Rotational energy term in the empirical formula for the yrast energies in even-even nuclei

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Outline

I. Introduction

II. Model and Interpretation

III. Results and Discussion

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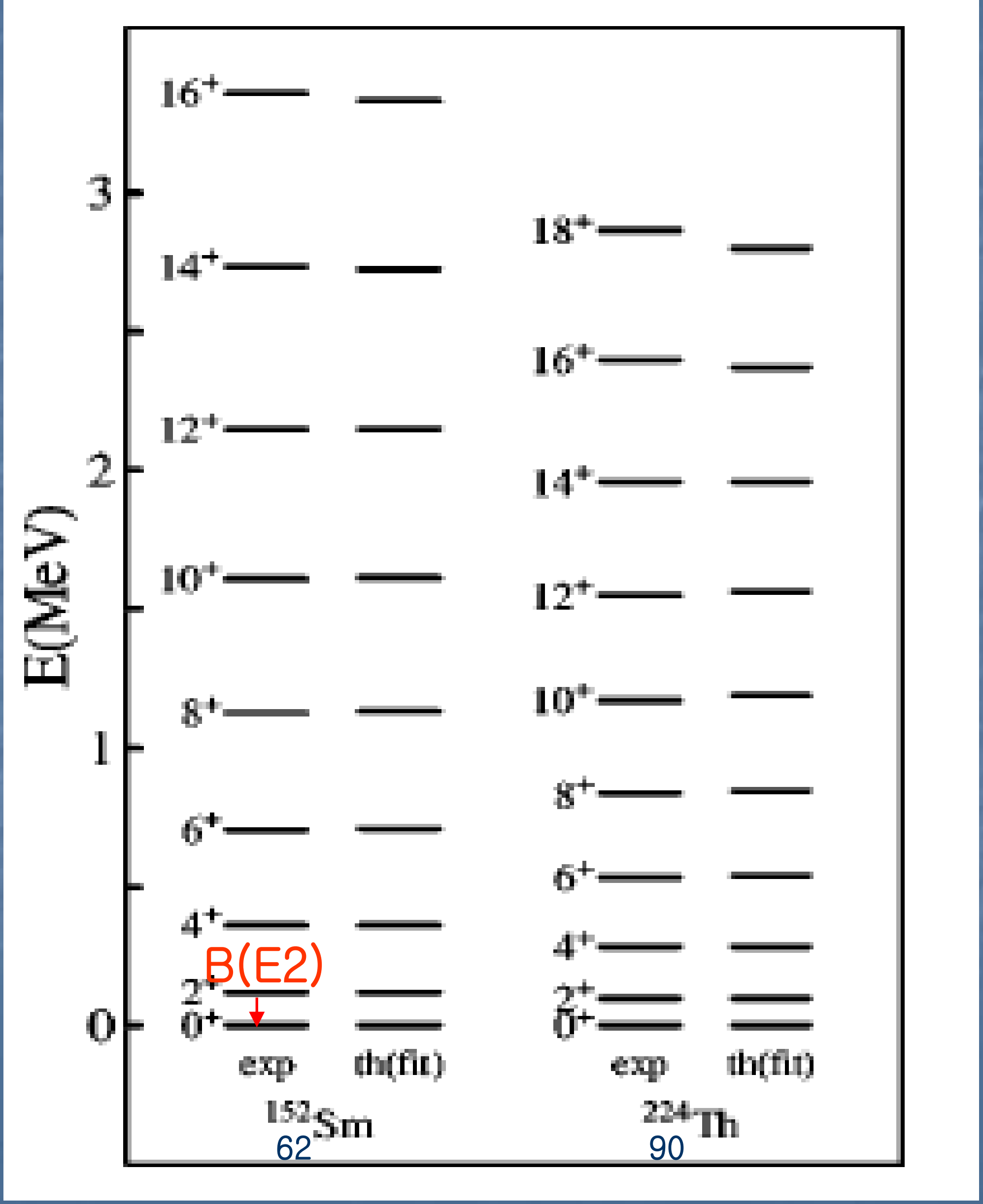
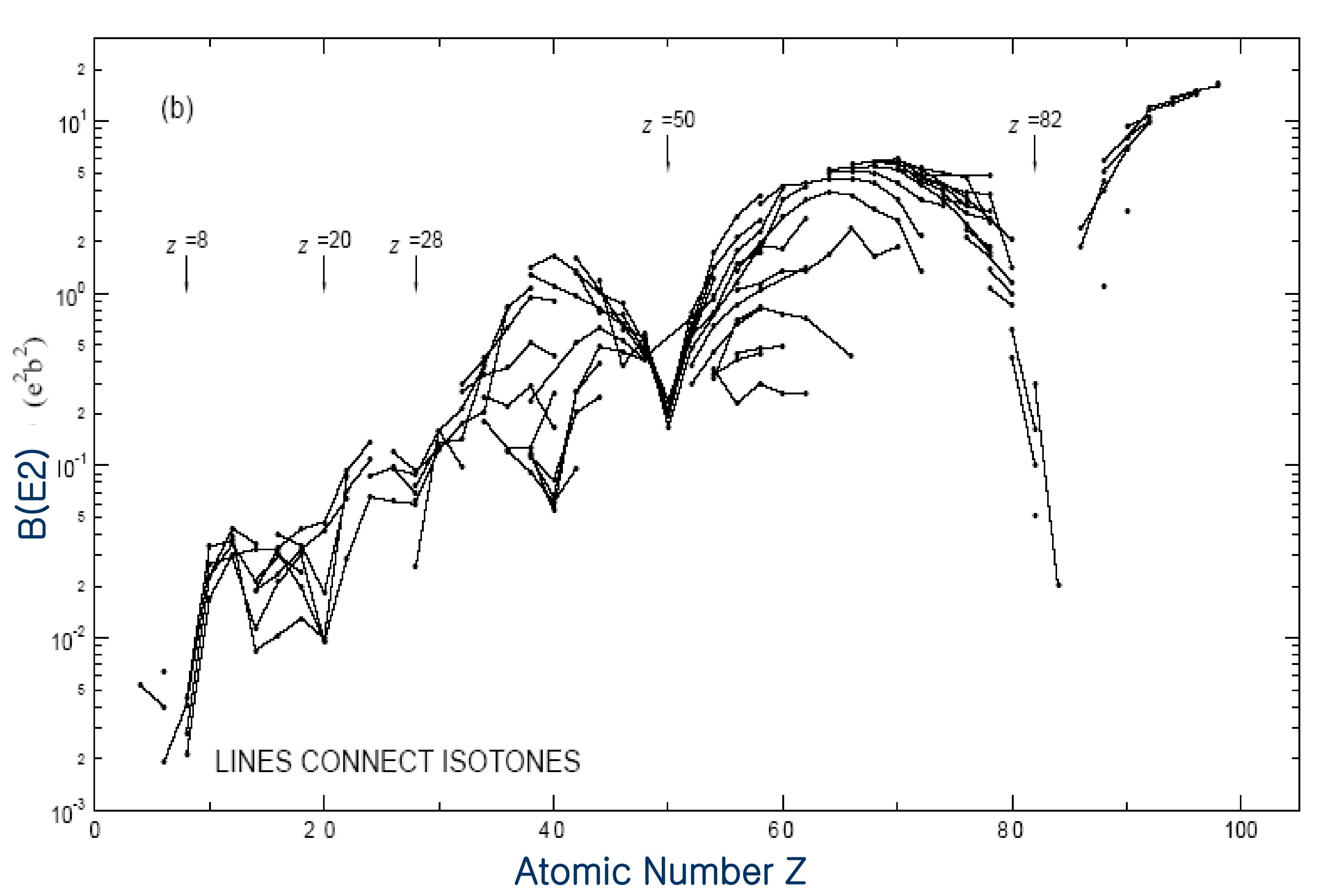
I. Introduction

❖ Yrast levels of even-even nuclei

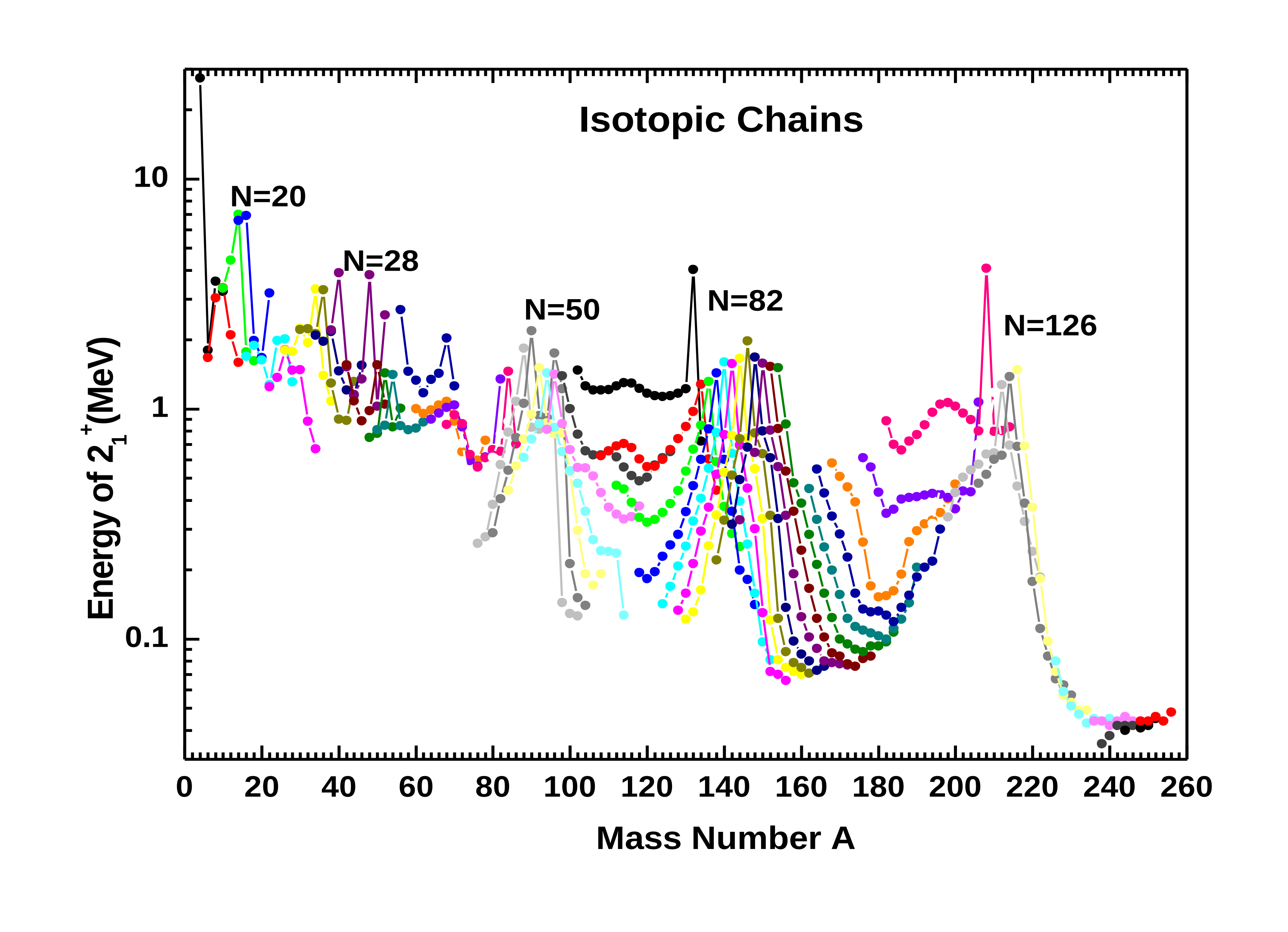


❖ Reduced transition probability B(E2; 2⁺→0⁺)



S. Raman, C. W. Nestor, Jr., and P. Tikkanen, At. Data Nucl. Data Tables 78, 1(2001)

❖ Energy of 2_1^+



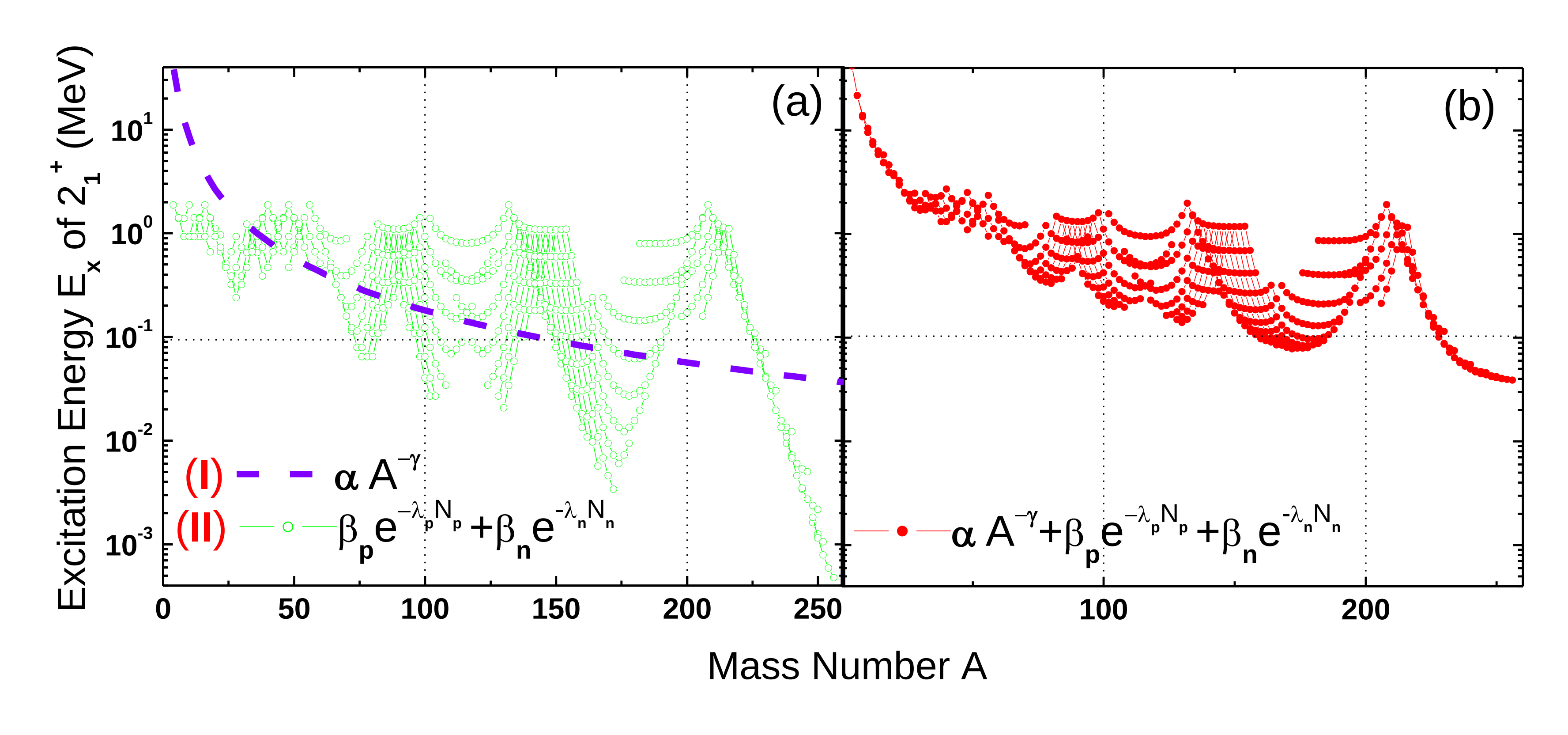
Isotopic Chains
N=20
N=28
N=50
N=82
N=126
10
1
0.1
0
20
40
60
80
100
120
140
160
180
200
220
240
260
Energy of 2_1^+(MeV)
Mass Number A

Our Empirical Formula

$$E_x = \boxed{\alpha A^{-\gamma}} + \boxed{\beta_p \exp(-N_p\lambda_p) + \beta_n \exp(-N_n\lambda_n)} \cdots (1)$$

(I) (II)

- mass number : A, valence proton (neutron) number : N_p (N_n)



Ref.[1] : D. Kim, E. Ha, and D. Cha, Nucl. Phys. A 799, 46 (2008)

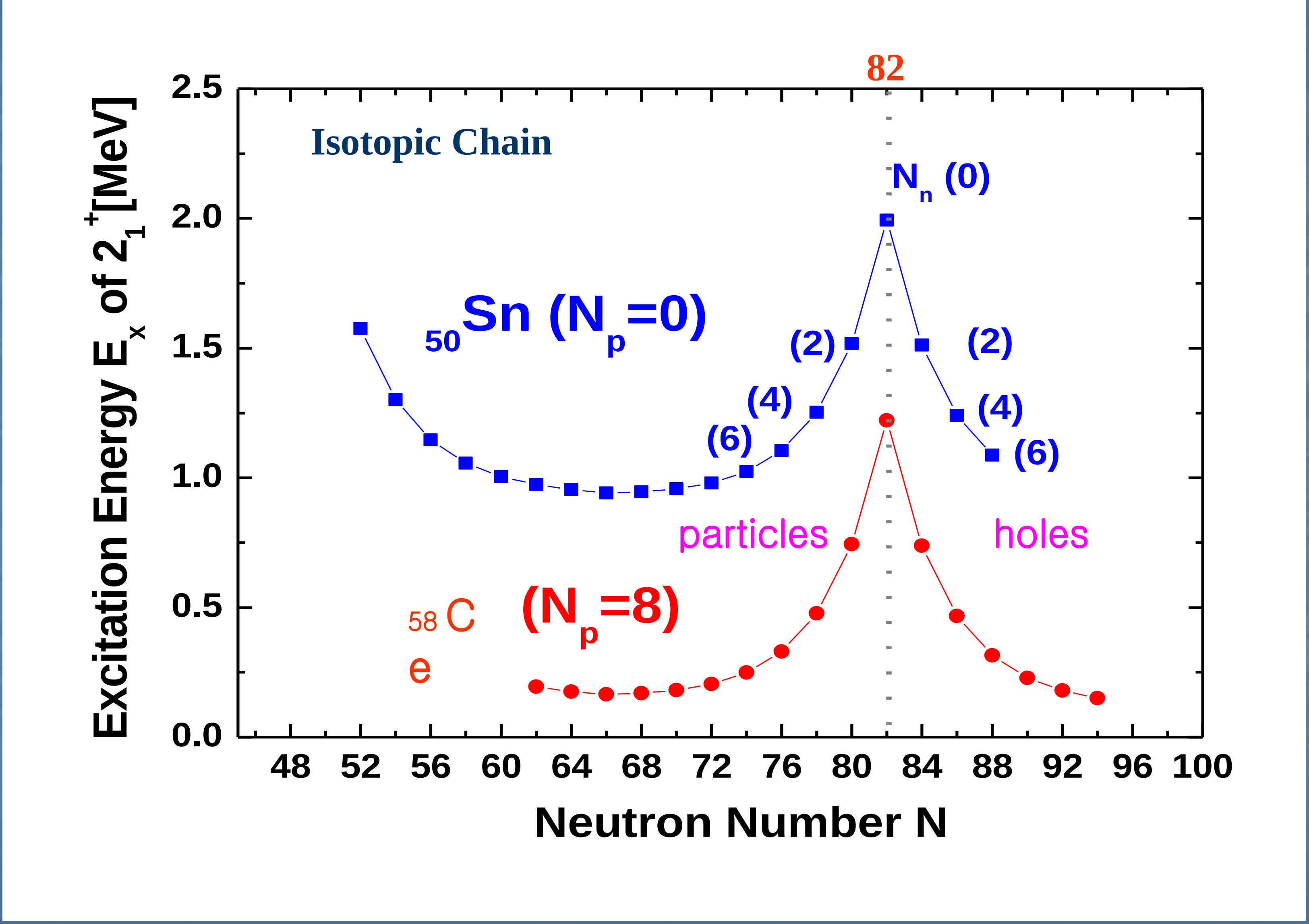
❖ Determination of N_p (N_n)

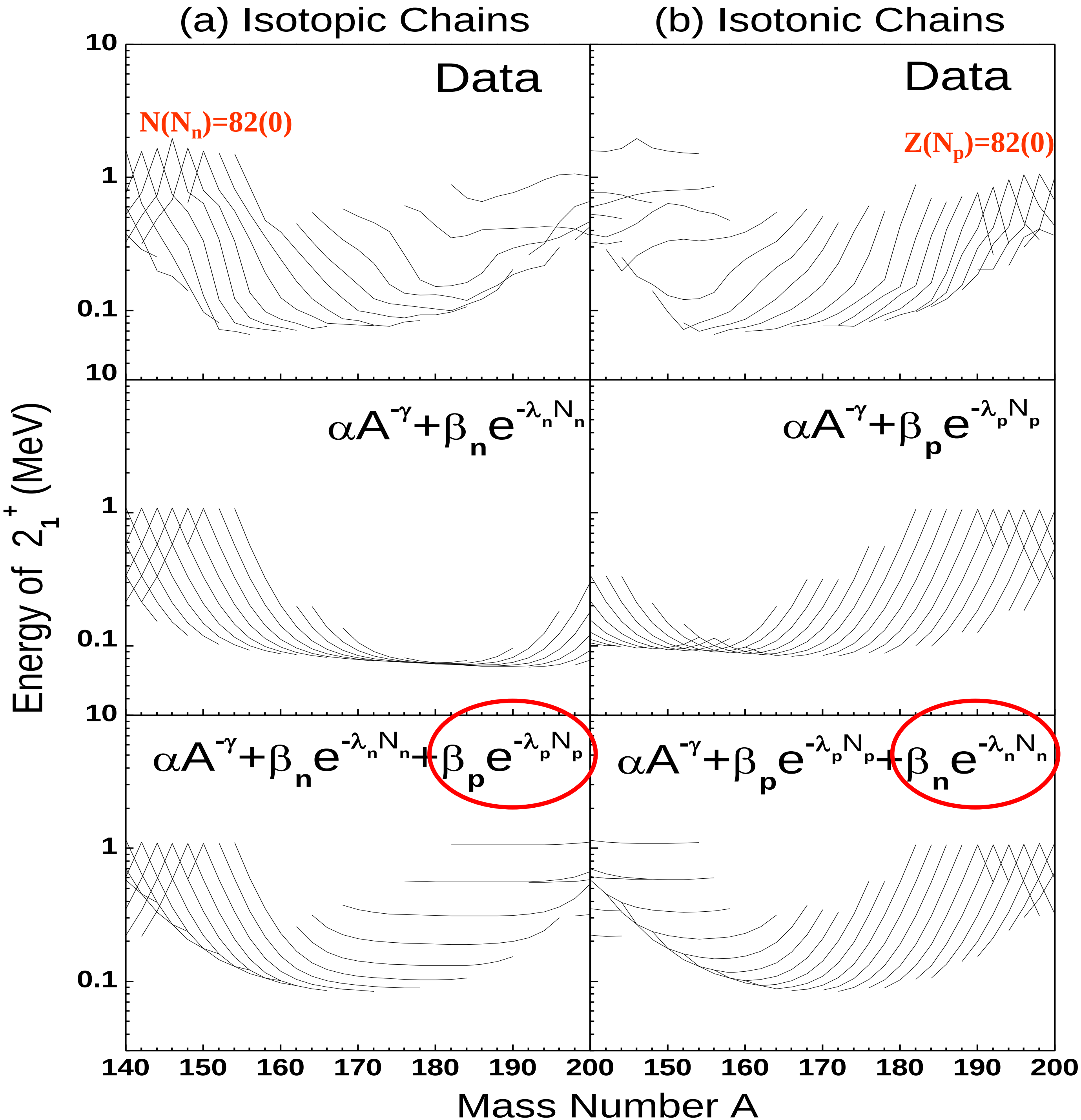
Z (N)	N_p (N_n)	Z (N)	N_p (N_n)
28	0	40	10
30	2	42	8
32	4	44	6
34	6	46	4
36	8	48	2
38	10	50	0

particles

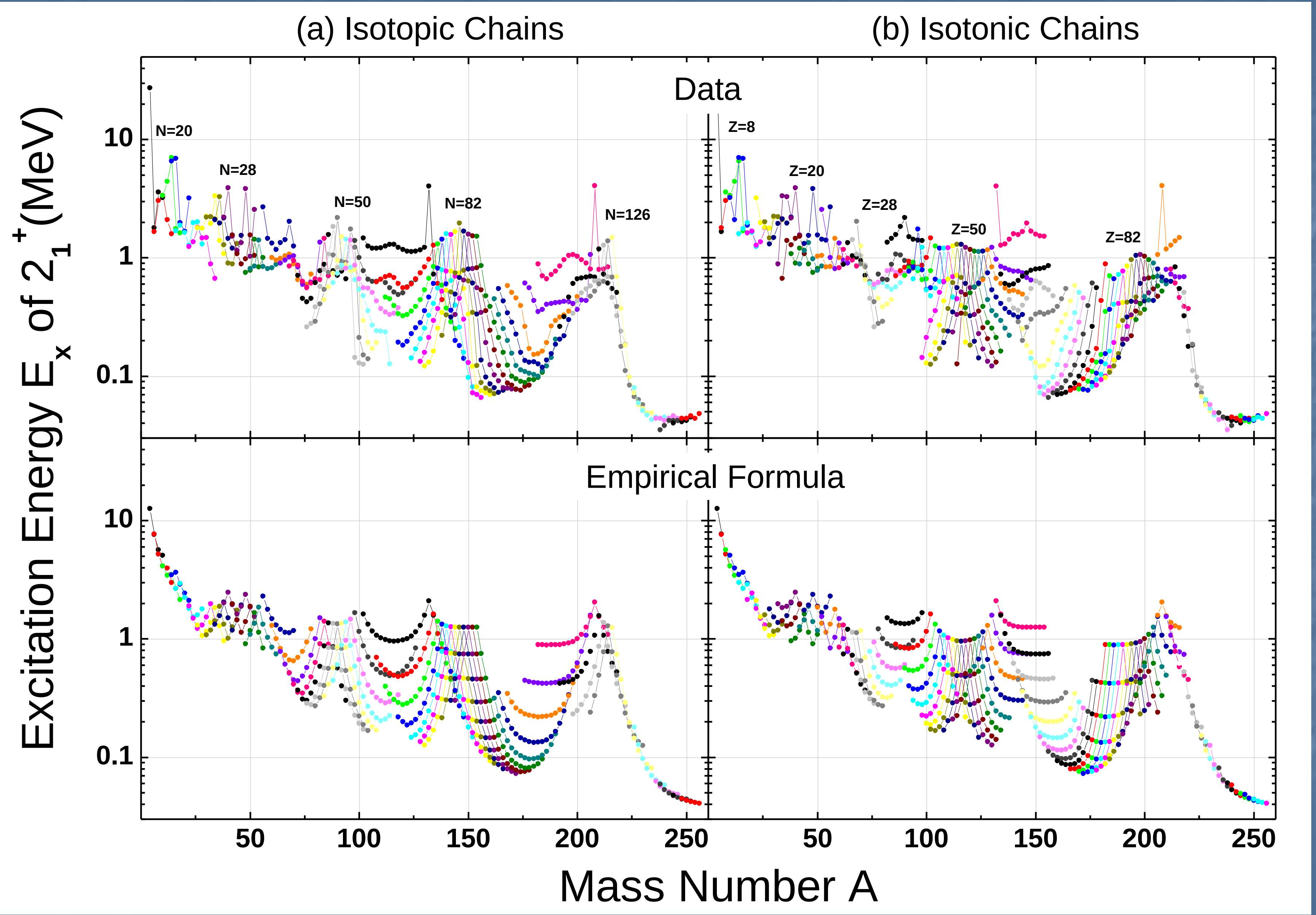
holes

- Example : $E_x = \alpha A^{-\gamma} + \beta_p \exp(-N_p\lambda_p) + \beta_n \exp(-N_n\lambda_n)$





(a) Isotopic Chains
(b) Isotonic Chains
Data
Data
N(N_n)=82(0)
Z(N_p)=82(0)
αA^-γ+β_n e^-λ_n N_n
αA^-γ+β_p e^-λ_p N_p
αA^-γ+β_n e^-λ_n N_n+β_p e^-λ_p N_p
αA^-γ+β_p e^-λ_p N_p+β_n e^-λ_n N_n
Energy of 2_1^+ (MeV)
Mass Number A
10
1
0.1
140
150
160
170
180
190
200


(a) Isotopic Chains
(b) Isotonic Chains
Data
Empirical Formula
N=20
N=28
N=50
N=82
N=126
Z=8
Z=20
Z=28
Z=50
Z=82
10
1
0.1
50
100
150
200
250
Excitation Energy E_x of 2_1^+ (MeV)
Mass Number A

The physical meaning of the empirical formula ?

Ⅱ. Model and Interpretation

In the deformed nuclei around doubly mid-shell region

$$E_x^{mid} \approx \alpha A^{-\gamma} \cdots\cdots (2)$$

Ref.[2] : G. Jin, D. Cha, and J-H Yoon, J. Korean Phys. Soc. 52, 1164(2008)

The rotational energy for the rotational bands such as $J^\pi = 2^+, 4^+, 6^+, \cdots$

$$E_{rot}(J^+) = \frac{J(J+1)\hbar^2}{2I} \cdots\cdots (3)$$

- J : total angular momentum
- I : effective moment of inertia

If a nucleus is a rigid body having the axial symmetry about the intrinsic 3 axis

$$I_{rig} = \frac{2}{5} M R_0^2 (1 + \frac{\delta}{3}) \cdots\cdots (4)$$

- nuclear mass $M = uA$
- nuclear mean radius $R_0 = 1.2A^{1/3}$
- distortion parameter $\delta \approx (R_3 - R_\perp)/R_0$

$\Rightarrow 0.3$ (for ${}^{174}_{70}Yb$)

Ref.[3] : A. Bohr and B. R. Mottelson, Nuclear Structure, Vol. II

The observed moments of inertia are smaller than the moment of inertia given by Eq (4), , I_{rig} around the doubly mid-shell

$$\boxed{I = kI_{rig} \cdots\cdots (5)} \qquad k = 1/2$$

By inserting Eq (5) into Eq (3)

$$\boxed{E_{rot}(J^{+}) = \alpha' A^{-\gamma'} \cdots\cdots (6)}$$

$$\alpha' = J(J+1)\,\alpha_0, \quad \gamma' = 5/3 \cong 1.67$$

$$\alpha_0 = \frac{5\,\hbar^2}{4\,k\,u\,1.2^2\,(1+\frac{\delta}{3})}$$

We perform the χ^2 analyses

i) fixed $\gamma = 1.40$ for $1.34 \sim 1.47 (2^+ \sim 10^+) \Leftrightarrow \alpha, \beta_p, \lambda_p, \beta_n$, and λ_n

ii) fixed α' and $\gamma' = 5/3 \quad \Leftrightarrow \beta_p, \lambda_p, \beta_n$, and λ_n

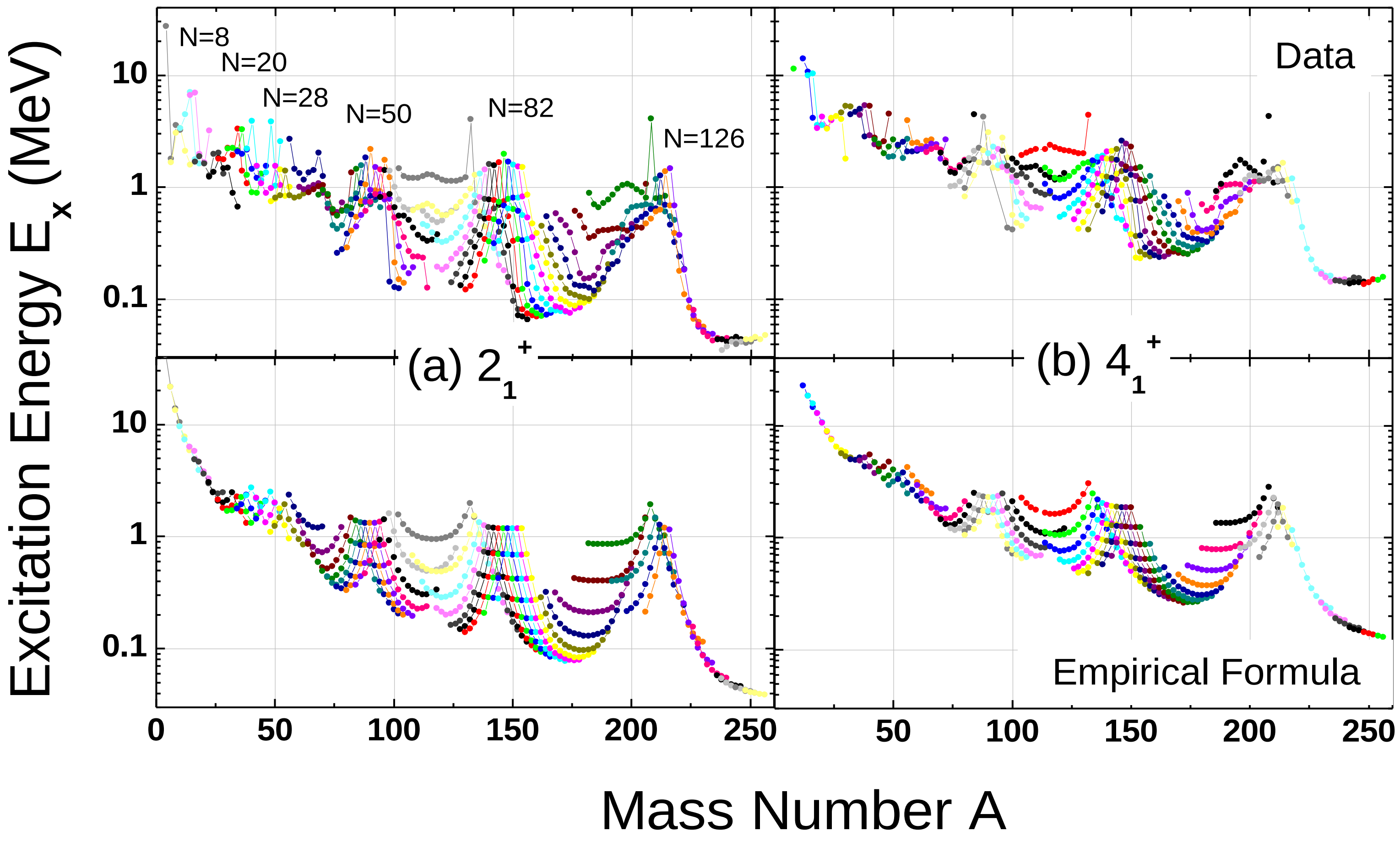
$$R_E(i) = \log[E_x^{cal}(i)] - \log[E_x^{\exp}(i)] \cdots\cdots (8)$$

$$\chi^2 = \frac{1}{N_0} \sum_{i=1}^{N_0} [R_E(i)]^2 \cdots\cdots (9)$$

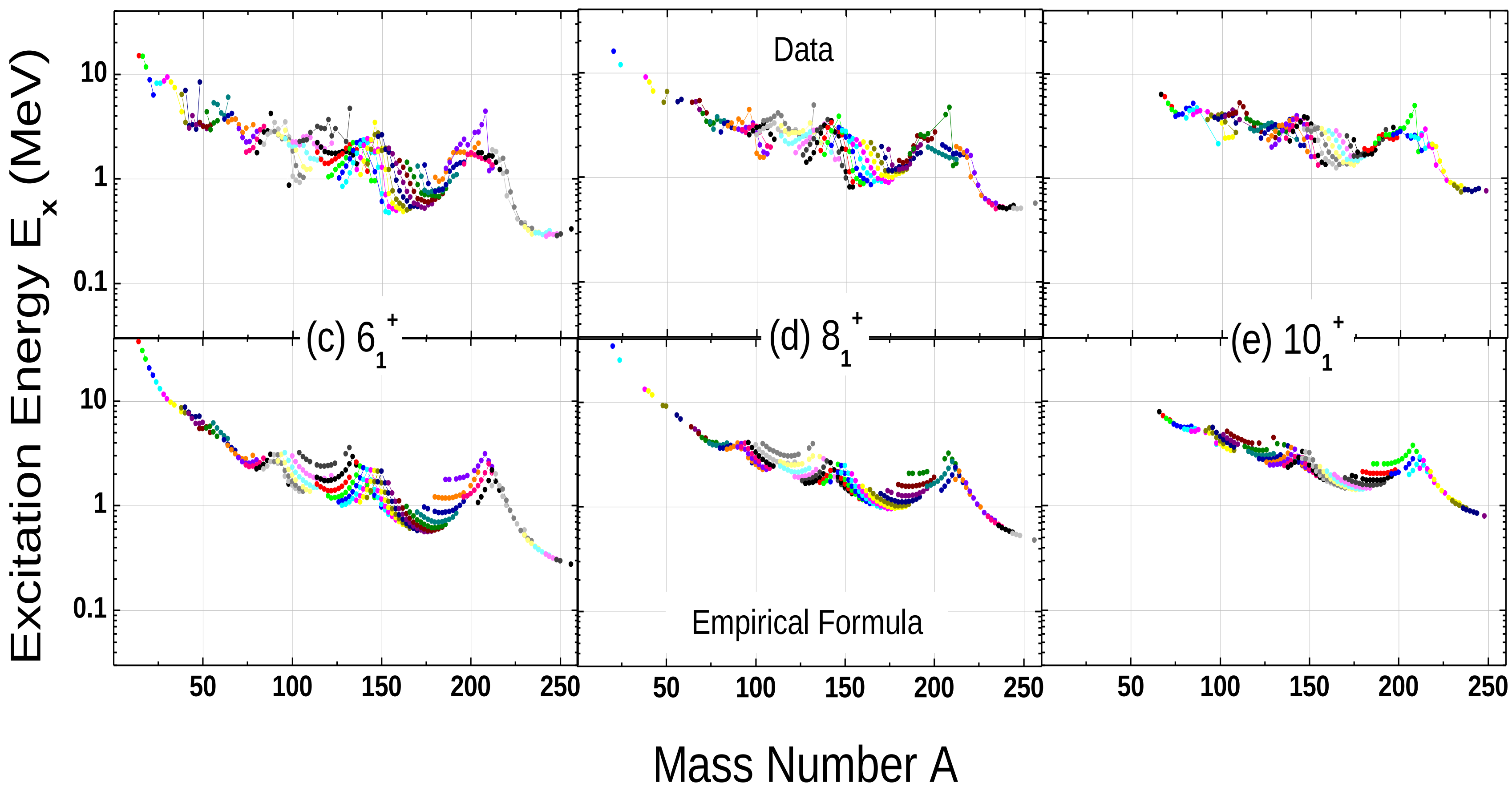
N_0 : Number of total data points in the fitting

Ⅲ. Results and Discussion

- Excitation energies of 2^+ and 4^+ Measured vs. Calculated



❖ Excitation energies of $6^+, 8^+, \text{and} 10^+$: Measured vs. Calculated

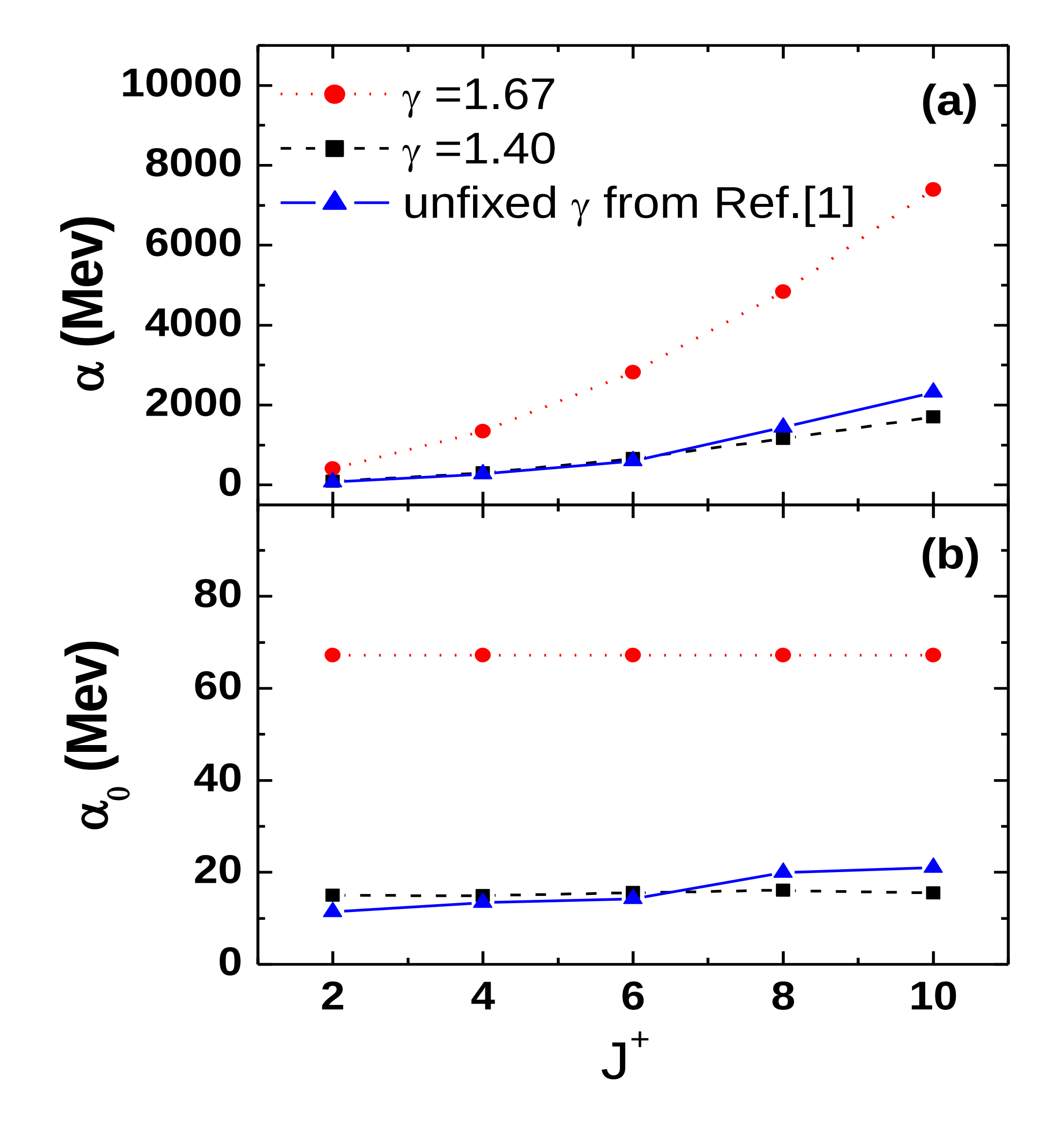


III. Results and Discussion

J_1^π	γ	α(MeV)	α_0(MeV)	β_p(MeV)	β_n(MeV)	λ_p	λ_n	χ^2	N_0
2_1^+	1.67	395.76	65.96	0.79	1.09	0.42	0.29	0.157	557
4_1^+	1.67	1319.20	65.96	1.12	1.54	0.34	0.24	0.094	430
6_1^+	1.67	2770.32	65.96	1.31	1.46	0.32	0.18	0.086	375
8_1^+	1.67	4749.12	65.96	1.27	1.34	0.26	0.17	0.060	309
10_1^+	1.67	7255.60	65.96	1.30	1.46	0.23	0.18	0.040	265
2_1^+	1.40	89.89	14.98	0.82	1.15	0.41	0.28	0.126	557
4_1^+	1.40	297.87	14.89	1.20	1.67	0.33	0.23	0.071	430
6_1^+	1.40	654.71	15.59	1.40	1.64	0.31	0.18	0.069	375
8_1^+	1.40	1155.90	16.05	1.34	1.50	0.26	0.15	0.053	309
10_1^+	1.40	1702.79	15.48	1.34	1.64	0.22	0.15	0.034	265
2_1^+	1.34	68.37	11.40	0.83	1.17	0.42	0.28	0.126	557
4_1^+	1.38	268.04	13.40	1.21	1.68	0.33	0.23	0.071	430
6_1^+	1.38	598.17	14.24	1.40	1.64	0.31	0.18	0.069	375
8_1^+	1.45	1438.59	19.98	1.34	1.50	0.26	0.15	0.053	309
10_1^+	1.47	2316.85	21.06	1.36	1.65	0.21	0.14	0.034	265

25% (for 2_1^+), 32% (for 4_1^+), 25% (for 6_1^+), 13% (for 8_1^+), and 18% (for 10_1^+)

❖ α and α_0 for the three different γ

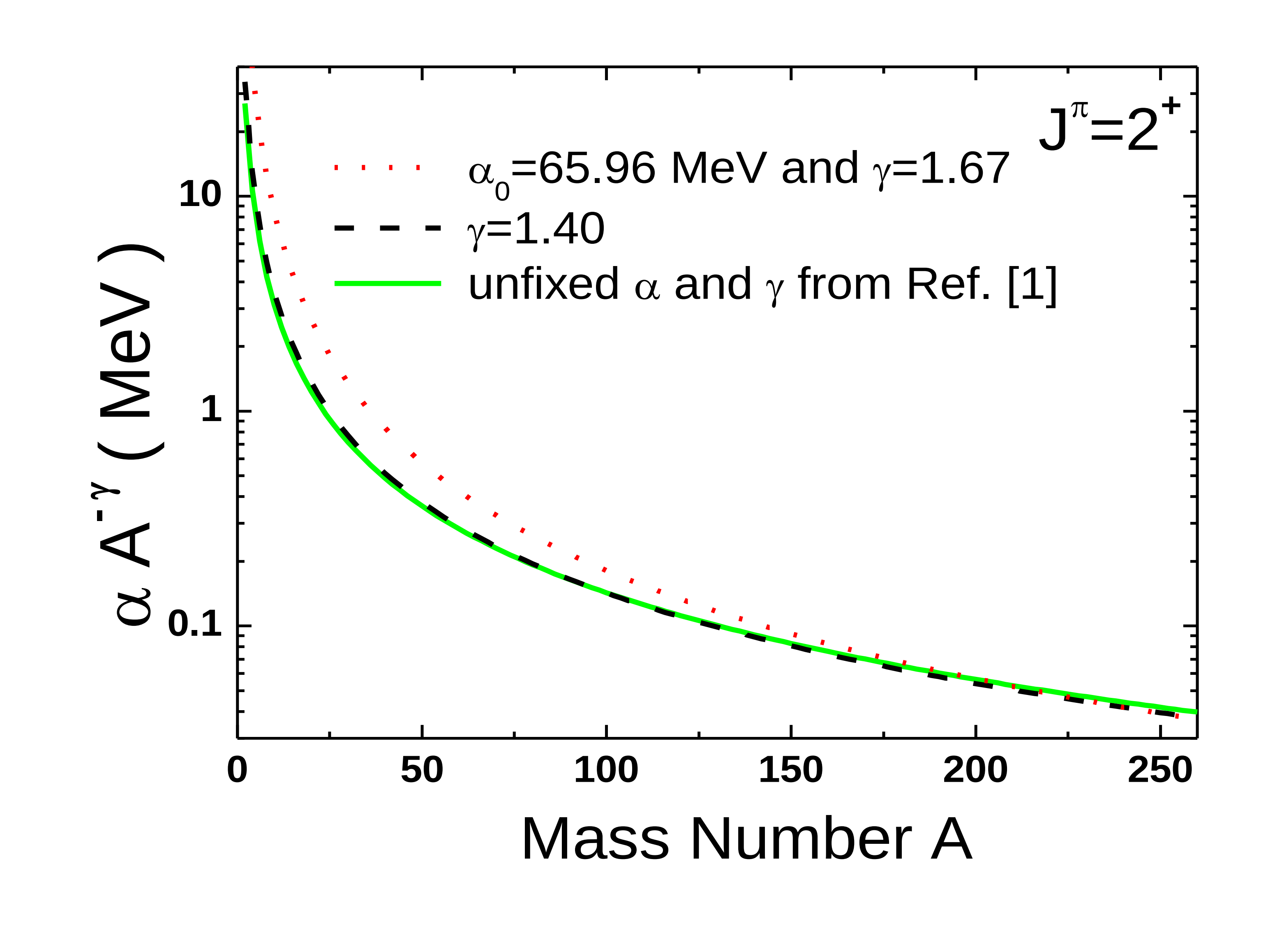


$$E_x = \alpha A^{-\gamma} + \cdots$$

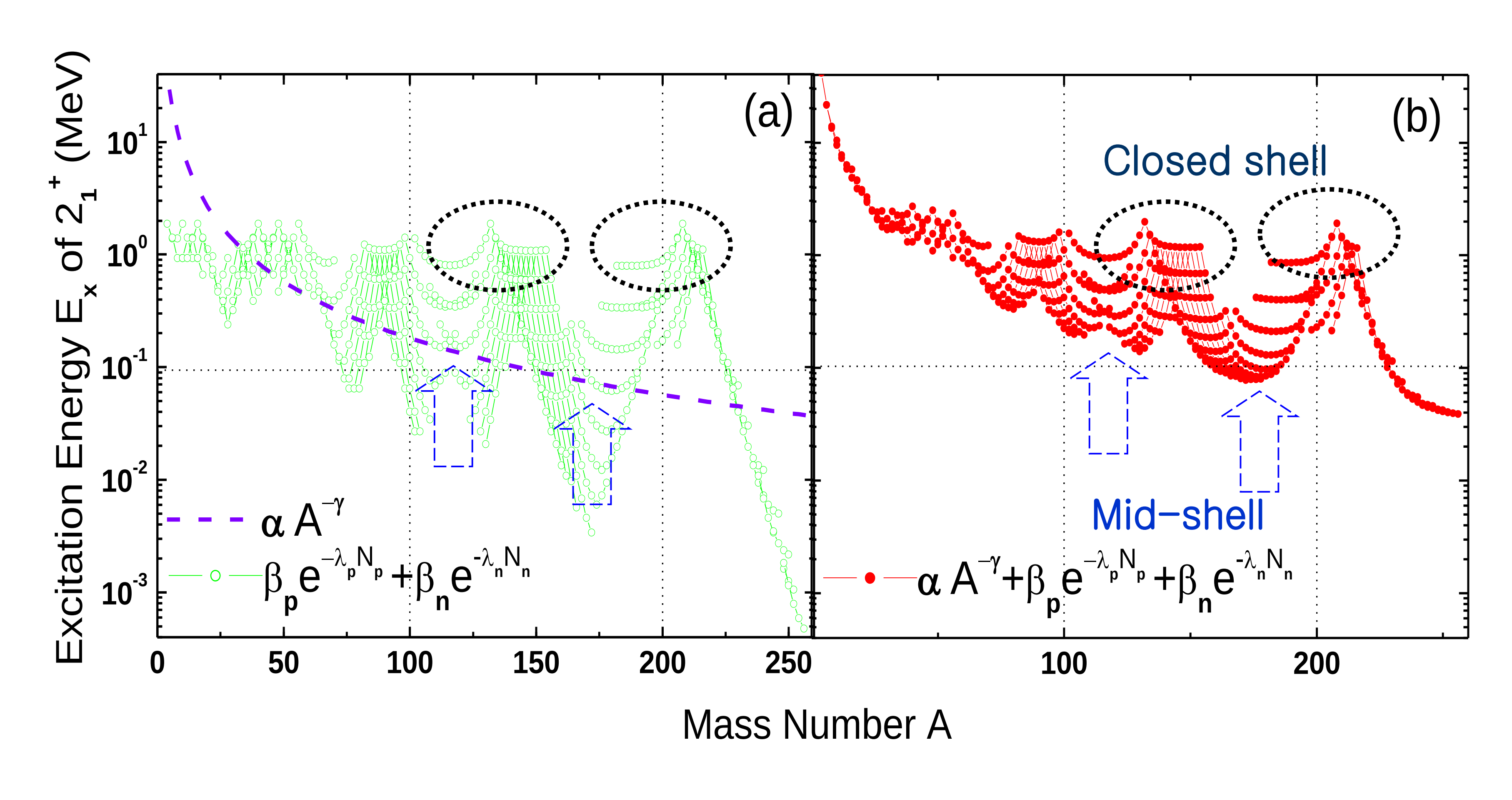
$$= \alpha_0 J(J+1) A^{-\gamma} + \cdots$$

$$\left\{ \frac{A^{(-1.40)}}{A^{(-1.67)}} \approx 4 \quad (\text{for } {}^{174}_{70}\text{Yb}) \right\}$$

❖ First term of the empirical formula for the three different



❖ Excitation energies of the 2_1^+ states for the first, two exponential, and all term



IV. Summary

- The term of $\alpha A^{-\gamma}$ can be obtained by considering the moment of inertia of a deformed nucleus can interpret well previous analyses.

- It is remarkable that the parameters α and γ extracted earlier research agree with those (α' and γ') obtained from the rotor model and that the previous values of α' divided by J(J+1) are almost constant as expected from the rotor model.

- The excitation energies calculated with constant α' and γ' can describe the main essential features of the measured first excitation energies for all of the natural parity even multipole states in even-even nuclei.

- The empirical formula can be well separated into the moment of inertia term $\alpha A^{-\gamma}$ and the two exponential terms which are thought to be related to the shell effect.