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INSTITUTE OF NUCLEAR AND PARTICLE PHYSICS, ASTRONOMY AND COSMOLOGY

The density curvature parameter and high density behavior of the symmetry energy

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- **The symmetry energy**
- **Current constraints on the symmetry energy**
  - **n-A elastic scattering and the symmetry potential**
  - **Symmetry energy at 0.11 fm^{-3}**
  - **High density behaviors**
- **Density curvature K_{sym} and the high density symmetry energy**
- **Summary and outlook**

"Heavy-Ion Meeting", Korea University, Seoul, May 24, 2013

Outline

- The symmetry energy
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The Symmetry Energy

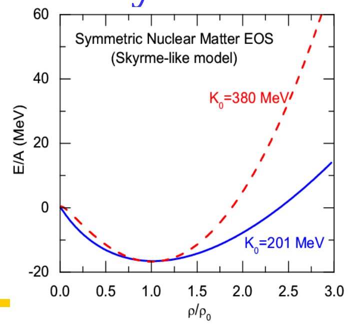
EOS of Isospin Asymmetric Nuclear Matter (Parabolic law)

$$E(\rho,\delta) = E(\rho,0) + E_{\rm sym}(\rho)\delta^2 + O(\delta^4), \quad \delta = (\rho_n - \rho_p)/\rho$$

Symmetric Nuclear Matter (relatively well-determined)

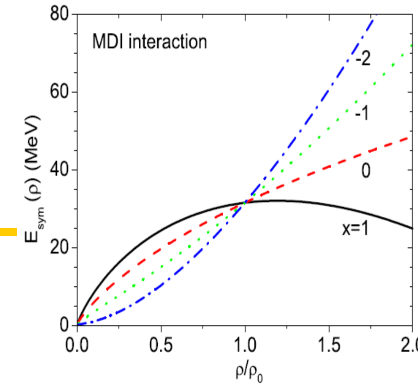
Isospin asymmetry

Symmetry energy term (poorly known)



The Nuclear Symmetry Energy

$$E_{\rm sym}(\rho) \equiv \frac{1}{2}\frac{\partial^2 E(\rho,\delta)}{\partial \delta^2}$$



$$E_{\rm sym}(\rho) = E_{\rm sym}(\rho_0) + \frac{L}{3}\left(\frac{\rho-\rho_0}{\rho_0}\right) + \frac{K_{\rm sym}}{18}\left(\frac{\rho-\rho_0}{\rho_0}\right)^2 + \cdots\cdots, \quad (\rho \approx \rho_0)$$

$E_{\rm sym}(\rho_0) \approx 30$ MeV (LD mass formula: *Myers & Swiatecki, NPA81; Pomorski & Dudek, PRC67*)

$$L \equiv 3\rho_0 \left.\frac{\partial E_{\rm sym}(\rho)}{\partial \rho}\right|_{\rho=\rho_0}$$ (Many-Body Theory: L : −50 ~ 200 MeV; Exp: ???)

$$K_{\rm sym} \equiv 9\rho_0^2 \left.\frac{\partial^2 E_{\rm sym}(\rho)}{\partial \rho^2}\right|_{\rho=\rho_0}$$ (Many-Body Theory: $K_{\rm sym}$: −700 ~ 466 MeV; Exp: ???)

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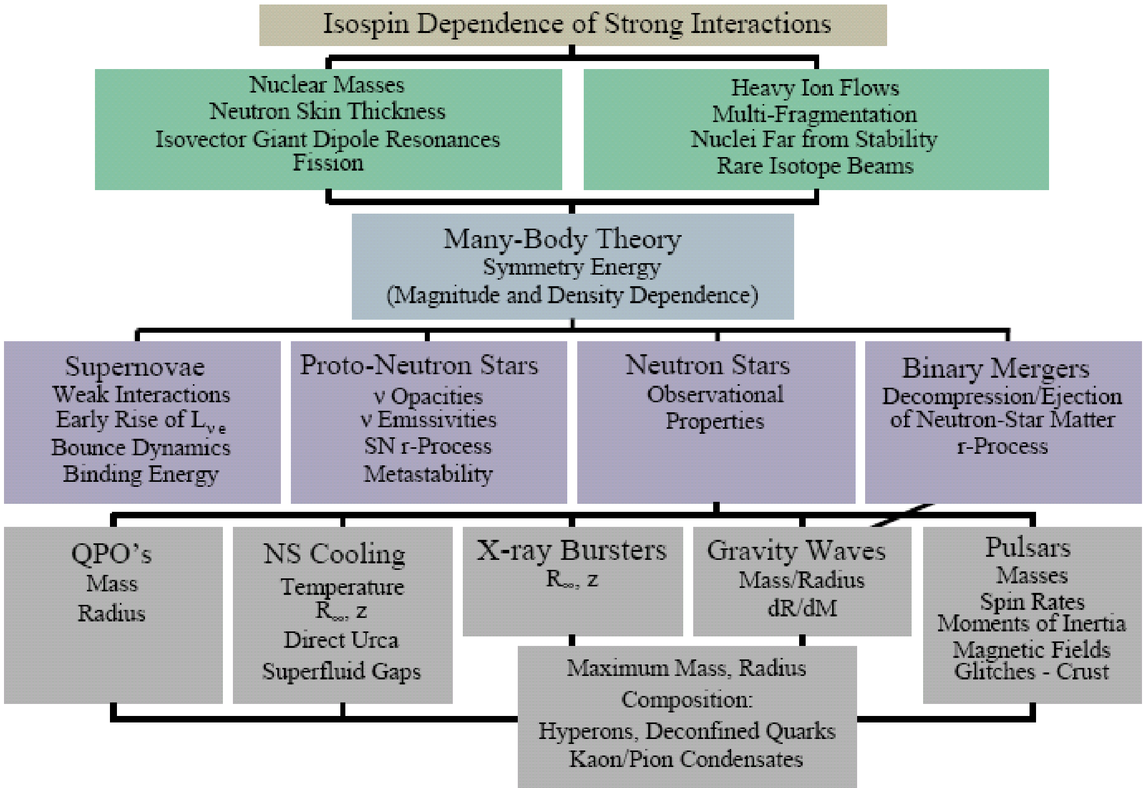
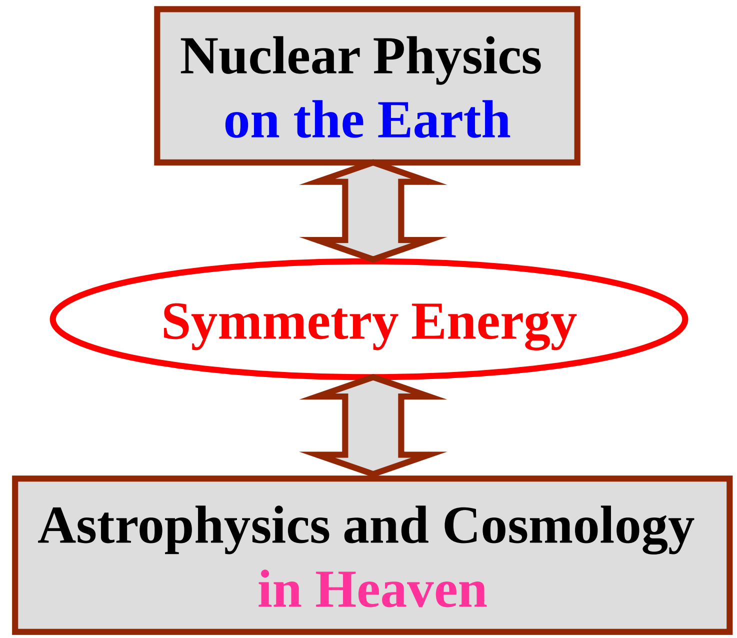
Why Symmetry Energy?

21.65.Ef

The multifaceted influence of the nuclear symmetry energy

A.W. Steiner, M. Prakash, J.M. Lattimer and P.J. Ellis, ***Phys. Rep. 411, 325 (2005).***





The symmetry energy is also related to some issues of fundamental physics:

1. **The precision tests of the SM through atomic parity violation observables (Sil et al., PRC05)**
2. **Possible time variation of the gravitational constant (Jofre et al. PRL06; Krastev/Li, PRC07)**
3. **Non-Newtonian gravity proposed in the grand unified theories (Wen/Li/Chen, PRL09)**
4. **Dark Matter Direct Detection (Hao Zheng and Lie-Wen Chen, in preparation, 2013)**

Phase Diagram of Strong Interaction Matter

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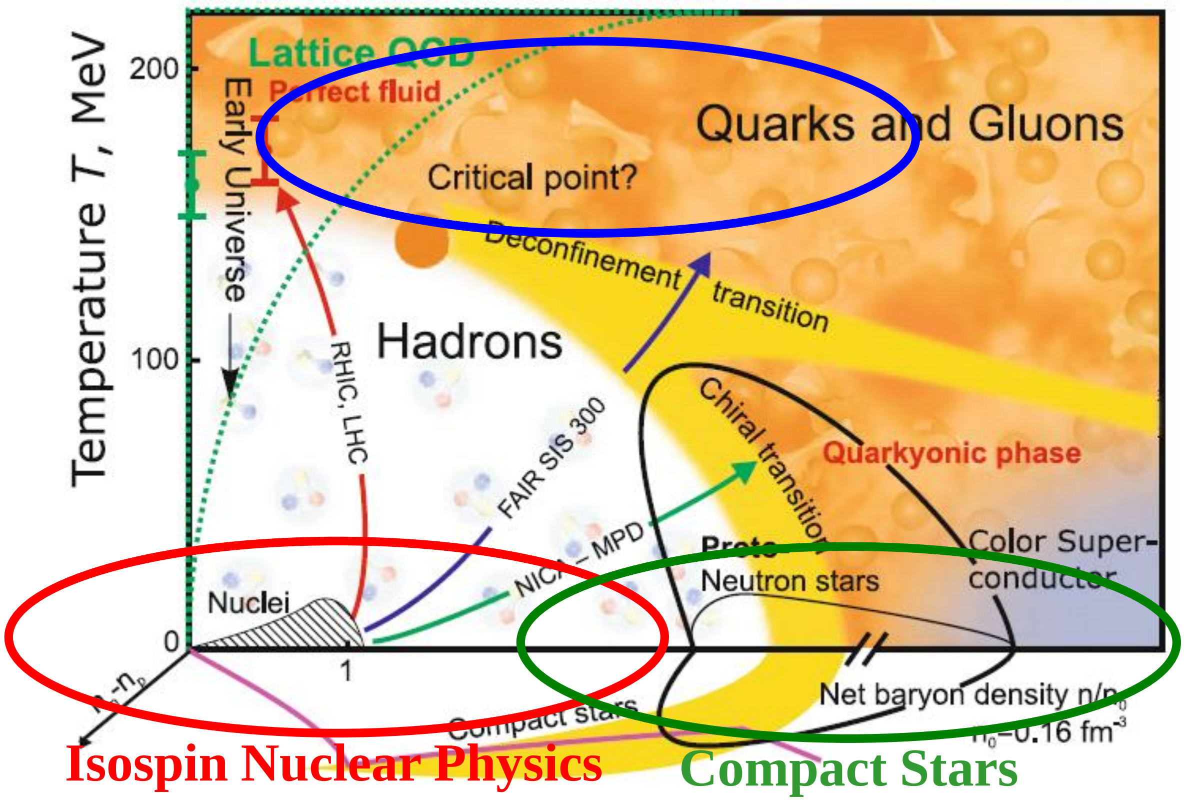
QCD Phase Diagram in 3D: density, temperature, and isospin

V.E. Fortov, Extreme States of Matter – on Earth and in the Cosmos, Springer-Verlag Berlin Heidelberg 2011

Physics of QGP



Isospin Nuclear Physics

Compact Stars

Holy Grail of Nuclear Physics

To Understand Strong Interaction Matter at Extreme, especially its EOS

1. Heavy Ion Collisions (Terrestrial Lab); 2. Compact Stars(In Heaven); …

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Facilities of Radioactive Beams

- **Cooling Storage Ring (CSR) Facility at HIRFL/Lanzhou in China (2008)**
  up to 500 MeV/A for ^{238}U
  http://www.impcas.ac.cn/zhuye/en/htm/247.htm
- **Beijing Radioactive Ion Facility (BRIF-II) at CIAE in China (2012)**
  http://www.ciae.ac.cn/
- **Radioactive Ion Beam Factory (RIBF) at RIKEN in Japan (2007)**
  http://www.riken.jp/engn/index.html
- **Texas A&M Facility for Rare Exotic Beams -T-REX (2013)**
  http://cyclotron.tamu.edu
- **Facility for Antiproton and Ion Research (FAIR)/GSI in Germany (2016)**
  up to 2 GeV/A for ^{132}Sn (NUSTAR - NUclear STructure, Astrophysics and Reactions)
  http://www.gsi.de/fair/index_e.html
- **SPIRAL2/GANIL in France (2013)**
  http://pro.ganil-spiral2.eu/spiral2
- **Selective Production of Exotic Species (SPES)/INFN in Italy (2015)**
  http://web.infn.it/spes
- **Facility for Rare Isotope Beams (FRIB)/MSU in USA (2018)**
  up to 400(200) MeV/A for ^{132}Sn
  http://www.frib.msu.edu/
- **The Korean Rare Isotope Accelerator (KoRIA-RAON(RISP Accelerator Complex) (Starting)**
  up to 250 MeV/A for ^{132}Sn, up to 109 pps

……

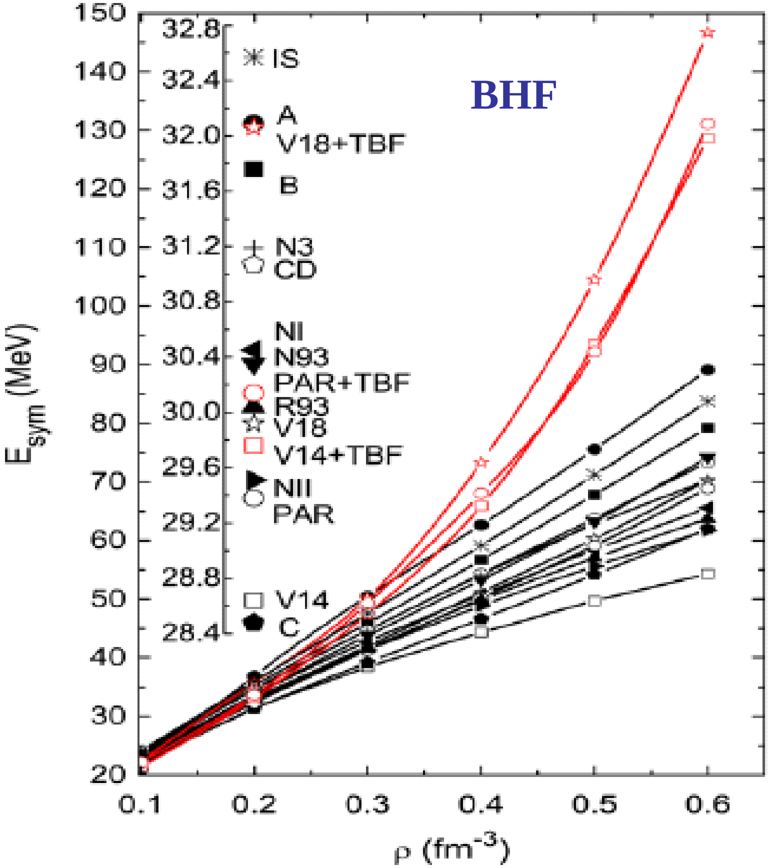
Nuclear Matter EOS: Many-Body Approaches

The nuclear EOS cannot be measured experimentally, its determination thus depends on theoretical approaches

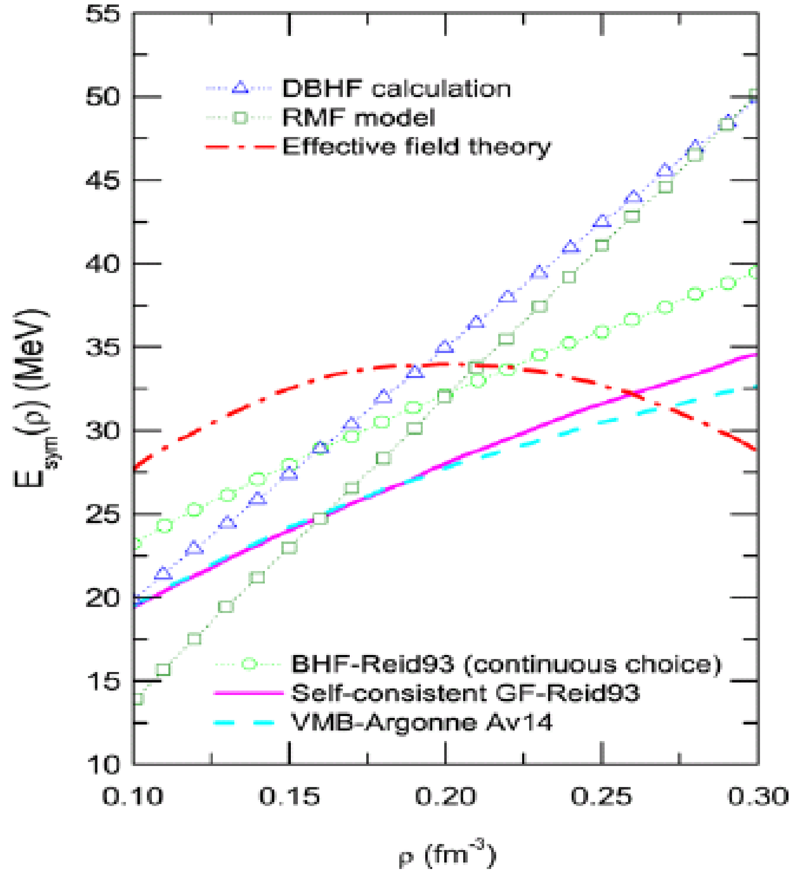
- **Microscopic Many-Body Approaches**
  - Non-relativistic Brueckner-Bethe-Goldstone (BBG) Theory
  - Relativistic Dirac-Brueckner-Hartree-Fock (DBHF) approach
  - Self-Consistent Green's Function (SCGF) Theory
  - Variational Many-Body (VMB) approach
  - Green's Function Monte Carlo Calculation
  - $V_{\mathrm{low}k}$ + Renormalization Group
- **Effective Field Theory**
  - Density Functional Theory (DFT)
  - Chiral Perturbation Theory (ChPT)
  - QCD-based theory
- **Phenomenological Approaches**
  - Relativistic mean-field (RMF) theory
  - Quark Meson Coupling (QMC) Model
  - Relativistic Hartree-Fock (RHF)
  - Non-relativistic Hartree-Fock (Skyrme-Hartree-Fock)
  - Thomas-Fermi (TF) approximations

Nuclear Matter Symmetry Energy

Z.H. Li et al., PRC74, 047304(2006)



Dieperink et al., PRC68, 064307(2003)



Transport Models

Ni + Au, E/A = 45 MeV/A

Central collisions

Transport Models for HIC's at intermediate energies:

N-body approaches
CMD, QMD,IQMD,IDQMD, ImQMD,ImIQMD,AMD,FMD

One-body approaches
BUU/VUU, BNV, LV, IBL

Relativistic covariant approaches
RVUU/RBUU,RQMD…

Broad applications of transport models
in astrophysics, plasma physics, electron transport in semiconductor and nanostructures, particle and nuclear physics, ……

Transport model for HIC's

Isospin-dependent BUU (IBUU) model

Phase-space distributions $f(\vec{r},\vec{p},t)$ satify the Boltzmann equation

$$\frac{\partial f(\vec{r},\vec{p},t)}{\partial t} + \vec{\nabla}_p \varepsilon \cdot \vec{\nabla}_r f - \vec{\nabla}_r \varepsilon \cdot \vec{\nabla}_p f = I_c(f,\sigma_{NN})$$

- Solve the Boltzmann equation using test particle method (**C.Y. Wong**)
- Isospin-dependent initialization
- Isospin- (momentum-) dependent mean field potential

$$V = V_0 + \frac{1}{2}(1-\tau_z)V_C + V_{\text{sym}}$$

EOS

- Isospin-dependent N-N cross sections
  a. Experimental free space N-N cross section σ_{exp}
  b. In-medium N-N cross section from the Dirac-Brueckner approach based on Bonn A potential $\sigma_{\text{in-medium}}$
  c. Mean-field consistent cross section due to m*
- Isospin-dependent Pauli Blocking

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Probes of the Symmetry Energy

Promising Probes of the $E_{sym}(\rho)$

(an incomplete list !)

At sub-saturation densities (亚饱和密度行为)

- Sizes of n-skins of unstable nuclei from total reaction cross sections
- Proton-nucleus elastic scattering in inverse kinematics
- Parity violating electron scattering studies of the n-skin in ^{208}Pb
- n/p ratio of FAST, pre-equilibrium nucleons
- Isospin fractionation and isoscaling in nuclear multifragmentation
- Isospin diffusion/transport
- Neutron-proton differential flow
- Neutron-proton correlation functions at low relative momenta
- t/^{3}He ratio
- Hard photon production
- Pigmy/Giant resonances
- Nucleon optical potential

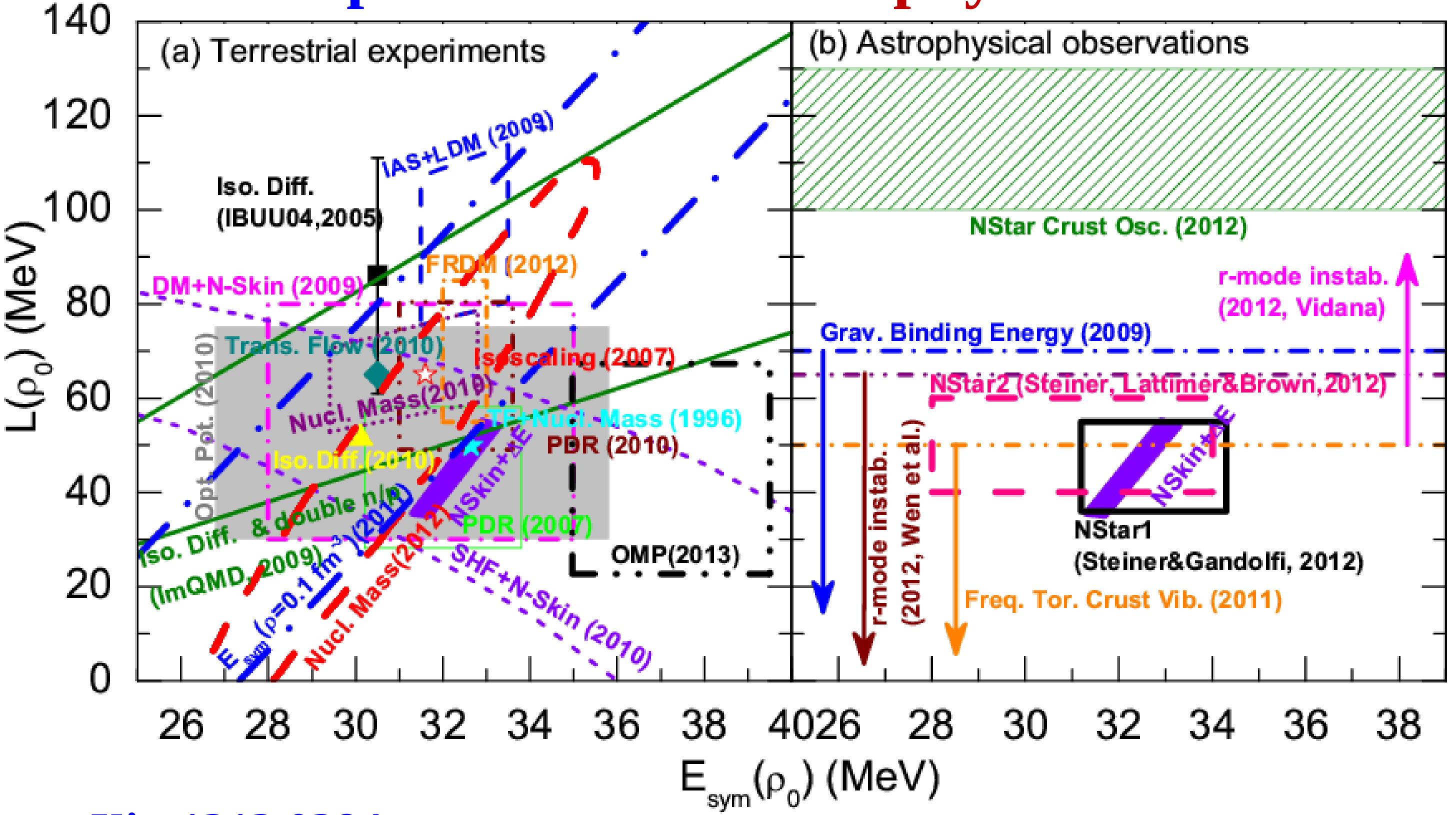
Towards high densities reachable at CSR/Lanzhou, FAIR/GSI, RIKEN, GANIL and, FRIB/MSU (高密度行为)

- π^-/π^+ ratio, K^+/K^0 ratio?
- Neutron-proton differential transverse flow
- n/p ratio at mid-rapidity
- Nucleon elliptical flow at high transverse momenta
- n/p ratio of squeeze-out emission

B.A. Li, L.W. Chen, C.M. Ko
Phys. Rep. 464, 113(2008)

E_{sym}: Around saturation density

Current constraints (totally 24) on E_{sym} (ρ_0) and L from terrestrial experiments and astrophysical observations



L.W. Chen, arXiv:1212.0284
B.A. Li, L.W. Chen, F.J. Fattoyev, W.G. Newton, and C. Xu, arXiv:1212.1178

n + A elastic scattering (This talk)
$E_{sym}(\rho_0)$=37.24 ± 2.26 MeV
L=44.98 ± 22.31 MeV

NSkin+ΔE(This talk)
$E_{sym}(\rho_0)$=32.3 ± 1.0 MeV
L=45.2 ± 10.0 MeV

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E_{sym}: Around saturation density

The current constraints on Esym are strongly model dependent, even around saturation density!!!

- Is there a general principle at some level, independent of the interaction and many-body theory, telling us what determines the $E_{sym}(\rho_0)$ and L?

- If possible, how to constrain separately each component of $E_{sym}(\rho_0)$ and L?

$E_{sym}(\rho)$ and $L(\rho)$ can be decomposed in terms of nucleon potential in asymmetric nuclear matter which can be extracted from Optical Model Potential from N-nucleus scattering

C. Xu, B.A. Li, L.W. Chen and C.M. Ko, NPA 865, 1 (2011)
C. Xu, B.A. Li, and L.W. Chen, PRC82, 054607 (2010)
R. Chen, B.J. Cai. L.W. Chen, B.A. Li, X.H. Li, and C. Xu, PRC85, 024305 (2012)
X.H. Li, B.J. Cai. L.W. Chen, R. Chen, B.A. Li, and C. Xu, PLB721, 101 (2013)

Decomposition of the Esym and L according to the Hugenholtz-Van Hove (HVH) theorem

C. Xu, B.A. Li, L.W. Chen and C.M. Ko, NPA 865, 1 (2011)
C. Xu, B.A. Li, and L.W. Chen, PRC82, 054607 (2010)
R. Chen, B.J. Cai. L.W. Chen, B.A. Li, X.H. Li, and C. Xu, PRC85, 024305 (2012).

$$U_\tau(\rho,\delta,k) = U_0(\rho,k) + \sum_{i=1,2,\cdots} U_{sym,i}(\rho,k)(\tau\delta)^i$$

$\tau = 1$ for neutrons and -1 for protons

$$= U_0(\rho,k) + U_{sym,1}(\rho,k)(\tau\delta)$$ ← The Lane potential (Symmetry potential)

$$+U_{sym,2}(\rho,k)(\tau\delta)^2 + \cdots,$$ ← Higher order in isospin asymmetry

$$t(k_{F_n}) + U_n(\rho,\delta,k_{F_n}) = \frac{\partial\varepsilon(\rho,\delta)}{\partial\rho_n}$$

$$t(k_{F_p}) + U_p(\rho,\delta,k_{F_p}) = \frac{\partial\varepsilon(\rho,\delta)}{\partial\rho_p}$$

Hugenholtz-Van Hove theorem

N. M. Hugenholtz, L. Van Hove, Physica **24**, 363 (1958)

$$E_{sym}(\rho) = \frac{1}{3}\frac{\hbar^2k^2}{2m_0^*}|_{k_F} + \frac{1}{2}U_{sym,1}(\rho,k_F),$$

K. A. Brueckner and J. Dabrowski, Phys. Rev. 134, B722 (1964)

$$L(\rho) = \frac{2}{3}\frac{\hbar^2k^2}{2m_0^*}|_{k_F} - \frac{1}{6}\left(\frac{\hbar^2k^3}{m_0^{*2}}\frac{\partial m_0^*}{\partial k}\right)|_{k_F} + \frac{3}{2}U_{sym,1}(\rho,k_F) + \frac{\partial U_{sym,1}}{\partial k}|_{k_F}\cdot k_F + 3U_{sym,2}(\rho,k_F),$$

$$m_0^*(\rho,k) = \frac{m}{1+\frac{m}{\hbar^2k}\frac{\partial U_0(\rho,k)}{\partial k}},$$

Constraining symmetry potentials from neutron-nucleus scattering data

X.H. Li, B.J. Cai. L.W. Chen, R. Chen, B.A. Li, and C. Xu, PLB721, 101 (2013)

- Is the second-order symmetry potential $U_{sym,2}$ (ρ,p) negligibly small compared to the first-order symmetry potential $U_{sym,1}$ (ρ,p) (Lane potential)?

- Both $U_{sym,1}$ (ρ,p) and $U_{sym,2}$ (ρ,p) at saturation density can be extracted from global neutron-nucleus scattering optical potentials

V

$$\frac{{}_{s}(r)}{|r}$$

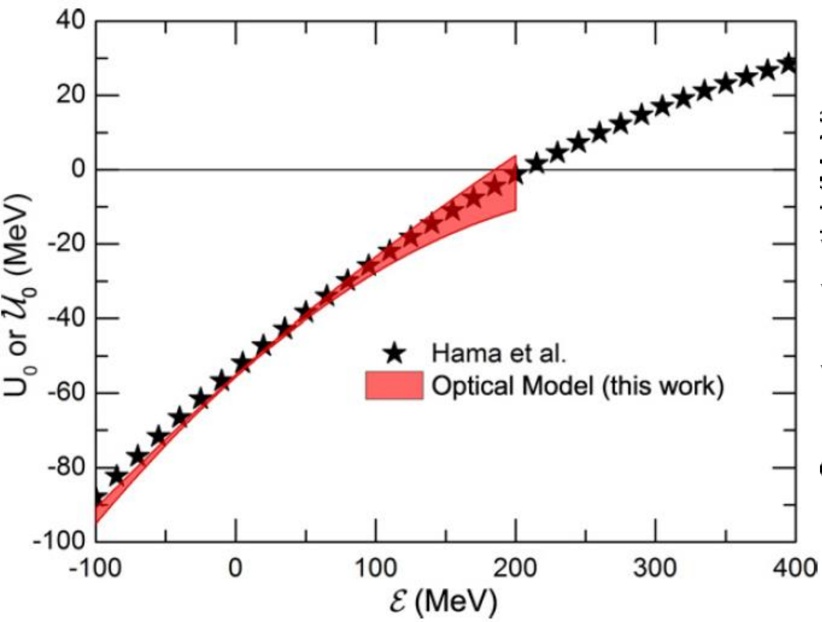
$$V_{\mathrm{v}} = V_0 + V_1\mathcal{E} + V_2\mathcal{E}^2 + (V_3 + V_{3\mathrm{L}}\mathcal{E})\frac{N-Z}{A} + (V_4 + V_{4\mathrm{L}}\mathcal{E})\frac{(N-Z)^2}{A^2}, \quad (2)$$

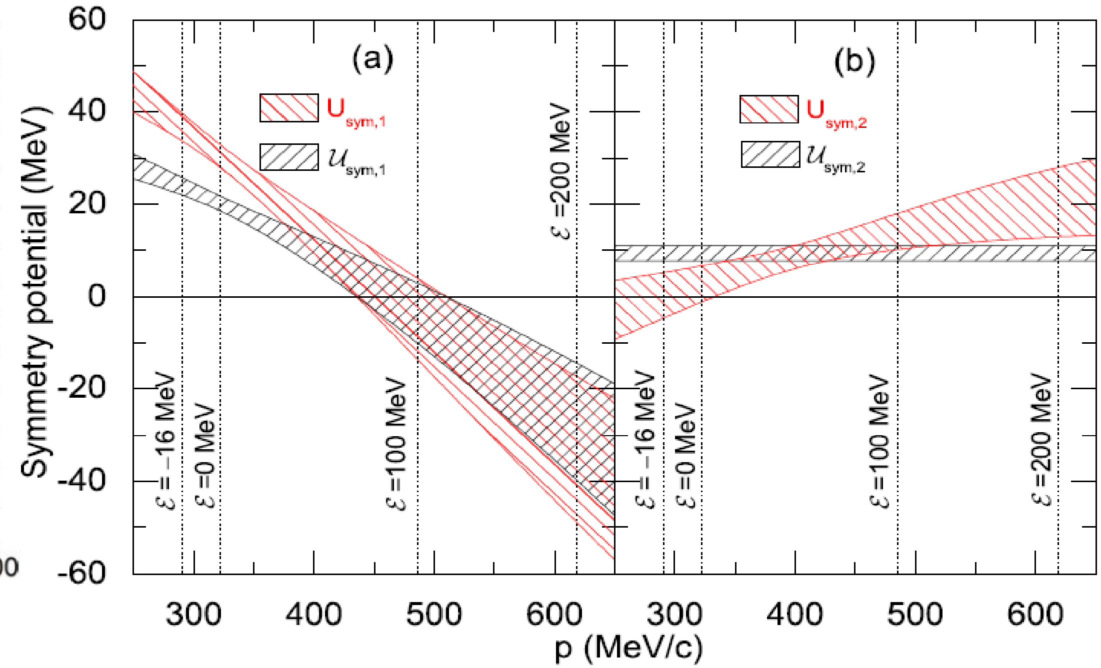
$$W_{\mathrm{s}} = W_{\mathrm{s}0} + W_{\mathrm{s}1}\mathcal{E} + (W_{\mathrm{s}2} + W_{\mathrm{s}2\mathrm{L}}\mathcal{E})\frac{N-Z}{A} + (W_{\mathrm{s}3} + W_{\mathrm{s}3\mathrm{L}}\mathcal{E})\frac{(N-Z)^2}{A^2}, \quad (3)$$

$$W_{\mathrm{v}} = W_{\mathrm{v}0} + W_{\mathrm{v}1}\mathcal{E} + W_{\mathrm{v}2}\mathcal{E}^2 + (W_{\mathrm{v}3} + W_{\mathrm{v}3\mathrm{L}}\mathcal{E})\frac{N-Z}{A} + (W_{\mathrm{v}4} + W_{\mathrm{v}4\mathrm{L}}\mathcal{E})\frac{(N-Z)^2}{A^2}, \quad (4)$$

Constraining symmetry potentials from neutron-nucleus scattering data

X.H. Li, B.J. Cai. L.W. Chen, R. Chen, B.A. Li, and C. Xu, PLB721, 101 (2013)





The second-order symmetry potential $U_{sym,2}$ (ρ,p) at saturation density is NOT so small as that we guess originally !

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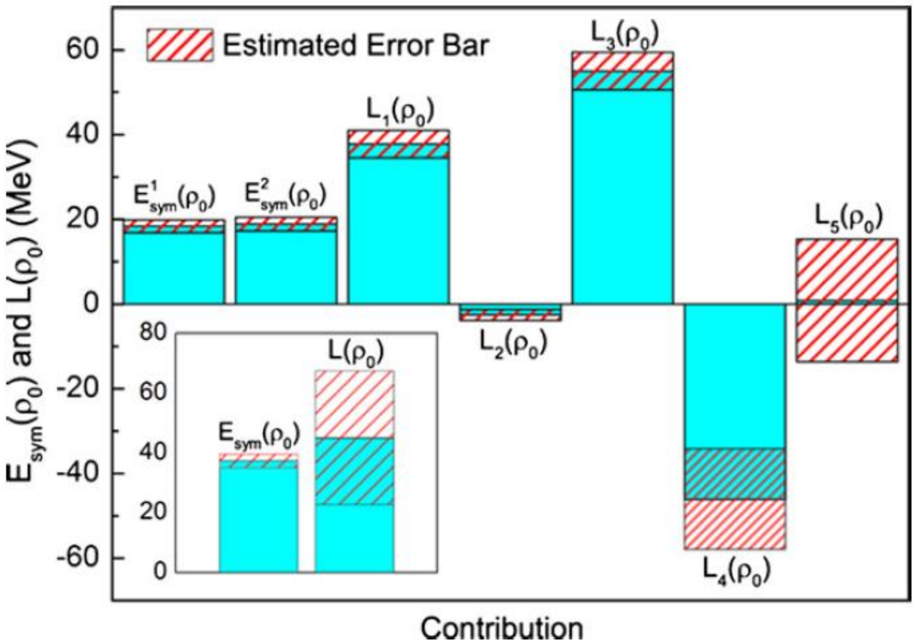
Constraints on L_n from n+A elastic scatterings

X.H. Li, B.J. Cai. L.W. Chen, R. Chen, B.A. Li, and C. Xu, PLB721, 101 (2013)



$$E_{sym}(\rho_0) = 37.24 \pm 2.26\ \text{MeV}$$
$$L(\rho_0) = 44.98 \pm 22.31\ \text{MeV}$$

$$E_1(\rho) = \frac{1}{3}\frac{\hbar^2 k_F^2}{2m_0^*(\rho, k_F)},$$

$$E_2(\rho) = \frac{1}{2}U_{\text{sym},1}(\rho, k_F)$$

$$L_1(\rho) = \frac{2}{3}\frac{\hbar^2 k_F^2}{2m_0^*(\rho, k_F)}$$

$$L_2(\rho) = -\frac{1}{6}\frac{\hbar^2 k_F^3}{m_0^{*2}(\rho, k_F)}\frac{\partial m_0^*(\rho, k)}{\partial k}|_{k_F}$$

$$L_3(\rho) = \frac{3}{2}U_{sym,1}(\rho, k_F)$$

$$L_4(\rho) = \frac{\partial U_{sym,1}(\rho, k)}{\partial k}|_{k_F} \cdot k_F$$

$$L_5(\rho) = 3U_{sym,2}(\rho, k_F).$$

The $U_{sym,2}$ (ρ_0,p) contribution to L is small ! (but with large uncertainty)

Extrapolation to negative energy (-16 MeV) from scattering state has been made, which may lead to some uncertainty on E_{sym} (ρ_0) but almost no influence on L (Dispersive OM may help?)

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Lorentz Covariant Self-energy Decomposition of the $E_{\rm sym}$ and L

B.J. Cai and L.W. Chen, PLB711, 104 (2012)

$$\Sigma^J(\rho,\alpha,|\mathbf{k}|) = \Sigma_{\rm S}^J(\rho,\alpha,|\mathbf{k}|) - \gamma_\mu \Sigma^{\mu,J}(\rho,\alpha,|\mathbf{k}|)$$
$$= \Sigma_{\rm S}^J(\rho,\alpha,|\mathbf{k}|) + \gamma^0 \Sigma_{\rm V}^J(\rho,\alpha,|\mathbf{k}|)$$
$$+ \boldsymbol{\gamma}\cdot\mathbf{k}^0 \Sigma_{\rm K}^J(\rho,\alpha,|\mathbf{k}|),$$

$$E_{\rm sym}(\rho) = \frac{|\mathbf{k}|^2}{6M^*_{0,\rm Lan}(\rho,|\mathbf{k}|)}\bigg|_{|\mathbf{k}|=k_{\rm F}} + E_{\rm sym}^{\rm 1st,K}(\rho) + E_{\rm sym}^{\rm 1st,S}(\rho) + E_{\rm sym}^{\rm 1st,V}(\rho)$$

$$L(\rho) = L^{\rm kin}(\rho) + L^{\rm mom}(\rho) + L^{\rm 1st}(\rho) + L^{\rm cross}(\rho) + L^{\rm 2nd}(\rho)$$

$$L^{\rm kin}(\rho) = \frac{k_{\rm F}k_{\rm F}^*}{6\mathcal{E}_{\rm F}^*} + \frac{k_{\rm F}^2 M_0^{*2}}{6\mathcal{E}_{\rm F}^{*3}}$$

$$L^{\rm mom}(\rho)$$
$$= \frac{k_{\rm F}^2 M_0^{*2}}{3\mathcal{E}_{\rm F}^{*2}}\frac{\partial \Sigma_{\rm K}^0}{\partial|\mathbf{k}|}\bigg|_{|\mathbf{k}|=k_{\rm F}}$$
$$+ \frac{k_{\rm F}^2}{6}\left[\frac{k_{\rm F}^*}{\mathcal{E}_{\rm F}^*}\frac{\partial^2 \Sigma_{\rm K}^0}{\partial|\mathbf{k}|^2} + \frac{M_0^*}{\mathcal{E}_{\rm F}^*}\frac{\partial^2 \Sigma_{\rm S}^0}{\partial|\mathbf{k}|^2} + \frac{\partial^2 \Sigma_{\rm V}^0}{\partial|\mathbf{k}|^2}\right]_{|\mathbf{k}|=k_{\rm F}}$$
$$+ \frac{k_{\rm F}}{6}\left[\frac{k_{\rm F}^*}{\mathcal{E}_{\rm F}^*}\frac{\partial \Sigma_{\rm K}^0}{\partial|\mathbf{k}|} + \frac{M_0^*}{\mathcal{E}_{\rm F}^*}\frac{\partial \Sigma_{\rm S}^0}{\partial|\mathbf{k}|} + \frac{\partial \Sigma_{\rm V}^0}{\partial|\mathbf{k}|}\right]_{|\mathbf{k}|=k_{\rm F}}$$
$$+ \frac{k_{\rm F}^2}{6\mathcal{E}_{\rm F}^{*3}}\left[M_0^{*2}\left(\frac{\partial \Sigma_{\rm K}^0}{\partial|\mathbf{k}|}\right)^2 + k_{\rm F}^{*2}\left(\frac{\partial \Sigma_{\rm S}^0}{\partial|\mathbf{k}|}\right)^2\right]_{|\mathbf{k}|=k_{\rm F}}$$
$$- \frac{k_{\rm F}^2 k_{\rm F}^* M_0^*}{3\mathcal{E}_{\rm F}^{*3}}\left[\frac{\partial \Sigma_{\rm S}^0}{\partial|\mathbf{k}|}\left(1 + \frac{\partial \Sigma_{\rm K}^0}{\partial|\mathbf{k}|}\right)\right]_{|\mathbf{k}|=k_{\rm F}},$$

$$L^{\rm 1st}(\rho)$$
$$= \frac{3}{2\mathcal{E}_{\rm F}^{*3}}\left[M_0^*\Sigma_{\rm K}^{\rm sym,1} - k_{\rm F}^*\Sigma_{\rm S}^{\rm sym,1}\right]^2$$
$$+ \frac{3}{2}\left[\frac{k_{\rm F}^*}{\mathcal{E}_{\rm F}^*}\Sigma_{\rm K}^{\rm sym,1} + \frac{M_0^*}{\mathcal{E}_{\rm F}^*}\Sigma_{\rm S}^{\rm sym,1} + \Sigma_{\rm V}^{\rm sy}\right.$$
$$+ \frac{k_{\rm F}M_0^{*2}\Sigma_{\rm K}^{\rm sym,1}}{\mathcal{E}_{\rm F}^{*3}} - \frac{k_{\rm F}k_{\rm F}^*M_0^*\Sigma_{\rm S}^{\rm sym,1}}{\mathcal{E}_{\rm F}^{*3}},$$

$$L^{\rm cross}(\rho)$$
$$= k_{\rm F}\left[\frac{k_{\rm F}^*}{\mathcal{E}_{\rm F}^*}\frac{\partial \Sigma_{\rm K}^{\rm sym,1}}{\partial|\mathbf{k}|} + \frac{M_0^*}{\mathcal{E}_{\rm F}^*}\frac{\partial \Sigma_{\rm S}^{\rm sym,1}}{\partial|\mathbf{k}|} + \frac{\partial \Sigma_{\rm V}^{\rm sym,1}}{\partial|\mathbf{k}|}\right]_{|\mathbf{k}|=k_{\rm F}}$$
$$- \frac{k_{\rm F}\Sigma_{\rm K}^{\rm sym,1}}{\mathcal{E}_{\rm F}^*}\left[\frac{k_{\rm F}^{*2}}{\mathcal{E}_{\rm F}^{*2}}\left(\frac{\partial \Sigma_{\rm K}^0}{\partial|\mathbf{k}|} + \frac{M_0^*}{k_{\rm F}^*}\frac{\partial \Sigma_{\rm S}^0}{\partial|\mathbf{k}|}\right) - \frac{\partial \Sigma_{\rm K}^0}{\partial|\mathbf{k}|}\right]_{|\mathbf{k}|=k_{\rm F}}$$
$$- \frac{k_{\rm F}\Sigma_{\rm S}^{\rm sym,1}}{\mathcal{E}_{\rm F}^*}\left[\frac{M_0^{*2}}{\mathcal{E}_{\rm F}^{*2}}\left(\frac{k_{\rm F}^*}{M_0^*}\frac{\partial \Sigma_{\rm K}^0}{\partial|\mathbf{k}|} + \frac{\partial \Sigma_{\rm S}^0}{\partial|\mathbf{k}|}\right) - \frac{\partial \Sigma_{\rm S}^0}{\partial|\mathbf{k}|}\right]_{|\mathbf{k}|=k_{\rm F}}$$

$$L^{\rm 2nd}(\rho) = 3\left[\frac{k_{\rm F}^*}{\mathcal{E}_{\rm F}^*}\Sigma_{\rm K}^{\rm sym,2} + \frac{M_0^*}{\mathcal{E}_{\rm F}^*}\Sigma_{\rm S}^{\rm sym,2} + \Sigma_{\rm V}^{\rm sym,2}\right].$$

Lorentz covariant nucleon self-energy can be obtained from Dirac phenomenology, QCD sum rules (KS Jeoug/SH Lee, EG Drukarev),……

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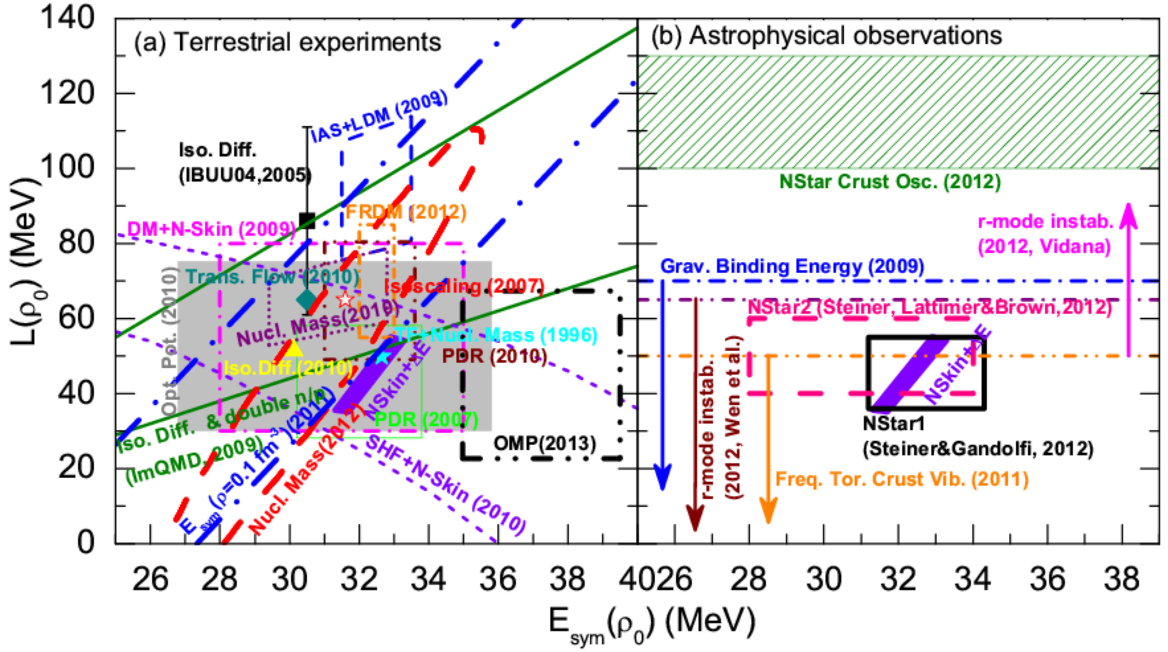
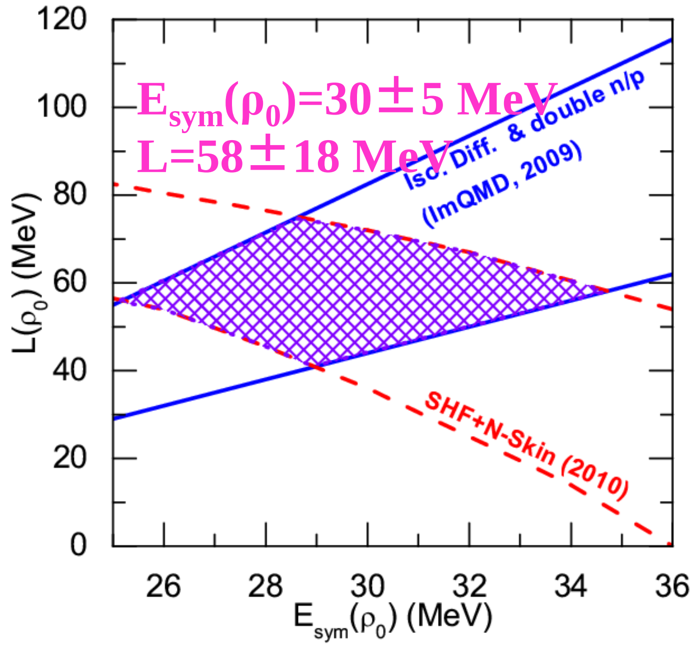
E_{sym}: Around saturation density

Current constraints (totally 24) on E_{sym} (ρ_0) and L from terrestrial experiments and astrophysical observations





Chen/Ko/Li/Xu, PRC82, 024321(2010)

Neutron skin constraint leads to a negative E_{sym}-L correlation!!!

But why???

The nskin is directly correlated with L(0.11 fm^{-3}) which leads to a negative E_{sym}-L correlation (at saturation density)

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Correlation analysis using macroscopic quantity input in Nuclear Energy Density Functional

Standard Skyrme Interaction:

$$
\begin{aligned}
V_{12}(\mathbf{R},\mathbf{r}) &= t_0(1+x_0P_\sigma)\delta(\mathbf{r}) \\
&+ \frac{1}{6}t_3(1+x_3P_\sigma)\rho^\sigma(\mathbf{R})\delta(\mathbf{r}) \\
&+ \frac{1}{2}t_1(1+x_1P_\sigma)(K'^2\delta(\mathbf{r})+\delta(\mathbf{r})K^2) \\
&+ t_2(1+x_2P_\sigma)\mathbf{K}'\cdot\delta(\mathbf{r})\mathbf{K} \\
&+ iW_0\mathbf{K}'\cdot\delta(\mathbf{r})[(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\times\mathbf{K}],
\end{aligned}
$$

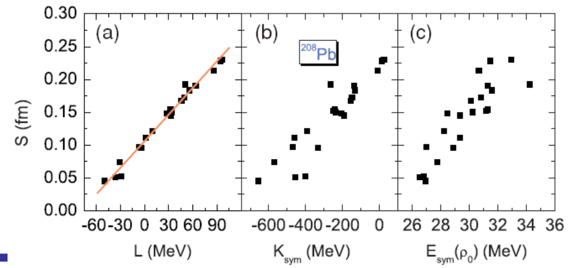
There are more than 120 sets of Skyrme-like Interactions in the literature

Agrawal/Shlomo/Kim Au PRC72, 014310 (2005)

with $\mathbf{r}=\mathbf{r}_1-\mathbf{r}_2$ and $\mathbf{R}=(\mathbf{r}_1+\mathbf{r}_2)/2$. In the above, the relative momentum operators $\mathbf{K}=(\nabla_1-\nabla_2)/2i$ and $\mathbf{K}'=-(\nabla_1-\nabla_2)/2i$ act on the wave function on the right and left, respectively. The quantities P_σ and σ_i denote, respectively, the spin exchange operator and Pauli spin matrices. The $\sigma, t_0-t_3, x_0-x_3, W_0$ are Skyrme interaction parameters that are chosen to fit the binding energies and radii of large number of nuclei in the periodic table.

Chen/Ko/Li/Xu PRC82, 024321(2010)

Yoshida/Sagawa PRC73, 044320 (2006)



9 Skyrme parameters: $\sigma,\ t_0-t_3,\ \text{and}\ \ x_0-x_3$

9 macroscopic nuclear properties:

$$\mathcal{H}_{fin}=\frac{G_S}{2}(\nabla\rho)^2-\frac{G_V}{2}(\nabla\rho_3)^2$$

$$\rho_0,\ E_0(\rho_0),\ K_0,\ m^*_{s,0},\ m^*_{v,0},\ E_{\rm sym}(\rho_0),\ L,\ G_S,\ \text{and}\ G_V$$

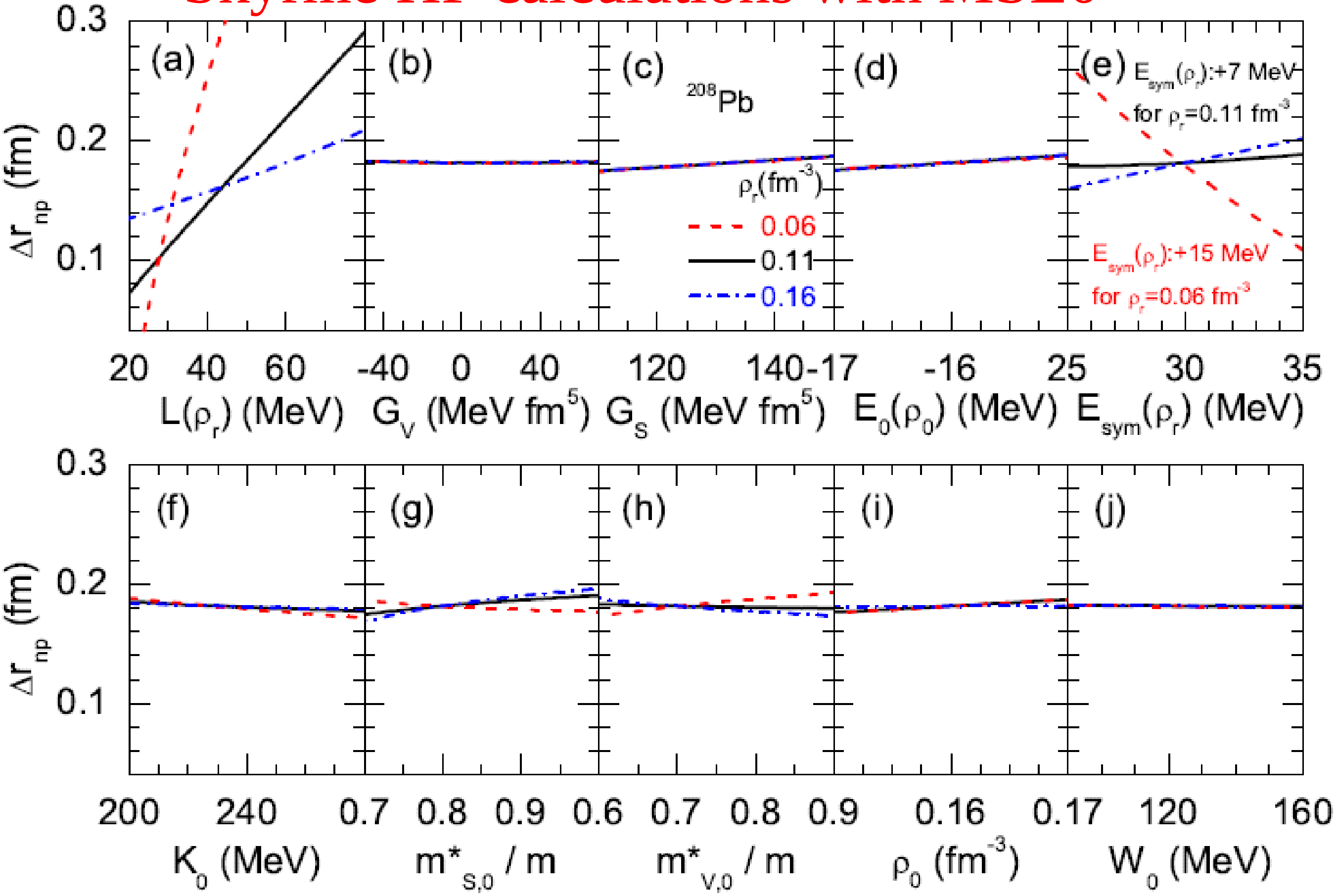
上海交通大學 SHANGHAI JIAO TONG UNIVERSITY

What really determine NSKin?

Zhen Zhang and Lie-Wen Chen ,arXiv:1302.5327

$$E_{\rm sym}(\rho) = E_{\rm sym}(\rho_r) + \frac{L(\rho_r)}{3}\chi_r + O(\chi_r^2)$$

Skyrme HF calculations with MSL0



- Neutron skin always increases with $L(\rho_r)$, but it can increase or decrease with $E_{sym}(\rho_r)$ depending on ρ_r

- When ρ_r =0.11 fm^{-3}, the neutron skin is essentailly only sensitive to $L(\rho_r)$!!!

The neutron skin of heavy nuclei ⟺ **$L(\rho_r)$ at ρ_r =0.11 fm^{-3}**

Determine L(0.11 fm^{-3}) from NSkin

Zhen Zhang and Lie-Wen Chen ,arXiv:1302.5327

- For a fixed L(0.11fm^{-3}) we

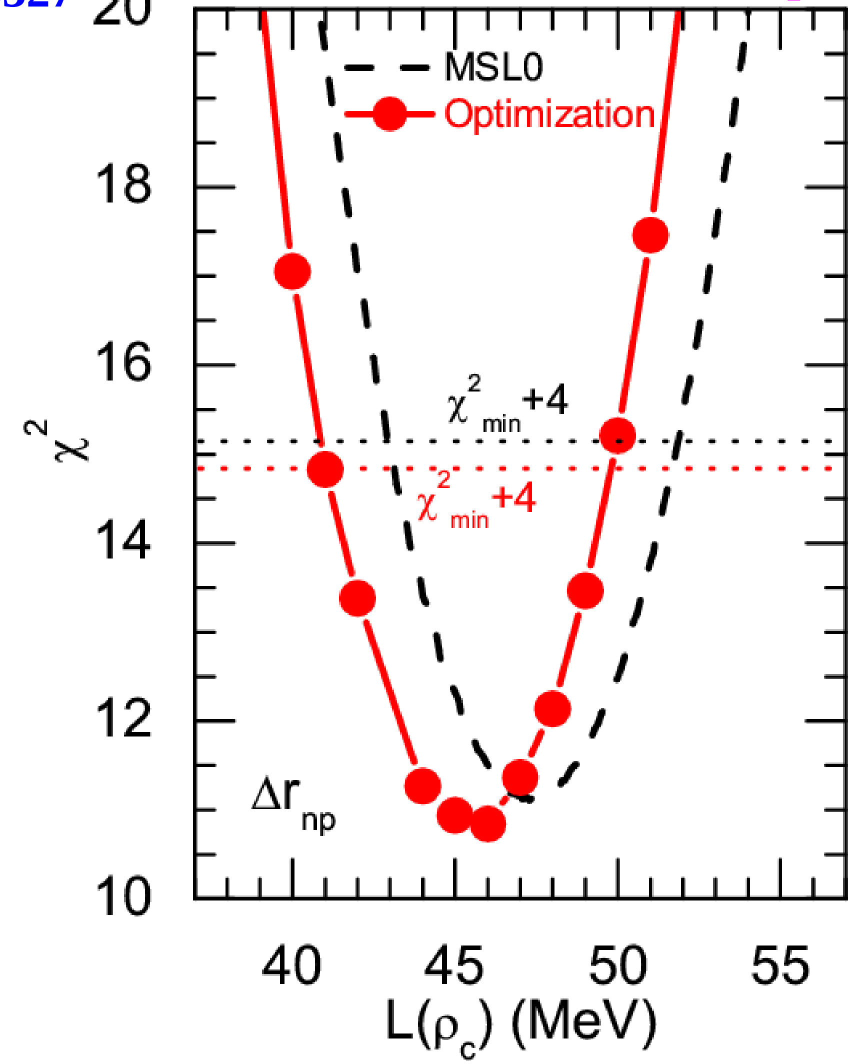
$$\chi^2_{op} = \chi^2_{EB} + \chi^2_{Rc} + \chi^2_{dE}$$

minimize to optimize other 9 MSL parameters.

- Result with the optimization is very close to that only fixing other parameters at MSL0

$$2\sigma : L(0.11\ \mathrm{fm}^{-3}) = 46.0 \pm 4.5\ \mathrm{MeV}$$

21 data of NSKin of Sn Isotope



What really determine ΔE?

$$E(N,Z) = a_{\rm vol} + a_{\rm surf}A^{-1/3} + a_{\rm sym}(A)\left(\frac{N-Z}{A}\right)^2 + a_{\rm Coul}\frac{Z(Z-1)}{A^{4/3}} + E_{\rm pair}.$$

Isotope binding energy difference (spherical even-even isotope pairs)

$$\Delta E = E(N+\Delta N, Z) - E(N,Z) \approx a_{\rm sym}(A)(2N-2Z+\Delta N)\Delta N/A^2 - a_{\rm Coul}\frac{Z(Z-1)}{A^{4/3}} \times \frac{4\Delta N}{3A}$$

($a_{sym}(A)$): Symmetry energy of finite nuclei)

$a_{sym}(A) \approx E_{sym}(\rho_c)$ with $\rho_c \approx 0.11$ fm^{-3} for heavy nuclei

$E_{sym}(\rho_c)$ at $\rho_c \approx 0.11$ fm^{-3}

M. Centelles et al., PRL102, 122502 (2009)
L.W. Chen, PRC83, 044308 (2011)

Binding energy difference of heavy isotope pair ⟺ $E_{sym}(\rho_c)$ at $\rho_c = 0.11$ fm^{-3}

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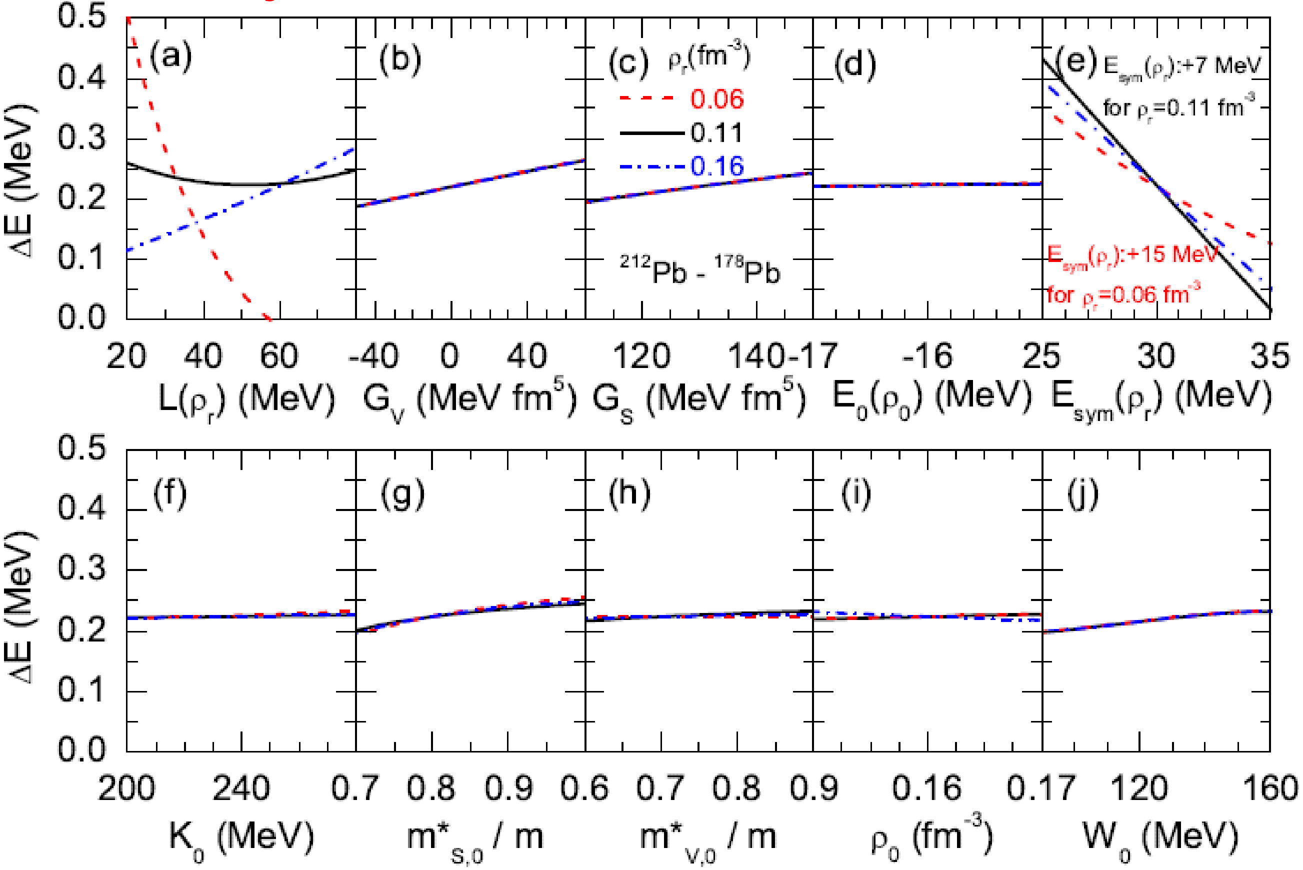
What really determine ΔE?

Zhen Zhang and Lie-Wen Chen ,
arXiv:1302.5327

Skyrme HF calculations with MSL0



- ΔE always decreases with $E_{sym}(\rho_r)$, but it can increase or decrease with $L(\rho_r)$ depending on ρ_r

- When ρ_r =0.11 fm^{-3}, ΔE is mainly sensitive to $E_{sym}(\rho_r)$!!!

Binding energy difference of heavy isotope pair ⟺ **$E_{sym}(\rho_c)$ at ρ_c =0.11 fm^{-3}**

Determine $E_{sym}(0.11\ fm^{-3})$ from ΔE

Zhen Zhang and Lie-Wen Chen ,arXiv:1302.5327

- For fixed $E_{sym}(0.11\mathrm{fm}^{-3})$ we minimize

$$\chi^2_{op} = \chi^2_{EB} + \chi^2_{Rc} + \chi^2_{NSkin}$$

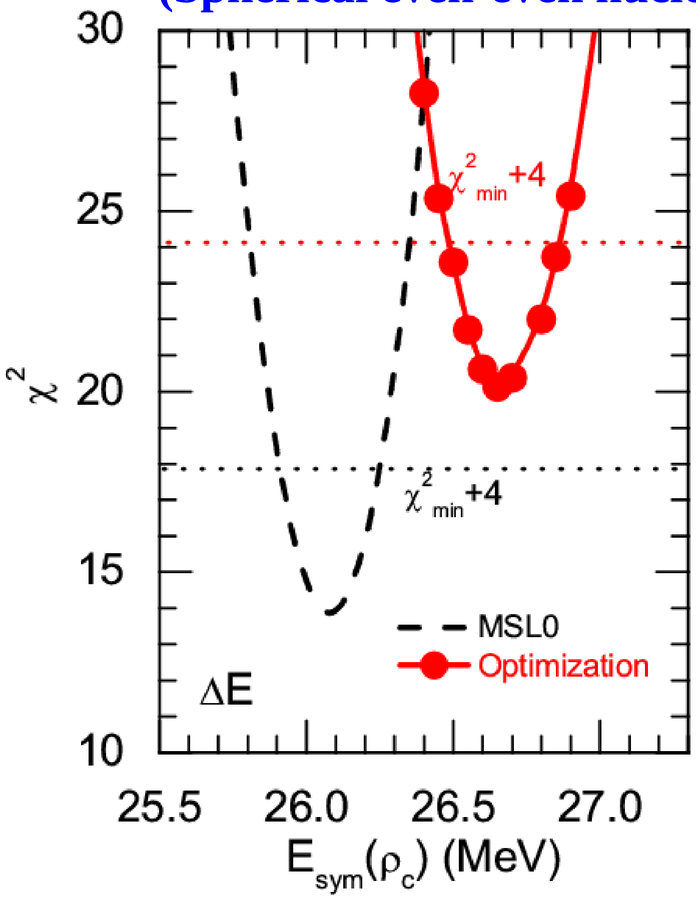
to optimize other 9 MSL parameters.

- With this set of parameters, we can calculate

$$\chi^2_{dE} = \sum_{i=1}^{12} \left(\frac{\Delta E_i^{exp} - \Delta E_i^{th}}{\sigma_i} \right)^2$$

and the result is shown in right figure.

19 data of Heavy Isotope Pairs
(Spherical even-even nuclei)



$$2\sigma : E_{sym}(0.11\ fm^{-3}) = 26.65 \pm 0.20\ MeV$$

Here, the χ^2 is not real since we use a theoretical error (The model is not good).

Our strategy: select a theoretical error (23%) to satisfy $\chi^2 / dof \sim 1$

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Symmetry energy around 0.11 fm^{-3}

The globally optimized parameters (MSL1)

TABLE I: Skyrme parameters in MSL1 (left side) and some corresponding nuclear properties (right side).

Quantity	MSL1	Quantity	MSL1
t_0 (MeV·fm^3)	−1963.23	ρ_0 (fm^{-3})	0.1586
t_1 (MeV·fm^5)	379.845	E_0 (MeV)	−15.998
t_2 (MeV·fm^5)	−394.554	K_0 (MeV)	235.12
t_3 (MeV·fm$^{3+3\sigma}$)	12174.9	$m^*_{s,0}/m$	0.806
x_0	0.320770	$m^*_{v,0}/m$	0.706
x_1	0.344849	$E_{sym}(\rho_c)$ (MeV)	26.67
x_2	−0.847304	$L(\rho_c)$ (MeV)	46.19
x_3	0.321930	G_S (MeV·fm^5)	126.69
σ	0.269359	G_V (MeV·fm^5)	68.74
W_0 (MeV·fm^5)	113.62	$E_{sym}(\rho_0)$ (MeV)	32.33
		$L(\rho_0)$ (MeV)	45.25

Zhen Zhang and Lie-Wen Chen
arXiv:1302.5327

Binding energy difference of heavy isotope pairs

$$E_{sym}(0.11\ fm^{-3}) = 26.65 \pm 0.2\ MeV$$

$$E_{sym}(0.11\ fm^{-3}) = 26.2 \pm 1.0\ MeV$$

Wang/Ou/Liu, PRC87, 034327 (2013)

(Fermi Energy Difference of Nuclei)

The neutron skin of Sn isotopes

$$L(0.11\ fm^{-3}) = 46.0 \pm 4.5\ MeV$$

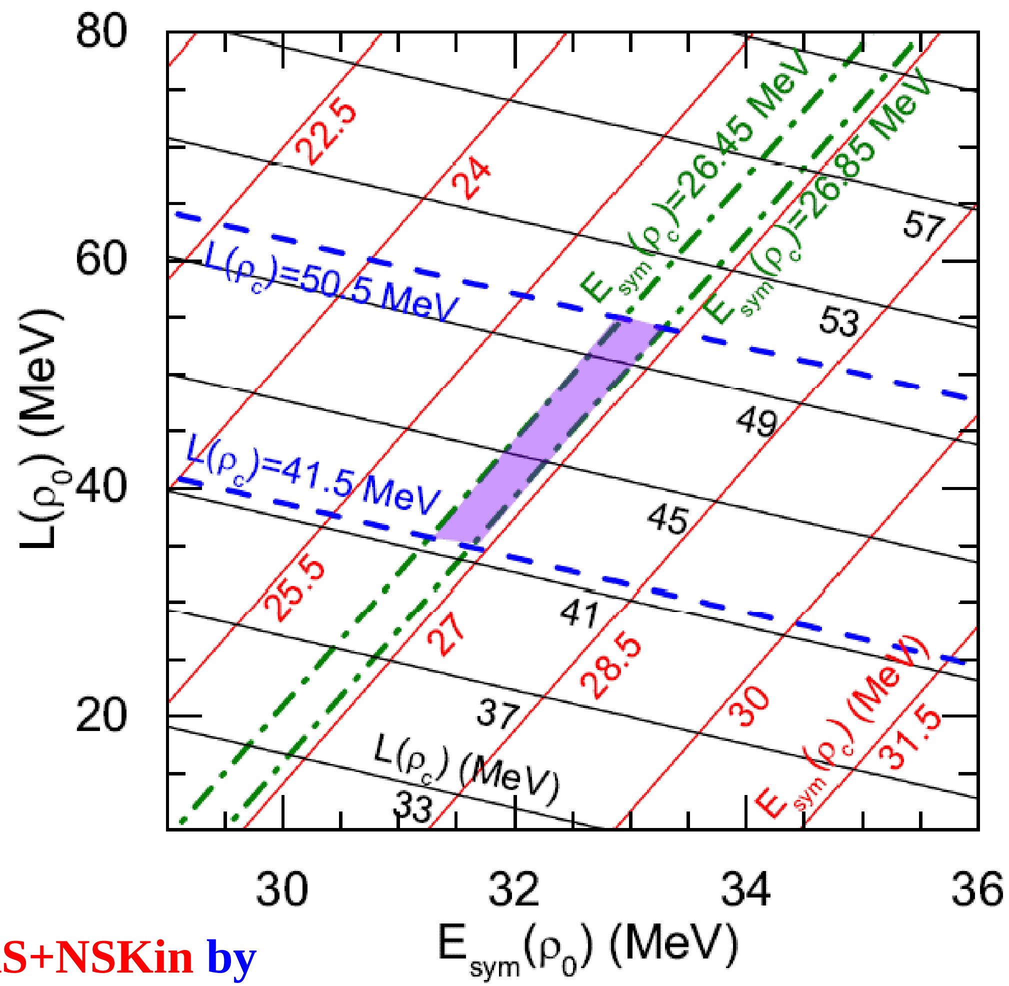
Extrapolation to ρ_0

A fixed value of $E_{sym}(\rho_c)$ at ρ_c =0.11 fm^{-3} leads to a positive $E_{sym}(\rho_0)$ -L correlation
A fixed value of $L(\rho_c)$ at ρ_c =0.11 fm^{-3} leads to a negative $E_{sym}(\rho_0)$ -L correlation

Zhen Zhang and Lie-Wen Chen
arXiv:1302.5327

- With the 8 parameters fixed, L and $E_{sym}(\rho_0)$ can be determined by $L(0.11\mathrm{fm}^{-3})$ and $E_{sym}(0.11\mathrm{fm}^{-3})$.

$$E_{sym}(\rho_0) = 32.3 \pm 1.0 \text{ MeV}$$
$$L(\rho_0) = 45.2 \pm 10.0 \text{ MeV}$$



Nicely agree with the constraints from IAS+NSKin by P. Danielewicz; IsospinD+n/p by Y Zhang and ZX Li

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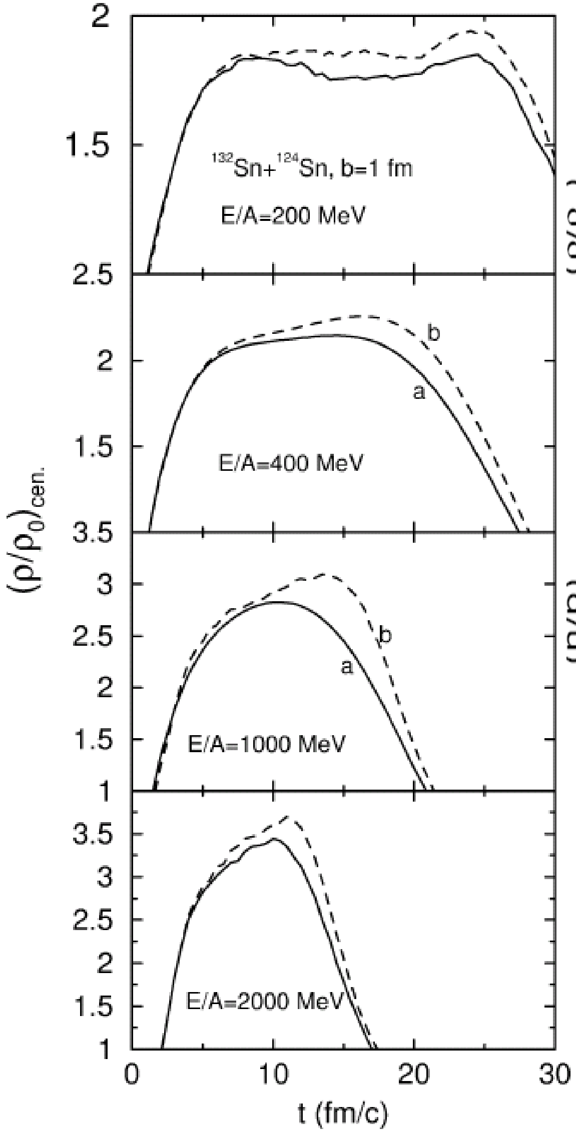
Outline

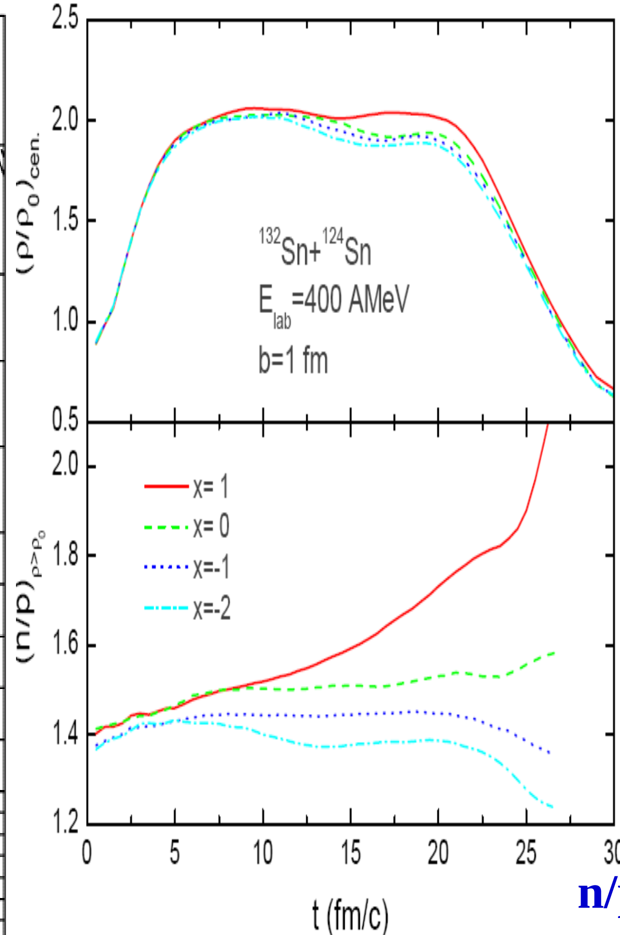
- The symmetry energy
- Current constraints on the symmetry energy
  - n-A elastic scattering and the symmetry potential
  - Symmetry energy at 0.11 fm^{-3}
  - High density behaviors
- Density curvature K_{sym} and the high density symmetry energy
- Summary and outlook

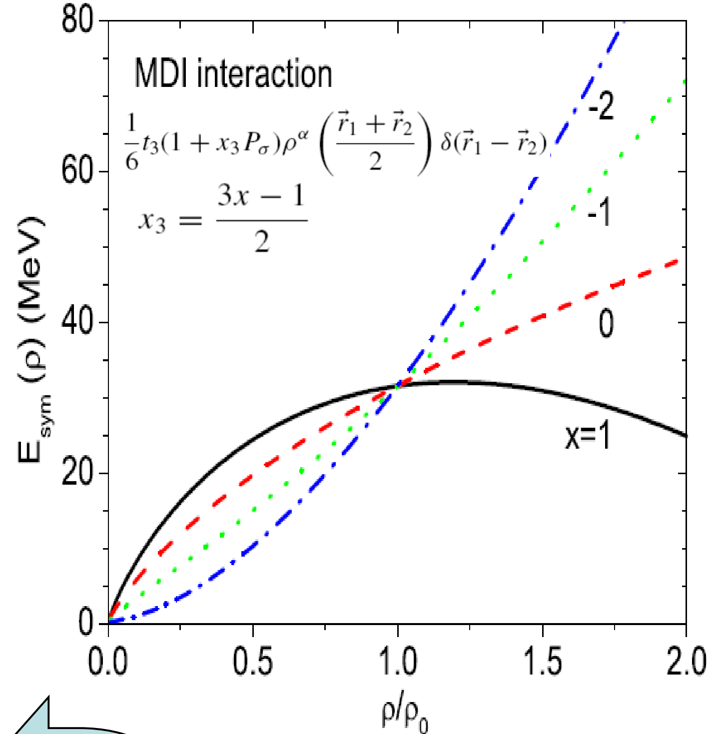
High Density Behaviors of E_{sym}: HIC

Heavy-Ion Collisions at Higher Energies

IBUU simu.







n/p ratio of the high density region

Isospin fractionation!

Xu/Tsang et al. PRL85, 716 (2000)
B.A. Li, PRL88, 192701(2002)

Particle Production in HIC

Besides protons, neutrons, deutrons, tritons, 3He, 4He, and on, the following particles can be produced in HIC at energies lower than their production threshold energies in NN collisions:

$$\pi: \quad \sim 289 \text{ MeV} \quad (NN \rightarrow NN\pi)$$

$$K^+(\Lambda,\Sigma): \quad \sim 1583 \text{ MeV} \quad (NN \rightarrow NYK^+)$$

$$K^-: \quad \sim 2513 \text{ MeV} \quad (NN \rightarrow NNK^+K^-)$$

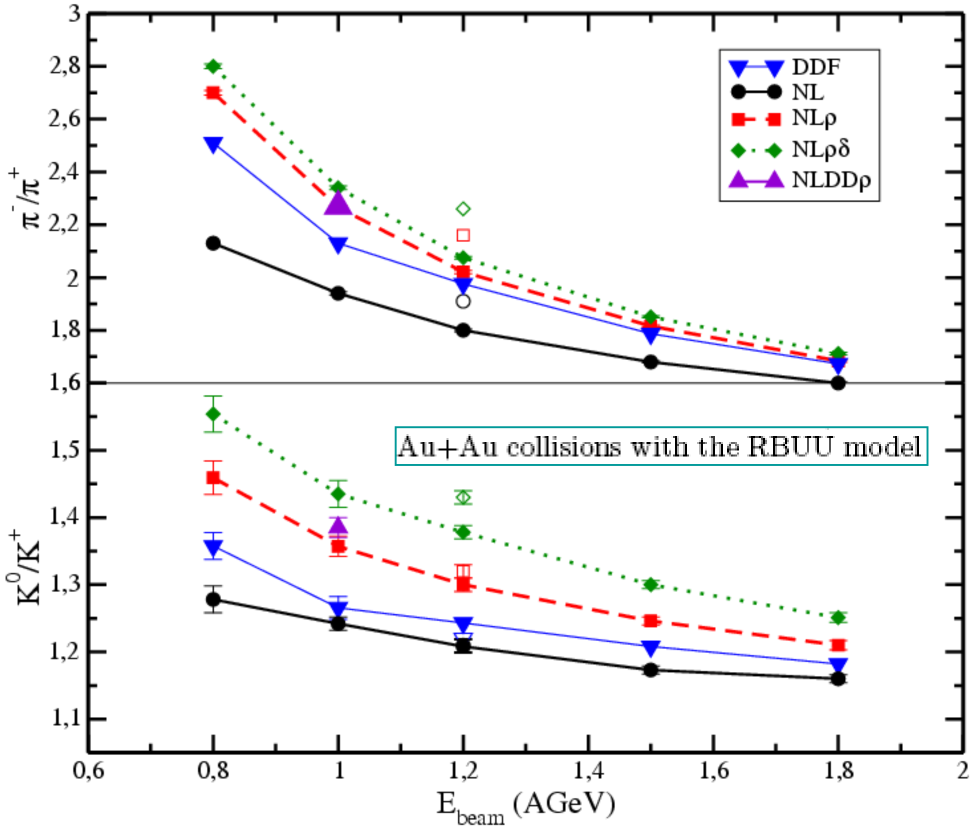
$$\Xi: \quad \sim 3740 \text{ MeV} \quad (NN \rightarrow N\Xi K^+K^+)$$

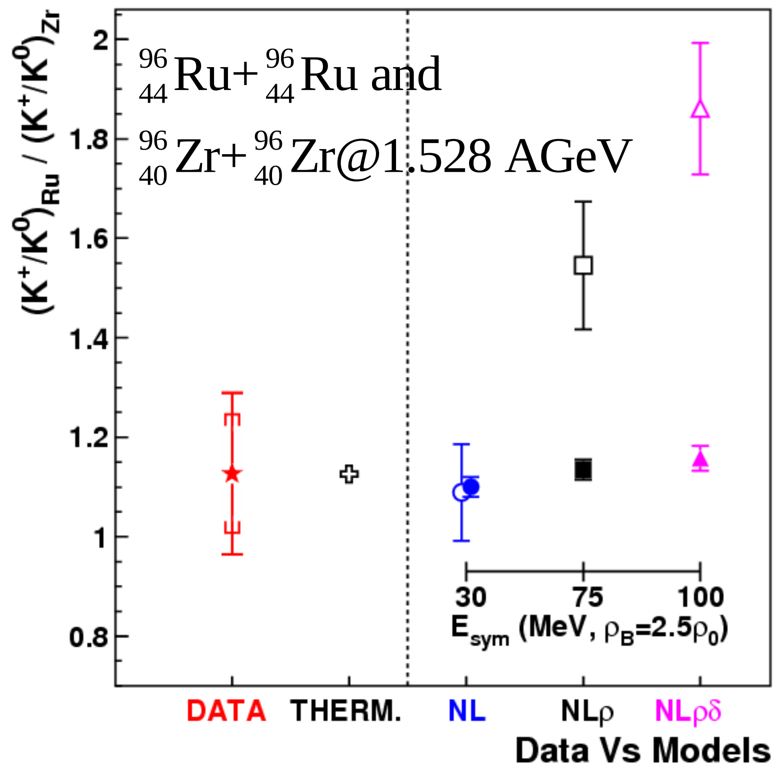
Particle subthreshold production in HIC provides an important way to explore nuclear matter EOS and hadron properties in nuclear medium

High density E_{sym}: Subthreshold kaon yield

Aichelin/Ko, PRL55, 2661 (1985): Subthreshold kaon yield is a sensitive probe of the EOS of nuclear matter at high densities (Kaons are produced mainly from the high density region and at the early stage of the reaction almost without subsequent reabsorption effects)

Theory: Famiano et al., PRL97, 052701 (2006) **Exp.: Lopez et al. FOPI, PRC75, 011901(R) (2007)**





Subthreshold K^0/K^+ yield may be a sensitive probe of the symmetry energy at high densities

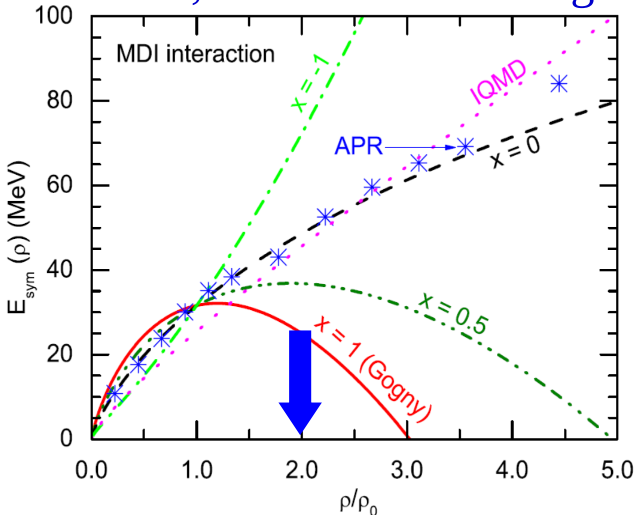
K^0/K^+ yield is not so sensitive to the symmetry energy! Lower energy and more neutron-rich system???

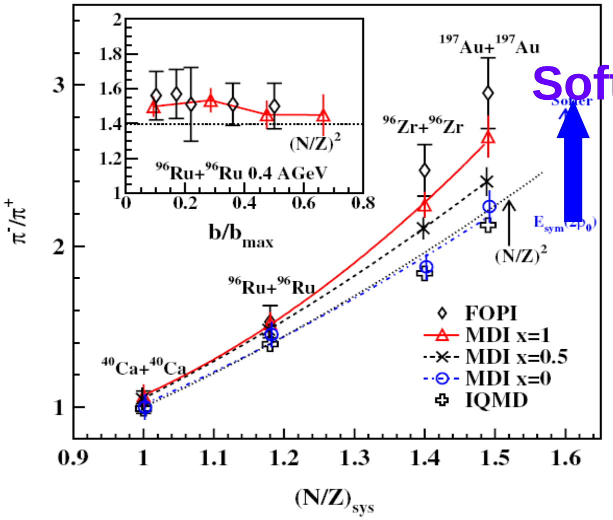
High density E_{sym}: pion ratio

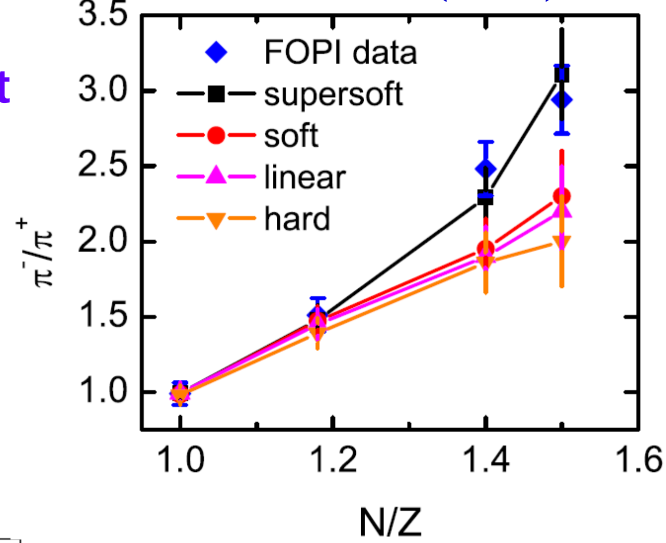
A Quite Soft Esym at supra-saturation densities ???

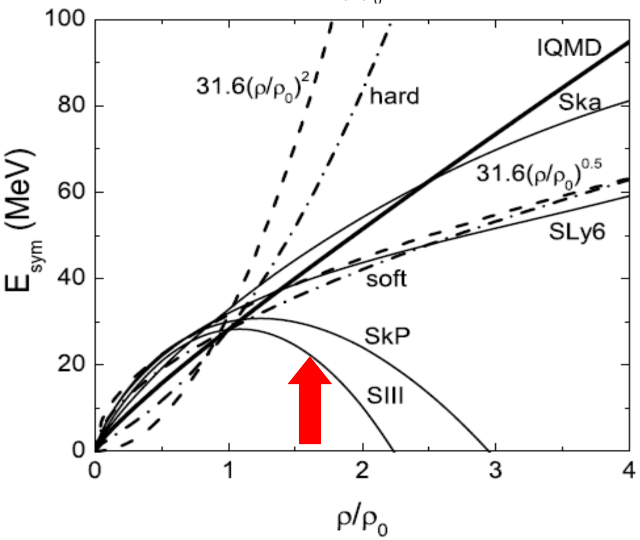
IBUU04, Xiao/Li/Chen/Yong/Zhang, PRL102,062502(2009)

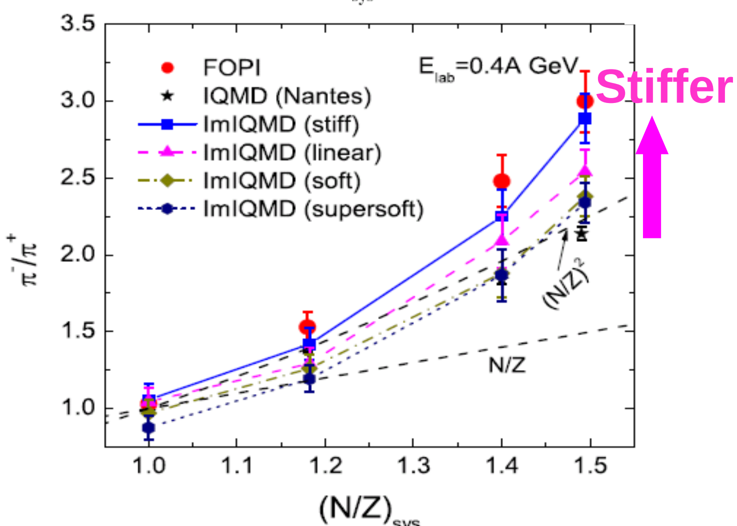
ImIBLE, Xie/Su/Zhu/Zhang, PLB718,1510(2013)











Pion Medium Effects?
Xu/Ko/Oh
PRC81, 024910(2010)

Threshold effects?
Δ resonances?
……

ImIQMD, Feng/Jin, PLB683, 140(2010)

High density E_{sym}: pion ratio

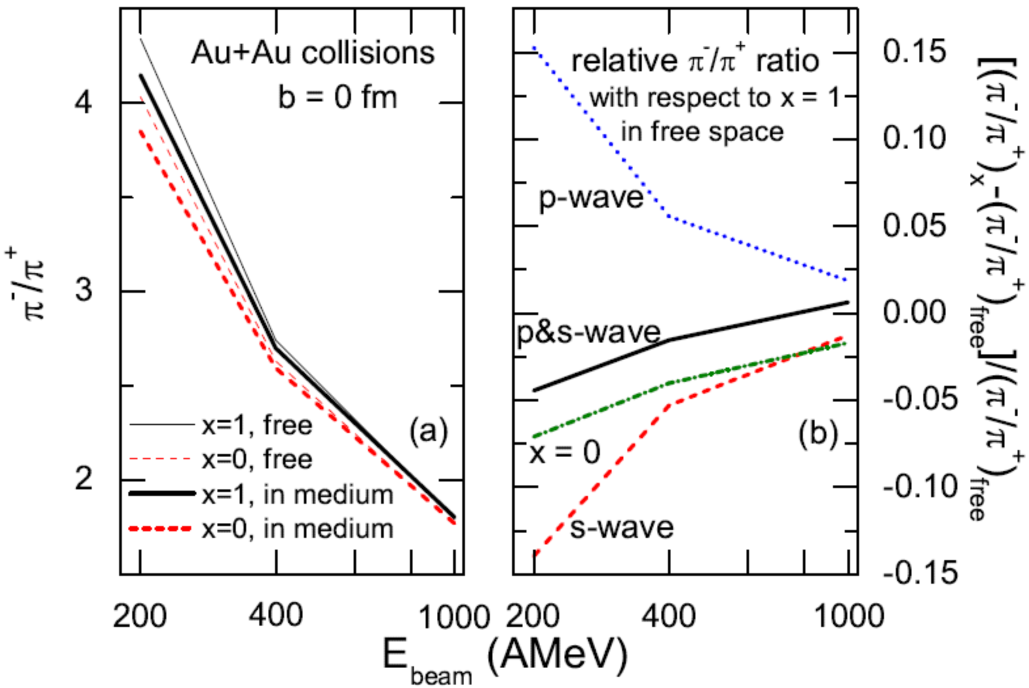
SHANGHAI JIAO TONG UNIVERSITY

Energy dependence of pion in-medium effects on π^-/π^+ ratio in heavy-ion collisions

arXiv:1305.0091
PRC, in press

Jun Xu,[1,*] Lie-Wen Chen,[2] Che Ming Ko,[3] Bao-An Li,[4,5] and Yu-Gang Ma[1]
[1] *Shanghai Institute of Applied Physics, Chinese Academy of Sciences, Shanghai 201800, China*

Within the framework of a thermal model with its parameters fitted to the results from an isospin-dependent Boltzmann-Uehling-Uhlenbeck (IBUU) transport model, we have studied the pion in-medium effect on the charged-pion ratio in heavy-ion collisions at various energies. We find that due to the cancellation between the effects from pion-nucleon s-wave and p-wave interactions in nuclear medium, the π^-/π^+ ratio generally decreases after including the pion in-medium effect. The effect is larger at lower collision energies as a result of narrower pion spectral functions at lower temperatures.



The pion in-meidum effects seem small in the thermal model !!!

But how about in more realistic dynamical model ???

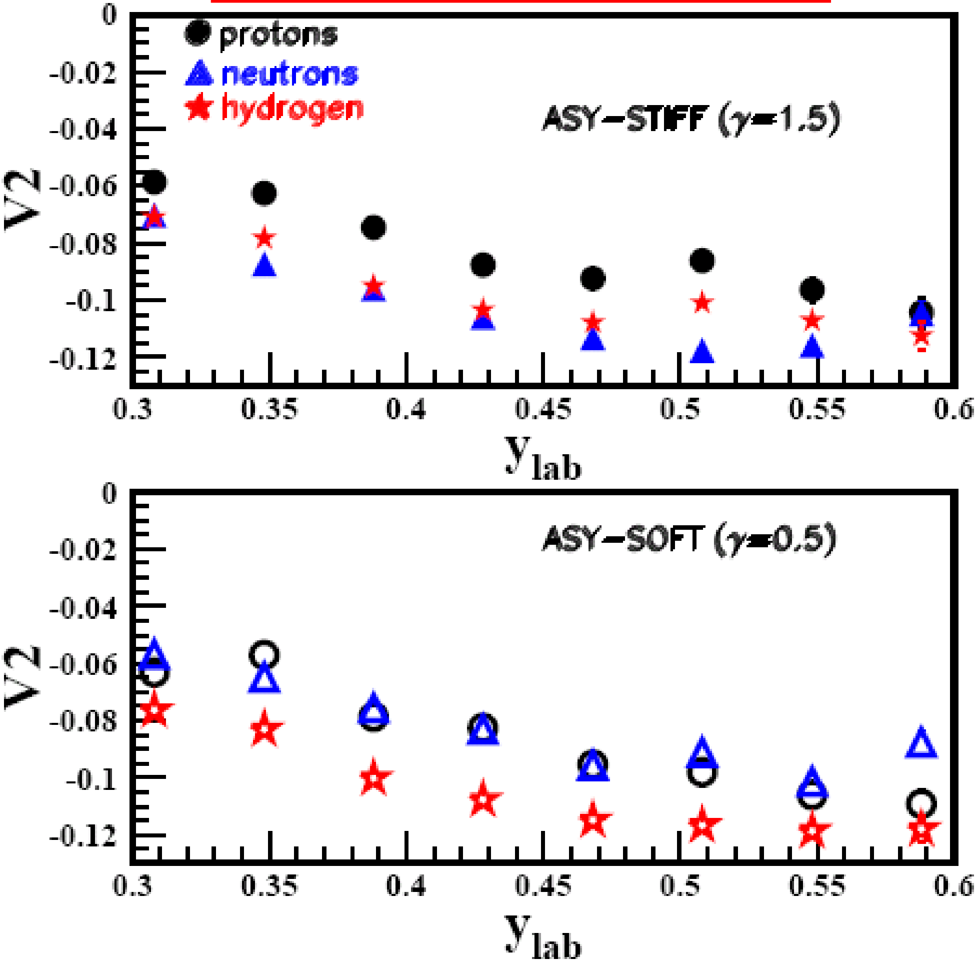
How to self-consistently teat the pion in-medium effects in transport model remains a big challenge !!!

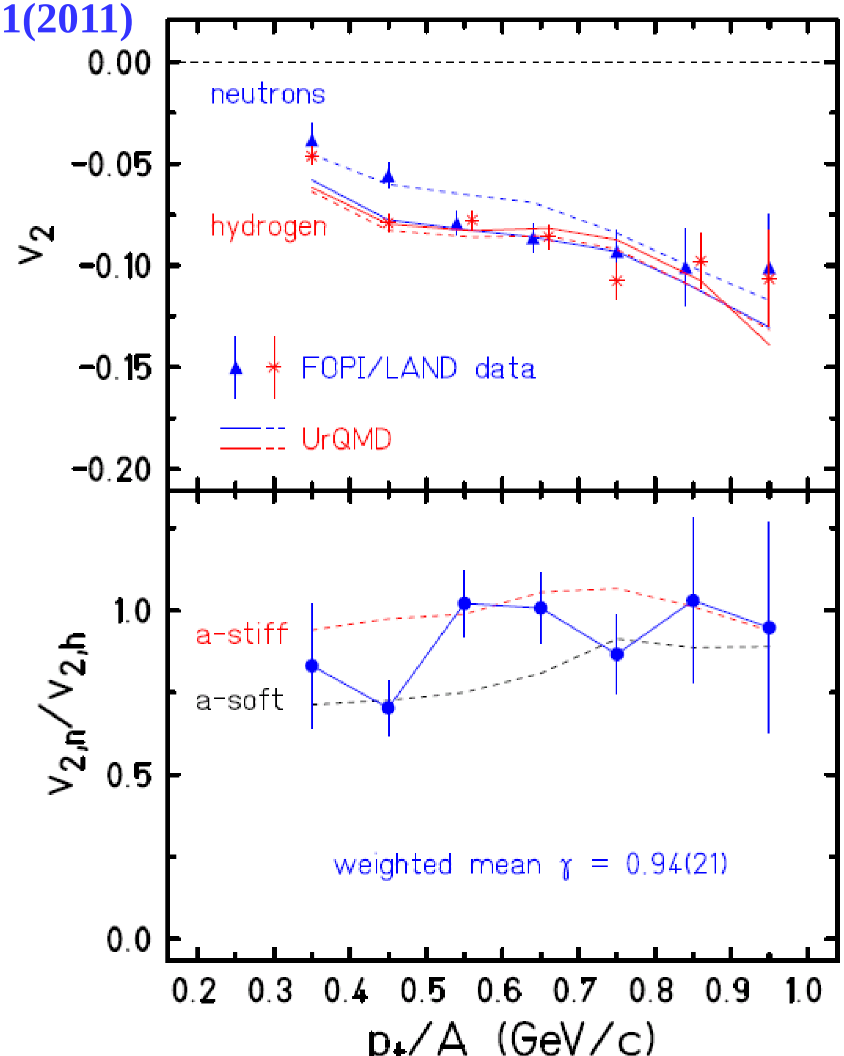
High density E_{sym}: n/p v2

A Soft or Stiff Esym at supra-saturation densities ???

P. Russotto, W. Trauntmann, Q.F. Li et al., PLB697, 471(2011)

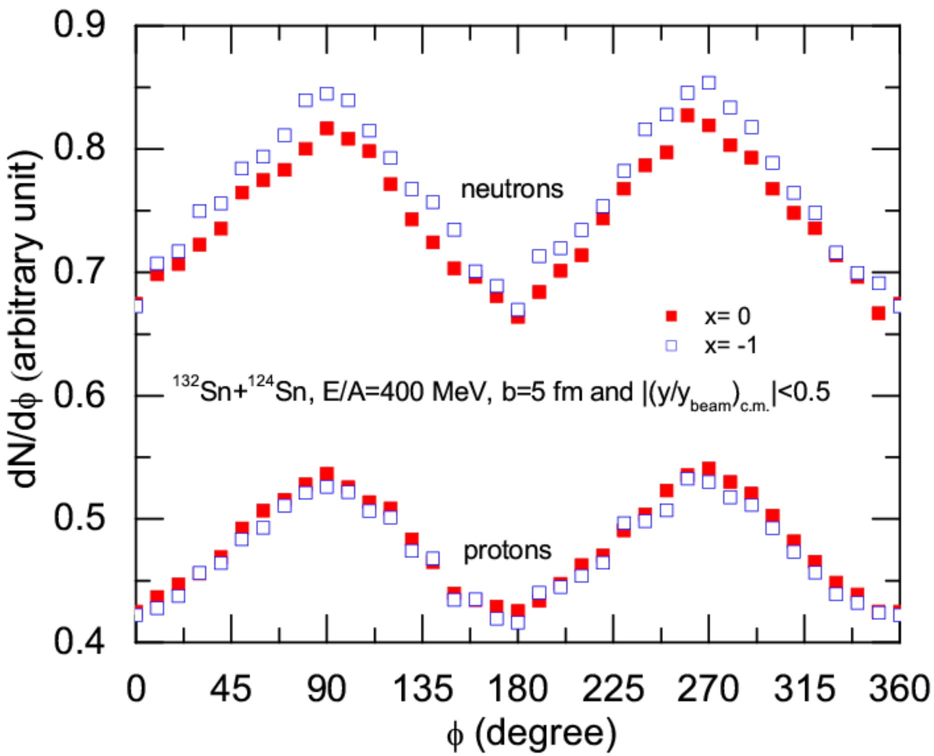
$(\rho/\rho_0)^{\gamma}$ with $\gamma = 0.9 \pm 0.4$

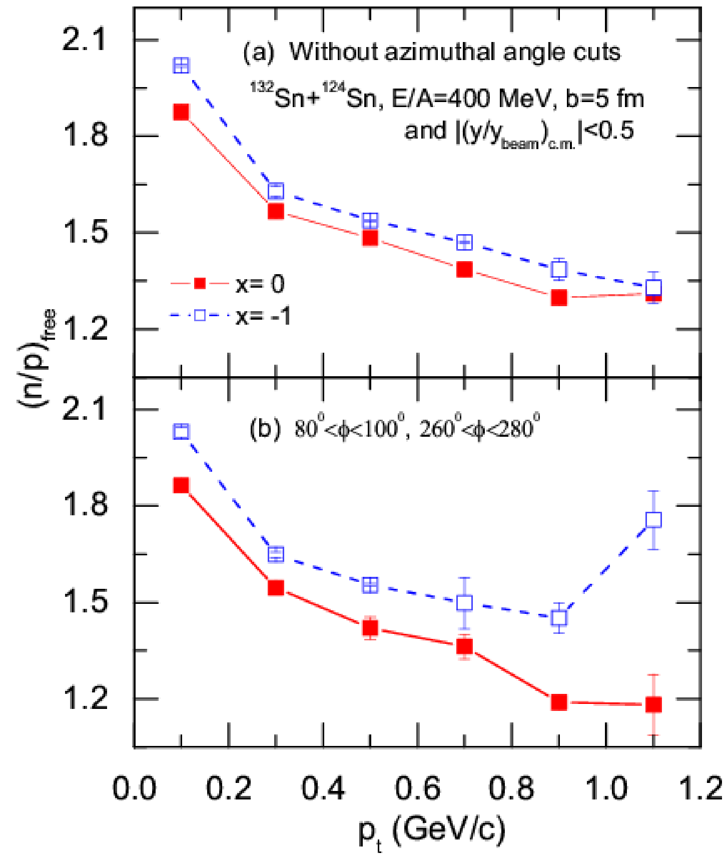




High density E_{sym}: n/p (t/^{3}He) ratio at squeeze-out direction

In the squeeze-out direction: nucleons emitted from the high density participant region have a better chance to escape without being hindered by the spectators. These nucleons thus carry more direct information about the high density phase of the reaction.

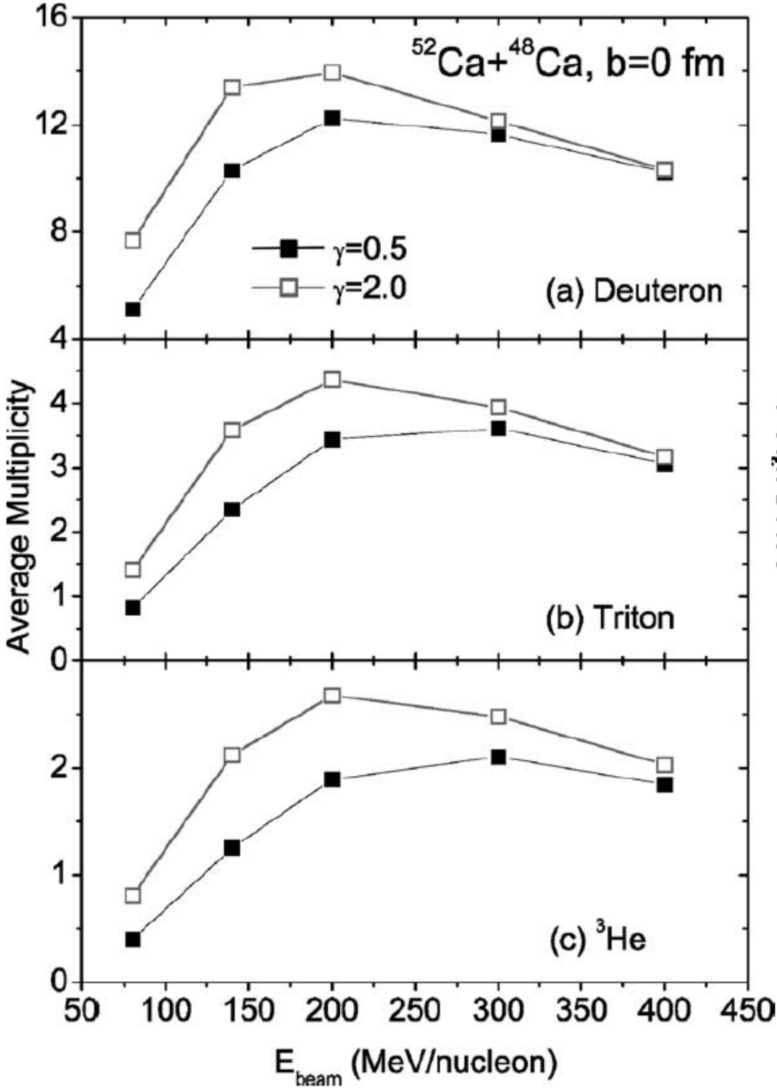




Yong/Li/Chen, PLB650, 344 (2007)

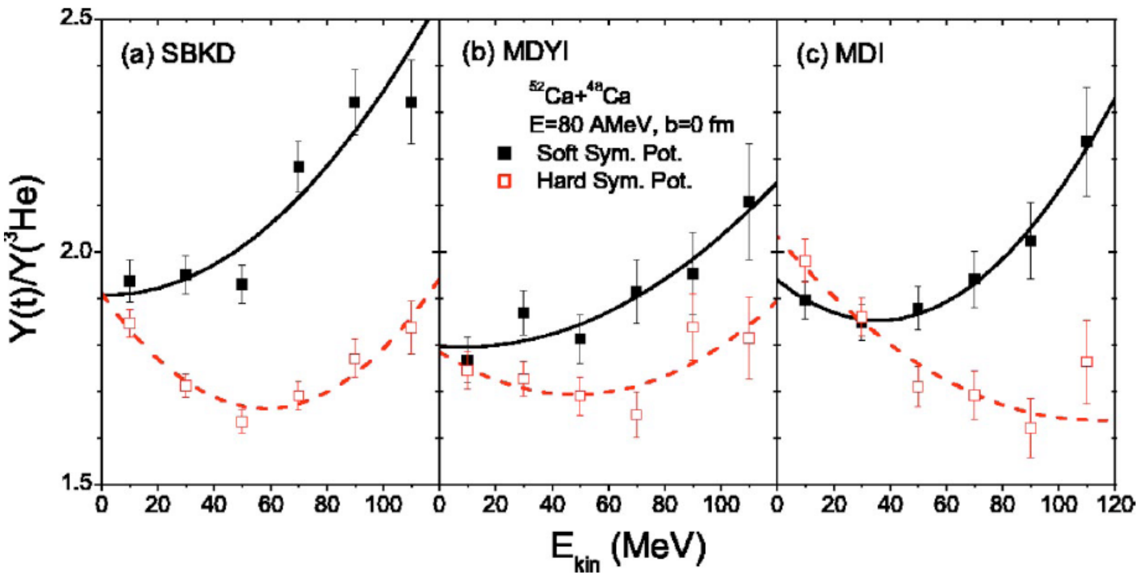
The effect can be 40% at higher p_T !
(RAON can make contribution)

High density E_{sym}: n/p (t/^{3}He) ratio at squeeze-out direction



Chen/Ko/Li, NPA729, 809 (2003)

IBUU04+Coalescence



Chen/Ko/Li, PRC69, 054606 (2004)

RAON can make contribution!!!

Outline

- The symmetry energy
- Current constraints on the symmetry energy
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High density E_{sym}: other ways?

The high behaviors of Esym are the most elusive properties of asymmetric nuclear matter!!!

- While high quality data and reliable models are in progress to constrain the high density Esym, can we find other ways to get some information on high density Esym?

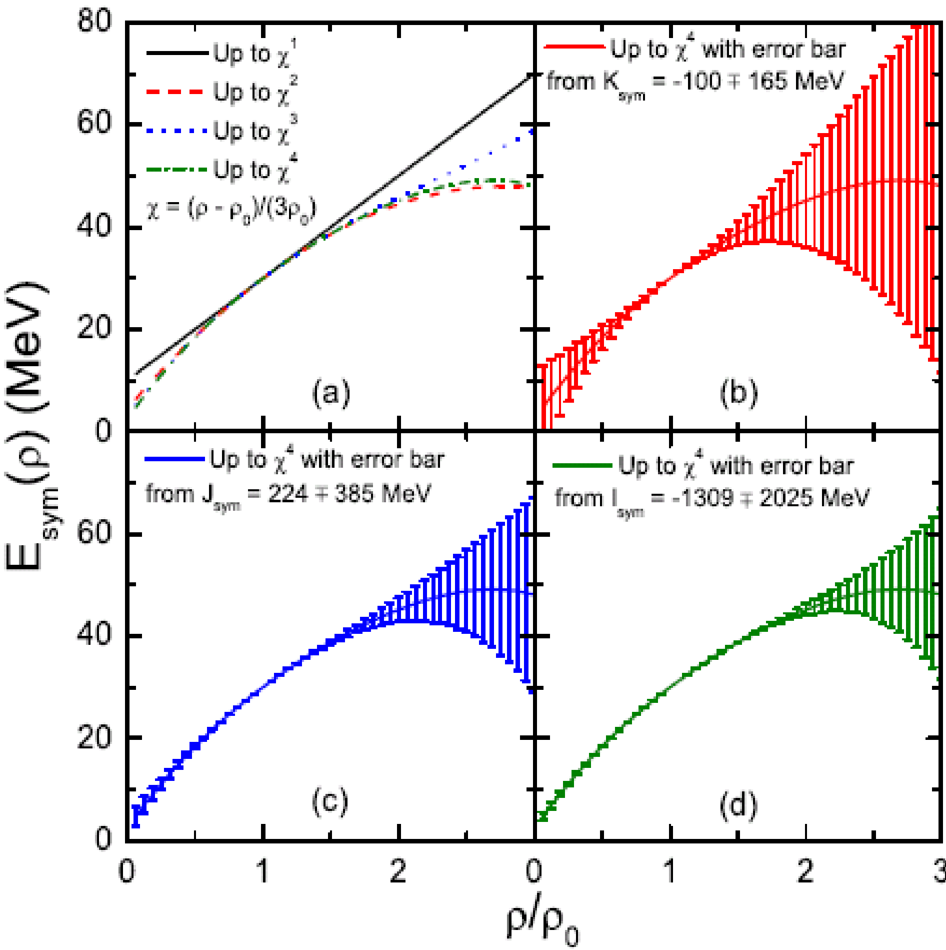
- Can we get some information on high density Esym from the knowledge of Esym around saturation density?

$$E_{sym}(\rho) = E_{sym}(\rho_0) + L\chi + \frac{K_{sym}}{2!}\chi^2 + \frac{J_{sym}}{3!}\chi^3 + \frac{I_{sym}}{4!}\chi^4 + O(\chi^5) \qquad \chi = \frac{\rho - \rho_0}{3\rho_0}$$

$E_{sym}(\rho_0)$, L, and K_{sym} ⇒ E_{sym} up to $2\rho_0$ or even higher densities!!!

High density E_{sym}: K_{sym} parameter?

$$E_{sym}(\rho) = E_{sym}(\rho_0) + L\chi + \frac{K_{sym}}{2!}\chi^2 + \frac{J_{sym}}{3!}\chi^3 + \frac{I_{sym}}{4!}\chi^4 + O(\chi^5) \qquad \chi = \frac{\rho - \rho_0}{3\rho_0}$$



The higher-order chracteristic parameters J_{sym}, I_{sym} et al seem only have tiny effects on $E_{sym}(\rho)$ below about $2\rho_0$ (Based on SHF)

$E_{sym}(\rho)$ up to about $2\rho_0$ is essentially determined by three characteristic parameters: $E_{sym}(\rho_0)$, L, and K_{sym}

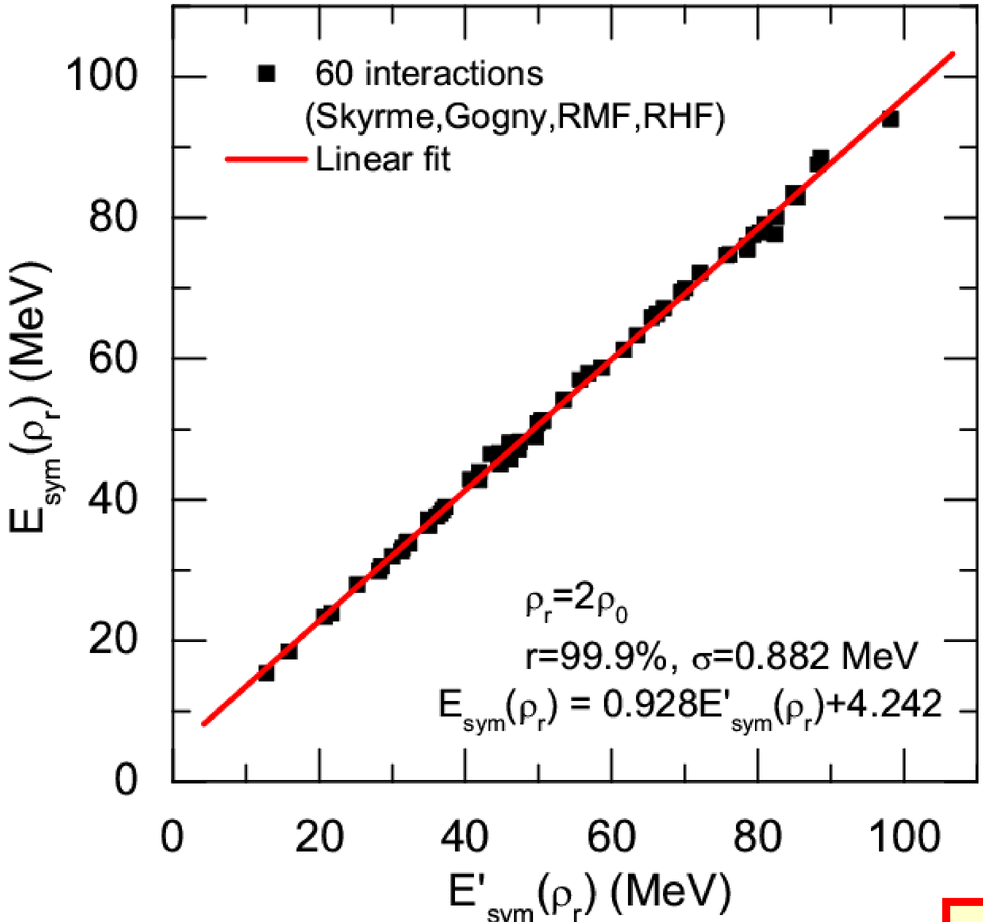
$E_{sym}(\rho_0)$, L, and K_{sym}

$E_{sym}(2\rho_0) = ?$

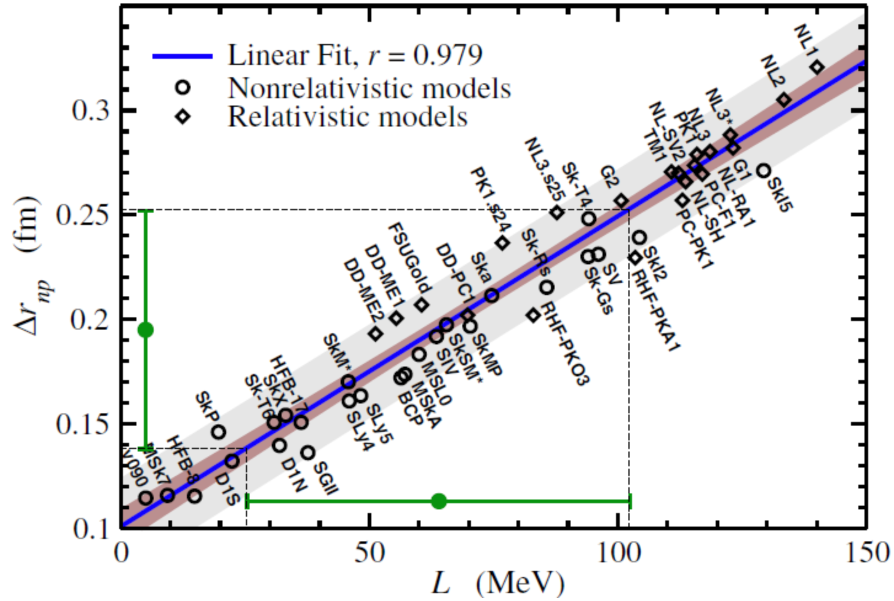
L.W. Chen, Sci. China Phys. Mech. Astron. 54, suppl. 1, s124 (2011) [arXiv:1101.2384]

High density E_{sym}: K_{sym} parameter?

$$E_{sym}(\rho) = E_{sym}(\rho_0) + L\chi + \frac{K_{sym}}{2!}\chi^2 + \frac{J_{sym}}{3!}\chi^3 + \frac{I_{sym}}{4!}\chi^4 + O(\chi^5) \qquad \chi = \frac{\rho-\rho_0}{3\rho_0}$$



$$E'_{sym}(2\rho_0) \equiv E_{sym}(\rho_0) + L/3 + K_{sym}/18$$



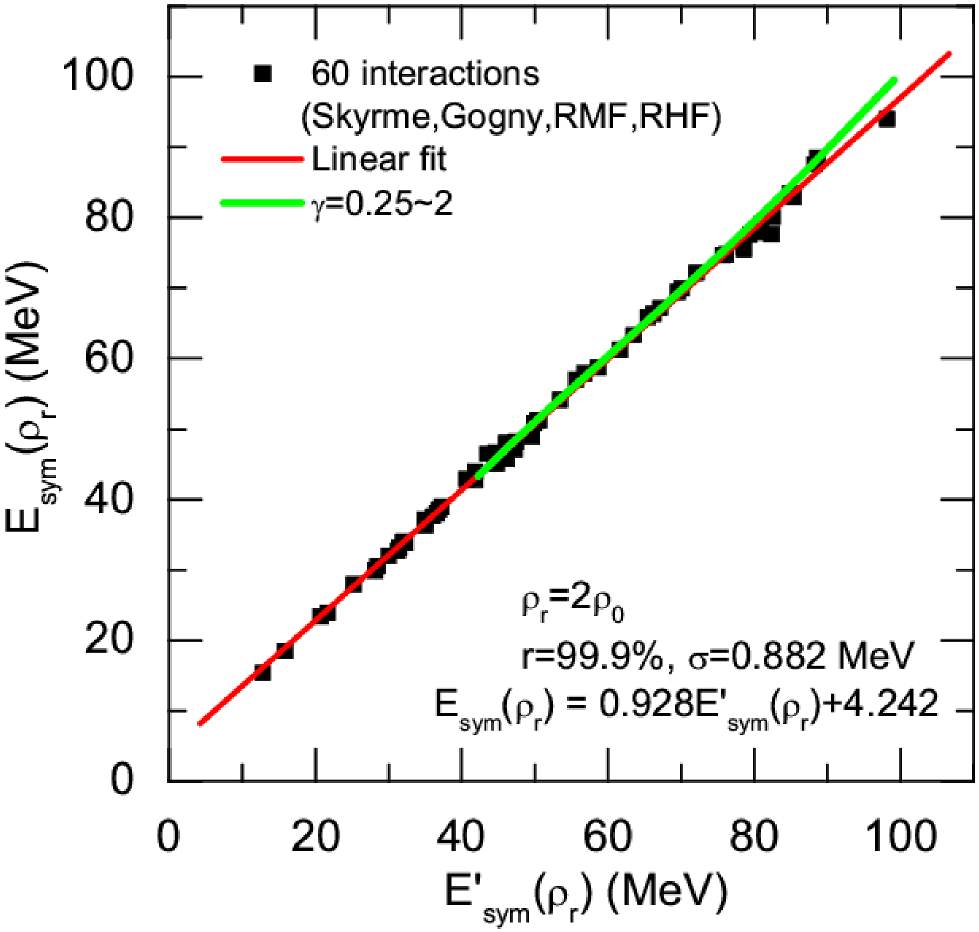
Roca-Maza et al., PRL106, 252501 (2011)

46 interactions +BSK18-21+MSL1+SAMi +SV-min+UNEDF0-1+TOV-min+IU-FSU +BSP+IU-FSU*+TM1*

$E_{sym}(\rho_0)$, L, and K_{sym} → $E_{sym}(2\rho_0) = ?$

High density E_{sym}: K_{sym} parameter?

$$E_{sym}(\rho) = E_{sym}(\rho_0) + L\chi + \frac{K_{sym}}{2!}\chi^2 + \frac{J_{sym}}{3!}\chi^3 + \frac{I_{sym}}{4!}\chi^4 + O(\chi^5) \qquad \chi = \frac{\rho - \rho_0}{3\rho_0}$$



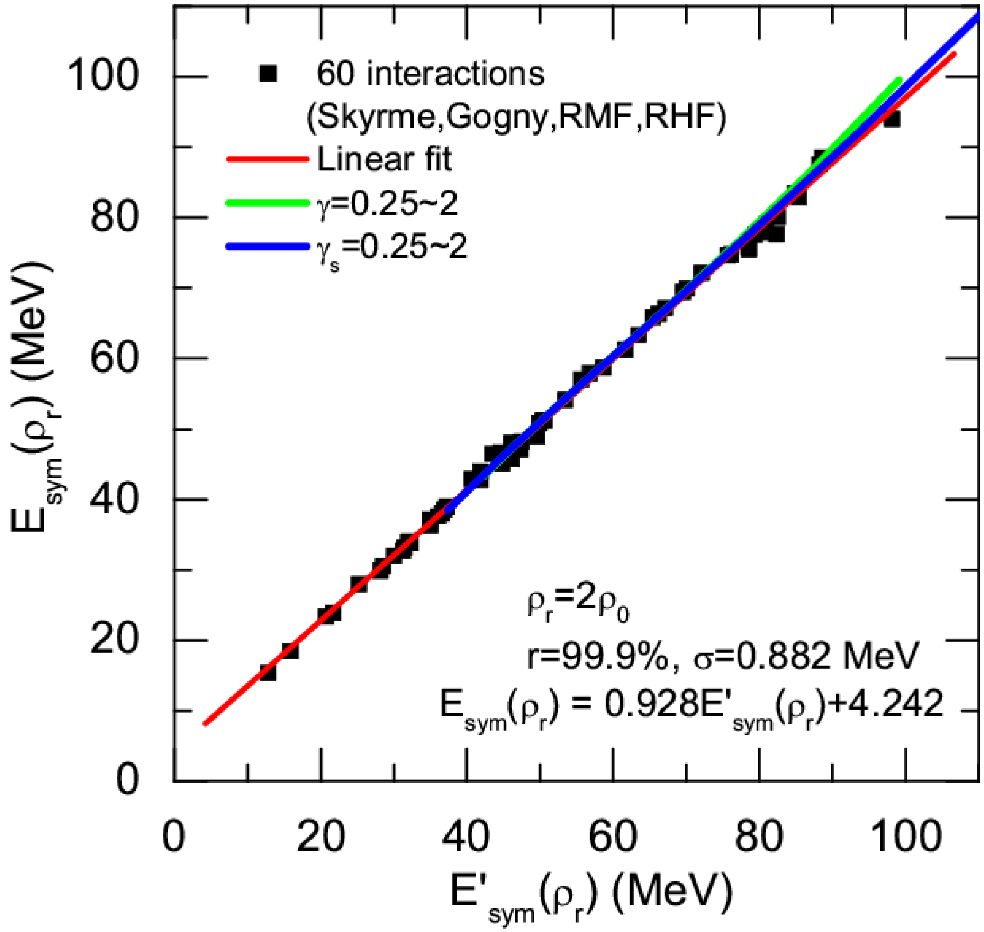
L.W. Chen, in preparation

$$E_{sym}(\rho) = 12.3\left(\frac{\rho}{\rho_0}\right)^{2/3} + 20\left(\frac{\rho}{\rho_0}\right)^{\gamma}$$

$$E'_{sym}(2\rho_0) \equiv E_{sym}(\rho_0) + L/3 + K_{sym}/18$$

High density E_{sym}: K_{sym} parameter?

$$E_{sym}(\rho) = E_{sym}(\rho_0) + L\chi + \frac{K_{sym}}{2!}\chi^2 + \frac{J_{sym}}{3!}\chi^3 + \frac{I_{sym}}{4!}\chi^4 + O(\chi^5) \qquad \chi = \frac{\rho - \rho_0}{3\rho_0}$$



L.W. Chen, in preparation

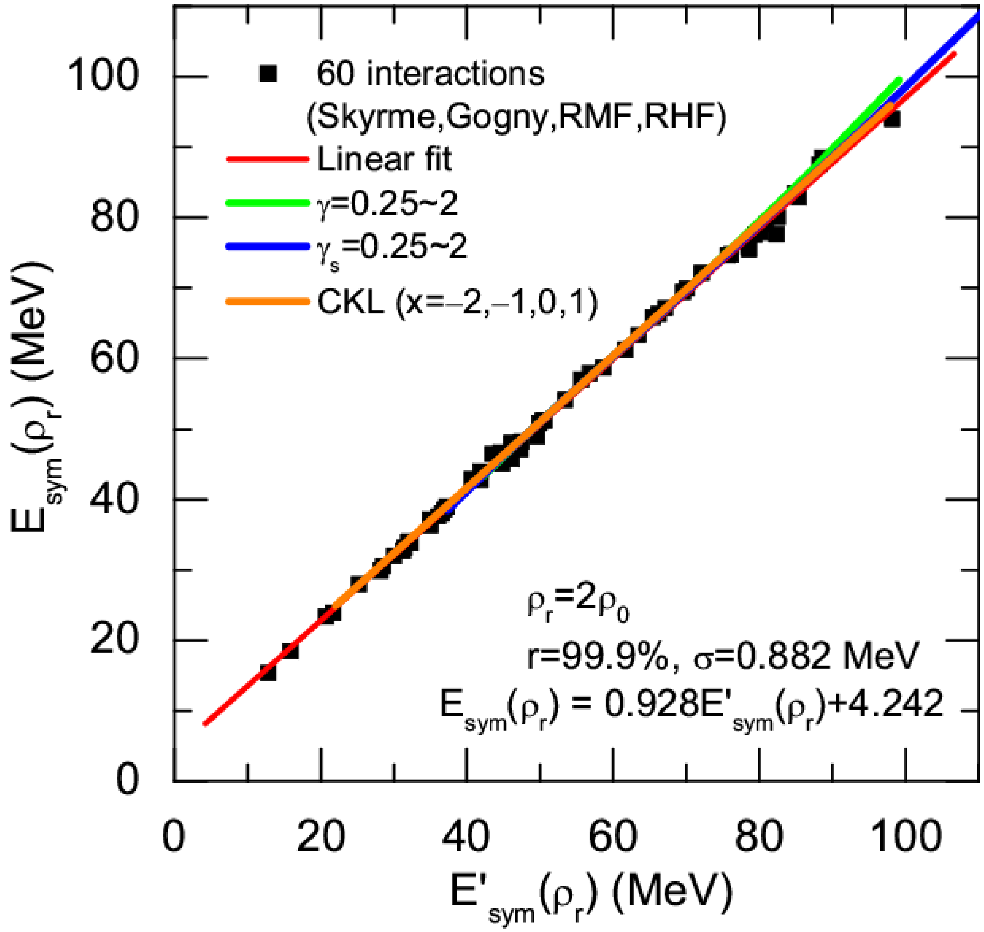
$$E_{sym}(\rho) = 12.3\left(\frac{\rho}{\rho_0}\right)^{2/3} + 20\left(\frac{\rho}{\rho_0}\right)^{\gamma}$$

$$E_{sym}(\rho) = 32.3\left(\frac{\rho}{\rho_0}\right)^{\gamma_s}$$

$$E'_{sym}(2\rho_0) \equiv E_{sym}(\rho_0) + L/3 + K_{sym}/18$$

High density E_{sym}: K_{sym} parameter?

$$E_{sym}(\rho) = E_{sym}(\rho_0) + L\chi + \frac{K_{sym}}{2!}\chi^2 + \frac{J_{sym}}{3!}\chi^3 + \frac{I_{sym}}{4!}\chi^4 + O(\chi^5) \qquad \chi = \frac{\rho-\rho_0}{3\rho_0}$$



$$E'_{sym}(2\rho_0) \equiv E_{sym}(\rho_0) + L/3 + K_{sym}/18$$

L.W. Chen, in preparation

$$E_{sym}(\rho) = 12.3\left(\frac{\rho}{\rho_0}\right)^{2/3} + 20\left(\frac{\rho}{\rho_0}\right)^{\gamma}$$

$$E_{sym}(\rho) = 32.3\left(\frac{\rho}{\rho_0}\right)^{\gamma_s}$$

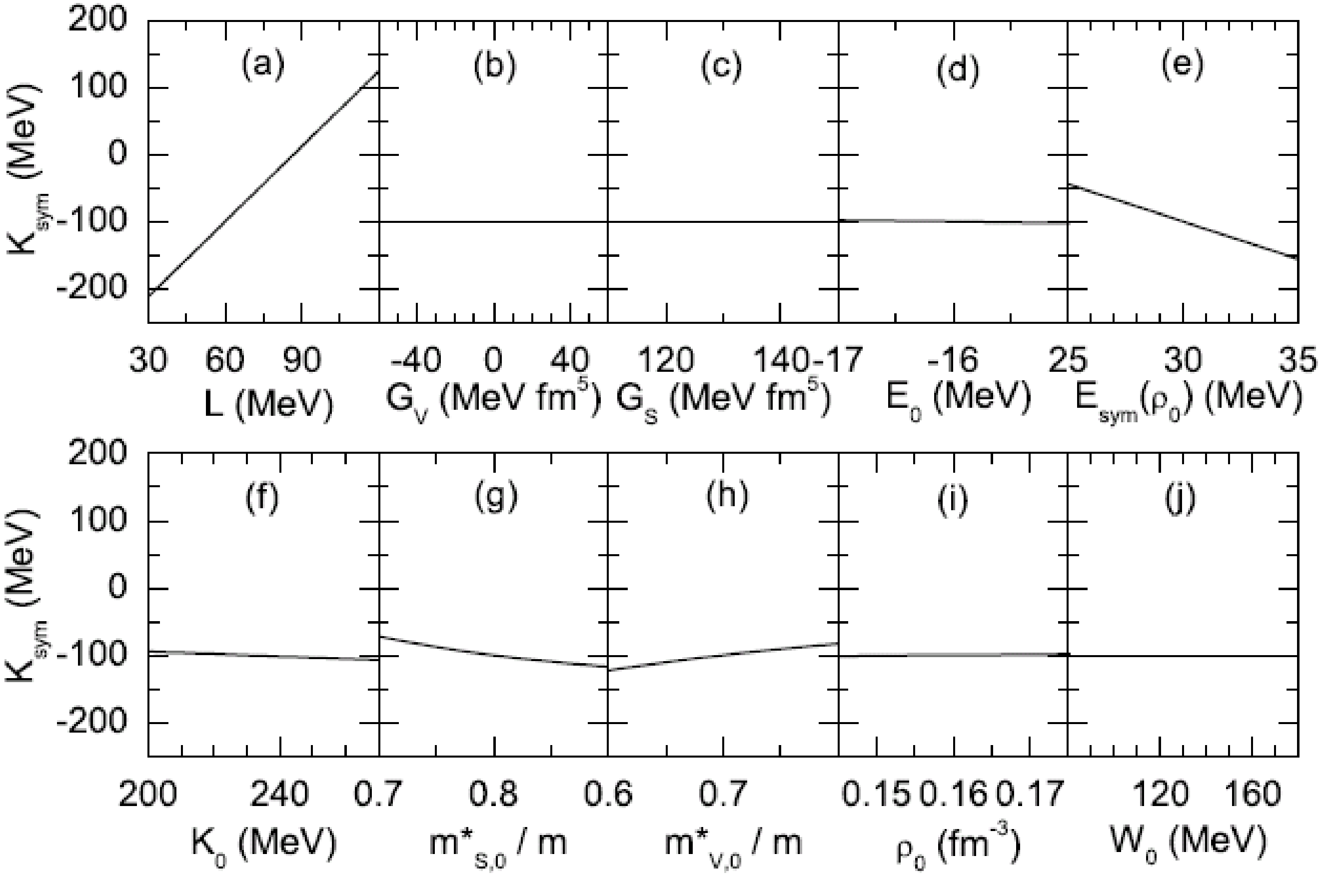
$$E_{sym}(\rho) = 13\left(\frac{\rho}{\rho_0}\right)^{2/3} + F(x)\left(\frac{\rho}{\rho_0}\right) + (18.6 - F(x))\left(\frac{\rho}{\rho_0}\right)^{G(x)}$$

(Chen/Ko/Li, PRL94, 032701(2005))

Model independent!

What's value of K_{sym}?

$$E_{sym}(\rho) = E_{sym}(\rho_0) + L\chi + \frac{K_{sym}}{2!}\chi^2 + \frac{J_{sym}}{3!}\chi^3 + \frac{I_{sym}}{4!}\chi^4 + O(\chi^5)$$



L.W. Chen, PRC83, 044308(2011)

$$K_{sym} = 3\gamma L + E_{sym}^{kin}(\rho_0)(3\gamma - 2) + 2D(5 - 3\gamma) - 9\gamma E_{sym}(\rho_0)$$

$$K_{sym} = (-100 \pm 165)\ \mathrm{MeV}$$

Based on SHF !

L.W. Chen, Sci. China Phys. Mech. Astron. 54, suppl. 1, s124 (2011) [arXiv:1101.2384]

K_{sym}: Symmetry energy of finite nuclei

Liquid-drop model

$$a_v A - a_s A^{2/3} - a_4 \frac{(N-Z)^2}{A} - a_c \frac{Z(Z-1)}{A^{1/3}} + a_p \frac{\Delta(N,Z)}{A^{1/2}}$$

Symmetry energy term

$$S_v \frac{(N_v - Z_v)^2}{A} + S_s \frac{(N_s - Z_s)^2}{A^{2/3}}$$

$$a_v A - a_s A^{2/3} - \frac{S_v}{1 + y_s A^{-1/3}} \frac{(N-Z)^2}{A} - a_c \frac{Z(Z-1)}{A^{1/3}} + a_p \frac{\Delta(N,Z)}{A^{1/2}}$$

Symmetry energy including surface diffusion effects ($y_s = S_v/S_s$

W. D. Myers, W.J. Swiatecki, P. Danielewicz, P. Van Isacker, A. E. L. Dieperink,……

K_{sym}: Symmetry energy of finite nuclei

Symmetry energy coefficient of finite nuclei in mass formula

$$a_{\rm sym}(A) = \frac{E_{\rm sym}(\rho_0)}{1+x_A} \quad \text{with} \quad x_A = \frac{9E_{\rm sym}(\rho_0)}{4Q}A^{-1/3}$$

Q: neutron-skin stiffness coefficient in the droplet model, it is also related to the surface symmetry energy, and can be obtained from asymmetric semi-infinite nuclear matter (ASINM) calculations

As a good approximation (See, e.g., L.W. Chen, PRC83, 044308 (2011)), we have

$$Q = \frac{9}{4}\frac{E^2_{\rm sym}(\rho_0)}{\varepsilon^e_\delta} \quad \text{with} \quad \varepsilon^e_\delta = \frac{2a}{r_{\rm nm}}\left(L - \frac{K_{\rm sym}}{12}\right)$$

$$\Rightarrow \quad x_A = (L - K_{\rm sym}/12)\frac{A^{-1/3}}{E_{\rm sym}(\rho_0)}$$

where $r_{\rm nm} = (\frac{4}{3}\pi\rho_0)^{-1/3}$ is the radius constant of nuclear matter and a is the diffuseness parameter in the Fermi-like function from the parametrization of nuclear surface profile of symmetric semi-infinite nuclear matter. Many calculations [25–27] have indicated $a \approx 0.55$ fm and then $2a/r_{\rm nm} \approx 1$.

M. Liu et al., PRC82, 064306 (2010)

$$a_{\rm sym}(A) = S_0(1+\kappa A^{-1/3})^{-1}$$

$$\kappa = \frac{9E_{\rm sym}(\rho_0)}{4Q} \approx \frac{L - K_{\rm sym}/12}{E_{\rm sym}(\rho_0)} \equiv \kappa'$$

ASINM calculations

3.2.3 *Surface Symmetry Energy and Neutron Skin*

In the case of a neutron excess, there are two ways of generalizing Eq. (3.20) to characterize the surface energy. They are discussed in details in Ref. [62]. They differ by the choice of the (volume) "reference energy" subtracted from the total energy when defining a surface quantity. The first one is to consider the energy of a system having the same total density as the actual one (here, a semi-infinite slab) but with a constant energy density e_0 equal to that of the uniform medium found in the asymptotic central region. One thus defines a surface tension $\sigma_e(I)$ by

$$\sigma_e(I) = \int [\mathcal{H} - e_0 \rho]\, dx. \qquad (3.52a)$$

The corresponding energy

$$\varepsilon_s^e = 4\pi r_0^2 \sigma_e(I) \qquad (3.52b)$$

will be referred to as the *e*-surface energy (r_0 is the nuclear radius constant in the case of a relative neutron excess I).

The alternative choice is to consider the energy of a system having the same neutron and proton densities as the actual one, but where all the neutrons (resp. protons) have an energy per particle equal to the neutron (resp. proton) chemical potential λ_n (resp. λ_p). This leads to defining a surface coefficient $\sigma_\lambda(I)$ by

$$\sigma_\lambda(I) = \int (\mathcal{H} - \lambda_n \rho_n - \lambda_p \rho_p)\, dx \qquad (3.53)$$

as the λ-surface energy coefficient.

The difference between Eqs. (3.52a) and (3.53) lies in the I-dependence of both quantities. More precisely, if one expands for small I's $\sigma_e(I)$ and $\sigma_\lambda(I)$,

$$\sigma_e(I) = \sigma + \sigma_\delta^e I^2, \qquad (3.54a)$$

$$\sigma_\lambda(I) = \sigma + \sigma_\delta^\lambda I^2, \qquad (3.55b)$$

one can show that

$$\sigma_\delta^\lambda = -\sigma_\delta^e < 0. \qquad (3.55)$$

This is done in Ref. [71]. By using the Euler equations, one obtains for the surface symmetry energy

$$\varepsilon_\delta^s = 4\pi r_{nm}^2 \sigma_\delta^e$$

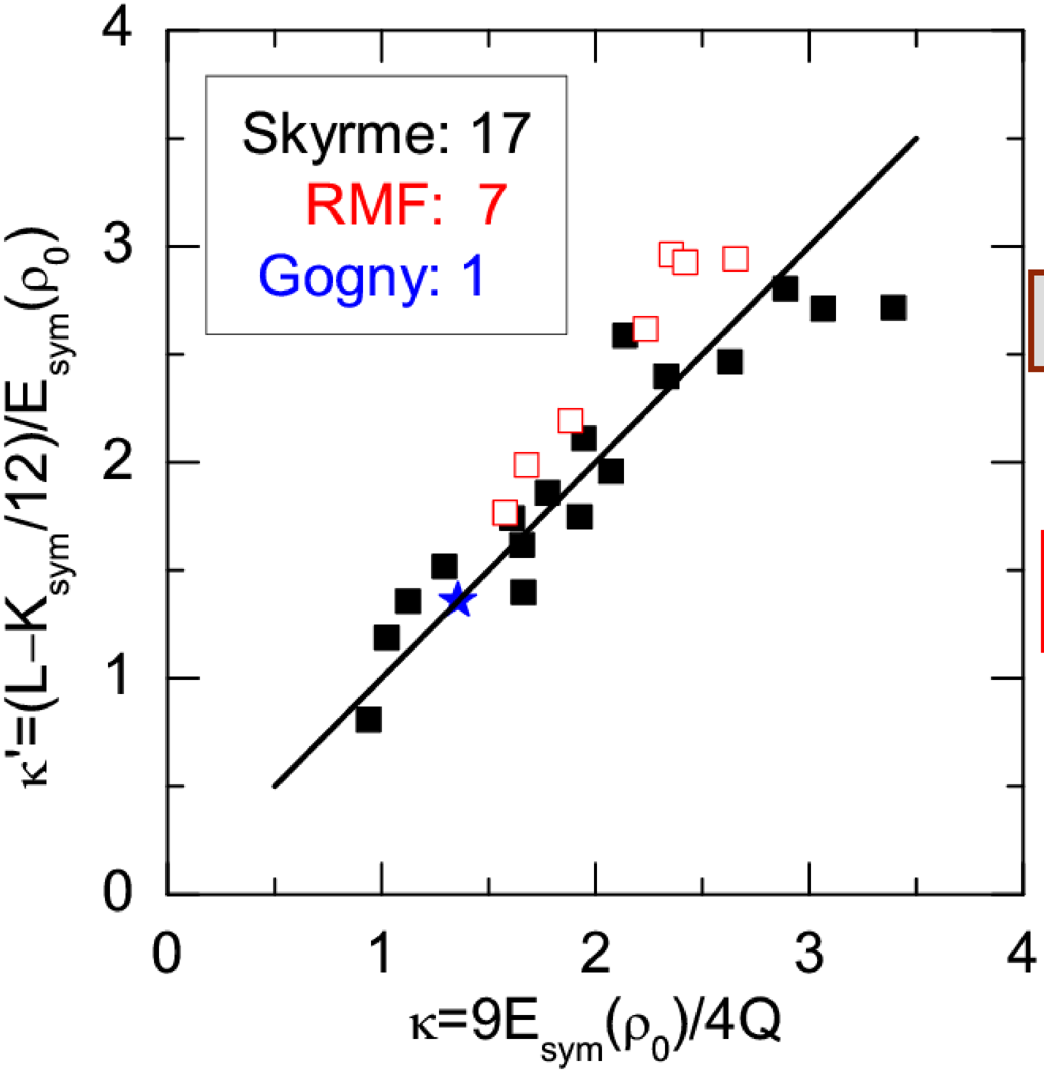
Treiner/Krivine, Ann. Phys. 170, 406(86)

$$\rho_n(x) = \rho_n^0 \Big/ \left(1 + \exp\frac{x - x_0}{a_n}\right)^{\nu_n}$$

$$\rho_p = \rho_p^0 \Big/ \left(1 + \exp\frac{x + x_0}{a_p}\right)^{\nu_p}$$

$$Q = \frac{9}{4}\frac{J^2}{\varepsilon_\delta^s}$$

k and k′ parameters



With about 20% uncertainty

$$\kappa = \frac{9E_{sym}(\rho_0)}{4Q} \approx \frac{L - K_{sym}/12}{E_{sym}(\rho_0)} \equiv \kappa'$$

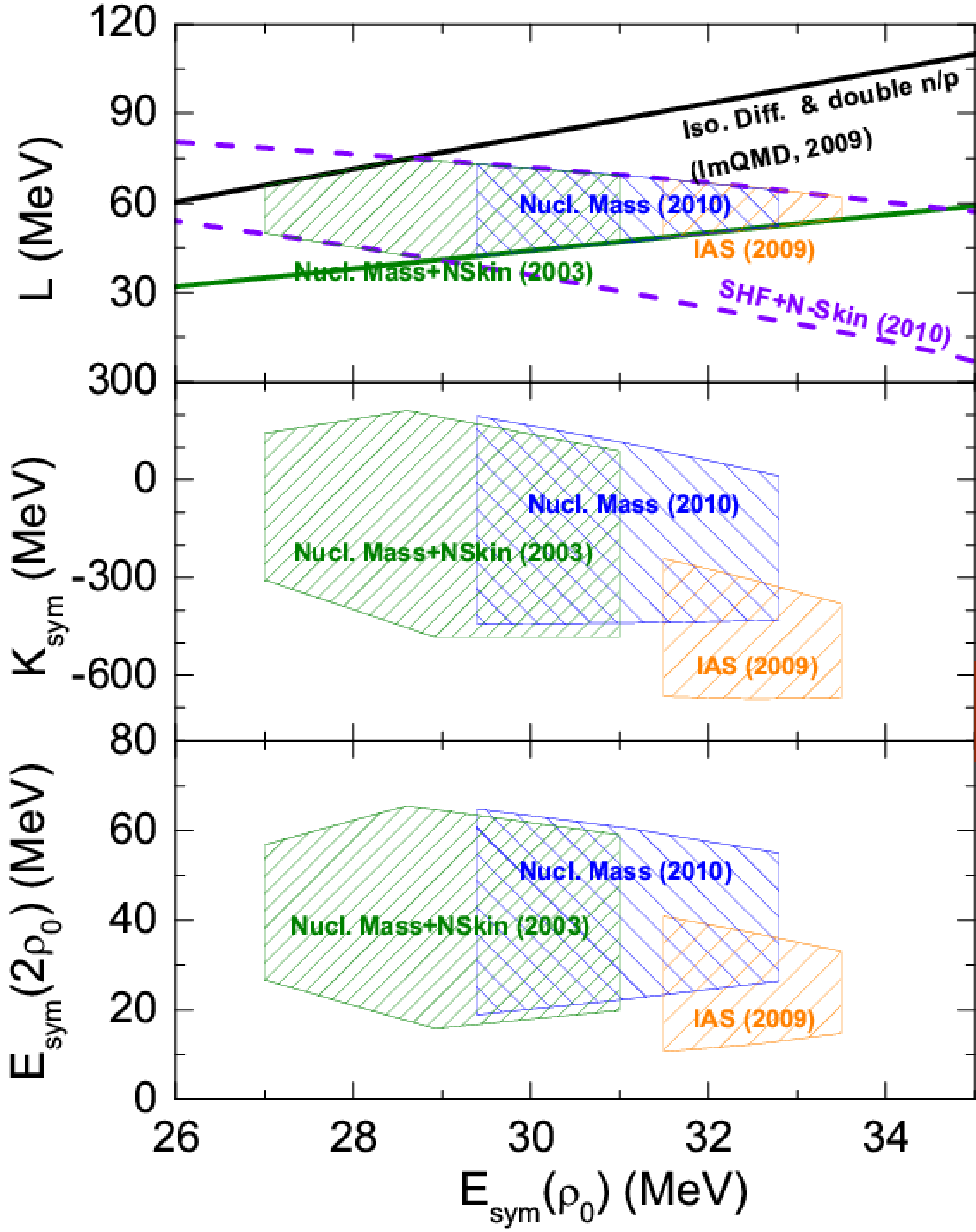
Esym of finite nuclei

$$a_{sym}(A) = \frac{E_{sym}(\rho_0)}{1+x_A} \quad \text{with} \quad x_A = \frac{9E_{sym}(\rho_0)}{4Q} A^{-1/3}$$

$E_{sym}(\rho_0)$, L, and κ'(or κ)

$K_{sym} = ?$

High density E_{sym} : $E_{sym}(2\rho_0)$?



$L, E_{sym}(\rho_0)$

Iso. Diff. & double n/p (ImQMD, 2009)
M.B. Tsang et al., PRL102, 122701 (2009)
SHF+N-Skin (2010)
L.W. Chen et al., PRC82, 024321 (2010)

$$(L - K_{sym}/12)/E_{sym}(\rho_0) \equiv \kappa' \approx \kappa$$

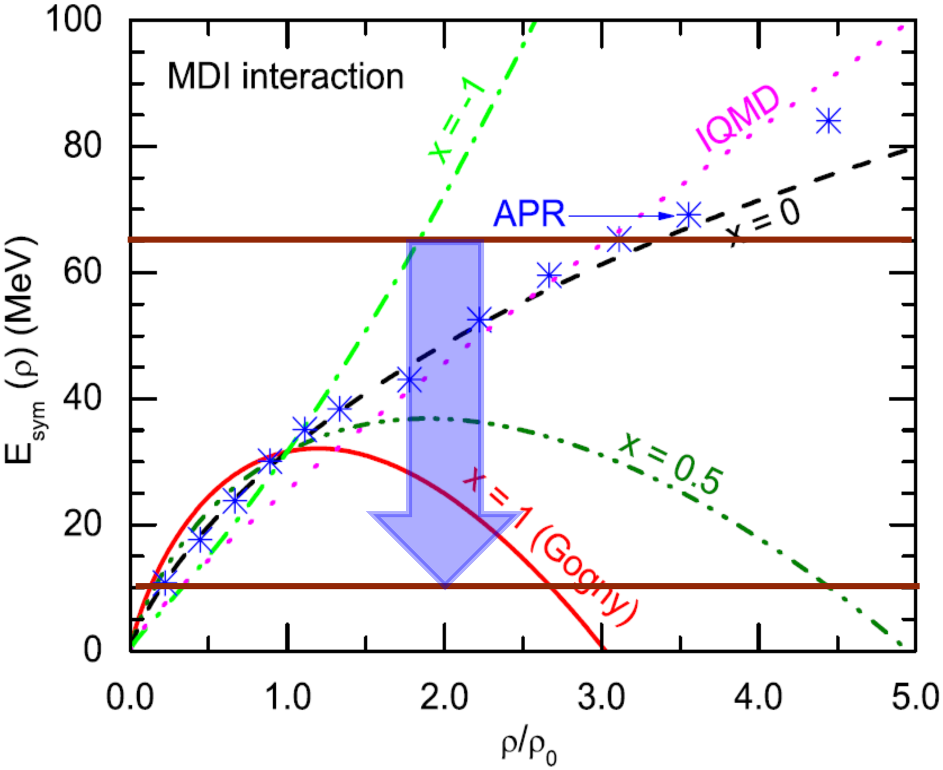
Nucl. Mass+Nskin (2003):
Danielewicz, NPA727, 233 (2003)
IAS (2009):
Danielewicz/Lee, NPA818, 36 (2009)
Nucl. Mass (2010):
M. Liu et al., PRC82, 064306 (2010)

$$K_{sym} \approx [-669, 213] \text{ MeV}$$

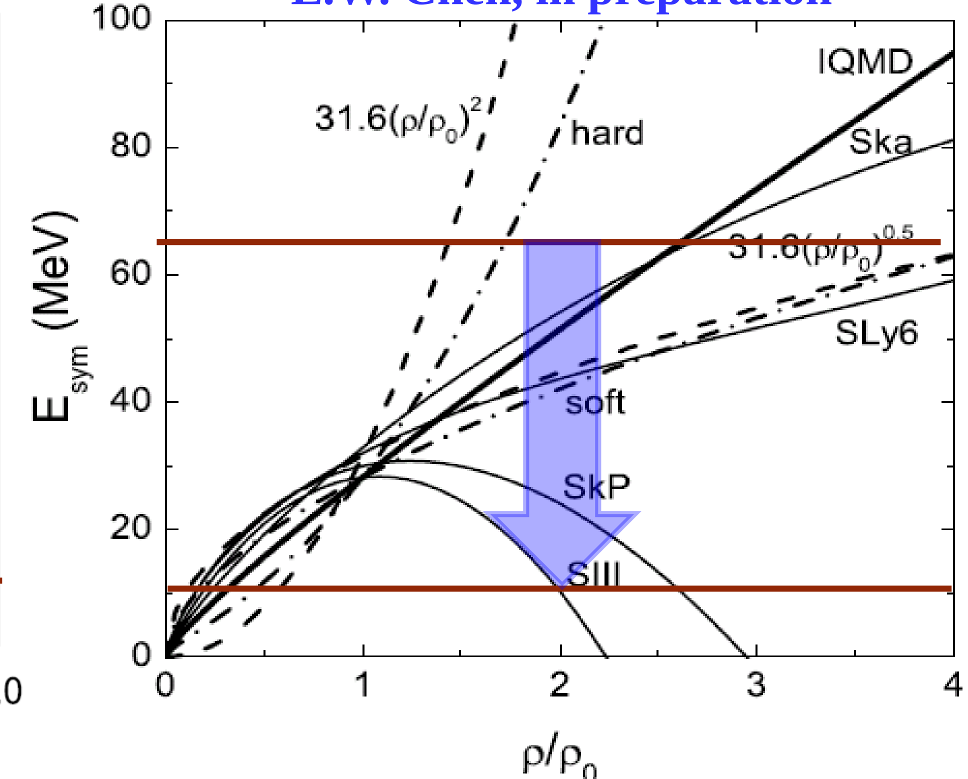
$$E_{sym}(2\rho_0) \approx [10, 65] \text{ MeV}$$

L.W. Chen, in preparation

High density E_{sym} : $E_{sym}(2\rho_0)$?



L.W. Chen, in preparation



$$E_{sym} = E_{sym}^{pot} + E_{sym}^{kin}$$
$$= 22\ \mathrm{MeV} \cdot (\rho/\rho_0)^{\gamma} + 12\ \mathrm{MeV} \cdot (\rho/\rho_0)^{2/3}$$

$\gamma = 0.9 \pm 0.4$

P. Russotto,W. Trauntmann, Q.F. Li et al., PLB697, 471(2011)

$E_{sym}(2\rho_0) \approx [10, 65]$ MeV

$\gamma \le 1.07$ (for $\rho \le 2\rho_0$)

Stiff symmetry energy ($\rho \le 2\rho_0$) is ruled out!!!

Outline

- The symmetry energy
- Current constraints on the symmetry energy
  - n-A elastic scattering and the symmetry potential
  - Symmetry energy at 0.11 fm^{-3}
  - High density behaviors
- Density curvature K_{sym} and the high density symmetry energy
- Summary and outlook

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Summary

- Neutron-nucleus scattering data provide new constraints on energy dependent symmetry potential (Lane potential $U_{sym,1}(\rho_0,p)$ and $U_{sym,2}(\rho_0,p)$), and we find $U_{sym,2}(\rho_0,p)$ is comparable with $U_{sym,1}(\rho_0,p)$. Furthermore, we obtain:

  $E_{sym}(\rho_0)$ =37.24±2.26 MeV and L=44.98±22.31 MeV

- The neutron skin is determined uniquely by $L(\rho_c)$ at ρ_c =0.11 fm^{-3}, and from the neutron skin of Sn isotopes, we obtain:

  L(0.11 fm^{-3}) =46.0±4.5 MeV

- The binding energy difference of heavy isotope pair is essentially determined uniquely by $E_{sym}(\rho_c)$ at ρ_c =0.11 fm^{-3}, and from a number of heavy isotope pairs, we obtain:

  E_{sym} (0.11 fm^{-3}) =26.65±0.2 MeV

- A fixed value of $E_{sym}(\rho_c)$ at ρ_c =0.11 fm^{-3} leads to a positive $E_{sym}(\rho_0)$-L correlation while a fixed value of $L(\rho_c)$ at ρ_c =0.11 fm^{-3} leads to a negative $E_{sym}(\rho_0)$-L correlation. From E_{sym} (0.11 fm^{-3}) and L(0.11 fm^{-3}), we obtain:

  $E_{sym}(\rho_0)$ =32.3±1.0 MeV and L=45.2±10.0 MeV

- $E_{sym}(2\rho_0)$ is essentially determined by $E_{sym}(\rho_0)$, L, and K_{sym}. From the surface symmetry energy in finite nuclei, we can obtain

  K_{sym}:[-669,213] MeV, and $E_{sym}(2\rho_0)$:[10,65] MeV

Outlook

1. Some promising probes for high density E_{sym} in heavy ion collisions

- **n/p**: spectra, flows, squeeze-out,…
  (direct probe to symmetry potential/energy)
- **t/^{3}He**: spectra, flows, ,squeeze-out,…
  (Semi-direct probe to symmetry potential/energy through nucleon coalescence)
- **π -/π+ ratio**:
  (Secondary probe to symmetry potential/energy)
- **K0/K+ ratio**:
  (Secondary+ probe to symmetry potential/energy, but suffers from much weak final state interactions compared with pions, ….)

2. Accurate constraints on E_{sym} around saturation density can help to limit the high density E_{sym}

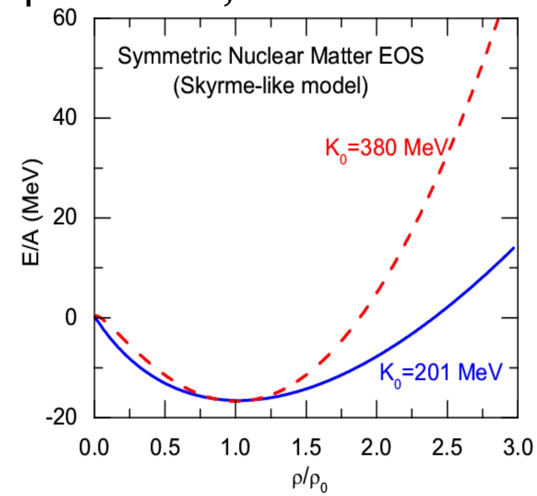
3. More accurate measurements on M-R of neutron stars ….

谢 谢！
Thanks！

Nuclear Matter EOS

The energy of per nucleon in a nuclear matter with density ρ, temperature T, and isospin asymmetry δ ($\equiv \frac{\rho_n - \rho_p}{\rho}$) can be expressed as

$$E/A = \varepsilon = \varepsilon(\rho, T, \delta) \text{ (Nuclear Matter EOS)}$$

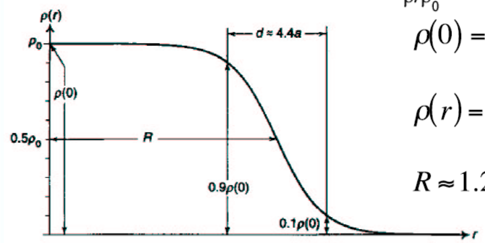


The pressure P of the nuclear matter can be expressed as

$$P(\rho, T, \delta) = \rho^2 \left(\frac{\partial \varepsilon}{\partial \rho}\right)_{T,N=\text{constant}}$$

The incompessibilty K of the nuclear matter can be expressed as

$$K(\rho, T, \delta) = 9 \left(\frac{\partial P}{\partial \rho}\right)_{T,N=\text{constant}}$$



$$\rho(0) = 0.17 \text{nucleons/fm}^3$$

$$\rho(r) = \rho_0 \left[1 + \exp\left(\frac{r-R}{a}\right)\right]^{-1}$$

$$R \approx 1.2 A^{1/3} \text{fm}, \quad a \approx 0.6 \text{fm}$$

Empirical values about the nuclear matter EOS:

Saturation density of symmetric nuclear matter at T=0 MeV: $\rho_0 \approx 0.16 \text{ fm}^{-3} (\approx 10^{14} \rho_{\text{Water}})$

Pressure of symmetric nuclear matter at T=0 MeV: $P_0(\rho_0) = 0 \text{ MeVfm}^{-3}$

The energy of per nucleon of symmetric nuclear matter at ρ_0 and T=0 MeV:

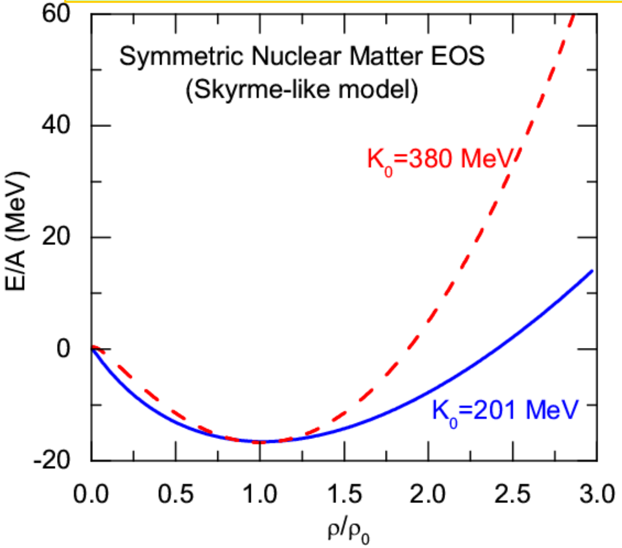
$\varepsilon_0 \approx -16$ MeV/nucleon

Incompessibilty of symmetric nuclear matter at T=0 MeV: $K_0 \approx 240 \pm 30$ MeV

EOS of Symmetric Nuclear Matter

(1) EOS of symmetric matter around the saturation density ρ_0

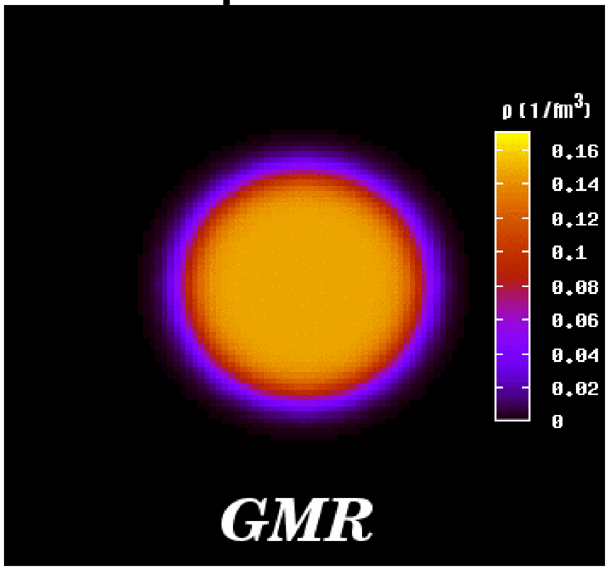
Incompressibility: $K_0 = 9\rho_0^2 \left(\frac{d^2E}{d\rho^2}\right)_{\rho_0}$



$$E_0(\rho) = E_0(\rho_0) + \frac{K_0}{2}\chi^2 + O(\chi^3)$$

$$\chi = \frac{\rho - \rho_0}{3\rho_0}$$

Giant Monopole Resonance



Frequency $f_{GMR} \propto \sqrt{K_0}$

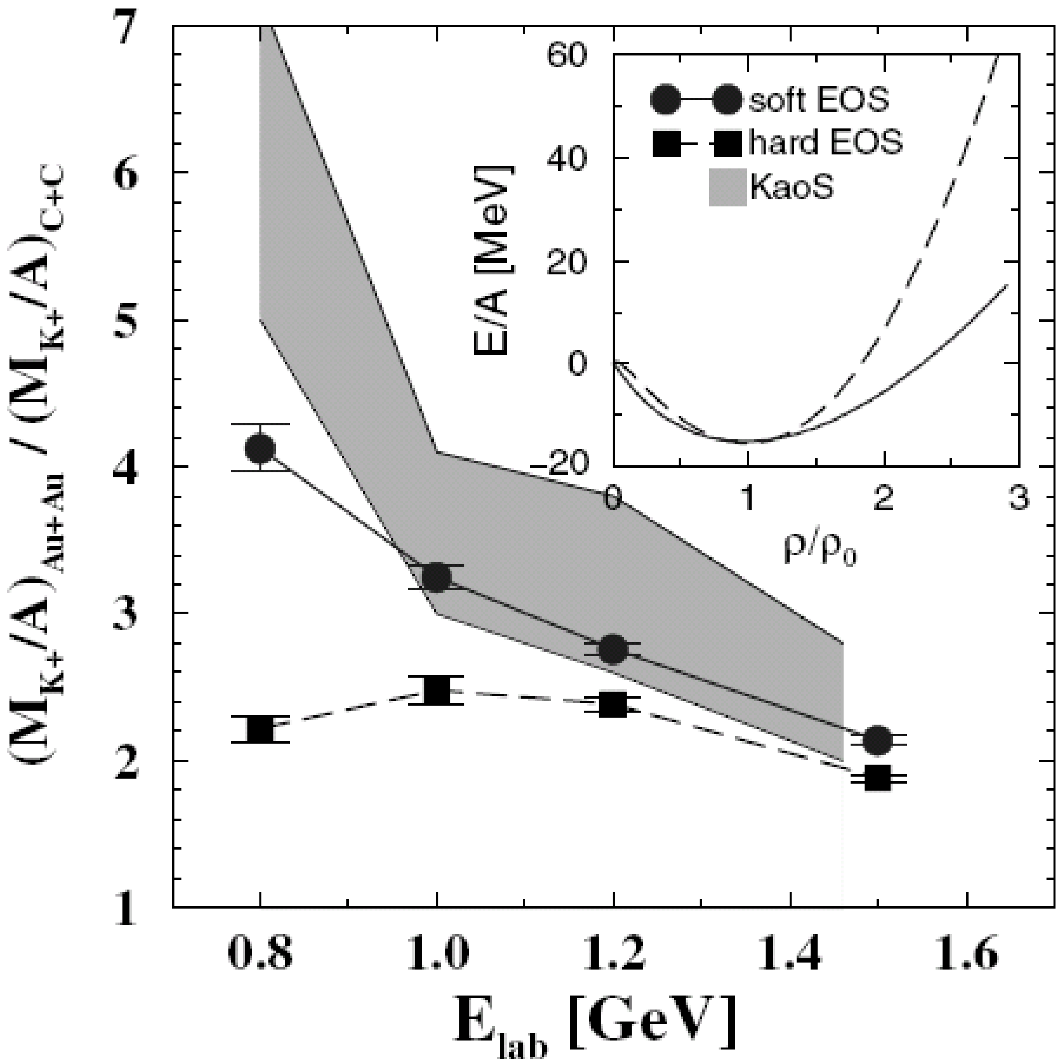
K_0=231±5 MeV
Yongblood/Clark/Lui, PRL82, 691 (1999)

Recent results:
K_0=240±20 MeV
G. Colo et al.
U. Garg et al.
S. Shlomo et al.

Uncertainty of the extracted K_0 is mainly due to the uncertainty of *L* (slope parameter of the symmetry energy) and m^*_0 (isoscalar nucleon effective mass)
(See, e.g., L.W. Chen/J.Z. Gu, JPG39, 035104(2012))

EOS of Symmetric Nuclear Matter

(2) EOS of symmetric matter for $1\rho_0 < \rho < 3\rho_0$ from K^+ production in HIC's



J. Aichelin and C.M. Ko, PRL55, (1985) 2661

C. Fuchs, Prog. Part. Nucl. Phys. 56, (2006) 1

C. Fuchs et al, PRL86, (2001) 1974

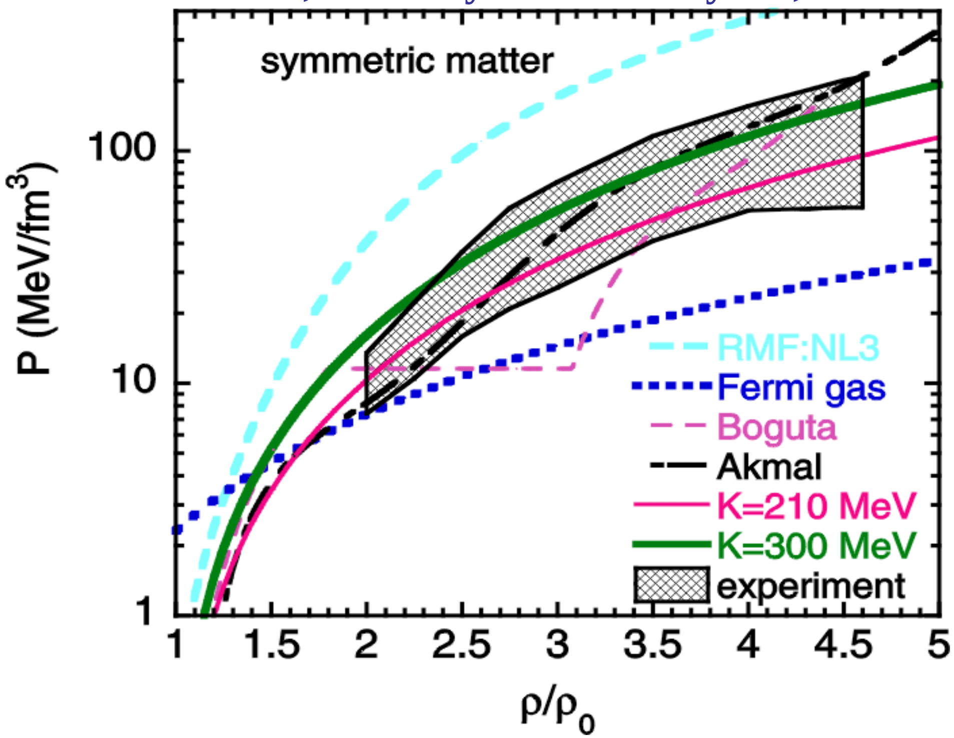
Transport calculations indicate that "results for the K^+ excitation function in Au + Au over C + C reactions as measured by the KaoS Collaboration strongly support the scenario with a **soft EOS**."

See also: C. Hartnack, H. Oeschler, and J. Aichelin, PRL96, 012302 (2006)

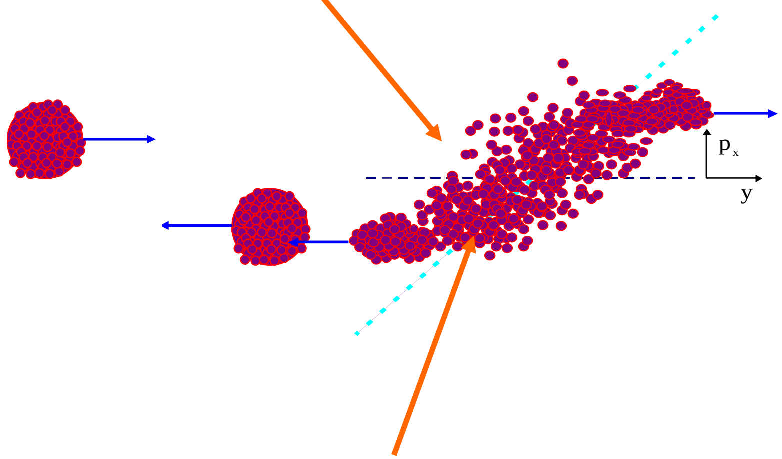
EOS of Symmetric Nuclear Matter

(3) Present constraints on the EOS of symmetric nuclear matter for $2\rho_0 < \rho < 5\rho_0$ using flow data from BEVALAC, SIS/GSI and AGS

P. Danielewicz, R. Lacey and W.G. Lynch, Science 298, 1592 (2002)



The highest pressure recorded under laboratory controlled conditions in nucleus-nucleus collisions



High density nuclear matter 2 to $5\rho_0$

- Use constrained mean fields to predict the EOS for symmetric matter
  - Width of pressure domain reflects uncertainties in comparison and of assumed momentum dependence.

$$\text{Pressure } P(\rho) = \rho^2 \left(\frac{\partial E}{\partial \rho} \right)_s$$

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k parameter: LDM+NSKin

Liquid Drop model + Neutron Skin

Danielewicz, NPA 727, 233 (2003)

Note: Actually, what they constrained are $E_{sym}(\rho_0)$ and the surface symmetry energy (k), rather than L

Abstract

Binding energy of symmetric nuclear matter can be accessed straightforwardly with the textbook mass-formula and a sample of nuclear masses. We show that, with a minimally modified formula (along the lines of the droplet model), the symmetry energy of nuclear matter can be accessed nearly as easily. Elementary considerations for a macroscopic nucleus show that the surface tension needs to depend on asymmetry. That dependence modifies the surface energy and implies the emergence of asymmetry skin. In the mass formula, the volume and surface and (a)symmetry energies combine as energies of two connected capacitors, with the volume and surface capacitances proportional to the volume and area, respectively. The net asymmetry partitions itself into volume and surface contributions in proportion to the capacitances. A combination of data on skin sizes and masses constrains the volume symmetry parameter to 27 MeV $\lesssim \alpha \lesssim$ 31 MeV and the volume-to-surface symmetry-parameter ratio to $2.0 \lesssim \alpha/\beta \lesssim 2.8$. In Thomas–Fermi theory, the surface asymmetry-capacitance stems from a drop of the symmetry energy per nucleon S with density. We establish limits on the drop at half of normal density, to $0.57 \lesssim S(\rho_0/2)/S(\rho_0) \lesssim 0.83$. In considering the feeding of surface by an asymmetry flux from interior, we obtain a universal condition for the collective asymmetry oscillations, in terms of the asymmetry-capacitance ratio.

$$\kappa \approx \frac{E_{\rm sym}(\rho_0)}{E^{Surf}_{\rm sym}(\rho_0)} = 2.0-2.8,\ E_{\rm sym}(\rho_0) \approx 27-31\ \rm MeV$$

k parameter: IAS+LDM

Isobaric Analog States + Liquid Drop model with surface symmetry energy

Danielewicz/Lee, NPA 818, 36 (2009)

Note: Actually, what they constrained are $E_{sym}(\rho_0)$ and the surface symmetry energy (k), rather than L

$$E_a = 4\, a_a(A) \frac{T(T+1)}{A} \qquad \frac{1}{a_a(A)} = \frac{1}{a_a^V} + \frac{A^{-1/3}}{a_a^S} \xrightarrow{A>20} a_a^V, a_a^S, L$$

tions, especially for the slope scaled with a_a^V. Thus, e.g. the analysis of excitation energies of isobaric analog states [97,98] yields independent values of a_a^V and a_a^S. While the volume symmetry coefficient from this type of analysis, $a_a^V \simeq (31.5\text{–}33.5)$ MeV, comes out quite in the middle of values found for the Skyrme interactions, the surface symmetry coefficient, $a_a^S \simeq (9.5\text{–}12)$ MeV, comes out right at the lower end of the values encountered for the Skyrme interactions. The coefficient ratio from that analysis is in the range $a_a^V/a_a^S \simeq (2.8\text{–}3.3)$. That ratio produces the effective surface displacement in the range of $\Delta_e R = (r_0/3)(a_a^V/a_a^S) \simeq (1.06\text{–}1.26)$ fm. Moreover, Figs. 14 and 15 yield the respective ranges of $\Delta R^0 \simeq (0.85\text{–}1.05)$ fm and $L/a_a^V \simeq (2.4\text{–}3.4)$ or $L \simeq (78\text{–}111)$ MeV. The analysis [97,98] is relatively model-independent, provided the curvature effects play little role for heavier nuclei. If the latter were not the case, though, a bit softer symmetry energy would need to be deduced.

$$\kappa \approx \frac{E_{sym}(\rho_0)}{E^{Surf}_{sym}(\rho_0)} = 2.8 - 3.3,\ E_{sym}(\rho_0) \approx 31.5 - 33.5 \text{ MeV}$$

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k parameter: LDM

Liquid Drop model with surface symmetry energy

Note: Actually, what they constrained are $E_{sym}(\rho_0)$ and the surface symmetry energy (k), rather than L

PHYSICAL REVIEW C **82**, 064306 (2010)

Nuclear symmetry energy at subnormal densities from measured nuclear masses

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[1] *Key Laboratory of Beam Technology and Material Modification of Ministry of Education, College of Nuclear Science and Technology, Beijing Normal University, Beijing 100875, China*

[2] *Department of Physics, Guangxi Normal University, Guilin 541004, China*

[3] *Beijing Radiation Center, Beijing 100875, China*

[4] *China Institute of Atomic Energy, Beijing 102413, China*

(Received 3 August 2010; published 13 December 2010)

The symmetry energy coefficients for nuclei with mass number $A = 20$–250 are extracted from more than 2000 measured nuclear masses. With the semiempirical connection between the symmetry energy coefficients of finite nuclei and the nuclear symmetry energy at reference densities, we investigate the density dependence of the symmetry energy of nuclear matter at subnormal densities. The obtained results are compared with those extracted from other methods.

$$\kappa \approx \frac{E_{sym}(\rho_0)}{E^{Surf}_{sym}(\rho_0)} = 2.31 \pm 0.38,\ E_{sym}(\rho_0) \approx 29.4 - 32.8\ \text{MeV}$$

Neutron skin of ^{208}Pb

Jefferson Lab (JLab):
^{208}Pb Radius EXperiments - PREX

K. Kumar, R. Michaels, P. A. Souder, and G. M. Urciuoli (spokespersons) [http://hallaweb.jlab.org/parity/prex].

The Lead Radius Experiment ("PREX"), experiment number E06002, uses the parity violating weak neutral interaction to probe the neutron distribution in a heavy nucleus, namely ^{208}Pb, thus measuring the RMS neutron radius to 1% accuracy, which has an important impact on nuclear theory.

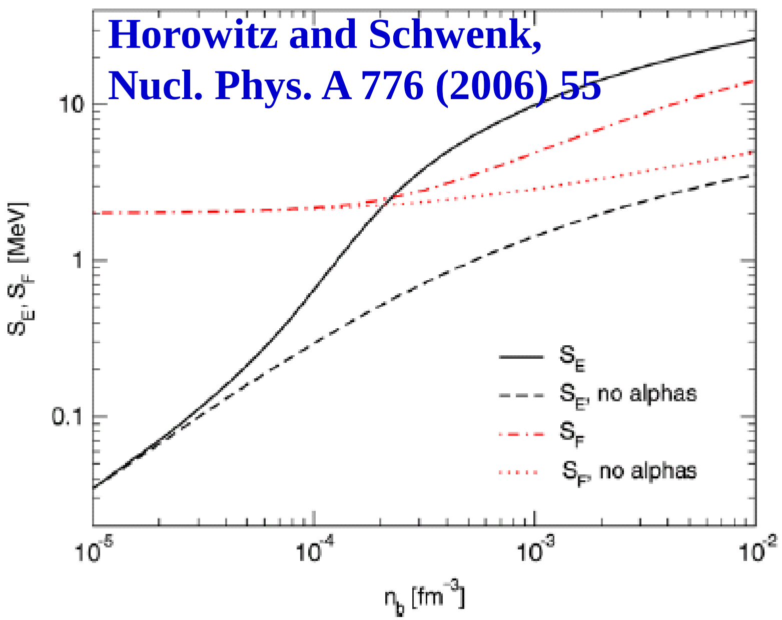
PRL 108, 112502 (2012) PHYSICAL REVIEW LETTERS week ending 16 MARCH 2012

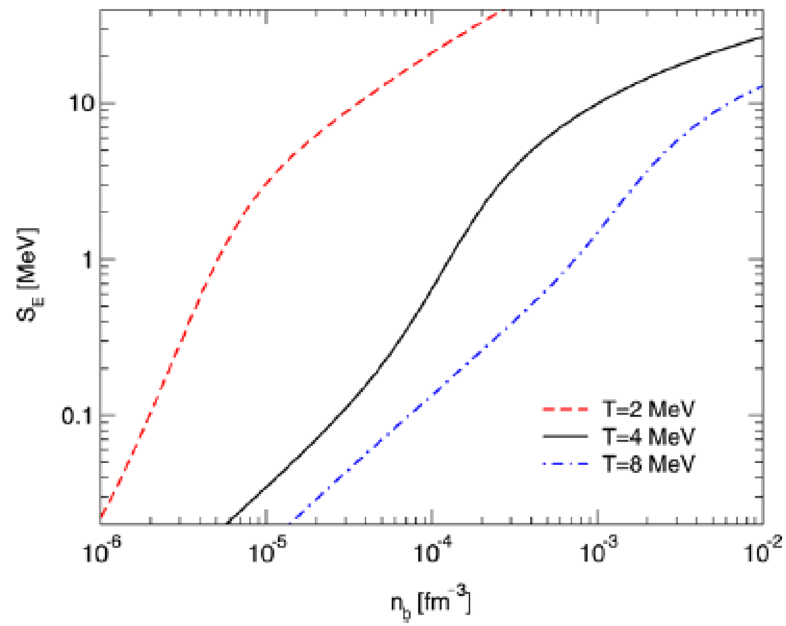
Measurement of the Neutron Radius of ^{208}Pb through Parity Violation in Electron Scattering

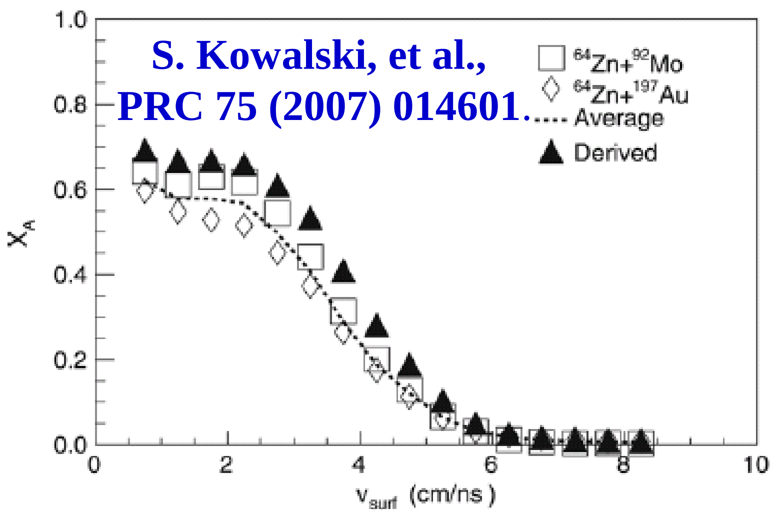
S. Abrahamyan,[38] Z. Ahmed,[29] H. Albataineh,[17] K. Aniol,[2] D. S. Armstrong,[5] W. Armstrong,[31] T. Averett,[5] B. Babineau,[19]

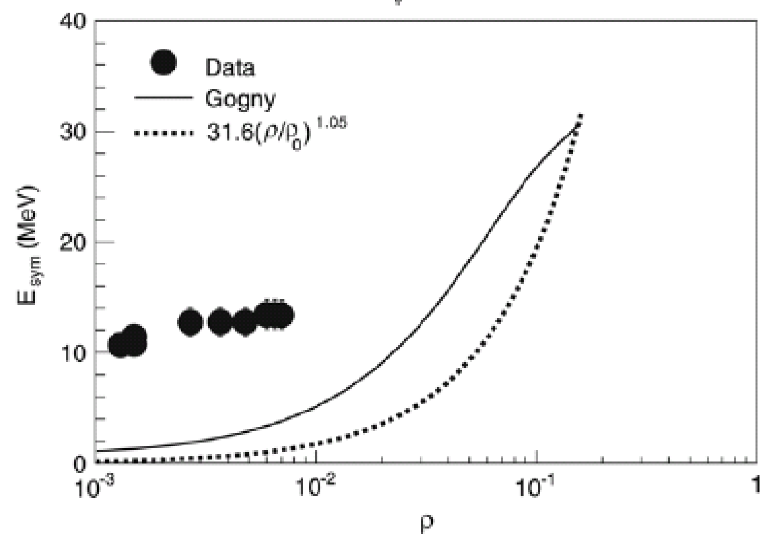
We report the first measurement of the parity-violating asymmetry A_{PV} in the elastic scattering of polarized electrons from ^{208}Pb. A_{PV} is sensitive to the radius of the neutron distribution (R_n). The result $A_{PV} = 0.656 \pm 0.060(\text{stat}) \pm 0.014(\text{syst})$ ppm corresponds to a difference between the radii of the neutron and proton distributions $R_n - R_p = 0.33^{+0.16}_{-0.18}$ fm and provides the first electroweak observation of the neutron skin which is expected in a heavy, neutron-rich nucleus.

E_{sym} at low densities: Clustering Effects









E_{sym} at low densities: Clustering Effects

Selected for a Viewpoint in *Physics*
PRL **104**, 202501 (2010) PHYSICAL REVIEW LETTERS week ending 21 MAY 2010

Symmetry Energy of Dilute Warm Nuclear Matter

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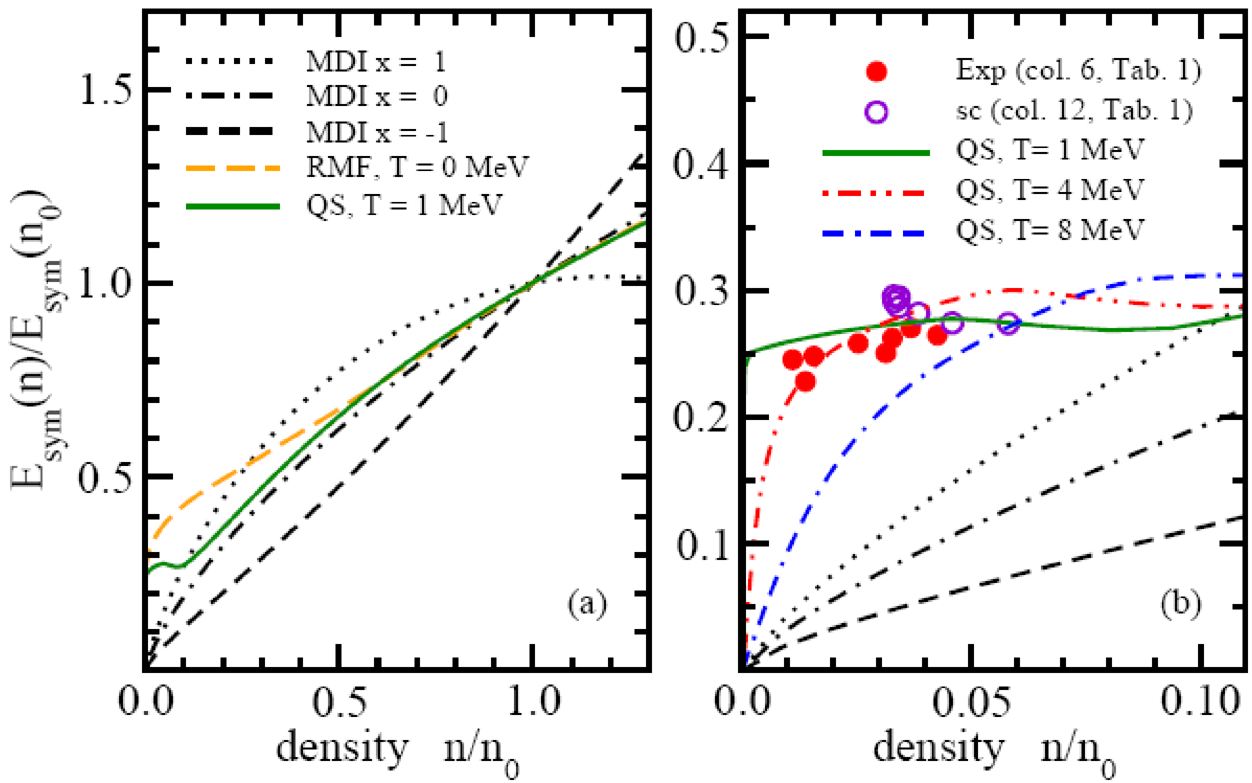
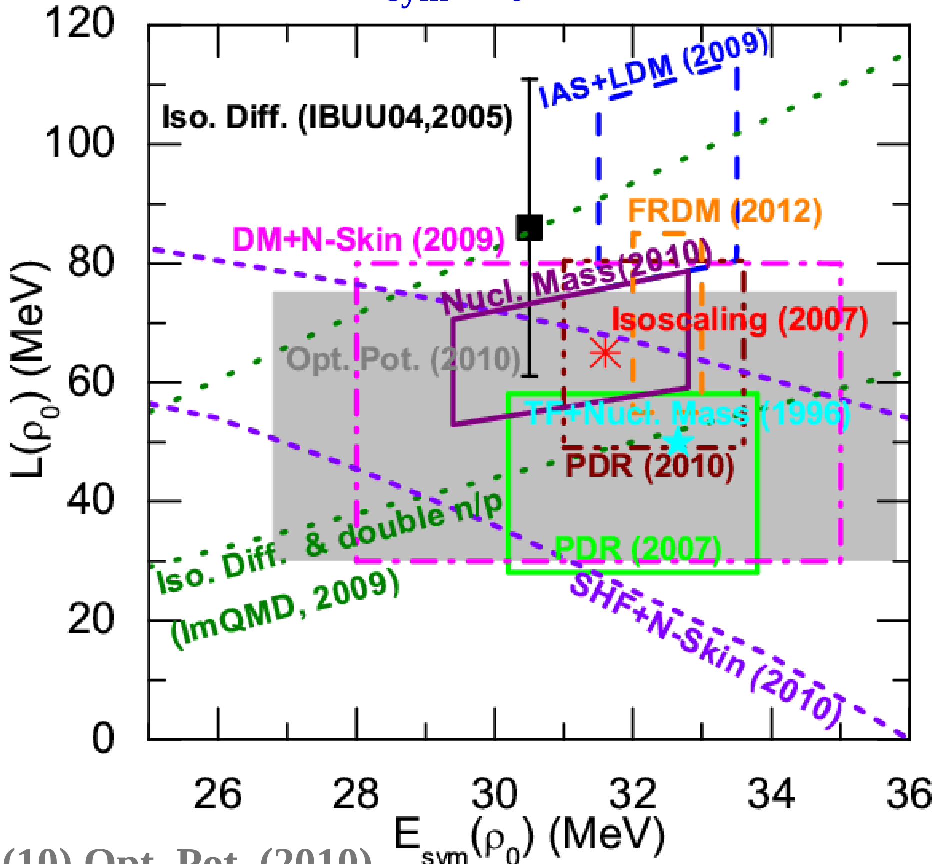


E_{sym}: Around saturation density

Constraints on $E_{sym}(\rho_0)$ and L from nuclear reactions and structures



(1) TF+Nucl. Mass (1996)
Myers/Swiatecki, NPA 601, 141 (1996)

(2) Iso. Diff. (IBUU04, 2005)
L.W. Chen et al., PRL94, 032701 (2005);
B.A. Li/L.W. Chen, PRC72, 064611(2005)

(3) Isoscaling (2007)
D. Shetty et al., PRC76, 024606 (2007)

(4) PDR in ^{130,132}Sn (2007) (LAND/GSI)
A. Klimkiewicz et al., PRC76, 051603(R)(2007)

(5) Iso. Diff. & double n/p (ImQMD, 2009)
M.B. Tsang et al., PRL102, 122701 (2009);

(6) IAS+LDM (2009)
Danielewicz/J. Lee, NPA818, 36 (2009)

(7) DM+N-Skin (2009)
M. Centelles et al., PRL102, 122502 (2009);
M. Warda et al., PRC80, 024316 (2009)

(8) PDR in ^{68}Ni and ^{132}Sn (2010)
A. Carbon et al., PRC81, 041301(R)(2010)

(9) SHF+N-Skin (2010)
L.W. Chen et al., PRC82, 024321 (2010)

(10) Opt. Pot. (2010)
C. Xu et al., PRC82, 054607 (2010)

(11) Nucl. Mass (2010)
M. Liu et al., PRC82, 064306 (2010)

(12) FRDM (2012)
P. Moller et al., PRL108, 052501 (2012)

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Optimization

The simulated annealing method (Agrawal/Shlomo/Kim Au, PRC72, 014310 (2005))

Taking other MSL parameters into account, we optimize other 9 parameters for a fixed $L(\rho_c)(E_{sym}(\rho_c))$, by minimizing

$$\chi_{op}^2 = \sum_i^N \left(\frac{M_i^{th} - M_i^{\exp}}{\sigma_i} \right)^2$$

Where, N is the number of experimental data points, M^{th} and M^{exp} is the theoretical and the corresponding experimental values and σ_i is the theoretical error

Experimental data

Binding energy per nucleon and charge rms radius of 25 spherical even-even nuclei (G.Audi et al., Nucl.Phy.A729 337(2003), I.Angeli, At.Data.Nucl.Data.Tab 87 185(2004))

$^{204,202,200,198,196,194,192,190}_{82}\mathrm{Pb}$, $^{124,122,120,118,116,112,108}_{50}\mathrm{Sn}$, $^{64,62,60,58}_{28}\mathrm{Ni}$, and $^{50,48,46,44,42,40}_{20}\mathrm{Ca}$.

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Optimization

Constraints:

- The neutron $3p_{1/2}$-$3p_{3/2}$ splitting in ^{208}Pb lies in the range of 0.8-1.0 MeV
- The pressure of symmetric nuclear matter should be consistent with constraints obtained from flow data in heavy ion collisions

  P. Danielewicz, R. Lacey and W.G. Lynch, Science 298, 1592 (2002)
- The binding energy of pure neutron matter should be consistent with constraints obtained the latest chiral effective field theory calculations with controlled uncertainties

  I. Tews, T. Kruger, K. Hebeler, and A. Schwenk, PRL 110, 032504 (2013)
- The critical density ρ_{cr}, above which the nuclear matter becomes unstable by the stability conditions from Landau parameters, should be greater than 2 ρ_0
- The isoscalar nucleon effective mass m^*_{s0} should be greater than the isovector effective mass m^*_{v0}, and here we set $m^*_{s0} - m^*_{v0} = 0.1m$ (m is nucleon mass in vacuum) to be consistent with the extraction from global nucleon optical potentials constrained by world data on nucleon-nucleus and (p,n) charge-exchange reactions and also dispersive optical model for Ca, Ni, Pb

C. Xu, B.A. Li, and L.W. Chen, PRC82, 054607 (2010); Bob Charity, DOM (2011)

Determine $E_{sym}(0.11\ fm^{-3})$ from ΔE

- To constrain $E_{sym}(\rho_c)$, we select 19 spherical isotope pairs and use their binding energy per nucleon.

G.Audi et al., Nucl.Phy.A729 337(2003)

- Similarly, we calculate

$$\chi^2_{dE} = \sum_{i=1}^{12}\left(\frac{\Delta E_i^{\exp} - \Delta E_i^{th}}{\sigma_i}\right)^2$$

Here σ_i is the theoretical error and we set it as 23% ΔE^{th}.

$2\sigma: E_{sym}(0.11\mathrm{fm}^{-3}) = 26.16^{+0.28}_{-0.27}\mathrm{MeV}$

constrain $E_{\rm sym}(\rho_c)$ from ΔE, here we select 19 heavy isotope pairs which are all spherical even-even nuclei, namely, $^{218,206}_{86}$Rn, $^{216,194}_{84}$Po, $^{214,178}_{82}$Pb, $^{212,178}_{82}$Pb, $^{210,178}_{82}$Pb, $^{208,178}_{82}$Pb, $^{206,178}_{82}$Pb, $^{206,172}_{80}$Hg, $^{136,106}_{52}$Te, $^{132,100}_{50}$Sn, $^{132,102}_{50}$Sn, $^{132,104}_{50}$Sn, $^{132,106}_{50}$Sn, $^{132,110}_{50}$Sn, $^{132,114}_{50}$Sn, $^{130,98}_{48}$Cd, $^{124,96}_{46}$Pd, $^{94,84}_{42}$Mo, and $^{94,82}_{40}$Zr.