AdS/QCD in Medium

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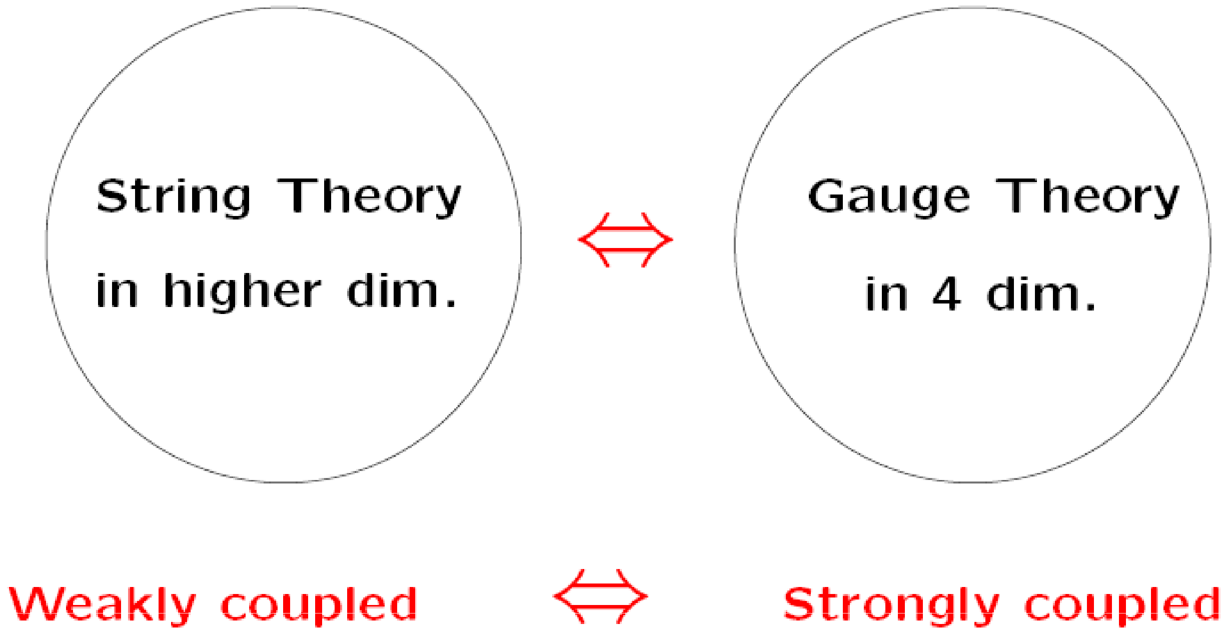
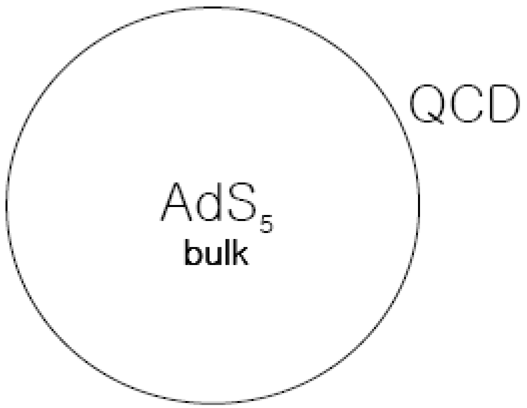
Outline

1. Bottom-up in medium.
2. Nucleon-nucleon potential in a top-down model.



Maldacena 98

AdS/QCD ?



$$\text{4D generating functional} : Z_4[\phi_0(x)] = \int \mathcal{D}[\Phi] \exp\{iS_4 + i\int \phi_0(x)\mathcal{O}\} \,,$$

$$\text{5D (classical) effective action} : \Gamma_5[\phi(x,z) = \phi_0(x)] \,; \phi_0(x) = \phi(x, z=0).$$

$$\text{AdS/CFT correspondence} : Z_4 = \Gamma_5.$$

Note that boundary value (or non-normalizable mode) of a bulk field is nothing but a 4D source term!!!

1. Bottom-up in medium

In a bottom-up approach, we first look at QCD and then attempts to guess its 5D holographic dual, through AdS/CFT dictionaries.

While, in top-down looking at QCD is nothing but choosing a suitable D-brane system: D3/D7, D4/D6, D4/D8, etc.

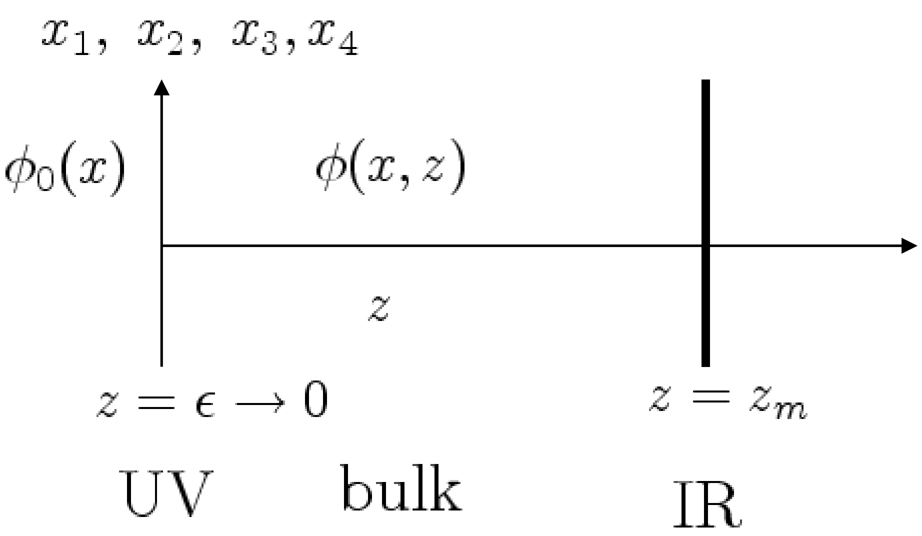
Hard wall model

J. Erlich, E. Katz, D. T. Son and M. A. Stephanov, Phys. Rev. Lett. **95**, 261602 (2005)

L. Da Rold and A. Pomarol, Nucl.Phys. **B721**, 79 (2005)

$$\mathrm{S_I} = \int d^4x dz \sqrt{g} \mathcal{L}_5 \,,$$

$$\mathcal{L}_5 = \mathrm{Tr}\left[-\frac{1}{4g_5^2}(L_{MN}L^{MN} + R_{MN}R^{MN}) + |D_M\Phi|^2 - M_\Phi^2|\Phi|^2 \right] ,$$



In the chiral limit, m_q=0, 3 unknowns in the model:

$$g_5^2 \longleftrightarrow 1/N_c,$$
$$z_m \longleftrightarrow m_\rho,$$
$$\langle \bar{q}q \rangle \longleftrightarrow m_{A_1}.$$

$$V_M \sim L_M + R_M\,, \quad A_M \sim L_M - R_M\,,$$
$$\Phi = S e^{iP/v(z)}, \quad v(z) \equiv \langle S \rangle\,,$$
$$\Phi \leftrightarrow X, \quad v(z) \leftrightarrow X_0\,.$$

$$ds_5^2 = \frac{1}{z^2}\left(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2\right)$$

Where is the chiral condensate?

Klebanov and Witten, 1999

$$\phi(x,z) \to z^{d-\Delta}\phi_0(x) + z^{\Delta}A(x) + \dots, z \to \epsilon\,,$$

where $\phi_0(x)$ is the source term of 4D operator $\mathcal{O}(x)$, and

$$A(x) = \frac{1}{2\Delta - d}\langle \mathcal{O}(x) \rangle\,.$$

For example, $\mathcal{O} = \bar{q}q$, $\phi(x,z) = v(z)$:

$$v(z) = c_1 z + c_2 z^3$$
$$c_1 \sim m_q, \quad c_2 \sim \langle \bar{q}q \rangle\,.$$

Note, however, that we cannot calculate the value of the condensate in bottom-up. So it will be fixed by an IR boundary condition ,as the quark mass is determined by a UV boundary condition.

$$
\begin{aligned}
\mathcal{S} &\sim \frac{1}{2}\int \sqrt{g}\partial_M \Phi \partial^M \Phi, \quad v(z) \equiv \langle S(x,z)\rangle \\
&\sim \frac{1}{2}\int \frac{1}{z^5}\partial_z v(z)\partial_z v(z) g^{zz} \\
&\sim \frac{1}{2}\int v(z)\partial_z \left(\frac{1}{z^3}\partial_z v(z)\right)(\rightarrow \text{EoM}) - \frac{1}{2}\int \partial_z \left(\frac{1}{z^3}v(z)\partial_z v(z)\right)(\rightarrow \text{Boundaryterm})\,.
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{S}_b &\sim \left(\frac{1}{z^3}v(z)\partial_z v(z)\right)_{z\rightarrow\epsilon}, \quad v(z) = m_q z + c z^3 \\
&\sim \frac{m_q^2}{2\epsilon^2} + 6 c m_q\,.
\end{aligned}
$$

$$
\begin{aligned}
\langle \bar{q} q\rangle &= \frac{\partial S_{\mathrm{QCD}}}{\partial_{m_q}} \\
&= \frac{\partial S_b}{\partial_{m_q}} \\
&= 3c, \quad m_q = 0\,.
\end{aligned}
$$

- 4D vector meson mass

$S \sim \int \sqrt{g}\, F_{MN} F^{MN} \qquad M,N = (\mu, z)$

$\sim \frac{1}{z^5} \Big[(\partial_\mu V_\nu - \partial_\nu V_\mu)(\mu \leftrightarrow \nu)\, \overbrace{g^{\mu\mu} g^{\nu\nu}}^{z^4}$

$+ 2 (\partial_z V_\mu - \partial_\mu V_z)(\mu \leftrightarrow z)\, \overbrace{g^{zz} g^{\mu\mu}}^{z^4} \Big]$

$\sim V \frac{1}{z} \partial_4^2 V + V \partial_z \left(\frac{1}{z} \partial_z V \right)$, $V_z = 0$ gauge

E.o.M

$\left[\partial_z^2 - \frac{1}{z} \partial_z - \partial_4^2 \right] V_{\mu\perp}(x, z) = 0$

$V_\mu(x, z) \sim f_n(z) \tilde{V}(x)$

$\hookrightarrow e^{iq \cdot x} \qquad q_\mu^2 = m^2$

$\left[\partial_z^2 - \frac{1}{z} \partial_z + m_n^2 \right] f_n^{(n)} = 0$

$f_n(0) = 0, \quad \partial_z f_n(z_m) = 0.$

$$V(x,z) = \sum f_v(z)\tilde{V}(x)$$

$$[\partial_z^2 - \frac{1}{z}\partial_z + q^2]f_v(z) = 0,\ \ q^2 = m_n^2$$

$$m_n \simeq (n - \frac{1}{4})\frac{\pi}{z_m}$$

$$m_1 = m_\rho\ ,\quad \frac{1}{z_m} \simeq 320\ \mathrm{MeV}.$$

• 4D Coupling : $g_{\rho\pi\pi}$

$$\rho \to \pi$$

$$(D_\mu \Phi)(D_\mu \Phi)^\dagger$$

$$\to \frac{1}{z} \operatorname{tr}\left(\partial_\mu \pi(x,z) \left[V_\mu(x,z) , \pi(x,z) \right] \right) + \cdots$$

$$\pi(x,z) \sim f_\pi(z) \tilde{\pi}(x)$$

$$S \sim \underbrace{\int d^4x\, dz \quad f_\pi(z)^2 f_\rho(z)}_{g_{\rho\pi\pi}} \cdot \operatorname{tr}\left(\partial \tilde{\pi} \left[\tilde{V}_\mu(x) , \tilde{\pi} \right] \right) + \cdots$$

$$\sim g_{\rho\pi\pi} \int d^4x \operatorname{tr}\left[\partial \tilde{\pi} \left[\tilde{V}_\mu , \tilde{\pi} \right] \right] + \cdots$$

Dense AdS/QCD

$$\text{QCD}: \;\mu_q \psi^\dagger \psi \; (= \mu_q \bar{\psi}\gamma_0\psi) \leftrightarrow \text{Gravity}: \; V_0(x,z) = \mu_q + \cdots, \; z \to 0\,.$$

$$\text{4D generating functional}: Z_4[\phi_0(x)] = \int \mathcal{D}[\Phi] \exp\{iS_4 + i\int \phi_0(x)\mathcal{O}\}\,,$$

$$\text{5D (classical) effective action}: \Gamma_5[\phi(x,z) = \phi_0(x)]\,; \phi_0(x) = \phi(x, z=0).$$

$$\text{AdS/CFT correspondence}: Z_4 = \Gamma_5.$$

Hard wall model with baryons in dense matter

YK, C. -H. Lee ,and H.-U. Yee, PRD(2008)

$$\mathcal{L}_{\rm int} \ni -g\left(\bar{N}_2 X N_1 + {\rm h.c.}\right) \ ,$$

D. K. Hong, T. Inami and H.-U. Yee, Phys. Lett. **B 646**, 165(2007).

$$f_{1L,R}(p,z)\,\psi_{1L,R}(p) = \int \ {\rm d}^4x\, N_{1L,R}(x,z)e^{ip\cdot x} \ ,$$

$$\begin{pmatrix} \partial_z - \frac{\Delta}{z} & -\frac{1}{2}g\sigma z^2 \\ -\frac{1}{2}g\sigma z^2 & \partial_z - \frac{4-\Delta}{z} \end{pmatrix} \begin{pmatrix} f_{1L} \\ f_{2L} \end{pmatrix} = -|p| \begin{pmatrix} f_{1R} \\ f_{2R} \end{pmatrix} \ ,$$

$$\begin{pmatrix} \partial_z - \frac{4-\Delta}{z} & \frac{1}{2}g\sigma z^2 \\ \frac{1}{2}g\sigma z^2 & \partial_z - \frac{\Delta}{z} \end{pmatrix} \begin{pmatrix} f_{1R} \\ f_{2R} \end{pmatrix} = |p| \begin{pmatrix} f_{1L} \\ f_{2L} \end{pmatrix} \ ,$$

Mean field approach:

to obtain density dependent chiral condensate

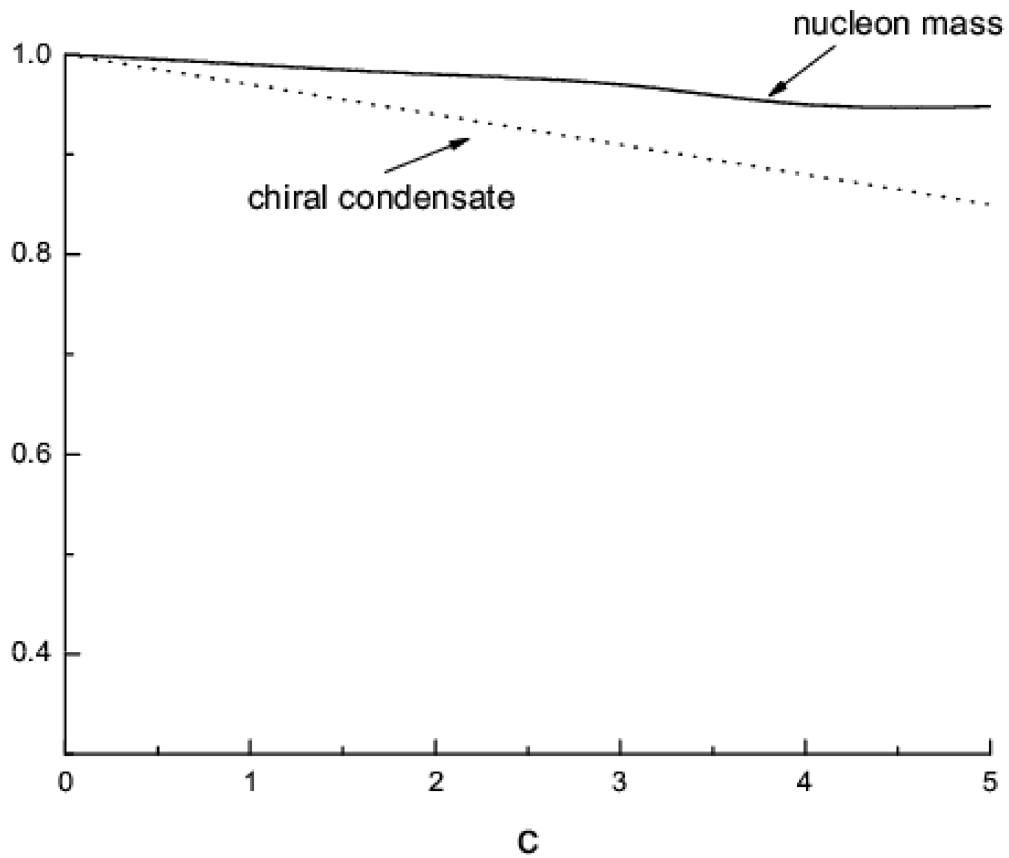
$$\langle \bar{N}(x,z)\gamma^0 N(x,z)\rangle = \sum f(z)^2 \langle \psi(x)^\dagger \psi(x)\rangle \,;\, \rho_B = \langle \psi(x)^\dagger \psi(x)\rangle$$

1. Chiral condensate $X_0 = \langle X \rangle$

$$[\partial_z^2 - \frac{3}{z}\partial_z + \frac{3}{z^2}]X_0 = \frac{1}{4}\frac{g}{z^2}(f_{2R}^2 - f_{1R}^2)\rho_s \quad \text{where } \rho_s \equiv \langle \bar{\psi}(x)\psi(x)\rangle .$$

$$X_0(z) = \frac{1}{2}m_q z + \frac{1}{2}\sigma z^3 ,$$

2. In-medium nucleon mass (iteratively)



$$\rho_B = c\rho_0$$

Back-reaction?

What about mean-field approach + standard AdS/CFT?

Up to some double counting, in-medium mass corrections might have two component: vacuum contribution and interaction effect.

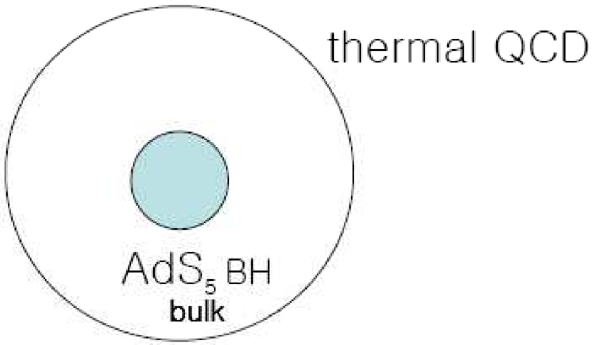
For more complete in-medium nucleon mass , see Kyung-il's talk.

Thermal AdS/QCD

E. Witten (1998)

$$ds_5^2 = \frac{1}{z^2}\left(f^2(z)dt^2 - (dx^i)^2 - \frac{1}{f^2(z)}dz^2\right),$$

$$f^2(z) = 1 - (\frac{z}{z_T})^4 \qquad T = \frac{1}{\pi z_T}$$



Transition between two backgrounds

1. thermal AdS: low T, confined , no way to extract T-dependence

$$ds^2 = L^2\left(\frac{dt^2 + d\vec{x}^2 + dz^2}{z^2}\right)$$

β' : the periodicity in the timedirection, (undetermined)

2. AdS black hole: high T, deconfined

$$ds^2 = \frac{L^2}{z^2}\left(f(z)dt^2 + d\vec{x}^2 + \frac{dz^2}{f(z)}\right) \qquad 0 \le t < \pi z_h$$

So, no temperature dependence in confined phase???

A way to introduce temperature dependence in the hard-wall model

YK, H. K. Lee, PRD (2008)

$$\left[\partial_z^2 - \frac{3}{z} + \frac{3}{z^2}\right] X_0 = 0, \quad X_0 = c_1 z + c_2 z^3 \, ,$$

Basic strategy is to impose the boundary conditions at a given temperature.
Then we have a temperature dependent chiral condensate.
Unfortunately, however, we cannot determine the temperature dependence of chiral condensate within the hard-wall model, since the chiral condensate is one of the integration constants to be fixed by the boundary conditions.

1. Vector and axial-vector meson masses

Kaluza-Klein reduction: $V_\mu(x,z) = \Sigma_n f_n^V(z) V_\mu^{(n)}(x)$

$$\left[\partial_z^2 - \frac{1}{z}\partial_z + m_n^2\right] f_n^V(z) = 0\,,$$

$$\left[\partial_z^2 - \frac{1}{z}\partial_z + m_n^2 - g_5^2\frac{v^2}{z^2}\right] f_n^A(z) = 0\,,$$

$$v(z) = \frac{1}{2}\sigma z^3$$

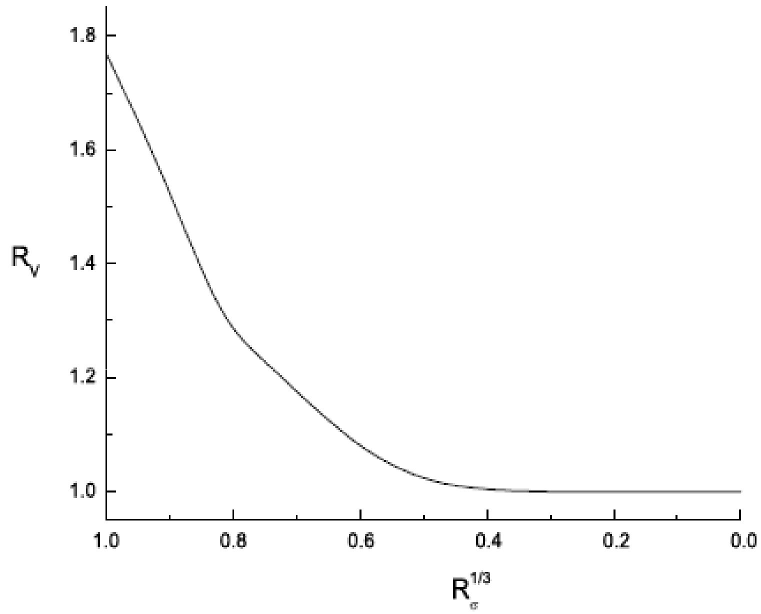


Figure 1: The scaling of a_1 mass normalized to m_ρ as a function of σ. Here $R_V \equiv m_{a_1}(\sigma)/m_\rho$ and $R_\sigma^{1/3} = (\sigma/\sigma_0)^{1/3}$.

2. Pion decay constant

$$f_\pi^2 = -\frac{1}{g_5^2}\frac{\partial_z A(0,z)}{z}\Big|_{z=z_0}\,,$$

where $A(0,z)$ is the solution of the following equation,

$$\left[\partial_z^2 - \frac{1}{z}\partial_z - 2\frac{v^2}{z^2}\right] A(0,z) = 0\,.$$

3. Nucleon mass

Hard wall model with baryons

D. K. Hong, T. Inami and H.-U. Yee, Phys. Lett. **B 646**, 165(2007).

$$\mathcal{L}_{\rm int} \ni -g\left(\bar{N}_2 X N_1 + {\rm h.c.}\right) ,$$

$$f_{1L,R}(p,z)\,\psi_{1L,R}(p) = \int {\rm d}^4x\, N_{1L,R}(x,z)e^{ip\cdot x} ,$$

$$\begin{pmatrix} \partial_z - \frac{\Delta}{z} & -\frac{gX_0}{z} \\ -\frac{gX_0}{z} & \partial_z - \frac{4-\Delta}{z} \end{pmatrix} \begin{pmatrix} f_{1L} \\ f_{2L} \end{pmatrix} = -m_N \begin{pmatrix} f_{1R} \\ f_{2R} \end{pmatrix} ,$$

$$\begin{pmatrix} \partial_z - \frac{4-\Delta}{z} & \frac{gX_0}{z} \\ \frac{gX_0}{z} & \partial_z - \frac{\Delta}{z} \end{pmatrix} \begin{pmatrix} f_{1R} \\ f_{2R} \end{pmatrix} = m_N \begin{pmatrix} f_{1L} \\ f_{2L} \end{pmatrix} ,$$

with the IR boundary condition $f_{1R}(z_m) = f_{2L}(z_m) = 0$. Here $\Delta = 9/2$, and $\langle X \rangle = \frac{1}{2}\sigma z^3$. The scaling behavior of m_N is shown in Fig. 2.

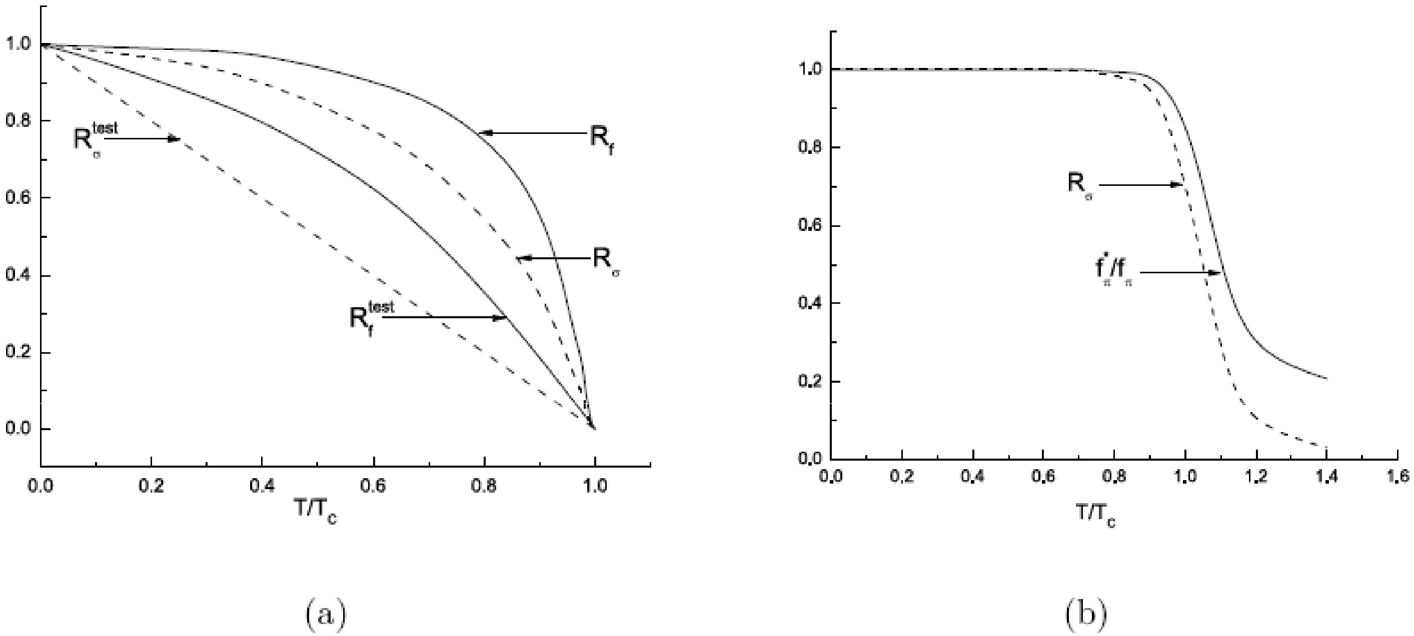


Figure 4: The temperature dependence of the pion decay constant. Here the temperature dependent chiral condensates are inputs: (a) $\mathrm{R}_\sigma^{\mathrm{test}} = 1 - \frac{T}{T_c}$ and R_σ [9], (b) R_σ [10],

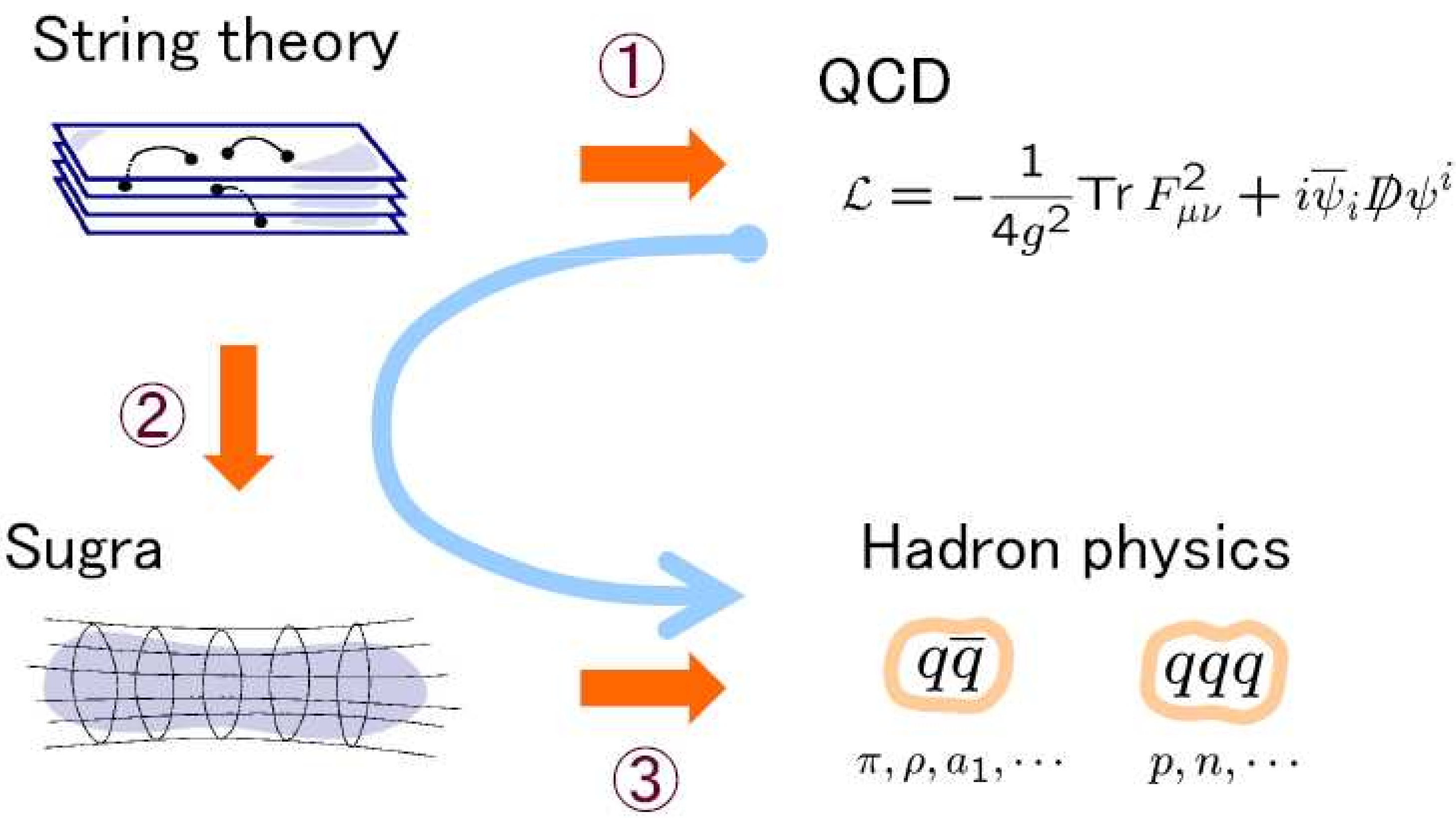
2. Nucleon-nucleon potential in a top-down model

YK, Sangmin Lee, and Piljin Yi, to appear

As a first step toward dense matter study in a 5D effective theory of the baryon based on the Sakai-Sugimoto model, we calculate a holographic nucleon-nucleon potential.

Sakai-Sugimoto model

Outline

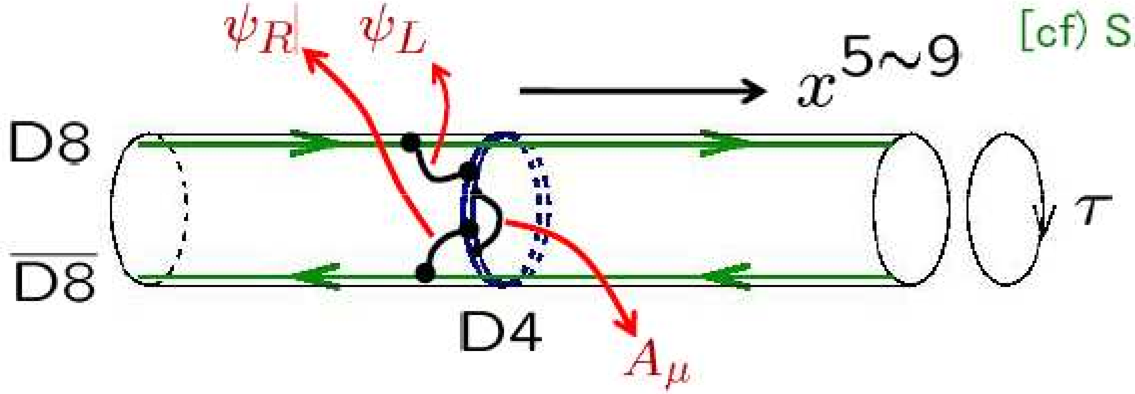


Sugimoto 2004

★ the brane configuration

		x^0	x^1	x^2	x^3	τ	x^5	x^6	x^7	x^8	x^9
D4	$\times N_c$	○	○	○	○	○	–	–	–	–	–
D8-$\overline{\text{D8}}$	$\times N_f$	○	○	○	○	–	○	○	○	○	○

[cf) S.S.–Takahashi 2004]

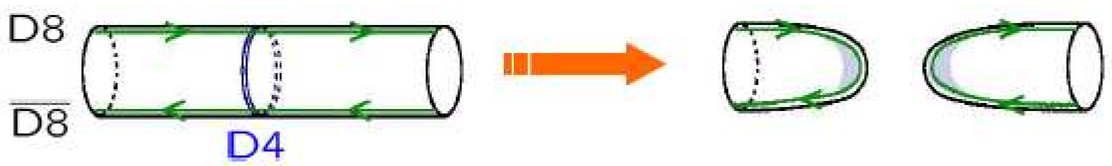


	D4 $U(N_c)$	D8 $U(N_f)_L$	$\overline{\text{D8}}$ $U(N_f)_R$
A_μ	adjoint	1	1
ψ_L	N_c	N_f	1
ψ_R	N_c	1	N_f

➡ 4 dim $U(N_c)$ QCD with N_f massless quarks

★ Chiral symmetry breaking

We replace D4 with the SUGRA sol.



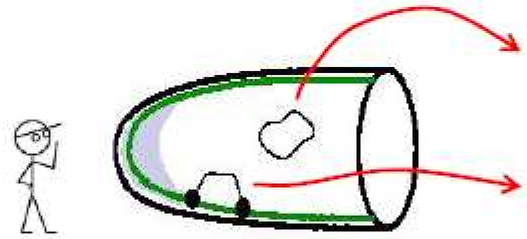
D8 and $\overline{\text{D8}}$ must be connected

in the D4 background.

➡ interpreted as the chiral symmetry breaking !

$$U(N_f)_L \times U(N_f)_R \to U(N_f)_V$$

★ Where are mesons?



closed string → glueballs

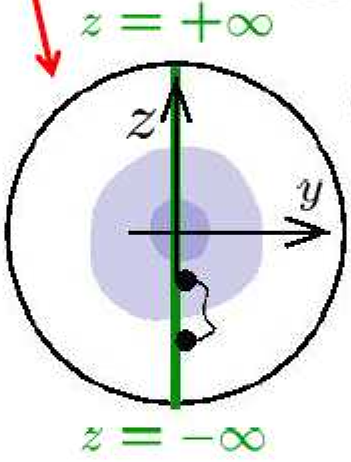
open string → mesons

The effective theory is a 9 dim $U(N_f)$ gauge theory

$$A_\mu(x^\mu, z, \theta^i),\ A_z(x^\mu, z, \theta^i),\ A_i(x^\mu, z, \theta^i)$$

(θ^i ↰ S^4)

Note: There is an $SO(5)$ symmetry ↷ S^4



$(u^3 = 1 + y^2 + z^2)$

※ Today, we only consider the $SO(5)$ singlet states, for simplicity.

(mesons in realistic QCD live in this sector)

➡ reduced to 5 dim $U(N_f)$ gauge theory $A_\mu(x^\mu, z),\ A_z(x^\mu, z)$

D8−brane action

- $S_{\rm D8}^{\rm DBI} \simeq -T\int d^9x\, e^{-\phi}\sqrt{-\det(g_{MN}+2\pi\alpha' F_{MN})}$

  $\sim \int d^9x\, e^{-\phi}\sqrt{-g}\,g^{MN}g^{PQ}F_{MP}F_{NQ}+\cdots$

Inserting the SUGRA solution,

$K(z)\equiv 1+z^2$

$$S_{\rm D8}^{\rm DBI} \sim \kappa\int d^4x dz\, {\rm Tr}\left(\frac{1}{2}K(z)^{-1/3}F_{\mu\nu}^2 + M_{\rm KK}^2 K(z)F_{\mu z}^2\right)$$

$\kappa \equiv \frac{\lambda N_c}{108\pi^3}$

- $S_{\rm D8}^{\rm CS} \simeq \int_9 C\wedge {\rm Tr}e^{F/2\pi} \sim \int_9 dC_3 \wedge \frac{1}{3!(2\pi)^3}\omega_5(A)+\cdots$

CS 5−form
$d\omega_5(A) = {\rm Tr}\,F^3$

D4 charge

$\frac{1}{2\pi}\int_{S^4} dC_3 = N_c$

$$S_{\rm D8}^{\rm CS} \sim \frac{N_c}{24\pi^2}\int_5 \omega_5(A)$$

This 5 dim YM−CS theory is considered as the effective theory of mesons.

Comparison of various stringy set-upz

- SS model(D4/D8/D8): chiral symmetry, massless quark (exponentially small quark mass or meson doubling, though), confinement.
- D3/D7: finite quark mass, no confinement, no chiral symmetry.
- D4/D6: finite quark mass, confinement, no chiral symmetry.

Baryons in SS model

- 5D instanton soliton is dual of 4D Skyrmion.
- The size of 5D instanton is a small object and so it may be treated as a point-like object.
- So we ay introduce Dirac field for the 5D baryons, and write down 5D effective action of the baryon.

The starting point is the five-dimensional effective action of isospin 1/2 baryons. With

$$\gamma^0 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \quad \gamma^i = \begin{pmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{pmatrix}, \quad \gamma^5 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

we have the following five-dimensional effective action,

$$\int d^4x dw \left[-i\bar{\mathcal{B}}\gamma^m D_m \mathcal{B} - i m_{\mathcal{B}}(w) \bar{\mathcal{B}}\mathcal{B} + \frac{2\pi^2 \rho_{baryon}^2}{3e^2(w)} \bar{\mathcal{B}}\gamma^{mn} F_{mn} \mathcal{B} \right]$$
$$- \int d^4x dw \frac{1}{4e^2(w)} \operatorname{tr} \mathcal{F}_{mn}\mathcal{F}^{mn},$$

where the covariant derivative is defined as $D_m = \partial_m - i(N_c \mathcal{A}_m^{U(1)} + A_m)$ with A_m in the fundamental representation of $SU(N_F = 2)$.

Hong-Rho-Yee-Yi (2007)

To obtain interactions between nucleons and mesons, we mode expand $\mathcal{B}(x^\mu, w) = B_+(x^\mu)f_+(w) + B_-(x^\mu)f_-(w)$ where $\gamma^5 B_\pm = \pm B_\pm$ and the profile functions $f_\pm(w)$ satisfy

$$\begin{aligned} \partial_w f_+(w) + m_{\mathcal{B}}(w) f_+(w) &= m_{\mathcal{N}} f_-(w) , \\ -\partial_w f_-(w) + m_{\mathcal{B}}(w) f_-(w) &= m_{\mathcal{N}} f_+(w) , \end{aligned}$$

Inserting this into the action we find the following structure of the four-dimensional nucleon action

$$\int dx^4 \, \mathcal{L}_4 = \int dx^4 \left(-i\bar{\mathcal{N}}\gamma^\mu \partial_\mu \mathcal{N} - i m_{\mathcal{N}} \bar{\mathcal{N}}\mathcal{N} + \mathcal{L}_{\text{vector}} + \mathcal{L}_{\text{axial}} \right) ,$$

where we have, schematically, the vector-like cubic couplings

$$\mathcal{L}_{\text{vector}} = -i\bar{\mathcal{N}}\gamma^\mu \beta_\mu \mathcal{N} - \sum_{k\geq 1} g_V^{(k)} \bar{\mathcal{N}}\gamma^\mu v_\mu^{(2k-1)} \mathcal{N} + \sum_{k\geq 1} g_{dV}^{(k)} \bar{\mathcal{N}}\gamma^{\mu\nu} \partial_\mu v_\nu^{(2k-1)} \mathcal{N} ,$$

and the axial cubic couplings to axial mesons,

$$\mathcal{L}_{\text{axial}} = -\frac{i g_A}{2} \bar{\mathcal{N}}\gamma^\mu \gamma^5 \alpha_\mu \mathcal{N} - \sum_{k\geq 1} g_A^{(k)} \bar{\mathcal{N}}\gamma^\mu \gamma^5 v_\mu^{(2k)} \mathcal{N} + \sum_{k\geq 1} g_{dA}^{(k)} \bar{\mathcal{N}}\gamma^{\mu\nu} \gamma^5 \partial_\mu v_\nu^{(2k)} \mathcal{N} .$$

Remarkably, even before we go into any detail, we have a prediction that all isospin singlet vectors and all axial-vectors have no derivative coupling in this approximation.

Holographic NN potential

- One natural way is: (1) obtain B=2 soliton solution and (2) then separate them to obtain a potential.
- One boson exchange type potential.

The interaction Lagrangians for boson-nucleon couplings are, for pseudoscalar mesons:

$$\mathcal{L}_P = -g_{\varphi\mathcal{N}\mathcal{N}}\bar{\mathcal{N}}(x)\gamma_5\varphi(x)\mathcal{N}(x)\,,$$

and for vector mesons:

$$\mathcal{L}_V = -g_{v\mathcal{N}\mathcal{N}}\bar{\mathcal{N}}(x)\gamma^\mu v_\mu(x)\mathcal{N}(x) + \frac{\tilde{g}_{v\mathcal{N}\mathcal{N}}}{2m_\mathcal{N}}\bar{\mathcal{N}}(x)\gamma^{\mu\nu}\partial_\mu v_\nu(x)\mathcal{N}(x)\,,$$

where $m_\mathcal{N}$ is the nucleon mass. For the D4-D8 holographic model, we saw that the derivative coupling is absent for the isospin singlet vectors such as ω.[#9] The same is true of axial vectors, so we have only

$$\mathcal{L}_A = -g_{a\mathcal{N}\mathcal{N}}\bar{\mathcal{N}}(x)\gamma^\mu\gamma_5 a_\mu(x)\mathcal{N}(x)\,.$$

The leading large N_c and large λ scaling is such that, for pseudo-scalars ($\varphi = \pi, \eta'$)

$$
\begin{aligned}
\frac{g_{\pi\mathcal{N}\mathcal{N}}}{2m_{\mathcal{N}}} M_{KK} &= \frac{g_A^{triplet}}{2f_\pi} M_{KK} \simeq \frac{2\cdot 3\cdot \pi}{\sqrt{5}} \times \sqrt{\frac{N_c}{\lambda}}, \\
\frac{g_{\eta'\mathcal{N}\mathcal{N}}}{2m_{\mathcal{N}}} M_{KK} &= \frac{N_c g_A^{singlet}}{2f_\pi} M_{KK} \simeq \sqrt{\frac{3^9}{2}}\pi^2 \times \frac{1}{\lambda N_c}\sqrt{\frac{N_c}{\lambda}},
\end{aligned}
$$

for vectors ($v = \rho^{(k)}, \omega^{(k)}$)

$$
\begin{aligned}
g_{\rho^{(k)}\mathcal{N}\mathcal{N}} &= \frac{g_V^{(k)triplet}}{2} \simeq \sqrt{2 \cdot 3^3 \cdot \pi^3}\,\hat{\psi}_{(2k-1)}(0) \times \frac{1}{N_c}\sqrt{\frac{N_c}{\lambda}}, \\
g_{\omega^{(k)}\mathcal{N}\mathcal{N}} &= \frac{N_c g_V^{(k)singlet}}{2} \simeq \sqrt{2 \cdot 3^3 \cdot \pi^3}\,\hat{\psi}_{(2k-1)}(0) \times \sqrt{\frac{N_c}{\lambda}}, \\
\frac{\tilde{g}_{\rho^{(k)}\mathcal{N}\mathcal{N}}}{2m_{\mathcal{N}}} M_{KK} &= \frac{g_{dV}^{(k)triplet} M_{KK}}{2} \simeq \sqrt{\frac{2^2 \cdot 3^2 \cdot \pi^3}{5}}\,\hat{\psi}_{(2k-1)}(0) \times \sqrt{\frac{N_c}{\lambda}},
\end{aligned}
$$

and for axial vectors ($a = a^{(k)}, f^{(k)}$),

$$
\begin{aligned}
g_{a^{(k)}\mathcal{N}\mathcal{N}} &\equiv \frac{g_A^{(k)triplet}}{2} \simeq \sqrt{\frac{2^2 \cdot 3^2 \cdot \pi^3}{5}}\,\hat{\psi}_{(2k)}{}'(0) \times \sqrt{\frac{N_c}{\lambda}}, \\
g_{f^{(k)}\mathcal{N}\mathcal{N}} &\equiv \frac{N_c g_A^{(k)singlet}}{2} \simeq \sqrt{\frac{3^9 \cdot \pi^5}{2}}\,\hat{\psi}_{(2k)}{}'(0) \times \frac{1}{\lambda N_c}\sqrt{\frac{N_c}{\lambda}}.
\end{aligned}
$$

N-N potential in the leading order

$$\frac{g_{\pi\mathcal{N}\mathcal{N}} M_{KK}}{2m_{\mathcal{N}}} \sim g_{\omega^{(k)}\mathcal{N}\mathcal{N}} \sim \frac{\tilde{g}_{\rho^{(k)}\mathcal{N}\mathcal{N}} M_{KK}}{2m_{\mathcal{N}}} \sim g_{a^{(k)}\mathcal{N}\mathcal{N}} \sim \sqrt{\frac{N_c}{\lambda}} \,,$$

$$V_{\pi}^{holographic} = \frac{1}{4\pi} \left(\frac{g_{\pi\mathcal{N}\mathcal{N}} M_{KK}}{2m_{\mathcal{N}}} \right)^2 \frac{1}{M_{KK}^2 r^3} \, S_{12} \, \vec{\tau}_1 \cdot \vec{\tau}_2 \, .$$

$$V_{\omega^{(k)}}^{holographic} \;=\; \frac{1}{4\pi} \; (g_{\omega^{(k)}\mathcal{N}\mathcal{N}})^2 \; m_{\omega^{(k)}} \; y_0(m_{\omega^{(k)}} r).$$

$$V_{\rho^{(k)}}^{holographic} \simeq$$

$$\frac{1}{4\pi} \left(\frac{\tilde{g}_{\rho^{(k)}\mathcal{N}\mathcal{N}} M_{KK}}{2m_{\mathcal{N}}} \right)^2 \frac{m_{\rho^{(k)}}^3}{3M_{KK}^2} [2y_0(m_{\rho^{(k)}} r)\vec{\sigma}_1 \cdot \vec{\sigma}_2 - y_2(m_{\rho^{(k)}} r) S_{12}(\hat{r})] \; \vec{\tau}_1 \cdot \vec{\tau}_2 \, .$$

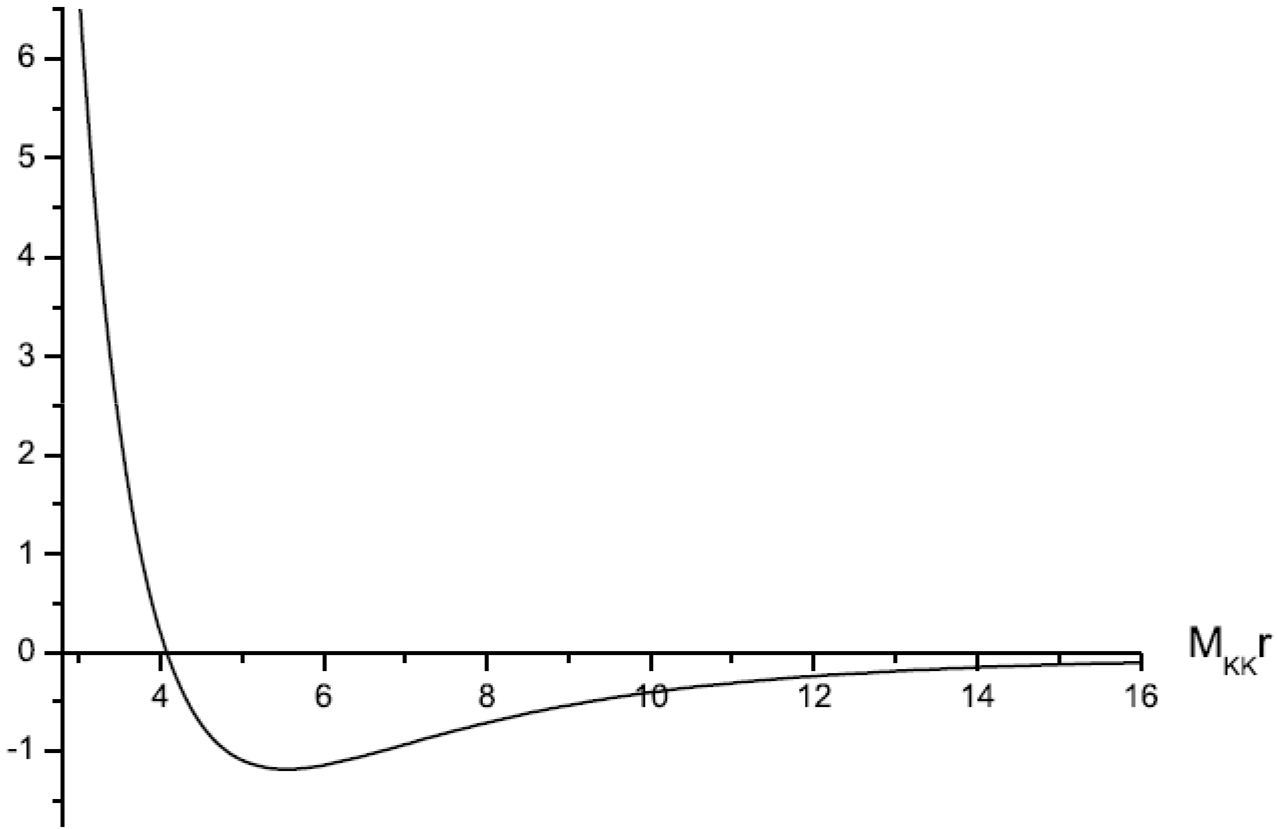
$$V_{a^{(k)}}^{holographic} \simeq$$

$$\frac{1}{4\pi} \; (g_{a^{(k)}\mathcal{N}\mathcal{N}})^2 \; \frac{m_{a^{(k)}}}{3} \; [-2y_0(m_{a^{(k)}} r)\vec{\sigma}_1 \cdot \vec{\sigma}_2 + y_2(m_{a^{(k)}} r) S_{12}(\hat{r})] \; \vec{\tau}_1 \cdot \vec{\tau}_2 \, .$$

$$g_{\omega\mathcal{N}\mathcal{N}} \simeq \xi_k \sqrt{\frac{N_c}{\lambda}}, \qquad \frac{\tilde{g}_{\rho\mathcal{N}\mathcal{N}}}{2m_{\mathcal{N}}} M_{KK} \simeq \zeta_k \sqrt{\frac{N_c}{\lambda}}, \qquad g_{a^{(k)}\mathcal{N}\mathcal{N}} \simeq \chi_k \sqrt{\frac{N_c}{\lambda}}.$$

Coefficients, ξ_k, ζ_k, χ_k, are determined by $\psi_{(2k-1)}(0)$ and $\psi'_{(2k)}(0)$, we list these values in the following table 1, together with the masses (in unit of M_{KK}) of the vector and the axial vector mesons.

k	$m_{\omega^{(k)}} = m_{\rho^{(k)}}$	$\hat{\psi}_{(2k-1)}(0)$	ξ_k	ζ_k	$m_{a^{(k)}}$	$\hat{\psi}'_{(2k)}(0)$	χ_k
1	0.818	0.5973	24.44	8.925	1.25	0.629	9.40
2	1.69	0.5450	22.30	8.143	2.13	1.10	16.4
3	2.57	0.5328	21.81	7.961	3.00	1.56	23.3
4	3.44	0.5288	21.64	7.901	3.87	2.02	30.1
5	4.30	0.5270	21.57	7.874	4.73	2.47	36.9
6	5.17	0.5261	21.52	7.860	5.59	2.93	43.8
7	6.03	0.5255	21.50	7.852	6.46	3.38	50.5
8	6.89	0.5251	21.48	7.846	7.32	3.83	57.3
9	7.75	0.5249	21.48	7.843	8.19	4.29	64.1
10	8.62	0.5247	21.47	7.840	9.05	4.74	70.9


6
5
4
3
2
1
0
-1
4
6
8
10
12
14
16
$M_{KK}r$