Instanton vacuum at finite density

Hyun-Chul Kim

Department of Physics
Inha University

S.i.N. and H.-Ch.Kim, *Phys. Rev. D* 77, 090014 (2008)

S.i.N., H.Y.Ryu, M.Musakhanov and H.-Ch.Kim, arXiv:0804.0056 [hep-ph]

S.i.N. and H.-Ch.Kim, *Phys. Lett. B 666, 324 (2008)*

Introduction

QCD, the underlying theory for the strong interaction.

Difficulties in QCD in the low energy region: Nonperturbative feature

Developing models guided by symmetries

Spontaneous Breakdown of Chiral Symmetry

Chiral bag, NJL, Skyrmion, HLS etc…

But… what is the origin of this interesting phenomena?

Highly nontrivial QCD vacuum

It also effects on QCD phase structure, represented by $\langle\bar{q}q\rangle$, $\langle qq\rangle$,…

Investigations on the vacuum itself: **Instanton**, dyon, caloron, etc.

Instanton

Classical ground state solution of the QCD in Euclidean space

QED (solid-state physics): electron & phonon vs. QCD : quark & instanton

Minimizing the YM action: Self-dual condition $F^a_{\mu\nu} = \tilde{F}^a_{\mu\nu}$

(Singular gauge) instanton solution

$$A^a_\mu(x) = \frac{2\bar{\eta}^\mu_{\nu a}\rho^2}{x^2(x^2+\rho^2)}$$

'‘t Hooft symbol

Instanton size ~ 1/3 fm

Nonperturbative part of gluons replaced by instantons

(Anti)quarks moving around this effective potential-like ensemble

$$[i\partial\!\!\!/ + A\!\!\!/_{I\bar{I}} + im_f]\,\Phi_{I\bar{I}} = 0$$

So called, quark zero-mode solution.

Instanton

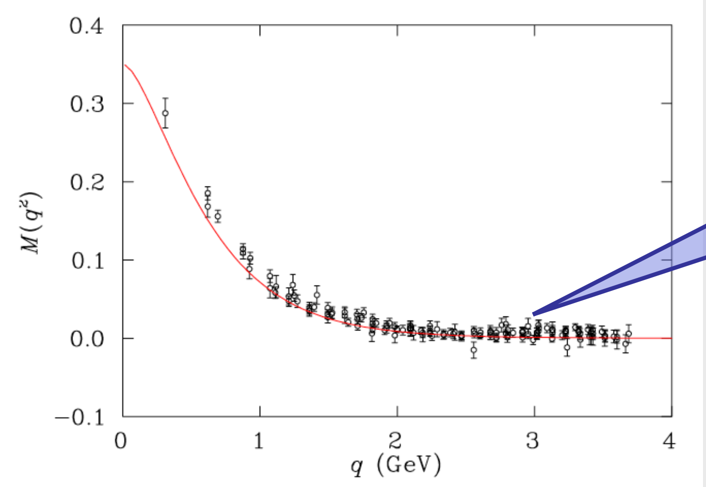
Quark propagator in the instanton effects

Fourier transform of the zero-mode solution

$$S^{-1}(i\partial) = i\partial\!\!\!/ + im_f + iM_0 F_f^2(i\partial)$$

$$F(k) = 2t\left[I_0(t)K_1(t) - I_1(t)K_0(t) - \frac{1}{t}I_1(t)K_1(t)\right], \quad t = \frac{k\bar{\rho}}{2}$$

Momentum-dependent quark mass via Fourier transformation of Φ



Lattice data via Extrapolation to the chiral limit (Improved staggered fermion)

P.Bowman et al.,hep-lat/0209129

Playing a role of a natural UV regulator in the framework

Ex) Non-zero chiral condensate, ⟨qq⟩ ≠ 0 → SBCS

Instanton

Merits of the instanton framework

· Preserving all relevant QCD symmetries

· $U(1)_A$ included via nonzero topological susceptibility

· Natural scale parameter: average instanton size & inter-distance

· No adjustable free parameters (at least for light quarks in leading *Nc*)

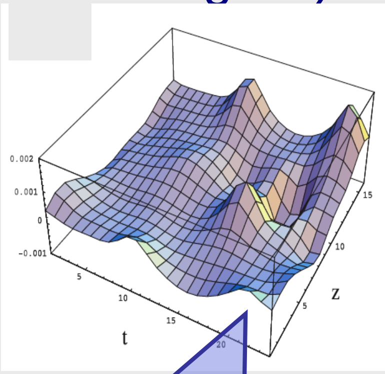
· Natural UV regulator: $M(k)$

· Nonlocal interaction between quarks

· Natural derivations of (almost) NJL and Skyrme mode

But No confinement

What is the instanton distribution? (δ-function)



Instanton fluctuation at zero-point (topological charge density)

M.-C.Chu et al.,PRD49,6039(1994)

S.i.N. and H.Ch.Kim, PRD77, 090014 (2008)

Pion Electromagnetic form factor

Pion Electromagnetic (EM) matrix element

$$\langle \mathcal{M}(P_f)|j_\mu^{\mathrm{EM}}(0)|\mathcal{M}(P_i)\rangle = (P_i + P_f)_\mu F_\mathcal{M}(q^2)$$

EM current

$$j_\mu^{\mathrm{EM}}(x) = \sum_{\mathrm{u,d,s}} e_f \psi_f^\dagger(x)\gamma_\mu\psi_f(x)$$

Normalization conditions corresponding to the Ward-Takahashi identity

$$F_{\pi^+}(0) = F_{K^+}(0) = 1, \quad F_{K^0}(0) = 0$$

EM charge radius $$\langle r^2\rangle_\mathcal{M} = -6\left[\frac{\partial\, F_\mathcal{M}(Q^2)}{\partial\, Q^2}\right]_{Q^2=0}$$

Pion electromagnetic form factor

Effective chiral action modified by external EM source

$$\mathcal{S}_{\rm eff}[\mathcal{M}^\alpha, V, m] = -\mathrm{Sp}_{c,f,\gamma} \ln \left[i\not{D} + im + i\sqrt{M(iD)}U^{\gamma_5}\sqrt{M(iD)} \right]$$

Momentum-dependent quark mass and covariant derivative

$$M(iD) = M_0 F^2(iD) \qquad iD_\mu = i\partial_\mu + V_\mu$$

Parameterized instanton form factor, Λ~ 600 MeV ~ 1/ρ

$$F(iD) = \frac{2\Lambda^2}{2\Lambda^2 - D^2}$$

M0 via the saddle-point equation

$$\frac{N}{V} = 4N_c \int \frac{d^4k}{(2\pi)^4} \frac{M^2(k)}{k^2 + M^2(k)}$$

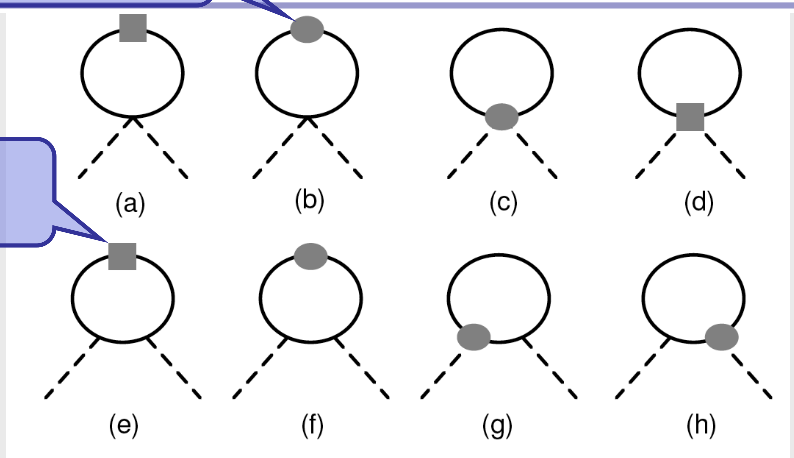
Pion electromagnetic form factor

Mesonic matrix element

$$\left(\frac{\delta^3 \mathcal{S}_{\rm eff}[\pi,V,m]}{\delta\pi^\alpha(y)\,\delta\pi^\beta(x)\,\delta V_\mu(0)}\right)_{V=0} = e_q N_c \mathrm{Tr}_{f,\gamma}\Bigg[\underbrace{\frac{1}{\mathcal{D}}X^{\alpha\beta}\frac{1}{\mathcal{D}}(\gamma_\mu+W_\mu-Z_\mu)}_{\rm a,b} - \underbrace{\frac{1}{\mathcal{D}}\left(W_\mu^{\alpha\beta}-Z_\mu^{\alpha\beta}\right)}_{\rm c}$$

$$\underbrace{\frac{1}{\mathcal{D}}X^{\alpha}\frac{1}{\mathcal{D}}X^{\beta}\frac{1}{\mathcal{D}}(\gamma_\mu+W_\mu-Z_\mu)\,\frac{1}{\mathcal{D}}X^{\beta}\frac{1}{\mathcal{D}}X^{\alpha}\frac{1}{\mathcal{D}}(\gamma_\mu+W_\mu-Z_\mu)}_{\rm e,f}$$

$$+\underbrace{\frac{1}{\mathcal{D}}X^{\alpha}\frac{1}{\mathcal{D}}\left(W_\mu^{\beta}-Z_\mu^{\beta}\right)}_{\rm g}+\underbrace{\frac{1}{\mathcal{D}}X^{\beta}\frac{1}{\mathcal{D}}\left(W_\mu^{\alpha}-Z_\mu^{\beta}\right)}_{\rm h}\Bigg],$$



$$X = i\sqrt{M(i\partial)}U^{\gamma_5}\sqrt{M(i\partial)}$$

$$\frac{\delta X}{\delta\pi^\alpha} = X^\alpha, \quad \frac{\delta^2 X}{\delta\pi^\alpha\delta\pi^\beta} = X^{\alpha\beta}$$

$$X = i\sqrt{M(i\partial)}U^{\gamma_5}\sqrt{M(i\partial)}$$

$$W_\mu = i\left(\frac{\partial}{\partial p_\mu}\sqrt{M(i\partial)}\right)U^{\gamma_5}\sqrt{M(i\partial)}$$

$$Z_\mu = i\sqrt{M(i\partial)}U^{\gamma_5}\left(\frac{\partial}{\partial p_\mu}\sqrt{M(i\partial)}\right)$$

Pion electromagnetic form factor

Analytic expression for the pion EM FF via NχQM

$$F_{\pi,K}(Q^2) = \frac{1}{F_{\pi,K}^2}\sum_{\text{flavor}}\frac{8e_q N_c}{(2p_i\cdot q + m_{\pi,K}^2)}\int_{-\infty}^{\infty}\frac{d^4k}{(2\pi)^4}\sum_{i=\text{a}}^{\text{h}}\mathcal{F}_i(k,q)$$

$$\mathcal{F}_{\text{a}}^{\text{L}} = \frac{\sqrt{M_b M_c}(\bar{M}_c k_{bd} + \bar{M}_b k_{cd})}{2(k_b^2+\bar{M}_b^2)(k_c^2+\bar{M}_c^2)},$$

$$\mathcal{F}_{\text{b}}^{\text{NL}} = \frac{\sqrt{M_b M_c}(\sqrt{M_c}\hat{M}_{bd} - \sqrt{M_b}\hat{M}_{cd})(k_{bc} - \bar{M}_b\bar{M}_c)}{(k_b^2+\bar{M}_b^2)(k_c^2+\bar{M}_c^2)}, \quad \mathcal{F}_{\text{c}}^{\text{NL}} = 0, \quad \mathcal{F}_{\text{d}}^{\text{NL}} = 0,$$

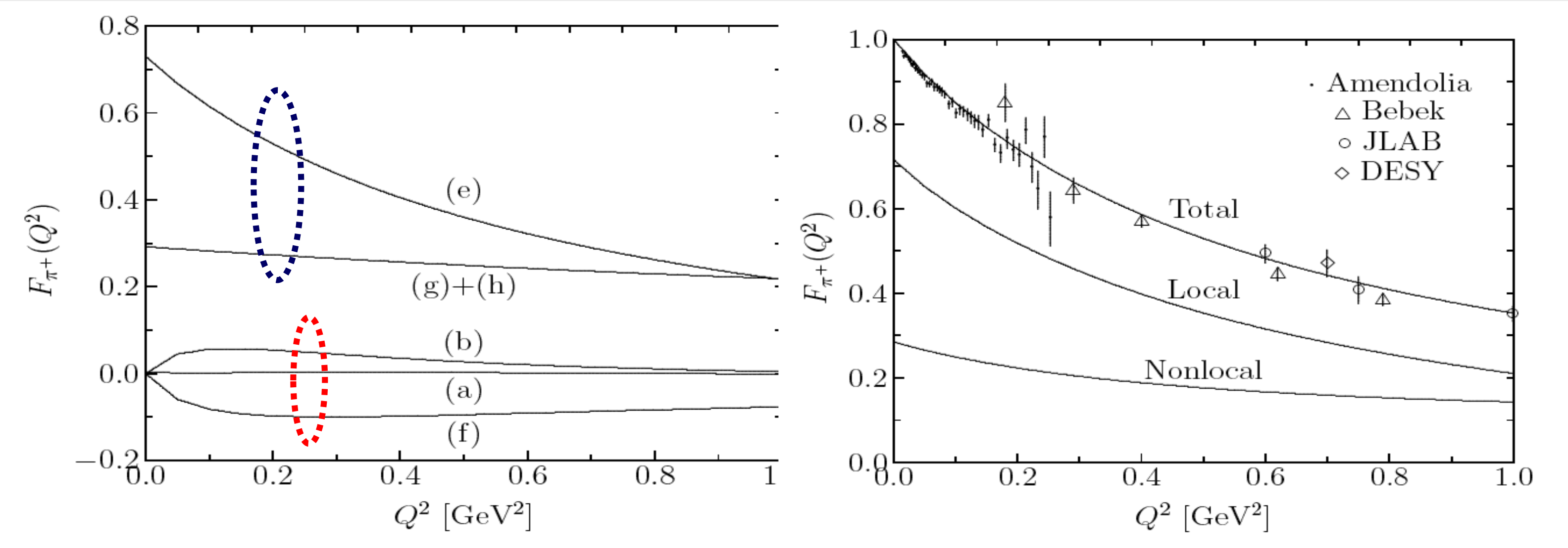
$$\mathcal{F}_{\text{e}}^{\text{L}} = \frac{M_a\sqrt{M_b M_c}(k_{ab}k_{cd} + k_{ac}k_{bd} - k_{bc}k_{ad} + \bar{M}_a\bar{M}_c k_{bd} + \bar{M}_a\bar{M}_b k_{cd} - \bar{M}_b\bar{M}_c k_{ad})}{(k_a^2+\bar{M}_a^2)(k_b^2+\bar{M}_b^2)(k_c^2+\bar{M}_c^2)},$$

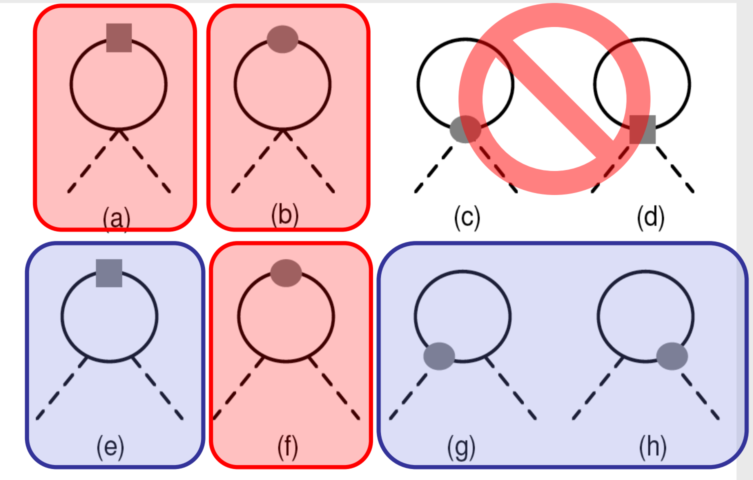
$$\mathcal{F}_{\text{f}}^{\text{NL}} = \frac{M_a\sqrt{M_b M_c}(\sqrt{M_b}\hat{M}_{cd} - \sqrt{M_c}\hat{M}_{bd})(\bar{M}_c k_{ab} + \bar{M}_b k_{ac} - \bar{M}_a k_{bc} + \bar{M}_a\bar{M}_b\bar{M}_c)}{(k_a^2+\bar{M}_a^2)(k_b^2+\bar{M}_b^2)(k_c^2+\bar{M}_c^2)},$$

$$\mathcal{F}_{\text{g}}^{\text{NL}} = \frac{\sqrt{M_a M_c}\left[\sqrt{M_b}\hat{M}_{ad} - \sqrt{M_a}\hat{M}_{bd}\right](k_{ac} + \bar{M}_a\bar{M}_c)}{2(k_a^2+\bar{M}_a^2)(k_c^2+\bar{M}_c^2)},$$

$$\mathcal{F}_{\text{h}}^{\text{NL}} = \frac{\sqrt{M_a M_b}\left[\sqrt{M_c}\hat{M}_{ad} - \sqrt{M_a}\hat{M}_{cd}\right](k_{ab} + \bar{M}_a\bar{M}_b)}{2(k_a^2+\bar{M}_a^2)(k_b^2+\bar{M}_b^2)},$$

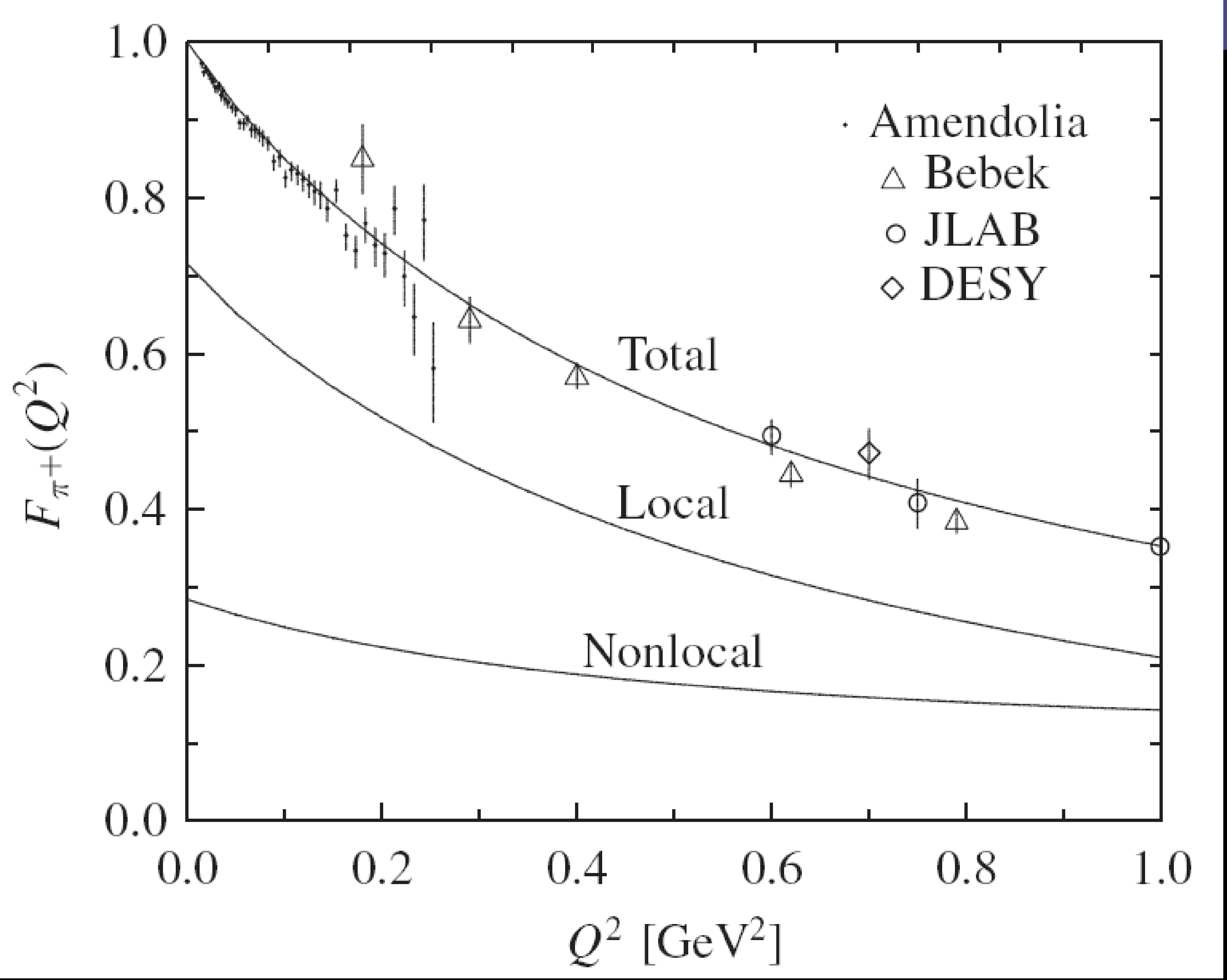
Pion electromagnetic form factor



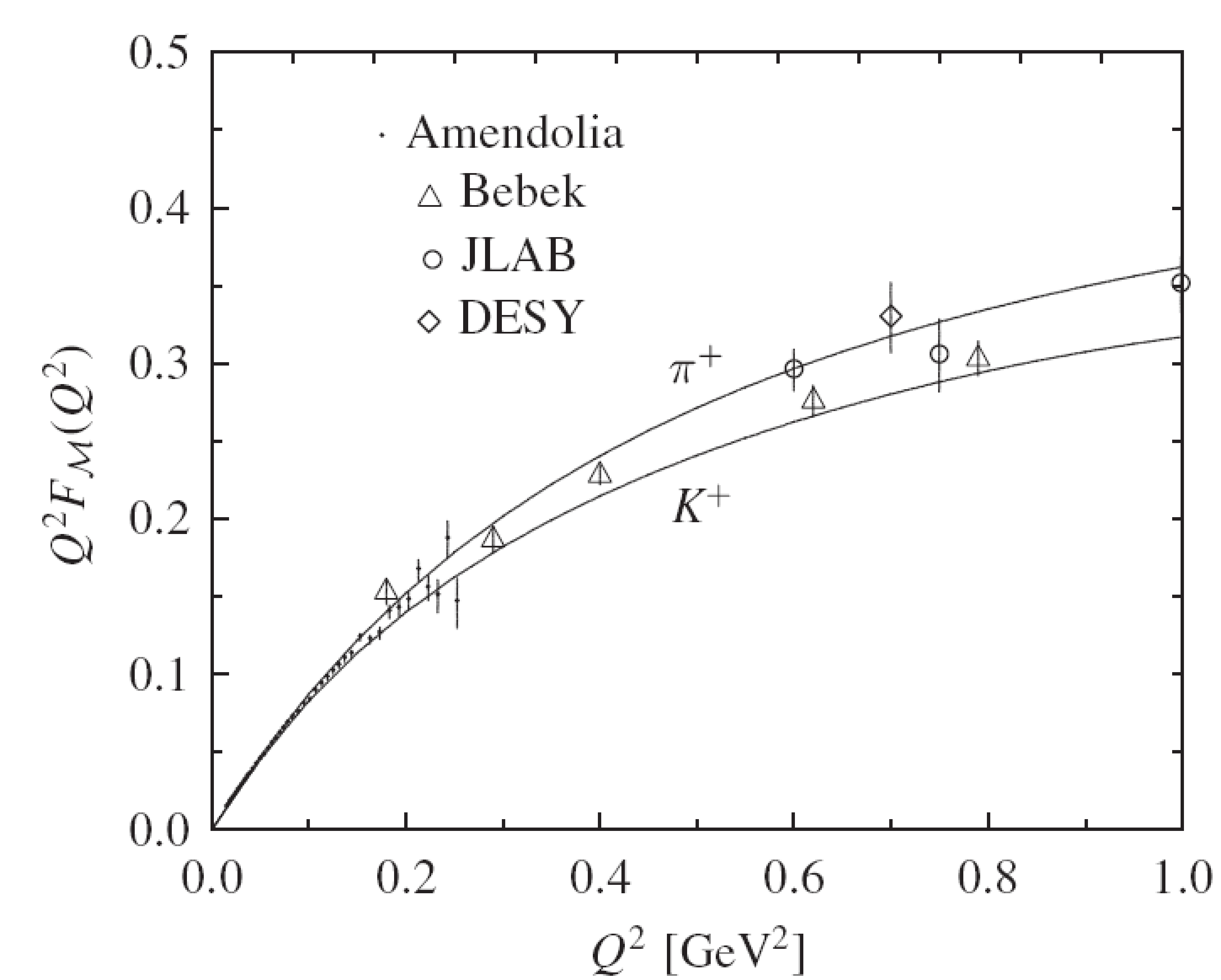


Charge Radius

Shape & strength



Pion electromagnetic form factor



Pion electromagnetic form factor

Pion electromagnetic form factor

Pion and kaon EM charge radii

$$\langle r^2\rangle_{\mathcal{M}} = -6\left[\frac{\partial F_{\mathcal{M}}(Q^2)}{\partial Q^2}\right]_{Q^2=0}$$

	Local	Nonlocal	Total	Exp. [63]
$\langle r^2\rangle^{1/2}_{\pi^+}$ [fm]	0.594	0.319	0.675	0.672 ± 0.008
$\langle r^2\rangle^{1/2}_{K^+}$ [fm]	0.658	0.318	0.731	0.560 ± 0.031
$\langle r^2\rangle_{K^0}$ [fm^2]	−0.044	−0.016	−0.060	-0.077 ± 0.010

Good agreement for the pion without explicit ρ-meson d.o.f.

A.E.Dorokhov,PRD70,094011 (2004)

Considerable disagreements for the kaon FFs?

Improper normalization in the model: smaller F_K ~ 108 MeV

S.i.N. and H.Ch.Kim, PRD75, 094011 (2007)

Meson-loop corrections necessary?

Instanton at finite quark chemical potential (μ)

Considerable successes in the application of the instanton

Extension of the instanton model to a system at finite ρ and T

QCD phase structure: nontrivial QCD vacuum: instanton

At finite density

· Simple extension to the quark matter: $p \to p + i\mu$

· Breakdown of the Lorentz invariance

At finite temperature (future work!!)

· Instanton at finite temperature: Caloron with Polyakov line

· Very phenomenological approach such as the pNJL

· Using periodic conditions (Matsubara sum)

Focus on the basic QCD properties at finite quark-chemical potential, μ

Instanton at finite μ

Modified Dirac equation with m

$$\left[i\partial\!\!\!/ - i\mu\!\!\!/ + A\!\!\!/_{I\bar{I}}\right]\Psi^{(n)}_{I\bar{I}} = \lambda_n \Psi^{(n)}_{I\bar{I}} \qquad \mu_\mu = (0,0,0,\mu_q)$$

Instanton solution (singular gauge)

$$A^\alpha_\mu(x) = \frac{2\bar{\eta}^{\alpha\nu}_\mu \bar{\rho}^2 x_\nu}{x^2(x^2+\bar{\rho}^2)}$$

Assumption:

No modification from μ

Low-energy dominated by zero-mode

$$\left[i\partial\!\!\!/ - i\mu\!\!\!/ + A\!\!\!/_{I\bar{I}}\right]\Psi^{(0)}_{I\bar{I}} = 0. \qquad S_0 = (i\partial\!\!\!/ - i\mu\!\!\!/)^{-1}$$

Quark propagator with m

$$S_{I\bar{I}} = \frac{1}{i\partial\!\!\!/ - i\mu\!\!\!/ + A\!\!\!/_{I\bar{I}}} \approx S_0 - \frac{\Psi^{(0)\dagger}_{I\bar{I}}\Psi^{(0)}_{I\bar{I}}}{im_q}$$

Instantons at finite μ

Quark propagator with the Fourier transformed zero-mode solution,

$$S = \frac{1}{i\not\partial - i\not\mu + iM(i\partial, \mu)}$$

$$M(p,\mu) = M_0(p+i\mu)^2\psi^2(p,\mu)$$

$$\psi_4(p,\mu) = \frac{\bar\rho^2}{8|\vec p|}\Big\{(|\vec p| - \mu_q - ip_4)\left[(2p_4 + i\mu_q)F^a_-(p,\mu) + i(|\vec p| - \mu_q - ip_4)F^b_-(p,\mu)\right]$$

$$+(|\vec p| + \mu_q + ip_4)\left[(2p_4 + i\mu_q)F^a_+(p,\mu) - i(|\vec p| + \mu_q + ip_4)F^b_+(p,\mu)\right]\Big\},$$

$$\vec\psi(p,\mu) = \frac{\bar\rho^2\hat p}{8|\vec p|}\Big\{(2|\vec p| - \mu_q)(|\vec p| - \mu_q - ip_4)F^a_-(p,\mu) + (2|\vec p| + \mu_q)(|\vec p| + \mu_q + ip_4)F^a_+(p,\mu)$$

$$+\left[2(|\vec p| - \mu_q)(|\vec p| - \mu_q - ip_4) - \frac{1}{|\vec p|}(\mu_q + ip_4)\left[p_4^2 + (|\vec p| - \mu_q)^2\right]\right]F^b_-(p,\mu)$$

$$+\left[2(|\vec p| + \mu_q)(|\vec p| + \mu + ip_4) + \frac{1}{|\vec p|}(\mu_q + ip_4)\left[p_4^2 + (|\vec p| + \mu_q)^2\right]\right]F^b_+(p,\mu)\Big\},$$

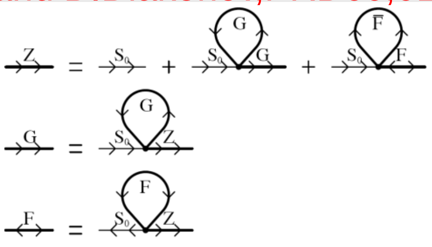
$$F^a_\pm = \frac{I_1(z_\pm)K_0(z_\pm) - I_0(z_\pm)K_1(z_\pm)}{z_\pm}, \quad F^b_\pm = \frac{I_1(z_\pm)K_1(z_\pm)}{z_\pm^2}$$

$$z_\pm = \frac{\bar\rho}{2}\sqrt{p_4^2 + (|\boldsymbol{p}| \pm \mu)^2}.$$

Schwinger-Dyson-Gorkov equation (N_f=2, N_c=3)

G.W.Carter and D.Diakonov,PRD60,016004 (1999)

$$
\begin{aligned}
Z(p) &= 1 - G(p)A(p,\mu)M_0, \\
G(p) &= Z(p)\psi^2(p)M_0, \\
F(p) &= 2Z(-p)\psi_\mu(p,\mu)\psi^\mu(-p,\mu)\Delta
\end{aligned}
$$



$$
\begin{aligned}
A(p,\mu) &= (p+i\mu)^2\psi^2(p,\mu), \\
B(p,\mu) &= (p^2+\mu^2)\psi_\mu(p,\mu)\psi^\mu(-p,\mu) + (p+i\mu)_\mu\psi^\mu(p,\mu)(p-i\mu)_\nu\psi^\nu(-p,\mu) \\
&- (p+i\mu)_\mu\psi^\mu(-p,\mu)(p-i\mu)_\nu\psi^\nu(p,\mu).
\end{aligned}
$$

$$
g(\mu) = \frac{\lambda M_0}{N_c^2-1}\int \frac{d^4p}{(2\pi)^4}\frac{\alpha(p,\mu)}{1+\alpha(p,\mu)M_0^2}, \quad f(\mu) = \frac{2\lambda\Delta}{N_c^2-1}\int \frac{d^4p}{(2\pi)^4}\frac{\beta(p,\mu)}{1+4\beta(p,\mu)\Delta^2},
$$

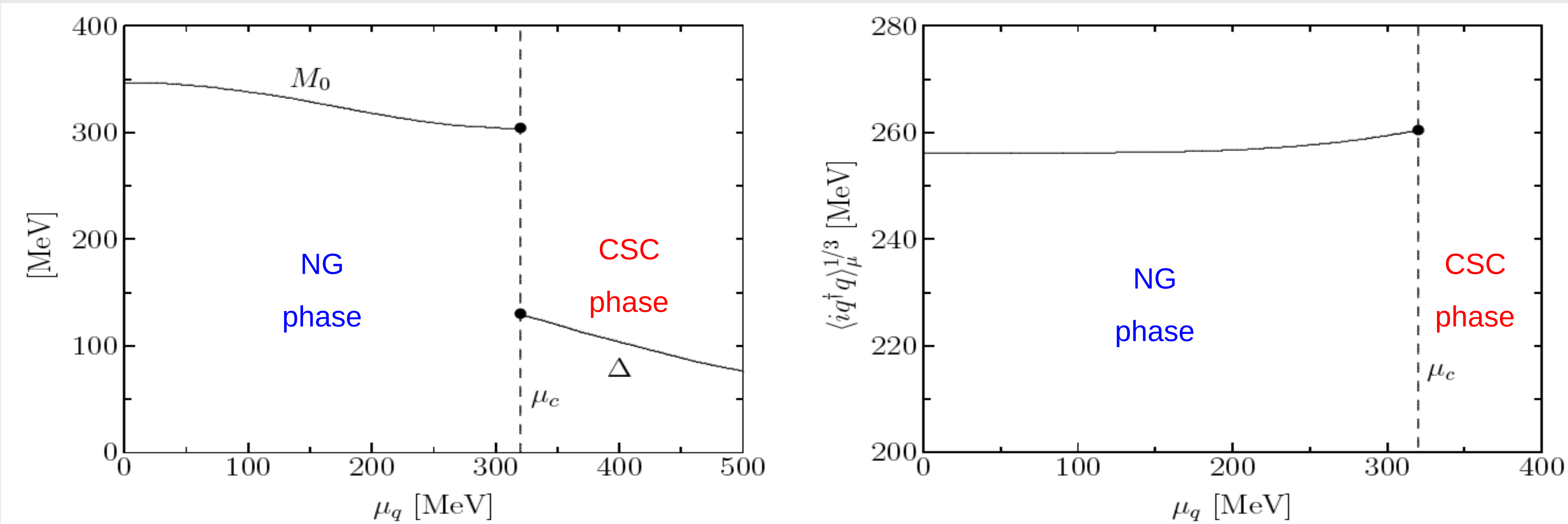
$$
\alpha(p,\mu) = A(p,\mu)\psi^2(p,\mu), \quad \beta(p,\mu) = B(p,\mu)\psi_\mu(p,\mu)\psi^\mu(-p,\mu)
$$

$$
M_0 = \left(2N_c - \frac{2}{N_c}\right) g(\mu), \quad \Delta = \left(1 + \frac{1}{N_c}\right) f(\mu)
$$

$$
\left.\frac{f(\mu)}{g(\mu)}\right|_{\mu=\mu_c} = \left[\frac{N_c(N_c-1)}{2}\right]^{\frac{1}{2}}
$$

M_0, Δ, and $\langle iq^\dagger q\rangle$ (N_f=2)

1st order phase transition occurred at μ ~ 320 MeV



Metastable state (mixing of σ and Δ) ignored for simplicity here

Chiral condensate with μ

$$\langle iq^\dagger q\rangle_\mu = 4N_c \int \frac{d^4k}{(2\pi)^4}\left[\frac{M(k,\mu)}{(k+i\mu)^2+M^2(k,\mu)}\right]$$

Magnetic susceptibility at finite μ

QCD magnetic susceptibility with externally induced EM field

$$\langle iq^\dagger \sigma_{\mu\nu} q \rangle_F = i e_q F_{\mu\nu} \langle iq^\dagger q \rangle_0 \chi$$

Only for the NG phase!!

Magnetic phase transition of the QCD vacuum

Effective chiral action with tensor source field *T*

$$\mathcal{S}_{\text{eff}}[T,\mu] = -\text{Sp}_{cf\gamma} \ln \left[i\not{D} - i\not{\mu} + iM(iD,\mu) + \sigma \cdot T \right]$$

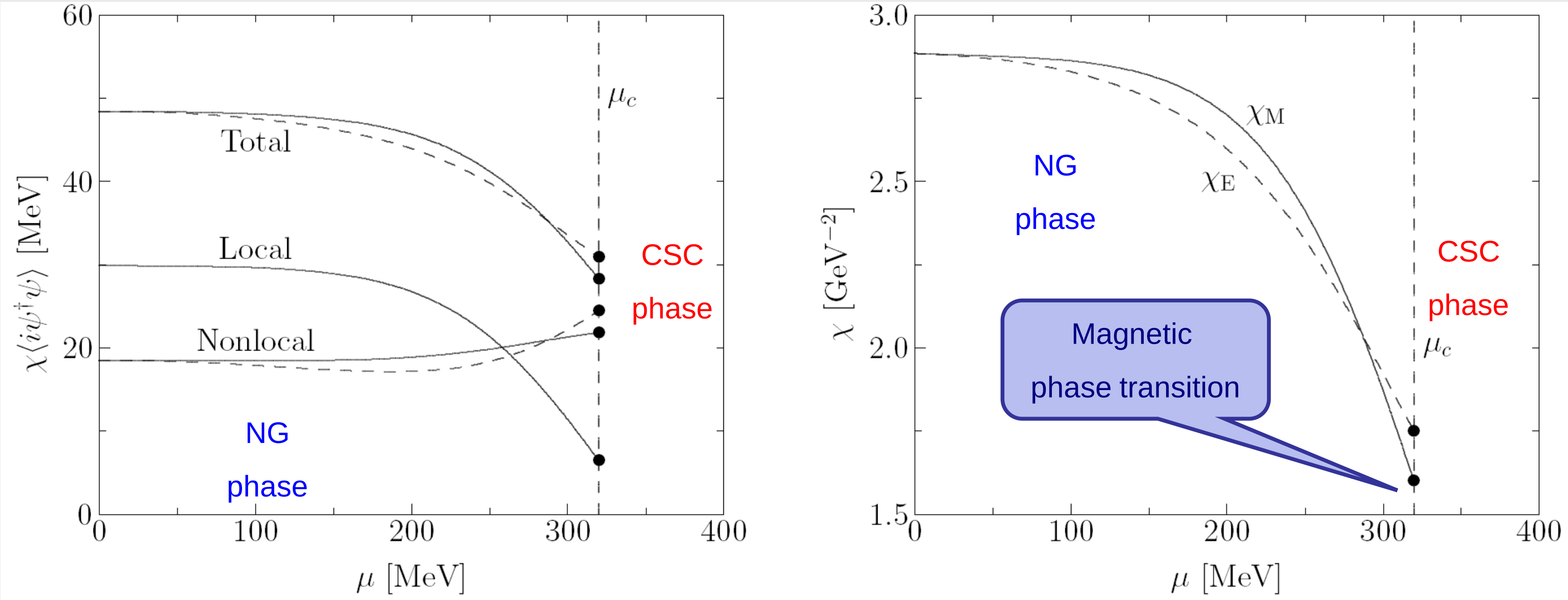
Evaluation of the matrix element

$$\left[\frac{\delta}{\delta T_{\mu\nu}} \mathcal{S}_{\text{eff}}[T,\mu] \right]_{T=0} = \langle 0 | \psi^\dagger \sigma_{\mu\nu} \psi | 0 \rangle_F = \text{Tr}_{cf\gamma} [S \sigma_{\mu\nu}]$$

Magnetic susceptibility at finite μ

$$\chi_{\rm M}\langle iq^\dagger q\rangle = 4N_c\int\frac{d^4p}{(2\pi)^4}\left[\frac{M(\bar p)-\bar p^2\tilde{M}'(\bar p)}{[\bar p^2+M^2(\bar p)]^2}\right]$$

$$\chi_{\rm E}\langle i\psi^\dagger\psi\rangle = \chi_{\rm M}\langle i\psi^\dagger\psi\rangle + \underbrace{4N_c\int\frac{d^4p}{(2\pi)^4}\left[\frac{\mu^2\tilde{M}'(\bar p)}{[\bar p^2+M^2(\bar p)]^2}\right]}_{\text{Breakdown of Lorentz invariance}}$$



1st order magnetic phase transition at μ_c

S.i.N. and H.Ch.Kim, arXiv:0805.0060 [hep-ph], accepted for publicaion in PLB

Pion weak decay constant at finite μ

Pion-to-vacuum transition matrix element

$$\langle 0|A_\mu^a(x)|\pi^b(P)\rangle = i\sqrt{2}F_\pi\delta^{ab}P_\mu e^{-iP\cdot x}$$

Separation for the time and space components

$$\langle 0|\boldsymbol{A}^a(x)|\pi^b(P)\rangle = i\sqrt{2}F_\pi^s\delta^{ab}\boldsymbol{P}e^{-iP\cdot x}, \quad \langle 0|A_4^a(x)|\pi^b(P)\rangle = i\sqrt{2}F_\pi^t\delta^{ab}P_4 e^{-iP\cdot x}$$

Effective chiral action with μ

$$\mathcal{S}_{\mathrm{eff}}[\pi,\mu] = -\mathrm{Sp}\ln\left[i\bar{\partial\!\!\!/} + i\sqrt{M(i\bar{\partial})}U_5\sqrt{M(i\bar{\partial})}\right]$$

Renormalized auxiliary pion field corresponding to the physical one

$$\pi_{\mathrm{phy}}^a = \frac{1}{C_r}\pi^a$$

Pion weak decay constant at finite μ

Effective chiral action with axial-vector source

$$\mathcal{S}_{\rm eff}[\pi,\mu,J^a_{5\mu}] \;=\; -{\rm Sp}\ln\left[i\bar{\partial\!\!\!/} + \gamma_5\gamma^\mu\frac{\tau^a}{2}J^a_{5\mu} + \sqrt{M(i\bar{\partial},J^a_{5\mu})}U_5\sqrt{M(i\bar{\partial},J^a_{5\mu})}\right]$$

Using the LSZ (Lehmann-Symanzik-Zimmermann) reduction formula

$$\begin{aligned} i\sqrt{2}\delta^{ab}F_\pi(q^2,\mu)q_\mu &= \mathcal{K}_\pi\int d^4x\langle 0|T\left[A^a_\mu(x)\pi^b_{\rm phy}(0)\right]|0\rangle\, e^{iq\cdot x} \\ &= \frac{\mathcal{K}_\pi}{C_r(\mu)}\int d^4x\langle 0|T\left[A^a_\mu(x)\pi^b(0)\right]|0\rangle\, e^{iq\cdot x} \end{aligned}$$

$$\langle 0|T[A^a_\mu(x)\pi^b(0)]|0\rangle = \frac{\delta^2\ln\mathcal{Z}_{\rm eff}[\pi,\mu,J^a_{5\mu}]}{\delta J^a_{5\mu}(x)\,\delta J^b_5(0)} = \int d^4z\,\frac{\delta^2\mathcal{S}_{\rm eff}[\pi,\mu,J^a_{5\mu}]}{\delta J^a_{5\mu}(x)\,\delta\pi^b(z)}\mathcal{K}^{-1}_\pi(z)$$

Pion weak decay constant at finite μ

Analytic expression for the pion weak decay constant

$$F_\pi(\mu)P_\mu = \frac{4N_c}{C_r F_0}\int\frac{d^4k}{(2\pi)^4}\Bigg[\underbrace{\frac{\sqrt{M(\bar{k})M(\bar{k}-P)}[(P_\mu-\bar{k}_\mu)M(\bar{k})+\bar{k}_\mu M(\bar{k}-P)]}{[\bar{k}^2+M^2(\bar{k})][(\bar{k}-P)^2+M^2(\bar{k}-P)]}}_{\text{local cont.}}$$

$$-\underbrace{\frac{M(\bar{k})\sqrt{M(\bar{k})}\sqrt{M(\bar{k}-P)}_\mu - M(\bar{k})\sqrt{M(\bar{k})}_\mu\sqrt{M(\bar{k}-P)}}{\bar{k}^2+M^2(\bar{k})}}_{\text{nonlocal cont.}}\Bigg],$$

$$\bar{k} = (\vec{k}, k_4 + i\mu)$$

Pion weak decay constant at finite μ

Local contributions

$$F^s_{\pi,\mathrm{L}}(\mu) = \frac{4N_c}{C_r F_0}\int\frac{d^4k}{(2\pi)^4}\frac{1}{(k^2+\mathcal{M}^2)^2}\left[\mathcal{M}^2-\frac{1}{2}k^2\mathcal{M}\tilde{\mathcal{M}}'-5\mu^2\,k_4^2\tilde{\mathcal{M}}'^2\right],$$

$$F^t_{\pi,\mathrm{L}}(\mu) = \frac{4N_c}{C_r F_0}\int\frac{d^4k}{(2\pi)^4}\frac{1}{(k^2+\mathcal{M}^2)^2}\left[\mathcal{M}^2-\frac{1}{2}k^2\mathcal{M}\tilde{\mathcal{M}}'-\mu^2\,k_4^2\tilde{\mathcal{M}}'^2\right],$$

Nonlocal contributions

$$F^s_{\pi,\mathrm{NL}}(\mu) = -\frac{4N_c}{C_r F_0}\int\frac{d^4k}{(2\pi)^4}\frac{1}{k^2+\mathcal{M}^2}\left[\mathcal{M}\tilde{\mathcal{M}}'+\frac{1}{2}k^2\mathcal{M}\tilde{\mathcal{M}}''-\frac{1}{2}k^2\tilde{\mathcal{M}}'^2-4\mu^2k_4^2\tilde{\mathcal{M}}'\tilde{\mathcal{M}}''\right]$$

$$F^t_{\pi,\mathrm{NL}}(\mu) = F^s_{\pi,\mathrm{NL}}(\mu).$$

When the density switched off,

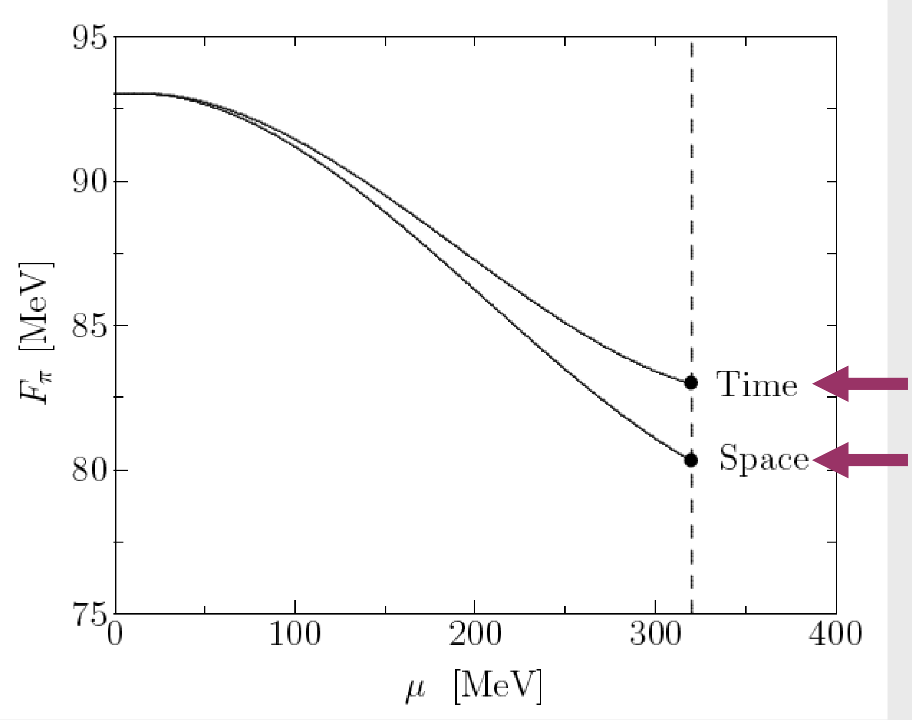
$$F_\pi(0) = \frac{4N_c}{C_r F_0}\int\frac{d^4k}{(2\pi)^4}\left[\frac{\mathcal{M}^2-\frac{1}{2}k^2\mathcal{M}\tilde{\mathcal{M}}'}{(k^2+\mathcal{M}^2)^2}-\frac{\mathcal{M}\tilde{\mathcal{M}}'+\frac{1}{2}k^2\mathcal{M}\tilde{\mathcal{M}}''-\frac{1}{2}k^2\tilde{\mathcal{M}}'^2}{k^2+\mathcal{M}^2}\right]$$

Pion weak decay constant at finite μ

Time component > Space component

$$F_\pi^s(\mu) \approx F_\pi^{\exp} + \mu^2 \left[\frac{N_c}{F_\pi^{\exp}} \int \frac{d^4k}{(2\pi)^4} \left(\frac{8k_4^2 \tilde{\mathcal{M}}' \tilde{\mathcal{M}}''}{k^2 + \mathcal{M}^2} - \frac{10 k_4^2 \tilde{\mathcal{M}}'^2}{[k^2 + \mathcal{M}^2]^2} \right) \right]$$

$$F_\pi^t(\mu) \approx F_\pi^s(\mu) + \mu^2 \left[\frac{N_c}{F_\pi^{\exp}} \int \frac{d^4k}{(2\pi)^4} \frac{8k_4^2 \tilde{\mathcal{M}}'^2}{[k^2 + \mathcal{M}^2]^2} \right],$$



Pion weak decay constant at finite μ

Comparison with the in-medium ChPT

K.Kirchbach and A.Wirzba, NPA616, 648 (1997)

$$F_\pi^s(\rho_0) = \left[1 + \frac{2c_3\rho_0}{(F_\pi^{\exp})^2}\right]\left[1 - \frac{\Sigma_{\pi N}\,\rho_0}{(F_\pi^{\exp})^2 m_\pi^2}\right]^{-1},$$

$$F_\pi^t(\rho_0) = \left[1 + \frac{2(c_2+c_3)\rho_0}{(F_\pi^{\exp})^2}\right]\left[1 - \frac{\Sigma_{\pi N}\,\rho_0}{(F_\pi^{\exp})^2 m_\pi^2}\right]^{-1}$$

$$c_3 < 0 \text{ and } c_2 > 0$$

Comparison with the QCD sumrule

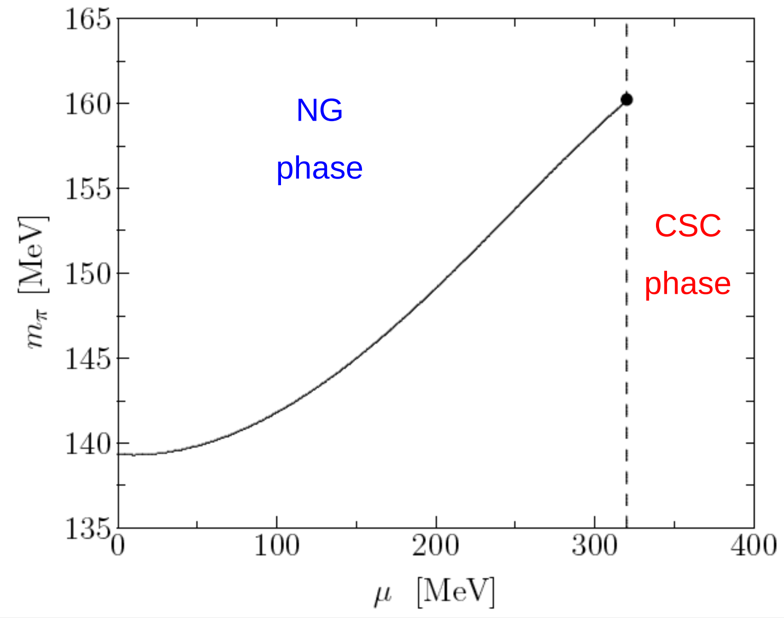
H.c.Kim and M.Oka, NPA720, 386 (2003)

m_π^* (MeV)	D_5 (GeV^2)	f_t/f_π	f_s/f_π
139	-0.019	0.79 (0.77)	0.78 (0.57)
159	-0.025	0.69 (0.63)	0.68 (0.37)

$F^s / F^t < 0.5$

Critical *p*-wave contribution

Pion weak decay constant at finite μ



GOR relation satisfied
within the present framework

$$(m_\pi^* F_\pi^t)^2 = 2m_q \langle i q^\dagger q \rangle^*$$

Changes of the pion properties with μ

	F_π^s	F_π^t	m_π
$\mu = 0$	93 MeV	93 MeV	139.33 MeV
$\mu = \mu_c \approx 320$ MeV	80.29 MeV	82.96 MeV	160.14 MeV
Modification	16% ↓	13% ↓	15% ↑

Summary and Conclusion

Instanton with finite μ applied to following problems

· Pion EM FF for free space as an example

· QCD magnetic susceptibility: 1st-order magnetic phase transition

· Pion weak decay constant at finite density: *p*-wave contribution?

Perspectives

Systematic studies for nonperturbative hadron properties with μ

Several works (effective chiral Lagrangian, LEC..) under progress

Extension to the finite *T* (Dyon with nontrivial holonomy, Caloron)

Thank you very much for your attention!!