Relativistic Approach to QQ Potential

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Introduction

- Relativistic Treatment of 2-Body Problem
  - Composite systems with light quarks
  - Systems with large coupling strength
- Dissociation and recombination of J/ψ

$$J/\psi + \pi \longleftrightarrow D + D^{+}$$

Useful information on the suppression or the enhancement of in high-energy heavy-ion collisions

- 2-body Dirac equations of constraint dynamics
  - successfully tested on relativistic 2-body bound states in QED, QCD, & 2-body NN scattering.

    (Horace W. Crater et al., PRD74, 054028 (2006))
  - Will be tested on heavy-quarkonium dissociation

2-Body Constraint Dynamics

- In a separable two-body basis
- Particles interacts pairwise through scalar and vector interactions.
- Spin-dependence is determined naturally.
- **Yields simple Schrodinger-type equation.**
- No cutoff parameter

2-Body Constraint Dynamics

- P. van Alstine et al., J. Math. Phys.23, 1997 (1982)
  - Two free spinless particles with the mass-shell constraint

$$H_1^0 \equiv p_1^2 + m_1^2 \approx 0, \qquad H_1^0 \equiv p_1^2 + m_1^2 \approx 0$$

  - Introducing the Poincare invariant world scalar $S(x, p_1, p_2)$

$$m_1 \rightarrow m_1 + S(x, p_1, p_2) \equiv M_1(x, p_1, p_2)$$
$$m_2 \rightarrow m_2 + S(x, p_1, p_2) \equiv M_2(x, p_1, p_2)$$

  - Dynamic mass-shell constraint

$$H_i \equiv p_i^2 + M_i^2 = p_i^2 + m_i^2 + V_i(x, p_1, p_2)$$

  - Schrodinger-like equation

$$H = \frac{\varepsilon_1 H_1 + \varepsilon_2 H_2}{w} = p^2 + V - b^2(w) \approx 0$$

where $b^2(w) = \varepsilon_w^2 - m_w^2$

with $\varepsilon_w = \dfrac{w^2 - m_1^2 - m_2^2}{2w}$ and $m_w = \dfrac{m_1 m_2}{w}$

2-Body Constraint Dynamics

- 2-body Dirac Equation $(p^2 + V_w)\psi = b^2(w)\psi$
- where $V_w = V_0 + V_D + V_{SO1} + V_{SO2} + V_{SO3} + V_{TSO} + V_{SS} + V_{TSS}$

$$V_0 = 2m_w S + S^2 + 2\varepsilon_w A - A^2$$

$$V_D = -\frac{2F'(\cosh 2K - 1)}{r} + K'^2 + F'^2 + \frac{2K'\sinh 2K}{r} - \nabla^2 F - \frac{2(\cosh 2K - 1)}{r} + m(r)$$

$$V_{SO1} = \vec{L}\cdot(\vec{\sigma}_1 + \vec{\sigma}_2)\left[-\frac{F'}{r} - \frac{F'(\cosh 2K - 1)}{r} - \frac{(\cosh 2K - 1)}{r^2} + \frac{K'\sinh 2K}{r}\right]$$

V_{SO2} & V_{SO3} =0 when m_1=m_2

$$V_{SO2} = \vec{L}\cdot(\vec{\sigma}_1 - \vec{\sigma}_2)[l'\cosh 2K - q'\sinh 2K]$$

$$V_{SO3} = i\vec{L}\cdot(\vec{\sigma}_1 \times \vec{\sigma}_2)[q'\cosh 2K + l'\sinh 2K]$$

$$V_{TSO} = (\vec{\sigma}_1\cdot\hat{r})(\vec{\sigma}_2\cdot\hat{r})\vec{L}\cdot(\vec{\sigma}_1 + \vec{\sigma}_2)\left[-\frac{K'(\cosh 2K - 1)}{r} + \frac{\sinh 2K}{r^2} - \frac{K'}{r} + \frac{F'\sinh 2K}{r}\right]$$

$$V_{SS} = \vec{\sigma}_1\cdot\vec{\sigma}_2\left[k(r) - \left(F' - K' + \frac{1}{r}\right)\frac{\sinh 2K}{r} - \left(F' - K' + \frac{1}{r}\right)\frac{(\cosh 2K - 1)}{r}\right]$$

$$V_{TSS} = (\vec{\sigma}_1\cdot\hat{r})(\vec{\sigma}_2\cdot\hat{r})\left[n(r) + \left(3F' - K' + \frac{1}{r}\right)\frac{\sinh 2K}{r} + \left(F' - 3K' + \frac{1}{r}\right)\frac{(\cosh 2K - 1)}{r} + 2F'K' - \nabla^2 K\right]$$

2-Body Constraint Dynamics

- Here
- $F=(L-3G)/2$ and $K=(G+L)/2$
  - $G(A)=-(1/2)\ln(1-2A/w)$
  - $L=(1/2)\ln[(1+2(S-A)/w+S_2/m_2)/(1-2A/w)]$
  - w : center of mass energy
- $l'(r)= -(1/2r)(E_2M_2-E_1M_1)/(E_2M_1+E_1M_2)(L-G)'$
- $q'(r)=(1/2r)\ )(E_1M_2-E_2M_1)/(E_2M_1+E_1M_2)(L-G)'$
- $m(r)=-(1/2)\nabla^2G+(3/4)G'^2+G'F'-K'^2$
- $k(r)=(1/2)\nabla^2G-(1/2)G'^2-G'F'-(1/2)(G'/r)+(K'/r)$
- $n(r)=\nabla^2K-(1/2)\nabla^2G-2K'F'+G'F'+(3G'/2r)-(3K'/r)$

2-Body Constraint Dynamics

- $V_{SO2}=0$ and $V_{SO3}=0$ when $m_1=m_2$
- For singlet state,

  no spin-orbit contribution $\leftarrow$ $\vec{L}\cdot(\vec{\sigma_1}+\vec{\sigma_2})=2\vec{L}\cdot\vec{S}=0$

  $V_D+V_{TSO}+V_{SS}+V_{TSS}=0$

  $H=p^2+2m_wS+S^2+2\varepsilon_wA-A^2$

Relativistic Application(I)

- Applied to the Binding Energy of chamonium
- Without spin-spin interaction
  - M(exp)=3.067 GeV
  - Compare the result with PRC 65, 034902 (2002)

T/Tc	BE(non-rel.)	BE(rel.)	Rel. Error(%)
0	-0.67	-0.61	9.0
0.6	-0.56	-0.52	7.1
0.7	-0.44	-0.41	6.8
0.8	-0.31	-0.30	3.2
0.9	-0.18	-0.17	4.0
1.0	-0.0076	-0.0076	0.0

  - At zero temperature, 10% difference at most!

Relativistic Application(2)

- With spin-spin interaction
  - M(S=0) = 3.09693 GeV

    M(S=1) = 2.9788 GeV
  - At T=0, relativistic treatment gives

    BE(S=0) = -0.682 GeV

    BE(S=1) = -0.586 GeV
  - Spin-spin splitting ~ 100 MeV

Overview of QQ Potential(I)

- Pure Coulomb : $A = -\frac{0.2}{r}$ and $S = 0$

BE=-0.0148 GeV for color-singlet

=-0.0129 GeV for color-triplet(no convergence)

- + Log factor : $A = -\frac{0.2}{r\frac{1}{2}\ln\left(e^2 + \frac{1}{\Lambda^2 r^2}\right)}$ and $S = 0$

BE=-0.0124 GeV for color-singlet

=-0.0122 GeV for color-triplet

- + Screening : $A = -\frac{0.2\,e^{-\mu r}}{r\frac{1}{2}\ln\left(e^2 + \frac{1}{\Lambda^2 r^2}\right)}$ and $S = 0$

BE=-0.0124 GeV for color-singlet

No bound state for color-triplet

Overview of QQ Potential(2)

- + String tension(with no spin-spin interaction)

$$A = -\frac{0.2\, e^{-\mu r}}{r \frac{1}{2} \ln\left(e^2 + \frac{1}{\Lambda^2 r^2}\right)} \quad \text{and} \quad S = -br$$

When b=0.17 BE=-0.3547 GeV

When b=0.2 BE=-0.5257 GeV

Too much sensitive to parameters!

QQ Potential

- Modified Richardson Potential

$$A = -\frac{4}{3}\alpha_s \frac{e^{-\mu r}}{r\frac{1}{2}\ln\left(e^2 + \frac{1}{\Lambda^2 r^2}\right)}$$

and

$$S = -\frac{8\pi}{27}\Lambda^2 r$$

$$\alpha_s = \frac{4\pi}{\left(11 - \frac{2}{3}n_f\right)} = \frac{12\pi}{27}$$

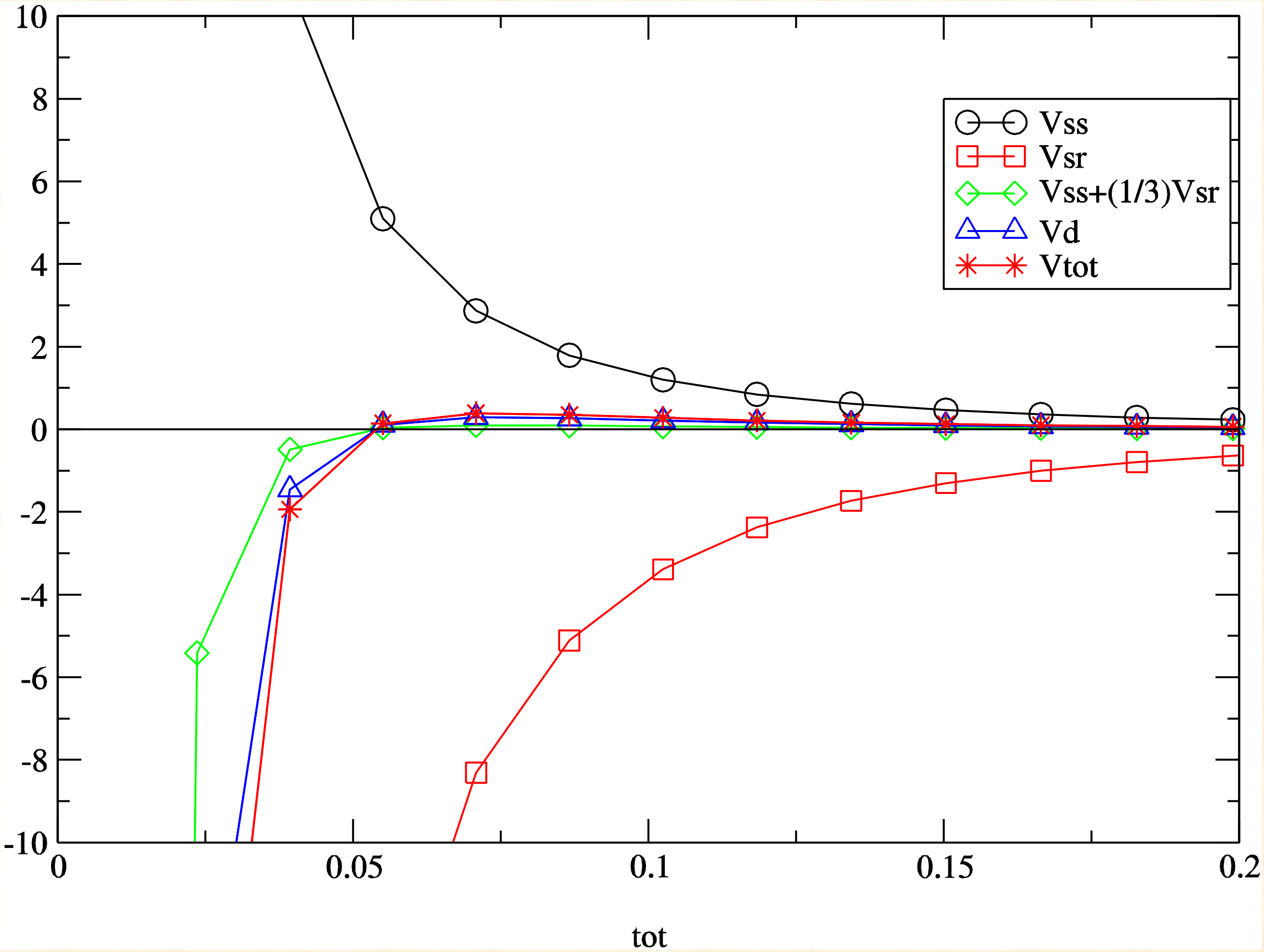
Parameters : μ, Λ

And mass=m(T)

A : color-Coulomb interaction with the screening

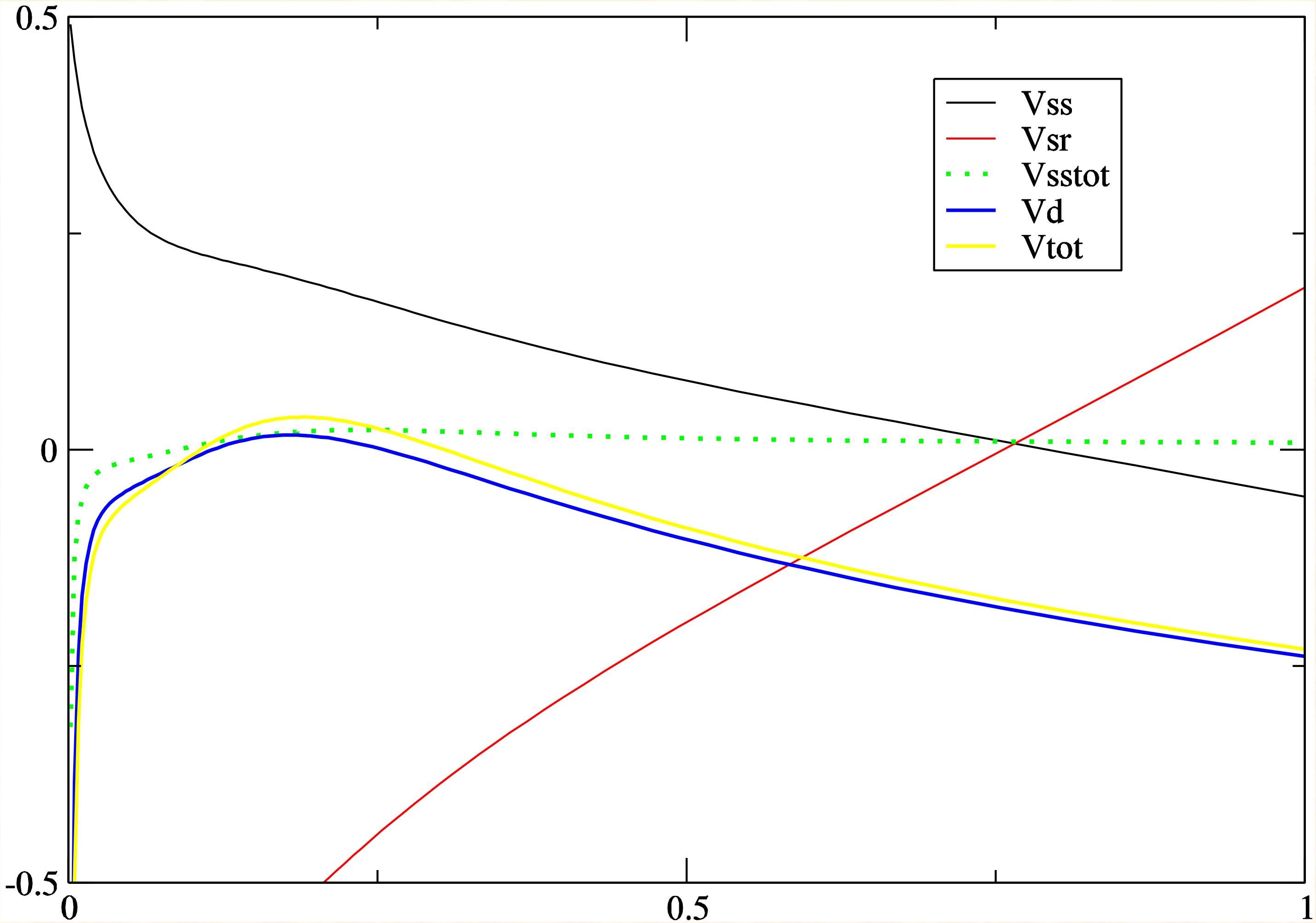
S : linear interaction for confinement

V(J/ψ)

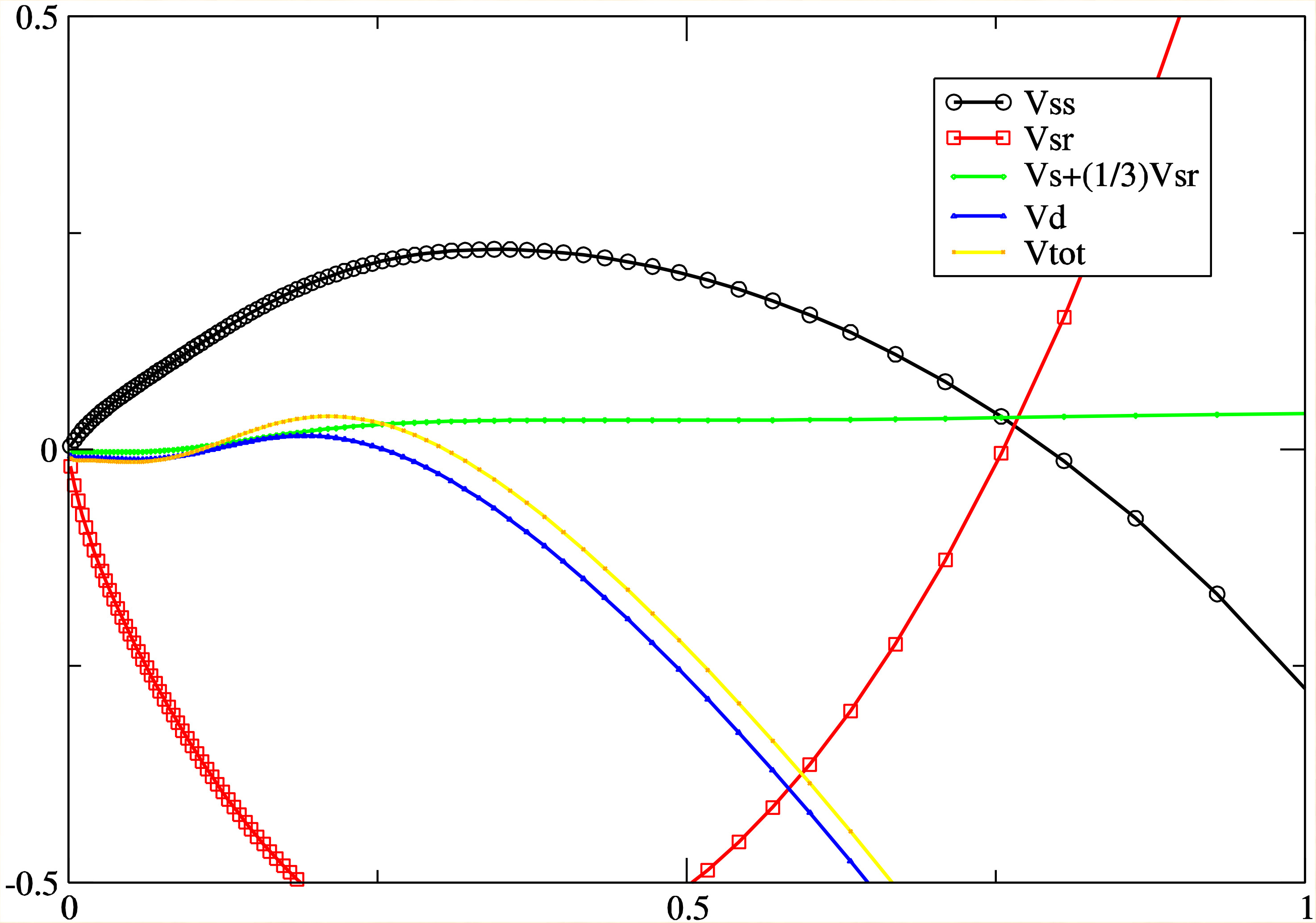


Too Much Attractive!

V(J/ψ) *r²



V(J/ψ) *r³



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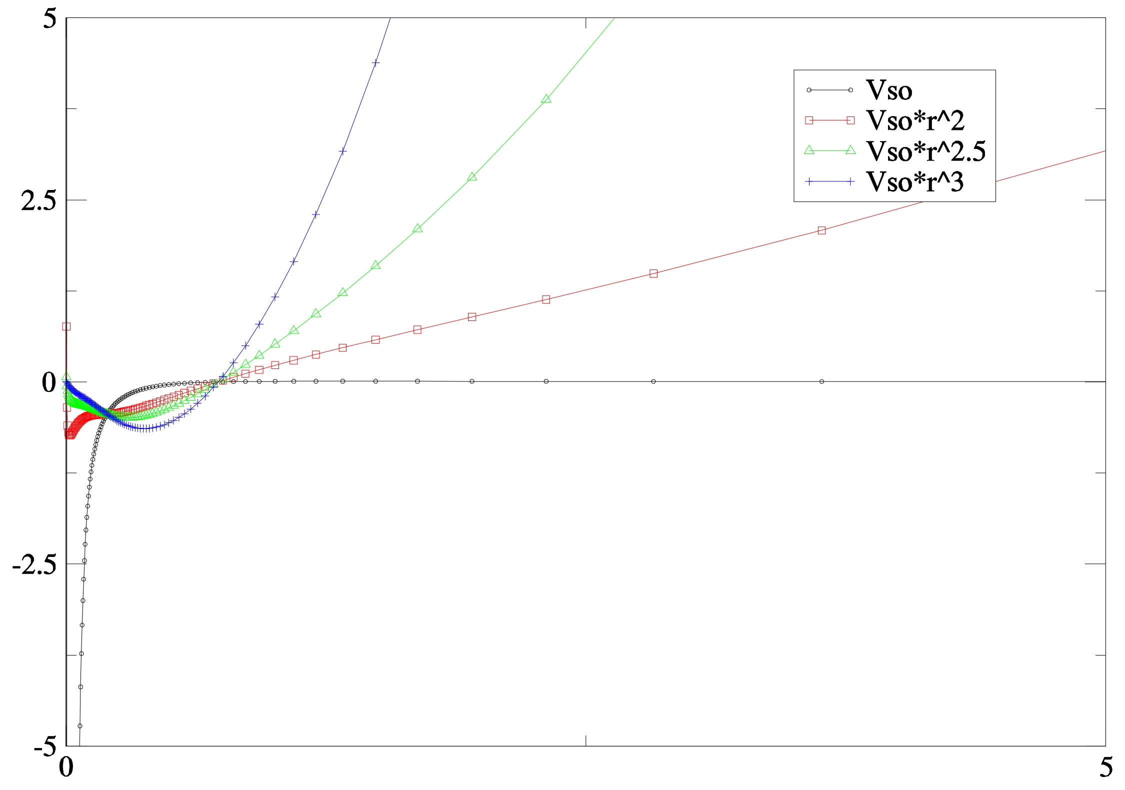
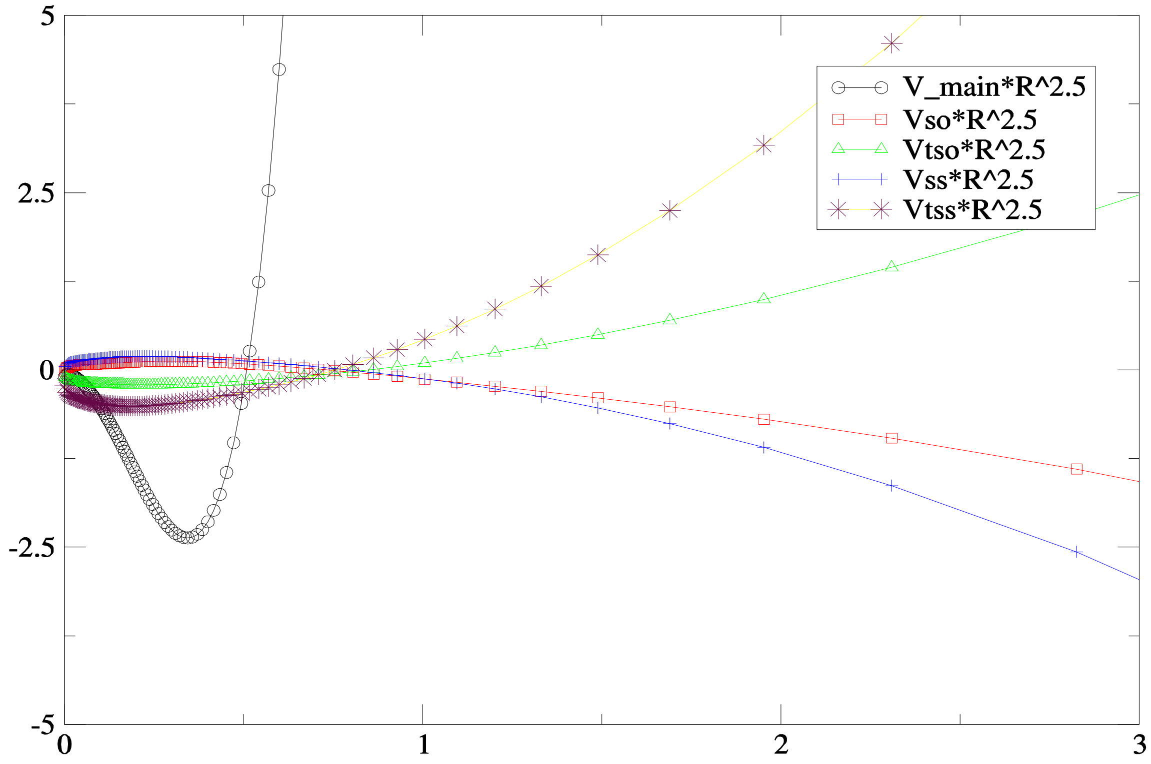
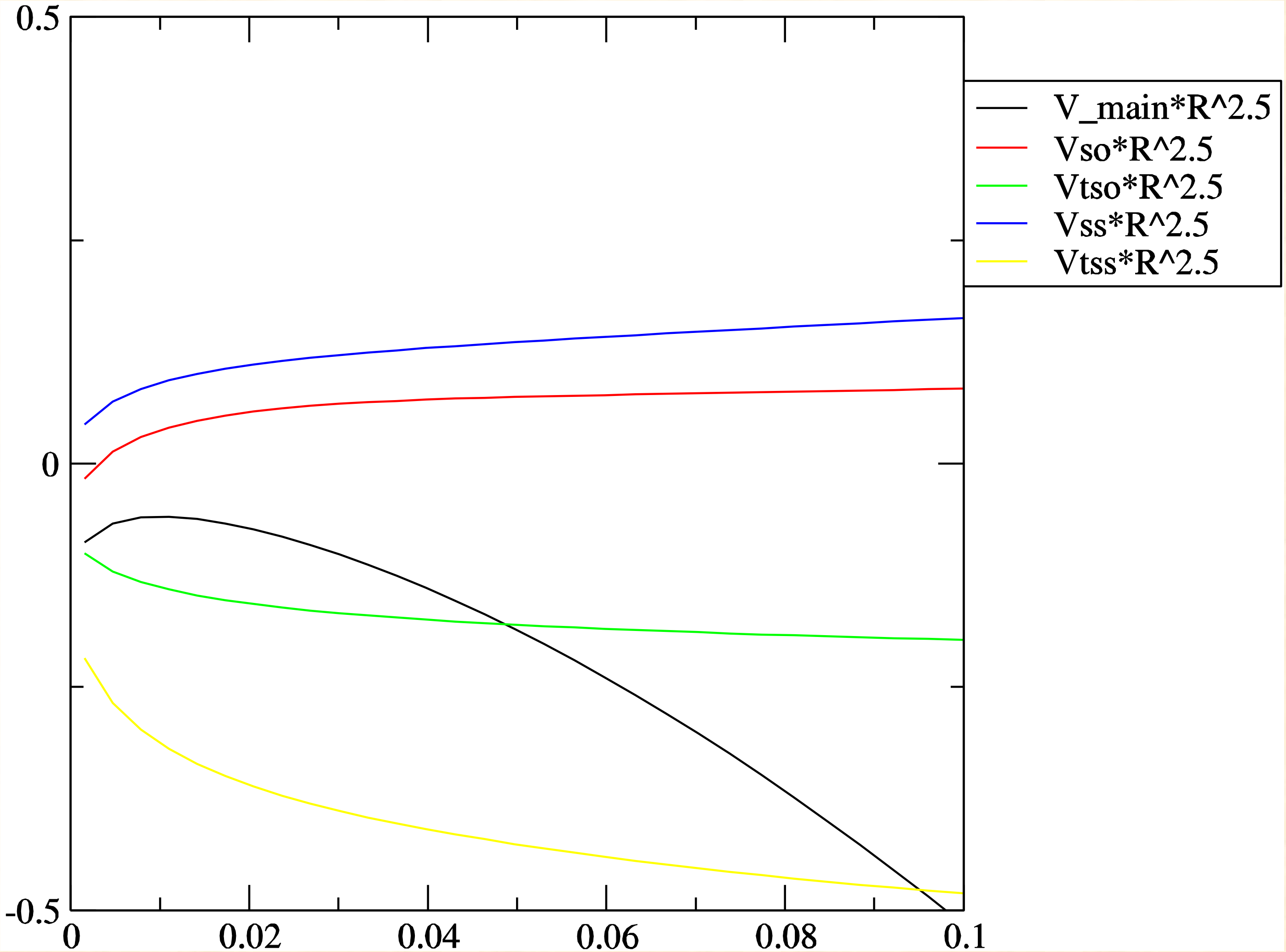
Vso(J/ψ)



$V(J/\psi)*r^{2.5}$



V(J/ψ) at small range

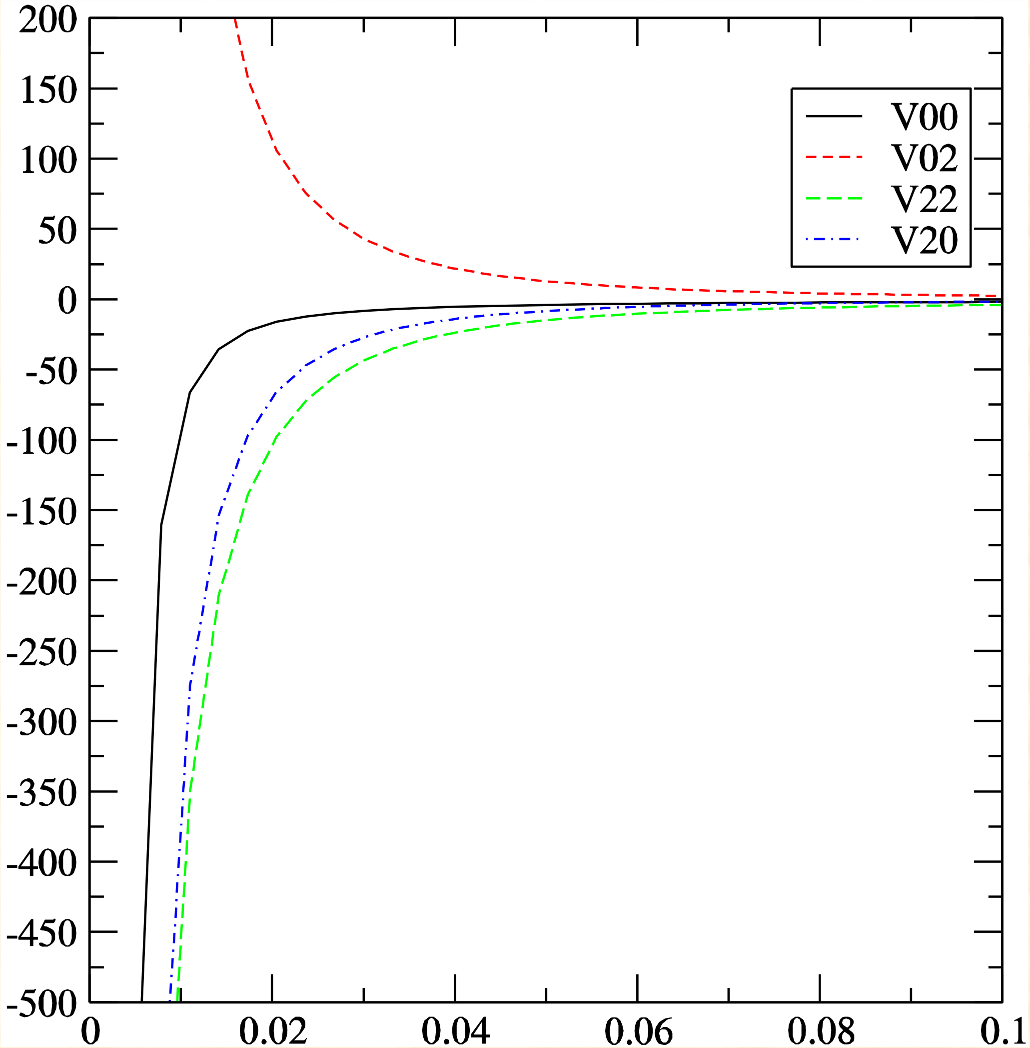


V(J/ψ) with mixing

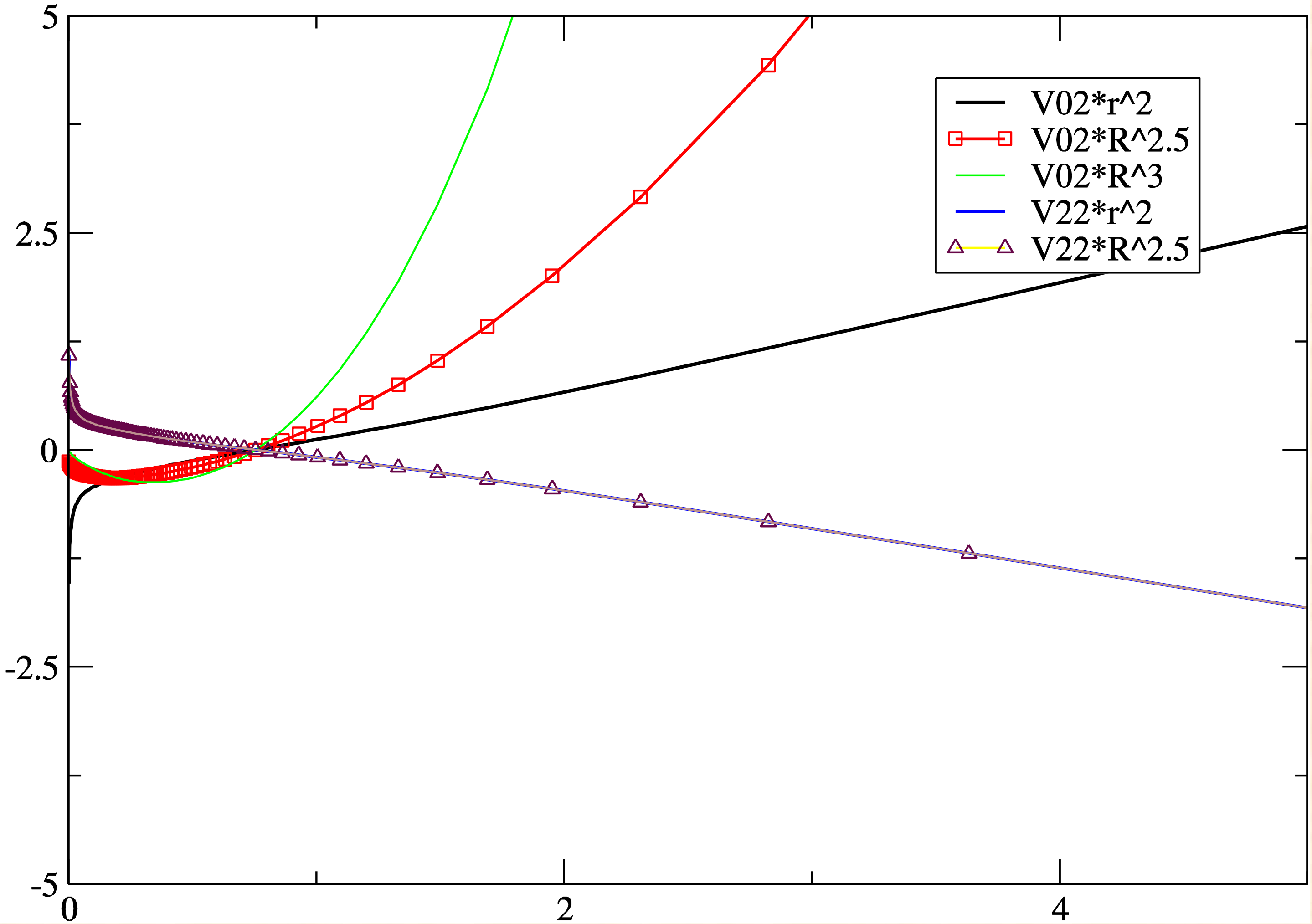
For J/ψ, S=1 and J=1.

Without mixing(L=0 only), splitting is reversed.

Therefore there has to be mixing between L=0 and L=2 states.



V(J/ψ) with mixing



Work on process

- To solve the S-eq. numerically,
- We introduce basis functions

  $\phi_n(r)=N_n r^l \exp(-n\beta^2 r^2/2)\ Y_{lm}$

  $\phi_n(r)=N_n r^l \exp(-\beta r/n)\ Y_{lm}$

  $\phi_n(r)=N_n r^l \exp(-\beta r/\sqrt{n})\ Y_{lm}$

  …

  - None of the above is orthogonal.
  - We can calculate $\langle p^2 \rangle$ analytically, but all the other terms has to be done numerically.
  - The solution is used as an input again → need an iteration
  - Basis ftns. depend on the choice of β quite sensitively and therefore on the choice of the range of r.

Future Work

- Find the QQ potential which describes the mass spectrum of mesons and quarkonia well.
- Extends this potential to non-zero temperature.
- Find the dissociation temperature and cross section of a heavy quarkonium.
- And more...

Thank You

For your attention.