Mesons and Baryons in AdS/CFT (Short Introduction)

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Introduction

- String theory
  - Fundamental object : one dimensional string

Open string

Closed string

  - Spacetime dimension : 10
  - D-branes : p-dimensional extended object
  - 5 different theory (type I, type IIA, type IIB, Heterotic SO(32), Heterotic $E_8 \times E_8$)
  - Contains gauge field and gravity in the theory

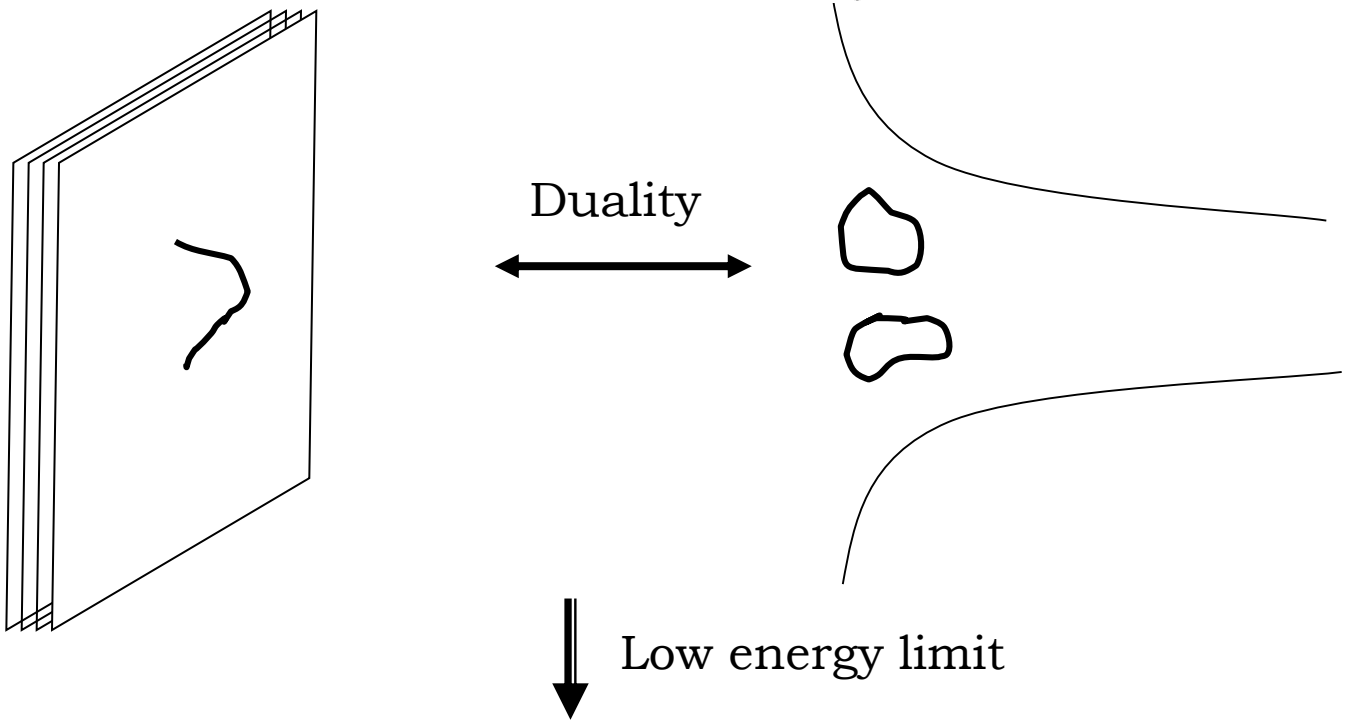
Open string

$$\alpha^{\mu}_{-1}|0>$$

Closed string

$$\alpha^{\mu}_{-1}\tilde{\alpha}^{\nu}_{-1}|0>$$

AdS/CFT Correspondence



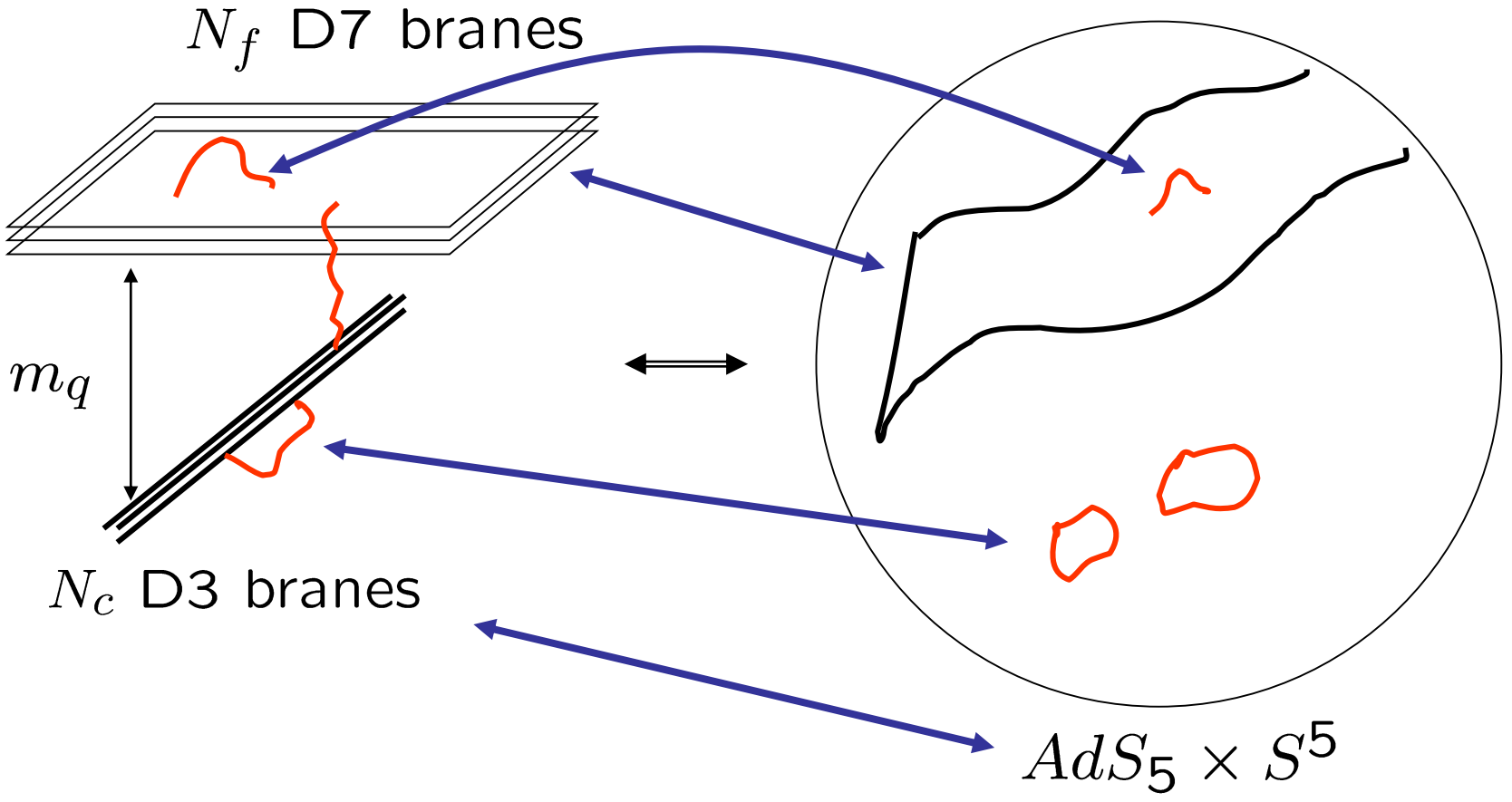
$\mathcal{N} = 4 SU(N_c)$ SYM theory in four dimensions($N_c \to \infty$)

Duality

Classical gravity on $AdS_5 \times S^5$

AdS/CFT Correspondence

- Adding Flavour : Introducing probe D-brane



Finite temperature

- Finite temperature : Introducing black hole

	x^0	x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9	
$D3$	○	○	○	○	–	–	–	–	–	–	N_c
$D7$	○	○	○	○	○	○	○	○	–	–	N_f

$$ds^2 = \frac{U^2}{R^2}\left[f(U)dt^2 + d\vec{x}^2\right] + R^2\left[\frac{dU^2}{f(U)U^2} + d\Omega_5^2\right]$$

$$f(U) = 1 - \left(\frac{U_0}{U}\right)^4 \qquad R^4 = 2\lambda l_s^4$$

- Temperature and horizon related by

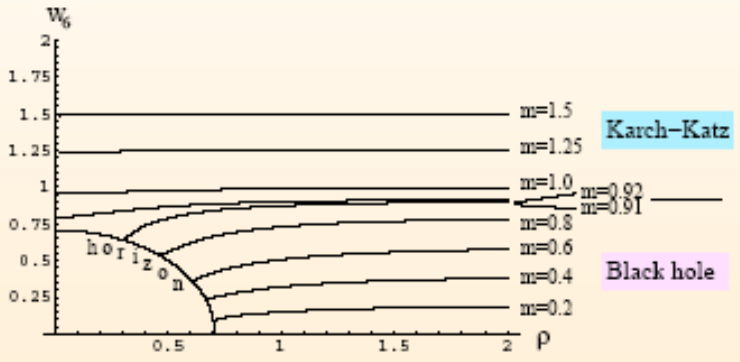
$$T = \frac{U_0}{\pi R^2} = \frac{U_0}{\sqrt{2\lambda}\pi l_s^2}$$

- DBI action

$$S_{DBI} = \mu_7 \int \sqrt{\det G}$$

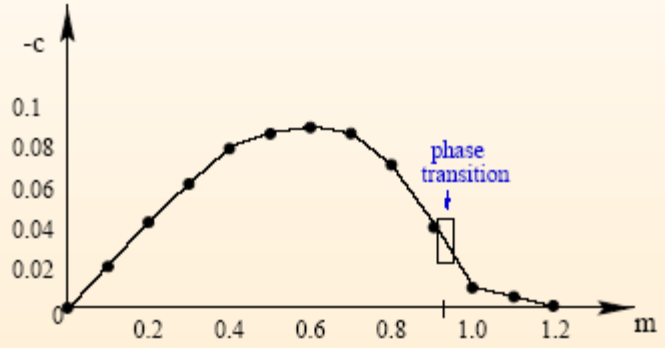
Finite Temperature

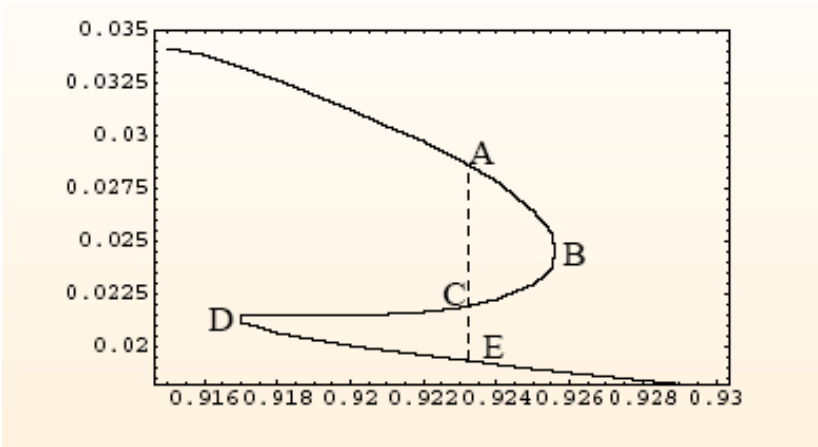
D7 brane embedding in black hole background



$$y \sim m_q + \frac{c}{\rho^2} + \cdots$$

Condensate c versus quark mass m (c, m normalized to T)





Erdmenger et al.

Finte Density

AdS/CFT

Boundary global symmetry	⟷	bulk guage symmetry

$U(1)_B$ $U(1)$ Gauge field on D-brane

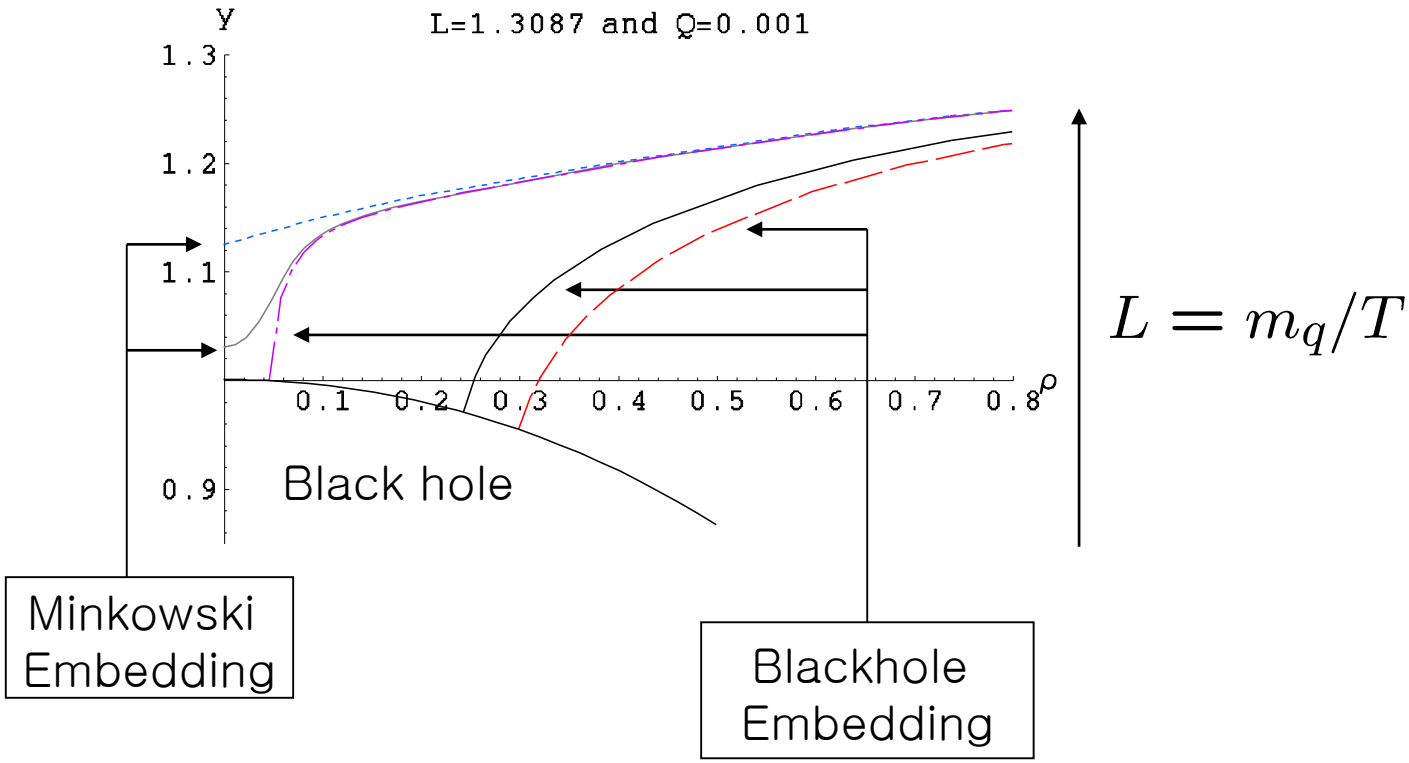
$$A_0(\rho, x) - A_0(\rho_{min}, x) = \mu + a\langle\psi^\dagger\psi\rangle\frac{1}{\rho^2} + \cdots$$

$$S_{DBI} = N_f\mu_7 \int dt d^3x d\rho d\Omega_3 \, (g_{xx}\, g_{\Omega\Omega})^{\frac{3}{2}} \sqrt{\left(g_{tt}g_{\rho\rho} - (2\pi\alpha' F_{\rho t})^2\right)}$$

$$= \tau_7 \int d\rho \;\; \rho^3\omega_+^{3/2}\sqrt{\frac{\omega_-^2}{\omega_+}(1+\dot{y}^2) - \tilde{F}_{\rho t}^2}$$

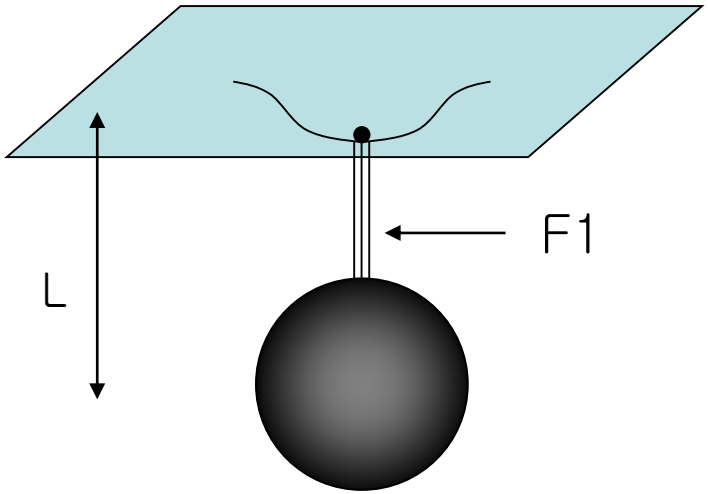
Finite Density

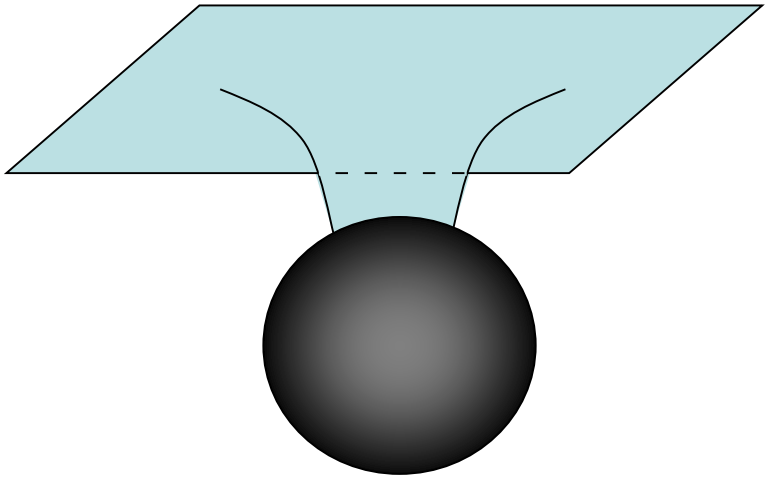
- Solution of DBI action



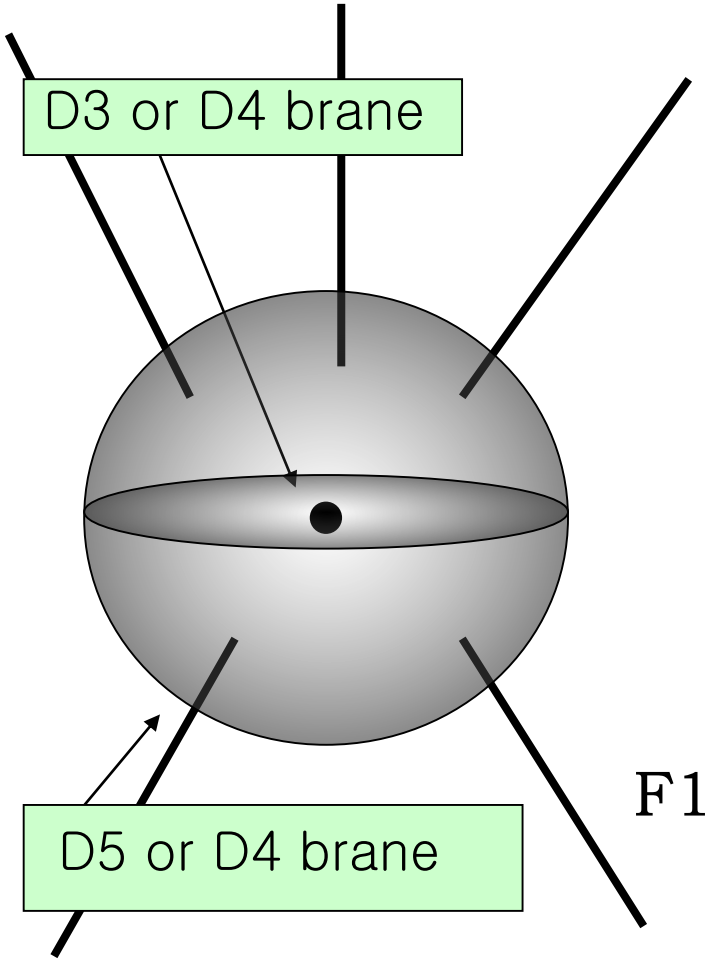
- Blackhole embeddings have lower energy

Adding source





Witten's Baryon Vertex



- Chern-Simons coupling gives flux on D5(or D4) brane
- To cancel the flux, we need N_c fundamental strings
- We need special background which provide confining phase

Dilaton background geometry

- Gubser's background

$$ds_{10}^2 = e^{\Phi/2}\left(\frac{r^2}{R^2}A^2(r)\eta_{\mu\nu}dx^\mu dx^\nu + \frac{R^2}{r^2}dr^2 + R^2 d\Omega_5^2\right)$$

$$A(r) = \left(1 - \left(\frac{r_0}{r}\right)^8\right)^{1/4}, \qquad e^{\Phi} = \left(\frac{(r/r_0)+1}{(r/r_0)-1}\right)^{\sqrt{3/2}}$$

- Deformation of D3 brane background $AdS_5 \times S^5$
- Supersymmetry and chiral symmetry are broken
- Special limit of 'Constable-Myers' background (δ=2)
- r_0 is proportional to the gluon condensation in boundary theory

Baryon vertex

- Baryons : D5 brane wrapped on an S^5 on which N_c F1's terminate
- The background 5-form field strength can couple to the world volume U(1) gauge field via Chern-Simons term
- Induced metric on D5 brane

$$ds^2_{D5} = e^{\Phi/2}\left[\frac{r^2}{R^2}A(r)^2dt^2 + \frac{R^2}{r^2}\left(r'^2 + r^2\right)d\theta^2 + R^2\sin^2\theta d\Omega_4^2\right]$$

- DBI action for single D5 brane with Nc F1's

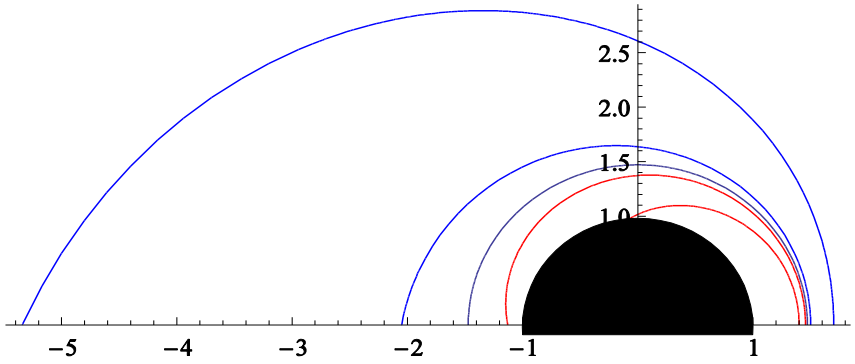
$$\begin{aligned} S_{D5} &= -\mu_5\int d^6\xi e^{-\Phi}\sqrt{\det(g+2\pi\alpha' F)} + \mu_5\int 2\pi\alpha' A\wedge G^{(5)} \\ &= \tau_5\int dtd\theta\sin^4\theta\left[-\sqrt{e^{\Phi}A(r)^2(r'^2+r^2)-\tilde{F}^2}+4\tilde{A}_t\right] \end{aligned}$$

Baryon vertex

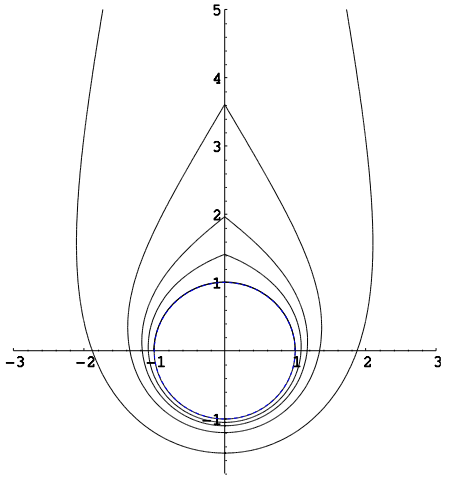
- Equation of motion

$$\frac{d}{d\theta}\left[\frac{r' A(r) e^{\Phi/2}\sqrt{D(\theta)^2+\sin^8\theta}}{\sqrt{r'^2+r^2}}\right] - \frac{\partial}{\partial r}\left(A(r)e^{\Phi/2}\sqrt{D(\theta)^2+\sin^8\theta}\right) = 0$$

- D5 brane solution



D4/D6(D3/D5)
YS, S Sin, JHEP 0804:010, 2008



- Force at cusp

$$\begin{aligned} F_{D5} &= -\frac{\partial \mathcal{H}_{D5}}{\partial r_c}\Big|_{fix\ other\ values} \\ &= -N_c T_F \frac{A(r) r' e^{\Phi/2}}{\sqrt{r'^2+r^2}}\Big|_{r=r_c} \end{aligned}$$

Probe D7 brane

- We consider D7 brane as a probe brane where the other end point of fundamental strings are attached

- End point can be regarded as point source on D7 brane

- Induced metric on D7 brane

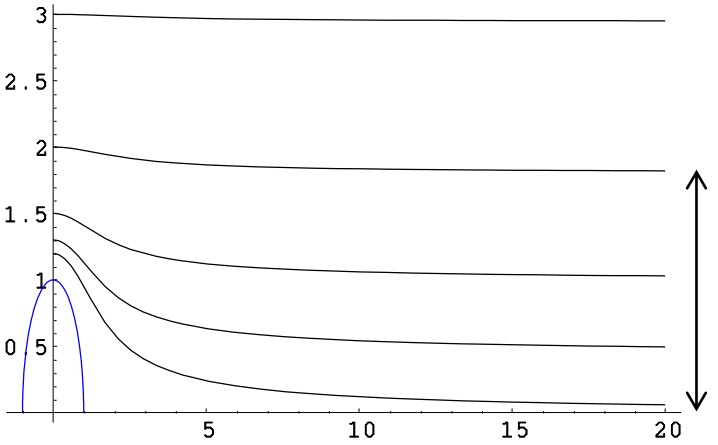
$$ds^2_{D7} = e^{\Phi/2}\left[\frac{r^2}{R^2}A^2(r)(dt^2+d\vec{x}^2)+\frac{R^2}{r^2}\left\{(1+\dot{y}^2)d\rho^2+\rho^2 d\Omega_3^2\right\}\right]$$

- DBI action

$$\begin{aligned} S_{D7} &= -N_f\mu_7\int d\xi^8 e^{-\Phi}\sqrt{\det(g+2\pi\alpha' F)} \\ &= -\tau_7\int dt d\rho A(r)^3\rho^3 e^{\Phi/2}\sqrt{e^{\Phi}A(r)^2(1+\dot{y}^2)-\tilde{F}^2} \end{aligned}$$

Probe D7 brane

- Solution of equation of motion for D7 brane(Q=0)



$$L \sim \frac{m_q}{< F^2 >}$$

- Force at the cusp of D7 brane(Q≠0)

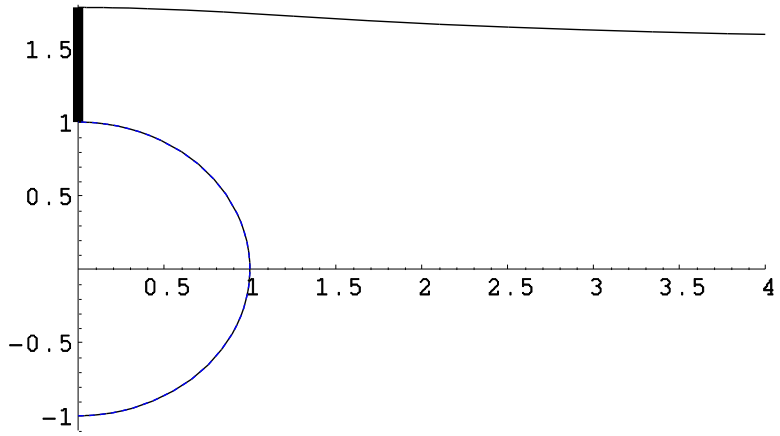
$$\begin{aligned} F_{D7} &= -\frac{\partial \mathcal{H}_{D7}}{\partial y_c}|_{fix\ other\ values} \\ &= -QT_F \frac{A(r)\dot{y}e^{\Phi/2}}{\sqrt{1+\dot{y}^2}}\Big|_{y=y_c} \end{aligned}$$

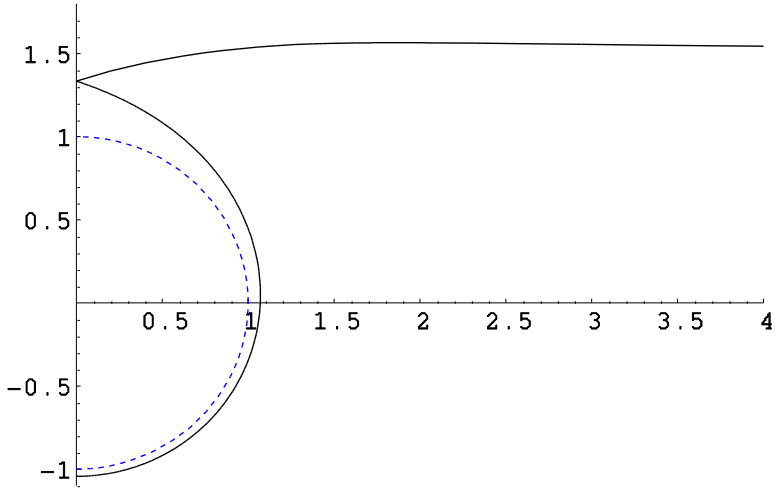
Force balance condition

- Total configuration(Baryon D5 + F1's +probe D7) can be stationary if there is force balance condition

- Force balance condition

$$F_{D7} = \frac{Q}{N_c} F_{D5} \quad \Longrightarrow \quad \dot{y}_c = \frac{r_c'}{y_c}$$



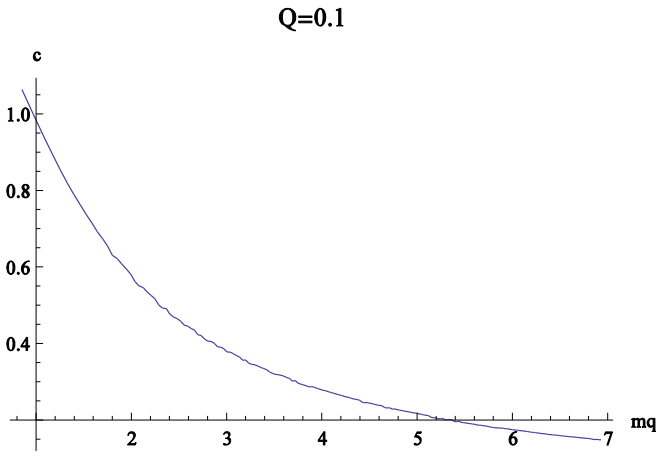


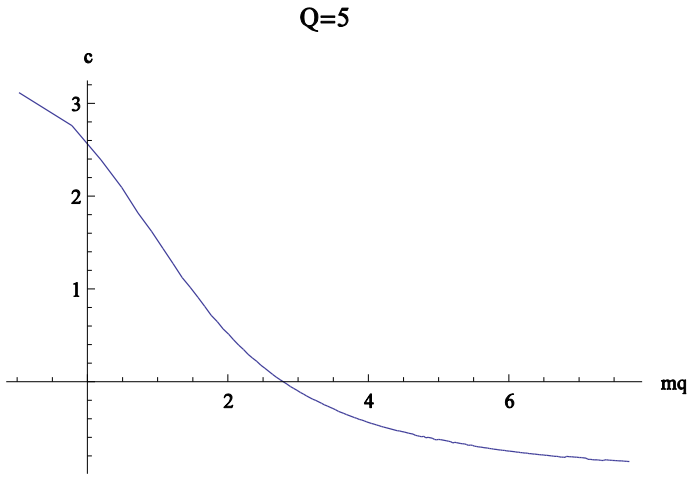
Chiral condensation

- Asymptotic behavior of D7 brane

$$y \sim m_q + \frac{c}{\rho^2} + \cdots$$

- Chiral condensation



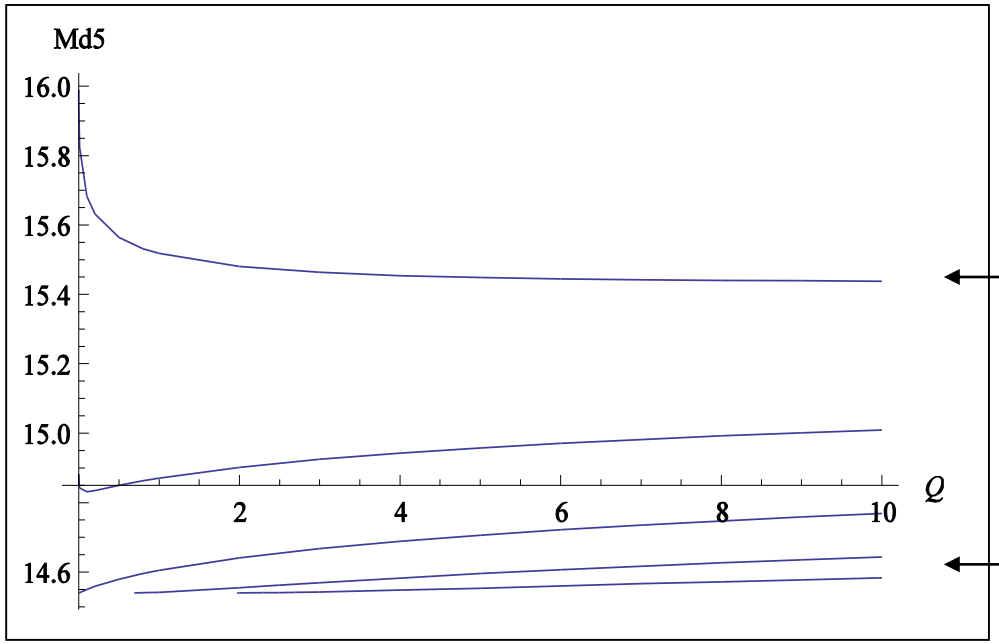


Baryon mass for Fixed quark mass

- Deformation of D-brane

Deformation of D5 brane	→	Baryon mass
Deformation of D7 brane	→	Baryon-baryon interaction

- Density dependence of baryon mass



Summary

- AdS/CFT provides a method of calculating properties of gaue theories at strong coupling
- Gauge theories at finite temperature can be studied by introducing black hole in the bulk
- Density and chemical potentials can be easily introduced by turn on gauge field on probe brane

- Meson spectrum : fluctuation of probe brane
- Baryon vertex: D5 brane wrapping D3 brane