BFKL equation at finite temperature

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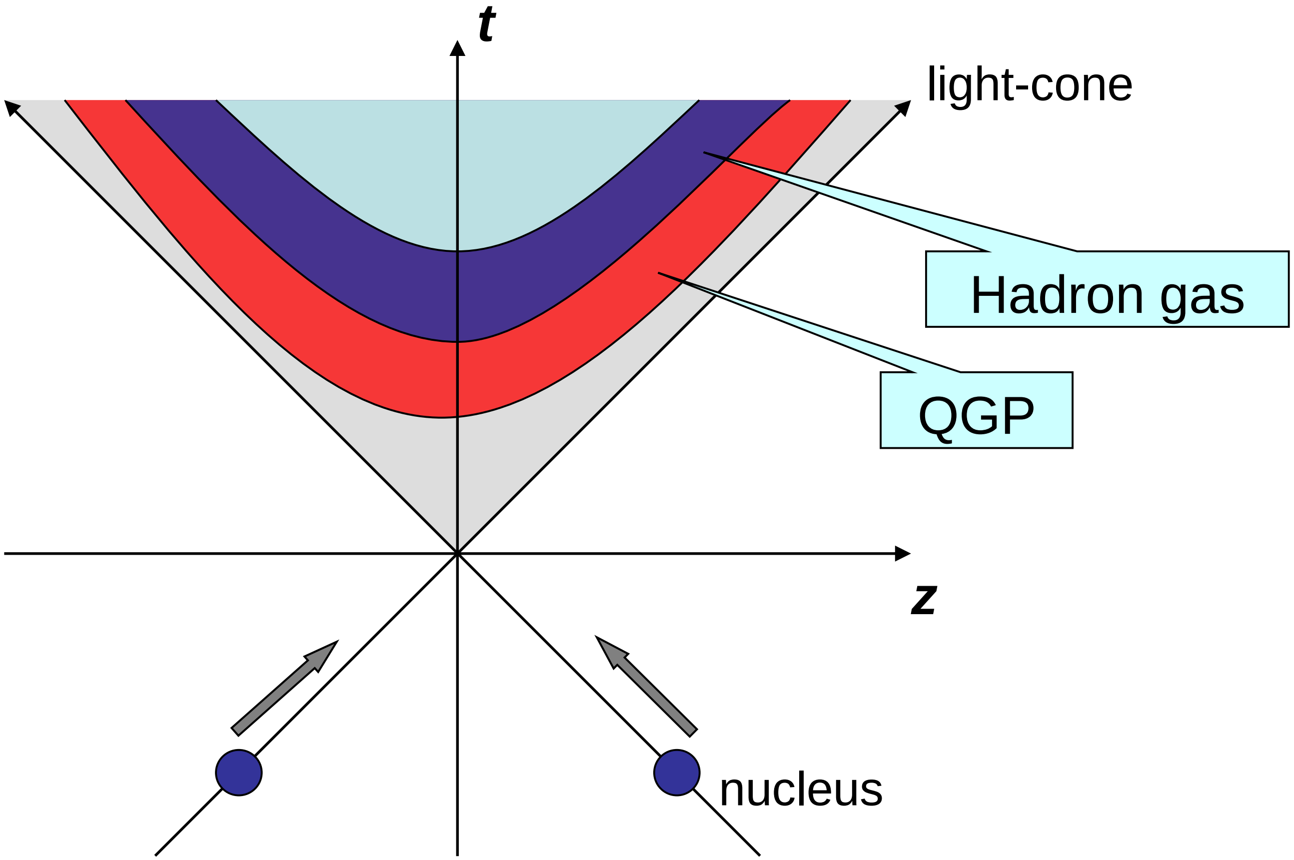
1. Introduction

Relativistic Heavy Ion Collision experiments (RHIC,LHC)

→ Quark-Gluon Plasma

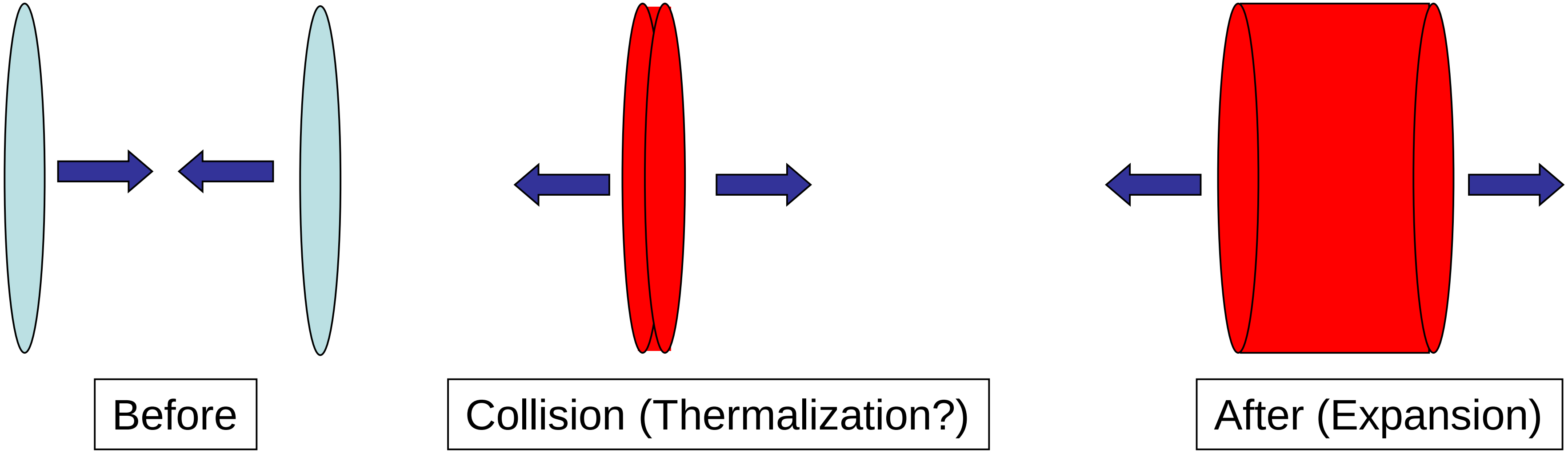


Relativistic Heavy Ion Collision experiments

✓Success of Hydrodynamic description

- Ideal fluid with no dissipation
- Early thermalization: Just after collision?

  (<1 fm/*c* at RHIC?   Even earlier at LHC?)



Before

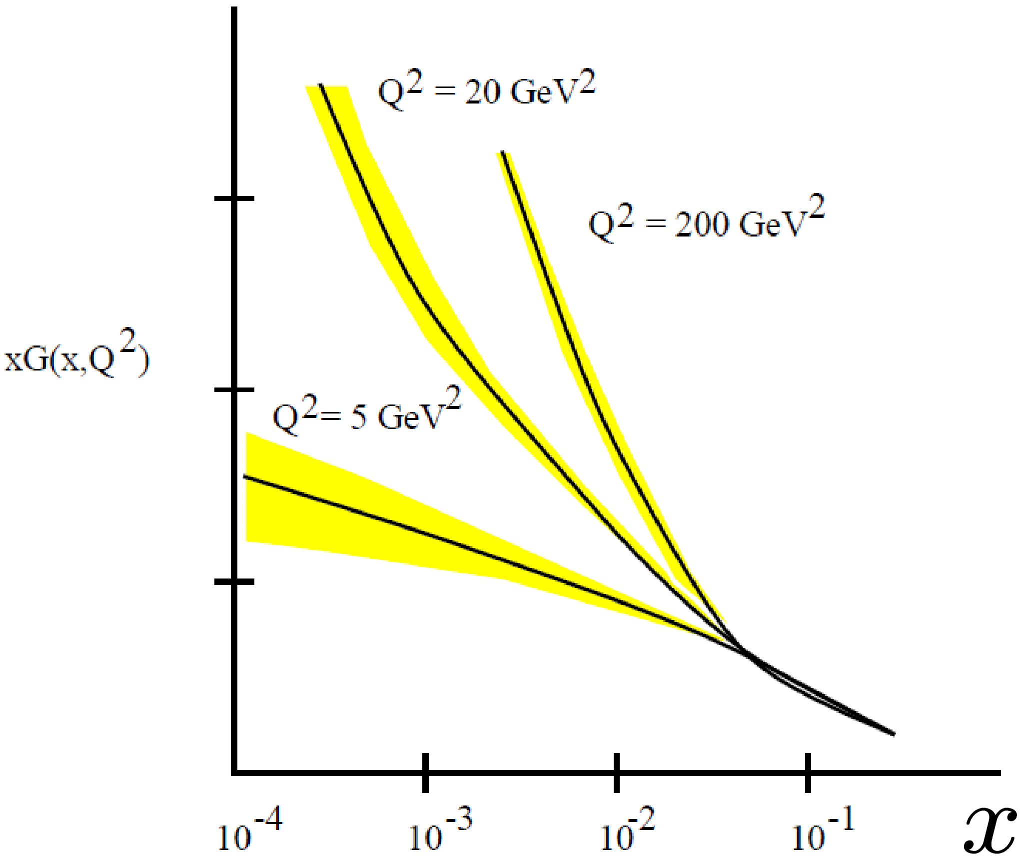
Collision (Thermalization?)

After (Expansion)

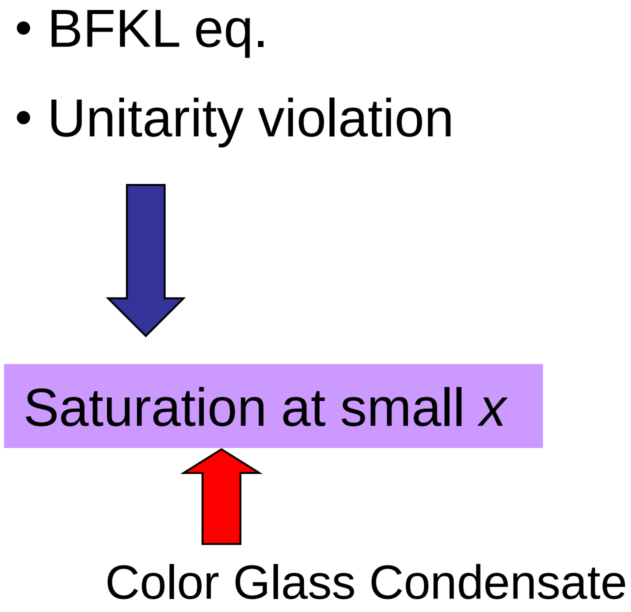
What is <u>Initial Condition</u> for Hydrodynamic expansion?

Small-*x* gluon distribution in Nucleon

Nucleon Structure ← DIS (Deep Inelastic Scattering)



Gluon distribution (Zeus data)



- BFKL eq.
- Unitarity violation

Saturation at small *x*

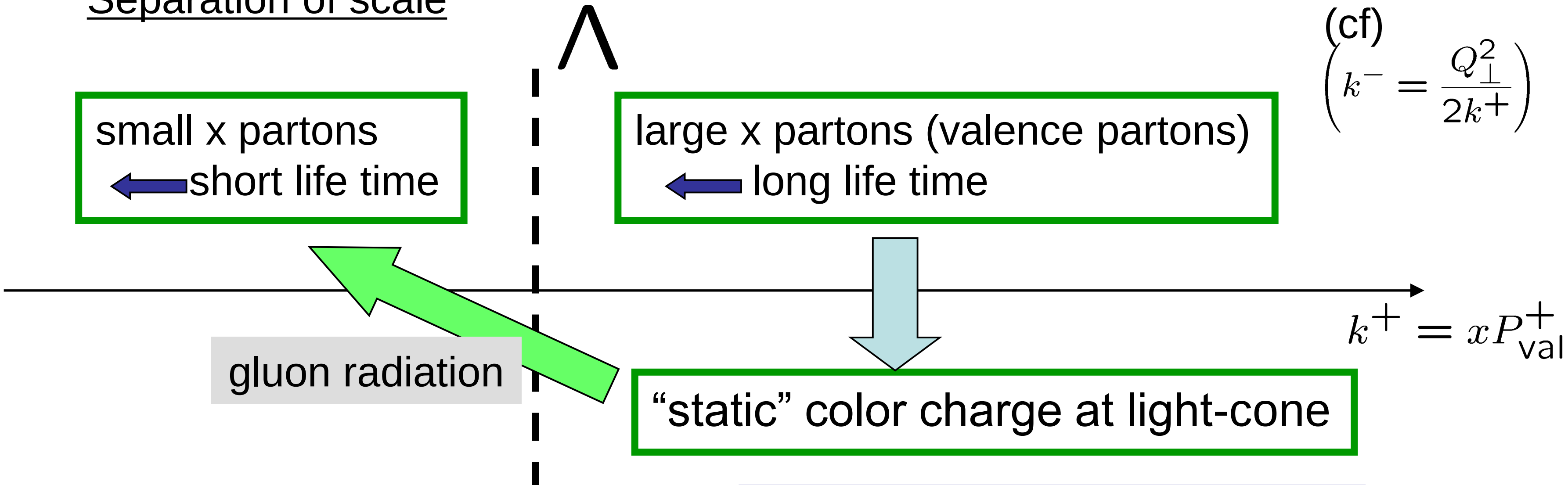
Color Glass Condensate

Color Glass Condensate

McLerran & Venugopalan (1994)
Jalilian-Marian, Kovner, Leonidov & Weigert (1997)
Iancu, Leonidov & McLerran(2001)

CGC is a successful framework to explain saturation

Separation of scale



Λ

small x partons
short life time

large x partons (valence partons)
long life time

(cf) $\left(k^- = \frac{Q_\perp^2}{2k^+}\right)$

$k^+ = xP^+_{\text{val}}$

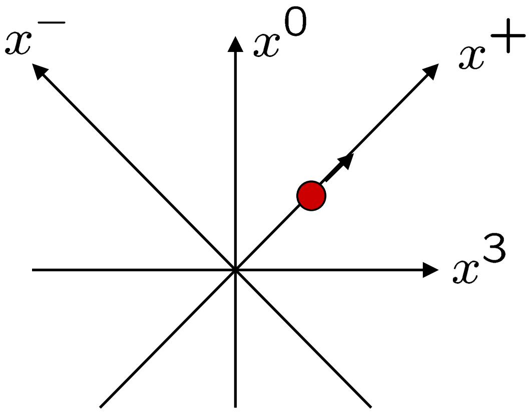
gluon radiation

“static” color charge at light-cone

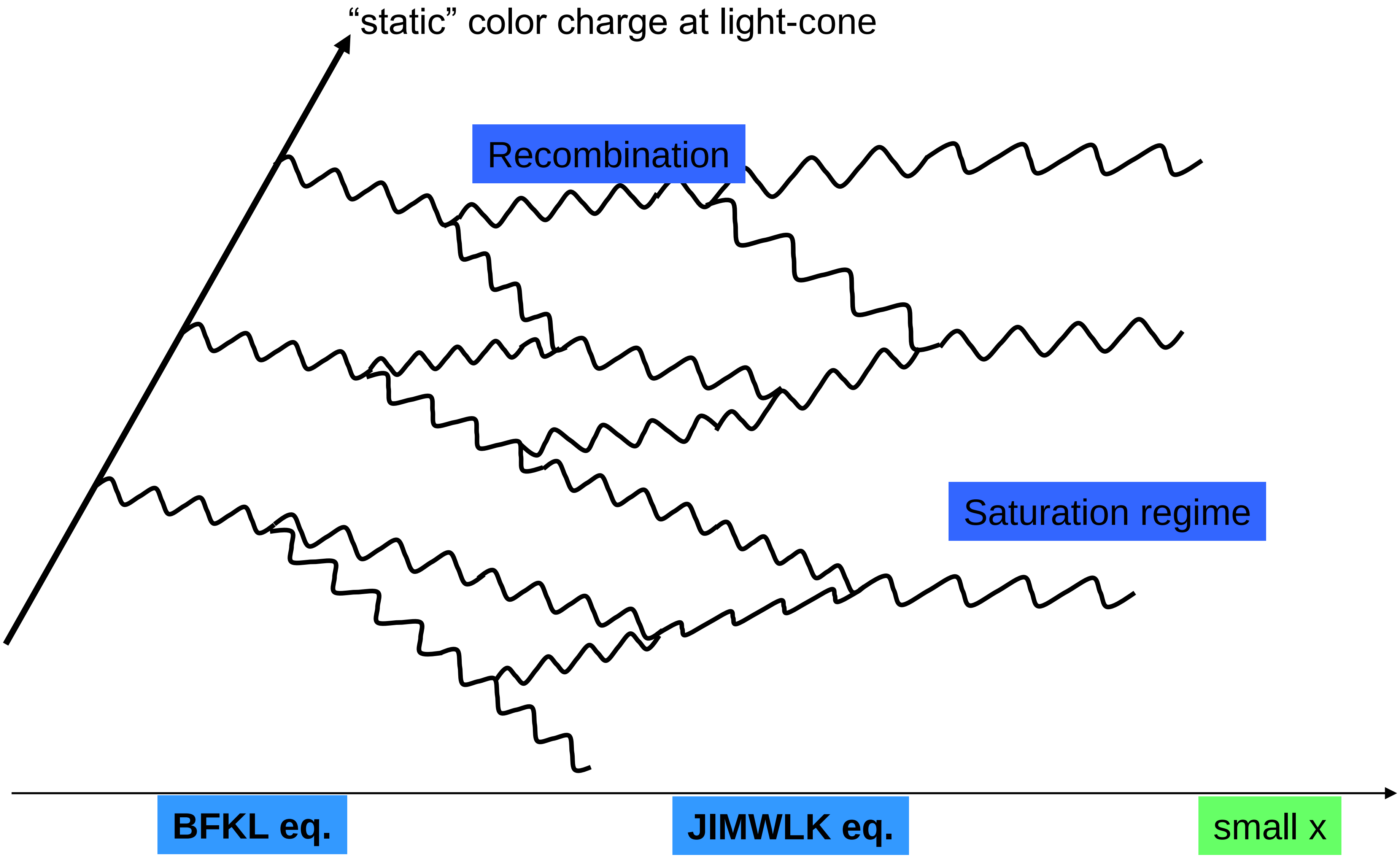
$$J^\mu(x) = \delta^{\mu +}\delta(x^-)\rho(x_\perp)$$

$$x^\pm = \frac{1}{\sqrt{2}}(x^0 \pm x^3)$$

$$k^\pm = \frac{1}{\sqrt{2}}(k^0 \pm k^3)$$



Color Glass Condensate



“static” color charge at light-cone

Recombination

Saturation regime

BFKL eq.

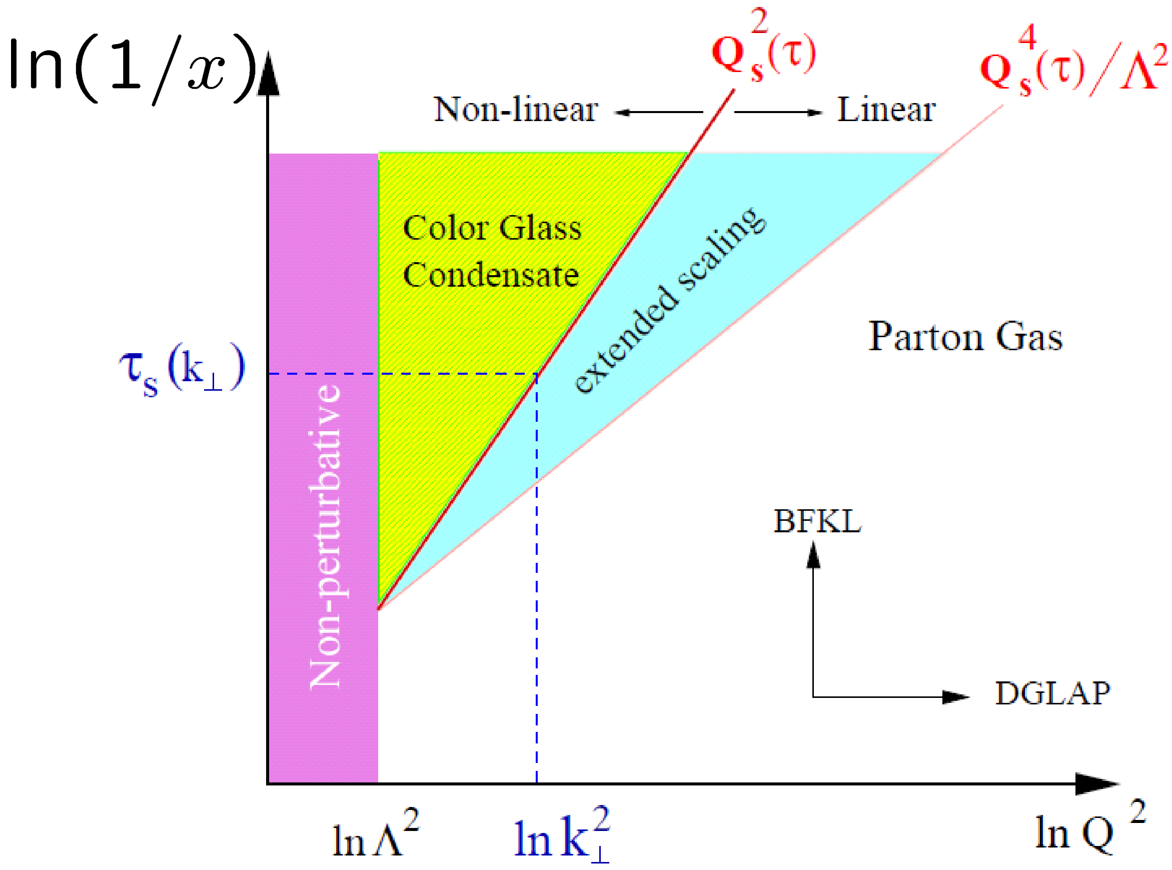
JIMWLK eq.

small x

(Jalilian-Marian,Iancu,McLerran,Wegert,Leonidov,Kovner)

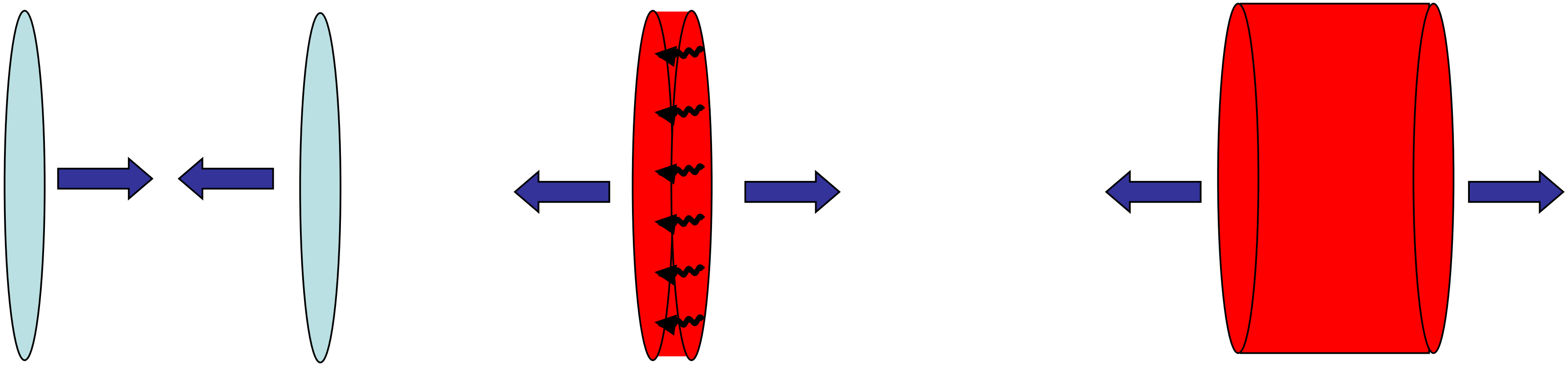
Color Glass Condensate

"Phase diagram" of high energy hadron



Relativistic Heavy Ion Collision experiments (revisited)

✓ **Bjorken picture:** Valence partons (large x partons) are intact and keep staying on light-cone



Before

Collision (Thermalization?)

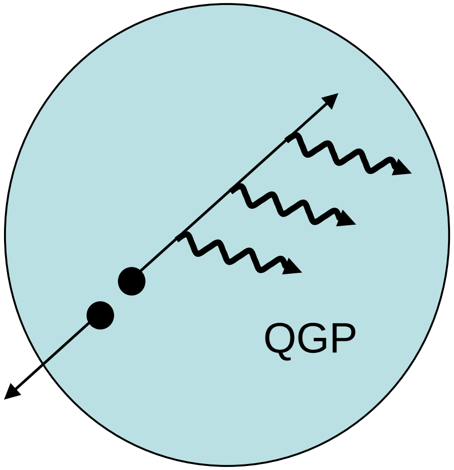
After (Expansion)

If thermalization takes place just after the collision, valence partons radiate soft gluons into finite temperature medium.

➡ CGC at finite temperature (Thermal BFKL & Thermal JIMWLK eq.)

➡ Initial Condition for Hydrodynamic expansion

Jet Quenching



⟺ High energy collision between jet and QGP

Jet Quenching

The energetic parton radiates soft gluons

(Casalderrey-Solana & X.N.Wang: arXiv:0705.1352)

⇓

We need thermal BFKL (JIMWLK) eq.

2. CGC at zero temperature

- Partition func. for CGC

$$\mathcal{Z}[j] = \underbrace{\int \mathcal{D}\rho\, W_\Lambda[\rho]}_{\text{Averaging with weight func for } \rho} \underbrace{\int^\Lambda \mathcal{D}A_a^\mu\, \delta\left(A_a^+\right) \mathrm{e}^{\mathrm{i}S[A,\rho]-\mathrm{i}\int j\cdot A}}_{\text{Partition func for soft gluon at fixed } \rho}$$

- Gluon distribution func.

1. Solve classical YM eq. at fixed ρ

$$\begin{cases} \mathcal{A}^i(x^-, x_\perp) = \theta(x^-)\dfrac{\mathrm{i}}{g} V \partial^i V^\dagger \\ V(x_\perp) \equiv \mathrm{P}\exp\left\{\mathrm{i}g \displaystyle\int \mathrm{d}z^- \frac{1}{\nabla_\perp^2}\rho(z^-, x_\perp)\right\} \end{cases}$$

(cf. Coulomb potential for QED)

← Static (time-independent) solution

2. Average $\mathcal{A}$ over ρ with $W_\Lambda[\rho]$

$$\left\langle A^i(x) A^j(y) \right\rangle_\Lambda = \int \mathcal{D}\rho\, W_\Lambda[\rho]\, \mathcal{A}^i(x)\mathcal{A}^j(y)$$

• RG equation for $W_\Lambda[\rho]$

If we integrate out the hard modes of momentum shell $(b\Lambda < p^+ < \Lambda)$, then the fluctuation is renormalized into $W_\Lambda[\rho]$

➡ RG eq. for $W_\Lambda[\rho]$

$$\frac{\partial W_\tau[\rho]}{\partial \tau} = \alpha_{\mathrm{s}} \left\{ \frac{1}{2} \frac{\delta^2}{\delta\rho_\tau(x)\delta\rho_\tau(y)} [W_\tau \chi_{xy}] - \frac{\delta}{\delta\rho_\tau(x)} [W_\tau \sigma_x] \right\}$$

$$\tau = \ln(1/x)$$

➡ BFKL eq. & JIMWLK eq.

BFKL eq.

$$\begin{cases} x\dfrac{\partial}{\partial x}\varphi(x,k_\perp) = 4\alpha_{\mathrm{s}} N_{\mathrm{c}} \displaystyle\int \frac{\mathrm{d}^2 p_\perp}{(2\pi)^2} \frac{k_\perp^2}{p_\perp^2 (p_\perp - k_\perp)^2} \left(\varphi(x,p_\perp) - \frac{1}{2}\varphi(x,k_\perp) \right) \\ \varphi(x,k_\perp) \equiv \langle \rho_a(k_\perp)\rho_a(-k_\perp) \rangle_\tau \end{cases}$$

:unintegrated gluon distribution

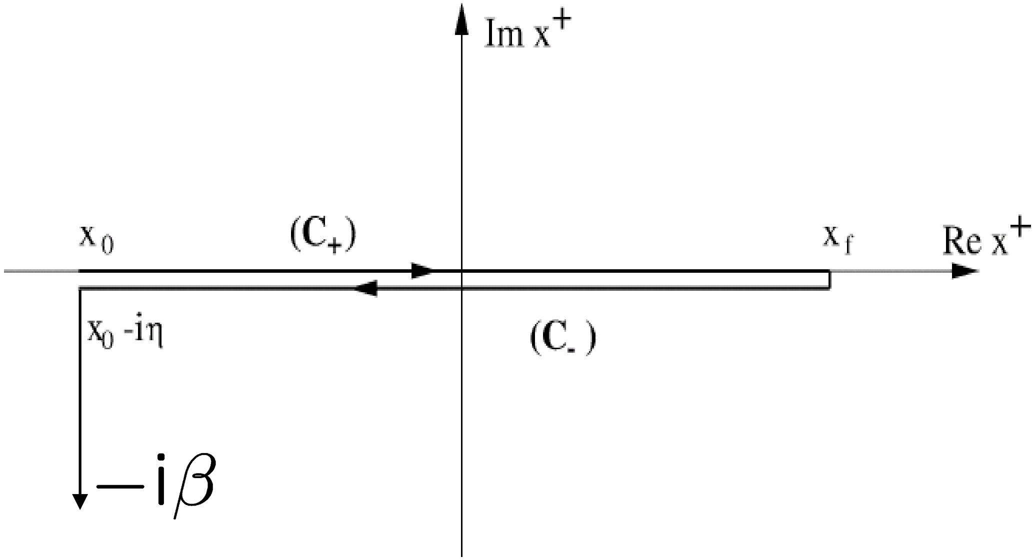
3. BFKL eq. at finite temperature

$$\boxed{\mathcal{Z}[j] = \int \mathcal{D}\rho W_{\Lambda}[\rho] \int^{\Lambda} \mathcal{D}A_a^{\mu} \delta\left(A_a^{+}\right) \mathrm{e}^{\mathrm{i}S[A,\rho] - \mathrm{i}\int j\cdot A}}$$

$$\begin{cases} S[A,\rho] &= S_{\mathrm{YM}} + S_W \\ S_{\mathrm{YM}} &= -\int_C \mathrm{d}^4x \frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu} \\ S_W &= \frac{\mathrm{i}}{gN_{\mathrm{C}}} \int \mathrm{d}^3\vec{x}\, \mathrm{Tr}\left[\rho(\vec{x}) W_C[A^-](\vec{x})\right] \end{cases}$$

Real-time formalism for finite *T* field theory (Alves,Das&Perez:PRD66(2002)125008)

C : complex time contour



$$iG_{++}(p) = \frac{i}{2p^+p^- - \omega_p^2 + i\epsilon} + 2\pi n_B(|u\cdot p|)\,\delta(2p^+p^- - \omega_p^2)$$

$$iG_{+-}(p) = 2\pi[\theta(-u\cdot p) + n_B(|u\cdot p|)]\,\delta(2p^+p^- - \omega_p^2)$$

$$iG_{-+}(p) = 2\pi[\theta(u\cdot p) + n_B(|u\cdot p|)]\,\delta(2p^+p^- - \omega_p^2)$$

$$iG_{--}(p) = -\frac{i}{2p^+p^- - \omega_p^2 - i\epsilon}$$

1. Classical solution

$\mathcal{A}^i(x^-, x_\perp)$: time independent → same as at zero temperature

2. RG eq. for $W_\Lambda[\rho]$

$$\frac{\partial W_\tau[\rho]}{\partial \tau} = \alpha_s \left\{ \frac{1}{2} \frac{\delta^2}{\delta\rho_\tau(x)\delta\rho_\tau(y)} [W_\tau \chi_{xy}] - \frac{\delta}{\delta\rho_\tau(x)} [W_\tau \sigma_x] \right\}$$

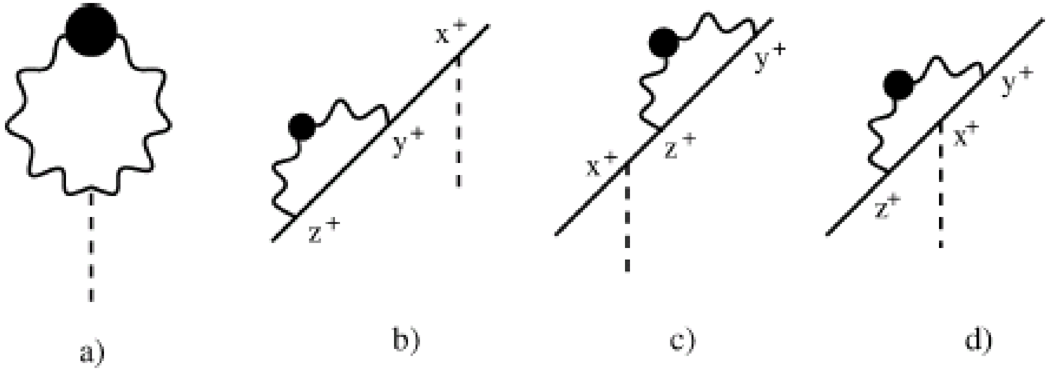
$$\left\{ \begin{array}{rcl} \sigma_a(\vec{x}) & \equiv & \langle \delta\rho_a(x) \rangle \\ \chi_{ab}(x,y) & \equiv & \langle \delta\rho_a(x)\delta\rho_b(y) \rangle \end{array} \right.$$

$$\delta\rho_a(x) \equiv - \left. \frac{\delta S}{\delta A_a^-(x)} \right|_{\mathcal{A}+a}$$

: induced charge

- Evaluation of induced charge correlators

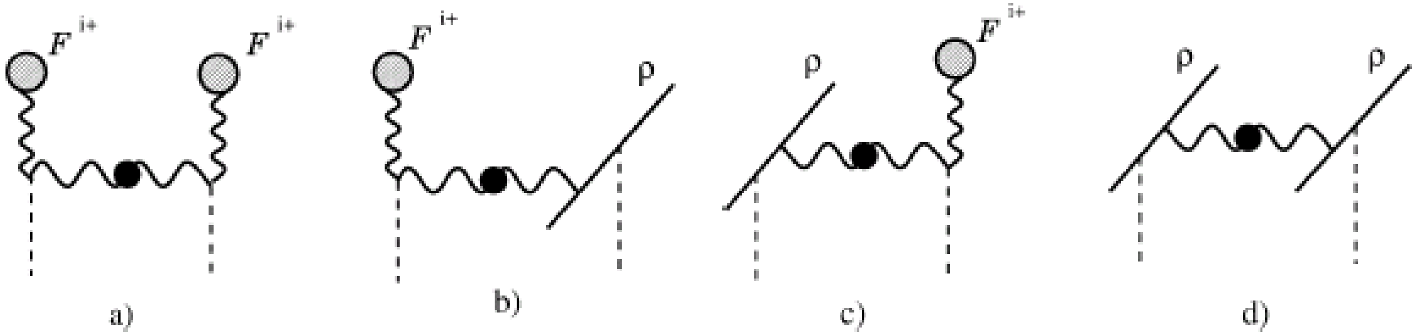
$\langle \delta\rho \rangle =$



$\equiv \sigma$

Virtual BFKL kernel

$\langle \delta\rho\delta\rho \rangle =$



$\equiv \chi$

Real BFKL kernel

Thermal BFKL eq.

$$x\frac{\partial}{\partial x}\varphi(x,k_\perp) = 4\alpha_{\rm s} N_{\rm c} \underbrace{\left(1+\frac{2}{\exp\left(\frac{P^-_{\rm val}}{\sqrt{2}Tx}\right)-1}\right)}_{\text{Bose enhancement factor}}\int\frac{{\rm d}^2p_\perp}{(2\pi)^2}\frac{k_\perp^2}{p_\perp^2(p_\perp-k_\perp)^2}\left(\varphi(x,p_\perp)-\frac{1}{2}\varphi(x,k_\perp)\right)$$

⇨ **Saturation regime is reached sooner than expected by vacuum BFKL eq.**

Note

de Vega & Lipatov: PLB578 (2004)335

(“BFKL pomeron at non-zero temperature and…”)

⇦ different from ours

4. Summary

- CGC at finite temperature → Initial Condition for heavy ion collision
  BFKL eq. at finite temperature
- Bose enhancement will increase soft gluons more rapidly than in vacuum.
- Thermal JIMWLK eq. will be interesting