Heavy Ion Meeting 2013-10,
Inha Univ., Korea, Nov.2, 2013

REVIEW OF RECENT DEVELOPMENT IN HYDRO

Tetsufumi Hirano
Sophia Univ.

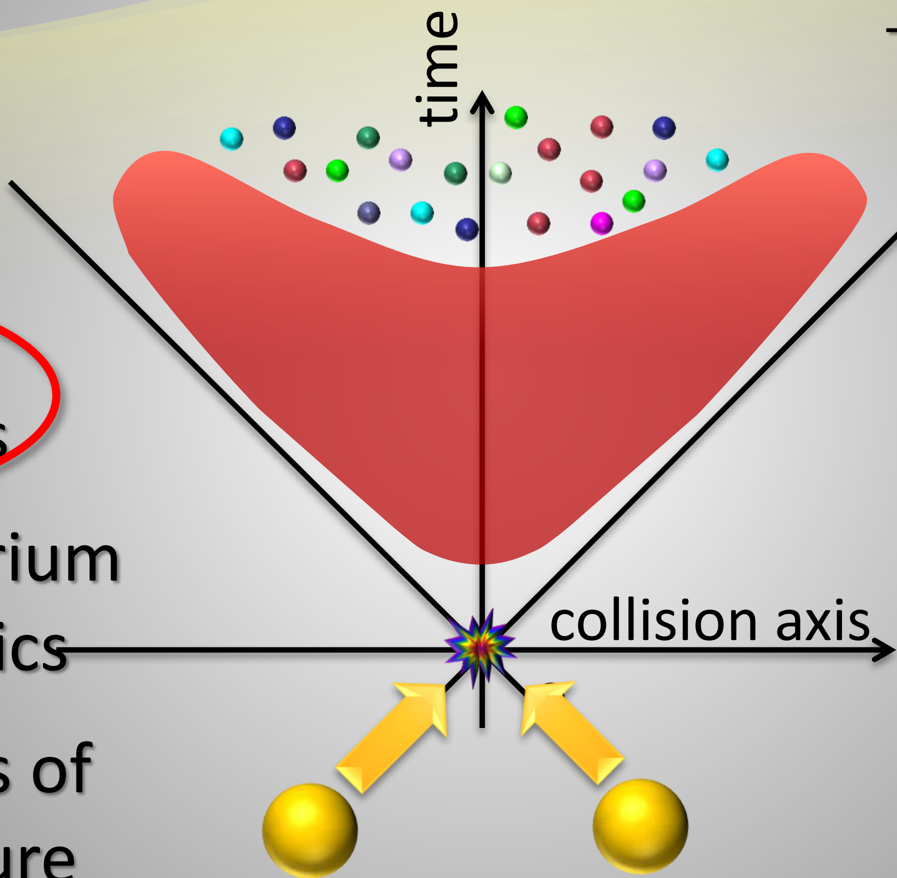
K.Murase, TH, arXiv:1304.3243
Y.Tachibana, TH, Nucl.Phys.A904-905(2013)1023c
Y.Tachibana, talk at Hard Probes 2013
M.Hongo, Y.Hirono, TH, arXiv:1309.2823

Outline

- Introduction
- Relativistic fluctuating hydrodynamics
- Medium response to jet propagation
- Anomalous hydrodynamics and chiral magnetic effect
- Summary

Introduction

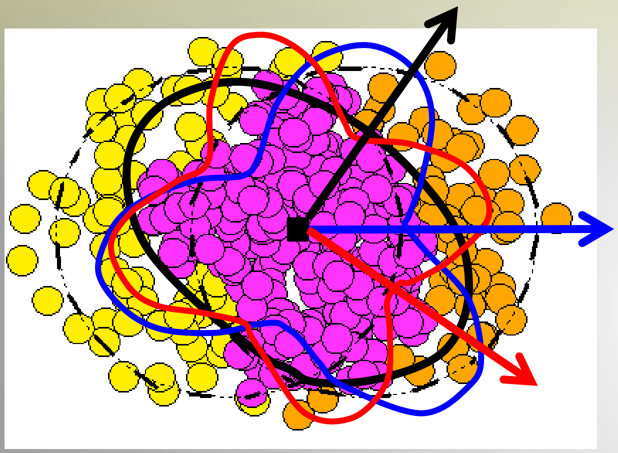
<u>Approach</u>	<u>"Form" of matter</u>
4. Relativistic kinetic theory	4. Hadron gas
3. Relativistic hydrodynamics	3. QGP fluid
2. Non-equilibrium statistical physics	2. Glasma
1. QCD aspects of nuclear structure	1. Color glass condensate

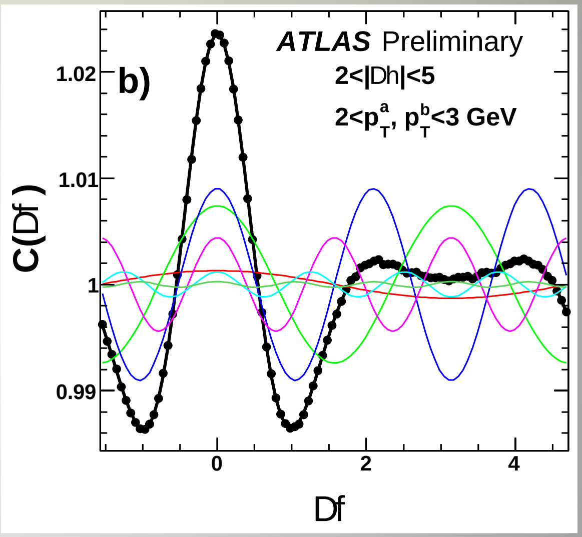


Hydrodynamics as a central framework of heavy ion collisions

RELATIVISTIC FLUCTUATING HYDRODYNAMICS

Higher Harmonics is Finite!



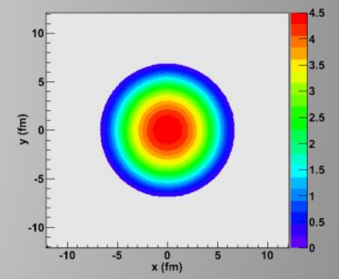
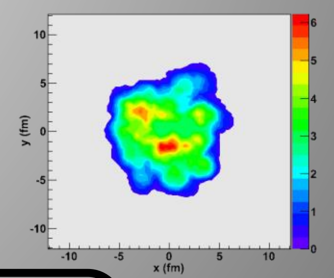


Figures adapted from talk by J.Jia (ATLAS) at QM2011

Two particle correlation function is composed solely of higher harmonics

See talks by Y.-S.Kim and M.-J. Kweon

Impact of Finite Higher Harmonics

	coarse graining size	initial profile
• Most of the people did not believe hydro description of the QGP (~ 1995)		
• Hydro at work to describe elliptic flow (~ 2001)	$d \lesssim 5\ \mathrm{fm}$	
• E-by-e hydro at work to describe higher harmonics (~ 2010)	$d \lesssim 1\ \mathrm{fm}$	

Is fluctuation important in such a small system?

Thermal Fluctuation

- Conventional hydro describes space-time evolution of (coarse-gained) thermodynamic quantities.
- Some of microscopic information must be lost through coarse-graining process.
- Does the lost information play an important role in dynamics on an e-by-e basis?

→ Thermal (Hydrodynamic) fluctuation!

J.Kapusta, B.Muller, M.Stephanov, PRC85(2012)054906.
J.Kapusta, J.M.Torres-Rincon, PRC86(2012)054911.
P.Kovtun, G.D.Moore, P.Romatschke, PRD84(2011)025006.
J.Peralta-Ramos, E.Calzetta, JHEP1202(2012)085.

Causal Hydrodynamics

Linear response to thermodynamic force

$$\Pi(t) = \int dt' G_R(t, t') F(t')$$

	Dissipative current Π	Themodynamic force F
Shear	Shear stress tensor	Gradient of flow
Bulk	Bulk pressure	Divergence of flow
Diffusion	Diffusion current	Gradient of chemical potential

Causal Hydrodynamics (contd.)

1. Delta function (acausal, 1st order hydro)

$$G_R(t,t') = \kappa\delta(t-t') \;\rightarrow\; \Pi(t) = \kappa F(t)$$

2. Retarded Green function (causal, 2nd order hydro)

$$G_R(t,t') = \frac{\kappa}{\tau}\exp\left(-\frac{t-t'}{\tau}\right)\theta(t-t')$$

Differential form

$$\dot{\Pi}(t) = -\frac{\Pi(t)-\kappa F(t)}{\tau}, \quad v_{\mathrm{signal}} = \sqrt{\frac{\kappa}{\tau}} < c$$

➡ Maxwell-Cattaneo Eq. (simplified Israel-Stewart Eq.)

K.Murase and TH, arXiv:1304.3243[nucl-th]

Relativistic Fluctuating Hydrodynamics (RFH)

Generalized Langevin Eq. for fields

$$\Pi(x) = \int d^4x' G_R(x,x')F(x') + \delta\Pi(x)$$

Fluctuation-Dissipation Relation (FDR)

$$\langle \delta\Pi(x)\delta\Pi(x')\rangle = TG^*(x,x')$$

G^* : Symmetrized correlation function

$\delta\Pi$: Hydrodynamic fluctuation

For non-relativistic case, see Landau-Lifshitz, Fluid Mechanics

K.Murase and TH, arXiv:1304.3243[nucl-th]

Colored Noise in Relativistic System

$$G_R(t,t') = \frac{\kappa}{\tau}\exp\left(-\frac{t-t'}{\tau}\right)\theta(t-t')$$

G^*: Extention to $t < t'$

Correlation in Fourier space

$$\left\langle \delta\Pi^*_{\omega,\boldsymbol{k}}\delta\Pi_{\omega',\boldsymbol{k}'}\right\rangle = 2\kappa\frac{(2\pi)^4\delta(\omega-\omega')\,\delta^{(3)}(\boldsymbol{k}-\boldsymbol{k}')}{1+\omega^2\tau^2}$$

→ Colored noise!

→ As a consequence of causality

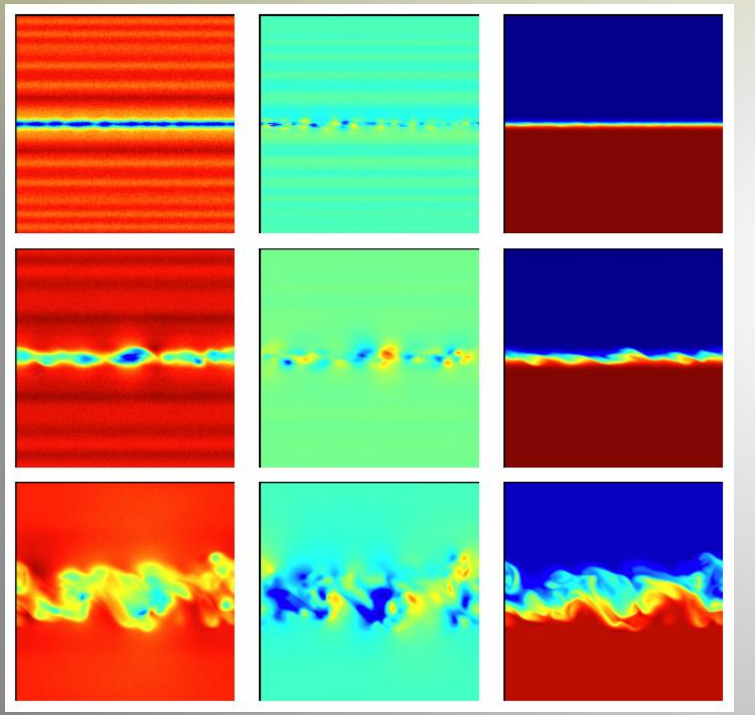
→ Note: white noise in differential form

cf.) C. Young, arXiv:1306.0472

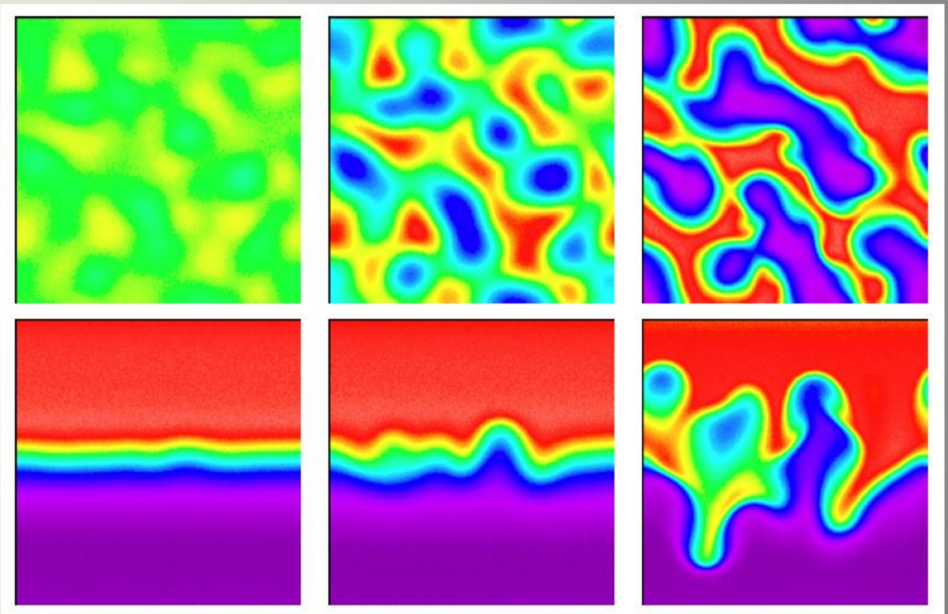
Figures from J.B.Bell *et al.*, ESAIM 44-5 (2010)1085

Fluctuation in Non-Linear Equation

Ex.) Seeds for instabilities



Kelvin-Helmhortz instability



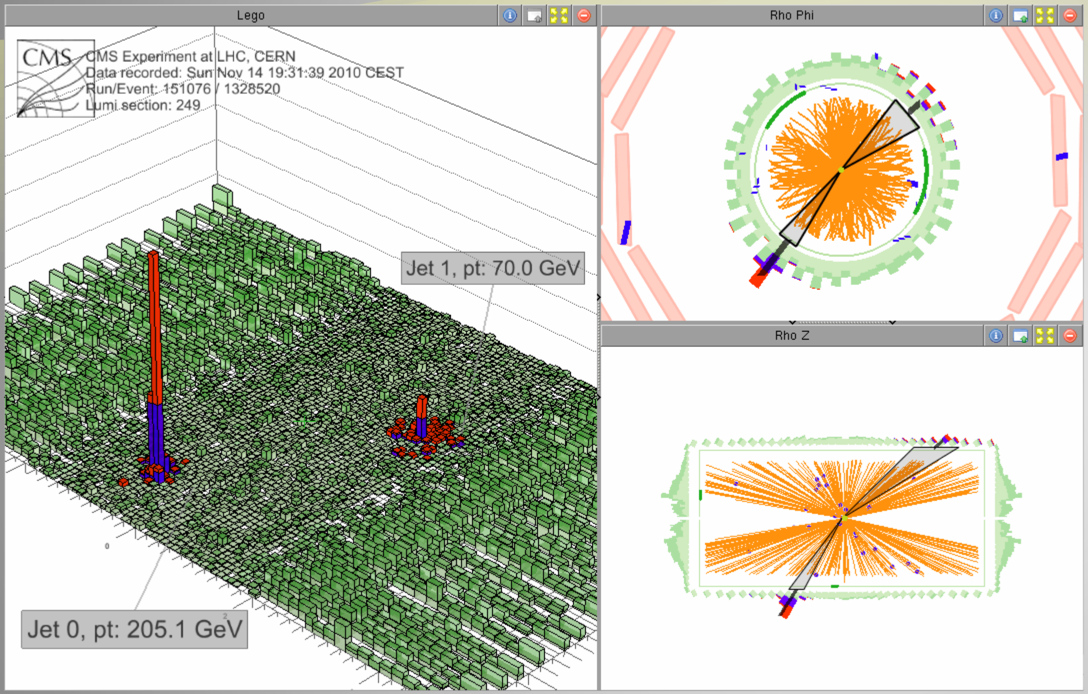
Rayleigh-Taylor instability

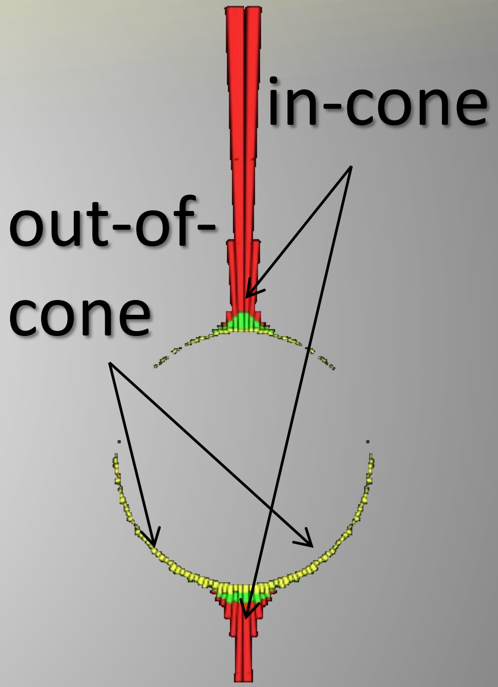
Need numerical implementation of fluctuating hydro

MEDIUM RESPONSE TO JET PROPAGATION

Figures adapted from talk
by C.Roland (CMS) at QM2011

Jet Quenching as Missing p_T





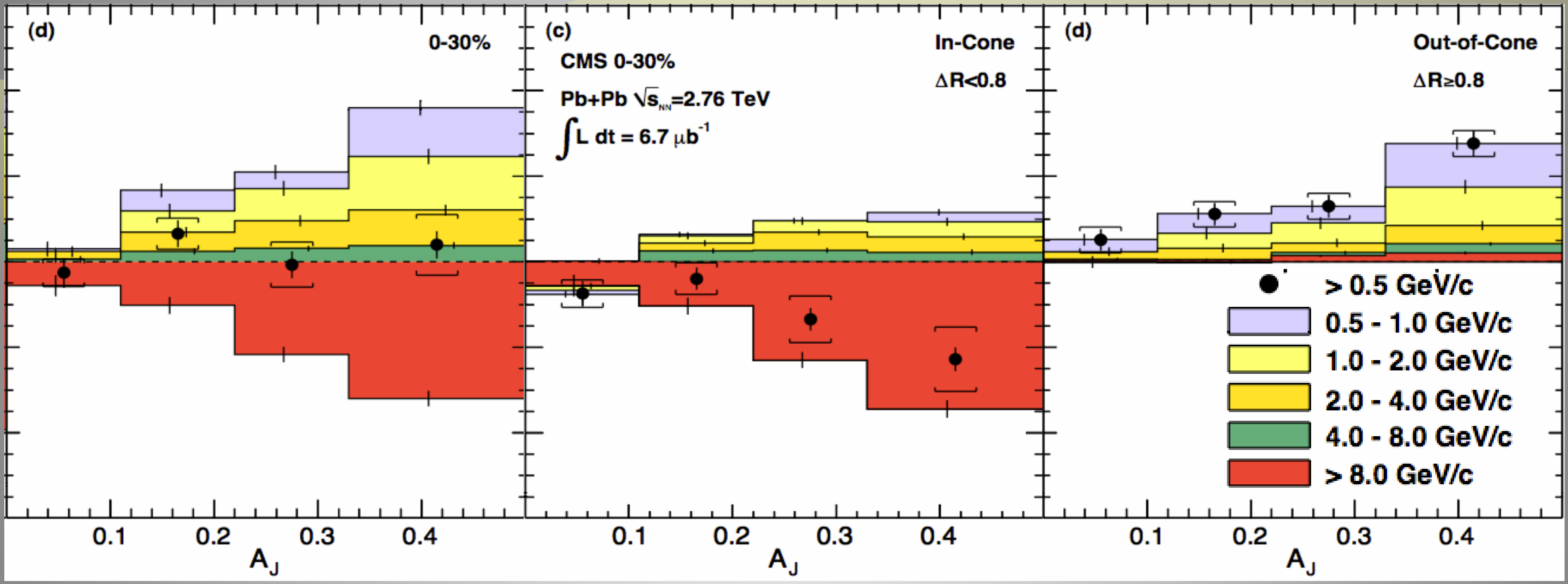
Jet momentum balanced by low p_T hadrons outside the jet cone!

→ Medium response?

See talk by Y.-S. Kim

Figure adapted from talk
by C.Roland (CMS) at QM2011

Momentum Balance Restored



Horizontal: Jet asymmetry

Vertical: Transverse momenta (anti-)parallel to jet axis

Lost energy → low p_T particles at large angle

Energy and Momentum Flowing into QGP

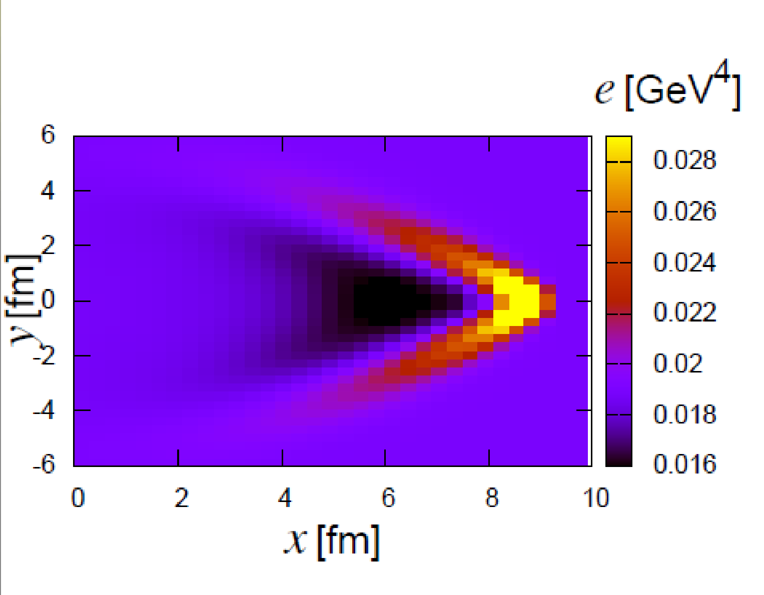
- Jet quenching could affect QGP expansion at LHC (and even at RHIC?)
- Relativistic hydrodynamics with a source term due to propagation of jets

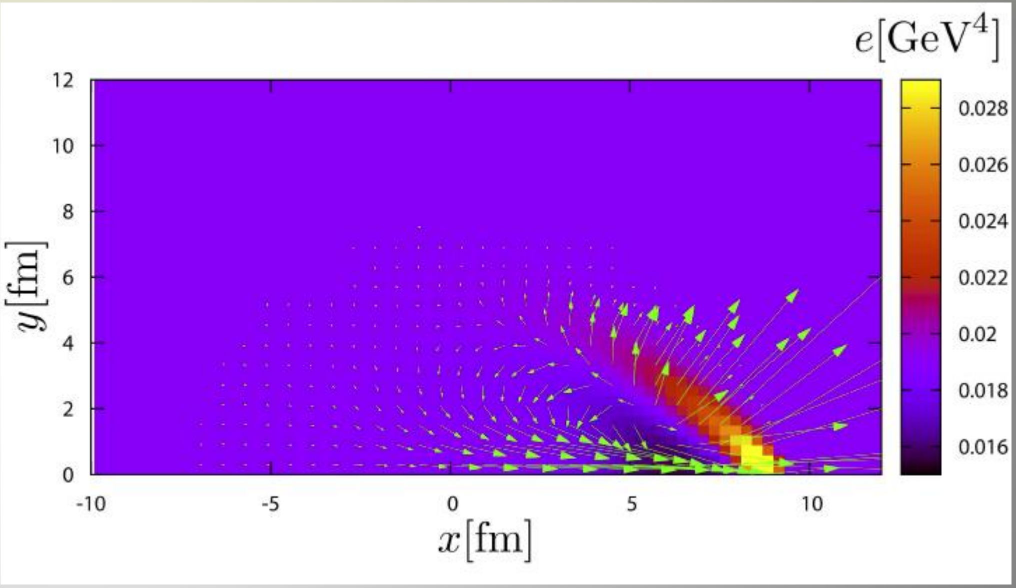
$$\partial_\mu T^{\mu\nu}_{\mathrm{fluid}} = J^\nu_{\mathrm{jet}}$$

$$J^0_{\mathrm{jet}}(x) = -\frac{dp_0}{dt}\delta^{(3)}(\boldsymbol{x} - \boldsymbol{x}(t))$$

$$J^i_{\mathrm{jet}}(x) = -\frac{dp_0}{dt}\frac{p^i}{p^0}\delta^{(3)}(\boldsymbol{x} - \boldsymbol{x}(t))$$

QGP Wake by Jet

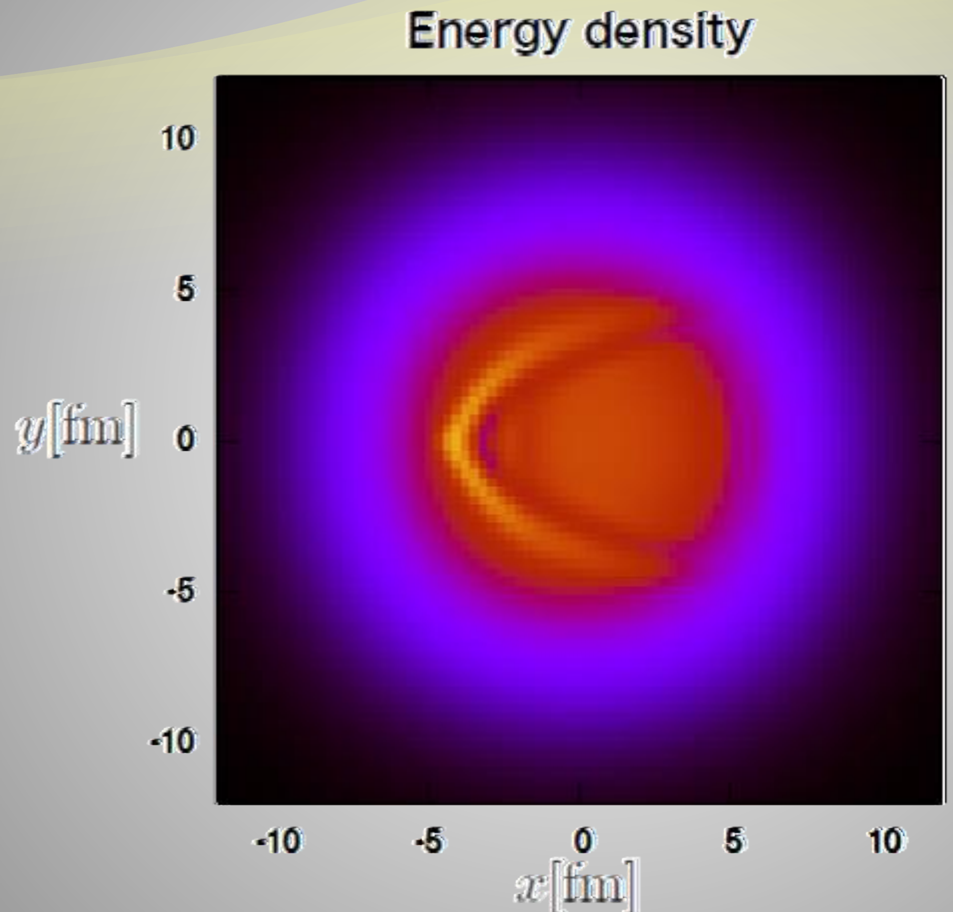




A 50 GeV jet traverses a background with T=0.5 GeV
→ Mach-cone like structure

Vortex ring behind a jet

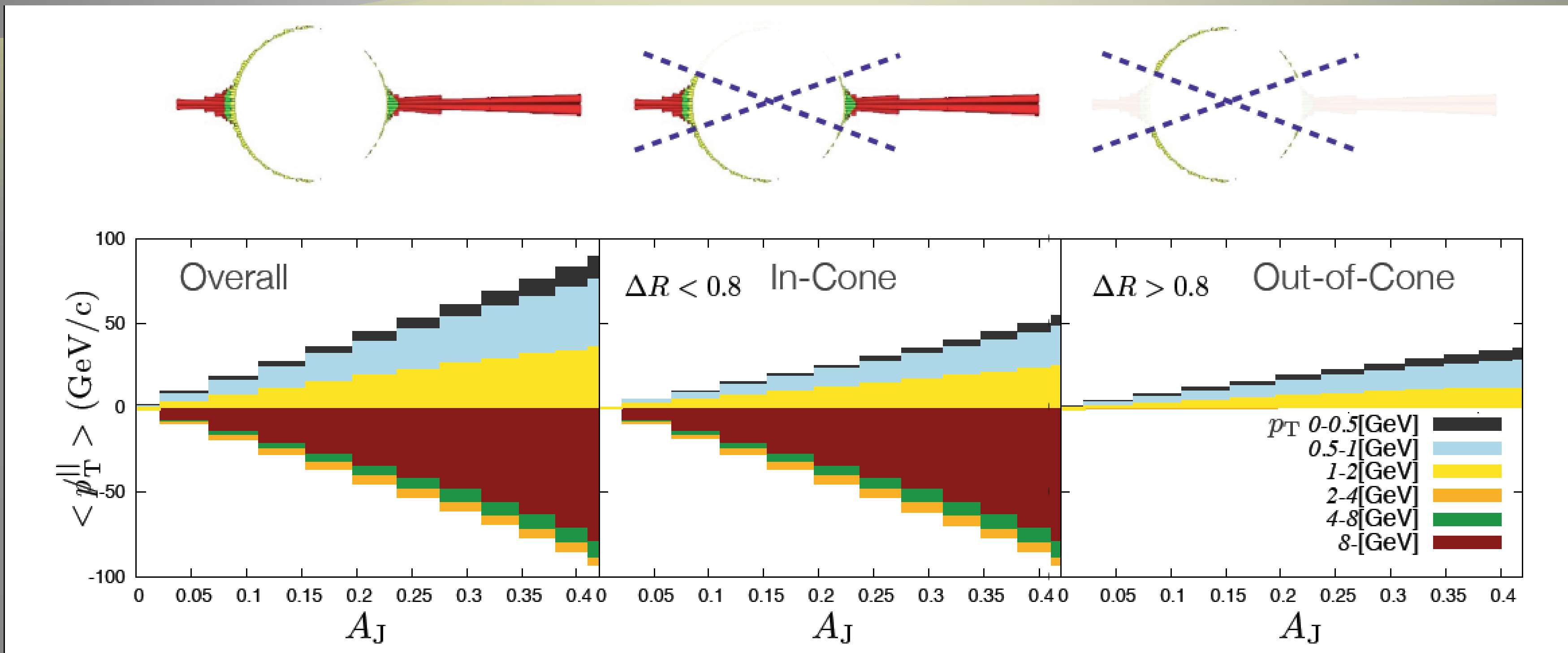
QGP Mach Cone Induced by a Jet



Central collision at LHC
Jet momentum: $p_x = -200\text{GeV}$

<u>Fully non-linear</u> and fully 3D hydro simulation of Mach cone

Energy Balance in and out-of Jet Cone in Hydro + Jet Model



Energy balance

Leading jet

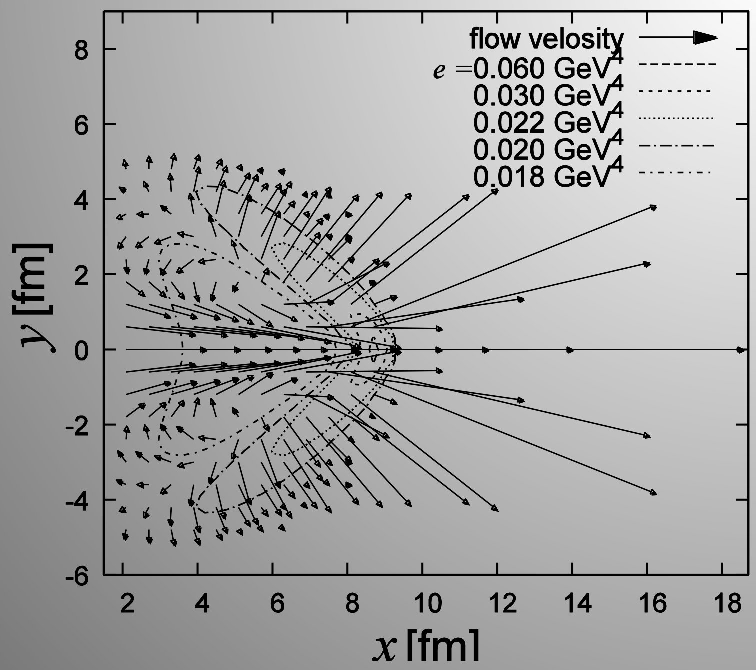
Soft particle ($p_T < 2$GeV)

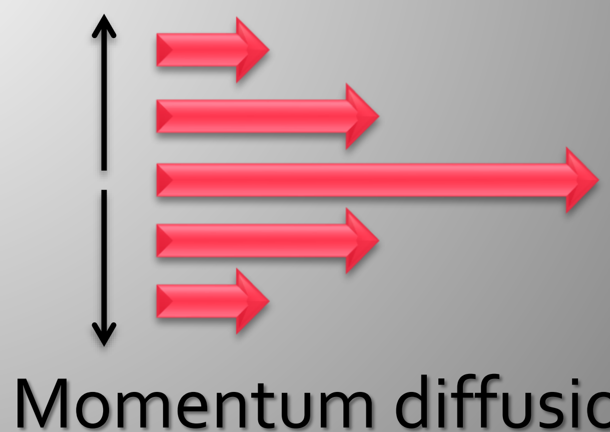
Qualitatively consistent with CMS results

Y.Tachibana and TH, Nucl.Phys.A904-905 (2013)1023c

Another Channel to Constrain Shear Viscosity?

- Jet propagation induces large flow velocity along jet axis.
- How much momentum diffuses perp. to jet axis?





ANOMALOUS HYDRODYNAMICS AND CHIRAL MAGNETIC EFFECT

Strong EM fields in Heavy Ion Collisions

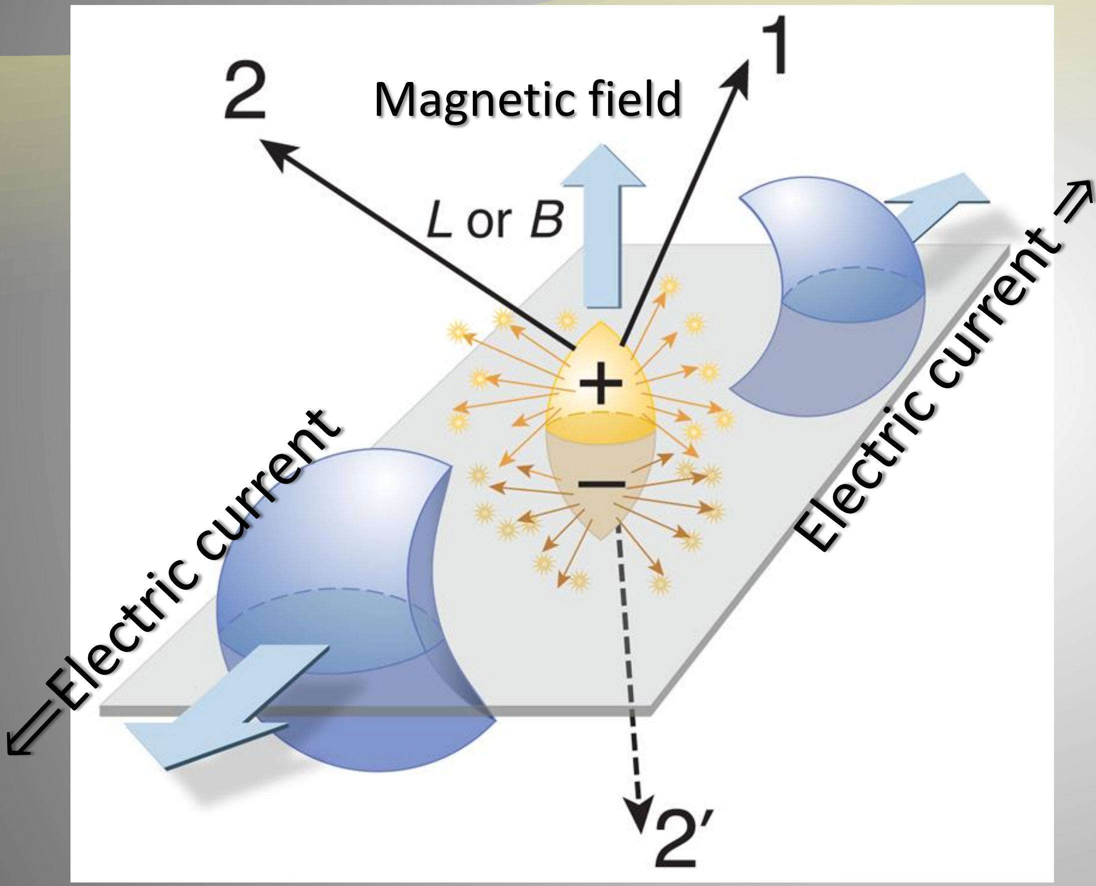


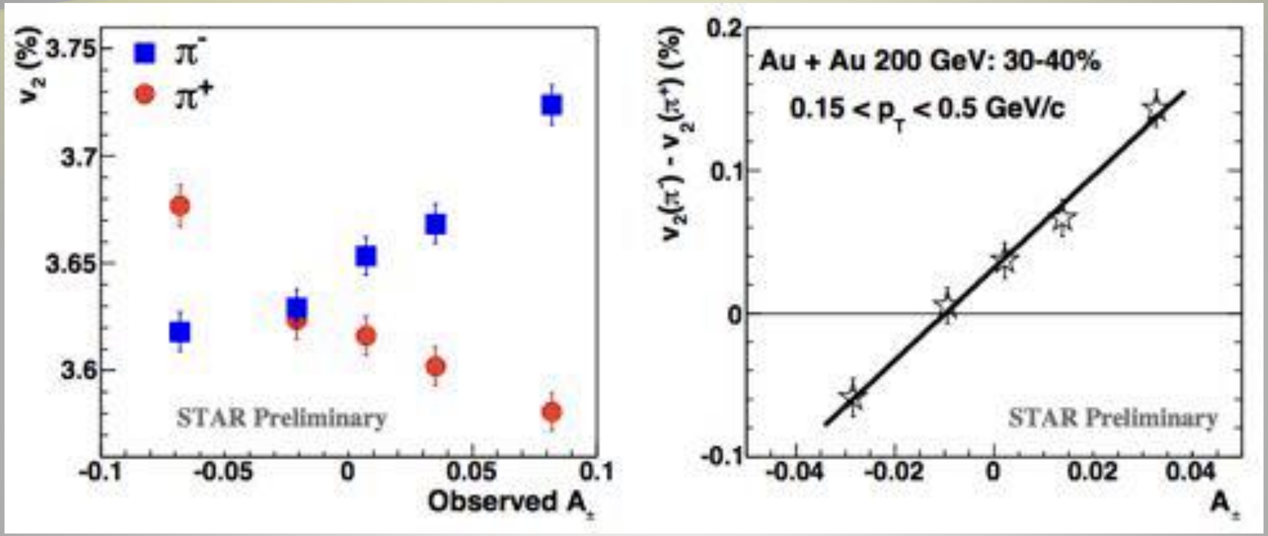
Figure taken from
http://physics.aps.org/articles/v2/104

Positively charged heavy ion as a source of electro magnetic field
→ Ampere's law

$eB \sim 10^{17}$ Gauss!!!

Charge Asymmetry of Pion v_2

G.Wang (STAR), arXiv:1210.5498



$$A^{\pm} = \frac{\overline{N_+} - \overline{N_-}}{\overline{N_+} + \overline{N_-}}$$

Charge asymmetic results indicate existence of electromagnetic effects of transport?

Chiral Magnetic Wave

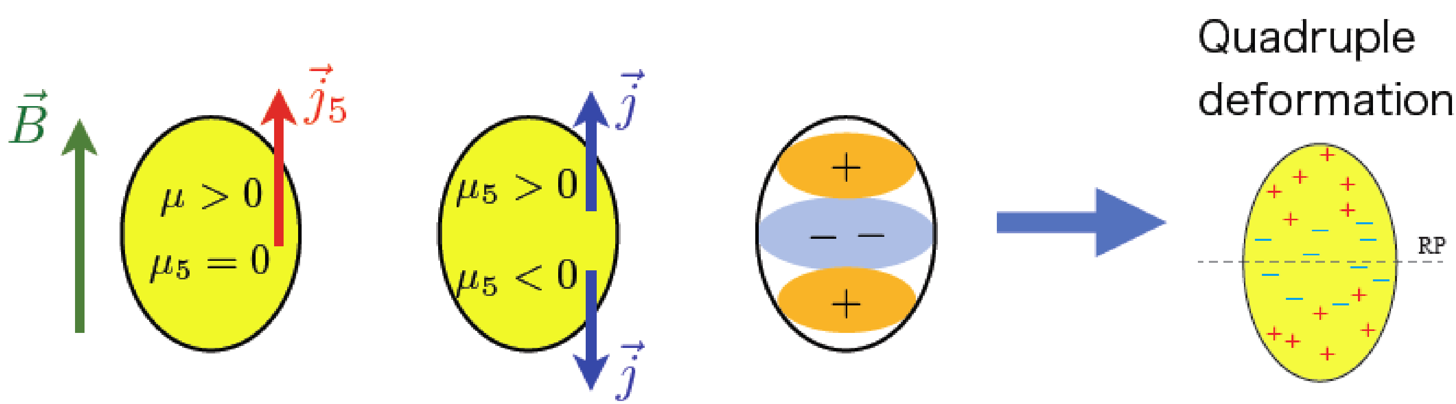
chiral separation effect

$$\vec{j}_5 = \frac{N_c e}{2\pi^2}\underline{\mu}\vec{B}$$

$$\vec{j} = \frac{N_c e}{2\pi^2}\underline{\mu_5}\vec{B}$$

chiral magnetic effect

◇ In the case of finite chemical potential (low energy collisions)



D.Kharzeev and H.U.Yee(2011);Y.Burnier et al.(2011)

Slide taken from a seminar talk by M.Hongo at BNL (2013)

For introduction of CME and CSE, see K.Fukushima, arXiv:1209.5064

Anomalous Hydro

Hydrodynamic eqs. under EM fields + anomaly

$$\partial_\mu T^{\mu\nu} = F^{\nu\lambda} j_\lambda$$

$$\partial_\mu j^\mu = 0, \partial_\mu j_5^\mu = -\frac{1}{2\pi^2} E_\mu B^\mu$$

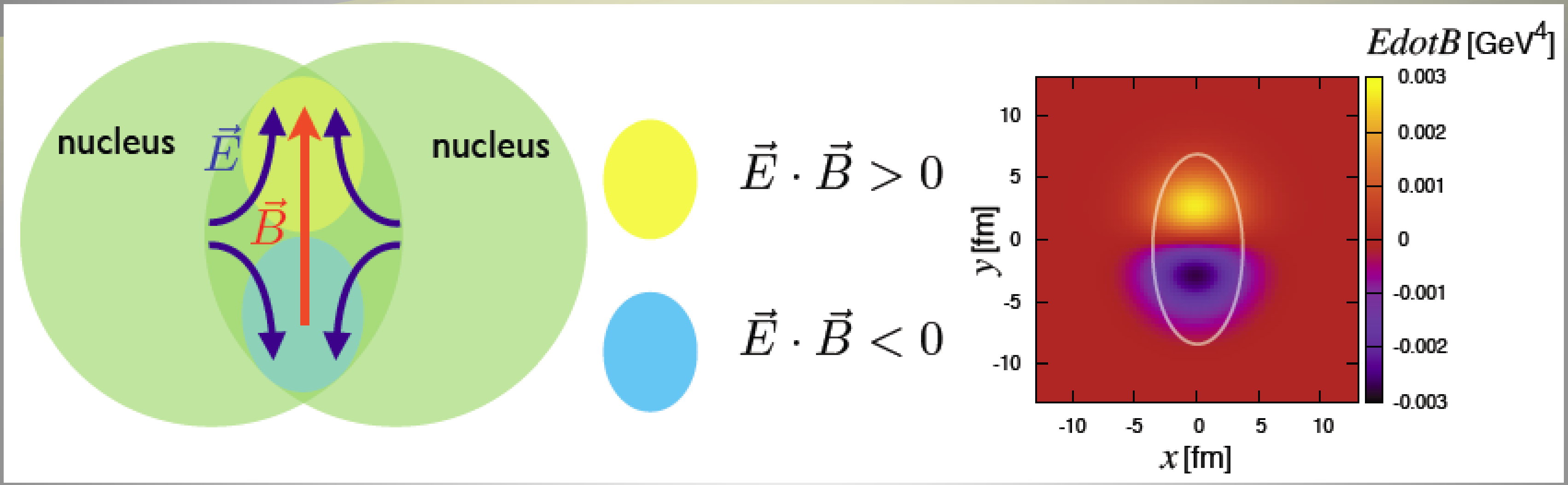
Constitutive eqs. incl. CME and CSE

$$j^\mu = n u^\mu + \kappa_B(\mu, \mu_5) B^\mu$$

$$j_5^\mu = n_5 u^\mu + \xi_B(\mu, \mu_5) B^\mu$$

D.T.Son and P.Surowka,PRL103(2009)191601

Source of Axial Charge

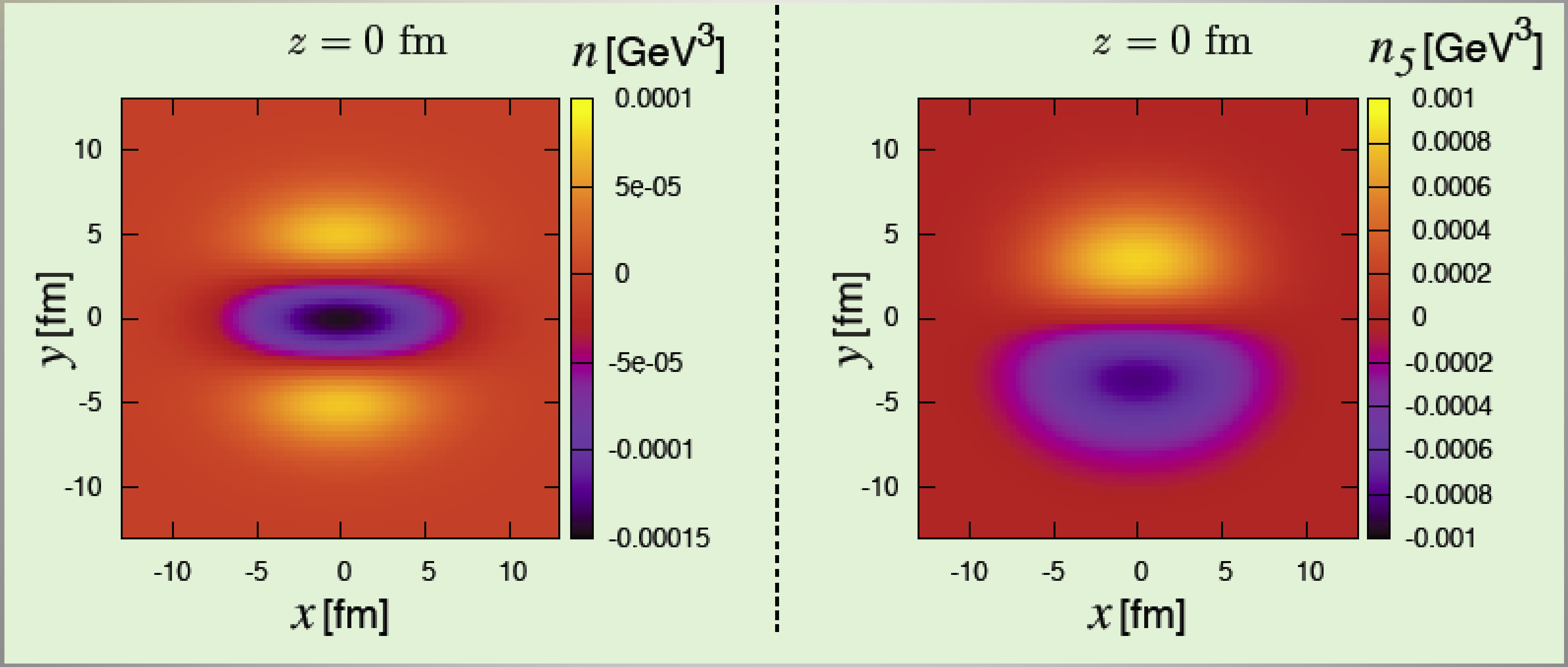


$$\partial_\mu j_5^\mu = -\frac{1}{2\pi^2} E_\mu B^\mu$$

→ Quantum anomaly term

M.Hongo, Y.Hirono, TH, arXiv:1309.2823

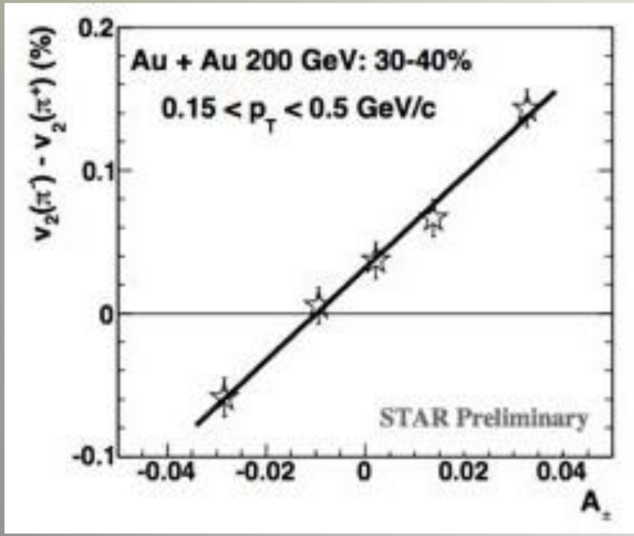
Charge and Axial Charge in Expanding Case



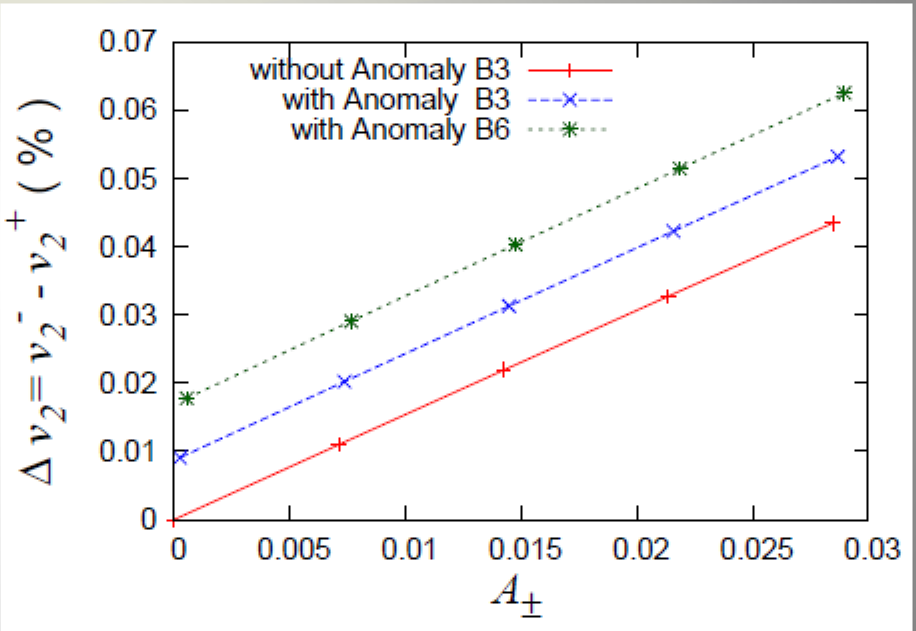
Charge density and axial charge density in the transverse plane at $t = 6\text{fm/c}$

M.Hongo, Y.Hirono, TH, arXiv:1309.2823

Difference of v_2 btw. Positive and Negative pions



STAR data



$\Delta v_2 \propto A_\pm$ without anomaly

Intercept is finite with anomaly

Need more quantitative analysis of anomalous hydro

Summary

Hydrodynamic framework/simulation extended in three directions

- Relativistic fluctuating hydrodynamics

$$\Pi(x) = \int d^4x' G_R(x, x') F(x') + \delta\Pi(x)$$

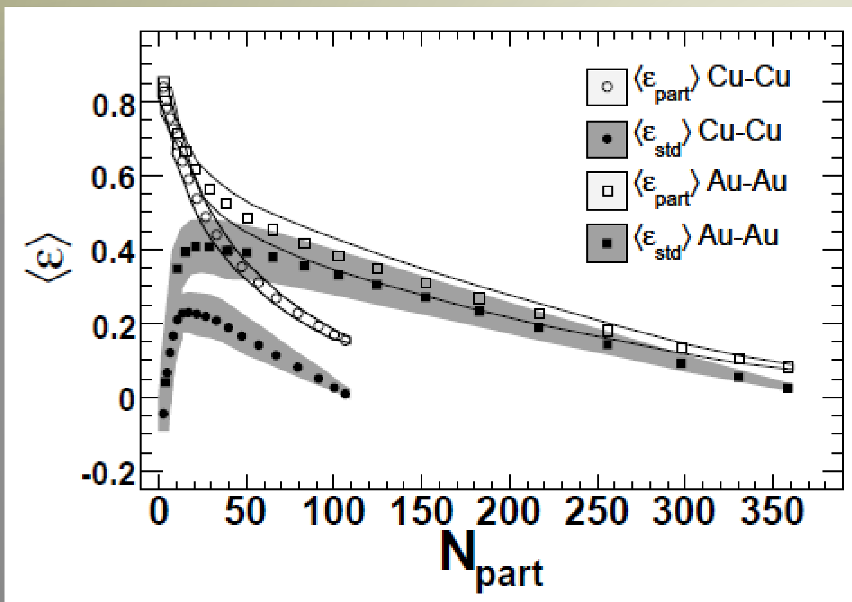
- Source term from jet propagation

$$\partial_\mu T^{\mu\nu}_{\text{fluid}} = J^\nu_{\text{jet}}$$

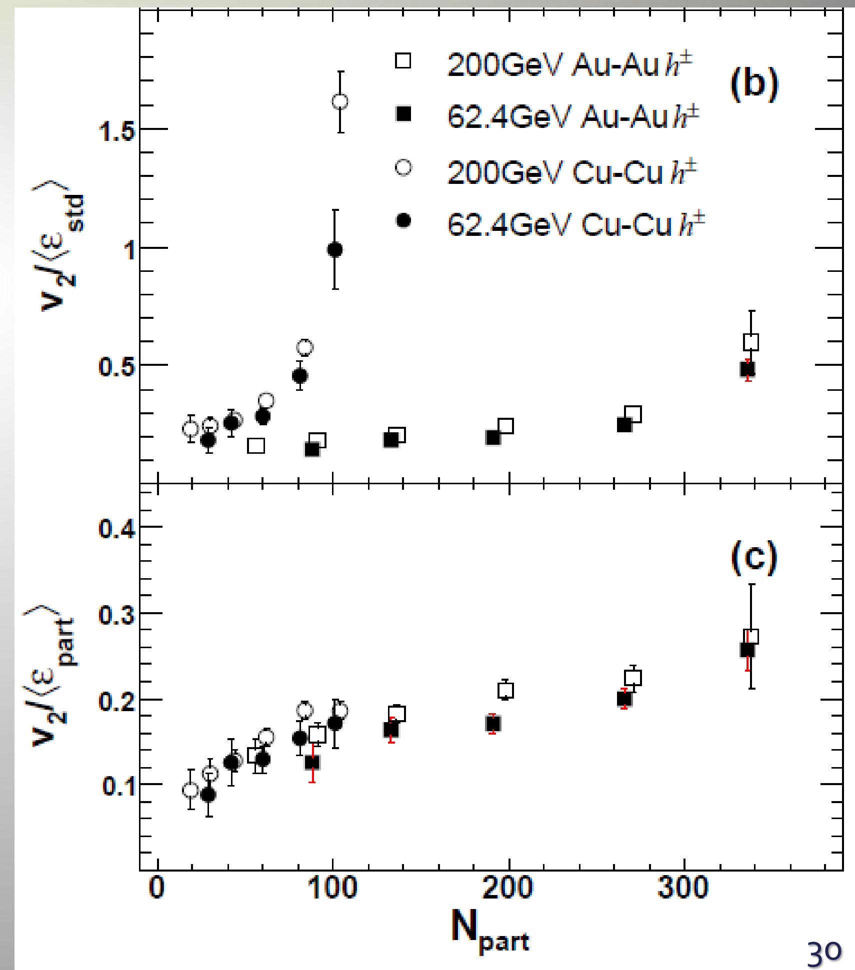
- Quantum anomalous effects

$$j^\mu \propto \mu_5 B^\mu, \qquad j_5^\mu \propto \mu B^\mu$$

Eccentricity Fluctuation in Small System

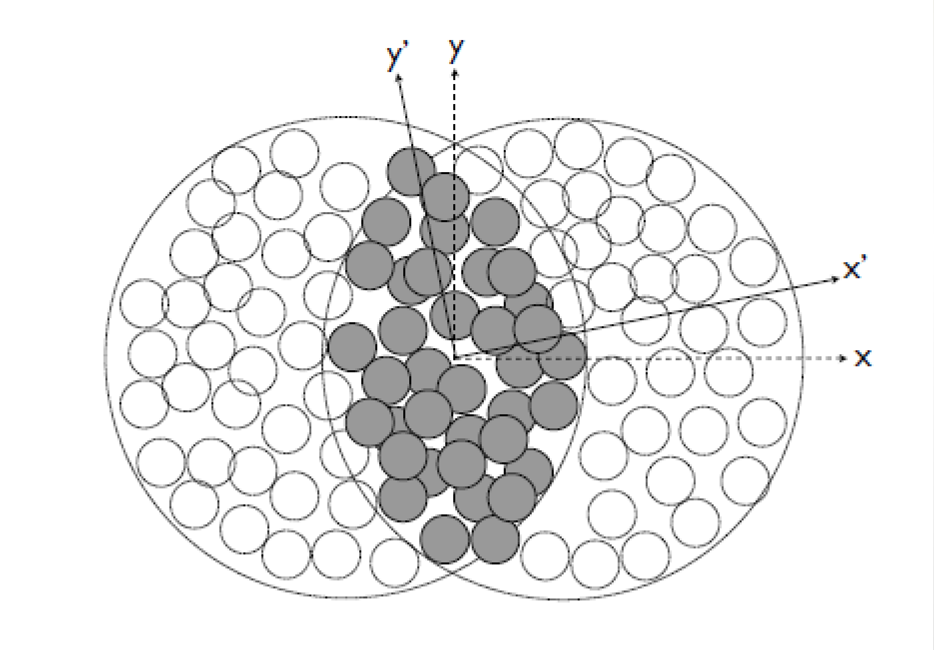


$$\varepsilon_{\mathrm{part}} > \varepsilon_{\mathrm{std}}$$



PHOBOS,
Phys. Rev. Lett. 98, 242302 (2007)

Fluctuation in Initial Conditions

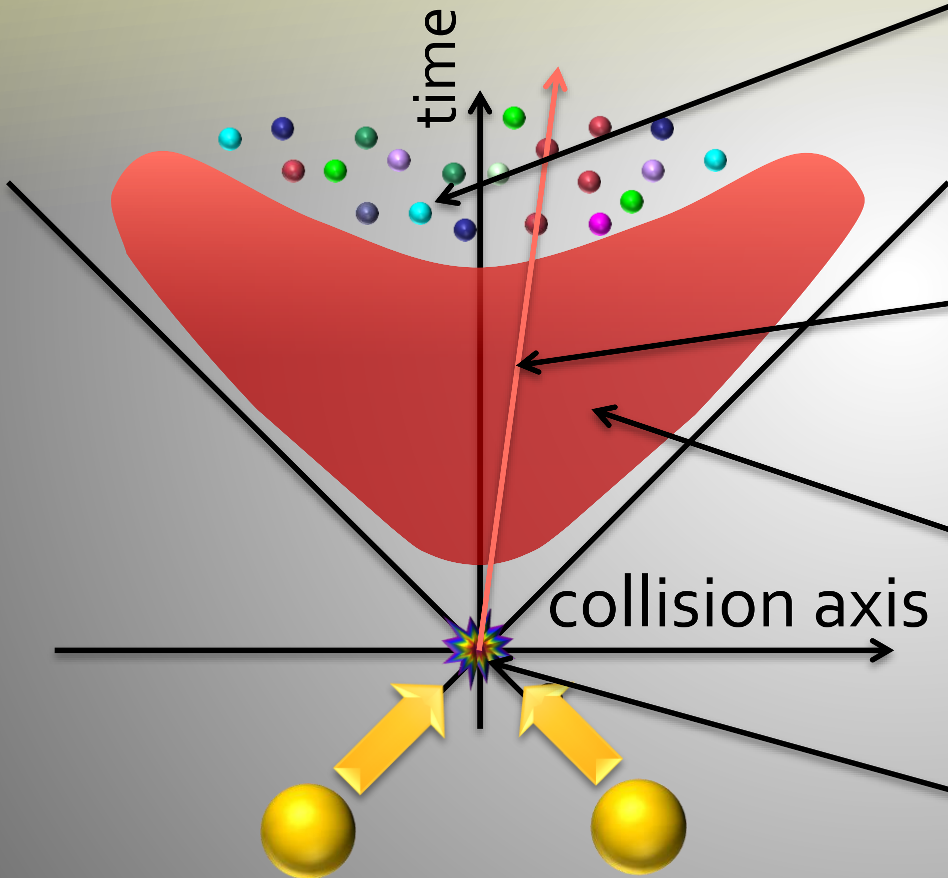


$$\varepsilon_n = \frac{\langle r^2 e^{in\phi} \rangle}{\langle r^2 \rangle}$$

$$\varepsilon_{n,\mathrm{std}} = \Re \varepsilon_n$$
$$\varepsilon_{n,\mathrm{part}} = |\varepsilon_n|$$

B.Alver *et al.*,
Phys. Rev. C 77, 014906 (2008)

Fluctuation Appears Everywhere



Finite number of hadrons

Propagation of jets

Hydrodynamic fluctuation

Initial fluctuation and particle production

Green-Kubo Formula

$$\eta = \lim_{\omega\to 0}\lim_{q\to 0}\frac{1}{2\omega}\int dtdx\, e^{i(\omega t - qx)}$$

$$\times \langle [T_{xy}(t,x), T_{xy}(0,0)] \rangle$$

Slow dynamics → How slow?

Macroscopic time scale ~ $1/\omega$ ← $t_{\text{"macro"}}$

Microscopic time scale ~ τ

cf.) Long tail problem (liquid in 2D, glassy system, super-cooling, etc.)

Relaxation and Causality

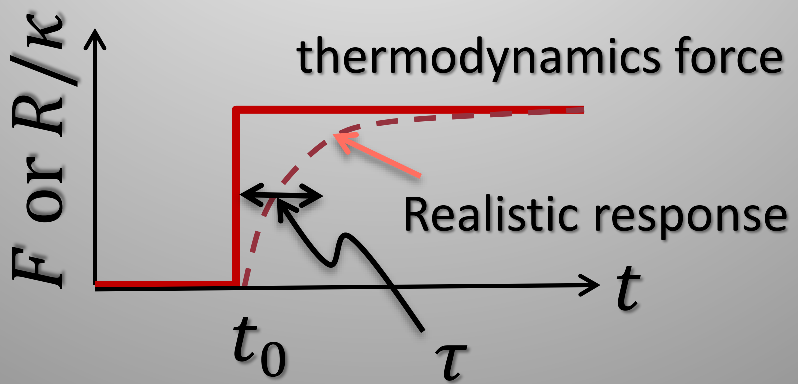
Constitutive equations at Navier-Stokes level

$$\pi^{\mu\nu} = 2\eta\partial^{<\mu}u^{\nu>},$$
$$\Pi = -\varsigma\partial_\mu u^\mu,$$
$$\dots$$

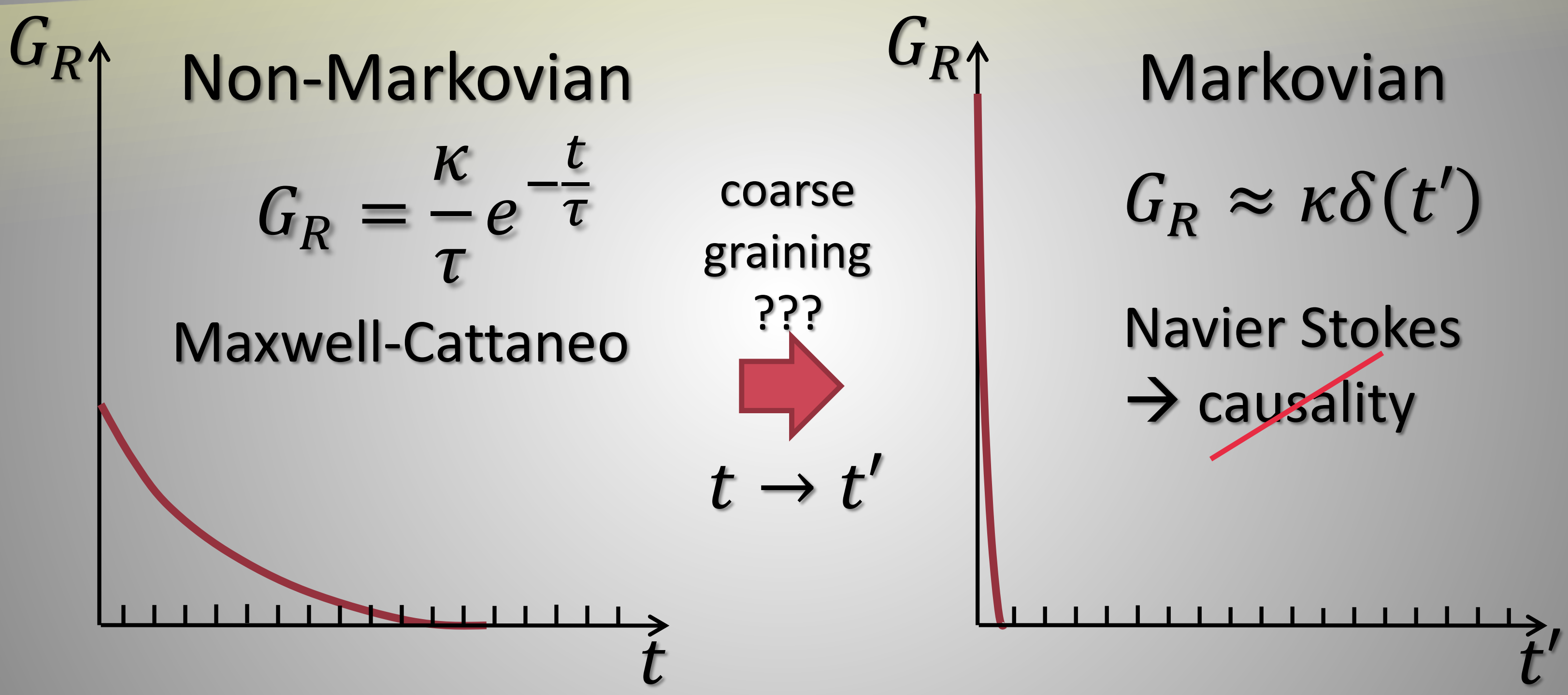
Instantaneous response violates causality

→ Critical issue in relativistic theory

→ Relaxation plays an essential role



Coarse-Graining in Time



Existence of upper bound in coarse-graining time
(or lower bound of frequency) in relativistic theory???

Outlook for RFH

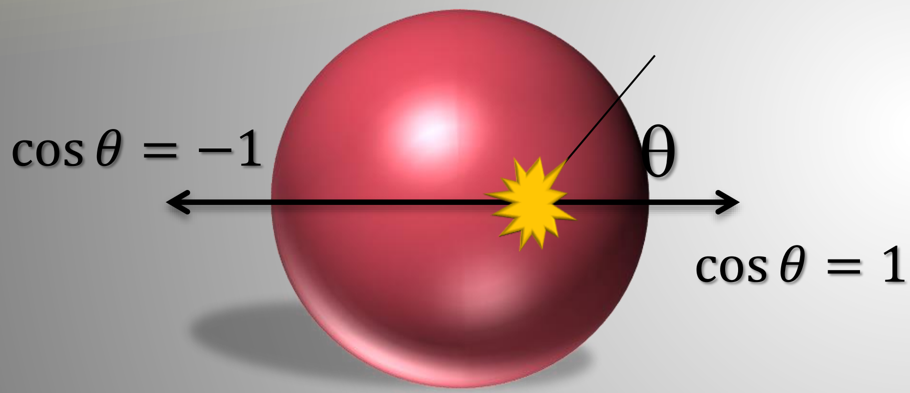
- Numerical implementation in full hydro equation (not linearized one) and its consequences in observables
- Langevin simulation in constrained system?
- Dynamic critical phenomena

  → Need further formulation of RFH

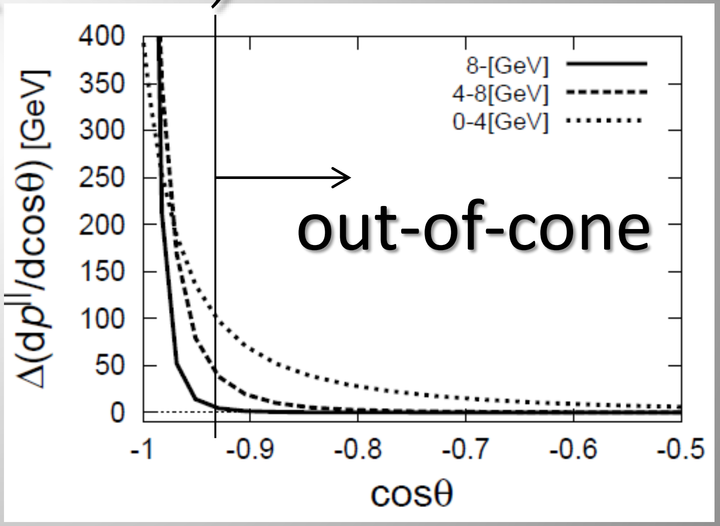
Y.Tachibana and TH, Nucl.Phys.A904-905 (2013)1023c

Large Angle Emission from Expanding Medium

$$\Delta\left(\frac{dp^{\parallel}}{d\cos\theta}\right) = \frac{dp^{\parallel}{}_{\mathrm{w.jet}}}{d\cos\theta} - \frac{dp^{\parallel}{}_{\mathrm{w.o.jet}}}{d\cos\theta}$$



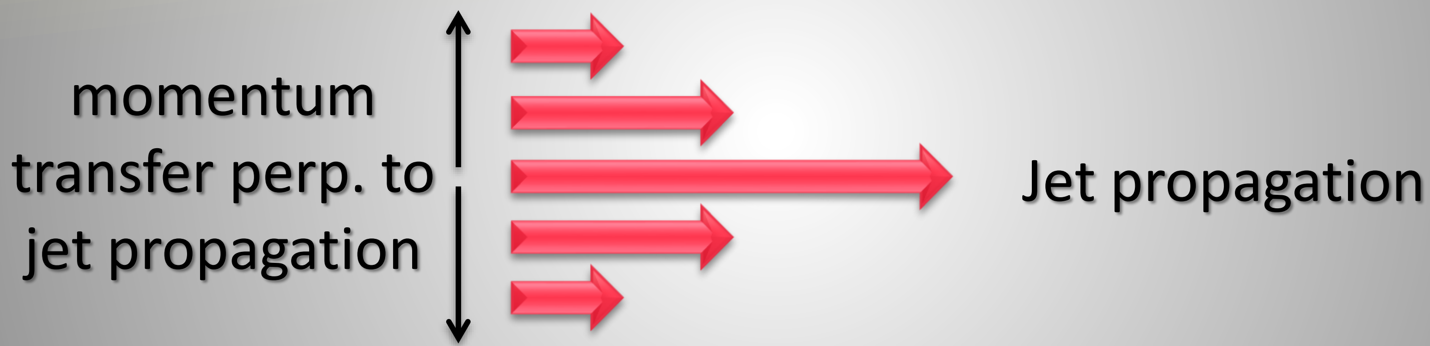
Jet pair created at off central position



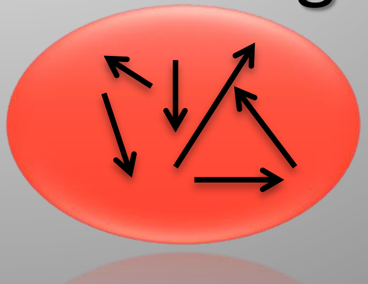
Enhancement of low momentum particles at large angle from jet axis

Outlook for Interplay between Soft and Hard

- Alternative constraint for shear viscosity through propagation of jets?



- Heat-up due to mini-jets propagation similar to neutrino reheating in Super Nova Explosion?



Anomalous Hydrodynamics

(Son, Surowka(2009))

◇ **Conservation law :**

$$\partial_\mu T^{\mu\nu} = eF^{\nu\lambda} j_\lambda, \quad \partial_\mu j^\mu = 0, \quad \partial_\mu j_5^\mu = -CE^\mu B_\mu$$

◇ **Constitutive equation :**

$$j^\mu = nu^\mu + \underbrace{\kappa_B B^\mu}_{\text{CME}} + \underbrace{\kappa_\omega \omega^\mu}_{\text{CVE}}$$

Chiral Magnetic/Vortical Effect(CME/CVE)

$$j_5^\mu = n_5 u^\mu + \underbrace{\xi_B B^\mu}_{\text{CSE}} + \underbrace{\xi_\omega \omega^\mu}_{\text{CVE}}$$

Chiral Separation Effect ... (CSE)

◇ **Coefficients :**

$$e\kappa_B = C\mu_5 \left(1 - \frac{\mu n}{\varepsilon + p}\right) \qquad e^2\kappa_\omega = 2C\mu\mu_5 \left(1 - \frac{\mu n}{\varepsilon + p}\right)$$

$$e\xi_B = C\mu \left(1 - \frac{\mu_5 n_5}{\varepsilon + p}\right) \qquad e^2\xi_\omega = C\mu^2 \left(1 - \frac{2\mu_5 n_5}{\varepsilon + p}\right)$$

Setup (1) Hydrodynamic equations

◇ Conservation law

$$\partial_\mu T^{\mu\nu} = eF^{\nu\lambda} j_\lambda, \quad \partial_\mu j^\mu = 0, \quad \partial_\mu j_5^\mu = -CE^\mu B_\mu \qquad \left(C = \frac{N_c e^2}{2\pi^2} \right)$$

◇ Constitutive equation (CME and CSE)

$$j^\mu = n u^\mu + \kappa_B B^\mu, \qquad j_5^\mu = n_5 u^\mu + \xi_B B^\mu$$

◇ EoS: ideal gas (Gluons +1-component Fermion)

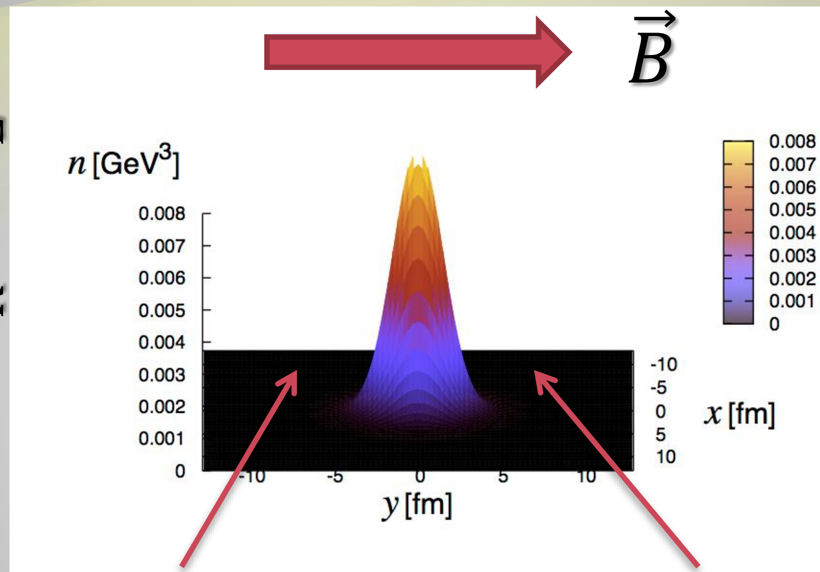
$$p = \frac{1}{3} e = \frac{g_{qgp}\pi^2}{90} T^4 + \frac{N_c}{6}\left(\mu^2 + \mu_5^2\right) T^2 + \frac{N_c}{12\pi^2}\left(\mu^4 + 6\mu^2\mu_5^2 + \mu_5^2\right)$$

$$n = \frac{N_c}{3\pi^2}\mu^3 + \frac{N_c}{3}\mu\left(T^2 + \frac{3}{\pi^2}\mu_5^2\right), \; n_5 = \frac{N_c}{3\pi^2}\mu_5^3 + \frac{N_c}{3}\mu_5\left(T^2 + \frac{3}{\pi^2}\mu^2\right)$$

M.Hongo, Y.Hirono and TH, arXiv:1309.2823

Full 3D Anomalous Hydro Simulation

$n = n_R + n_L$



n_L n_R

$eB_y = 1.0\text{GeV}^2$

Background temperature

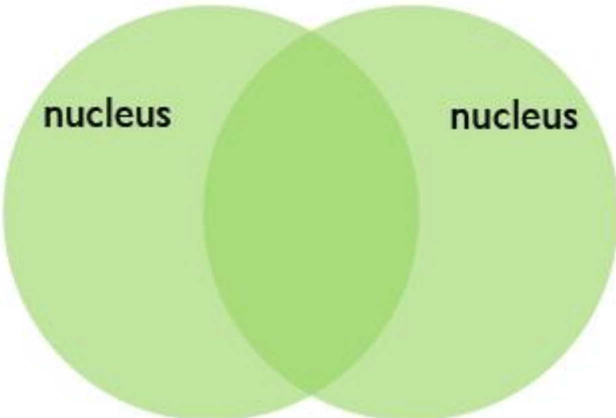
$T = 0.5\text{GeV}$

"Group velocity" of chiral magnetic wave ~ 0.58

→ CME under expanding background?

→ Source of charge asymmetric v_2?

Setup (2) Initial condition

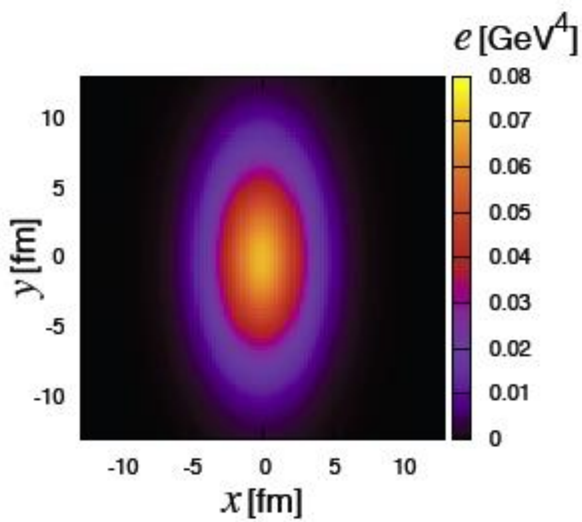


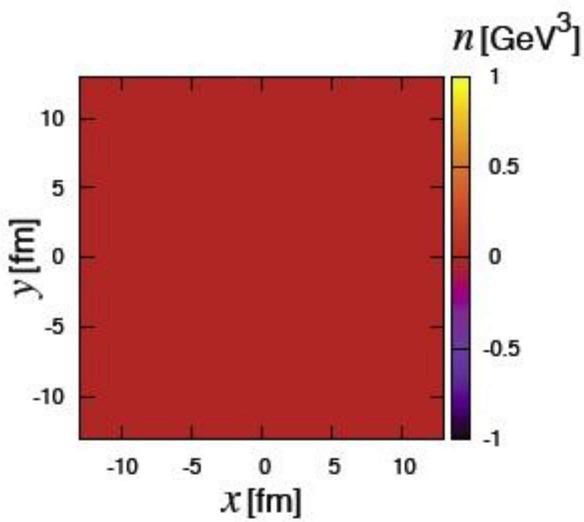
Parameters

$$T\left(\vec{r}\right) = T_0 \exp\left(-\frac{r_y^2}{2\sigma_T^2}\right) \qquad T_0 = 0.3\ \mathrm{GeV},\quad \sigma_T = 5\ \mathrm{fm}$$

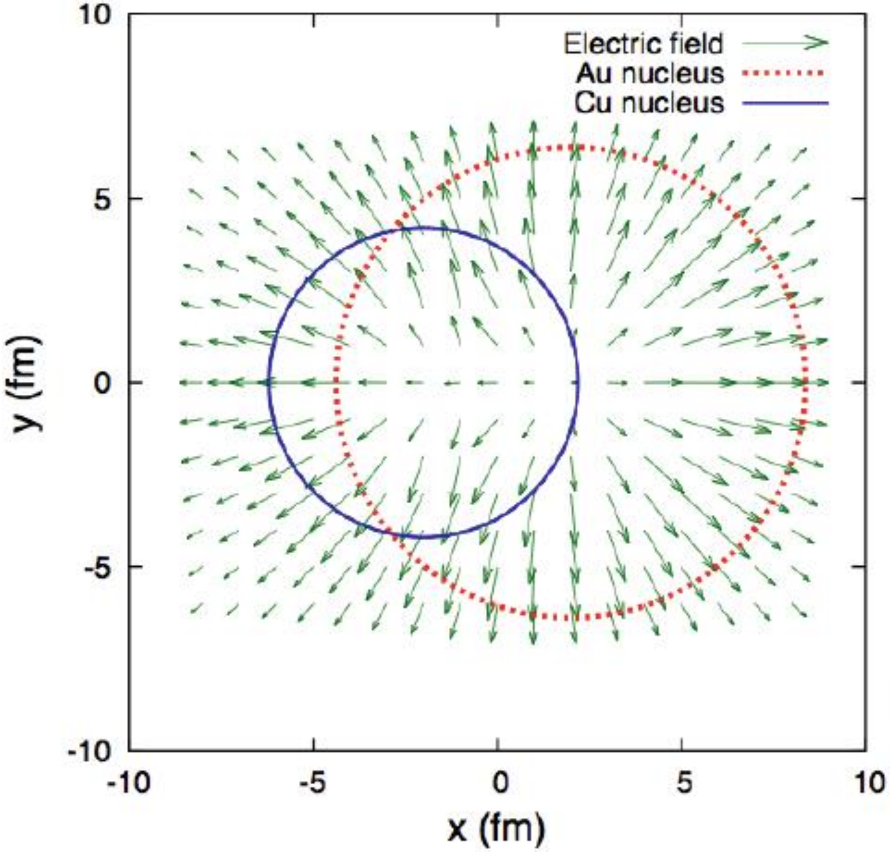
$$\mu\left(\vec{r}\right) = \mu_0 \exp\left(-\frac{r_y^2}{2\sigma_\mu^2}\right) \qquad \mu_0 = 0\ \mathrm{GeV},\quad \sigma_\mu = 5\ \mathrm{fm}$$

$$r_y = \sqrt{x^2 + \left(y/2\right)^2 + z^2}$$





Event-averaged electric fields



b = 4 fm
# of events: 10000