Heavy Ion Meeting 20313-05 , Asan Science Hall, Korea University, May 24 2013

Symmetry energy in the era of advanced gravitational wave detectors

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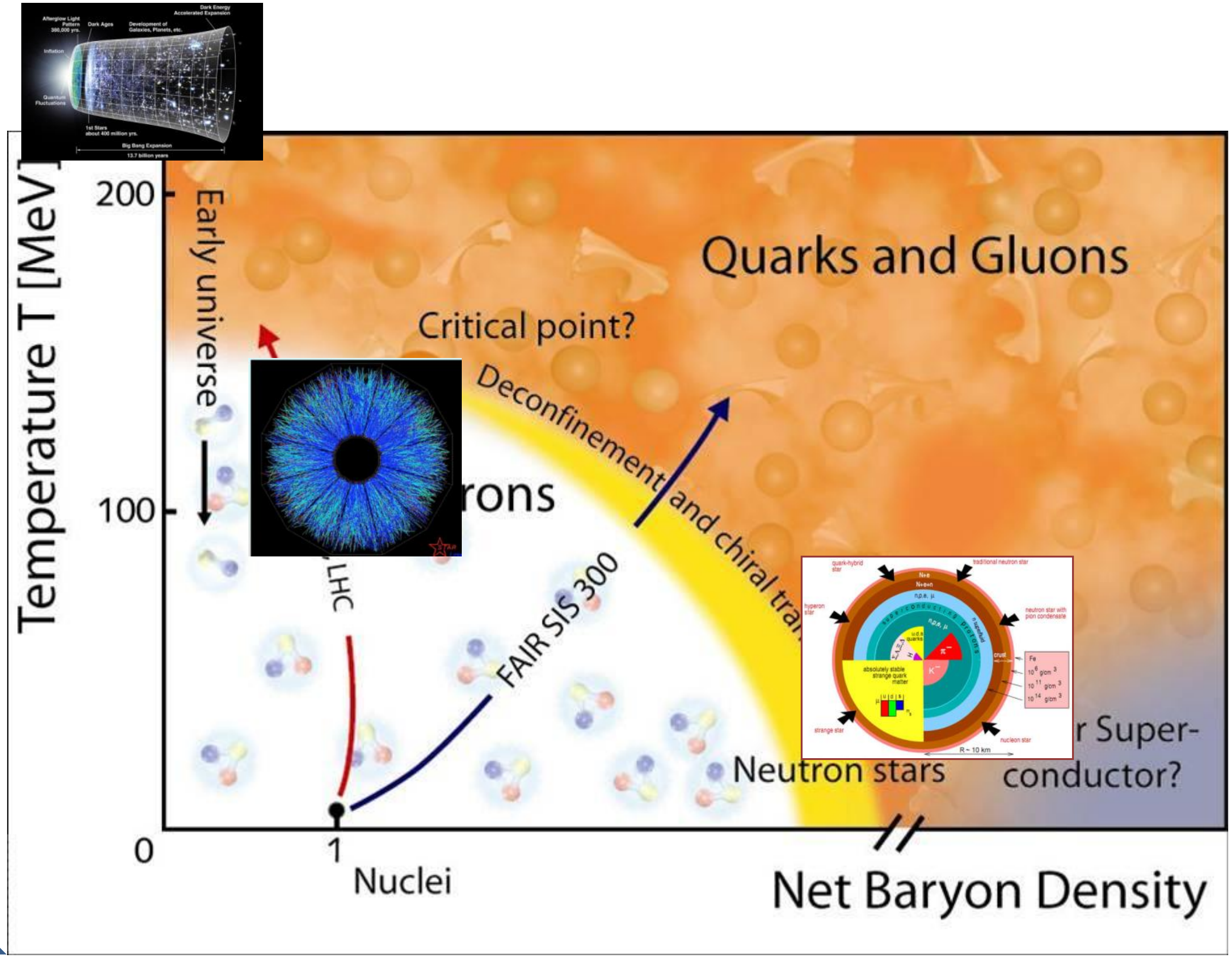
Hanyang University

극한조건에서의 강입자 물질 연구사업단
Hadronic Matter under Extreme Conditions

0. Introduction

Temperature frontier



Density frontier

High baryon number density

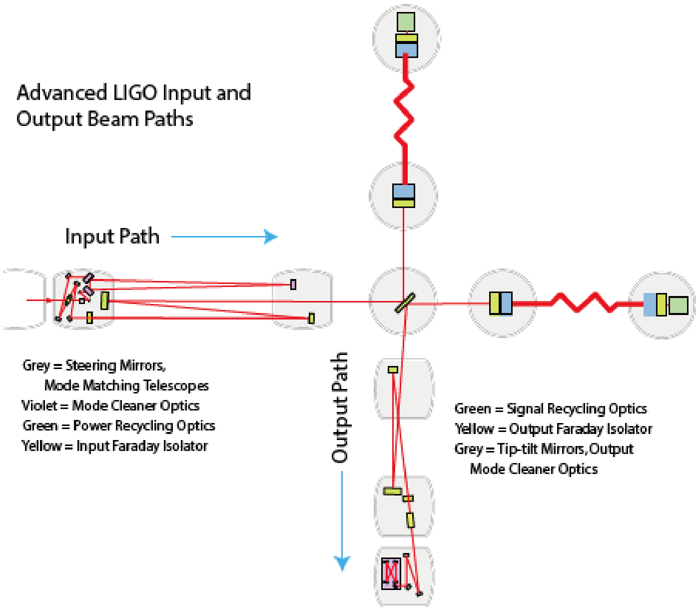
- RIB Machines(FAIR, RAON, ..): $n > n_0$
- Neutron star

  1 solar mass inside 10km :

  $n \sim 10^{15} g/cm^3 \gg n_0$

→ Binary coalescence : Gravitational wave

I. Advanced LIGO: 2015 →



The Advanced LIGO beam layout. Note the folded input and output paths.

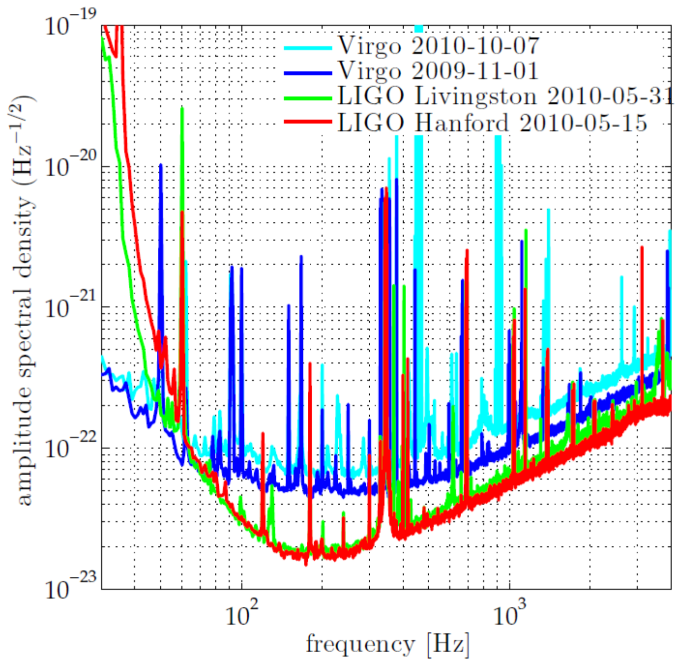
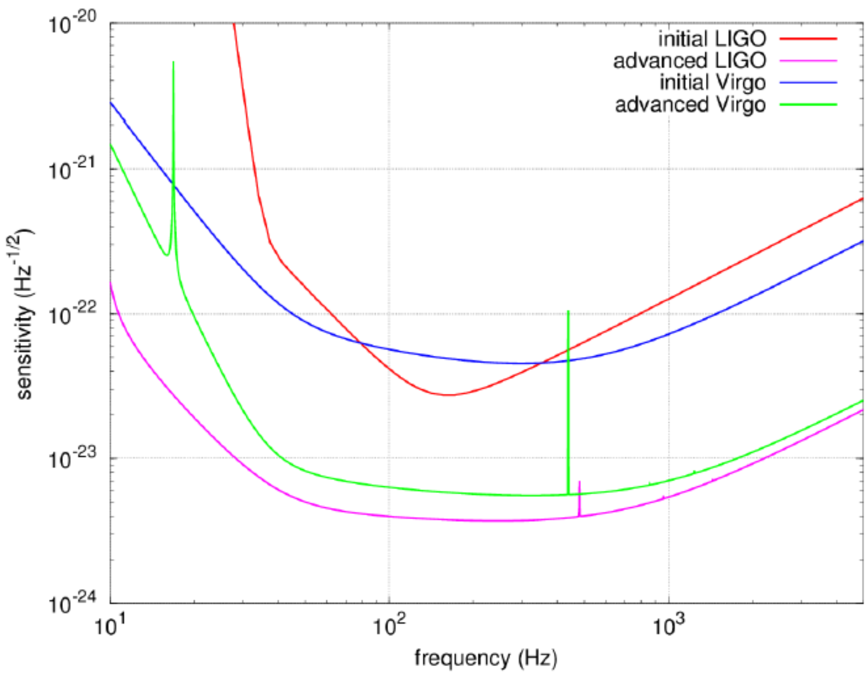


FIG. 1.— Best strain noise spectra from the LIGO and Virgo detectors during the 2009-2010 science runs.

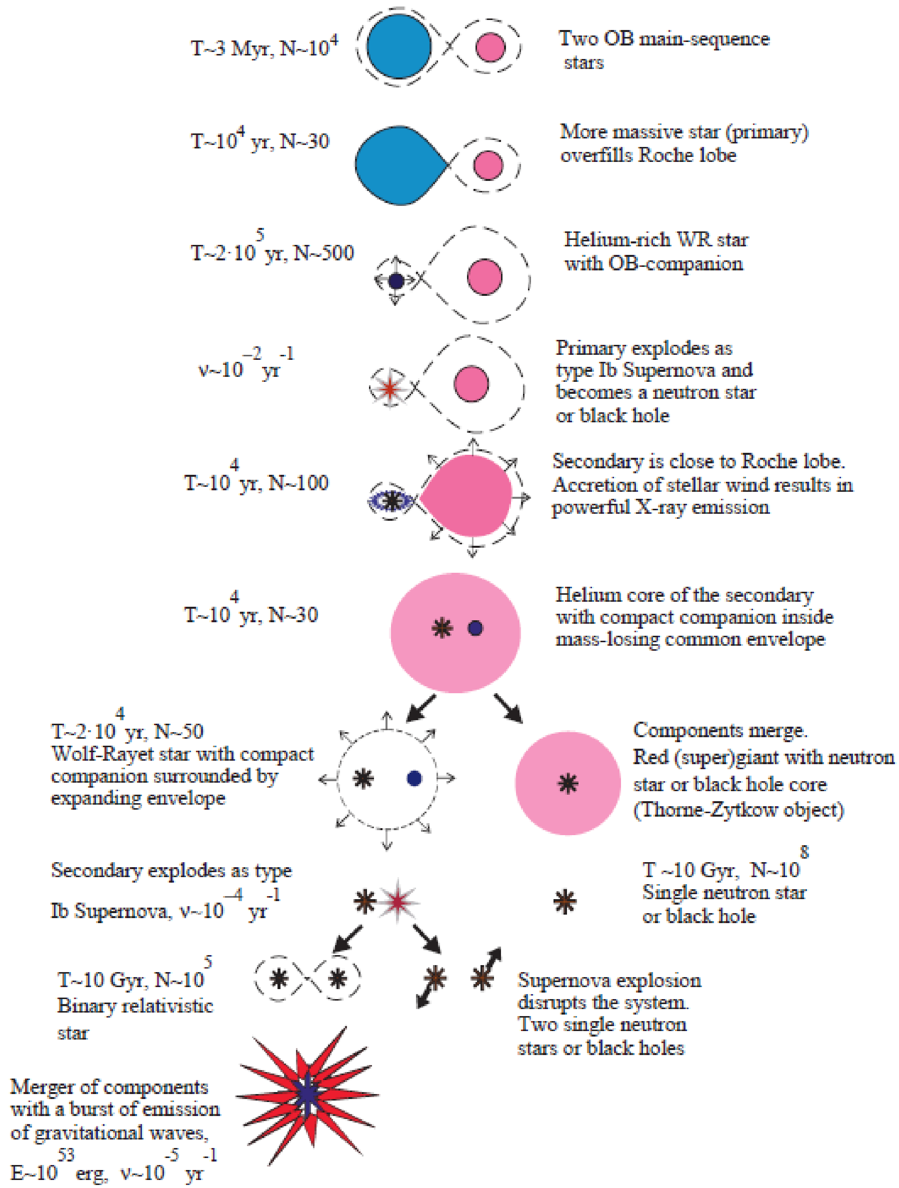


LIGO, 1205.2216

Eric Chassande-Mottin, 1210.7173

Improvement of Sensitivity : 10-times

- Formation and merging of binary compact stars



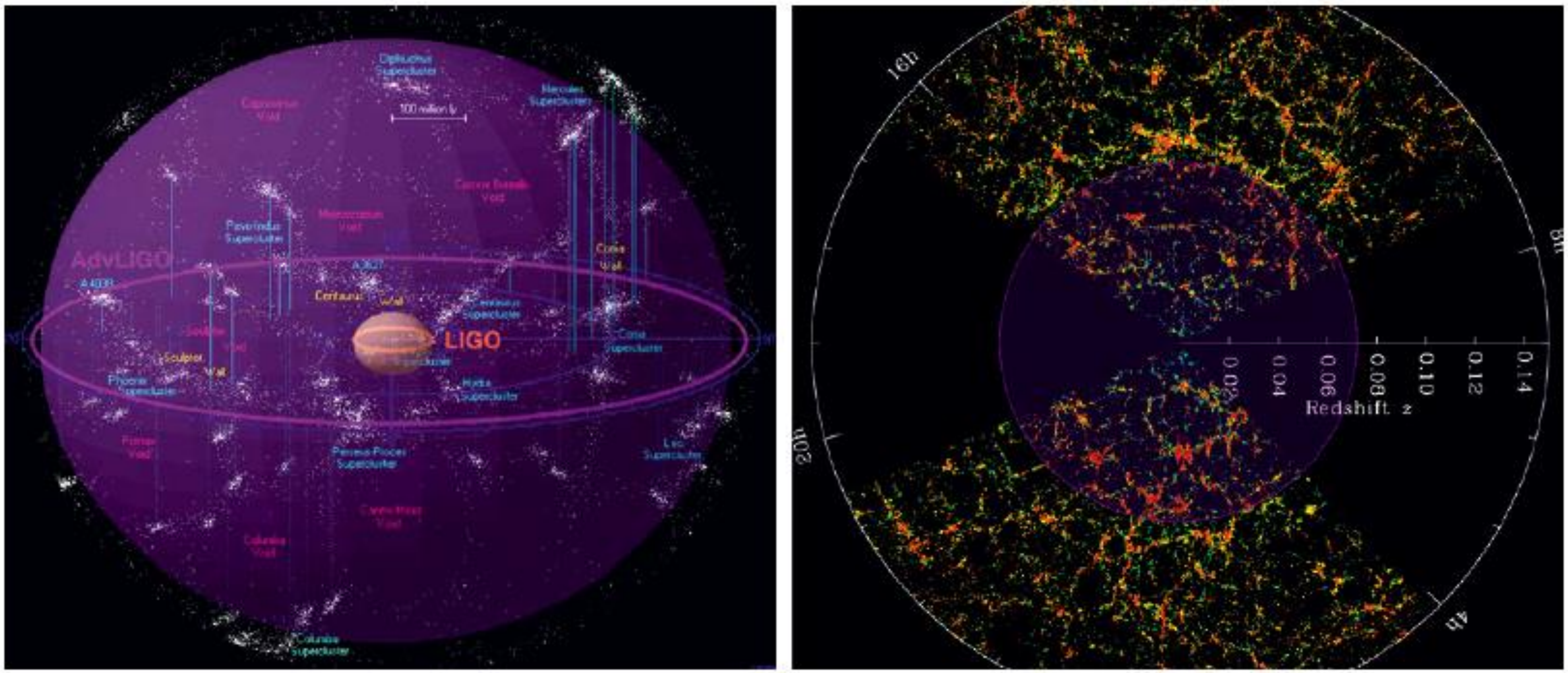
- Higher rate of GW detection
  NS-NS coalescence : 0.02/yr → 40/yr

Table 5. Detection rates for compact binary coalescence sources.

IFO	Source[a]	$\dot{N}_{low}$ yr^{-1}	$\dot{N}_{re}$ yr^{-1}	$\dot{N}_{high}$ yr^{-1}	$\dot{N}_{max}$ yr^{-1}
	NS–NS	2×10^{-4}	0.02	0.2	0.6
	NS–BH	7×10^{-5}	0.004	0.1	
Initial	BH–BH	2×10^{-4}	0.007	0.5	
	IMRI into IMBH			<0.001[b]	0.01[c]
	IMBH-IMBH			10^{-4}[d]	10^{-3}[e]
	NS–NS	0.4	40	400	1000
	NS–BH	0.2	10	300	
Advanced	BH–BH	0.4	20	1000	
	IMRI into IMBH			10[b]	300[c]
	IMBH-IMBH			0.1[d]	1[e]

J.Abadie, et al. Class. Quantum Grav. 27,173001(2010)

GW amplitude detection: increase of 10 times sensitivity
→ increase of 10^3 times volume for observation.

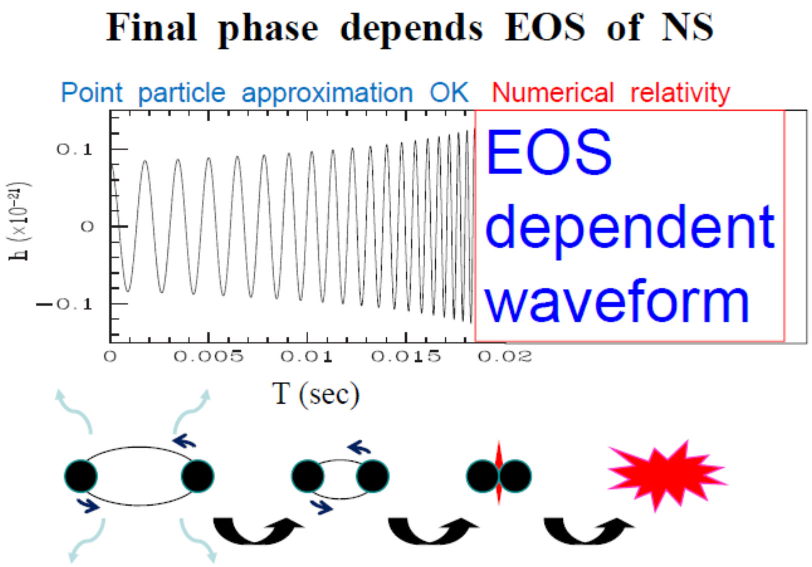


The superclusters within 300 Mpc (left) and the Sloan Digital Sky Survey (right)

- Current and future GW interferometric detectors

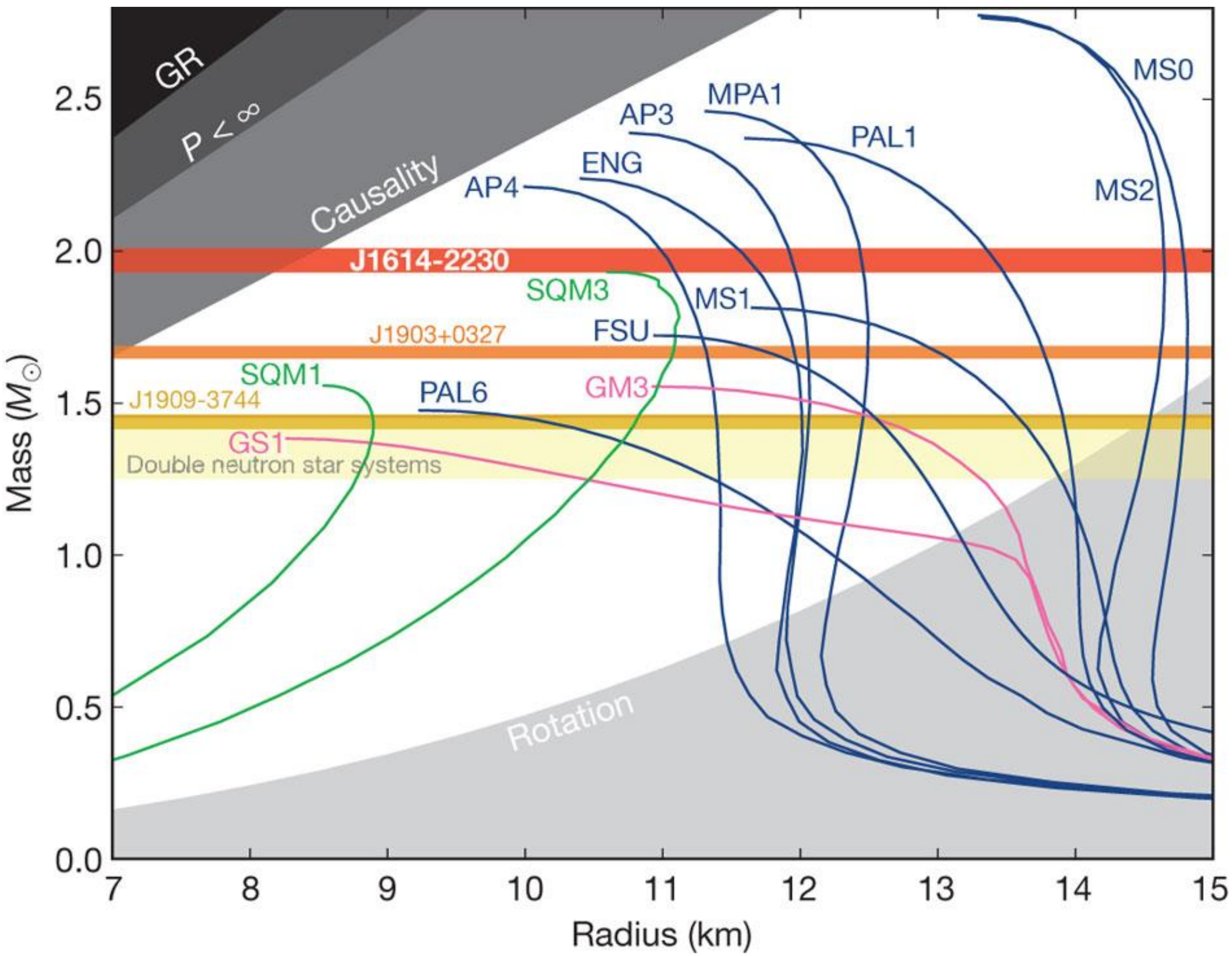
Eric Chassande-Mottin, 1210.7173

- EoS in Binary merger
  a. inspiral period: mass
  b. coalescing period : EoS (soft or stiff)



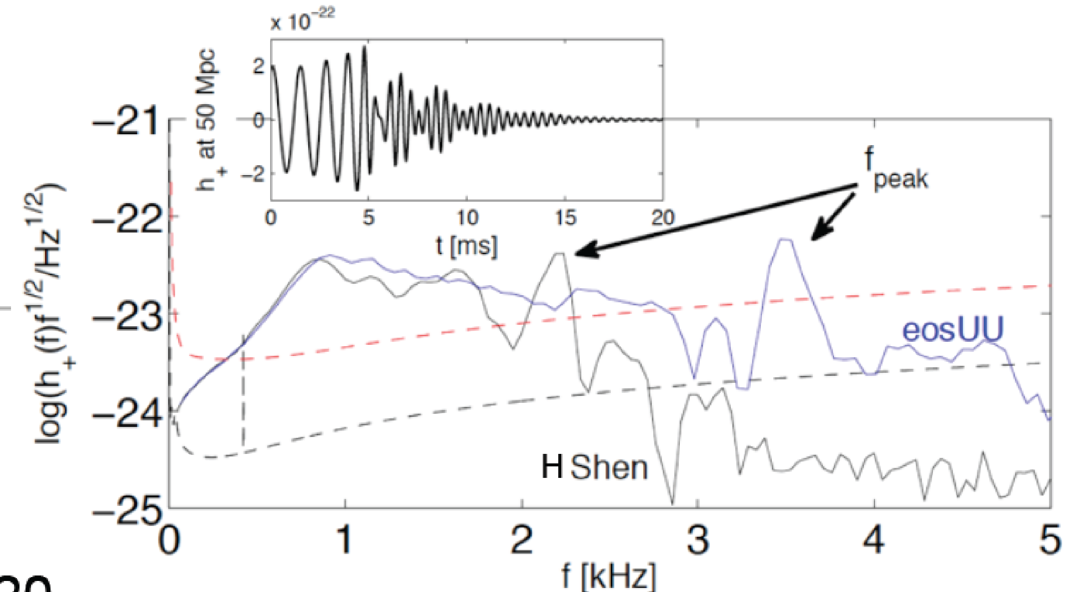
Shibata, 2011

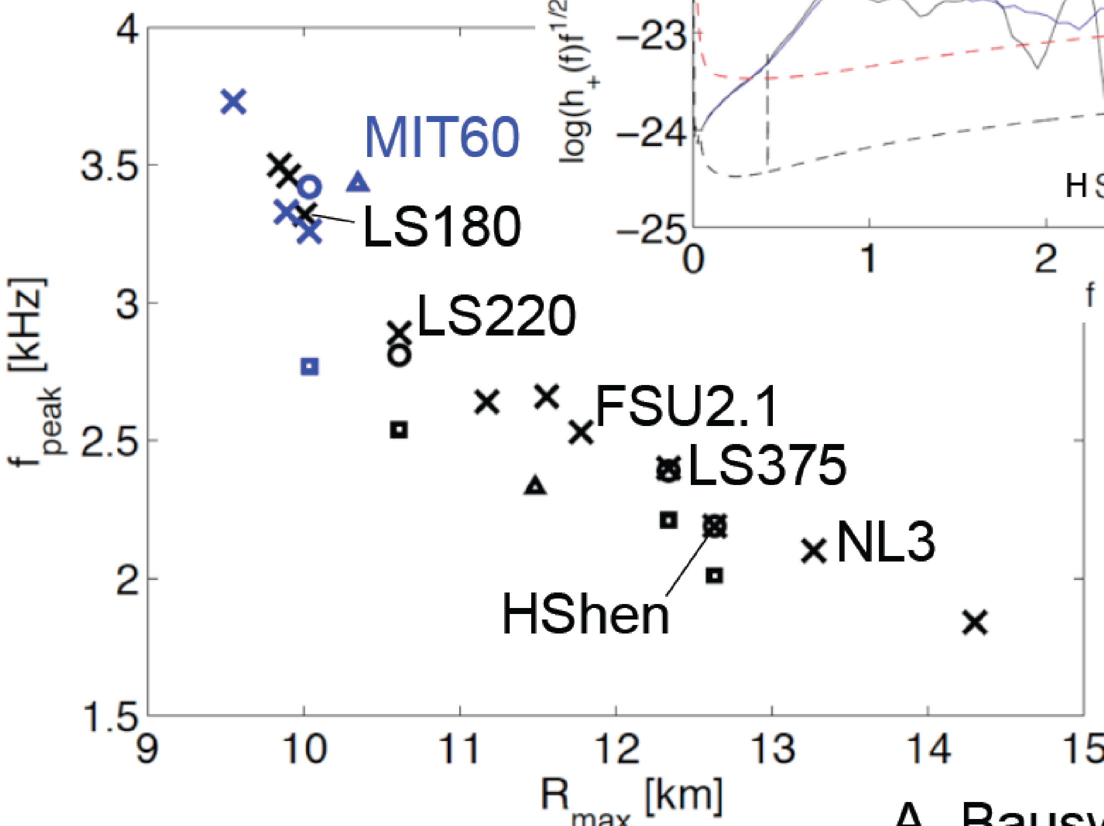
constraint on EoS from NS observations



Gravitational Waves from NS mergers and EOS

Merger of two
1.35 M_{sun} NS





Frequency of peak GW signal correlated with radius of max. mass star.

A. Bauswein + H. T. Janka

Effect of hyperons

Sekiguchi, Kiuchi, Kyutoku, Schibata PRL 107, 211101(2011)

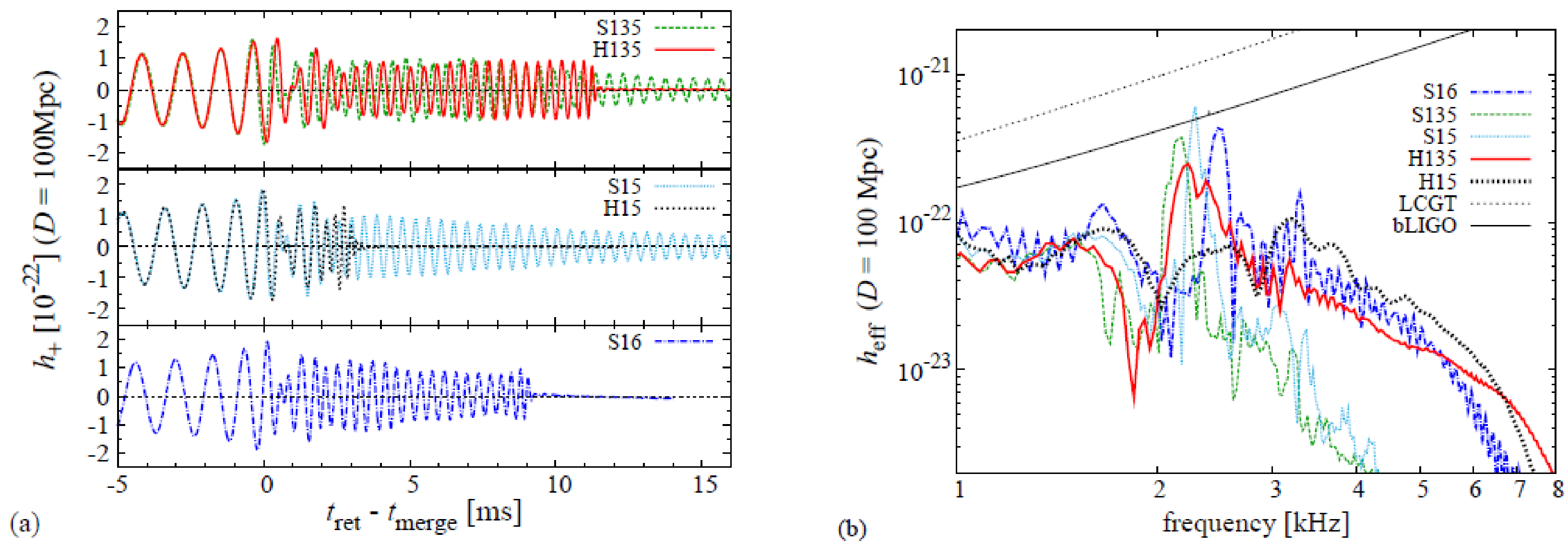


FIG. 4: (a) GWs observed along the axis perpendicular to the orbital plane for the hypothetical distance to the source $D = 100$ Mpc. (b) The effective amplitude of GWs defined by $0.4f|h(f)|$ as a function of frequency for $D = 100$ Mpc. The noise amplitudes of a broadband configuration of Advanced Laser Interferometer Gravitational wave Observatories (bLIGO), and Large-scale Cryogenic Gravitational wave Telescope (LCGT) are shown together.

II. Symmetry energy

Nuclei

Mass formula (Weizsäcker, 1935; Bethe and Bacher, 1936)
Binding energy of a nucleus ($A = Z + N$)

$$\mathcal{B}(N,Z) = b_{\rm vol}A - b_{\rm surf}A^{2/3} - \frac{1}{2}b_{\rm sym}\frac{(N-Z)^2}{A} - \frac{3}{5}\frac{Z^2e^2}{R_c}$$

Nuclear matter

$$x = \frac{n_p}{n}$$

$$E(n,x) = m_N + \frac{3}{5}E_F^0\left(\frac{n}{n_0}\right)^{2/3} + S(n)(1-2x)^2 + V(n)$$

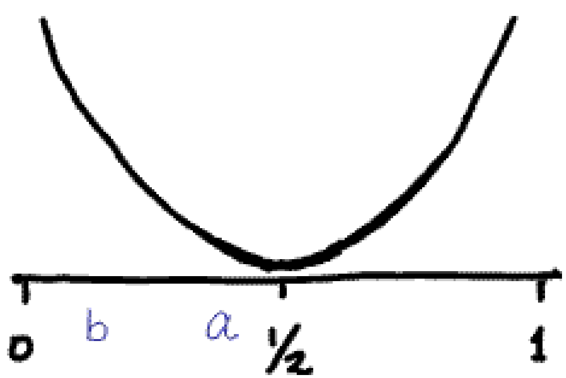
Symmetry energy:
measure of n-p asymmetry in nuclear matter

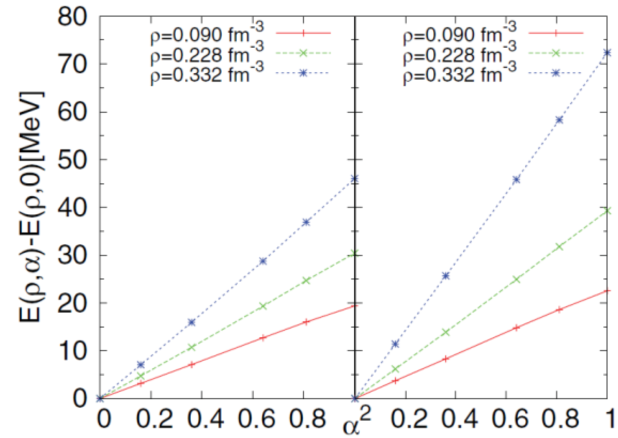
- Driving force toward symmetric matter

$$E(n,x) \cong E(n, x=1/2) + (1-2x)^2 S_{1/2}(n)$$

$$S_{1/2} = \frac{1}{8}\frac{\partial^2 E(n,x)}{\partial x^2}\Big|_{x=1/2}$$

$$E_{symm}(n,x) = (1-2x)^2 S(n)$$





J.M. Lattimer and M. Prakash, Phys. Rep. (2000),
H.Dong, T.T.S. Kuo, R. Machleitd, PRC 83(2011)

Iso-spin symmetry and symmetry energy

Consider a system of nucleon with N_p protons and N_n neutrons

$$|N_p, N_n >= \Sigma_I C_I |I : N_p, N_n >$$

where $|\vec{I}|^2 = I(I+1)$.
Energy of the state

$$\begin{aligned} E(N, x) =< N, x|H|N, x > &= \Sigma_I |C_I(n, x)|^2 E_I(n) \\ &= (1-2x)^2 S(n) + \cdots \end{aligned}$$

E_I is an reduced matrix element of the Hamiltonian, $H = H_0 + H_{int}$,

$$E_I =< I||H||I >$$

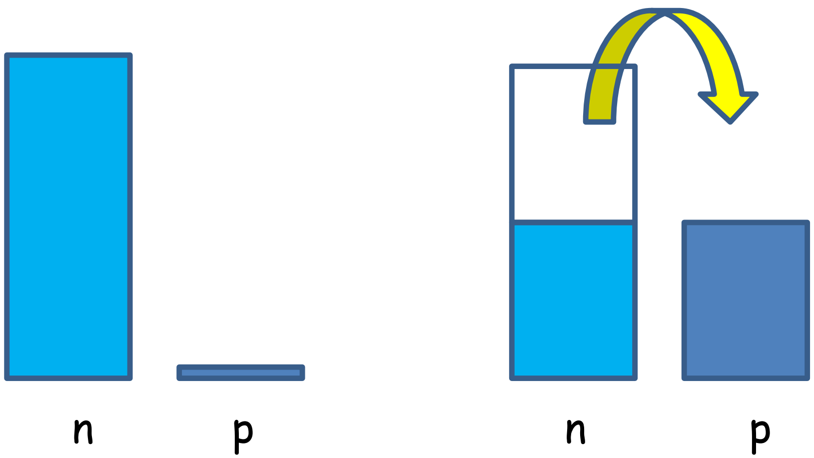
which is independent of I_3, and depends on the details of the strong interactions for each I-channel.

example: SU(2) Casimir operator and Pauli principle

- Tensor force

$$V_M^T(r) = S_M \frac{f_{NM}^2}{4\pi} m_M \tau_1 \cdot \tau_2 S_{12} \left(\left[\frac{1}{(m_M r)^3} + \frac{1}{(m_M r)^2} + \frac{1}{3 m_M r} \right] e^{-m_M r} \right)$$

Pauli exclusion principle for nucleon

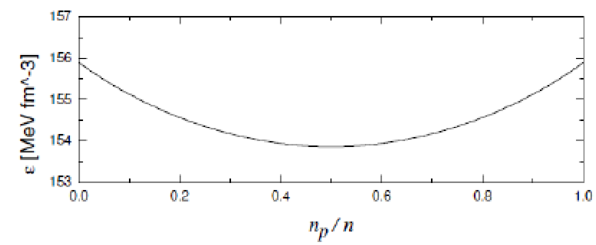


E(x~0) > E(x=1/2)

- Non-interacting n-p system

$$\epsilon_n = \frac{8\pi}{(2\pi)^3}\int_0^{p_F} \left(p^2 + m_n^2\right)^{1/2} p^2 dp$$

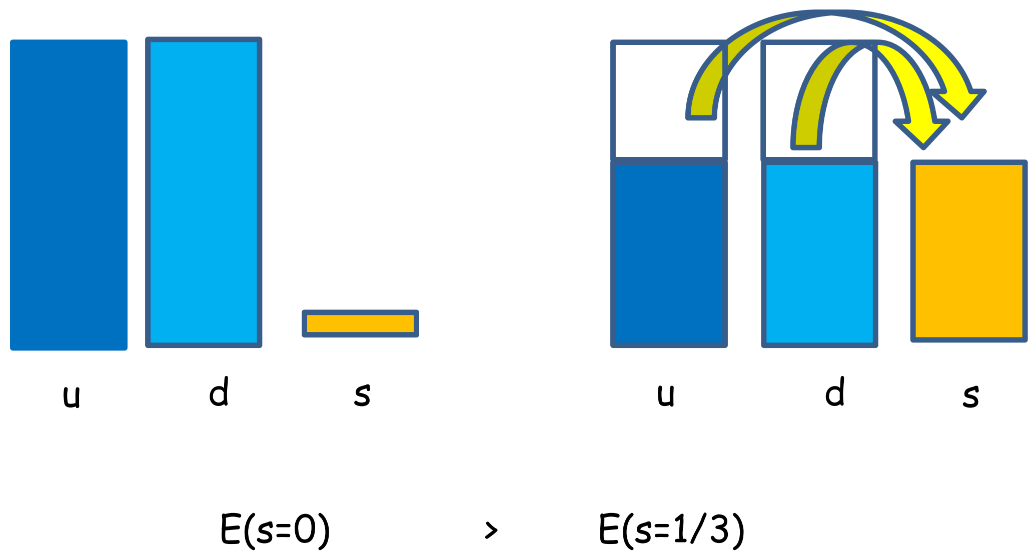
$$n_n = \frac{8\pi}{(2\pi)^3}\int_0^{p_F} p^2 dp$$



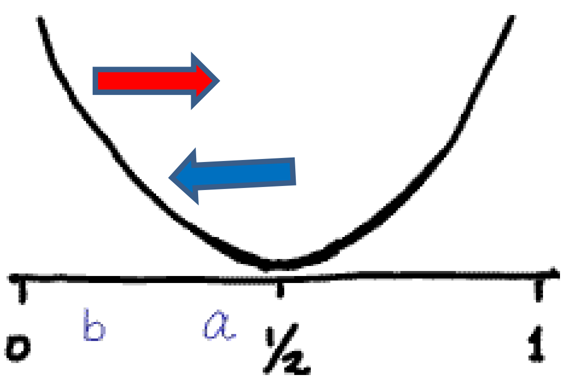
$$S_{free}(n) = \left(2^{2/3} - 1\right)\frac{3}{5}E_F^0\left(\frac{n}{n_0}\right)^{2/3}$$

→ additional degrees of freedom reduce energy of the baryonic matter

- Strange quark matter (SQM) Witten(1984)
  - More stable than nucleon matter at higher density
  - Energy for u, d and s quark matter is lower than u and d quark matter → SQM



- Examples of asymmetric configuration
  a. Heavier nuclei with N> Z
    Coulomb interaction due to proton
      → less proton number
        (asymmetric config.)
    Strong interaction
      symmetry energy → symmetric config.



x ~ 1/2

b. Neutron star matter

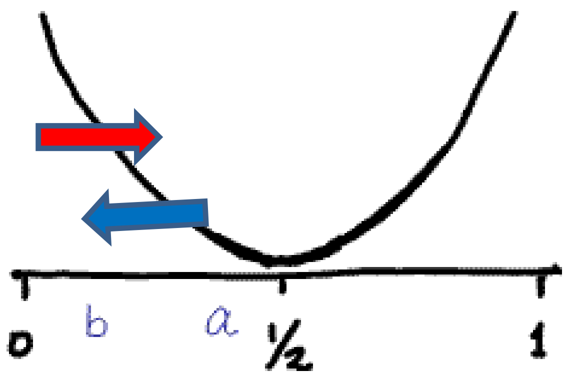
N >> Z → N ~ Z

Strong interaction

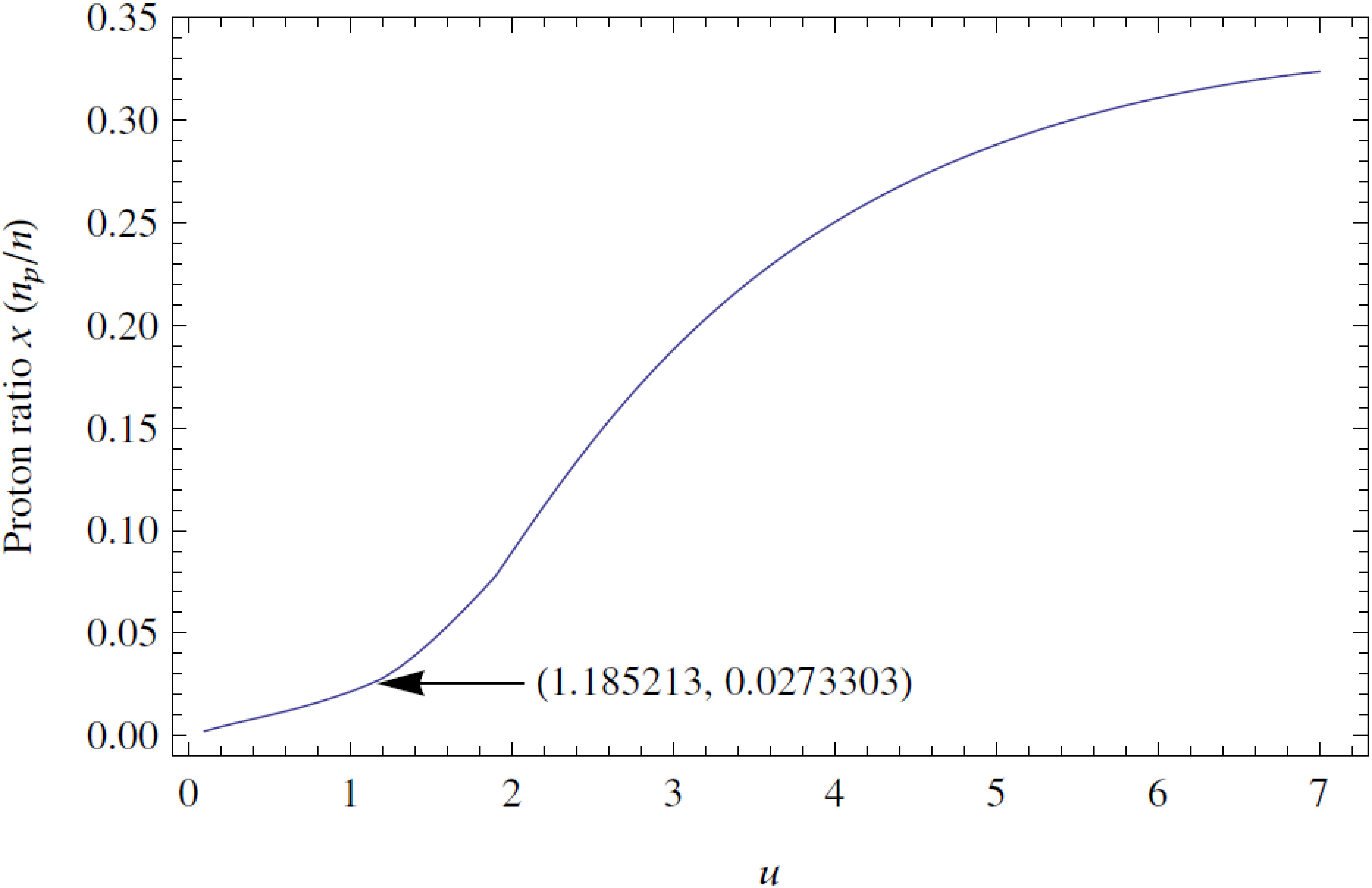
Symmetry energy → more protons

Charge neutrality and Weak interaction :

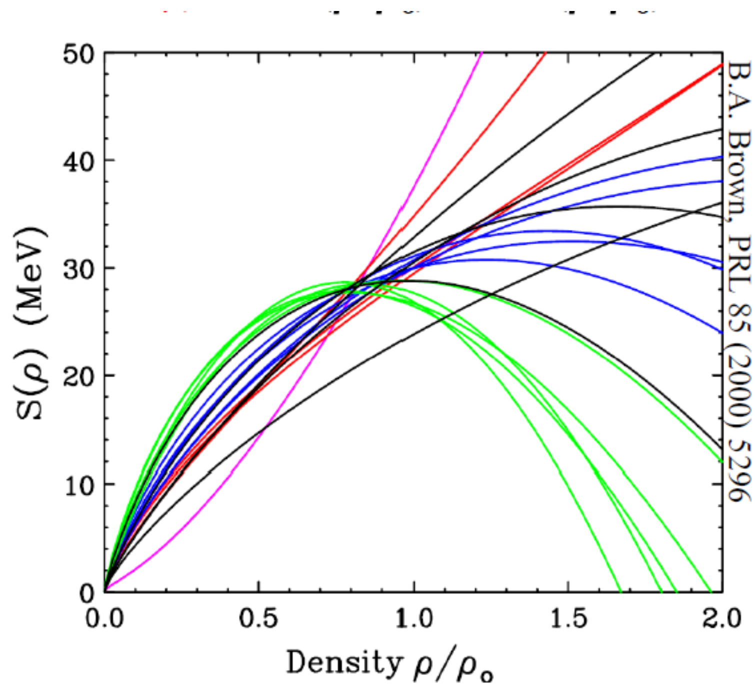
→ Weak equilibrium

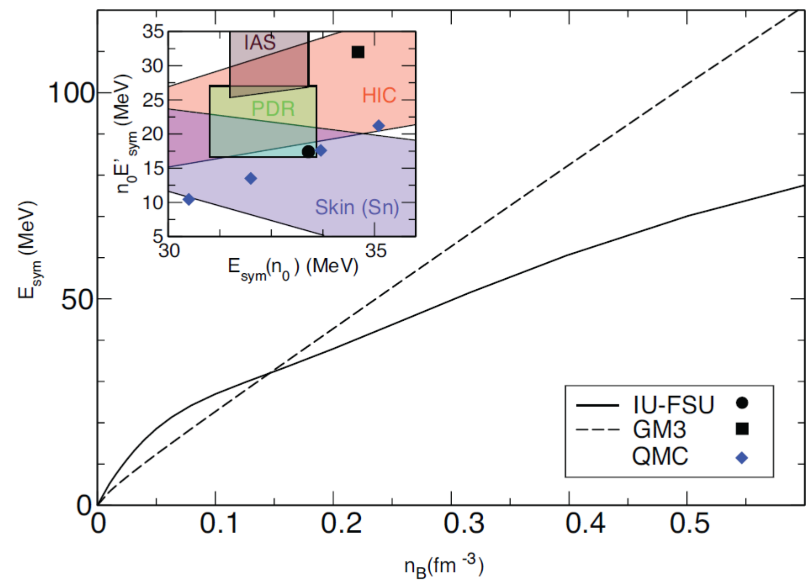


x ~ 0 - 1/2



0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
Proton ratio x (n_p/n)
(1.185213, 0.0273303)
0
1
2
3
4
5
6
7
u





L.F. Roberts et al. PRL 108, 1103(2012)

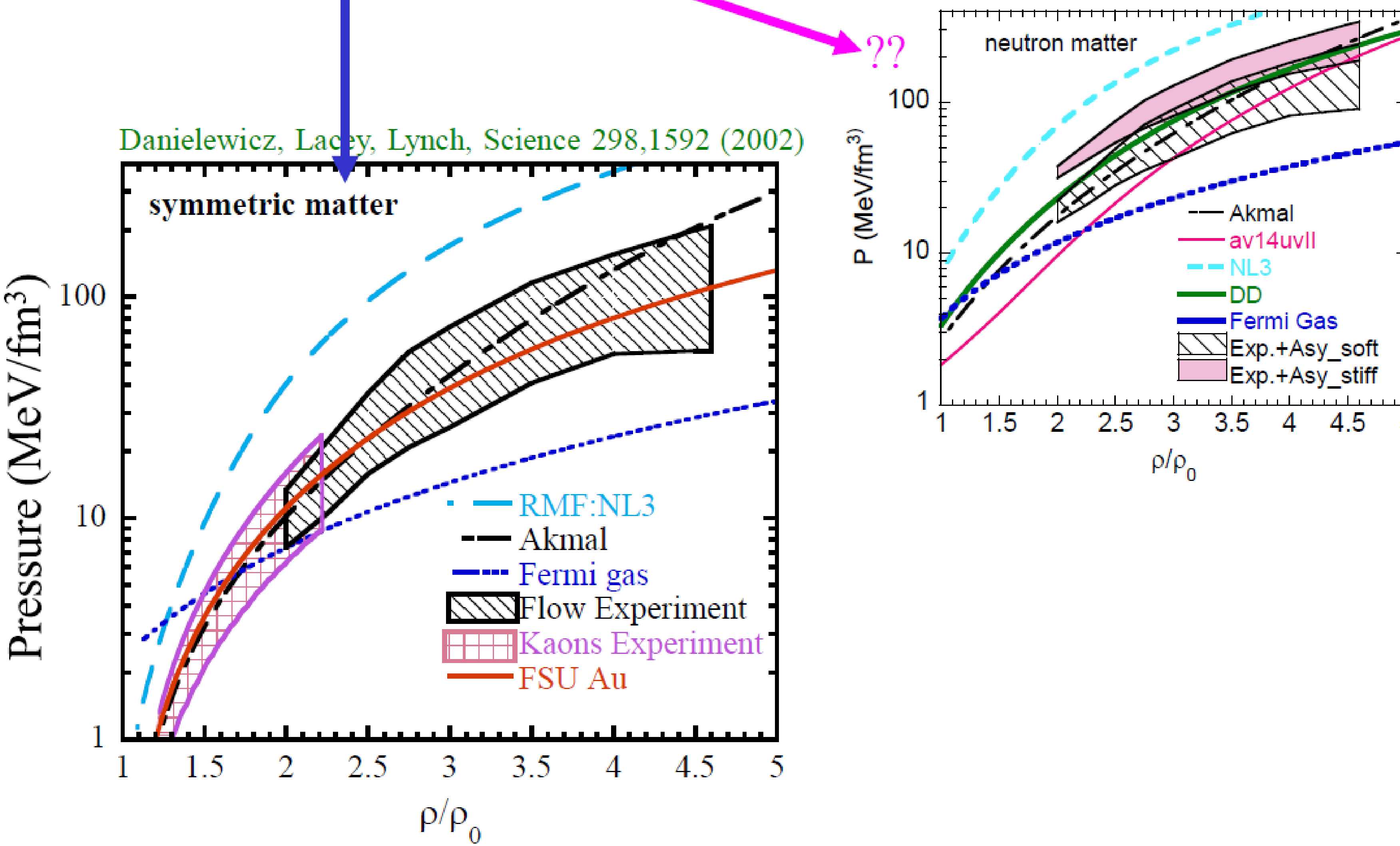
S(n) beyond n_0: ?

Density dependence of Symmetry Energy

$E/A(\rho,\delta) = E/A(\rho,0) + \delta^2 \cdot S(\rho)$; $\delta = (\rho_n - \rho_p)/(\rho_n + \rho_p) = (N-Z)/A$

??

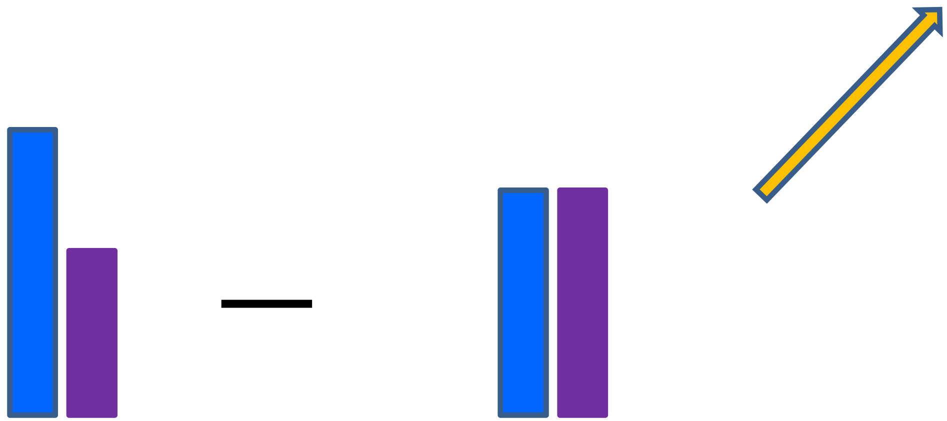
Danielewicz, Lacey, Lynch, Science 298,1592 (2002)



- Anatomy of symmetry energy

HKL and M. Rho 2013, arXiv:1201. 6486

$$\epsilon(n,x) = \epsilon(n, x=1/2) + \epsilon_{sym}(n,x)$$



$$\epsilon(n,x) = \epsilon(n, x=1/2) + nS(n)(1-2x)^2$$

$$\Delta_{nn}(n) = \epsilon(n, 0) - 2\epsilon(n/2, x = 0)$$

$$\Delta_{np}(n) = \epsilon(n, x = 1/2) - 2\epsilon(n/2, x = 0)$$

$$\epsilon_{sym}(n, 0) = \Delta_{nn}(n) - \Delta_{np}(n)$$

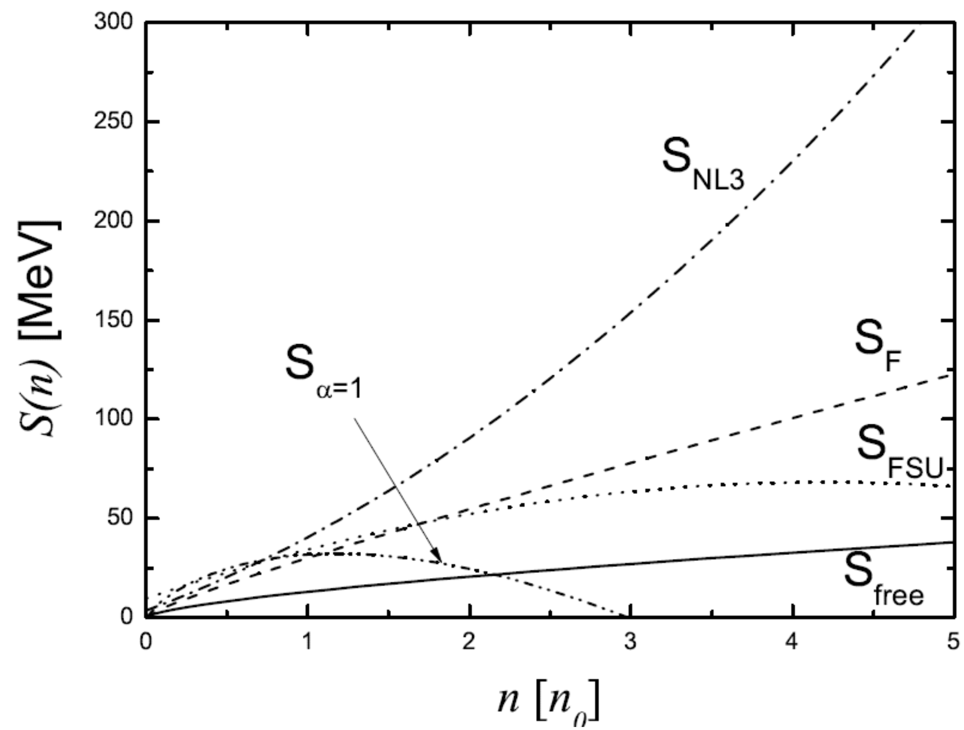
$$\boxed{S(n) = [\Delta_{nn}(n) - \Delta_{np}(n)]/n}$$

Phenomenological forms of symmetry energy

$$S_F(n) = (2^{2/3} - 1)\frac{3}{5}E_F^0\left[\left(\frac{n}{n_0}\right)^{2/3} - F(n)\right] + S_0 F(n)$$

$$S_\alpha = (2^{2/3} - 1)\frac{3}{5}E_F^0\left(\frac{n}{n_0}\right)^{2/3} + A(\alpha)\frac{n}{n_0} + [18.6 - A(\alpha)]\left(\frac{n}{n_0}\right)^{B(\alpha)}$$

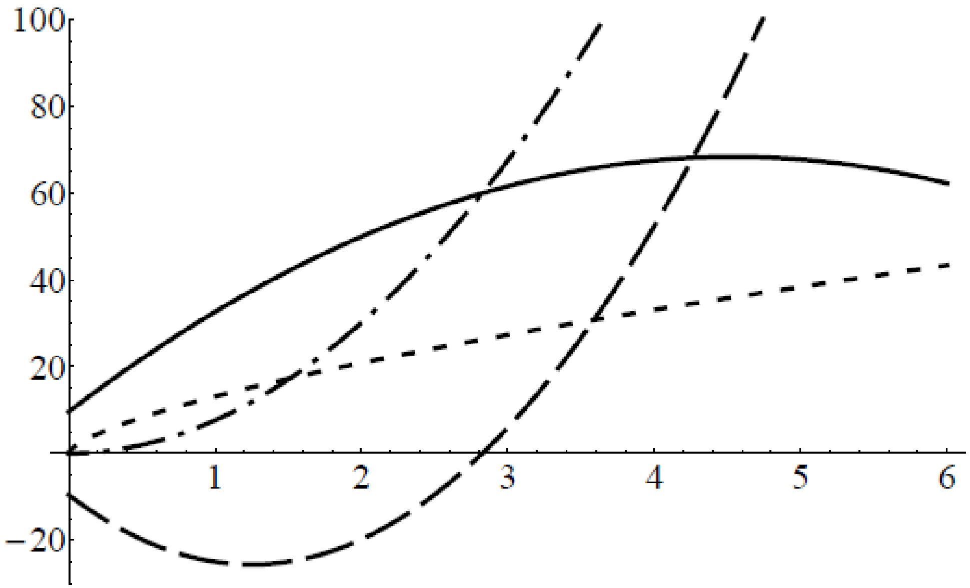
$$S_3(n) \simeq S_0^* + L\rho + \frac{1}{2}K\rho^2$$



Model	J	L	$K_{\rm sym}$	K_0	ϵ_0
FSU	32.59	60.5	−51.3	230	−16.30
NL3	37.29	118.2	100.9	271.5	−16.24

$$\Delta_{nn} = n\left[\frac{1}{2}K_0(\tilde{u}^2 - \bar{u}^2) + L(\tilde{u} - \bar{u}) + \frac{1}{2}K_{\rm sym}(\tilde{u}^2 - \bar{u}^2)\right]$$

$$\Delta_{np} = n\left[\frac{1}{2}K_0(\tilde{u}^2 - \bar{u}^2) - \left(J + L\bar{u} + \frac{1}{2}K_{\rm sym}\bar{u}^2\right)\right] ,$$



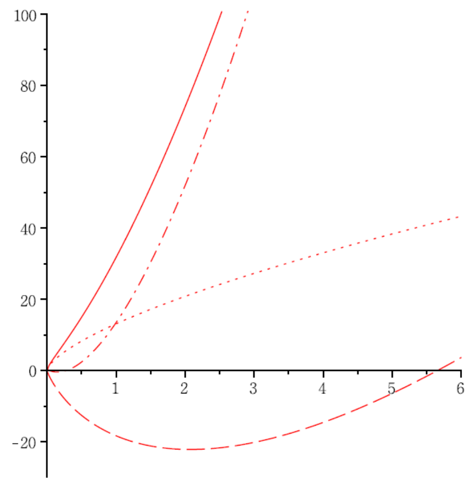


Figure 1: The symmetry energy factor $S(n)$ (thick line), $\Delta np/n$ (dashed), Δ_{nn}/n(dash-dotted) and the symmetry energy for non-interacting nucleons (dotted) for the LCK model with $\alpha = -1$.

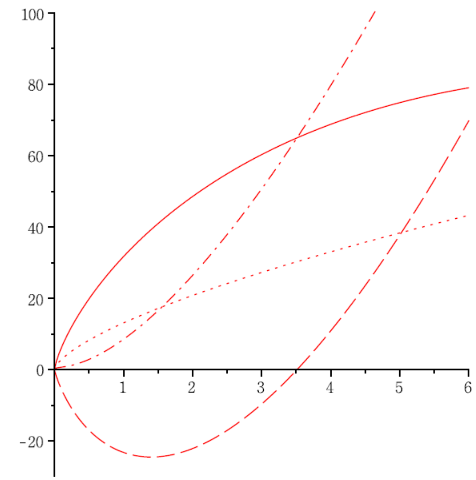


Figure 2: Same as Fig. 1 for the LCK model with $\alpha = 0$.

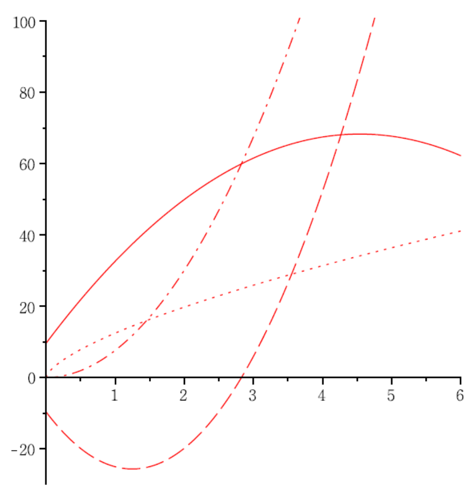


Figure 4: Same as Fig. 1 for the FSU model.

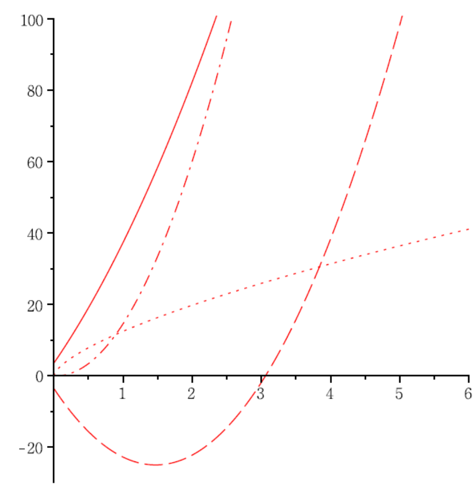


Figure 5: Same as Fig. 1 for the NL3 model.

- Tensor force

$$V_M^T(r) = S_M \frac{f_{NM}^2}{4\pi} m_M \tau_1 \cdot \tau_2 S_{12} \left(\left[\frac{1}{(m_M r)^3} + \frac{1}{(m_M r)^2} + \frac{1}{3 m_M r} \right] e^{-m_M r} \right)$$

III. Half skyrmion and new scaling(BLPR)

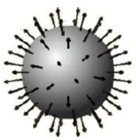
single hedgehog skyrmion

1960, T. H. R. Skyrme

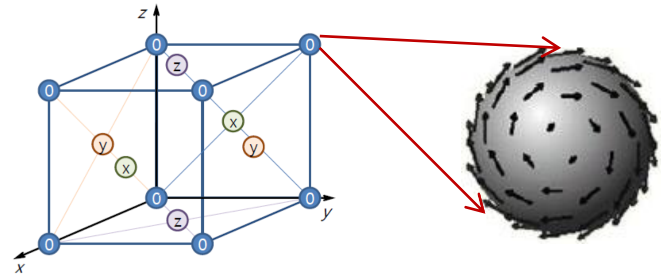
$$\mathcal{L} = \frac{f_\pi^2}{4}\,\mathrm{Tr}(\partial_\mu U^\dagger \partial^\mu U) + \frac{1}{32e^2}\,\mathrm{Tr}[U^\dagger \partial_\mu U, U^\dagger \partial_\nu U]^2$$

U(x̄) : mapping from R^3-{∞}=S^3 to *SU(2)*=S^3

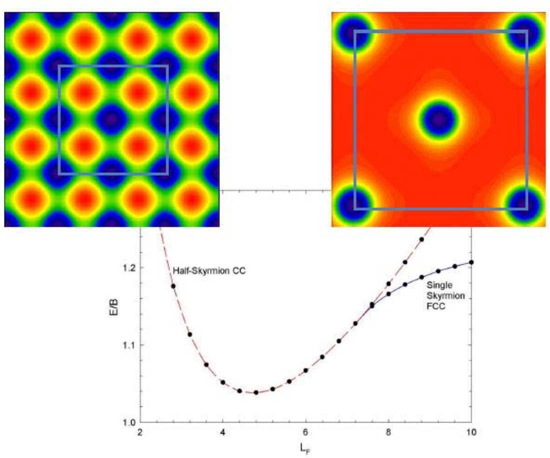
→ topological soliton



R ~ 1 f m → baryon ?
M ~ 1.5 GeV

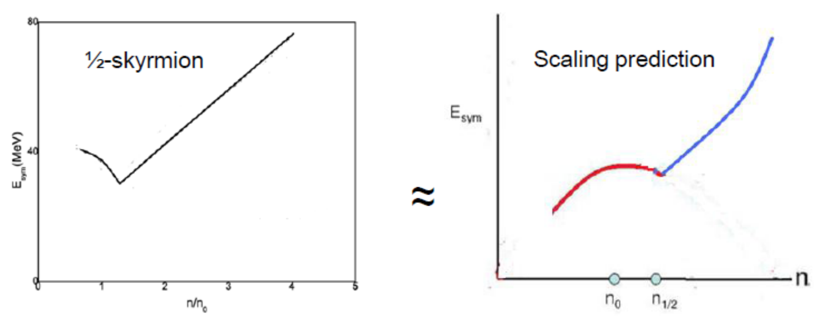


Skyrmion Crystal



Goldhaber and Manton(1987)
Byung-yoon Park et al. (2003), …

Effect on symmetry energy



HKL, Park and Rho, PRC83, 025206(2011)

(a) Suppression of rho-mediated tensor force

$$V_M^T(r) = S_M \frac{f_{NM}^2}{4\pi} m_M \tau_1 \cdot \tau_2 S_{12} \left(\left[\frac{1}{(m_M r)^3} + \frac{1}{(m_M r)^2} + \frac{1}{3 m_M r} \right] e^{-m_M r} \right)$$

$$M = \pi, \rho, \; S_{\rho(\pi)} = +1(-1)$$

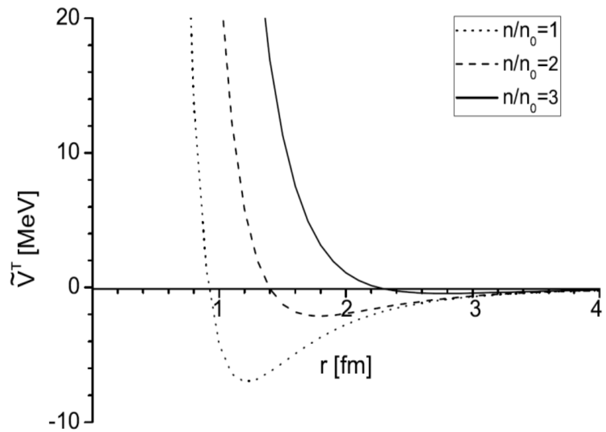


FIG. 1: Sum of π and ρ tensor forces $\tilde{V}^T \equiv (\tau_1 \cdot \tau_2 S_{12})^{-1}(V_\pi^T + V_\rho^T)$ in units of MeV for densities n/n_0 =1, 2 and 3 with the "old scaling," $\Phi \approx 1 - 0.15n/n_0$ and $R \approx 1$ for all n.

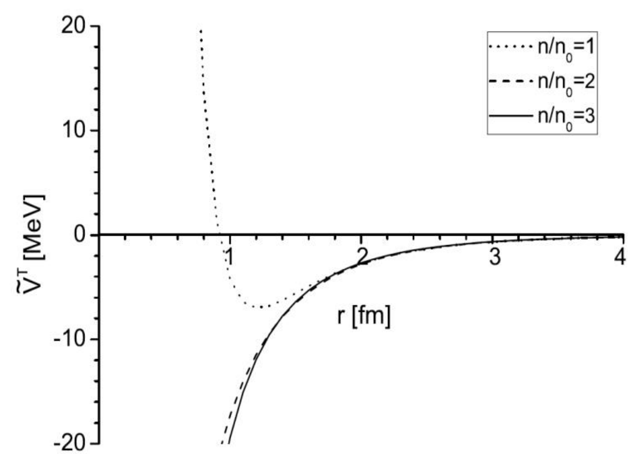
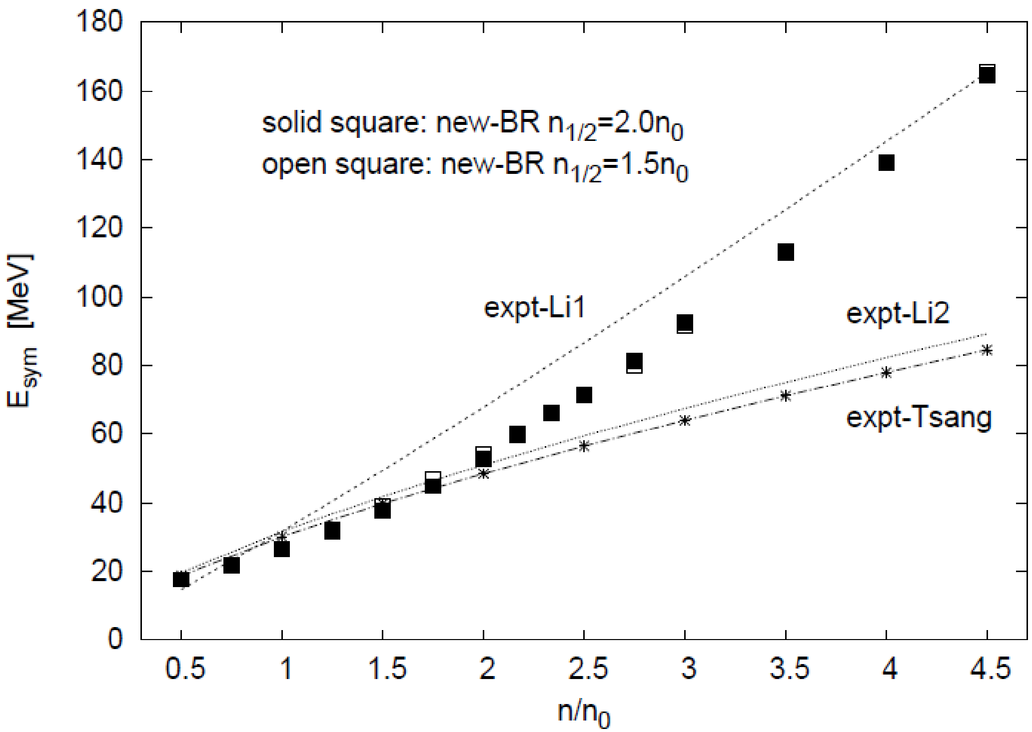


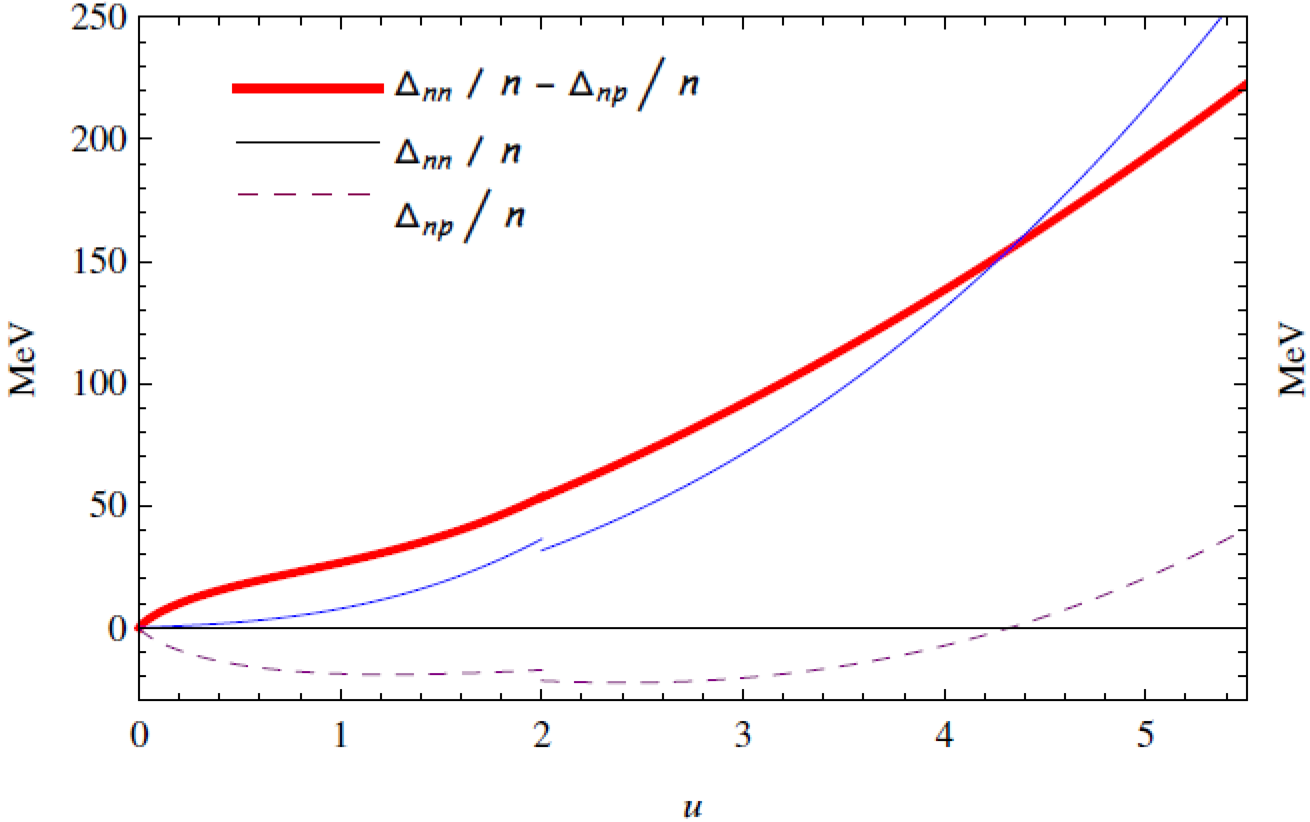
FIG. 2: The same as Fig. 1 with the "new scaling," $\Phi \approx 1 - 0.15n/n_0$ with $R \approx 1$ for $n < n_{1/2}$ and $R \approx \Phi^2$ for $n > n_{1/2}$, assuming $n_0 < n_{1/2} < 2n_0$.

(b) stronger attraction in n-p channel
→ stiffer symmetry energy S(n)

$$S(n) = E(n, n_p = 0) - E(n, n_n = n_p)$$

Dong, Kuo, HKL,Machleidt, Rho(12)

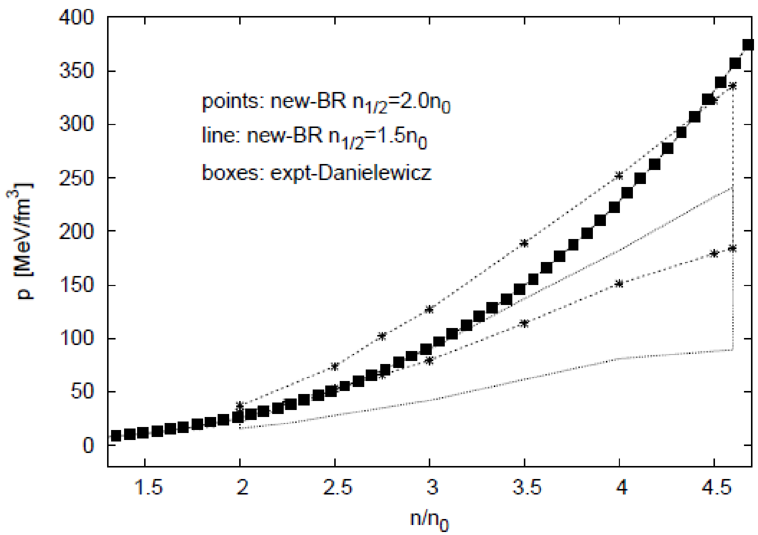


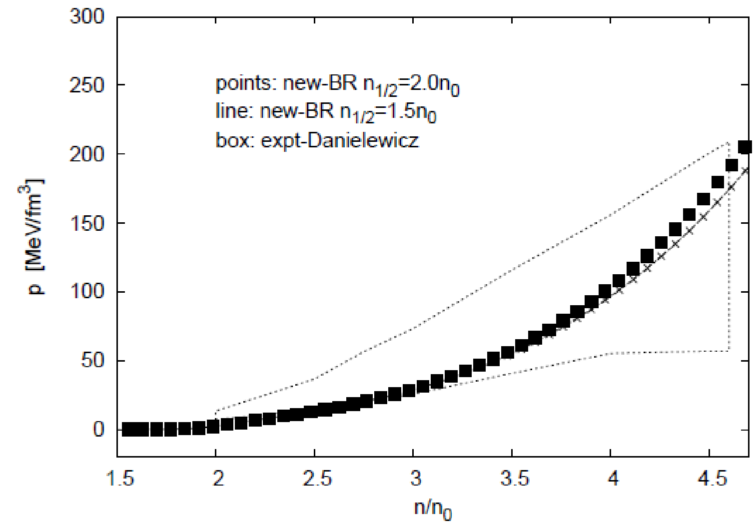


250
200
150
100
50
0
MeV
MeV
0
1
2
3
4
5
u
Δnn / n − Δnp / n
Δnn / n
Δnp / n

(c) EoS with new scaling, BLPR

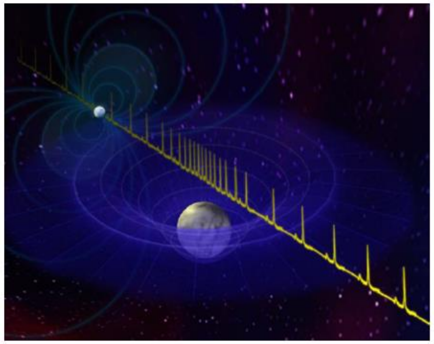
Dong, Kuo, HKL, Machleidt, Rho, 1207.0429



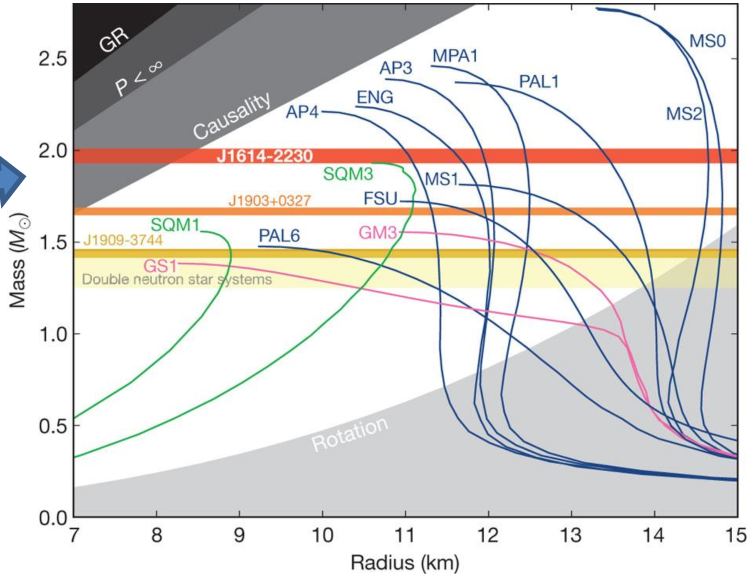


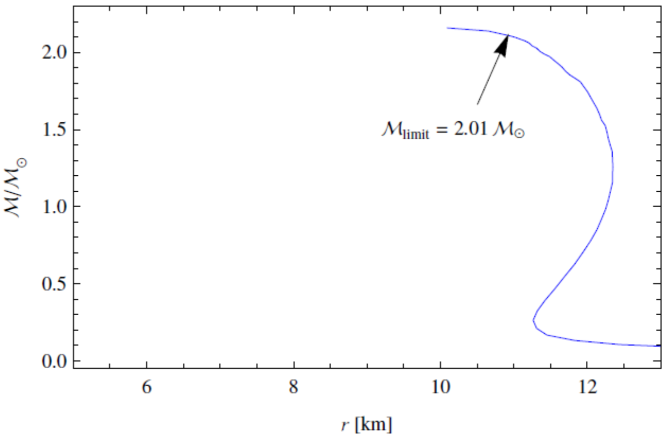
Stiffer EoS for massive neutron star

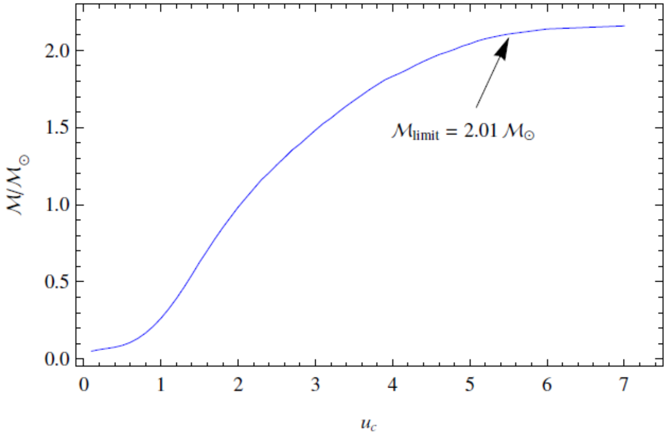
massive neutron star



PSR J1614-2230
$(1.97 \pm 0.04) M_{\odot}$







Remark I

- How does low density nuclear matter (laboratory experiment-nuclear physics), constrains the high density hadronic matter(compact star-observations)?
- EM observations of neutron stars(mass and radius)
- Detection of gravitational waves
- Laboratory for QCD at high density

  EoS → GW waveform

Remark II

Symmetry energy S(n)

(1) thresholds for new degree of freedom

$$\mu_n - \mu_p \geq m_i$$
$$\mu_n - \mu_p = 4(1-2x)S(n)$$

(2) composition of degrees of freedom in star matter