Exotic mesons in holographic QCD

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Hyun-Chul Kim and YK,
JHEP 0810:011,2008 (mixed condensate);
"**Hybrid** exotic **meson with J^{PC}=1^{-+} in AdS/QCD,**"
arXiv:0811.0645 [hep-ph]

Contents

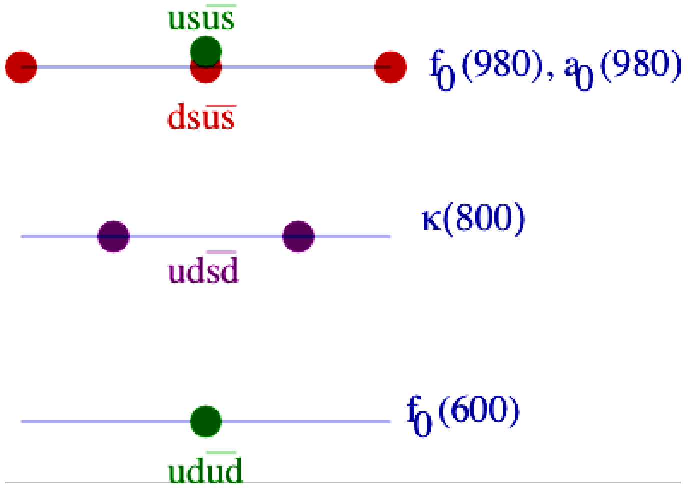
- Exotic mesons?
- Exotic mesons in hQCD
- Summary

Exotic mesons?

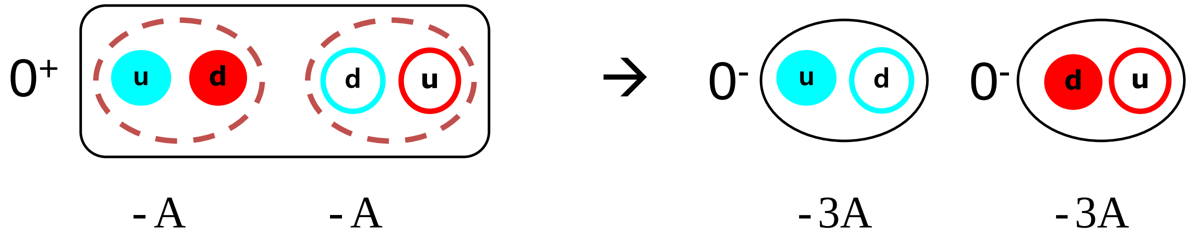
Mesons with exotic quantum numbers are non-$\bar{q}q$ objects. They may be hybrid mesons $(q\bar{q}g)$, multiquark states $(q\bar{q}q\bar{q}\ldots)$, multimeson states $(M_1M_2\ldots)$ or, possibly, glueballs. Exotic quantum numbers are, e.g.

$$J^{PC}_{\text{dexotics}} = 0^{--}, 0^{+-}, 1^{-+}, 2^{+-}, 3^{-+}\cdots$$

Scalar tetra-quark (Jaffe 76)



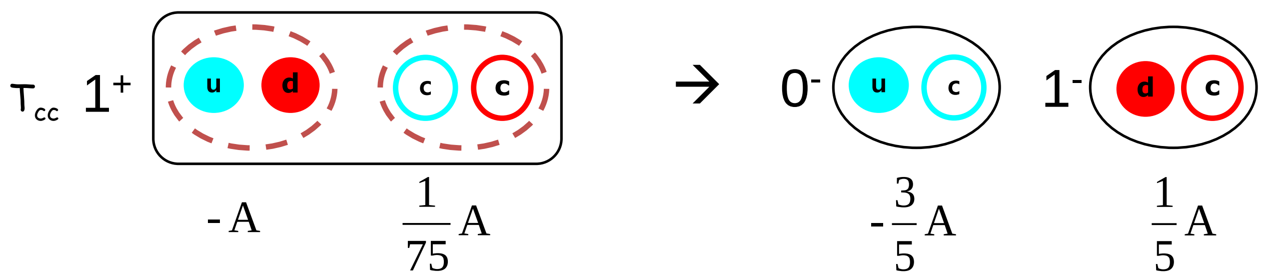
States with diquark - anti-diquark (Tetra-quark)



Binding $= -4A$

$\rightarrow$ not bound

Binding = (Mass of 2 Mesons) – (Mass of Tetraquark)



binding $= \frac{44}{75}A$

T_{cc} found to be stable in QCD sum rules and quark model

S.H. Lee

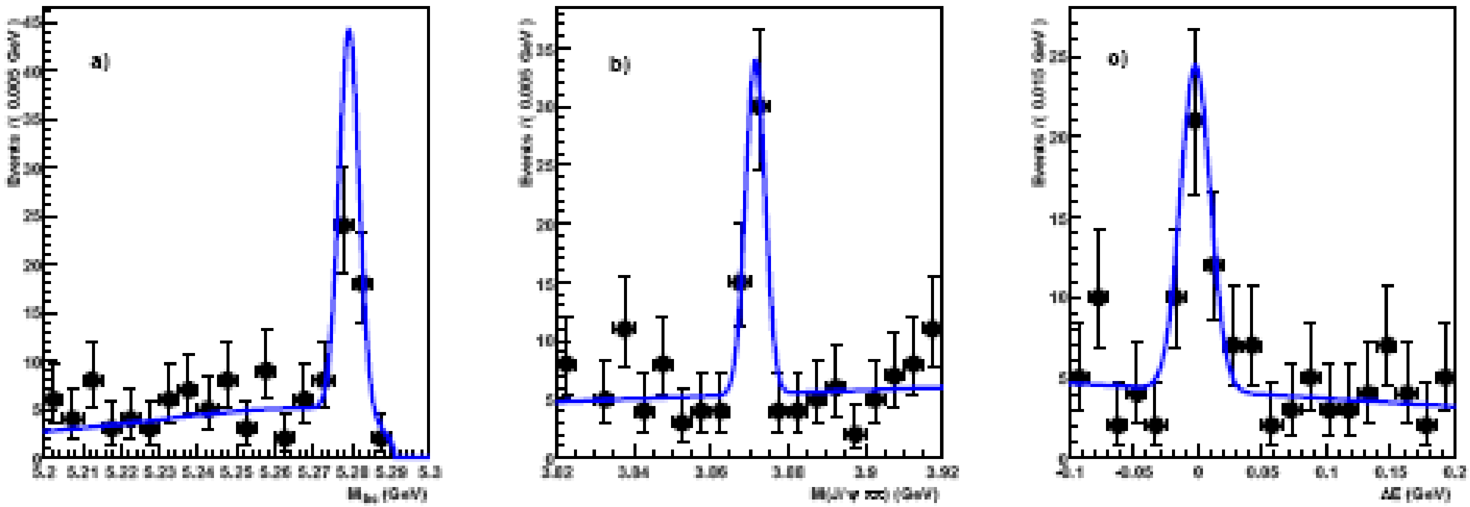


FIG. 17: X(3872) in the $J/\psi\pi^+\pi^-$ channel from Ref. [150].

X(3872), Y(4260),

Z(4430)→ $\psi'\pi$

Z(4051),Z(4248)→ $\chi_{c1}\pi$ (arXiv:0806.4098)

Must contain $c\bar{c}$?

Z(4248) ?

Tetraquark ?

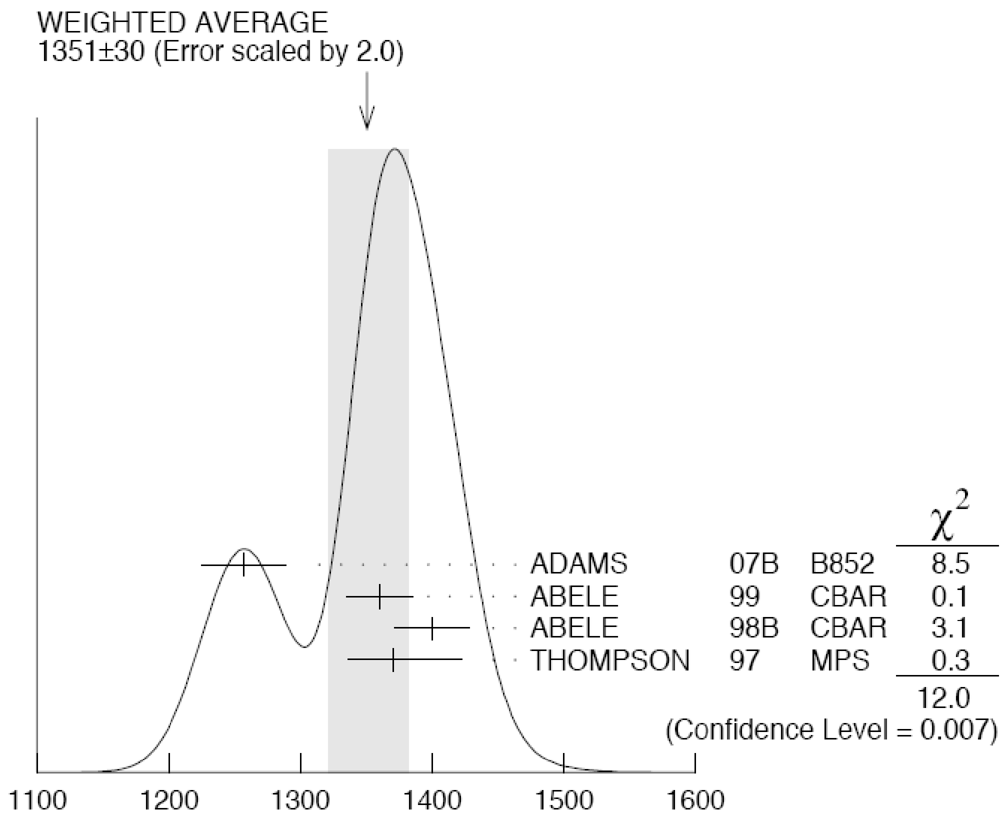
S.H. Lee

We observe two more charmonium-like states, Z_1^+ and Z_2^+, with non-zero charge where both decay into $\chi_{c1}\pi^+$. Their quark content is $c\bar{c}u\bar{d}$. The $X(3872)$ decaying into $\pi^+\pi^- J/\psi$ has been well established, while its underlying nature has not been conclusively identified. The favored interpretations are either a mesonic molecule or a diquark-diantiquark tetraquark meson. Some additional properties of the $X(3872)$ are measured. The mass of the $X(3872)$

"Observation of candidate exotic mesons **containing heavy quarks with Belle,"** S.-K. Choi, for the Belle Collaboration ,e-Print: **arXiv:0810.3546** [hep-ex]

Quark-gluon hybrid states

$q\bar{q}G$



$\pi_1(1400)$

$I^G(J^{PC}) = 1^-(1^{-+})$

Prediction of various models for the mass of the lightest hybrid meson

Model	Mass, Gev/c^2
Bag model	1.3– 1.4
Flux-tube model	1.8–2.0
Sum rules	1.3–1.9
Lattice QCD	1.8–2.3
Effective Hamiltonian	2.0–2.2

E. Klempt, A. Zaitsev / Physics Reports 454 (2007) 1–202

Exotic mesons in hQCD

A holographic model for non-exotic mesons

AdS/CFT Dictionary

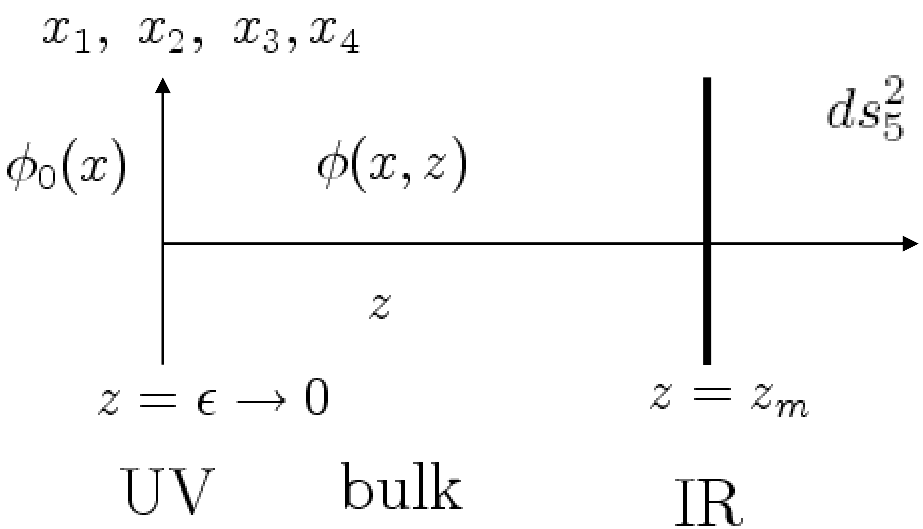
- 4D CFT (QCD) ←→ 5D AdS
- 4D generating functional ←→ 5D (classical) effective action
- Operator ←→ 5D bulk field
- [Operator] ←→ 5D mass
- Current conservation ←→ gauge symmetry
- Large Q ←→ small z
- Confinement ←→ Compactified z
- Resonances ←→ Kaluza-Klein states

Hard wall model

$$
\begin{aligned}
S_5 &= \int d^4x \int dz \mathcal{L}_5 \\
&= \int d^4x \int dz \sqrt{g}\ \mathrm{Tr}[-\frac{1}{4g_5^2}(F_L^2 + F_R^2) + |DX|^2 + 3|X^2|\ ]
\end{aligned}
$$

J. Erlich, E. Katz, D. T. Son and M. A. Stephanov, Phys. Rev. Lett. **95**, 261602 (2005)

L. Da Rold and A. Pomarol, Nucl.Phys. **B721**, 79 (2005)



$$ds_5^2 = \frac{1}{z^2}\left(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2\right)$$

★ 5D field contents

Operator → 5D bulk field

$$\bar{q}_R q_L \longrightarrow \Phi(x,z)$$
$$\bar{q}_L \gamma^\mu q_L \longrightarrow L_M(x,z)$$
$$\bar{q}_R \gamma^\mu q_R \longrightarrow R_M(x,z)$$

[Operator] → 5D mass

$$(\Delta - p)(\Delta + p - 4) = m_5^2 \qquad m_\phi^2 = -3$$

★ 5D Symmetry

Current conservation → gauge symmetry

$SU(2)_L X SU(2)_R$ gauge symmetry in AdS_5

★

Background: AdS_5

$$ds_5^2 = \frac{1}{z^2}\left(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2\right)$$

Chiral condensate?

Klebanov and Witten, 1999

$$\phi(x,z) \to z^{d-\Delta}\phi_0(x) + z^{\Delta}A(x) + \dots\,, z \to \epsilon\,,$$

where $\phi_0(x)$ is the source term of 4D operator $\mathcal{O}(x)$, and

$$A(x) = \frac{1}{2\Delta - d}\langle \mathcal{O}(x)\rangle\,.$$

For example, $\mathcal{O} = \bar{q}q$, $\phi(x,z) = v(z)$:

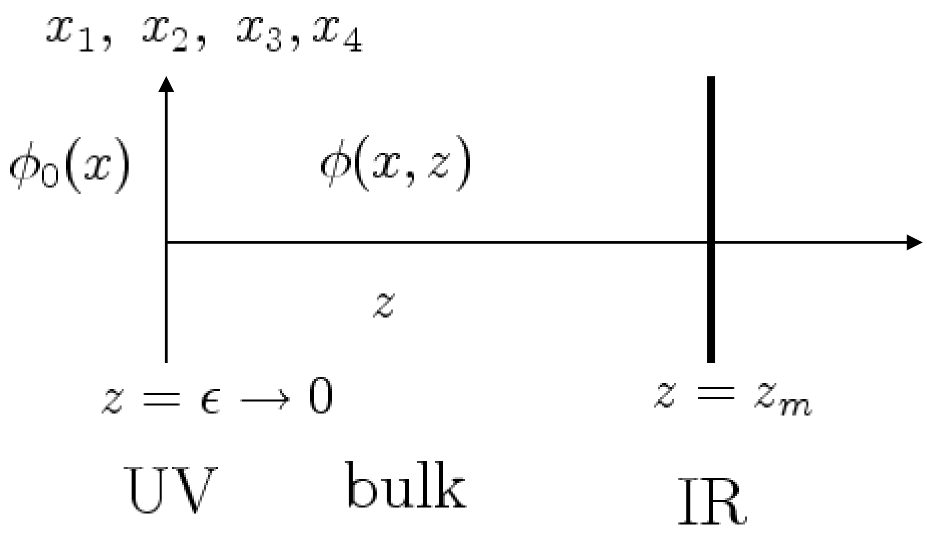
$$v(z) = c_1 z + c_2 z^3$$

$$c_1 \sim m_q, \quad c_2 \sim \langle \bar{q}q \rangle\,.$$

★ Confinement

Polchinski & Strassler, 2000

Confinement → IR cutoff in 5th direction



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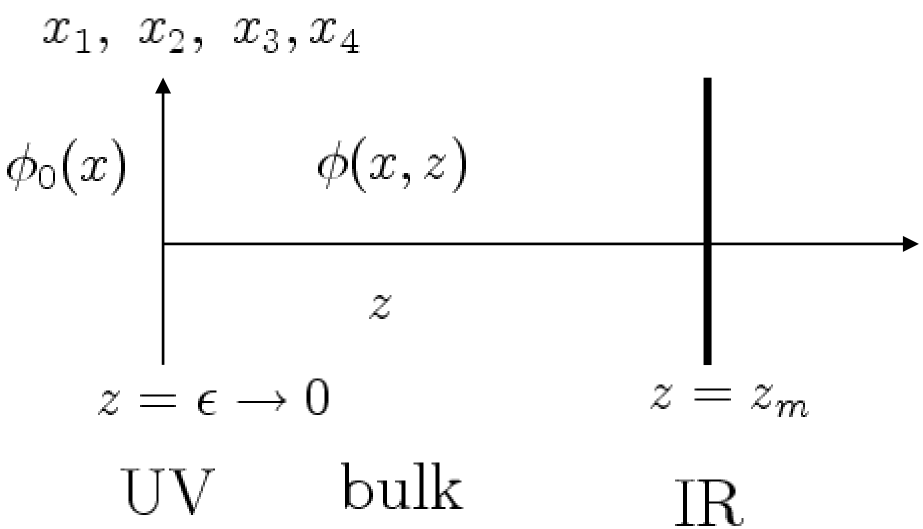

V~ L+R
A~ L-R

5d coupling and N_c

In QCD

$$\Pi(Q) = -\frac{N_c}{24\pi^2} \ln Q^2 + \dots$$

In AdS$_5$:

$$\Pi(Q^2) = -\frac{1}{2g_5^2} \ln Q^2 + \dots$$

Thus

$$g_5^2 = \frac{12\pi^2}{N_c}$$

Large N_c $\Leftrightarrow$ small coupling

M. Stephanov

A holographic model for quark-gluon hybrid mesons

$$\begin{aligned}
\text{non}-\text{exotic}\ (\bar{q}q) \quad &: \quad \text{quark}-\text{gluon mixed}\ (\bar{q}Gq) \\
\bar{q}t^a\gamma_\mu q \sim V_\mu^a(x,z)\ (\rho,\ldots) \quad &\Longleftrightarrow \quad \bar{q}t^a G_{\mu\nu}\gamma^\nu q \sim \tilde{V}_\mu^a(x,z)\ (\pi_1(1400),\ldots) \\
\langle\bar{q}q\rangle \sim \langle X(x,z)\rangle,\ \text{chiral condensate} \quad &\Longleftrightarrow \quad \langle\bar{q}G_{\mu\nu}\sigma^{\mu\nu}q\rangle \sim \langle\Phi(x,z)\rangle,\ \text{mixed condensate} \\
g_5^2 \sim 1/N_c \quad &\Longleftrightarrow \quad \tilde{g}_5^2 \sim 1 \\
z_m,\ \text{IR cutoff} \quad &\Longleftrightarrow \quad z_m,\ \text{IR cutoff}
\end{aligned}$$

A holographic model for mixed condensate

Hyun-Chul Kim and YK, JHEP 0810:011,2008

$$\langle\bar{\psi}\sigma^{\mu\nu}G_{\mu\nu}\psi\rangle = m_0^2\langle\bar{\psi}\psi\rangle$$

4D operators: $\mathcal{O}(x)$	5D fields: $\phi(x,z)$	p	Δ	m_5^2
$\bar{q}_L\gamma^\mu t^a q_L$	$A^a_{L\mu}$	1	3	0
$\bar{q}_R\gamma^\mu t^a q_R$	$A^a_{R\mu}$	1	3	0
$\bar{q}^\alpha_R q^\beta_L$	$(2/z)X^{\alpha\beta}$	0	3	-3
$\bar{q}^\alpha_R\sigma_{\mu\nu}G^{\mu\nu}q^\beta_L$	$(1/z^3)\Phi^{\alpha\beta}$	0	5	5

$$S \;=\; \int d^5x\,\sqrt{g}\,\mathrm{Tr}\left[|DX|^2+3|X|^2-\frac{1}{4g_5^2}(F_L^2+F_R^2)+|D\Phi|^2-5\Phi^2\right]$$

$$\begin{aligned} X_0(x,z) \;=\; \langle X(x,z)\rangle \;&=\; \frac{1}{2}(\hat{m}z+\sigma z^3),\\ \Phi_0(x,z) \;=\; \langle \Phi(x,z)\rangle \;&=\; \frac{1}{6}(c_1 z^{-1}+\sigma_M z^5) \end{aligned}$$

We are now in a position to derive the classical equations of motion for the axial-vector and pion. Introducing $v = m_q z + \sigma z^3$, $w = (\sigma_M/3)z^5$, and $(A_\mu)_{||} = \partial_\mu \phi$, we obtain

$$\begin{aligned} \left[\partial_z\left(\frac{1}{z}\partial_z A_\mu\right)+\frac{q^2}{z}A_\mu - g_5^2\frac{1}{z^3}(v^2+w^2)A_\mu\right]_\perp &= 0\,,\\ \partial_z\left(\frac{1}{z}\partial_z\phi^a\right)+g_5^2\frac{1}{z^3}v^2(\pi^a-\phi^a) &= 0\,,\\ -q^2\partial_z\phi^a+g_5^2\frac{1}{z^2}(v^2+w^2)\partial_z\pi^a &= 0\,. \end{aligned}$$

	Model I	Model II	Model III	Model IV	Experiment
m_{q}	1.6	3.7	2.3	2.3	$\cdots$
σ	$(0.1\,\mathrm{GeV})^3$	$(0.25\,\mathrm{GeV})^3$	$(0.307\,\mathrm{GeV})^3$	$(0.308\,\mathrm{GeV})^3$	$\cdots$
m_0^2	$13.32\,\mathrm{GeV}^2$	$0.72\,\mathrm{GeV}^2$	$0.006\,\mathrm{GeV}^2$	0	$\cdots$
m_ρ	775.8	775.8	775.8	832	775.49 ± 0.34
m_{a_1}	1230	1244	1246	1220	1230 ± 40
f_π	75.9	80.5	85.5	84.0	92.4 ± 0.35
$F_\rho^{1/2}$	330	330	330	353	345 ± 8
$F_{a_1}^{1/2}$	460	459	446	440	433 ± 13
m_π	138	139.3	137.5	141	139.57 ± 0.00035
$g_{\rho\pi\pi}$	8.27	4.87	4.87	5.29	6.03 ± 0.07
g_{A4}	1.71	1.69	1.71	1.88	$\cdots$

A holographic model for pi_1 (1400)

Hyun-Chul Kim and YK, **arXiv:0811.0645** [hep-ph]

$$J^a_\mu(x) = \bar\psi(x)T^a G_{\mu\alpha}(x)\gamma^\alpha\psi(x).$$

$$S = \int d^4x dz \sqrt{g}\,\mathrm{Tr}\left[-\frac{1}{4\tilde g_5^2}F_{MN}F^{MN} + \frac{1}{2}m_5^2 V_M V^M\right]$$

$$m_5^2 = (5-1)(5+1-4) = 8.$$

Vector-vector correlator

$$\left[\partial_z\left(\frac{1}{z}\partial_z V_\mu^a(q,\,z)\right)+(\frac{q^2}{z}-C_5^2/z^3)V_\mu^a(q,\,z)\right]_\perp = 0\,,\quad C_5^2\equiv m_5^2\tilde{g}_5^2.$$

$$V(Q,z)=c_1 z I_n(Qz)+c_2 z K_n(Qz)\,,\quad n = \sqrt{1+C_5^2},\ \text{and}\ Q^2 = -q^2.$$

UV boundary condition $V(Q,\epsilon) \sim 1/\epsilon^2$,

$$n=3\ (C_5^2=8),\ \text{and}\quad \tilde{g}_5^2=1$$

$$\phi(x,z) = z^{4-\Delta-p}\left[\phi_0(x)+O(z^2)\right]+z^{\Delta-p}\left[A(x)+O(z^2)\right],\qquad V(z) = c_1\frac{1}{z^2}+c_2 z^4.$$

m_{π_1}

$$\left(\partial_z^2 - \frac{1}{z}\partial_z + m_n^2 - \frac{8}{z^2}\right) f_n(z) = 0\,, \qquad f_n(z) = a_1 z J_3(m_n z) + a_2 z Y_3(m_n z)\,.$$

the IR boundary condition $\partial_z f_n(z)|_{z=z_m} = 0$: $m_n z_m J_2(m_n z_m) - 2J_3(m_n z_m) = 0$.

the mass of π_1 to be ~ 1476 MeV

the experimental value $m_{\pi_1} = 1351 \pm 30$ MeV

Decay constant

$$(F_{\pi_1} m_{\pi_1}^3)^2 = \frac{c^2}{\tilde{g}_5^2} \left(\frac{f_1'(\epsilon)}{\epsilon^3} \right)^2 , \quad F_{\pi_1} = 10.6 \text{ MeV.}$$

Summary

- Experiments are discovering (candidates of) exotic mesons.
- Quark-gluon mixed condensate is encoded in the hard wall model
- Predicted mass and decay constant of $Pi_1(1400)$ are reasonable.
- What about charmed tetra-quark mesons in hQCD? ~ 4.83 GeV for a vector state [Kyung-il Kim, YK, S. H. Lee, in progress].