Dense Matter in Astrophysics

Probing Neutron Star EOS
in Gravitational Waves & Gamma-ray Bursts

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- Neutron Star Binary System
  - (astrophysical part) In-Spiral & Mass Transfer(RLOF)
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- Numerical Result
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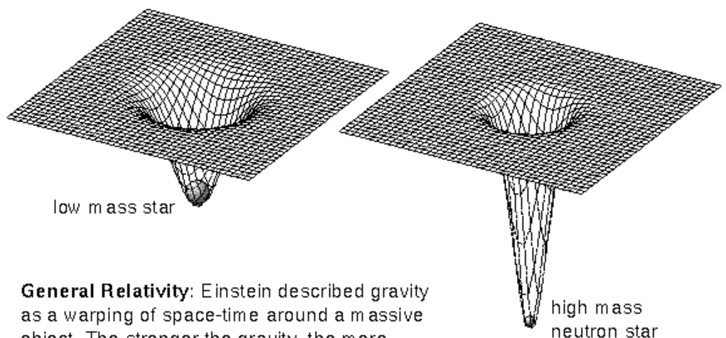
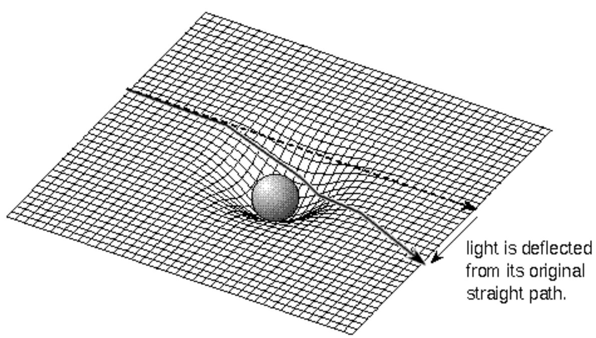
  - Polarization amplitude of GWR in case by case
  - Comparison between normal NS and Quark star
- Conclusion & Outlook

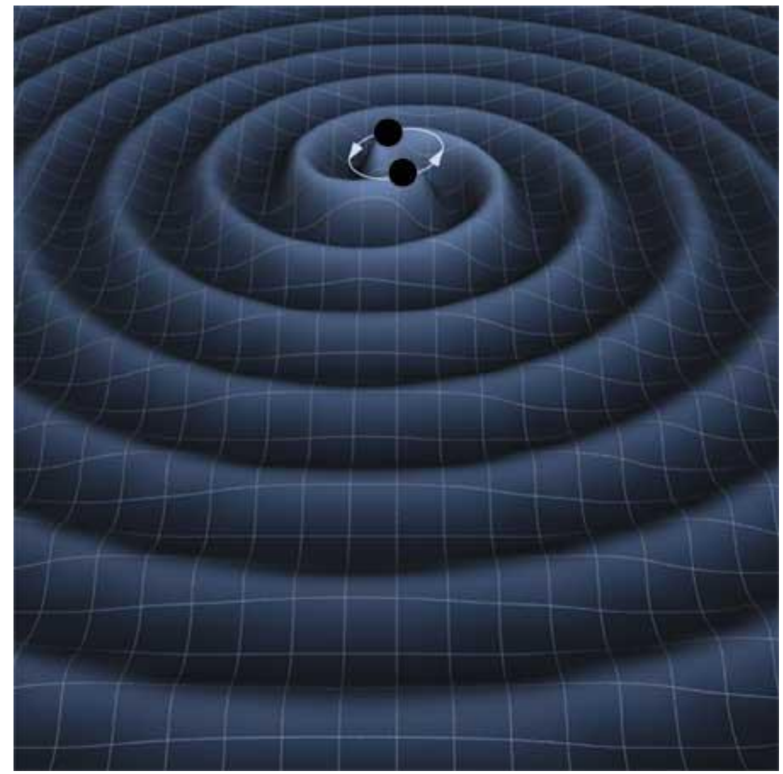
What is the GWR?



General Relativity: Einstein described gravity as a warping of space-time around a massive object. The stronger the gravity, the more space-time is warped.



General Relativity: Light travels along the curved space taking the shortest path between two points. Therefore, light is deflected toward a massive object! The stronger the local gravity is, the greater the light path is bent.



Ripples in the Fabric of the Space-Time

Gravitational radiation

Einstein Field Equation

$$G^{\mu\nu} = R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R^{\alpha}_{\alpha} = -\frac{8\pi G}{c^4} T^{\mu\nu}$$

$$g^{\mu\nu} = \eta^{\mu\nu} + f^{\mu\nu}$$

Linearized field equation $\partial_{\mu} h^{\mu\nu} = 0$

$$\partial^{\lambda}\partial_{\lambda}\left(f^{\mu\nu} - \frac{1}{2}\eta^{\mu\nu} f^{\alpha}_{\alpha}\right) = \partial^{\lambda}\partial_{\lambda} h^{\mu\nu} = -\kappa T^{\mu\nu}$$

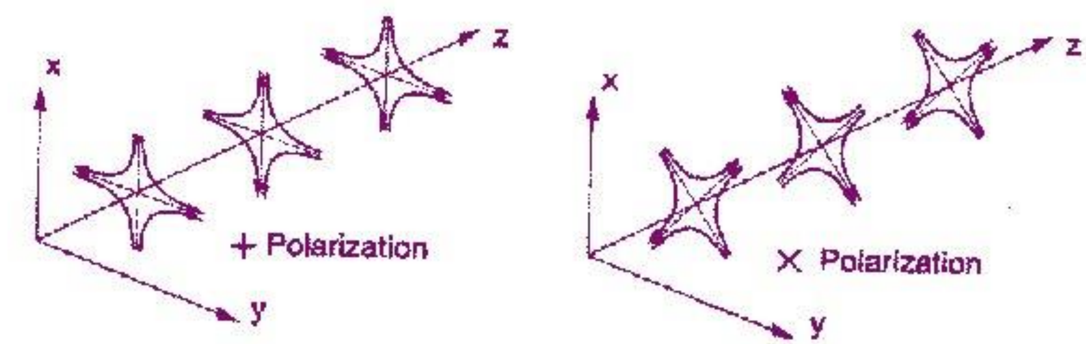
Wave Equation

$$h^{\mu\nu} = \frac{4\pi}{c^4} \int \frac{T^{\mu\nu}(t - |\vec{x} - \vec{x}'|/c, \vec{x}')}{|\vec{x} - \vec{x}'|} d^3\vec{x}'$$

Gravitational radiation

$$h^{\mu\nu} = \frac{4\pi}{c^4}\int \frac{T^{\mu\nu}(t-|\vec{x}-\vec{x}'|/c,\vec{x}')}{|\vec{x}-\vec{x}'|}d^3\vec{x}' \xrightarrow{|\vec{x}-\vec{x}'|\cong r=|\vec{x}|} \frac{2G}{rc^4}\ddot{Q}_{ij}(t-\frac{r}{c}) \longrightarrow \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+ & h_\times & 0 \\ 0 & h_\times & -h_+ & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$L_{GW} = \frac{1}{5}\frac{G}{c^5}\left\langle \dddot{Q}_{ij}\dddot{Q}_{ij}\right\rangle \longrightarrow \frac{32}{5}\frac{G^4}{c^5}\frac{M^5}{a^5}\frac{q^2}{(1+q)^4}$$



$$\dot{J}_{GW} = \frac{32}{5}\frac{G^{7/2}M^{9/2}}{c^5 a^{7/2}}\frac{q^2}{(1+q)^4}$$

Polarization amplitude for compact binary system

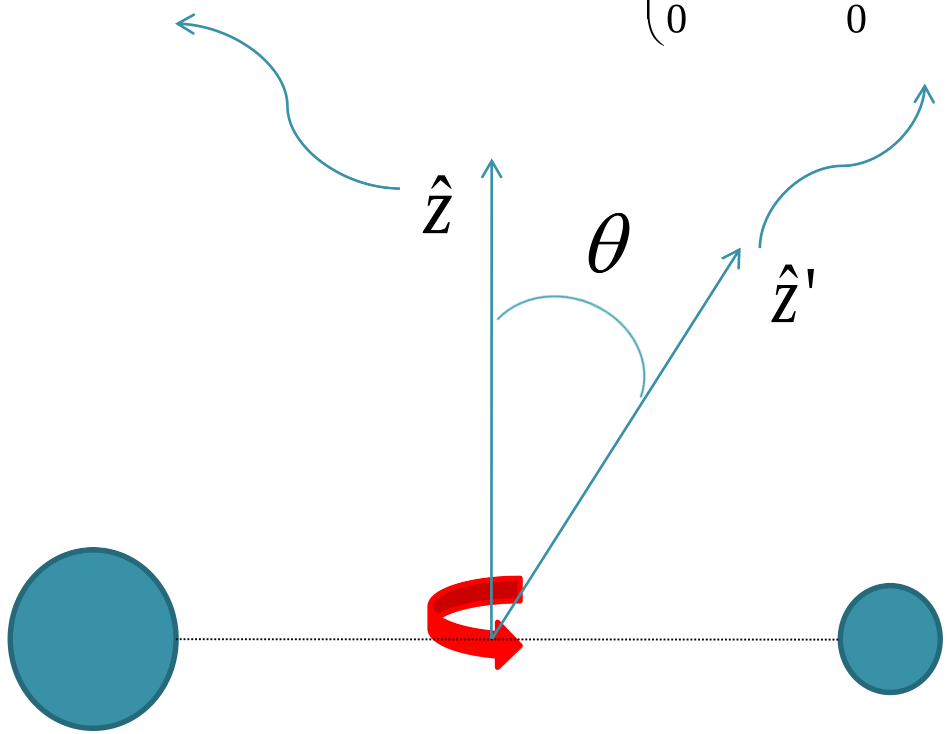
$$h_+(t) = \frac{4}{r}\frac{G^2M^2}{ac^4}\frac{q}{(1+q)^2}\cos 2\omega(t-r)$$

Angle dependence

$$h_{ij}^{TT}(t) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+ & h_\times & 0 \\ 0 & h_\times & -h_+ & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

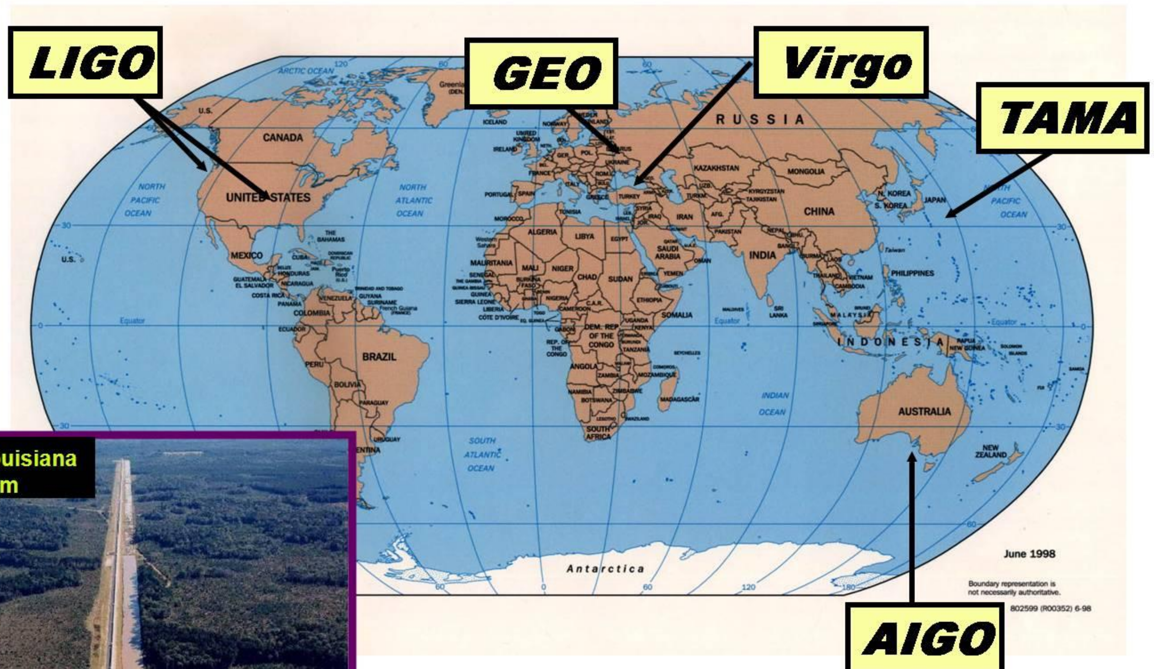
Rotate axis →

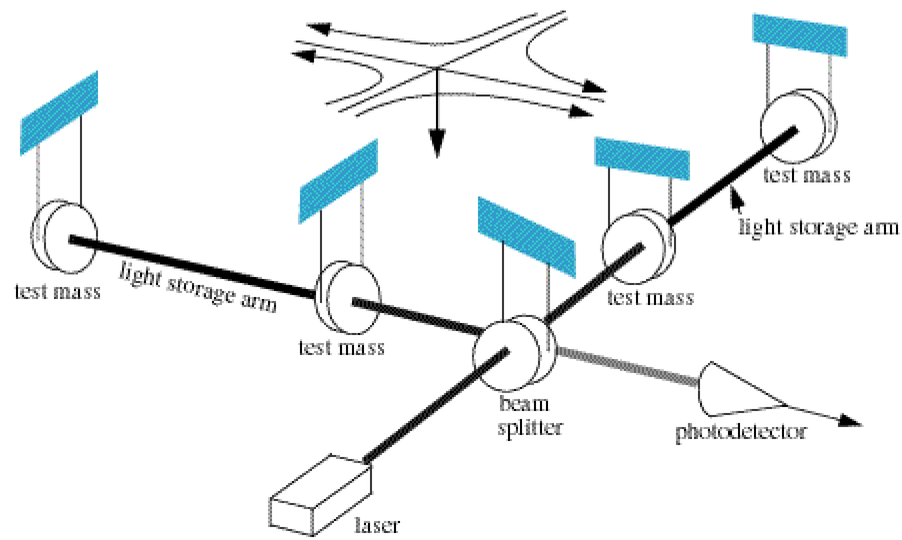
$$h_{ij}^{TT}(t) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2}(1+\cos^2\theta)h_+ & \cos\theta h_\times & 0 \\ 0 & \cos\theta h_\times & -\frac{1}{2}(1+\cos^2\theta)h_+ & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$



Detection of GWR

Network of Interferometers

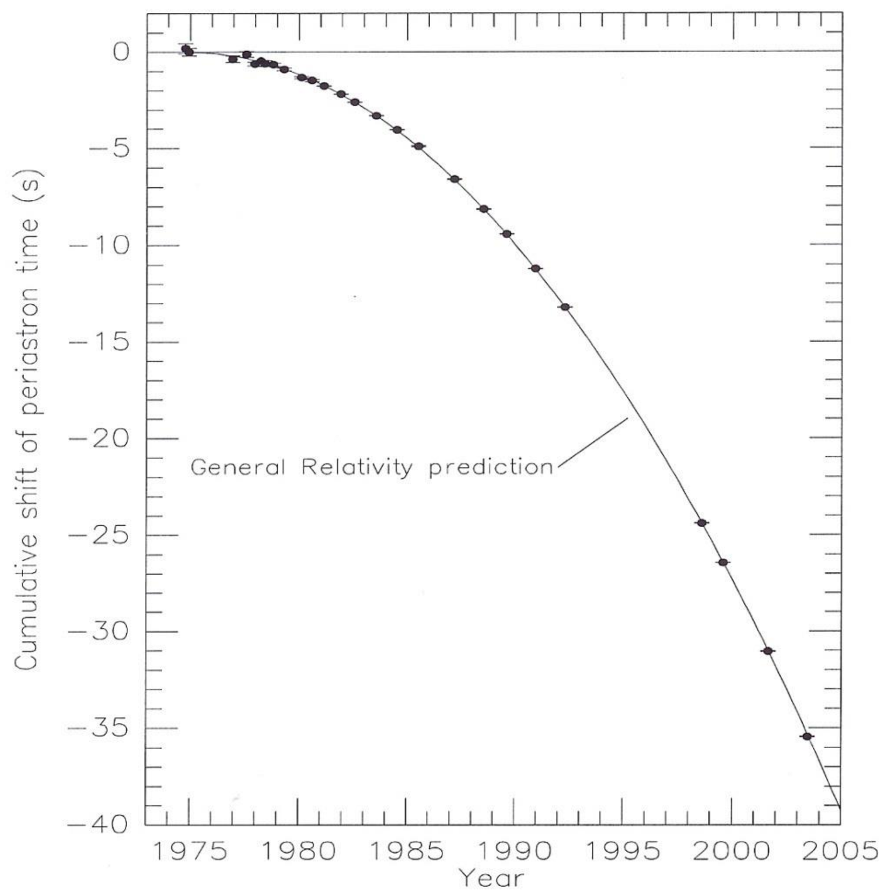




Gravitational wave from NS binary

B1913+16
Hulse & Taylor (1975)

→ 1993 Nobel Prize



Cumulative shift of periastron time decay due to the effect of Gravitational Wave Radiation

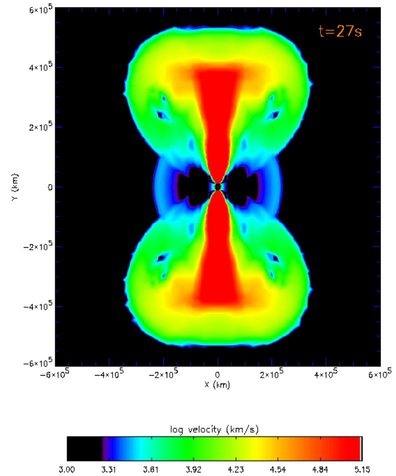
Sources of the GWR

Compact Star binary

Neutron Star-Neutron Star
Neutron Star-Black Hole
Black Hole-Black Hole

Source of GRB ,too

GRB~10^{51}erg
SN~10^{40}erg
Sun~10^{33}erg
H Bomb~10^{20}erg
Nuclear Power Plant~10^{15}erg
Light Bulb~10^{8}erg

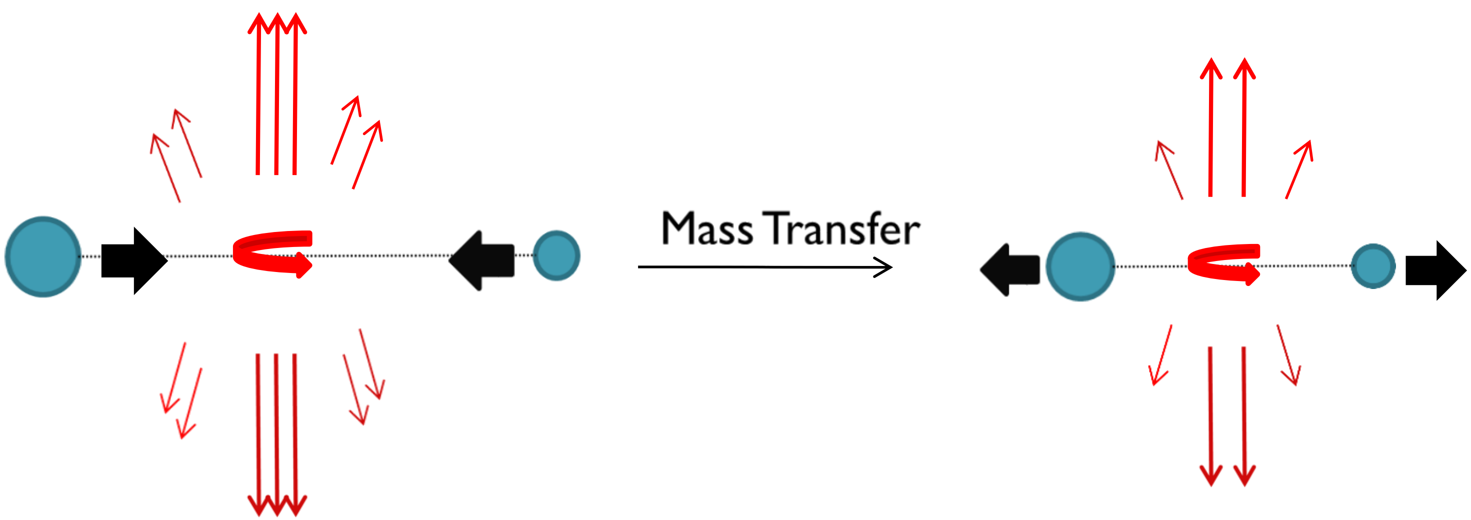


Callapsar: Woosley et al.

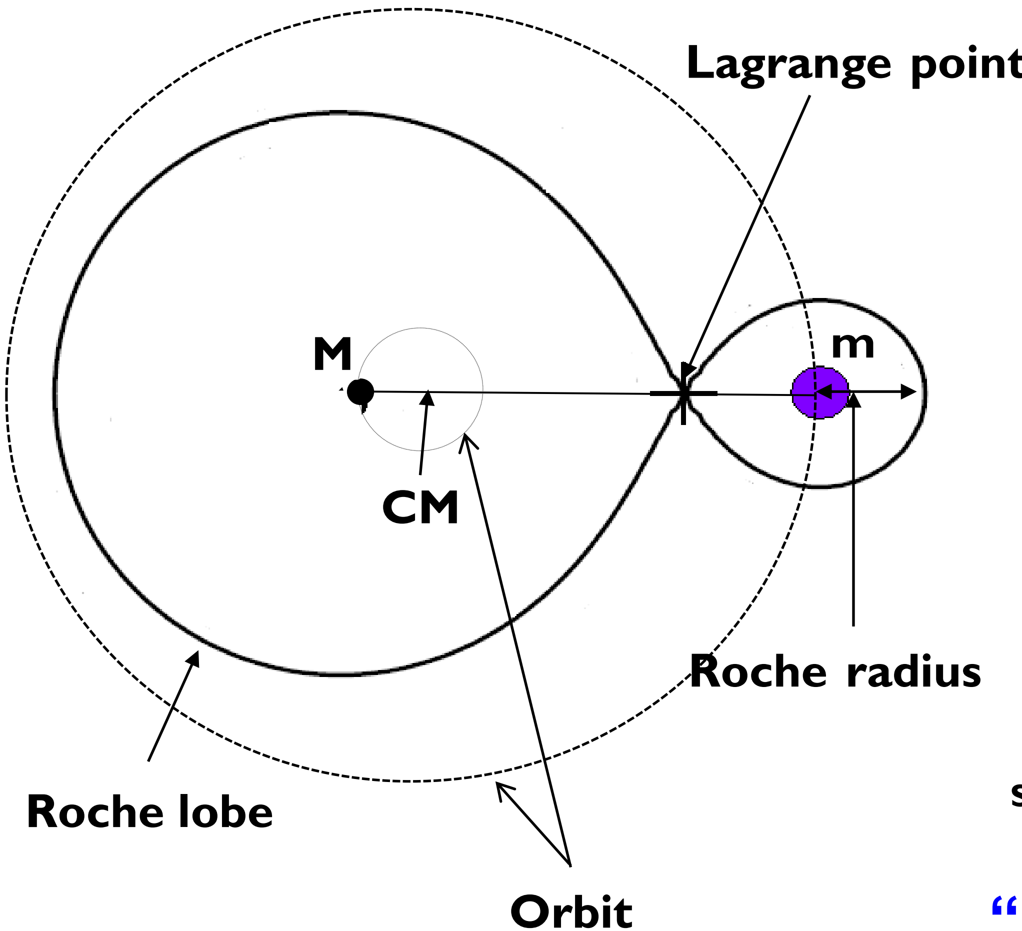
In-spiral & Mass transfer

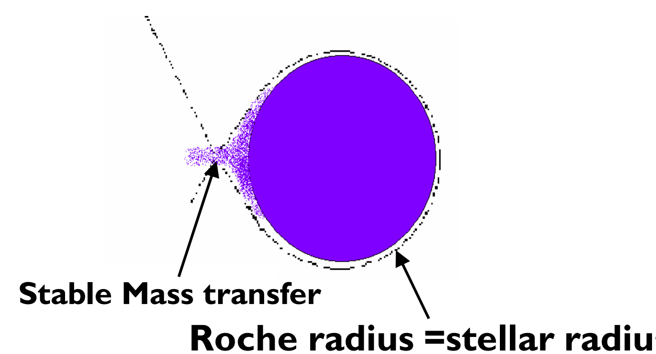
Orbit shrinks due to the gravitational radiation

Orbit increases due to the conservation of AM and mass transfer by Roche lobe over flow



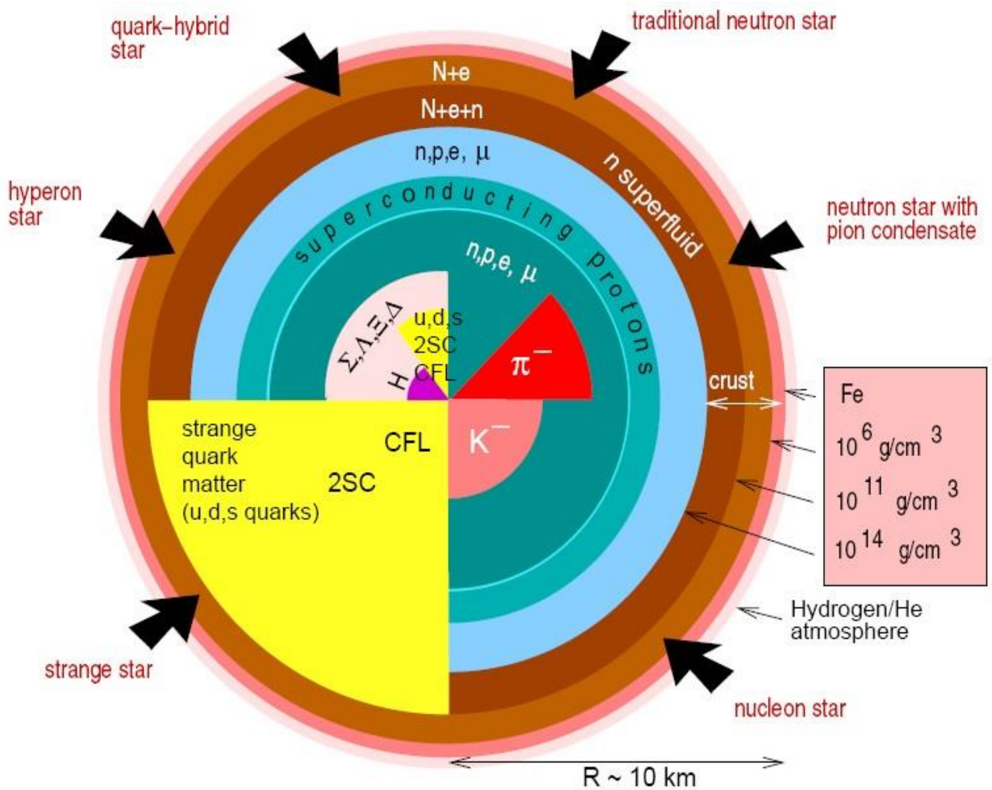
Roche Lobe OverFlow





"Roche lobe overflow"

Neutron Star structure



Calculated By C.Y. Ryu
@ Sungkyunkwan Univ.

TOV equation

Nuclear matter

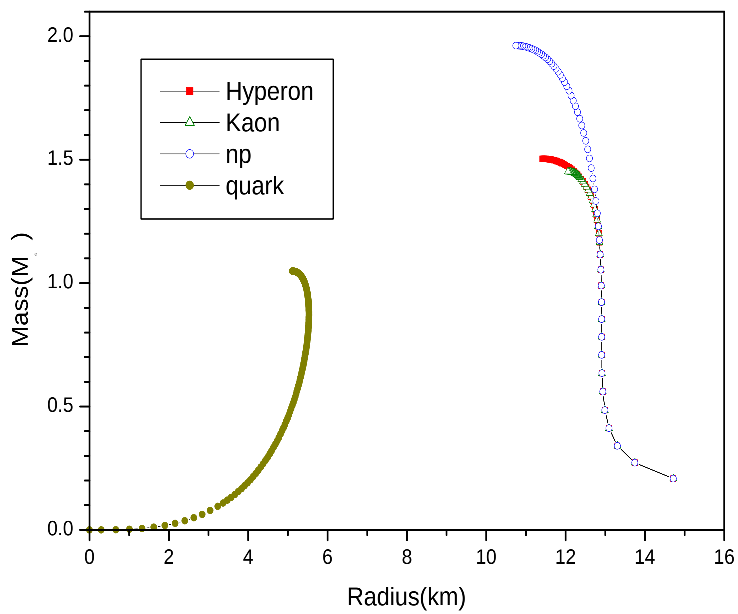
1) The properties of nuclear matter
2) N-N interaction
3) RMF models
   - Baryon octet
   - Kaon condensation

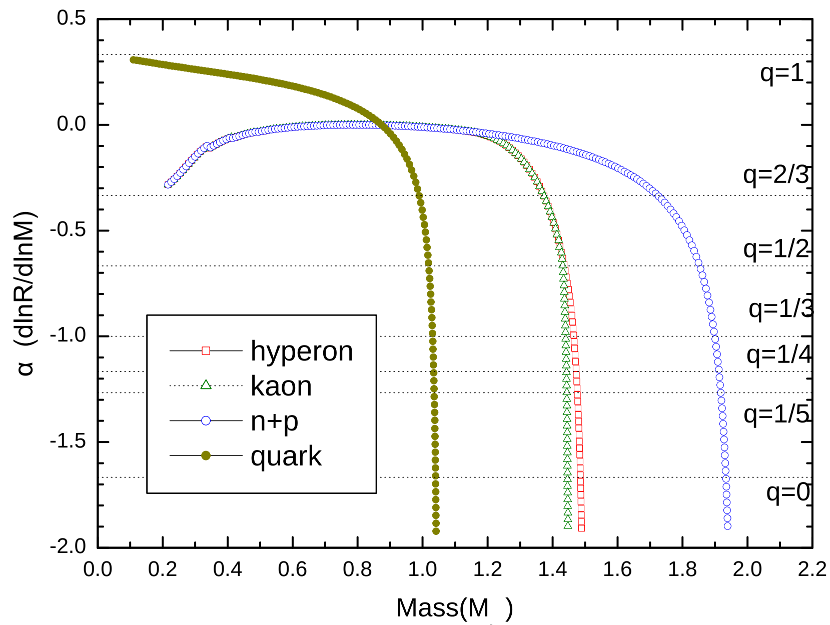
Quark matter

- MIT bag model

Neutron Star structure

Mass-Radius relation of Neutron Star





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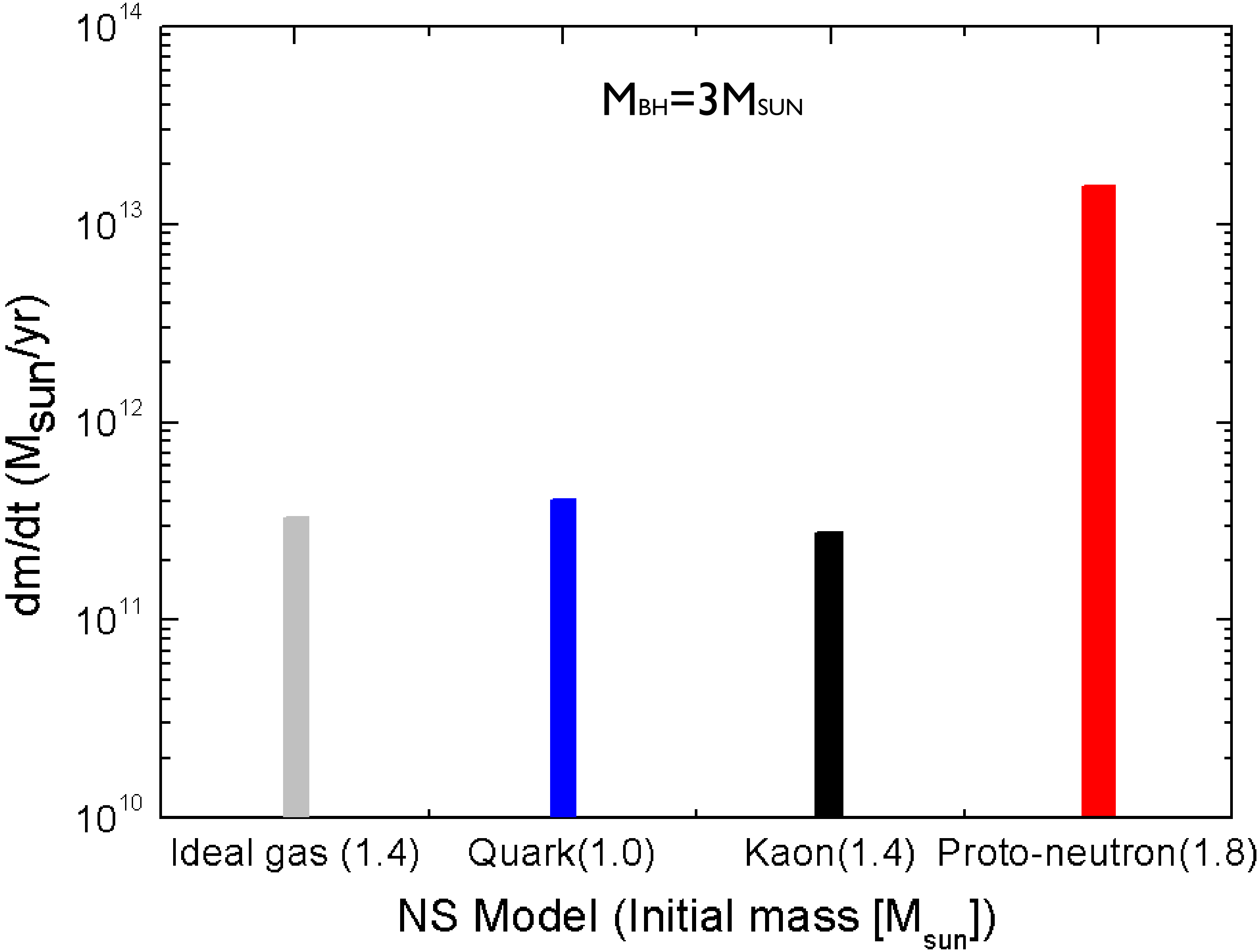
Initial mass transfer rate



$$10^{11} \sim 10^{13}$$

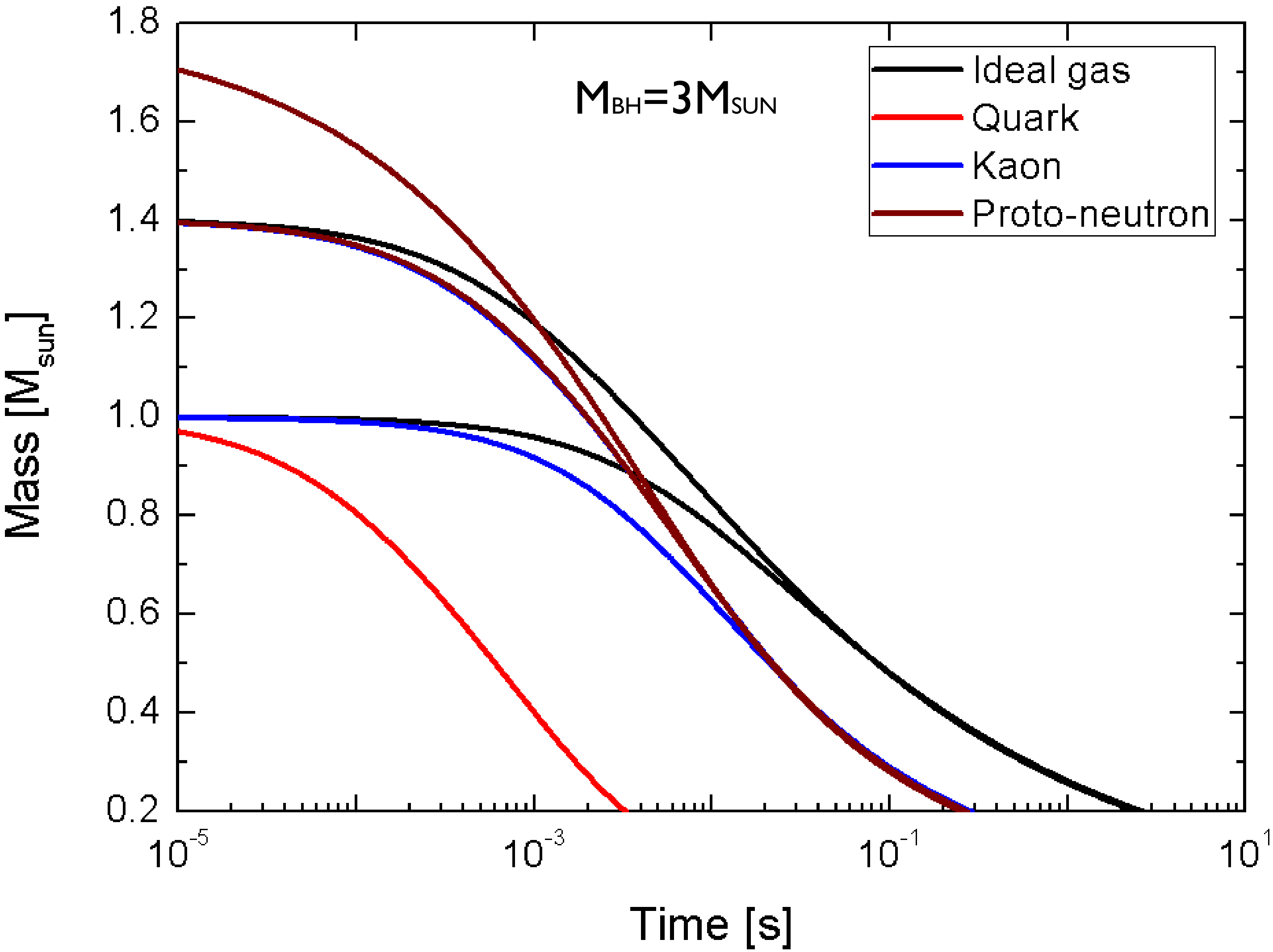
X-ray binary

$$\sim 10^{-8}$$

BH-WD

$$\sim 10^{-1}$$

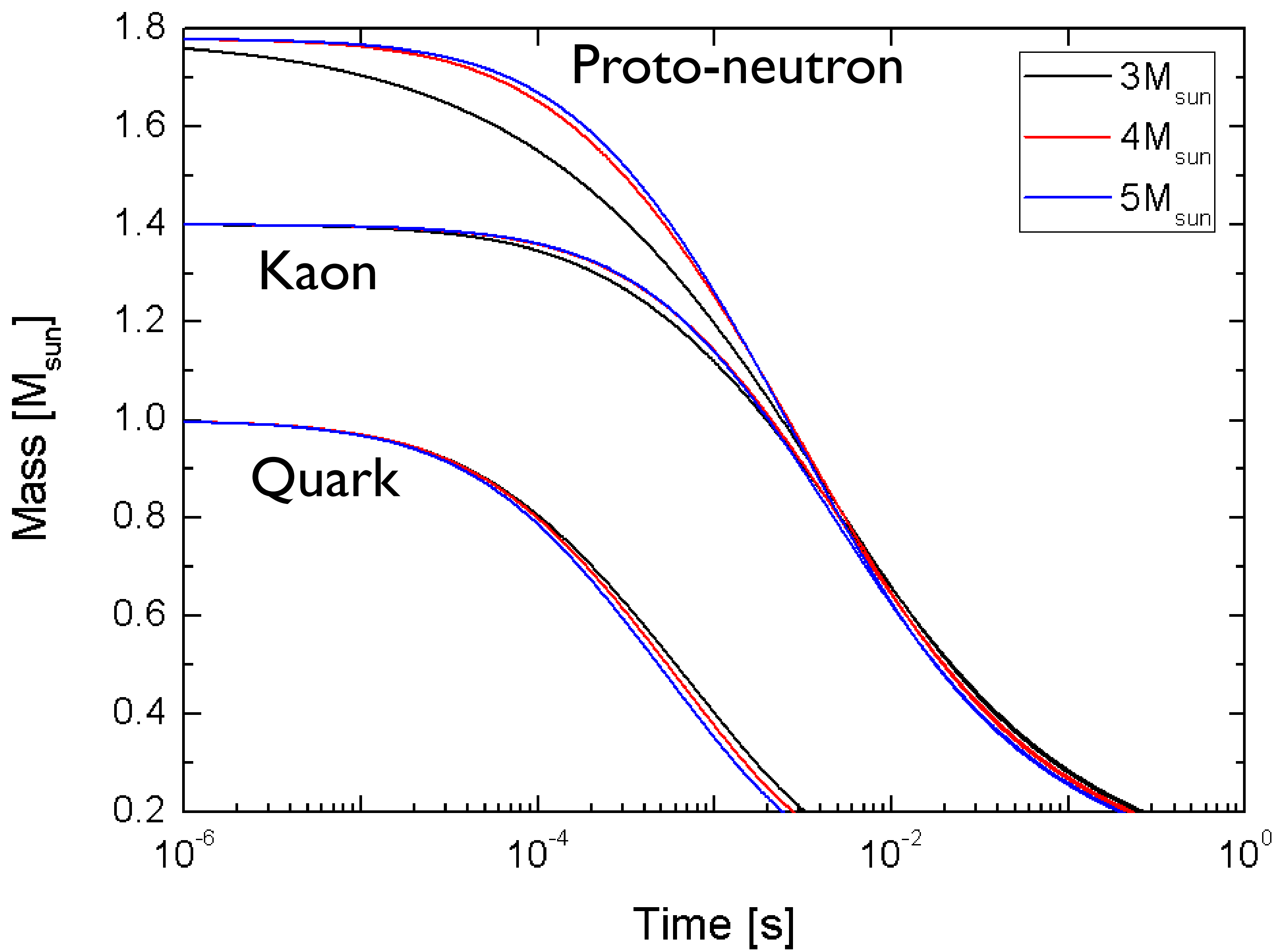
Mass transfer time scale



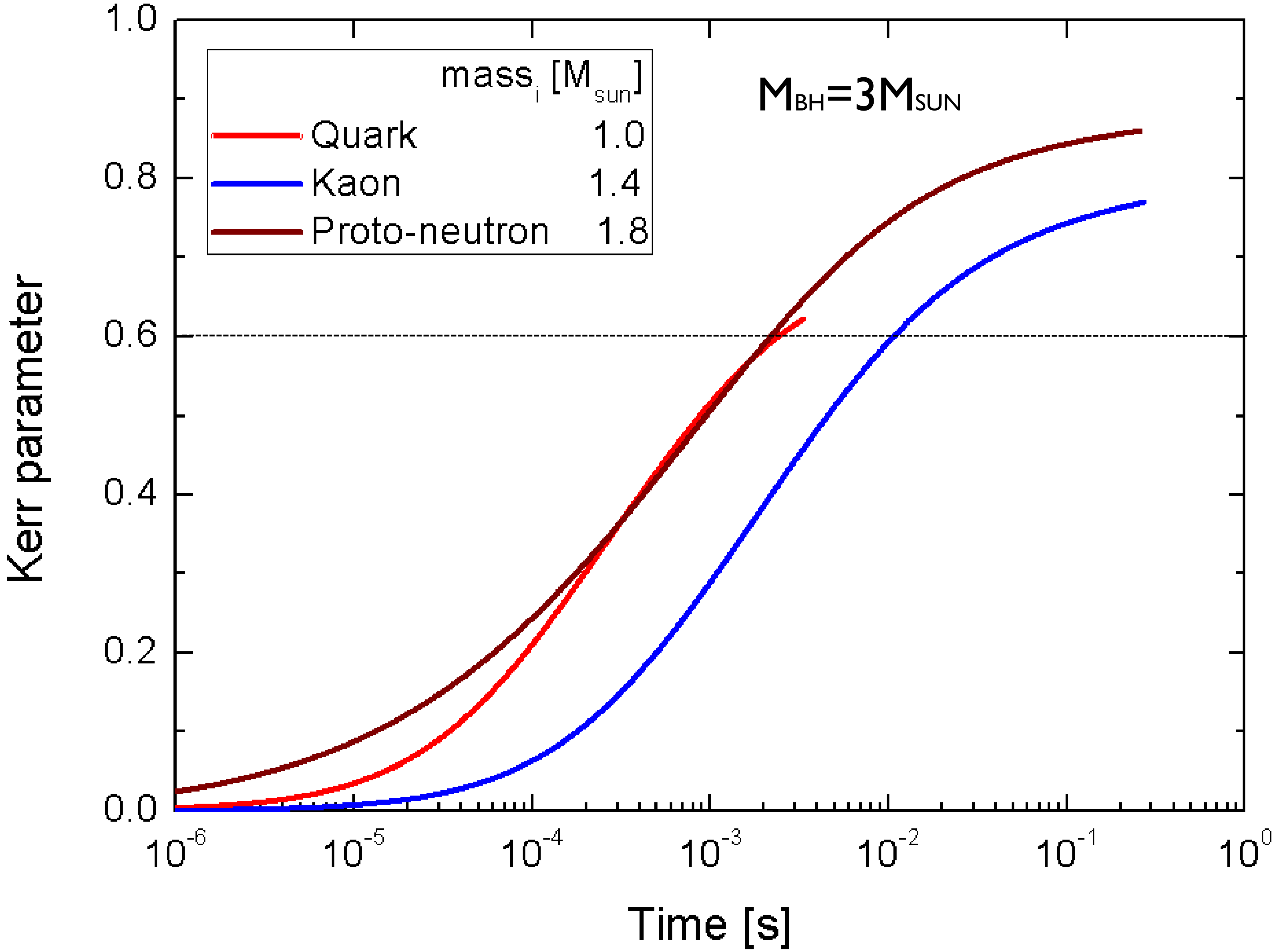
Merging time
~2 s

SHB duration
~2 s

Mass transfer time scale



BH spin up



BH spin energy

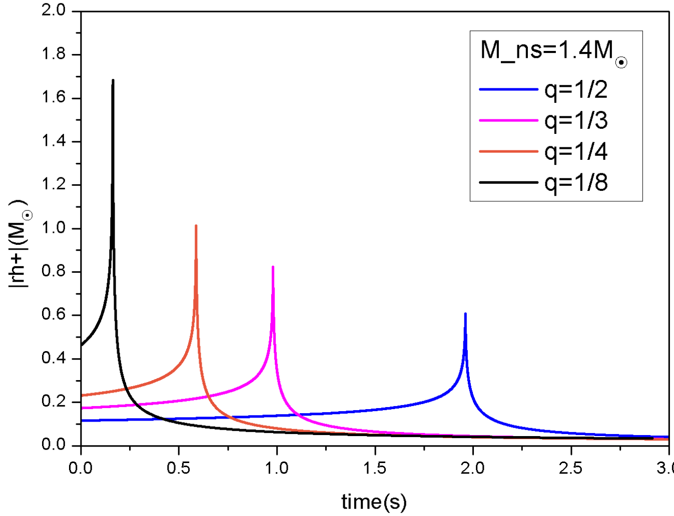
$\sim 10^{53}$ ergs

SHB energy

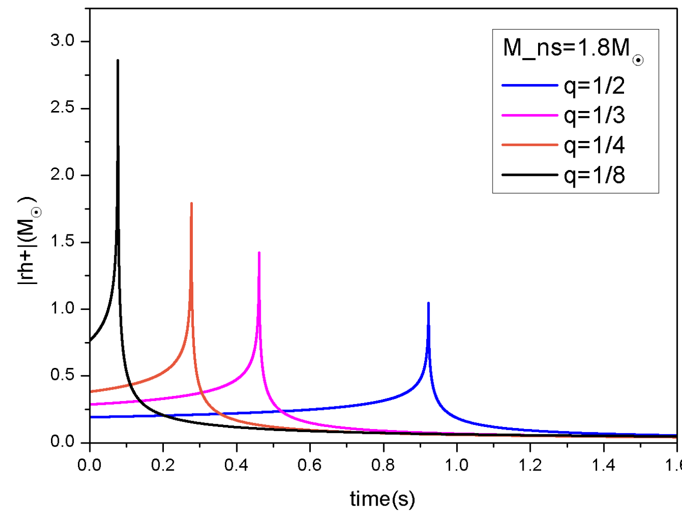
$10^{51} \sim 10^{53}$ ergs

Polarization amplitude of GW

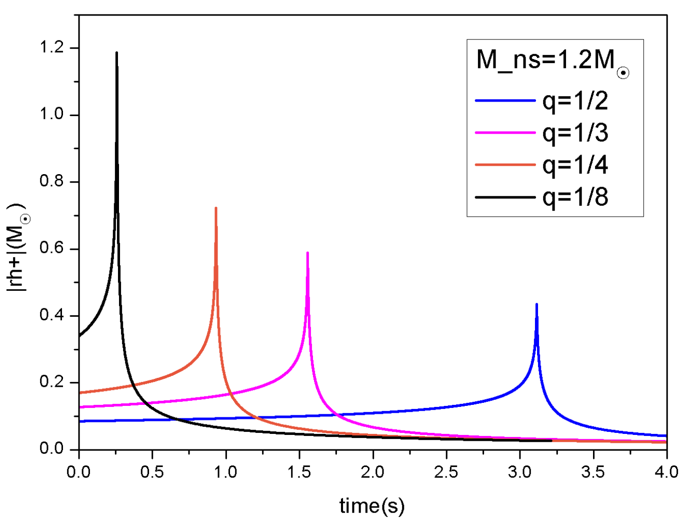
Kaon model



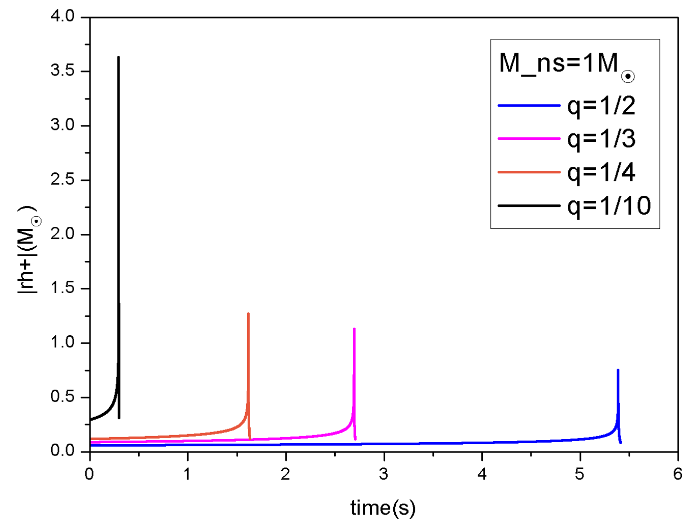
NP model



Hyperon model



Quark model

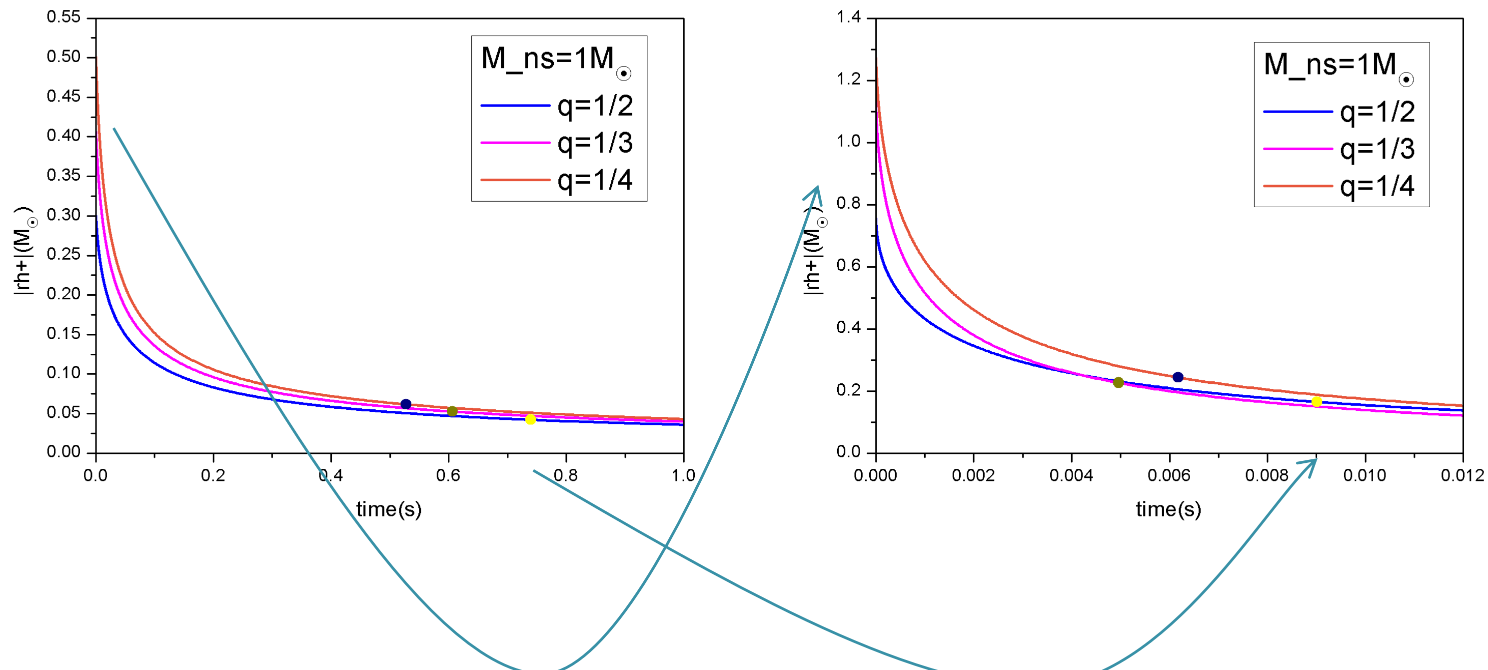


Normal NS vs. Quark Star

(kaon vs. quark)

Polarization amplitude($M_\odot$)   (After mass transfer occur)

Kaon model   Quark model

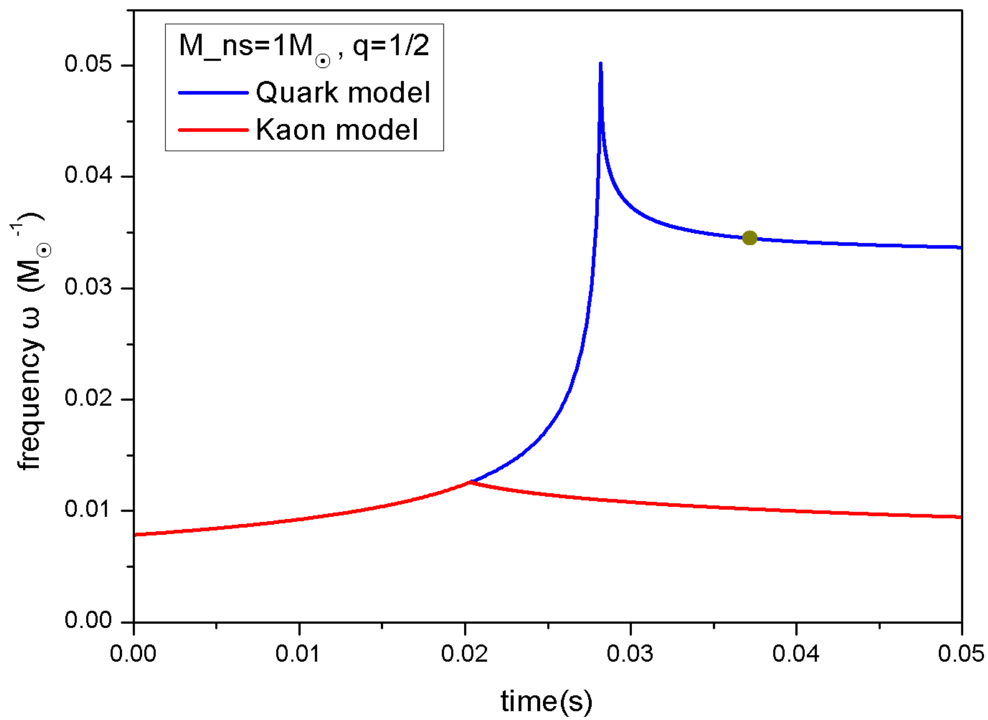


~ 2 times higher

~ 100 times quickly

Normal NS vs. Quark Star
(kaon vs. quark)

Frequency



Quark Star

Higher than Normal NS

Nearly constant
after mass transfer

Conclusions & Outlook

- Possibility of probing NS EOS in GW & GRBs. (At least, may be able to exclude some EOS)
- Need to consider the spin & eccentricities of NS-BH binaries
- And something more??