlich and S.Wheaton Phys. Rev. C73, 034905 (2006)

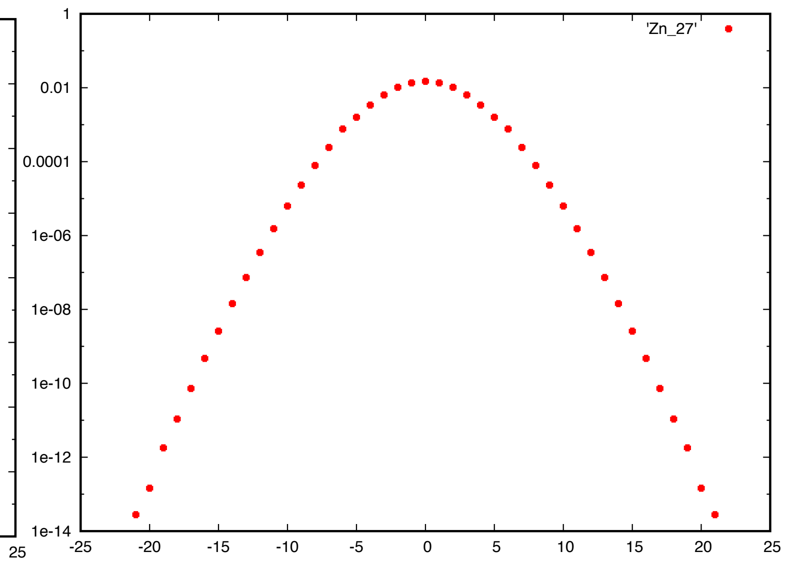
Z_n from RHIC data



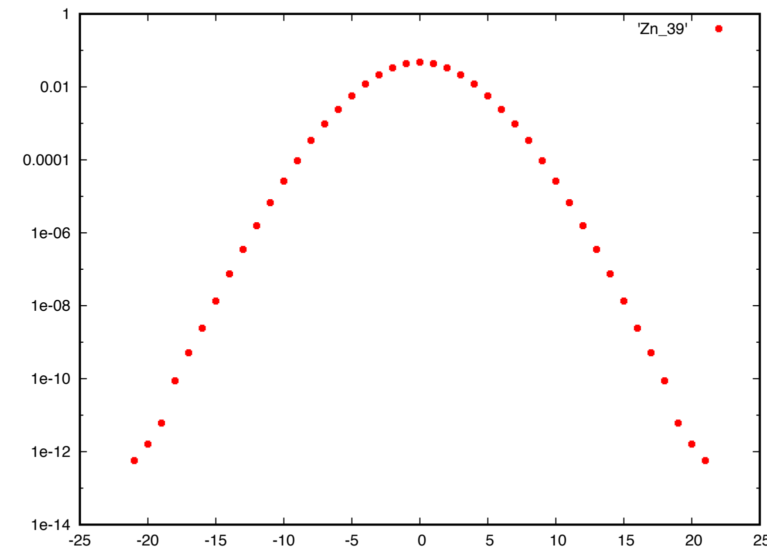
$\sqrt{s} = 19.6\mathrm{GeV}$



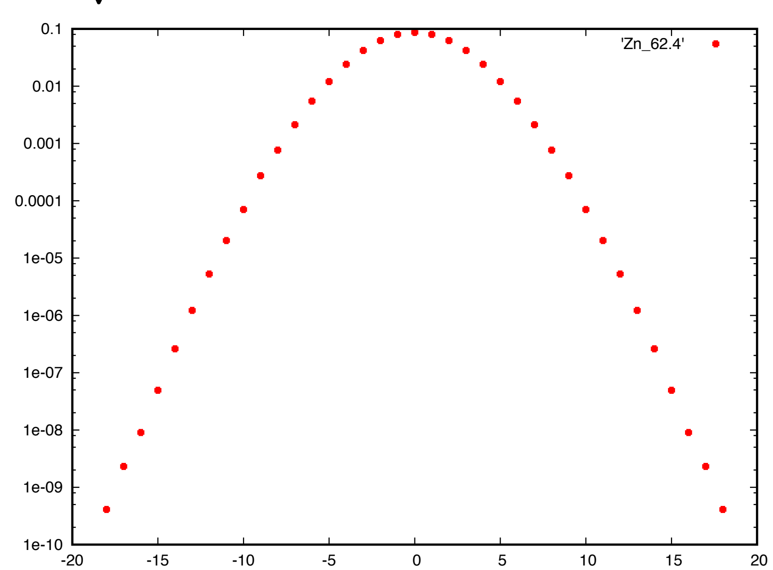
$\sqrt{s} = 27\mathrm{GeV}$



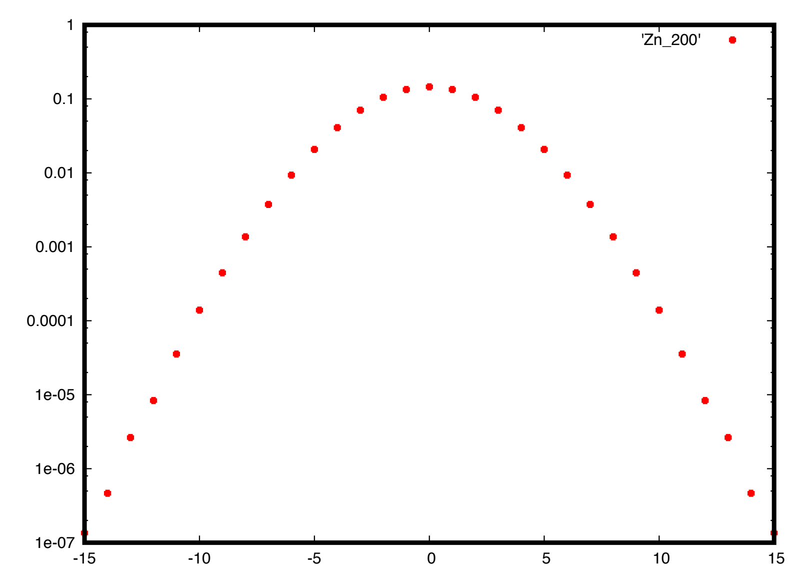
$\sqrt{s} = 39\mathrm{GeV}$



$\sqrt{s} = 62.4\mathrm{GeV}$



$\sqrt{s} = 200\mathrm{GeV}$



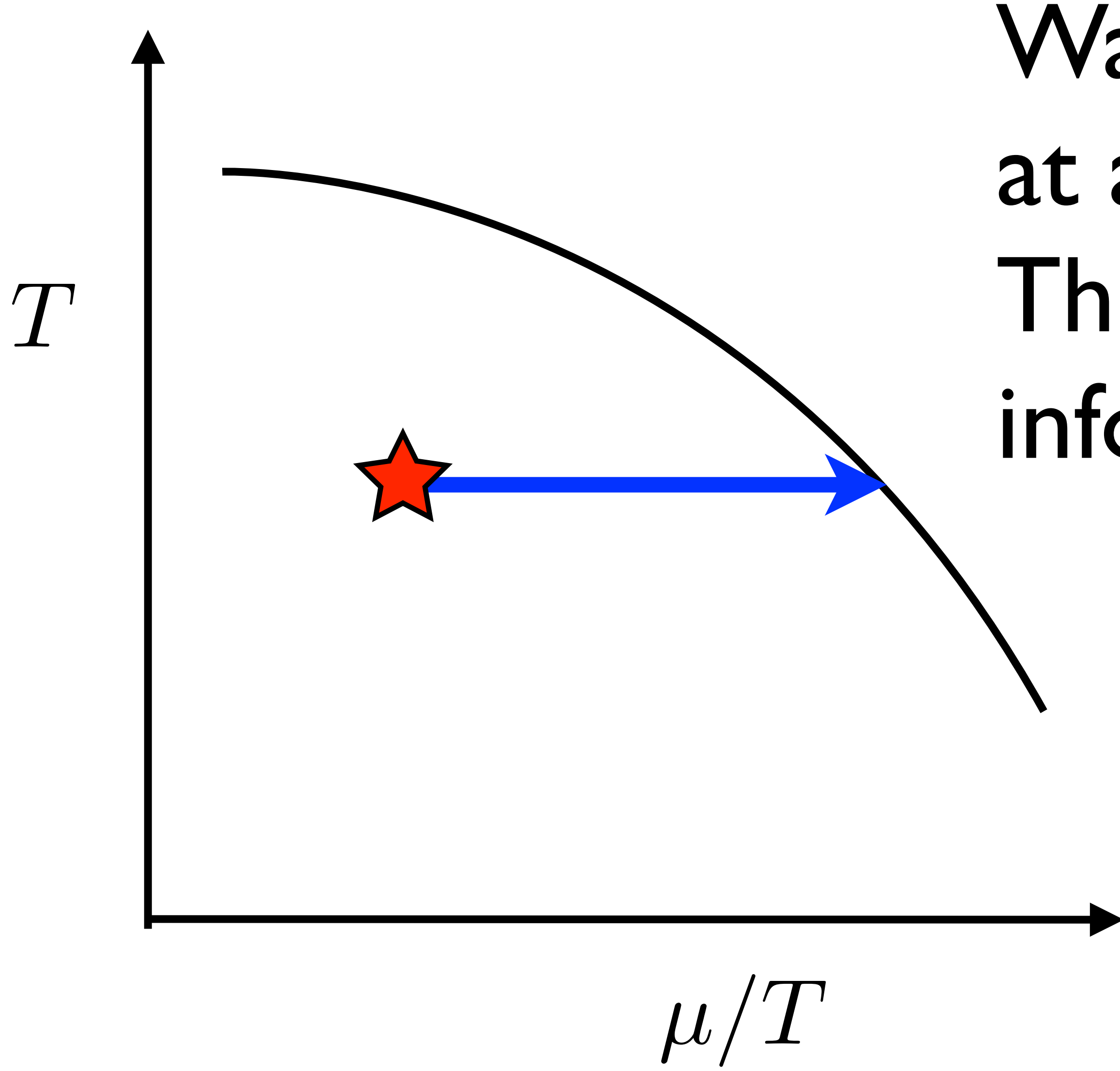
2. Z as a function of μ

Why you are happy to get Z_n

$$Z(\xi, T) = \sum_n Z_n(T)\,\xi^n$$

Now we have Zn of RHIC data
(sqrt(s)= 10.5,19.6, 27, 39, 62.4, 200 GeV)

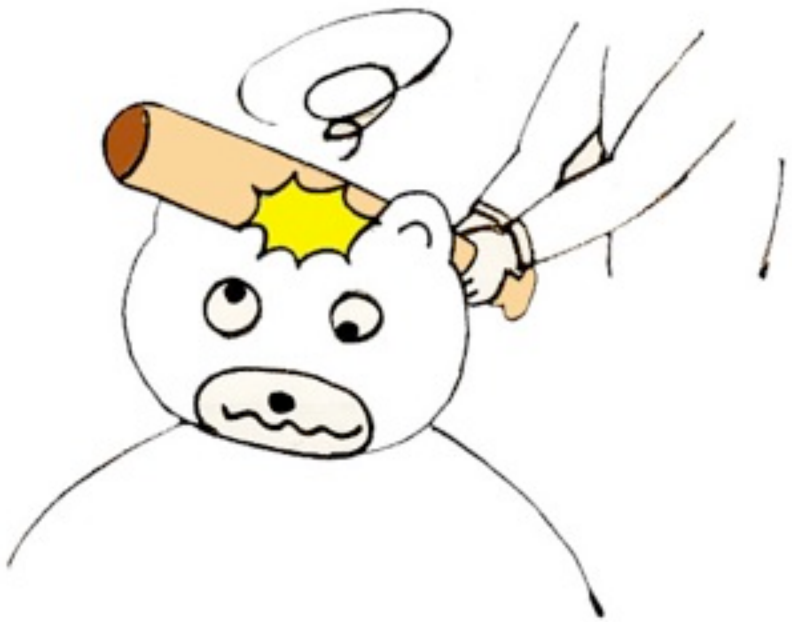
Wao ! We can calculate
at any density !
This includes all QCD Phase
information !



$(\xi \equiv e^{\mu/T})$

$$Z(\xi, T) = \sum_{n=-N_{max}}^{+N_{max}} Z_n(T)\,\xi^n$$

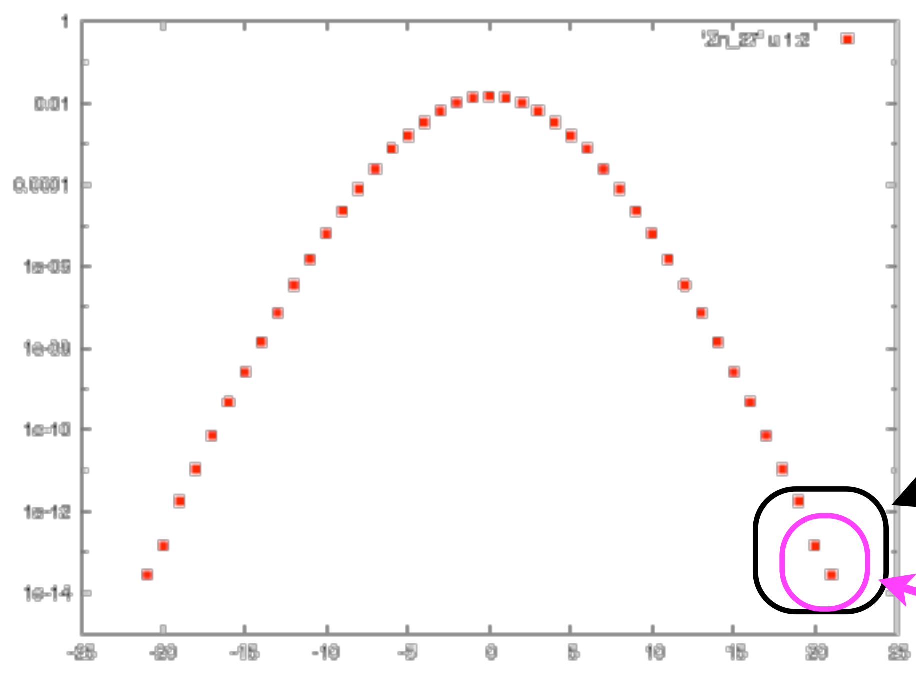
Do not forget that your *n* is finite !



Moments λ_k

$$Z(\xi, T) = \sum_{n=-N_{max}}^{+N_{max}} Z_n(T)\, \xi^n$$

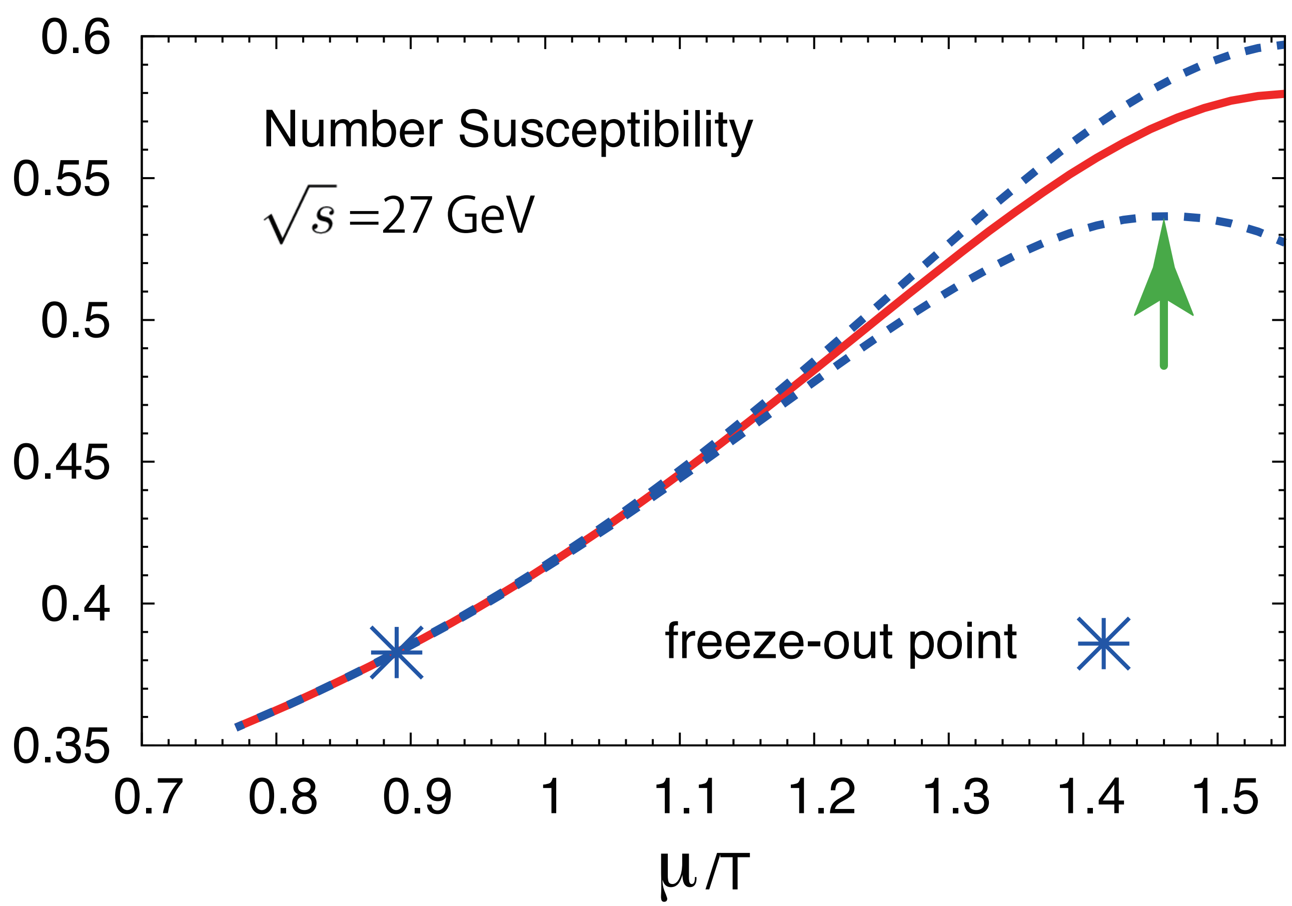
$$\lambda_k \equiv \left(T \frac{\partial}{\partial \mu} \right)^k \log Z$$

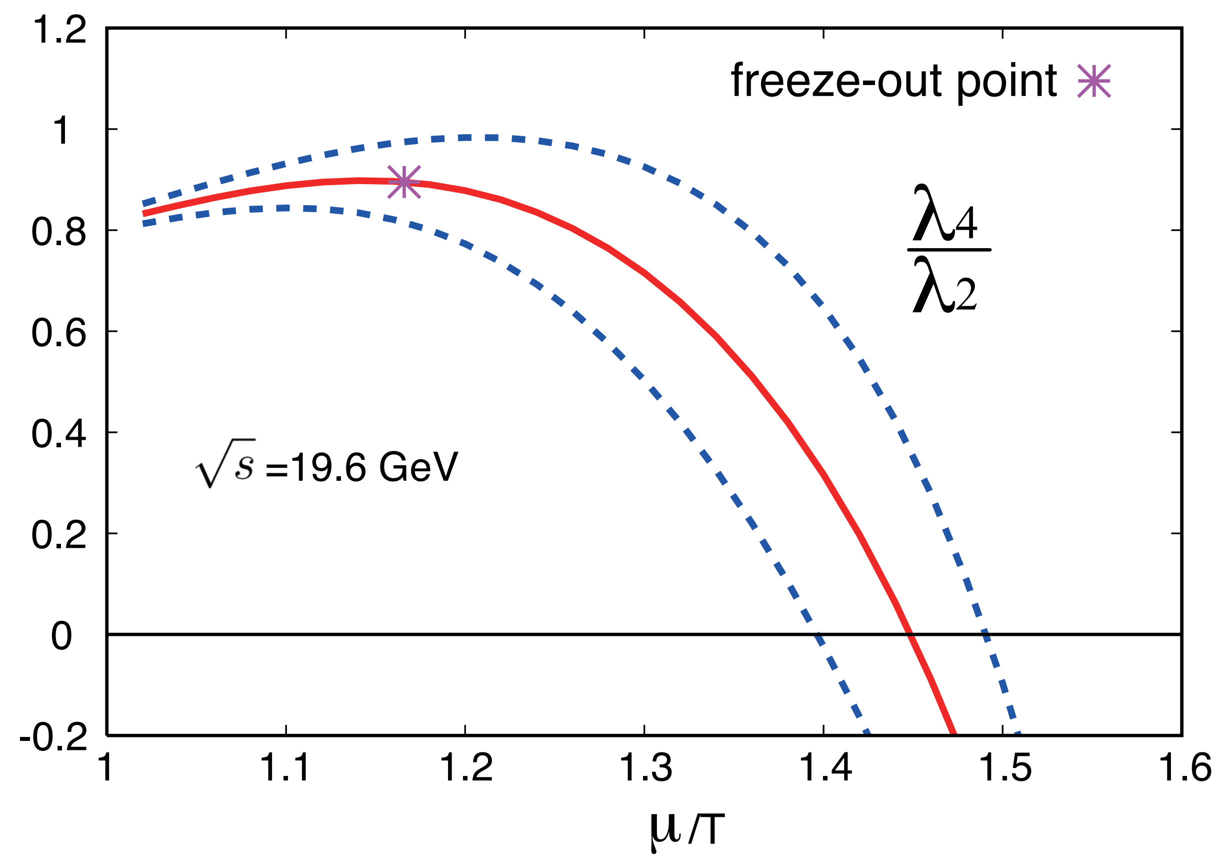


What happens ?

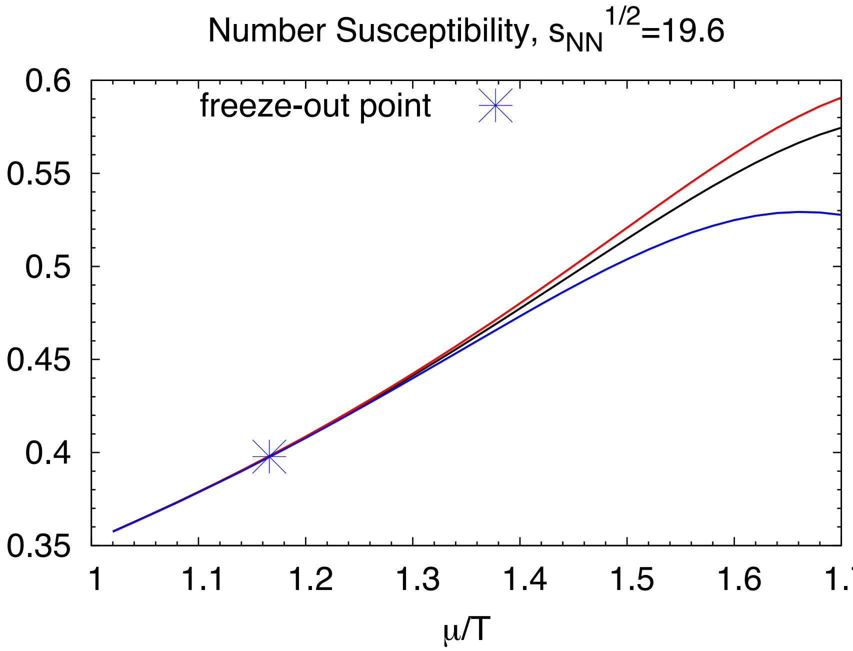
if we increase
these points 15%

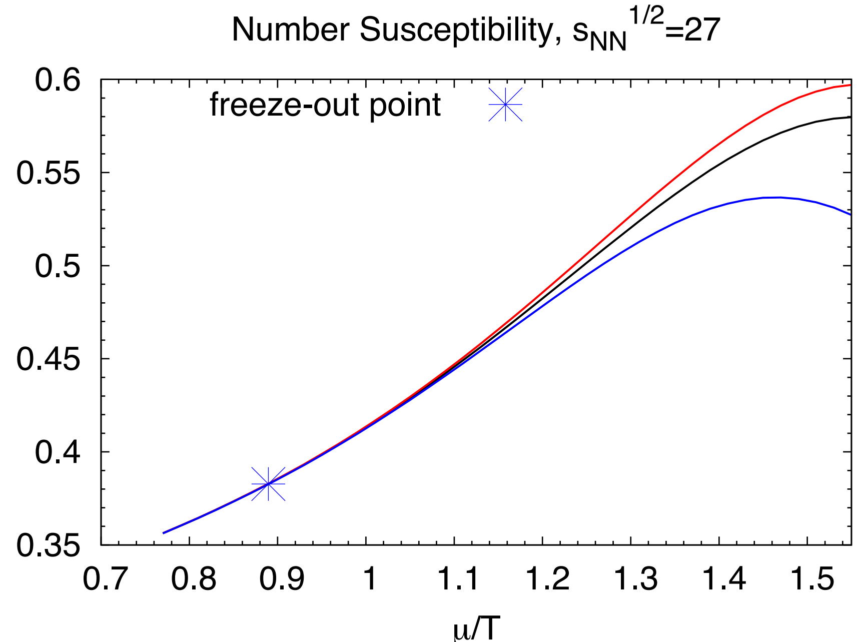
if we drop
these points

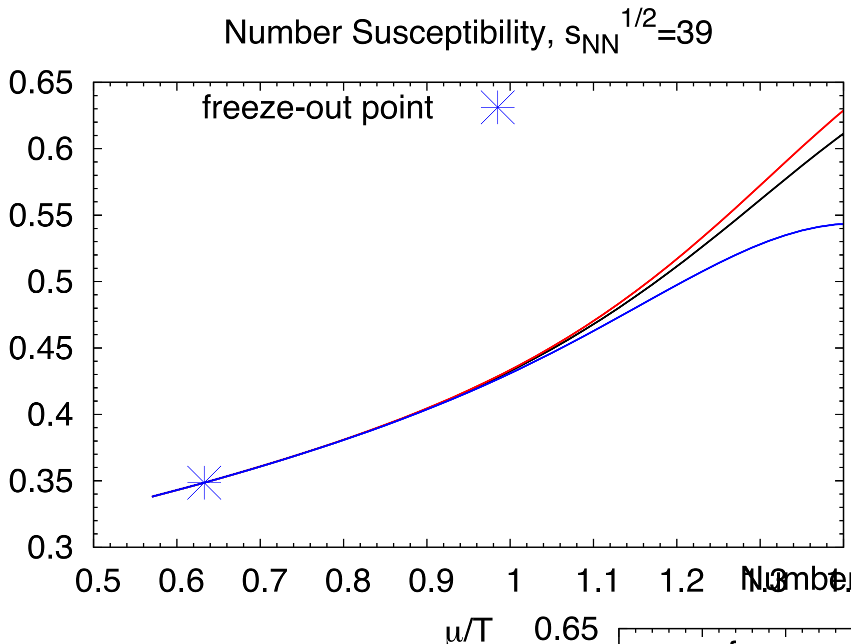


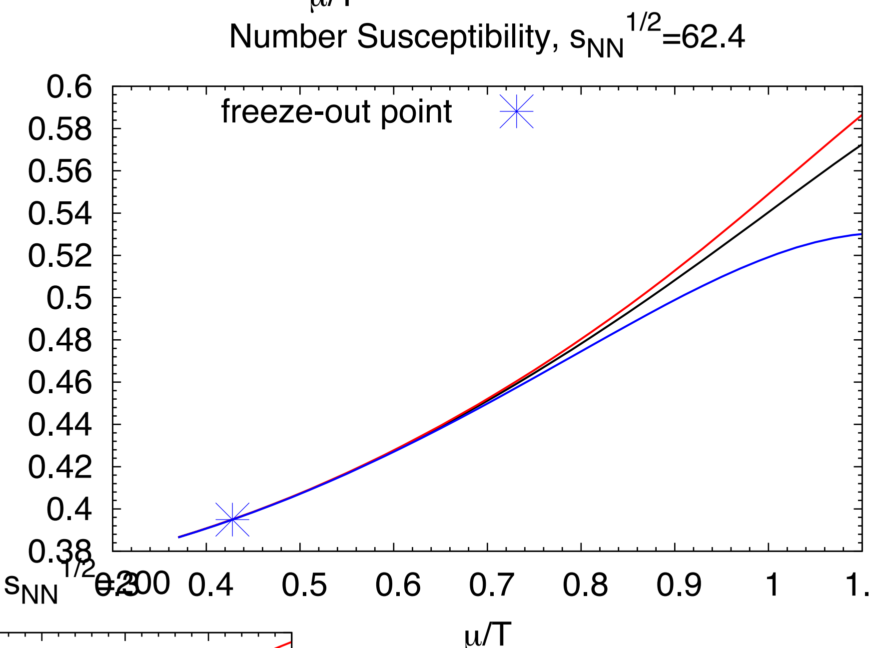


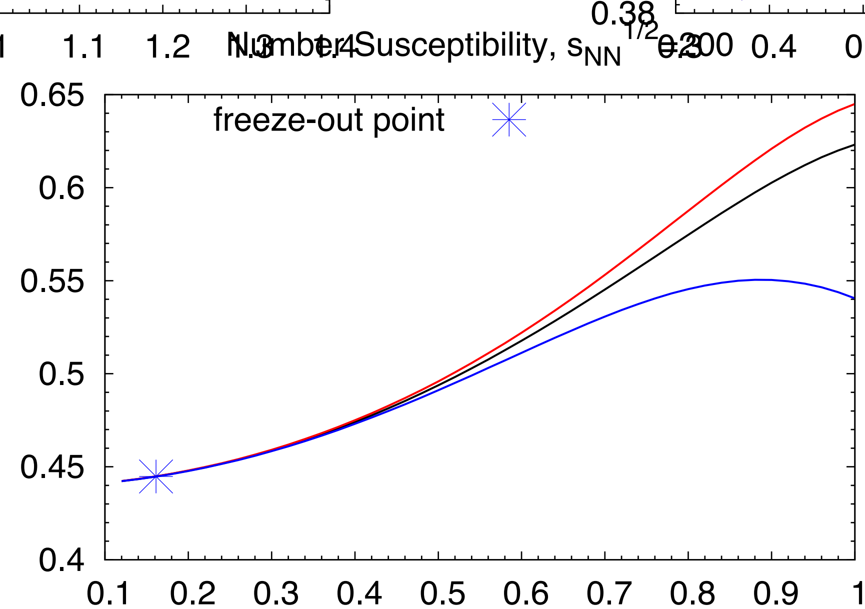
Susceptivility



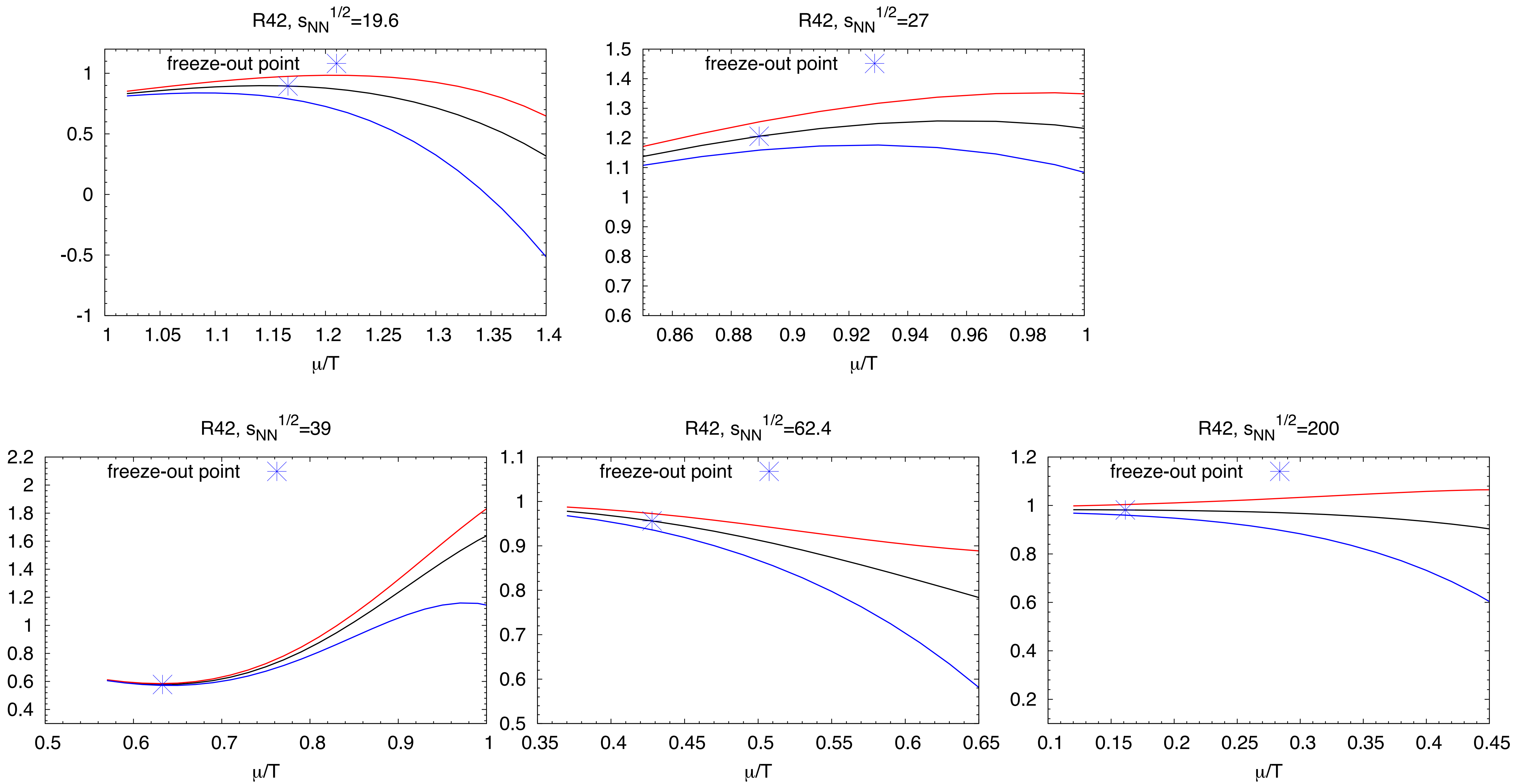








Kurtosis

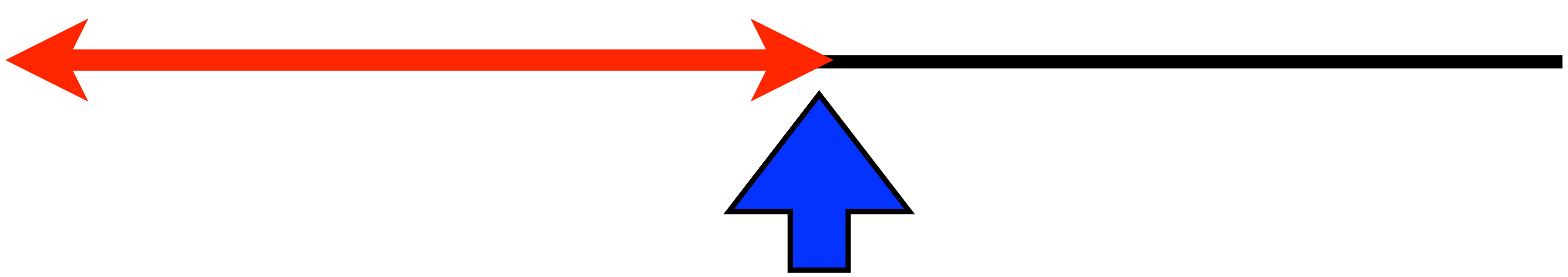

R42, $s_{NN}^{1/2}$=19.6
freeze-out point
μ/T
R42, $s_{NN}^{1/2}$=27
freeze-out point
μ/T
R42, $s_{NN}^{1/2}$=39
freeze-out point
μ/T
R42, $s_{NN}^{1/2}$=62.4
freeze-out point
μ/T
R42, $s_{NN}^{1/2}$=200
freeze-out point
μ/T

Hunting the QCD Phase Transition Regions

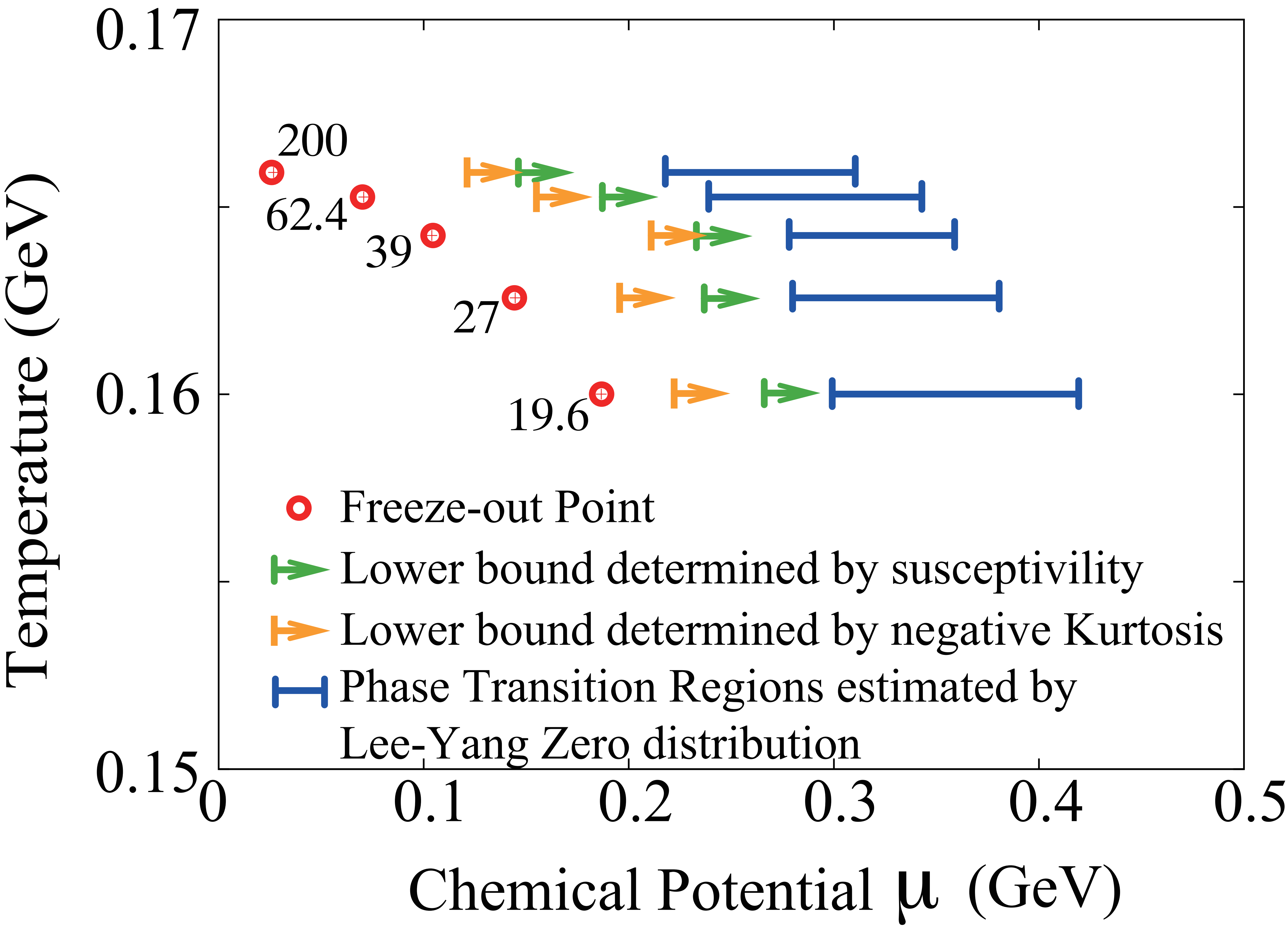
Find Rooms where No Criminal.

→ The Target is in other Room.

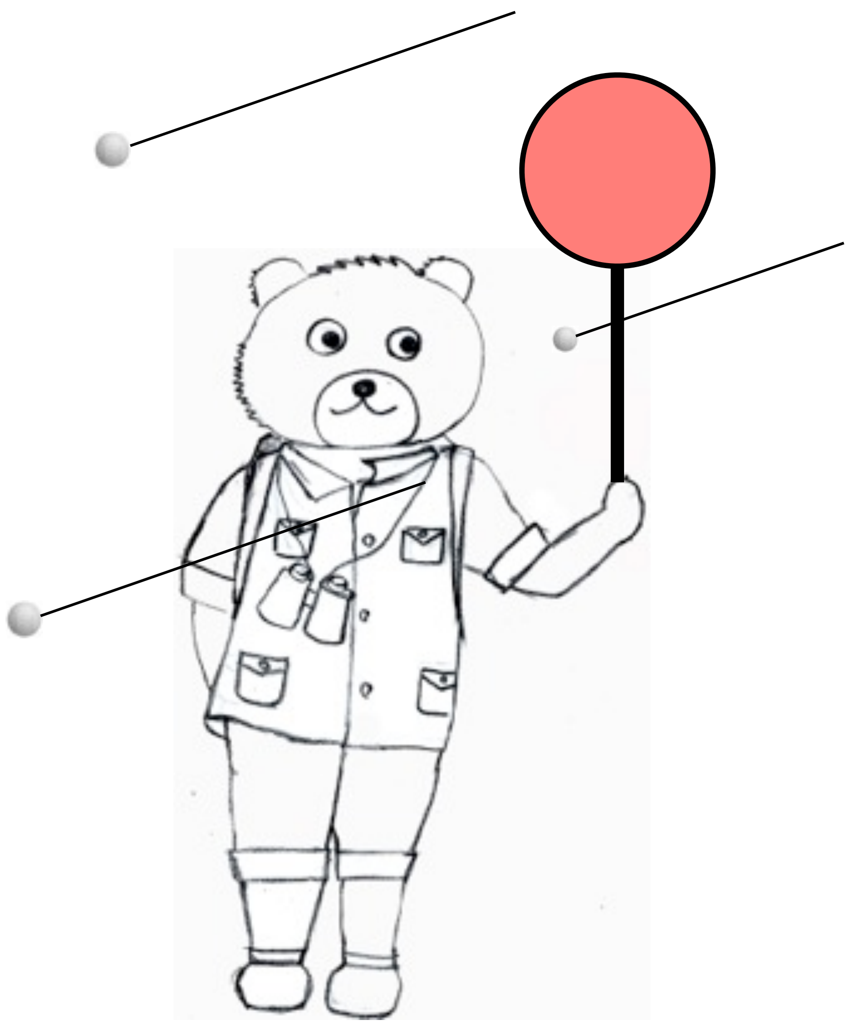
Not here ! Then, ..

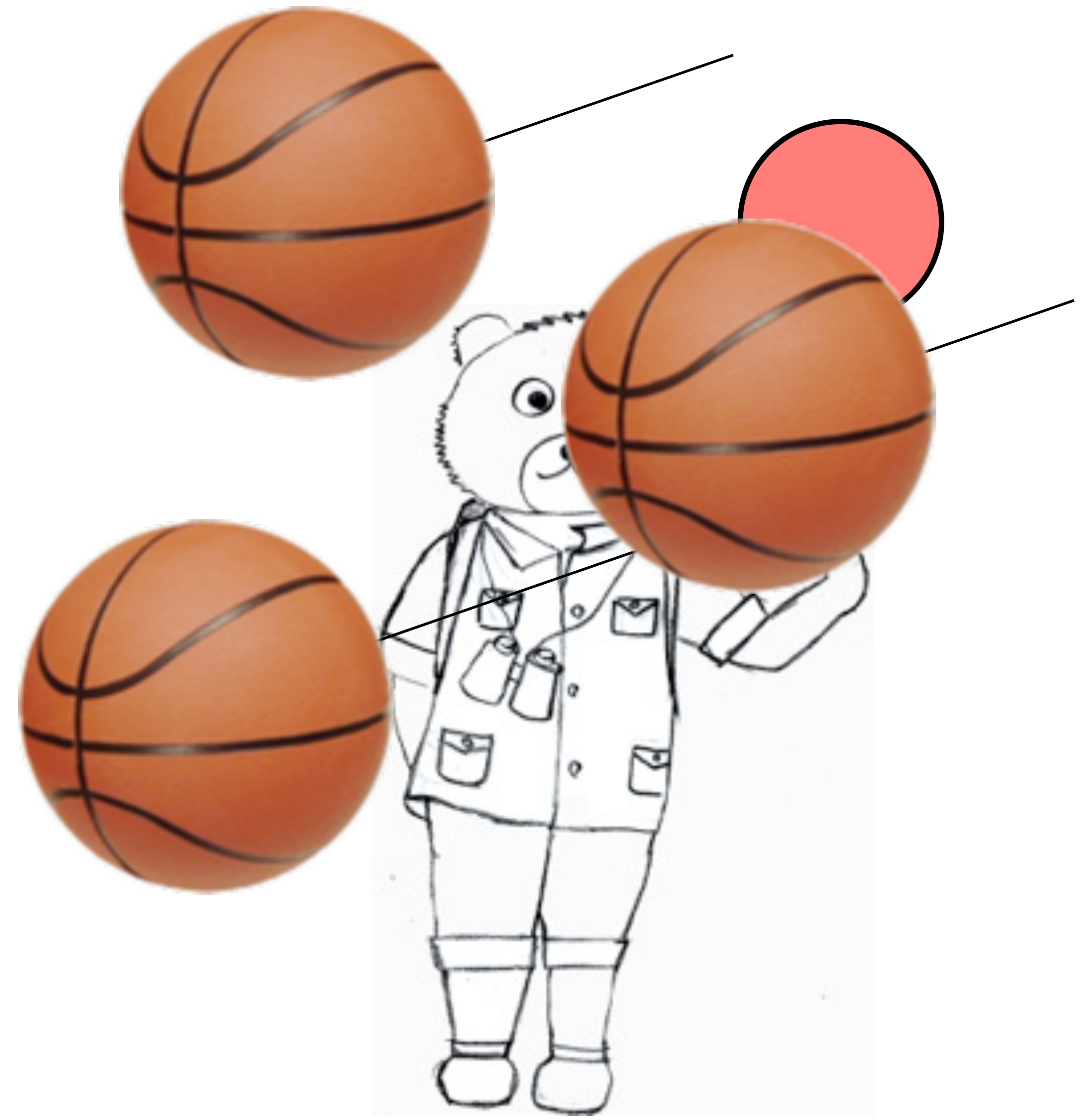


Lower Bound


0.17
0.16
0.15
Temperature (GeV)
0
0.1
0.2
0.3
0.4
0.5
Chemical Potential μ (GeV)
200
62.4
39
27
19.6
Freeze-out Point
Lower bound determined by susceptivility
Lower bound determined by negative Kurtosis
Phase Transition Regions estimated by
Lee-Yang Zero distribution

Summary of Sec.2





3.Lee-Yang Zeros

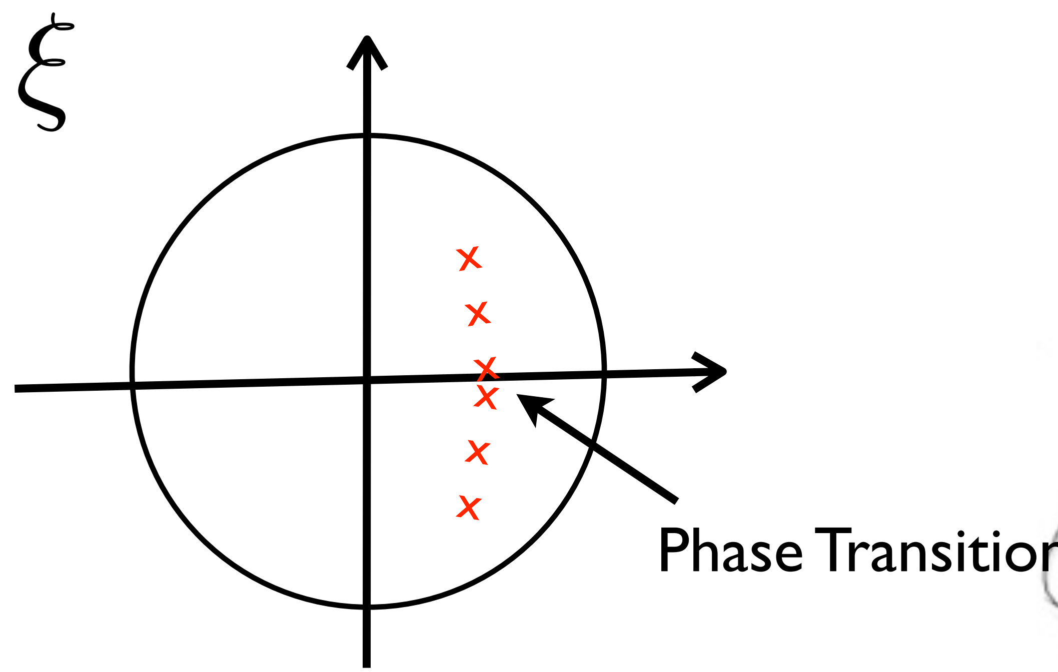
$$Z(\xi, T) = \sum_{n=-N_{max}}^{+N_{max}} Z_n(T)\,\xi^n$$

Lee-Yang Zeros (1952)

Zeros of $Z(\xi)$ in Complex Fugacity Plane.

$$Z(\alpha_k) = 0$$

Great Idea to investigate a Statistical System

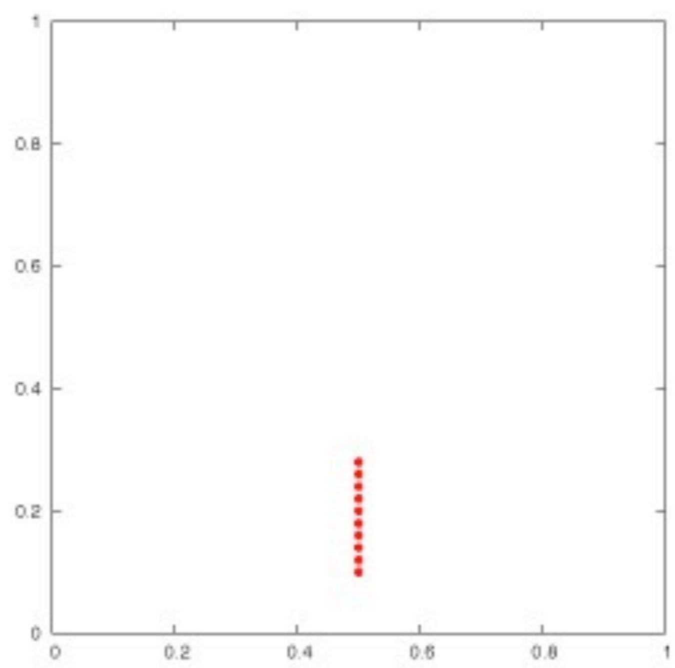


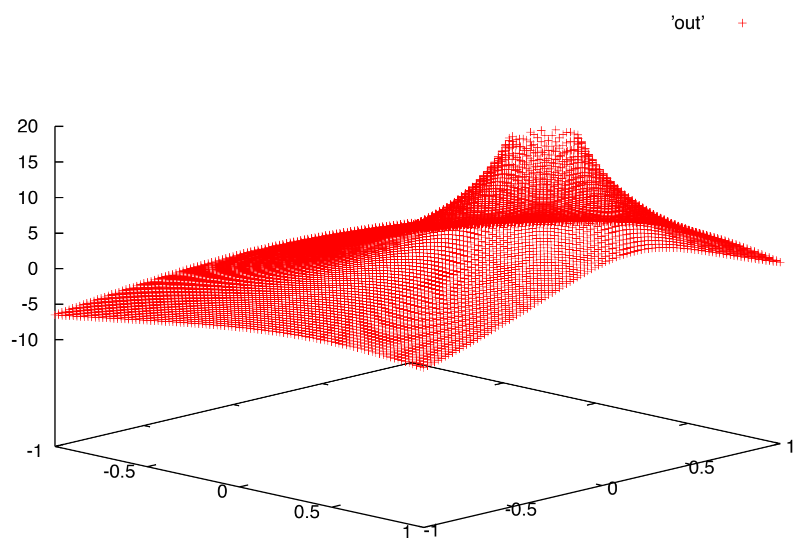
Lee-Yang Zeros

Non-trivial to obtain.
But once they are got, it is easy to figure out the Free-energy

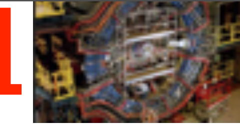
$$Z(\xi, T) = e^{-F/T}$$

Lee-Yang zeros	2-d Electro-Magnetic
F: Free-energy	F: Potential
α_k: zeros	α_k:Point charge





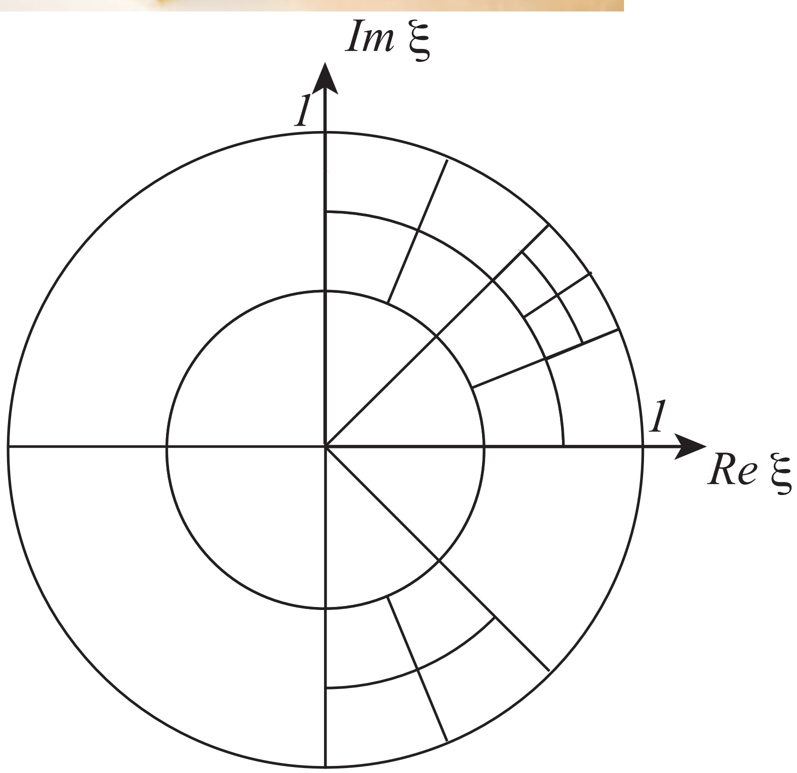
$$F(\xi) = -\sum_k \log(\xi - \alpha_k)$$

 and 

cut Baum-Kuchen (cBK) Algorithm

$$f(\xi) = \prod_k (\xi - \alpha_k)$$

$$\frac{f'}{f} = \sum_k \frac{1}{\xi - \alpha_k}$$

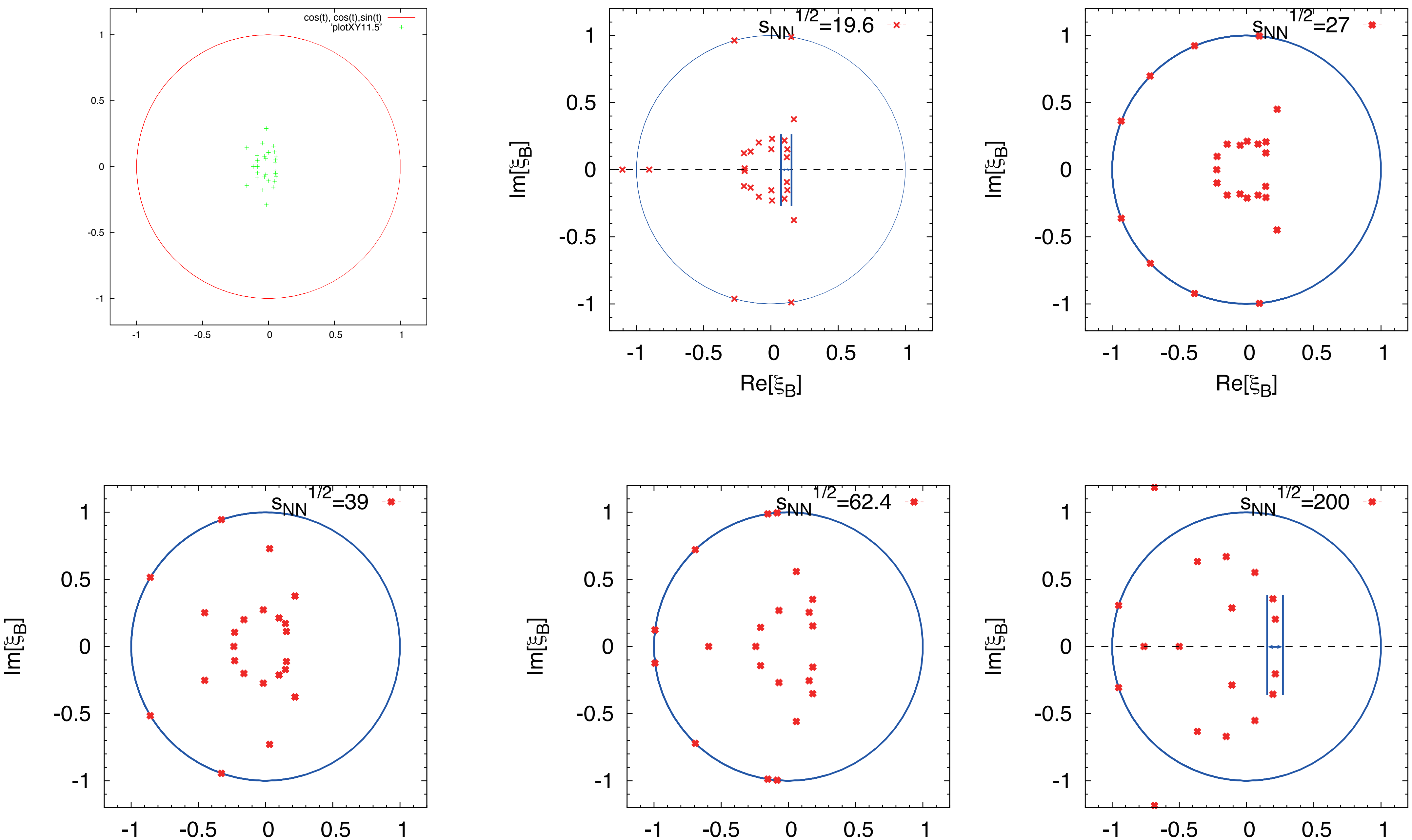


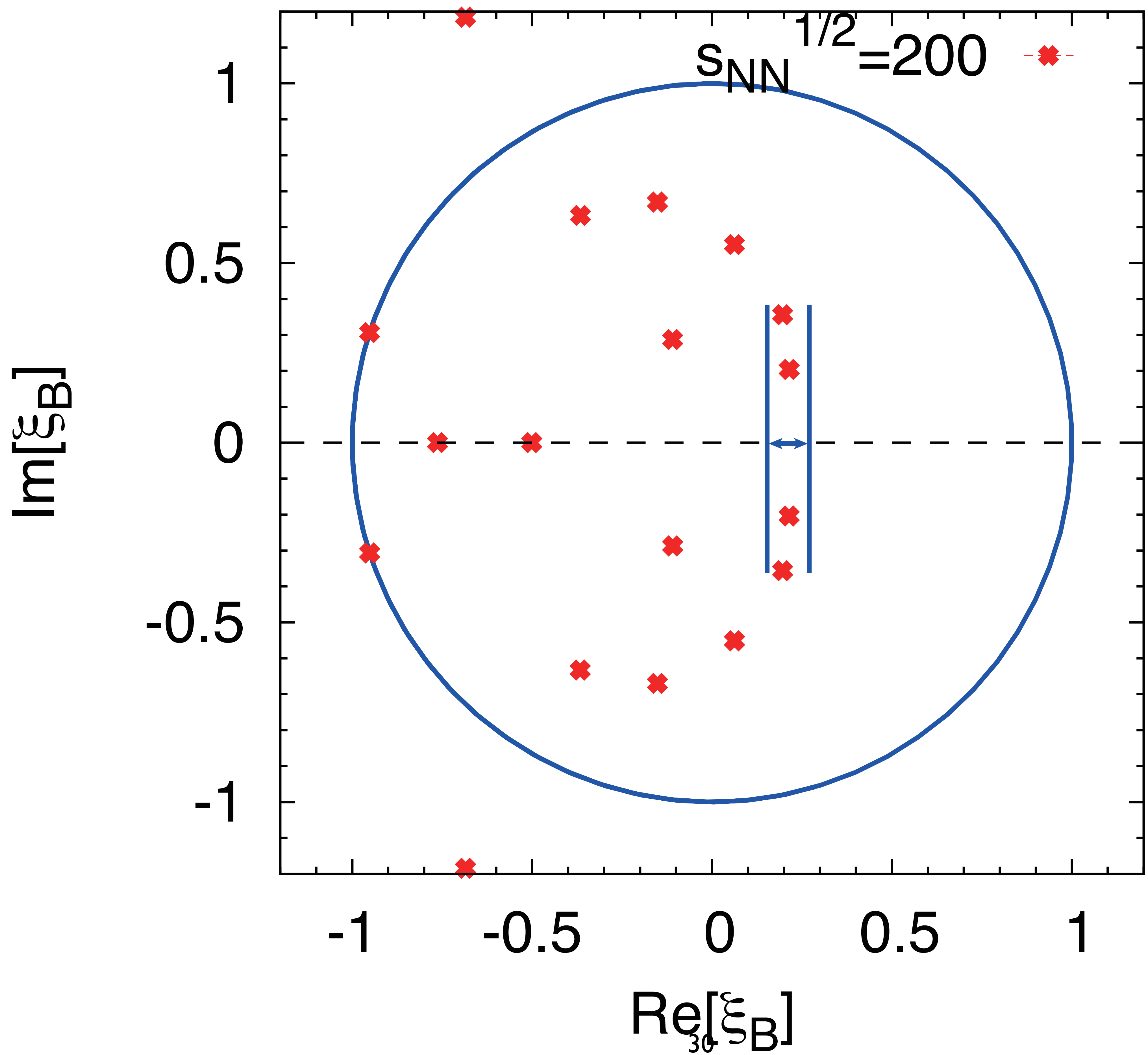
$$\frac{1}{2\pi i} \oint_C \frac{f'}{f} d\xi = \left(\begin{array}{c} \text{Number of} \\ \text{Zeros in} \\ \text{Contour } C \end{array} \right)$$

50 - 100 number of significant digits

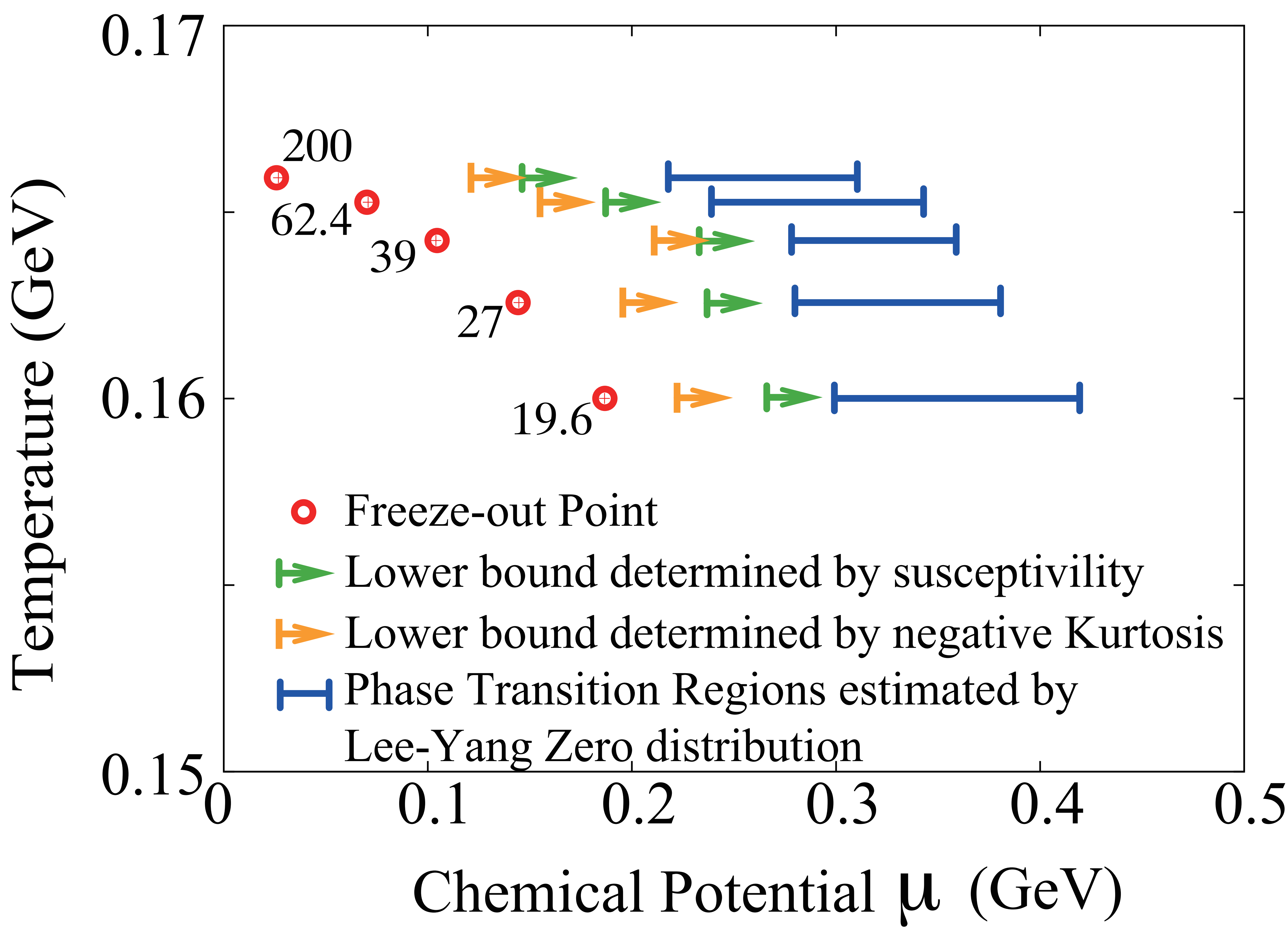
A Coutour is cut into four pieces if there are zeros inside.

Lee-Yang Zeros: RHIC Experiments





$s_{NN}^{1/2}$=200
1
0.5
0
-0.5
-1
Im[ξ_B]
-1
-0.5
0
0.5
1
Re[ξ_B]



13年11月2日土曜日

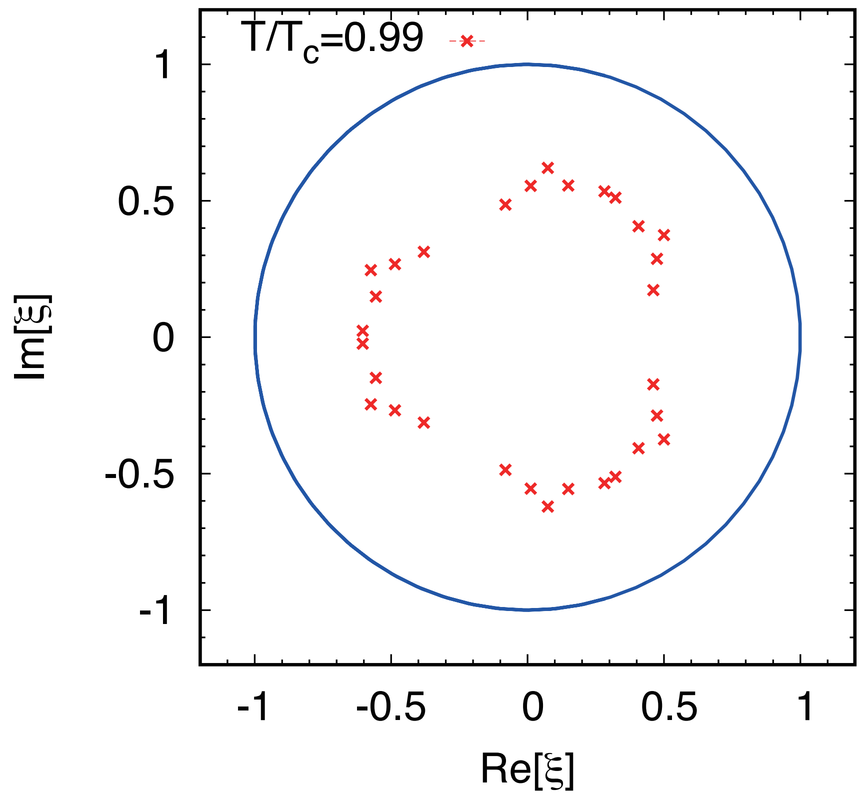
Lee-Yang Zeros
Lattice QCD

$$Z(\xi) = \sum_m Z_{3m} \times \xi^{3m}$$

$Z_n = 0$ **unless** $n \neq 3m$

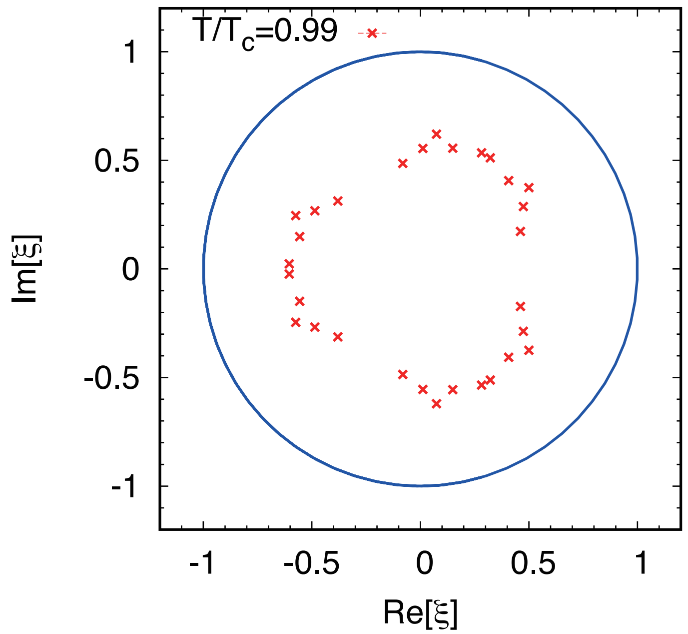
Periodicity $\theta = \frac{2\pi}{3}$

$(\xi = e^{i\theta})$

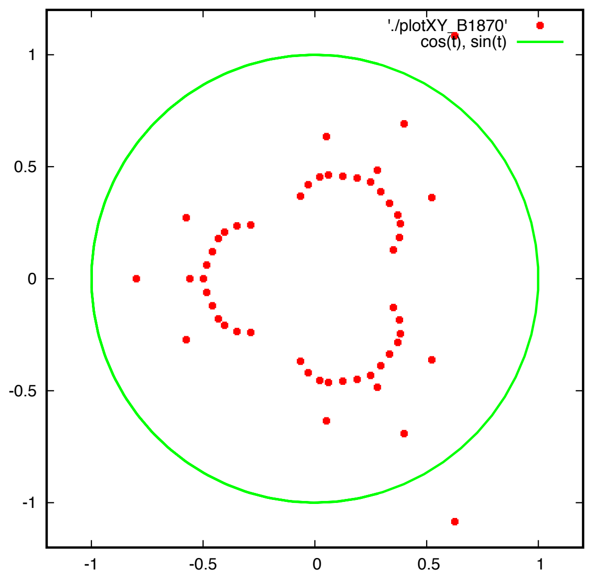


$\beta = 1.85$

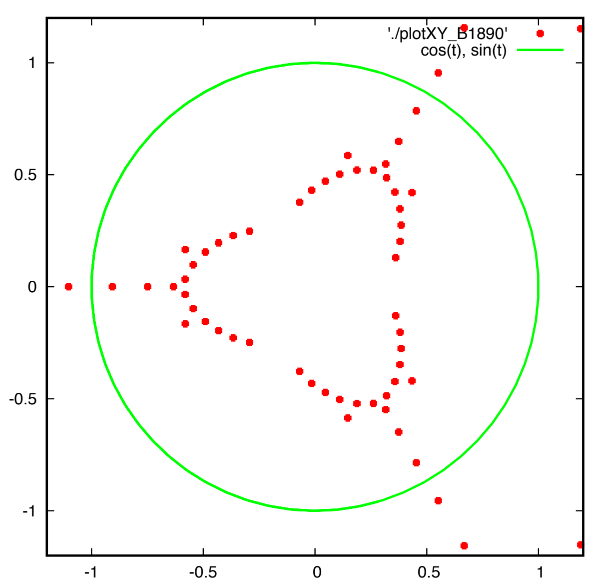
$T/T_c \sim 0.99$



$$\beta = 1.85 \qquad T/T_c \sim 0.99$$

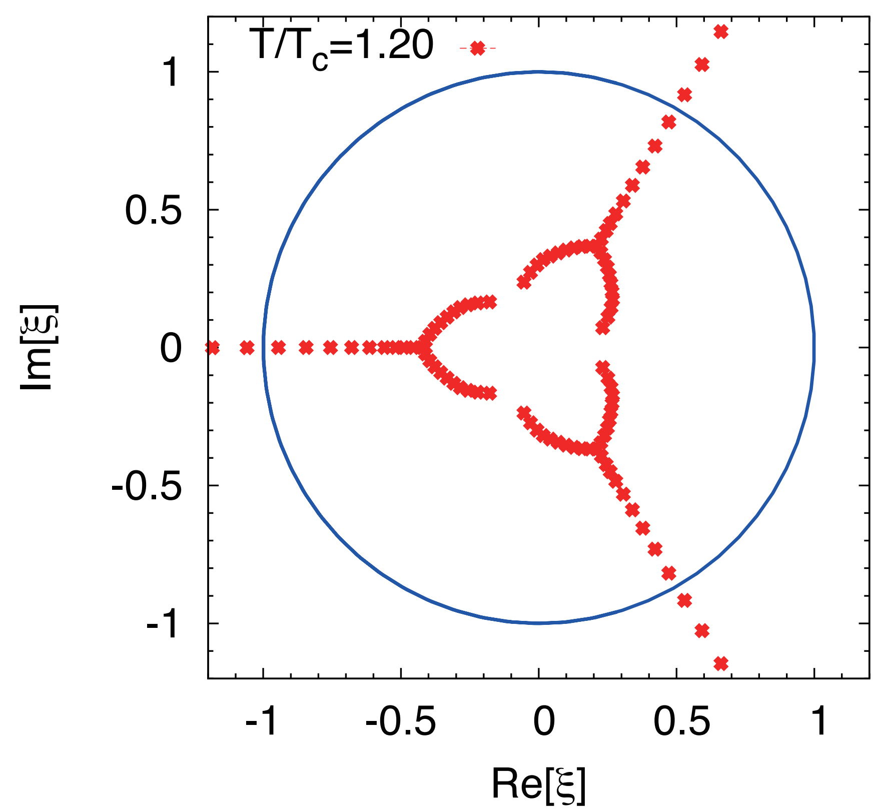


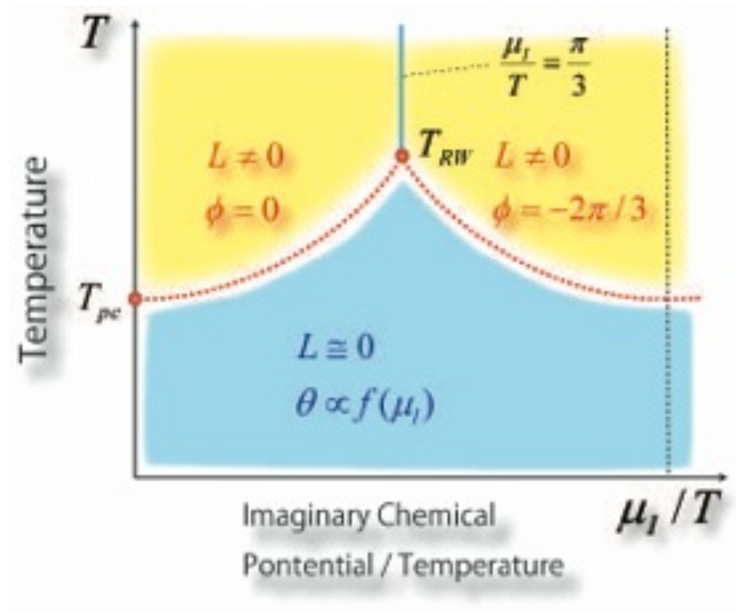
$$\beta = 1.87 \qquad T/T_c \sim 1.01$$



$$\beta = 1.89 \qquad T/T_c \sim 1.04$$

Lee-Yang Zeros
Lattice QCD





$$\xi = e^{\mu/T}$$

The Unit Cirle in ξ

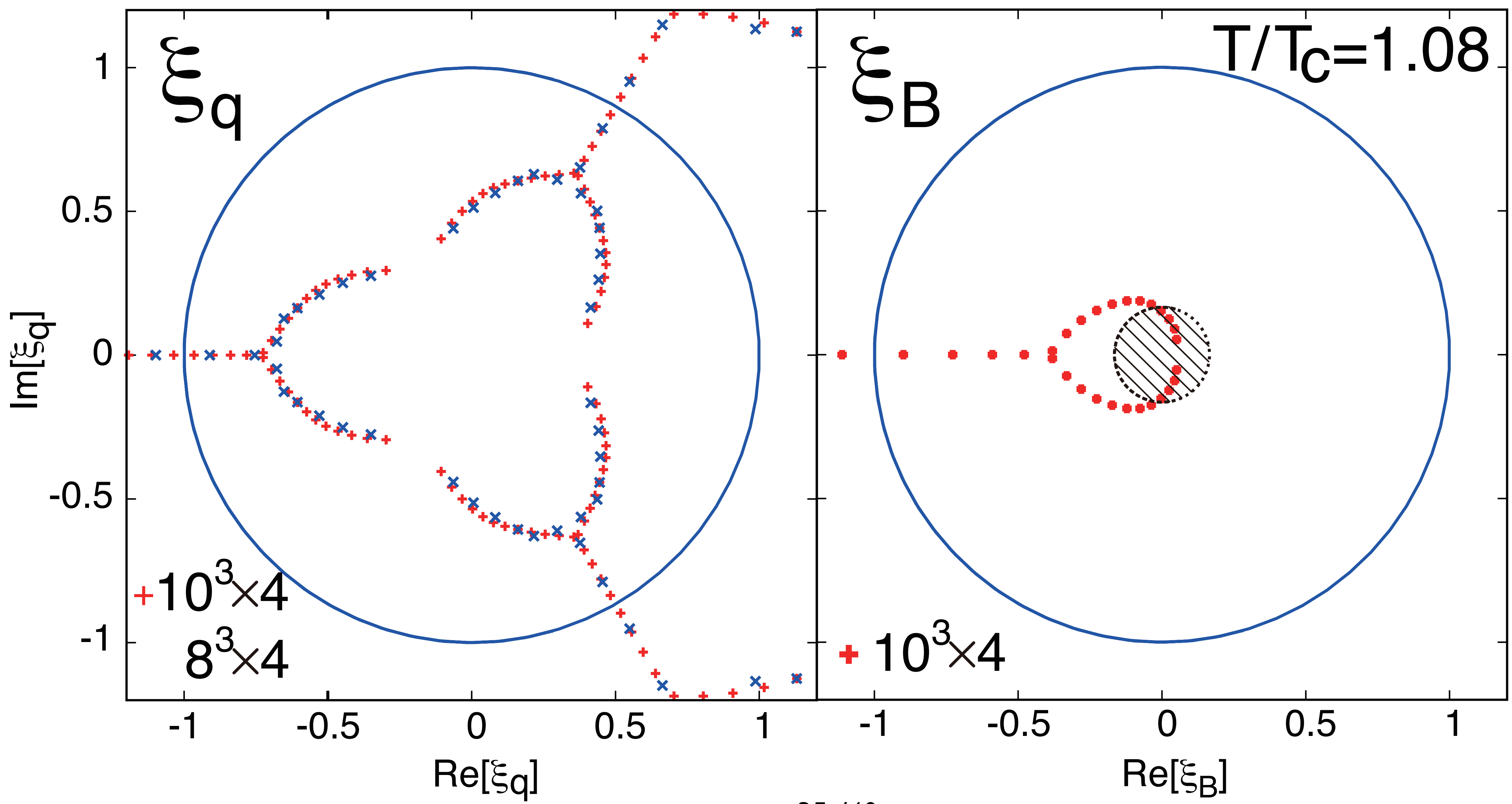
Imaginary μ

Roberge-Weise
Transition !

$$T/T_c \sim 1.20$$

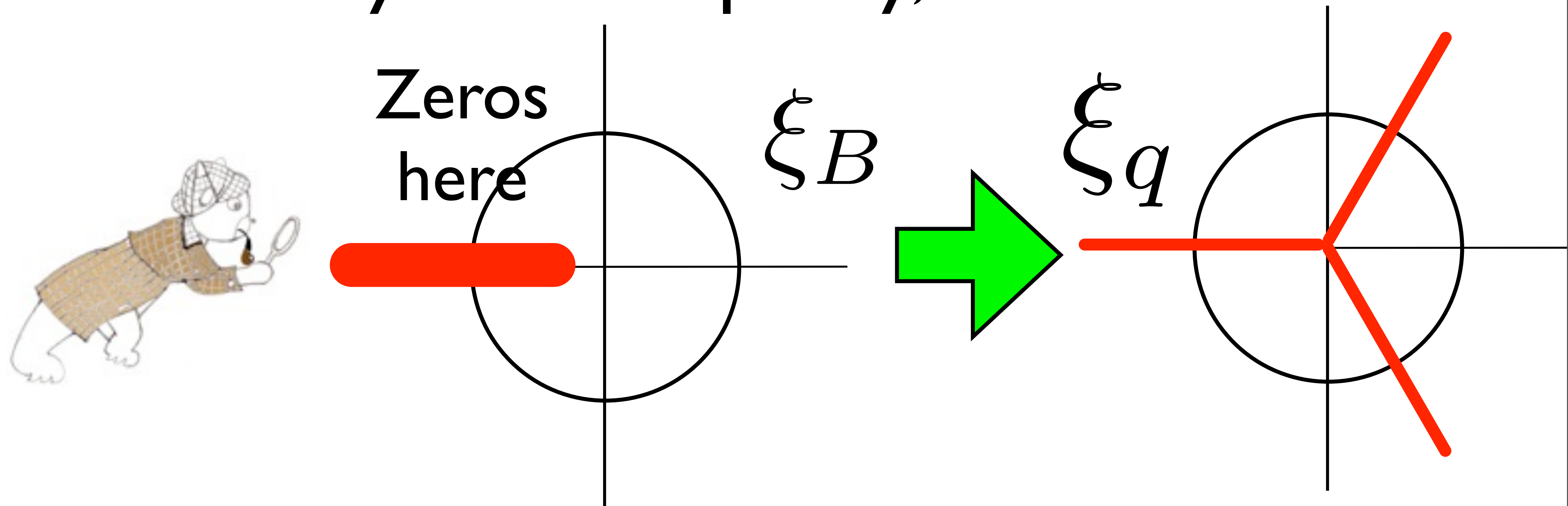
$$(\xi \equiv e^{\mu/T})$$

$\xi_q \Rightarrow \xi_B = \xi_q{}^3$



A Message to Experimentalists

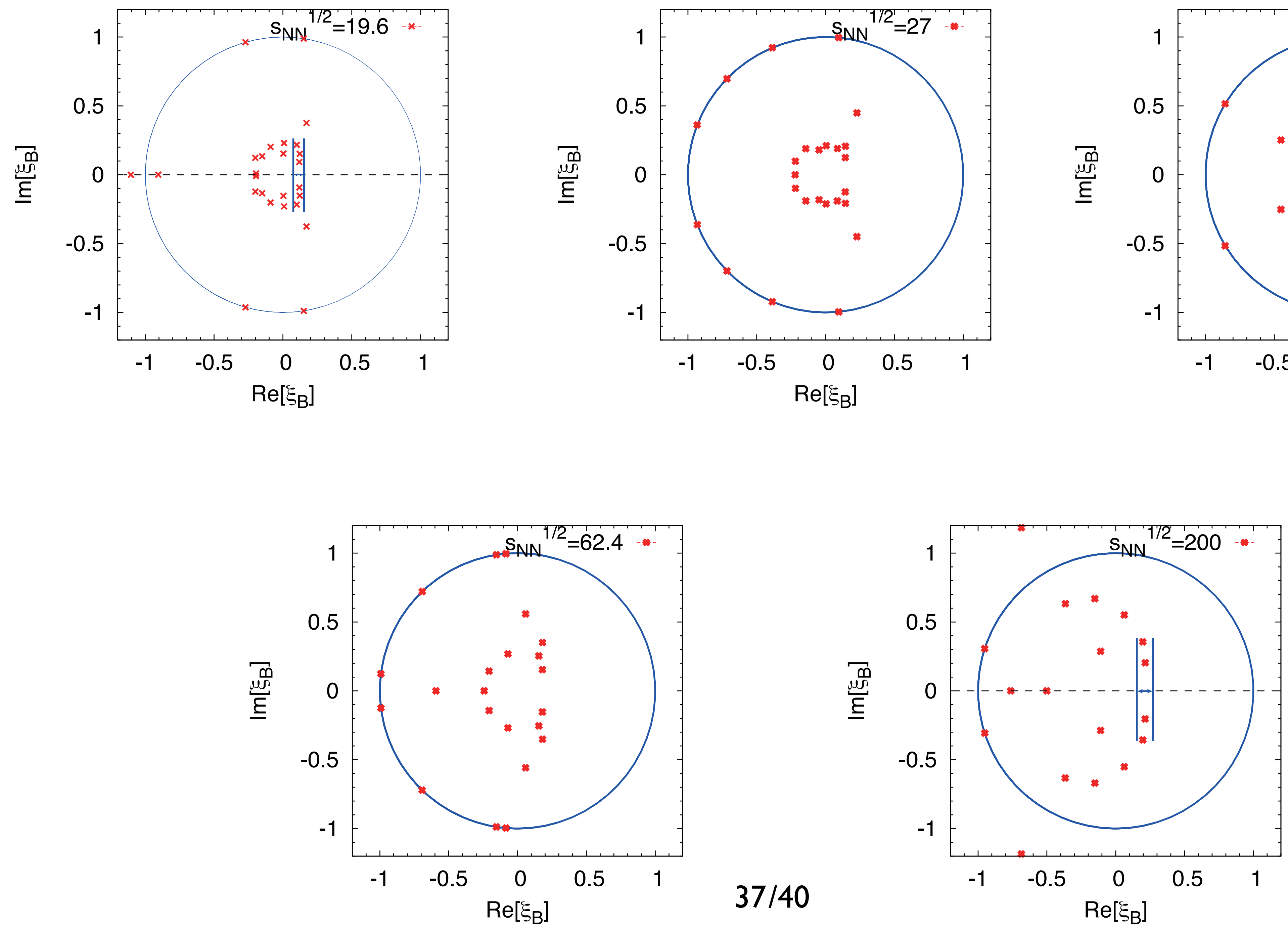
In the Lee-Yang Diagram constructed from your multiplicity,



No Roberge-Weise Transition

Your Temperature $T \leq T_{RW} \sim 1.2 T_c$

Lee-Yang Zeros: RHIC Experiments



Example of Lee-Yang zero
Skellam Model

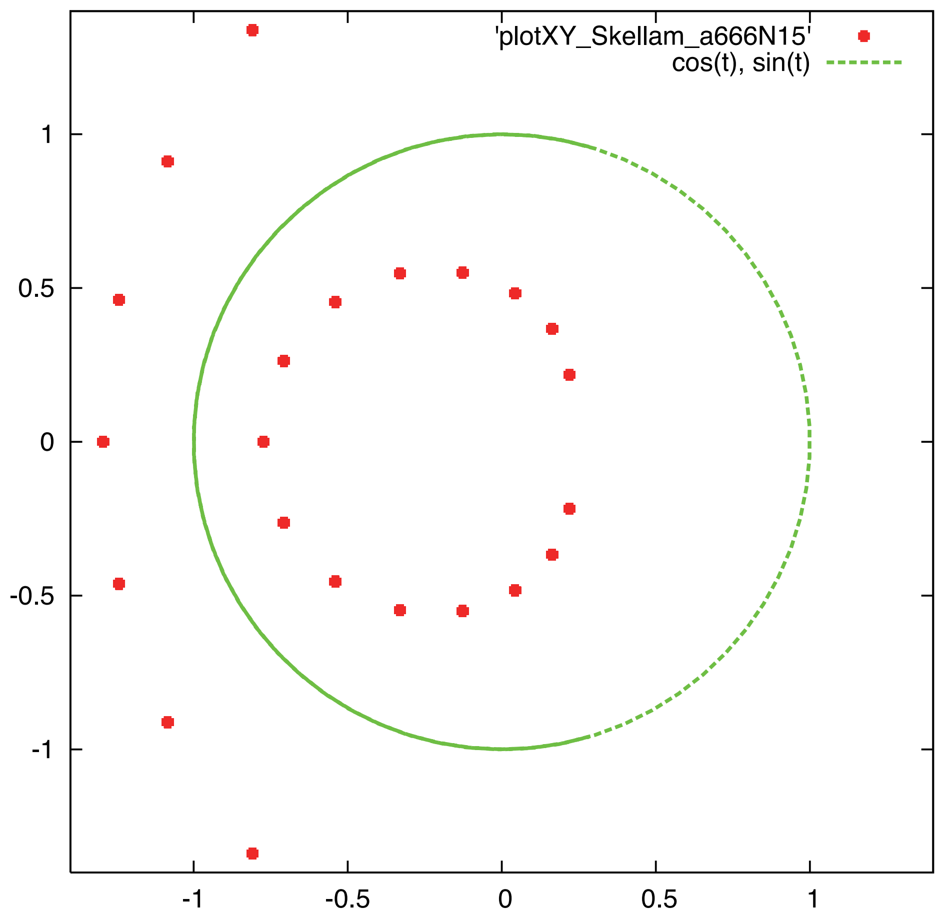
- Difference of two Poisson distributed variables.
  In our case proton and anti-proton

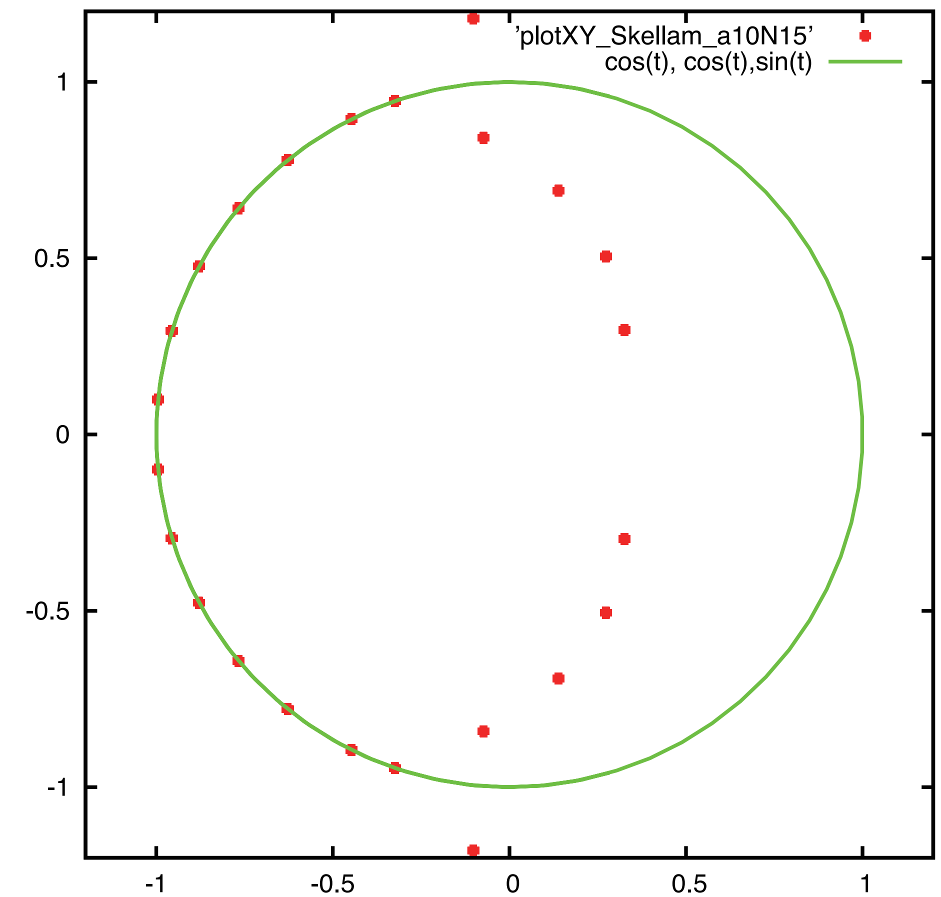
LYZ : Skellam

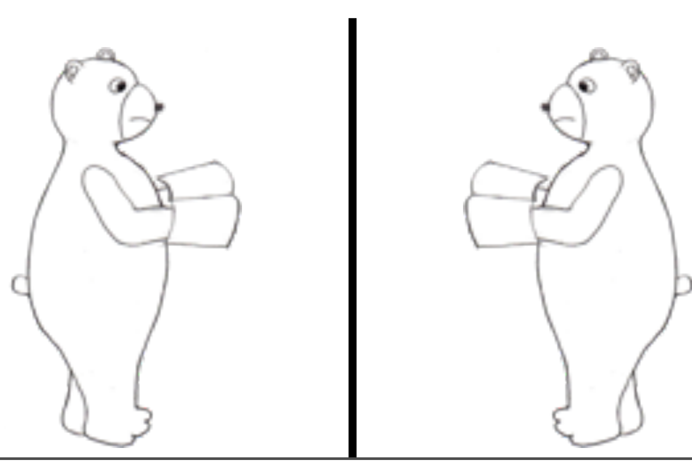
$+\mu \leftrightarrow -\mu$

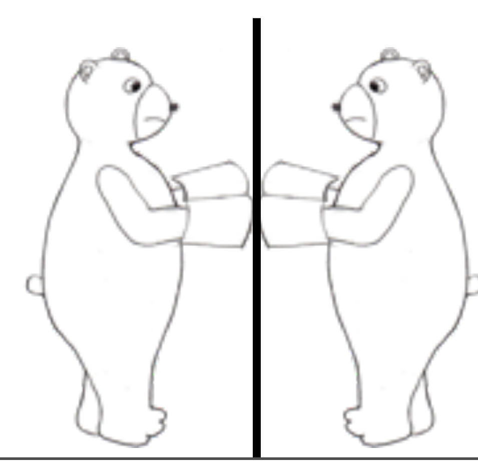
$e^{+\mu/T} \leftrightarrow e^{-\mu/T}$

Symmetric w.r.t. Unit Circle



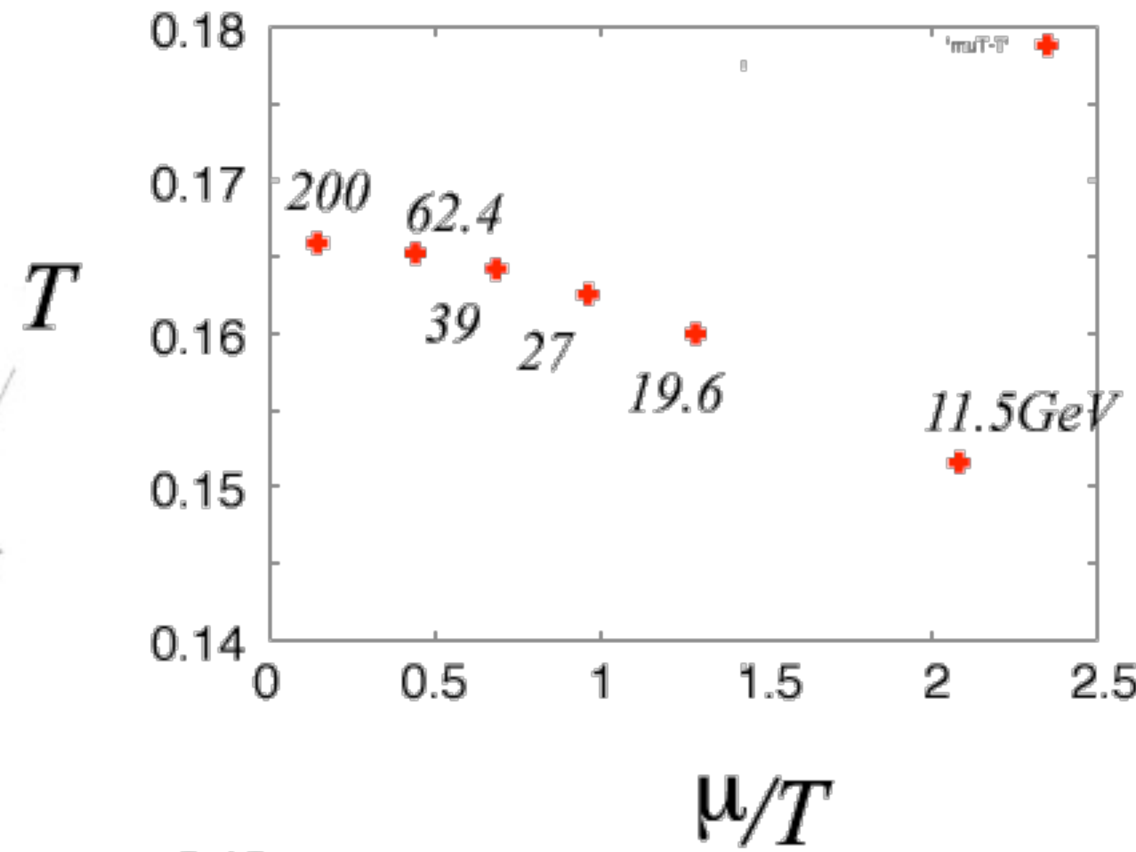






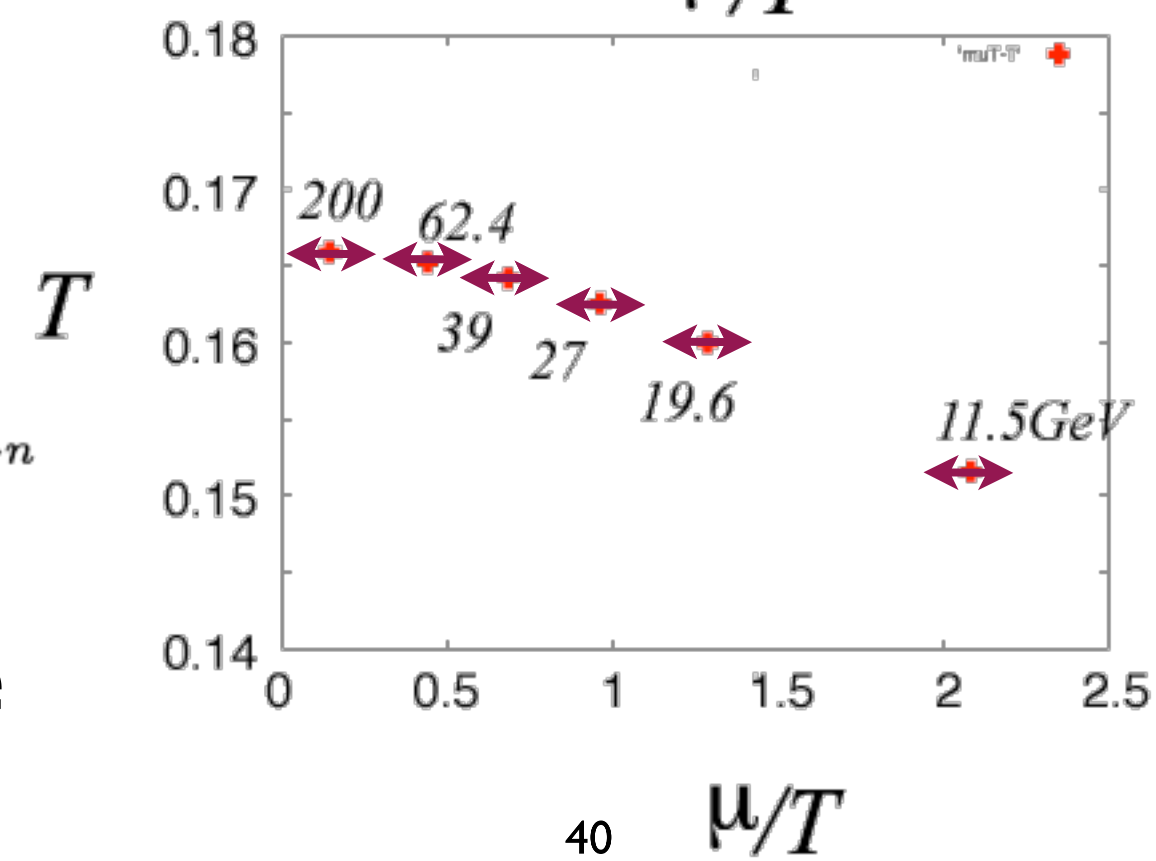
My Main Message

Multiplicity tells us



Not only Freeze -out points

Information of wider regions



$$Z(\xi,T) = \sum_{n=-N_{max}}^{+N_{max}} Z_n(T)\,\xi^n$$

Nmax →large

Wider

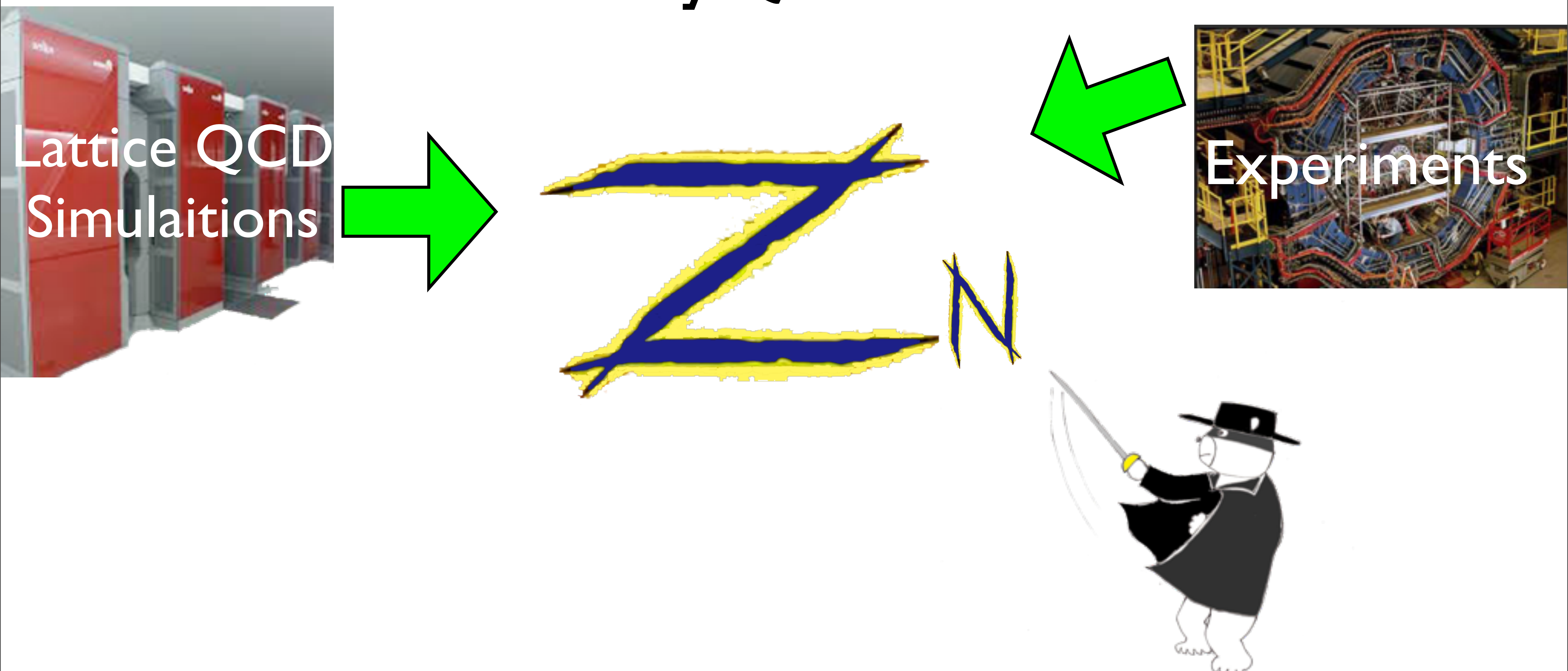
Other Messages

- Net proton multiplicity is Not a conserved quantity.
- Baryon multiplicity is perfect
- Can you estimate Baryon multiplicity from that of Proton ?
- Another conserved quantity is the Charge multiplicity. It should work as well.

You have a Big Chance to find QCD phase Transition !

Backup Slide

Canonical Partition Functions
is a Bridge
between Two Approaches
to Study QCD Phase



Lattice QCD
Canonical Approach

- Miller and Redlich
  - Phys. Rev. D35 (1987) 2524
- A.Hasenfratz and Toussaint
  - Nucl. Phys.B371 (1992) 539
- Barbour and Bell
  - Nucl. Phys. B372 (1992) 385
- Engels, Kaczmarek, Karsch and Laermann
  - Nucl.Phys. B558 (1999) 307
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- A.Li, Meng, Alexandru, K-F. Liu
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- Danzer and Gattringer
  - arXiv:1204-1020 Europian Journal

Lattice: How to Calculate

$$Z(\mu, T) = \int \mathcal{D}U \ \left(\det \Delta(\mu)\right)^{\mathrm{N_f}} \exp(-S_G)$$

$$\det \Delta(\mu) \to \left(\frac{\det \Delta(\mu)}{\det \Delta(0)}\right) \det \Delta(0)$$

$$\det \Delta(\mu) \propto \xi^{-N_{\mathrm{red}}/2} \prod_{n=1}^{N_{\mathrm{red}}} (\lambda_n + \xi)$$

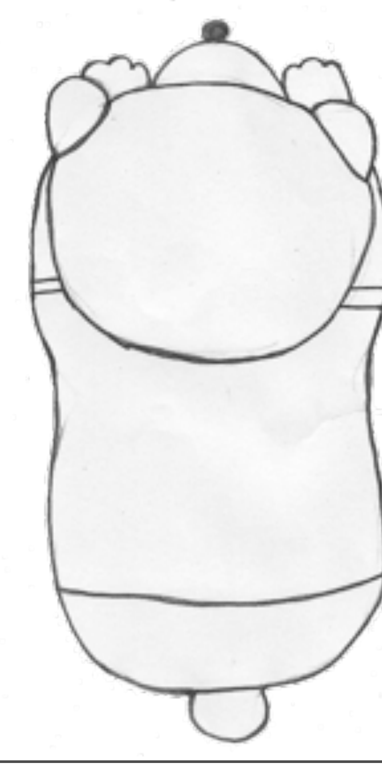
$$\propto \sum_{n=-N_{\mathrm{red}}/2}^{N_{\mathrm{red}/2}} c_n \xi^n$$

Fugacity Expansion
Nagata and A. Nakamura,
Phys. Rev. D82 (2010) 094027

Alexandru and Wenger,
Phys.Rev.D83 (2011) 034502

Four Excuses why not Baryon Multiplicities

1. This is a formulation. Let's wait until Experimentalists measure Baryon multiplicities
2. After the Freeze-out, the proton number is essentially constant.
3. Expect the proton multiplicity is similar to the baryon multiplicity
4. By some event generators or models, let us calculate the proton and baryon multiplicity. From that data, we can estimate the baryon multiplicity.



$$Z_{GC}(\mu) = \int DU \left[\frac{C_0 \sum c_n \xi^n}{\det \Delta(0)} \right]^{N_f} (\det \Delta(0))^{N_f} e^{-S_G}$$

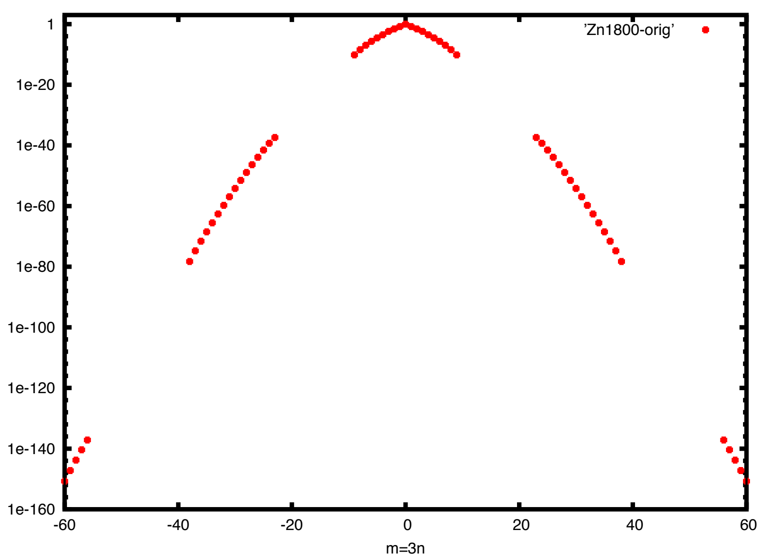
$$= \int DU (\sum a_n \xi^n)(\det \Delta(0))^{N_f} e^{-S_G}$$

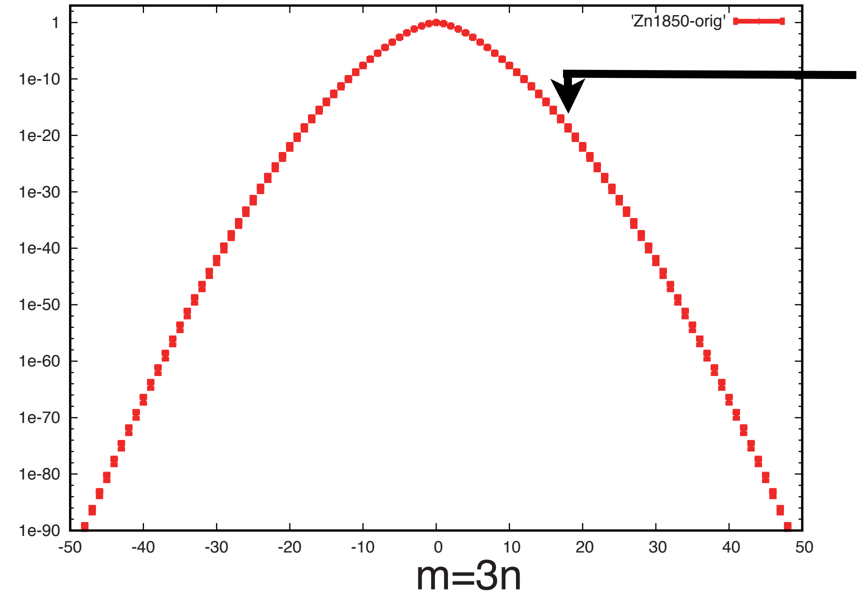
$$= \int DU (\sum a_n \xi^n)(\det \Delta(0))^{N_f} e^{-S_G}$$

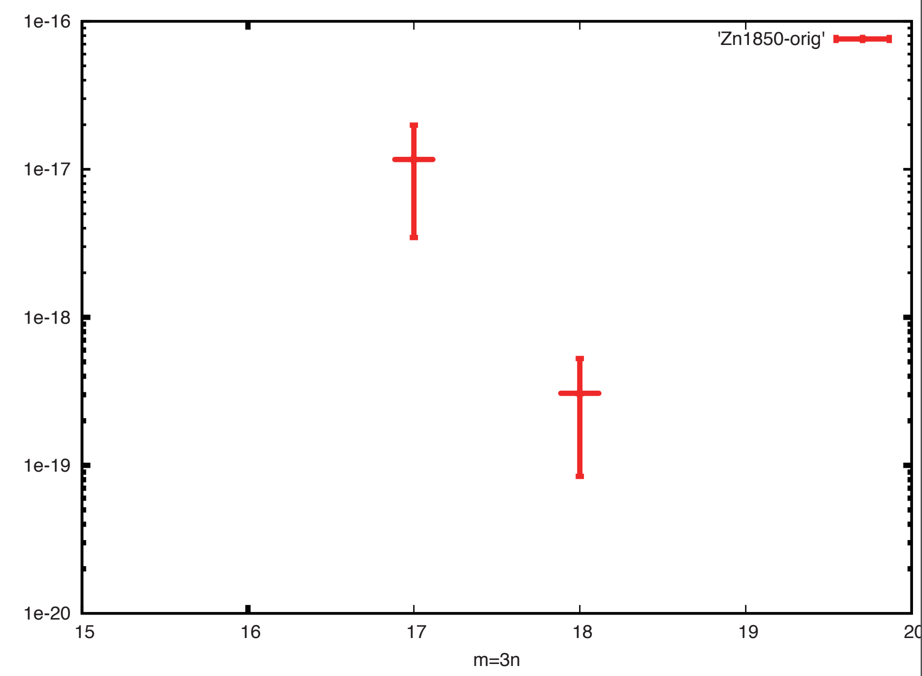
$$= \sum_n \xi^n \int DU a_n (\det \Delta(0))^{N_f} e^{-S_G}$$

$$Z_{GC}(\mu) = \sum Z_n \xi^n$$

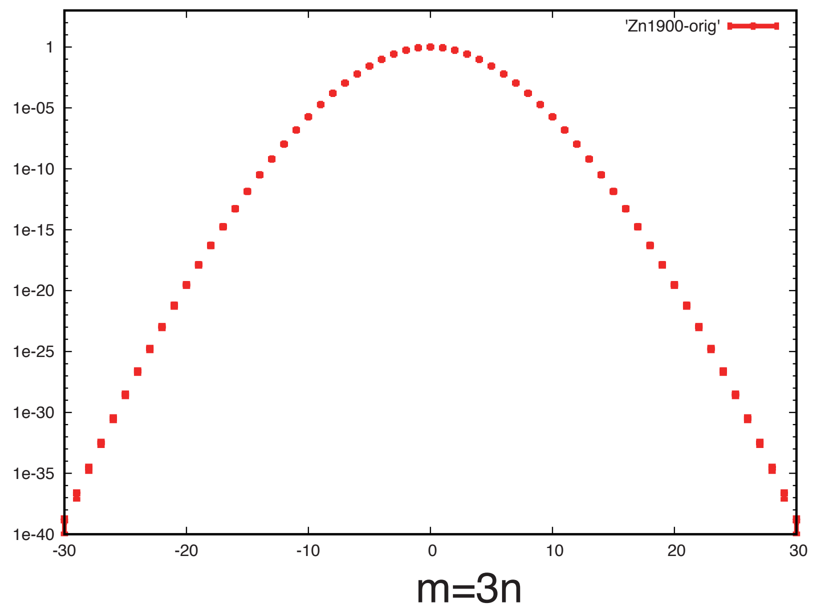
Z_n from lattice QCD

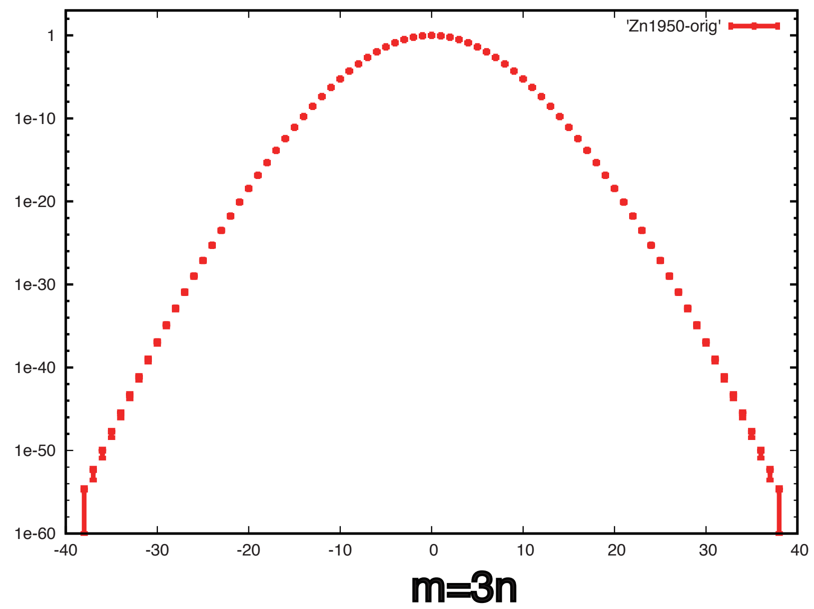


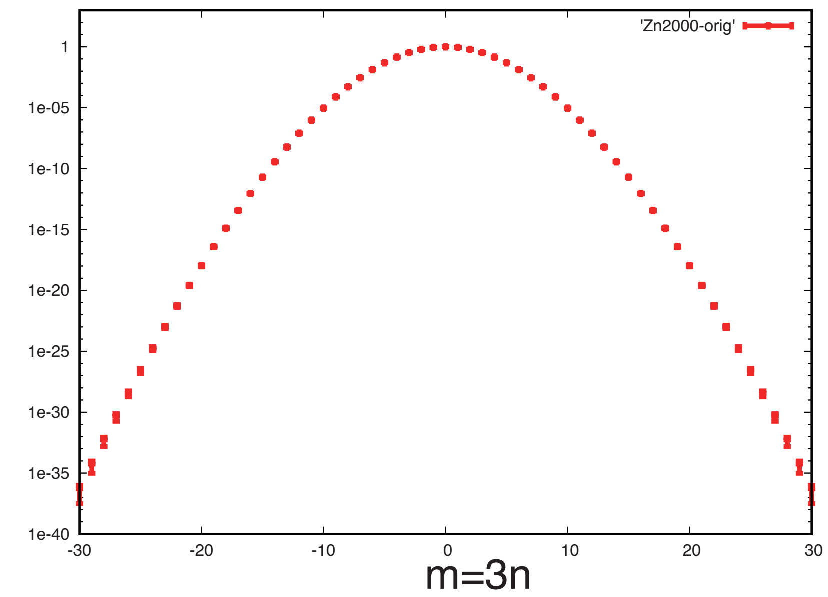




$Im(Z_n)$ are used as an error







A Strange Fact

- There are Lee-Yang Zeros on the unit circle.
  - Theoretically, a bit annoying.
  - Phenomenologically, very natural

$$Z(\mu) = \int \prod_f \det \Delta(m_f, \mu_f) e^{-S_G}$$

$\det \Delta(m, \mu)$ is REAL

if μ is pure Imaginary.

↑

On the unit circle in complex ξ plane.

$(\xi = e^{\mu/T})$

$\det \Delta(m,\mu)$ is REAL and Positive,

if μ is pure Imaginary
and *m* is sufficiently large.

⇒

$$Z(\mu_N) = \int \det \Delta(\text{Nucleon}) e^{-S_G} > 0$$

Lee-Yang zeros on the unit circle tell us that Nucleon is a composite.

Current lattice QCD simulations assumes

$$m_u = m_d$$

$$Z(\mu_N) = \int (\det \Delta(m_q, \mu_q))^2 \times \det \Delta(m_s, \mu_s) \cdots e^{-S_G}$$

$Z(\mu_N)$ can not take zero.

$$m_u > m_d$$

$$\mu_p = 2\mu_u + \mu_d$$

Pure imaginary μ_p does not mean

μ_u and μ_d are pure imaginary.