Pion properties from the instanton vacuum in free space and at finite density

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HIM, Feb. 23, 2009

Outline

Quantum Chromodynamics

⇩

Instanton vacuum

QCD vacuum
chiral symmetry &
its spontaneous br.

⇩

Effective Chiral Action

Chiral Quark-(Soliton) Model

⇩

Structure of hadrons

⇩

Confronting with experiments

Experiments & Theories

- **Elastic scattering** ⟶ **Form factors**
- **DI scattering** ⟶ **Parton distributions**
- **DVCS & HEMP** ⟶ **GPDs**
- **Hadronic reactions** ⟶ **Coupling constants**
- **Weak decays** ⟶ **Weak coupling consts.**
- **New spectroscopy** ⟶ **Quantum Nr.& masses**

Accelerators: Spring-8, JLAB, MAMI, ELSA,
GSI (FAIR: PANDA), COSY, J-PARC, LHC……

From QCD to An effective theory

QCD Lagrangian

$$\mathcal{L}_{\rm QCD} = -\frac{1}{4g^2} G^a_{\mu\nu} G^{a\mu\nu} + \sum_f \bar{\psi}_f (i\partial\!\!\!/ + A\!\!\!/ - m_f)\psi_f, \quad f = u,\, d,\, s \cdots$$

flavor	u	d	s
m_f [MeV]	5	10	180

50-100 MeV from ChPT

Running coupling constant → Non-perturbative in low-energy regime

$$\alpha_s^{(2)}(Q^2) = \frac{4\pi}{\beta_0}\left[\frac{1}{\ln\frac{Q^2}{\Lambda^2} + \frac{\beta_1}{\beta_0^2}\ln\left(1 + \frac{\beta_0^2}{\beta_1}\ln\frac{Q^2}{\Lambda^2}\right)}\right]$$

$$\beta_0 = 11 - (2/3)N_f, \qquad \beta_1 = 102 - (38/3)N_f$$

From QCD to An effective theory

Light quark systems: QCD in the Chiral limit, i.e. Quark masses → 0

$$L_{\mathrm{QCD}} = -\frac{1}{4g^2} G^a_{\mu\nu} G^{a\mu\nu} + \bar{\psi}(i\gamma^\mu \partial_\mu + A)\psi$$

$$SU(2): \quad \psi = \begin{pmatrix} \psi_u \\ \psi_d \end{pmatrix}$$

$$SU(3): \quad \psi = \begin{pmatrix} \psi_u \\ \psi_d \\ \psi_s \end{pmatrix}$$

Global Symmetries:

Invariance: Vector: $\psi \to \psi' = \exp(+\frac{i}{2}\alpha^A \tau^A)\psi$

$\bar{\psi} \to \bar{\psi}' = \bar{\psi}\exp(-\frac{i}{2}\alpha^A \tau^A)$

Axial Vector: $\psi \to \psi' = \exp(+\frac{i}{2}\alpha^A \tau^A \gamma_5)\psi$

$\bar{\psi} \to \bar{\psi}' = \bar{\psi}\exp(+\frac{i}{2}\alpha^A \tau^A \gamma_5)$

τ^A = Pauli matrices

λ^A = Gell-Mann matrices

From QCD to An effective theory

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Requires all current quark masses to be equal in order to be exactly fulfilled

Axial Vector: $\psi \rightarrow \psi' = \exp(+\frac{i}{2}\alpha^A \tau^A \gamma_5)\psi$

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Requires all current quark masses to be zero in order to be exactly fulfilled

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From QCD to An effective theory

General wisdom about QCD

Vector: $\psi \to \psi' = \exp(+\frac{i}{2}\alpha^A \tau^A)\psi$

$\bar{\psi} \to \bar{\psi}' = \bar{\psi}\exp(-\frac{i}{2}\alpha^A \tau^A)$

Axial Vector: $\psi \to \psi' = \exp(+\frac{i}{2}\alpha^A \tau^A \gamma_5)\psi$

$\bar{\psi} \to \bar{\psi}' = \bar{\psi}\exp(+\frac{i}{2}\alpha^A \tau^A \gamma_5)$

- **Irred. Representations**
- Octet
- Decuplet
- Antidecuplet...

- **Chiral symmetry**
- No multiplets
- Spontaneous breakdown of chiral symmetry
- dynamically generated quark mass m→M
- dynamical mass M~(350–400) MeV
- Massless Goldstone Bosons (pions)
- Chiral quark condensate

$$L_{QCD}(\psi_f, A) \to L_{eff}(\psi, \pi^a)$$

From QCD to An effective theory

Chiral symmetry and its spontaneous breaking

Banks-Casher theorem $\longrightarrow$ Zero-mode spectrum $\nu(0)$

$$\det(i\nabla + im) = \exp\left[\frac{1}{2}\int_{-\infty}^{\infty} d\lambda \nu(\lambda)\ln(\lambda^2 + m^2)\right]_{m\to 0}$$

$$\langle\bar{\psi}\psi\rangle = -\frac{1}{V}\int_{-\infty}^{\infty} d\lambda\bar{\nu}(\lambda)\frac{m}{\lambda^2+m^2}\bigg|_{m\to 0}$$
$$= -\frac{1}{V}\mathrm{sign}\pi\bar{\nu}(0)$$

This zero mode can be realized from the instanton vacuum

Simplest effective chiral Lagrangian

Vector: $\psi \to \psi' = V\psi$ $\quad V = \exp(+\frac{i}{2}\alpha^A\tau^A)$

$\bar\psi \to \bar\psi' = \bar\psi\, V^{-1}$ $\quad L_{eff} = L'_{eff}$

Axial $\psi \to \psi' = A\psi$ $\quad A = \exp(+\frac{i}{2}\alpha^A\tau^A\gamma_5)$

$\bar\psi \to \bar\psi' = \bar\psi\, A$ $\quad \bar\psi\psi \to \bar\psi'\psi' = \bar\psi AA\psi \neq \bar\psi\psi$

$$L_{eff} = \bar\psi(i\gamma^\mu\partial_\mu - M)\psi$$

Does not work

Vector: $\psi \to \psi' = V\psi$ $\quad U \to U' = VUV^{-1}$

$\bar\psi \to \bar\psi' = \bar\psi\, V^{-1}$ $\quad L_{eff} = L'_{eff}$

Axial $\psi \to \psi' = A\psi$ $\quad U \to U' = A^{-1}UA^{-1}$

$\bar\psi \to \bar\psi' = \bar\psi\, A$ $\quad L_{eff} = L'_{eff}$

$$L_{eff} = \bar\psi(i\gamma^\mu\partial_\mu - MU)\psi$$

Does work

Chiral Quark Model (ChQM):

$$L_{eff} = \bar\psi(i\gamma^\mu\partial_\mu - MU)\psi \qquad U(x) = \exp(\frac{i}{f_\pi}\tau^A\pi^A(x))$$

Any model that has SCSB must have this kind of a form!!!

Eff. Chiral Action from the instanton vacuum

Derivation of ChQM from QCD via Instantons by Diakonov and Petrov → Extended by **M.Musakhanov**,HChK, M.Siddikov

$$L_{Mink} = \overline{\psi}(i\partial - MU)\psi \qquad L_{Euk} = \psi^{\dagger}(i\partial + iMU)\psi$$

non-local M (from instantons):

$$Z=\int DU\int D\psi^{\dagger}D\psi \exp\{\int d^4x[\psi^{\dagger}(x)i\partial\psi(x) + i\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}e^{i(k_1-k_2)x}\psi^{\dagger}(k_1)\sqrt{M(k_1)}U(x)\sqrt{M(k_2)}\psi(k_2)$$

Dynamical quark mass (Constituent quark mass)

Model-dependent, gauge-dependent

Eff. Chiral Action from the instanton vacuum

Effective QCD action via instanton

Instanton induced partition function (instanton ensemble)

$$\mathcal{Z} = \int D\psi D\psi^\dagger \exp\left(\int d^4x \sum_{f=1}^{N_f} \bar{\psi}_f i\partial\!\!\!/ \psi^f\right) \left(\frac{Y_{N_f}^+}{VM_1^{N_f}}\right)^{N_+} \left(\frac{Y_{N_f}^-}{VM_1^{N_f}}\right)^{N_-}$$

t'Hooft 2Nf-interaction

$$Y_{N_f}^+ = \int d\rho\, d(\rho) \int dU \prod_{f=1}^{N_f} \left\{ \int \frac{d^4k_f}{(2\pi)^4} [2\pi\rho F(k_f\rho)] \int \frac{d^4 l_f}{(2\pi)^4} [2\pi\rho F(l_f\rho)] \right.$$

$$\left. \cdot (2\pi)^4 \delta(k_1 + \ldots + k_{N_f} - l_1 - \ldots - l_{N_f}) \cdot U_{i'_f}^{\alpha_f} U_{\beta_f}^{\dagger j'_f} \epsilon^{i_f i'_f} \epsilon_{j_f j'_f} \left[i\psi_{Lf\alpha_f i_f}^\dagger(k_f) \psi_L^{f\beta_f j_f}(l_f) \right] \right\}.$$

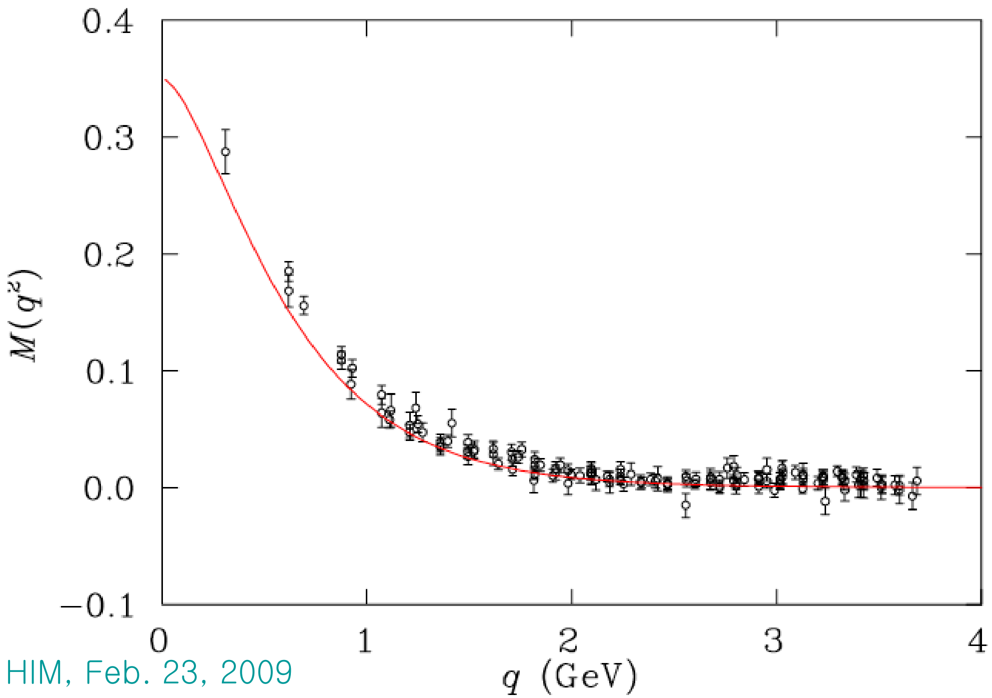
$d(\rho)$: instanton distribution, U: Color orientation, x: coordinate

Eff. Chiral Action from the instanton vacuum

Momentum-dependent quark mass M(k) induced via quark-instaton

Fourier transform of the zero mode solution $\rightarrow F(k)$ Diakonov & Petrov

$$F(k\rho) = 2t\left[I_0(t)K_1(t) - I_1(t)K_0(t) - \frac{1}{t}I_1(t)K_1(t)\right]_{t=\frac{k\rho}{2}}$$

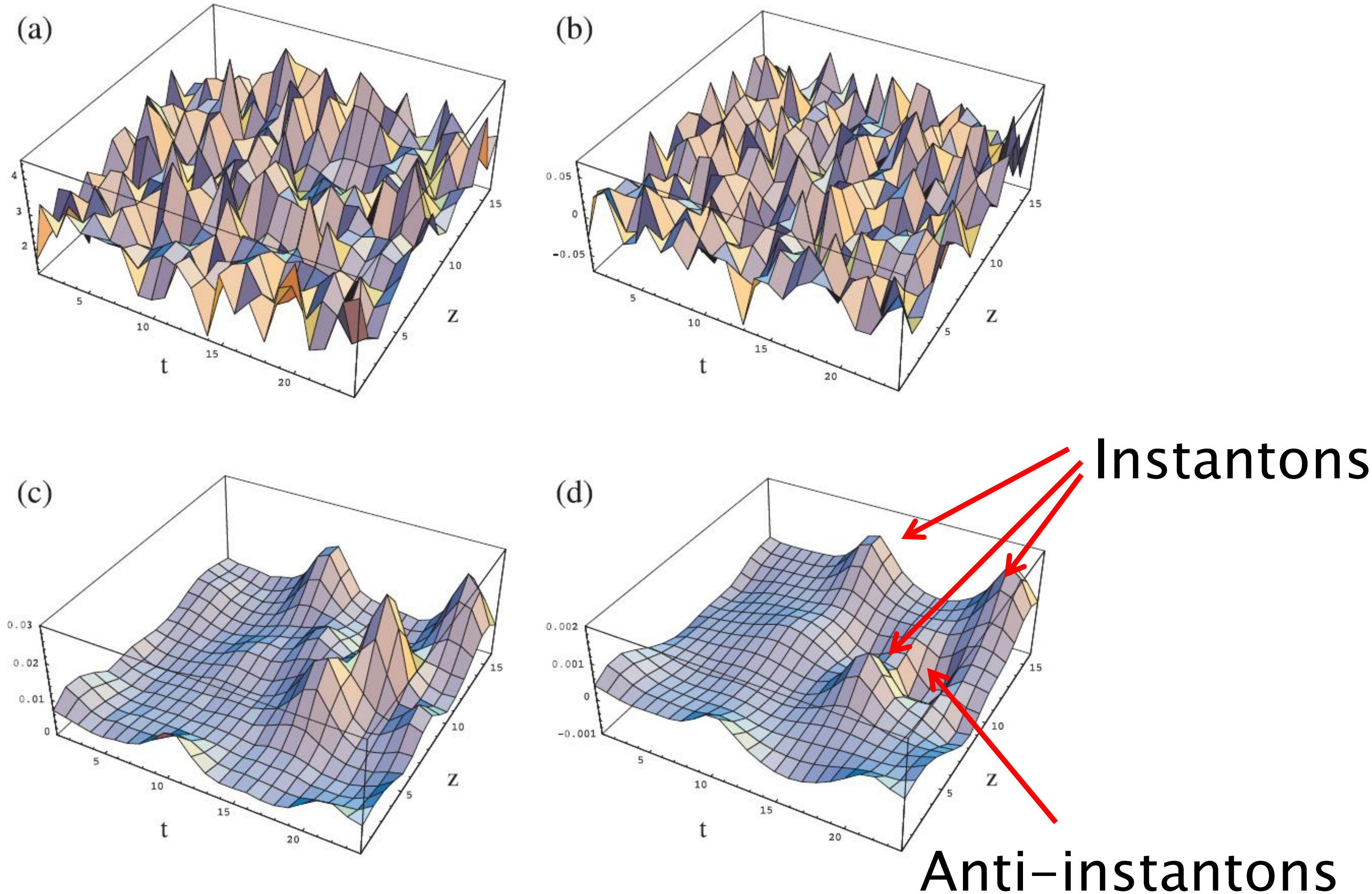


Lattice data from

P.O. Bowman et al., Nucl. Phys. Proc. Suppl. 128, 23 (2004)

From lattice QCD: After cooling

Chu et al., PRL 70, 225 (1993)



Low-energy effective QCD partition function

Effective Low-energy Partition function from the instanton vacuum

$$\mathcal{Z} = \int D\pi^A \int D\psi^\dagger D\psi \exp \int d^4x \left\{ \psi_f^\dagger(x) i\partial\!\!\!/ \psi^f(x) + i \int \frac{d^4k d^4l}{(2\pi)^8} e^{i(k-l,x)} \sqrt{M(k)M(l)} \right.$$

$$\left. \cdot \left[\psi_{f\alpha}^\dagger(k) \left(U_g^f(x) \frac{1+\gamma_5}{2} + U_g^{\dagger f}(x) \frac{1-\gamma_5}{2} \right) \psi^{g\alpha}(l) \right] \right\},$$

$$U_g^f(x) = \left(\exp i\pi^A(x)\lambda^A \right)_g^f.$$

This partition function is our starting point!

There is basically no free parameter:

Λ QCD=280 MeV

$$\bar{\rho} \simeq 0.48/\Lambda_{\overline{\rm MS}} \simeq 0.35\,{\rm fm},$$

$$\bar{R} = \left(\frac{N}{V}\right)^{-\frac{1}{4}} \simeq 1.35/\Lambda_{\overline{\rm MS}} \simeq 0.95\,{\rm fm}$$

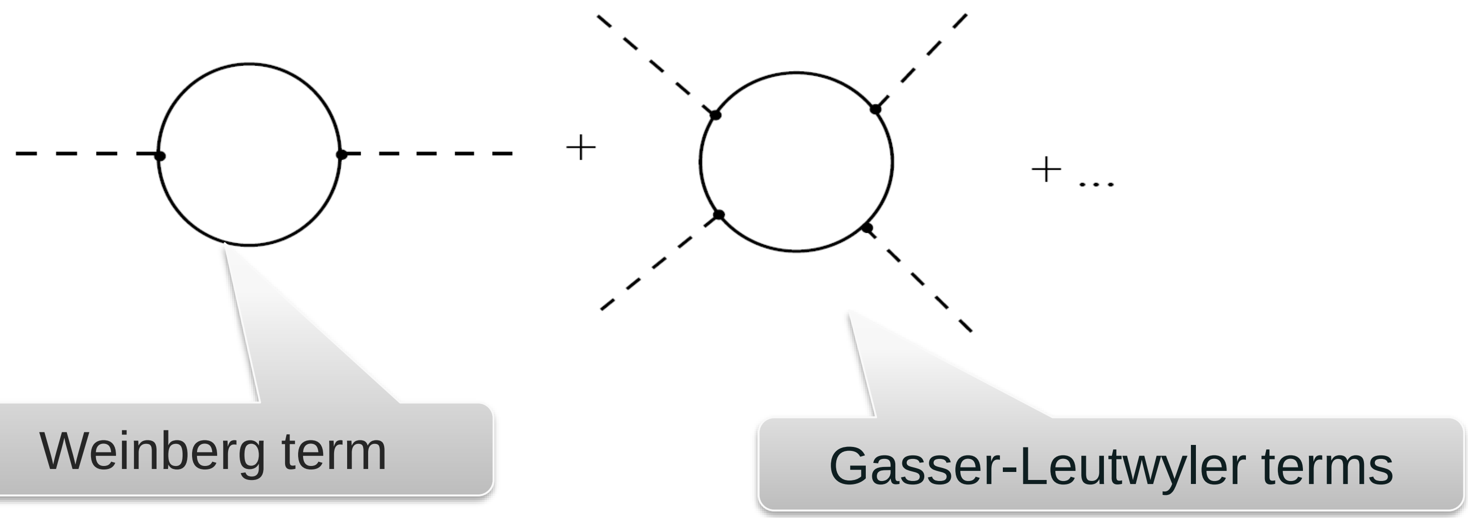
ρ/R = 1/3

Effective chiral action

Effective Chiral Lagrangian and LECs

$$S_{\rm eff} = -N_c {\rm Tr} \ln(i\partial\!\!\!/ + i\sqrt{M(i\partial)}U^{\gamma_5}\sqrt{M(i\partial)})$$

Derivative expansions



H.A. Choi and HChK, PRD 69, 054004 (2004)

Effective chiral action

Weinberg Lagrangian

$\mathcal{O}(p^2)$

$$\mathrm{Re}S_{\mathrm{eff}}^{(2)}[\pi^a] - \mathrm{Re}S_{\mathrm{eff}}^{(2)}[0] = \int d^4x \mathcal{L}^{(2)}$$

$$\mathcal{L}^{(2)} = \frac{F_\pi^2}{4}\left\langle D^\mu U^\dagger D_\mu U \right\rangle + \frac{F_\pi^2}{4}\left\langle \mathcal{X}^\dagger U + \mathcal{X} U^\dagger \right\rangle$$

$$F_\pi{}^2 = 4N_c \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2+M^2)^2}\left[M^2 - \frac{1}{2}kMM' + \frac{1}{4}k^2 M'^2\right]$$

H.A. Choi and HChK, PRD 69, 054004 (2004)

Effective chiral action

Gasser-Leutwyler Lagrangian

$\mathcal{O}(p^4)$

$$\mathcal{L}^{(4)} = L_1 \langle L_\mu L_\mu \rangle^2 + L_2 \langle L_\mu L_\nu \rangle^2 + L_3 \langle L_\mu L_\mu L_\nu L_\nu \rangle$$

- $L_2 = 2L_1$ (large N_c limit : Gasser &Leutwyler)
- $2L_2 + L_3 \neq 0$ (with constant M, $2L_2 + L_3 = 0$)

$$\Delta = -\frac{2L_2 + L_3}{L_2} \qquad \Delta = -3\frac{a_2^2}{a_2^0} + \mathcal{O}(m_\pi^2)$$

$$M_\sigma < 665[1 + 0.44\Delta + 0.33\Delta^2 + \mathcal{O}(\Delta^3)]\mathrm{MeV}$$

M.Polyakov and Vereshagin, Hep-ph/0104287

H.A. Choi and HChK, PRD 69, 054004 (2004)

Effective chiral action

Gasser-Leutwyler Lagrangian

	M_0(MeV)	Λ(MeV)	$L_1(\times 10^{-3})$	$L_2(\times 10^{-3})$	$L_3(\times 10^{-3})$
local χQM	350	1905.5	0.79	1.58	-3.17
DP	350	611.7	0.82	1.63	-3.09
Dipole	350	611.2	0.82	1.63	-2.97
Gaussian	350	627.4	0.81	1.62	-2.88
GL			0.9 ± 0.3	1.7 ± 0.7	-4.4 ± 2.5
Bijnens			0.6 ± 0.2	1.2 ± 0.4	-3.6 ± 1.3
Arriola			0.96	1.95	-5.21
VMD			1.1	2.2	-5.5
Holdom(1)			0.97	1.95	-4.20
Holdom(2)			0.90	1.80	-3.90
Bolokhov et al.			0.63	1.25	2.50
Alfaro et al.			0.45	0.9	-1.8

H.A. Choi and HChK, PRD 69, 054004 (2004)

Effective chiral action

Gasser-Leutwyler Lagrangian

Ratio Δ and the upper limit of M_σ

$\Delta = -(2L_2 + L_3)/L_2$

	$2L_2 + L_3 (\times 10^{-3})$	Δ	$\leq M_\sigma$(MeV)
local χQM	0	0	665
DP	1.67	-0.103	637.2
Dipole	0.29	-0.178	619.9
Gaussian	0.387	-0.243	606.9
Arriola	-1.31	0.672	960.7
VMD	-1.1	0.5	866.2
Holdom(1)	-0.3	0.154	715.3
Holdom(2)	-0.3	0.167	$720.$
Bolokhov et al.	0	0	665
Alfaro et al.	0	0	665

H.A. Choi and HChK, PRD 69, 054004 (2004)

Effective chiral action

- We also derived the effective weak chiral Lagrangians ($\Delta S=0,1,2$)

M. Franz, HChK, K. Goeke, Nucl. Phys. B 562, 213 (1999)
M. Franz, HChK, K. Goeke, Nucl. Phys. A 699, 541 (2002)
H.J. Lee, C.H. Hyun, C.H. Lee, HChK, EPJC45, 451(2006)

QCD vacuum properties

Quark condensates

Order parameter for spontaneous chiral symmetry breaking ⟨iq^+q⟩

$$\langle\bar{q}q\rangle_f = \frac{1}{V}\frac{\delta\ln\mathcal{Z}}{\delta m_f} = -iN_c\int\frac{d^4k}{(2\pi)^4}\mathrm{tr}_\gamma\left[\frac{\not{k}+i[m_f+M_f(k)]}{k^2+[m_f+M_f(k)]^2}-\frac{\not{k}+im_f}{k^2+m_f^2}\right]$$

$$= 4N_c\int\frac{d^4k}{(2\pi)^4}\left[\frac{m_f+M_f(k)}{k^2+[m_f+M_f(k)]^2}-\frac{m_f}{k^2+m_f^2}\right].$$

$$\langle\bar{\psi}\psi\rangle = -\frac{1}{V}\int_{-\infty}^{\infty}d\lambda\bar{\nu}(\lambda)\frac{m}{\lambda^2+m^2}\Big|_{m\to 0}$$

$$= -\frac{1}{V}\mathrm{sign}\pi\bar{\nu}(0)$$

S.i.Nam and HChK, Phys.Lett. B 647, 145 (2007)

QCD vacuum properties

Gluon condensate

Nonzero due to nonperturbative vacuum fluctuation

$$\langle G_{\mu\nu}G^{\mu\nu}\rangle_f = 32\pi^2 N/V \qquad N/V = 200^4\ \mathrm{MeV}^4$$

Saddle-point (self-consistent) equation

$$\begin{aligned}
\frac{\delta \ln \mathcal{Z}}{\delta\lambda} &= -\frac{N}{\lambda} + N_c V \int \frac{d^4k}{(2\pi)^4}\mathrm{tr}_\gamma \frac{iM_f(k)/N_c}{-\not{k} + im_f + iM_f(k)} = 0, \\
\frac{N}{\lambda} &= N_c V \int \frac{d^4k}{(2\pi)^4}\mathrm{tr}_\gamma \frac{iM_f(k)[-\not{k} - im_f - iM_f(k)]}{k^2 + [m_f + M_f(k])^2}, \\
\frac{N}{V} &= 4N_c \int \frac{d^4k}{(2\pi)^4}\frac{M_f(k)[m_f + M_f(k)]}{k^2 + [m_f + M_f(k)]^2}.
\end{aligned}$$

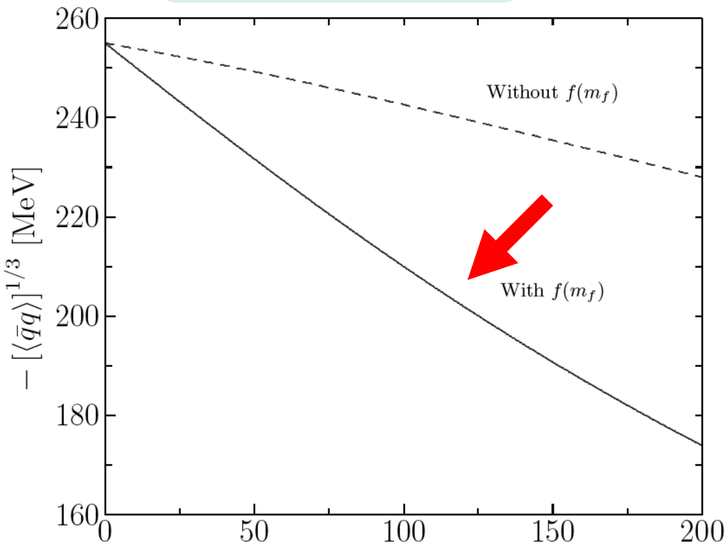
From this constraint, $M_f(0)=M_0$=350 MeV in the chiral limit

S.i.Nam and HChK, Phys.Lett. B 647, 145 (2007)

QCD vacuum properties

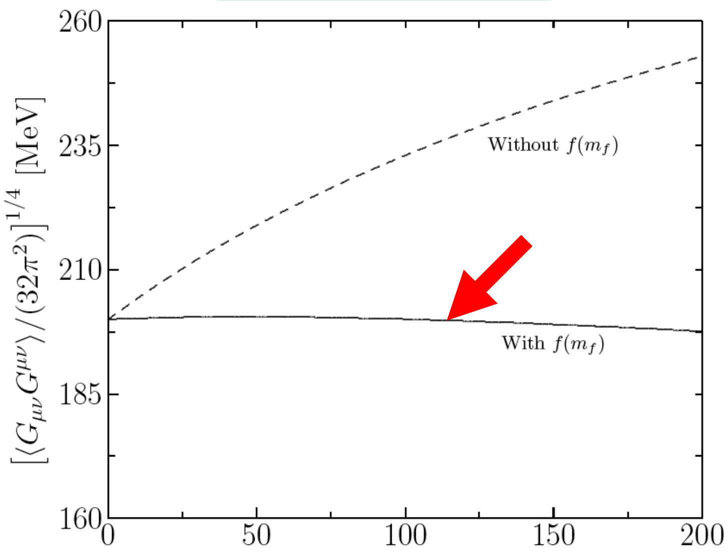
Numerical results

Quark condensate



	With $f(m_f)$	without $f(m_f)$
$\langle \bar{u}u \rangle$	-253^3	-254^3
$\langle \bar{s}s \rangle$	-191^3	-235^3

Gluon condensate



	With $f(m_f)$	without $f(m_f)$
$\langle G_{\mu\nu}G^{\mu\nu} \rangle_u/32\pi^2$	200^4	202^4
$\langle G_{\mu\nu}G^{\mu\nu} \rangle_s/32\pi^2$	199^4	243^4

S.i.Nam and HChK, Phys.Lett. B 647, 145 (2007)

QCD vacuum properties

Quark-gluon mixed condensate

Another order parameter: $\langle\bar{\psi}\sigma^{\mu\nu}G_{\mu\nu}\psi\rangle = m_0^2\langle\bar{\psi}\psi\rangle$

Related to $\langle k_\perp\rangle$ of the pseudoscalar meson wf.

Color-flavor matrix convoluted with single instanton (singular gauge)

$$G^a_{\pm\mu\nu}(x,x',U) = \frac{1}{2}\left[\lambda^a U\lambda^b U^\dagger\right] G^b_{\pm\mu\nu}(x'-x)$$

$$F^b_{\pm\mu\nu}(x) = \frac{8\rho^2}{(x^2+\rho^2)^2}\left[(\eta^\mp)^b_{\rho\nu}\frac{x_\rho x_\mu}{x^2} + (\eta^\mp)^b_{\mu\rho}\frac{x_\rho x_\nu}{x^2} - \frac{1}{2}(\eta^\mp)^b_{\mu\nu}\right]$$

Gluon field strength in terms of quark and instanton

$$\hat{G}^a_{\pm\mu\nu} = \frac{iN_cM}{4\pi\bar{\rho}^2}\int d^4x\int dU G^a_{\pm\mu\nu}(x,x',U)Y_{\pm,1}(x,U)$$

Quark-gluon Yukawa vertex $\rightarrow$ instanton-quark

S.i.Nam and HChK, Phys.Lett. B 647, 145 (2007)

QCD vacuum properties

Quark-gluon mixed condensate

$$\langle \bar{q}\sigma_{\mu\nu}G^{\mu\nu}q\rangle_f = 2N_c\bar{\rho}^2 \int \frac{d^4k_1}{(2\pi)^4}\int \frac{d^4k_2}{(2\pi)^4}\frac{\sqrt{M_f(k_1)M_f(k_2)}G(k_1,k_2)N(k_1,k_2)}{[k_1^2+[m_f+M_f(k_1)]^2][k_2^2+[m_f+M_f(k_2)]^2]}$$

$$G(k_1,k_2) = 32\pi^2\left[\frac{K_0(t)}{2}+\left[\frac{4K_0(t)}{t^2}+\left(\frac{2}{t}+\frac{8}{t^3}\right)K_1(t)-\frac{8}{t^4}\right]\right],$$

$$N(k_1,k_2) = \frac{1}{(k_1-k_2)^2}\left[8k_1^2k_2^2-6(k_1^2+k_2^2)k_1\cdot k_2+4(k_1\cdot k_2)^2\right]$$

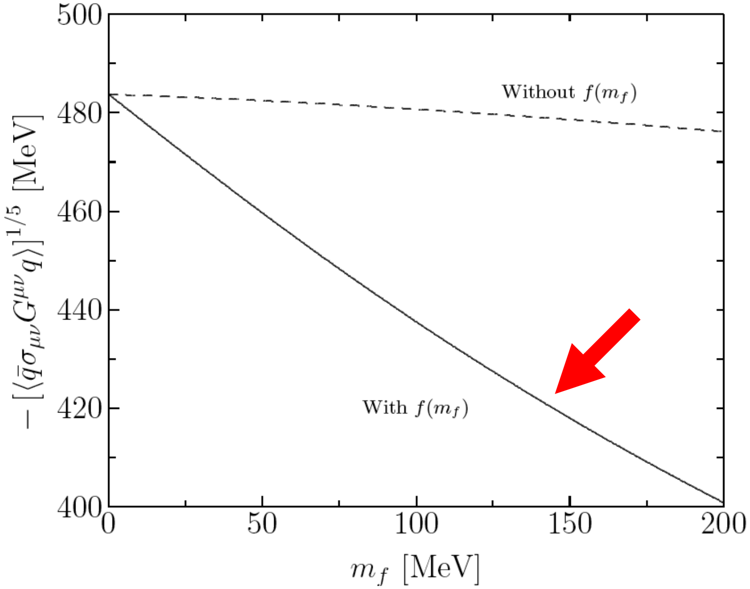
$$M_f(k) = M_0F_f^2(k)\left[\sqrt{1+\frac{m_f^2}{d^2}}-\frac{m_f}{d}\right] \quad d=\sqrt{\frac{0.08385}{2N_c}\frac{8\pi\bar{\rho}}{R^2}}\simeq 0.198\,\mathrm{GeV},$$

S.i.Nam and HChK, Phys.Lett. B 647, 145 (2007)

QCD vacuum properties

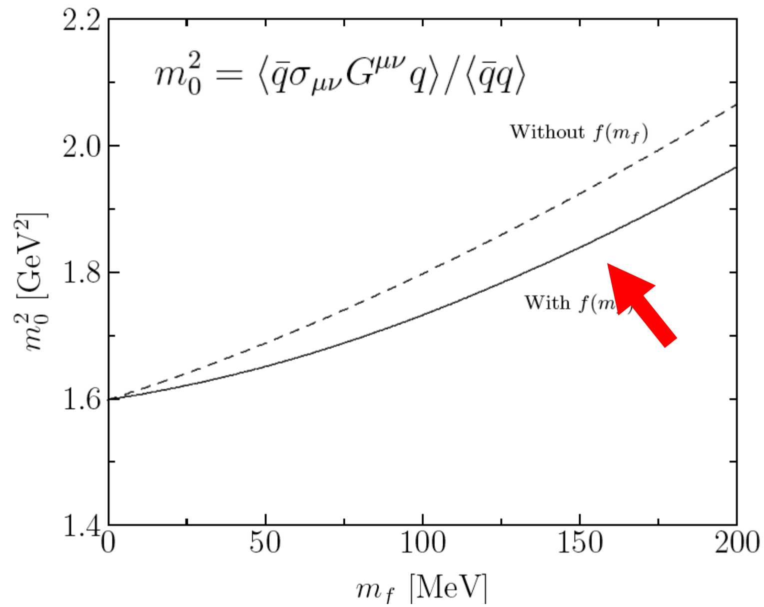
Numerical results

Mixed condensate



	With $f(m_f)$	without $f(m_f)$
$\langle\bar{u}\sigma_{\mu\nu}G^{\mu\nu}u\rangle$	-481^5	-484^5
$\langle\bar{s}\sigma_{\mu\nu}G^{\mu\nu}s\rangle$	-418^5	-483^5

Ratio of condensates



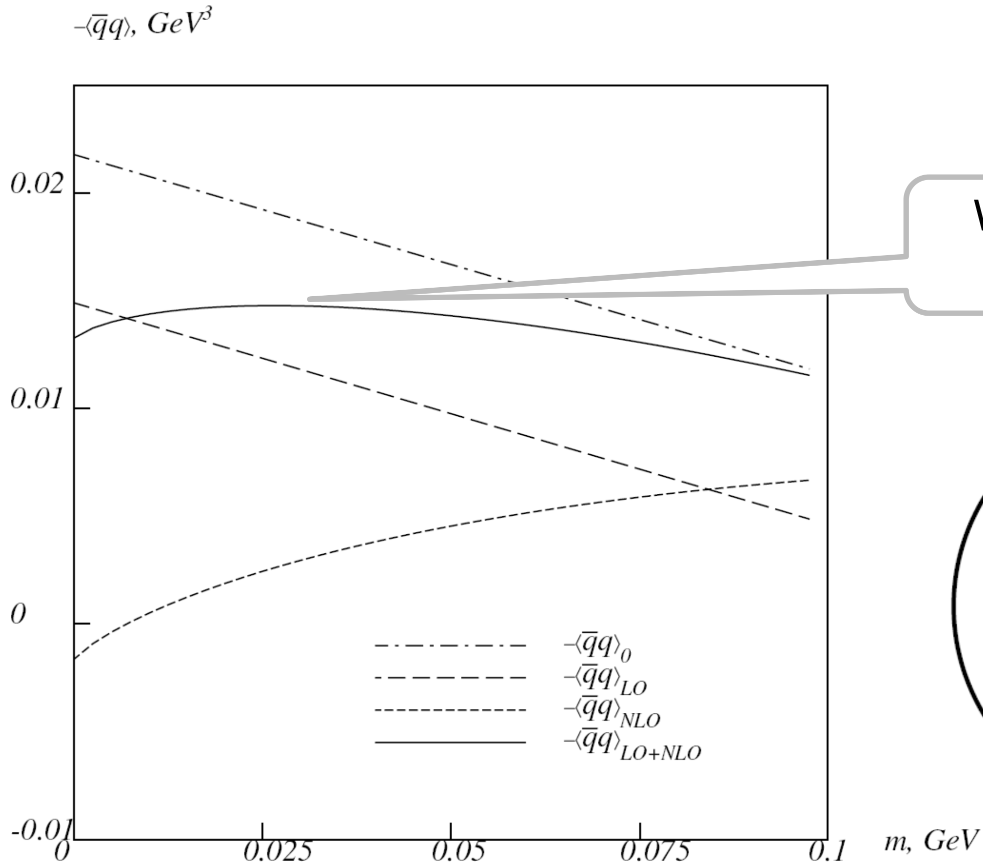
$$\langle\bar{\psi}\sigma^{\mu\nu}G_{\mu\nu}\psi\rangle = m_0^2\langle\bar{\psi}\psi\rangle$$

S.i.Nam and HChK, Phys.Lett. B 647, 145 (2007)

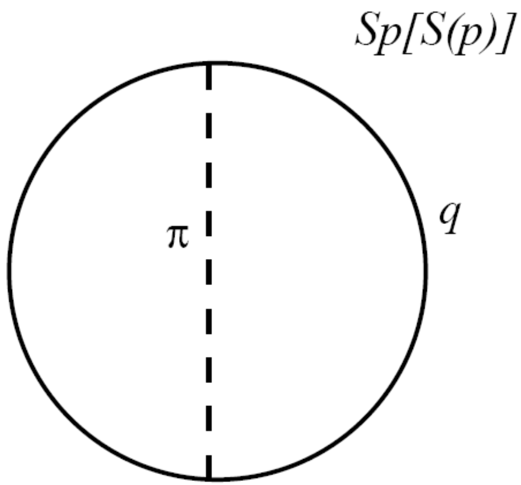
QCD vacuum properties

Quark condensate with meson-loop corrections

$$\langle \bar{q}q \rangle = \frac{1}{2}\frac{\partial \Gamma_{eff}}{\partial m} = -\frac{1}{2}\mathrm{Tr}\left(\frac{i}{\hat{p}+i\mu(p)} - \frac{i}{\hat{p}+im}\right) + \frac{1}{2}\int \frac{d^4q}{(2\pi)^4} V^{\bar{q}q}(q)\tilde{\Pi}_i(q)$$



With meson-loop corrections



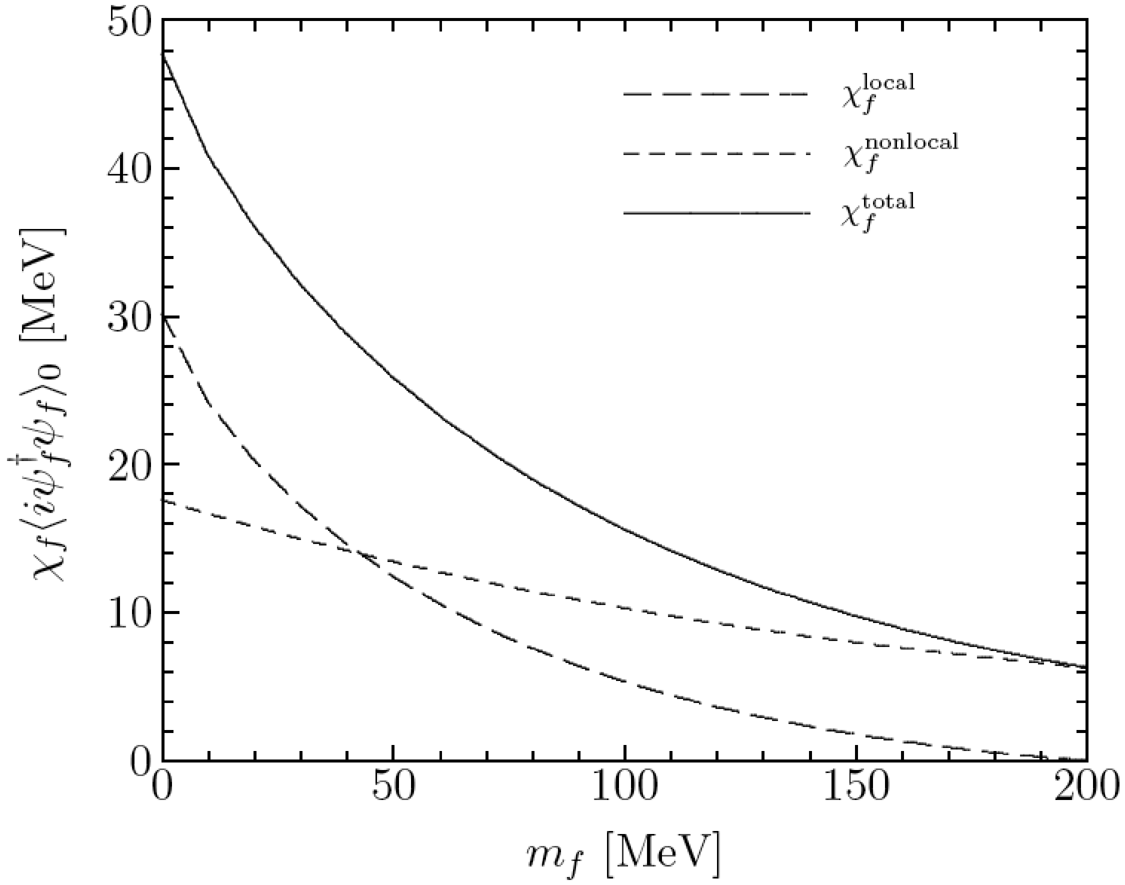
HChK, M.Musakhanov, M.Siddikov, PLB 633, 701 (2006)

QCD vacuum properties

Magnetic susceptibility of QCD vacuum

Magnetic response of the QCD vacuum to the external EM field

Related to the normalization of photon light-cone W.F.

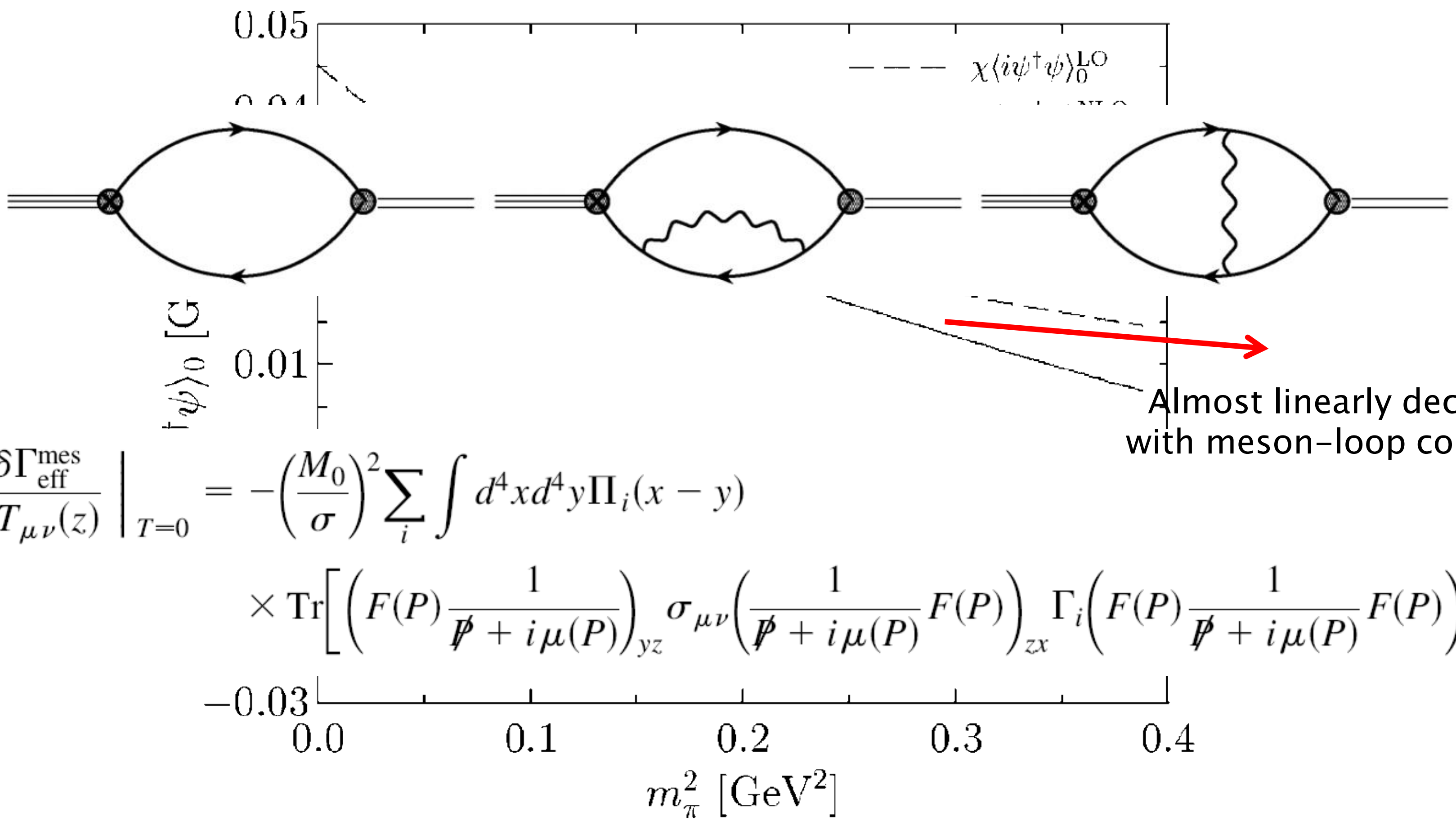


HChK, M.Musakhanov, M.Siddikov, PLB 608, 95 (2005)

QCD vacuum properties

Magnetic susceptibility of QCD vacuum

with meson-loop corrections



$$\frac{\delta\Gamma^{\mathrm{mes}}_{\mathrm{eff}}}{\delta T_{\mu\nu}(z)}\bigg|_{T=0} = -\left(\frac{M_0}{\sigma}\right)^2 \sum_i \int d^4x d^4y \Pi_i(x-y)$$

$$\times \mathrm{Tr}\left[\left(F(P)\frac{1}{\not P + i\mu(P)}\right)_{yz}\sigma_{\mu\nu}\left(\frac{1}{\not P + i\mu(P)}F(P)\right)_{zx}\Gamma_i\left(F(P)\frac{1}{\not P + i\mu(P)}F(P)\right)_{xy}\Gamma_i\right],$$

Goeke, Siddikov, Musakhanov, HChK, PRD, 76, 116007 (2007)

Mesonic properties

Meson electromagnetic form factors

Important physical quantity for understanding meson structure

$$(p_i + p_f)_\mu F_{\mathcal{M}}(q^2) = i \int d^4x \, e^{-iq\cdot x} \langle \mathcal{M}_f | j_\mu^{\rm EM}(x) | \mathcal{M}_i \rangle$$

EM current for the pseudoscalar meson

$$j_\mu^{\rm EM}(x) = \sum_{\rm flavor} e_f \, q_f^\dagger(x) \gamma_\mu q_f(x)$$

Charge radius for the pseudoscalar meson

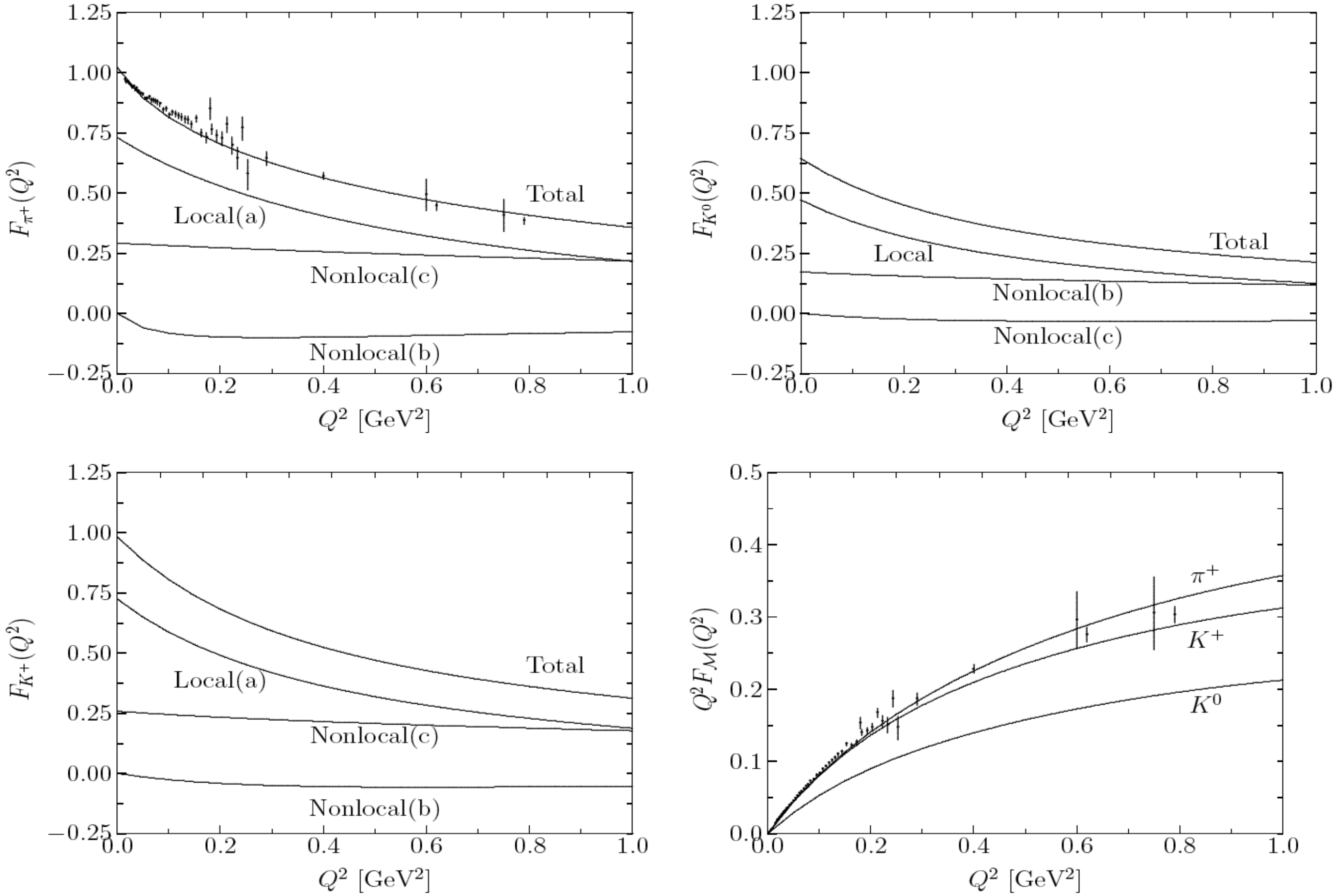
$$\langle r^2 \rangle_{\rm EM} = -6 \left[\frac{\partial \, F_{\mathcal{M}}(Q^2)}{\partial \, Q^2} \right]_{Q^2=0}$$

SiNam and HChK, PRD 77, 094014 (2008)

Mesonic properties

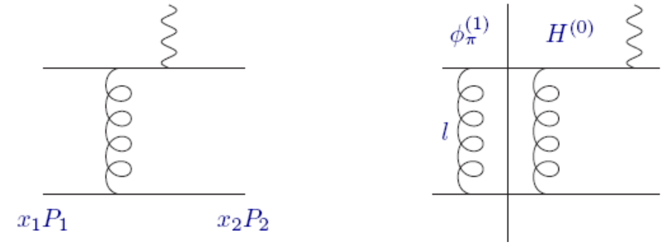
Meson electromagnetic form factors

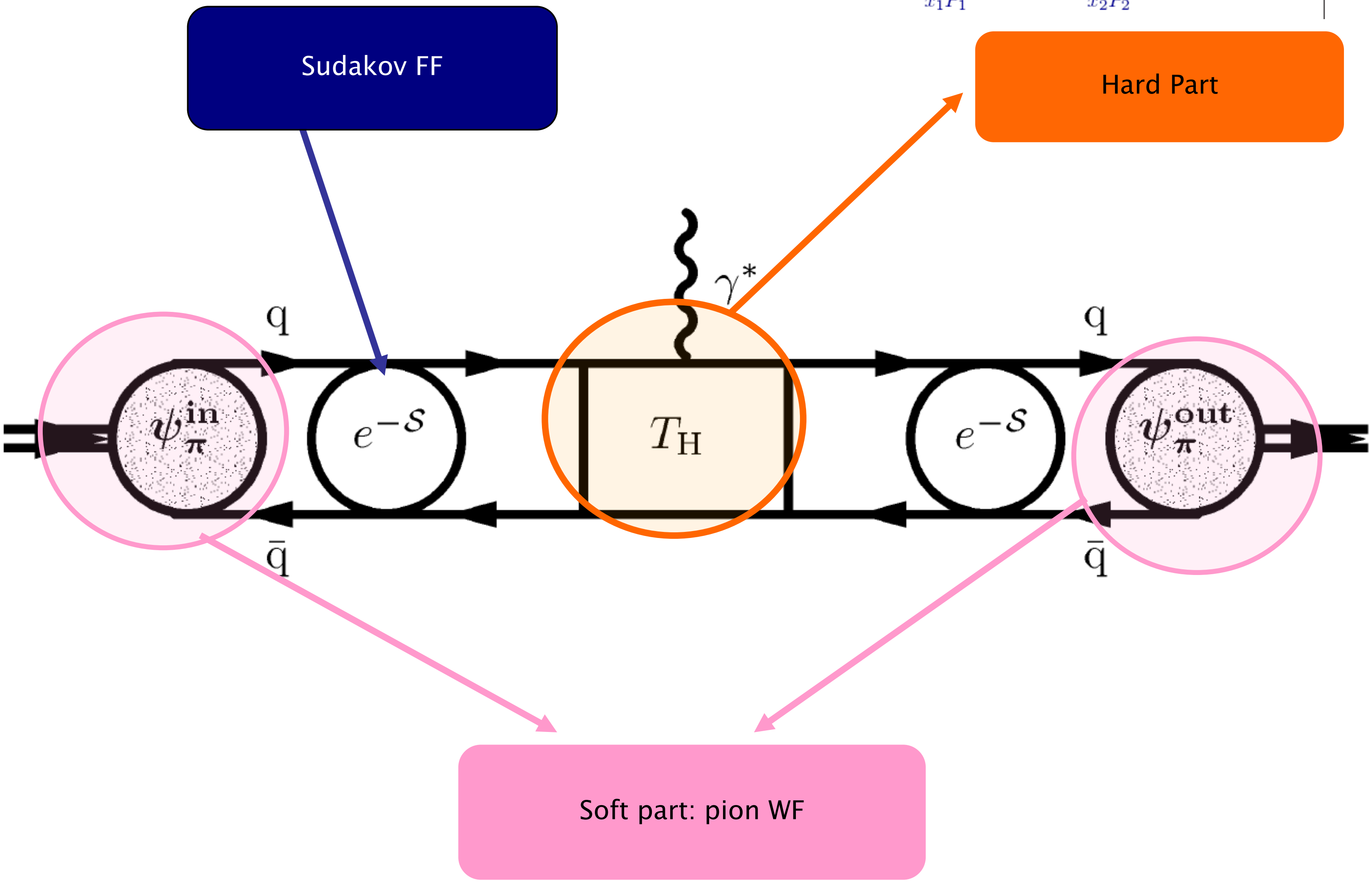
Numerical results for π^+, K^+ and K^0



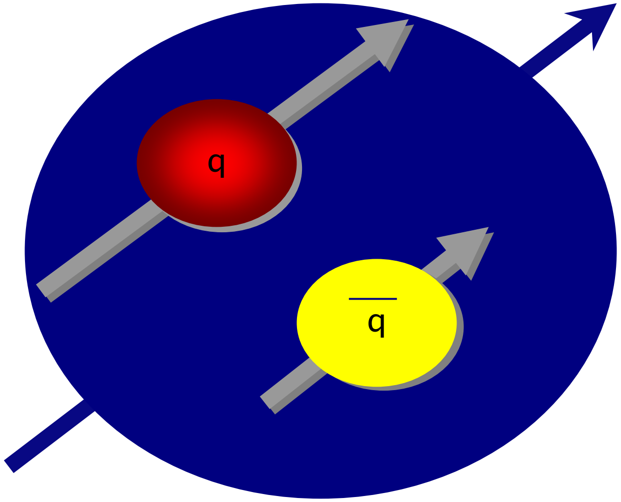
SiNam and HChK, PRD 77, 094014 (2008)

Pion EM form factor





Mesonic properties



How the quark and antiquark carry the momentum fraction of the mesons in $|q\bar{q}\rangle$

Leading-twist meson DAs

$$\Phi_\phi(u) = \frac{1}{i\sqrt{2}F_\phi}\int_{-\infty}^{\infty}\frac{dz}{2\pi}e^{-i(2u-1)z\cdot P}\langle 0|\bar{q}_f(z)\not{n}\gamma_5 \exp\left[ig\int_{-z}^{z}dz'^{\mu}A_\mu(z')\right]q_g(-z)|\phi(P)\rangle$$

Mesonic properties

Leading-twist meson DAs

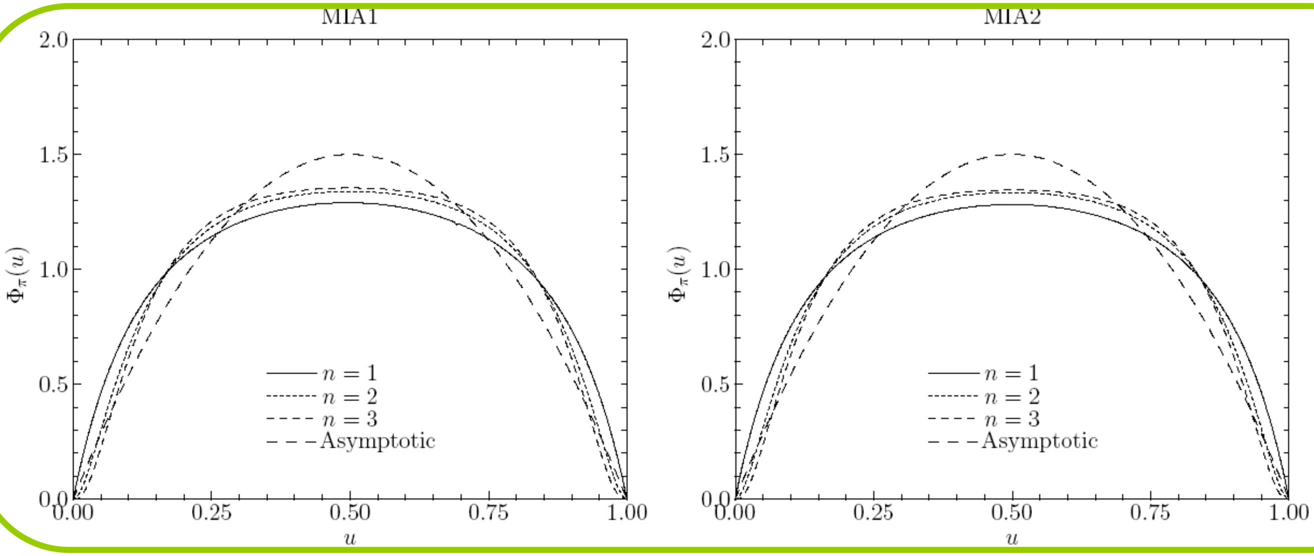
$$\Phi_\phi(u) = \frac{1}{i\sqrt{2}F_\phi}\int_{-\infty}^{\infty}\frac{dz}{2\pi}e^{-i(2u-1)z\cdot P}\langle 0|\bar{q}_f(z)\not{\hat{n}}\gamma_5 \exp\left[ig\int_{-z}^{z}dz'^\mu A_\mu(z')\right]q_g(-z)|\phi(P)\rangle$$

Two-particle twist-three meson DAs

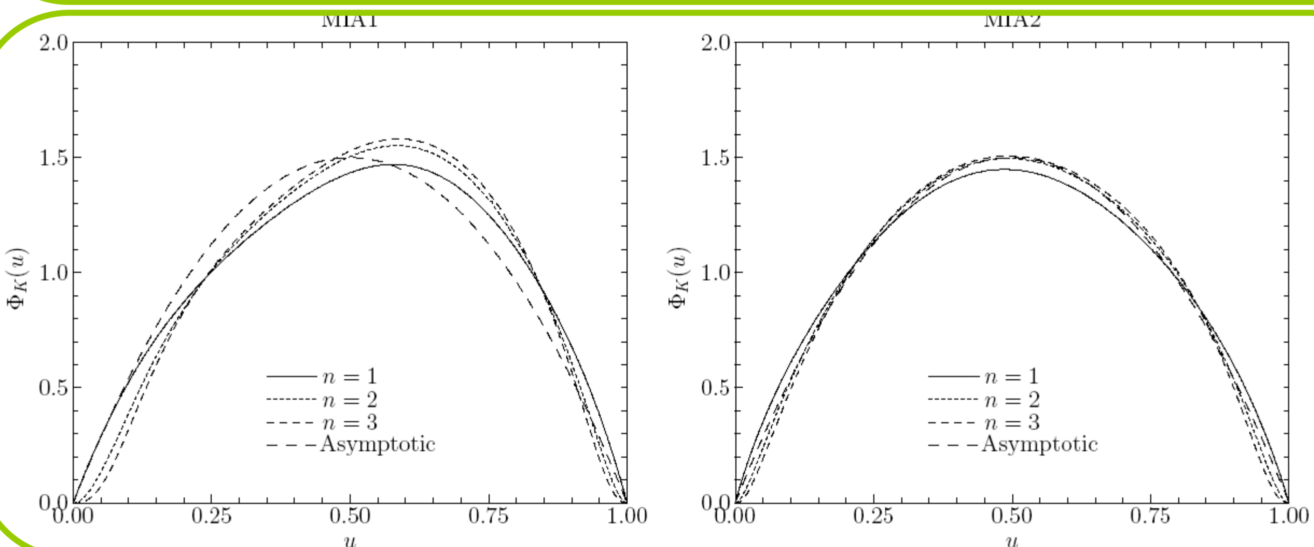
$$\phi^p_{\mathcal{M}}(u) = \frac{\sqrt{2}(P\cdot\hat{n})(m_f+m_g)}{m^2_{\mathcal{M}}F_{\mathcal{M}}}\int_{-\infty}^{\infty}\frac{d\tau}{\pi}\,e^{-i(2u-1)\tau P\cdot\hat{n}}\langle 0|\bar{\psi}_f(\tau\hat{n})i\gamma_5\psi_g(-\tau\hat{n})|\mathcal{M}(P)\rangle,$$

$$\phi^\sigma_{\mathcal{M}}(u) = -\frac{6\sqrt{2}(m_f+m_g)}{m^2_{\mathcal{M}}F_{\mathcal{M}}}\int_{-\infty}^{\infty}\frac{d\tau}{\pi}\int_0^u dv\,e^{-i(2v-1)\tau P\cdot\hat{n}} \times\langle 0|\bar{\psi}_f(\tau\hat{n})i(\not{P}\not{\hat{n}}-P\cdot\hat{n})\gamma_5\psi_g(-\tau\hat{n})|\mathcal{M}(P)\rangle,$$

Mesonic properties



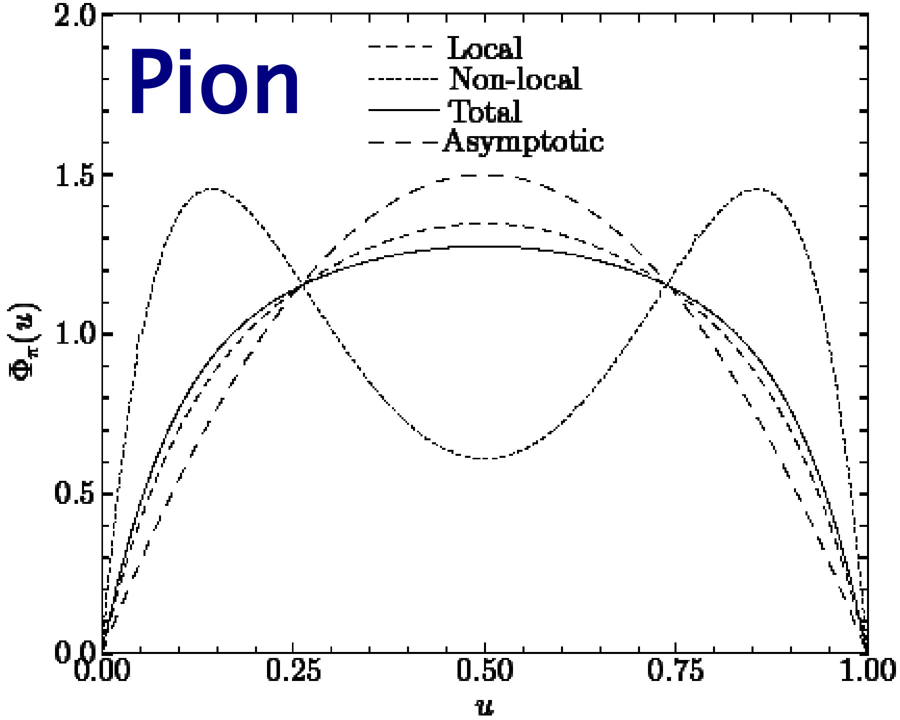
Pion DA

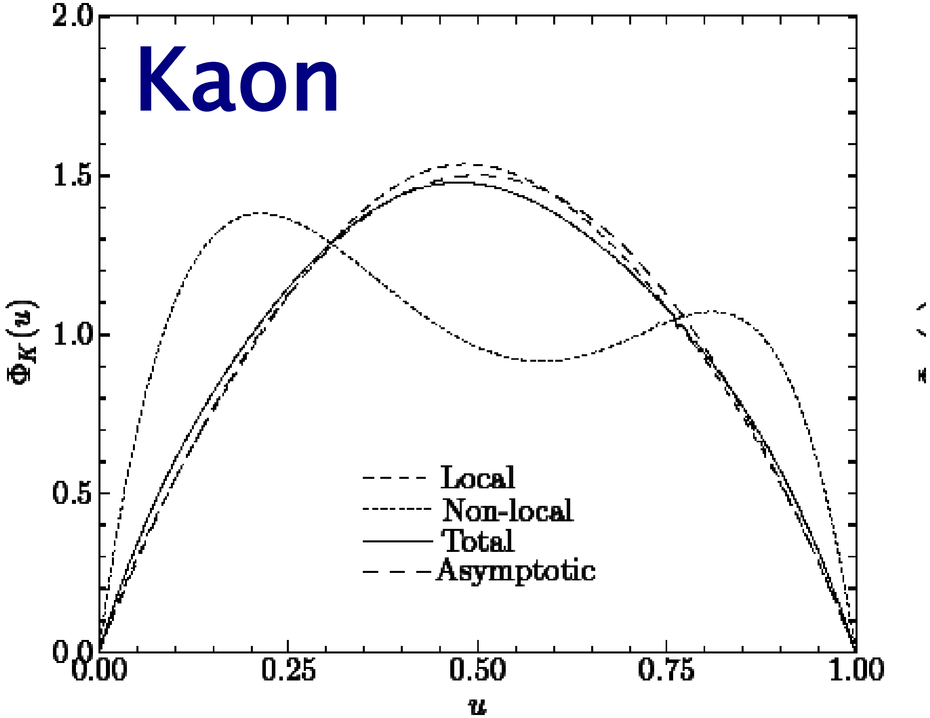


Kaon DA

SiNam, HChK, A.Hosaka, M.Musakhanov,
Phys.Rev.D74,014019 (2006)

Mesonic properties





SiNam and H.-Ch.Kim, Phys.Rev.D74, 076005 (2006)

Mesonic properties

Schmedding and Yakovlev scale: at 2.4 GeV

CLEO Experiment: PRD 57, 33 (1998)

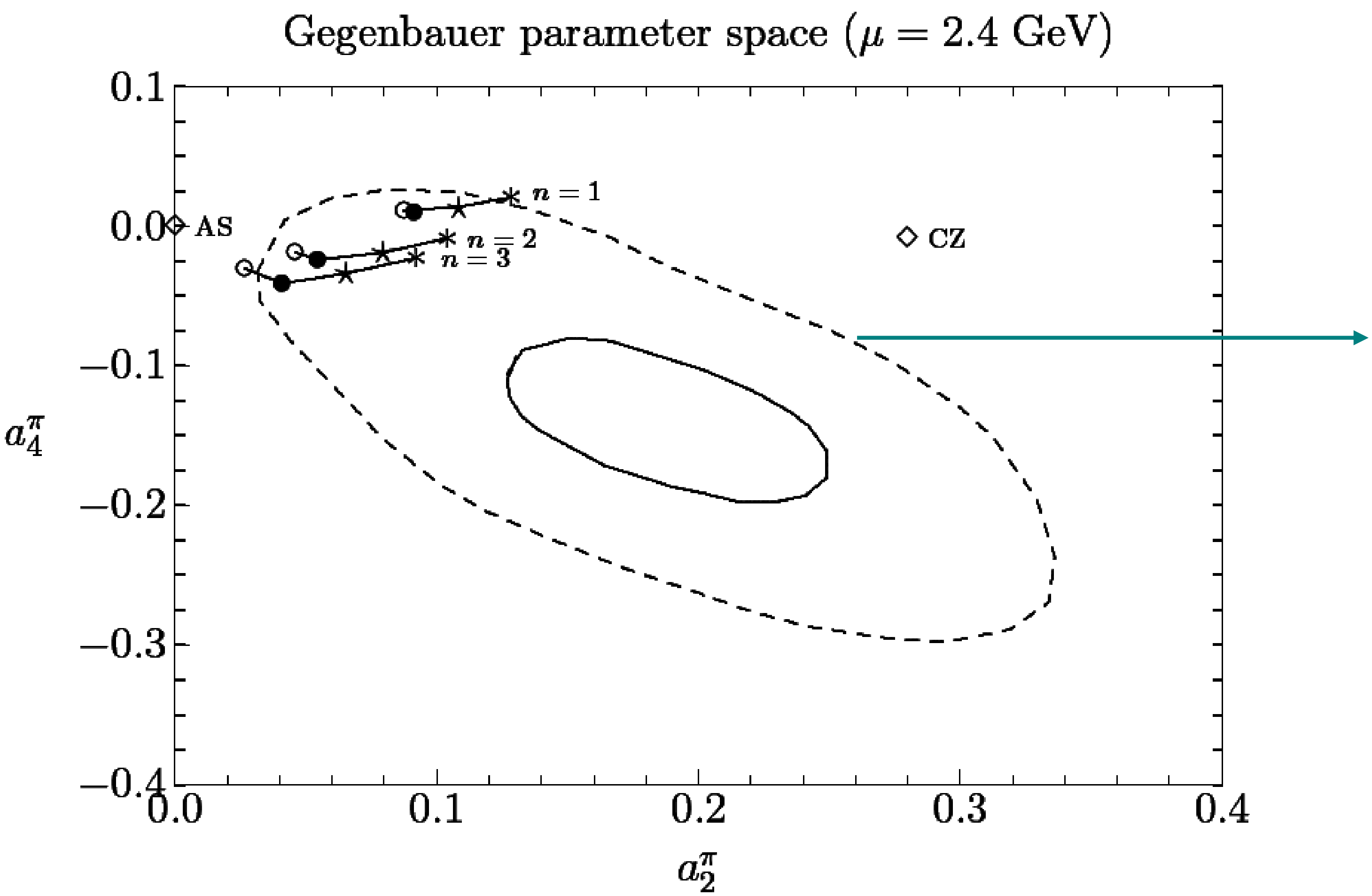
1-loop QCD evolution equation

$$a_m(\Lambda_1) = a_m(\Lambda_2)\left[\frac{\alpha(\Lambda_1)}{\alpha(\Lambda_2)}\right]^{\gamma_m^{(0)}/(2\beta_0)} \sim a_m(\Lambda_2)\left[\frac{\ln[\Lambda_2/\Lambda_{\rm QCD}]}{\ln[\Lambda_1/\Lambda_{\rm QCD}]}\right]^{\gamma_m^{(0)}/(2\beta_0)}$$

$$\gamma_m^{(0)} = -\frac{8}{3}\left[3 + \frac{2}{(m+1)(m+2)} - 4\sum_{m'=1}^{m+1}\frac{1}{m'}\right]$$

SiNam and H.-Ch.Kim, Phys.Rev.D74, 076005 (2006)

Mesonic properties



2s ellipsis

SiNam and H.-Ch.Kim, Phys.Rev.D74, 076005 (2006)

Mesonic properties

Kaon Semileptonic decay (K_{l3})

Decay process in terms of electroweak interaction

Flavor changing process for SU(3) symmetry breaking (s→d,u)

Useful in testing validity of models concerning low energy theorems

$$
\begin{aligned}
K^+(p_K) &\to \pi^0(p_\pi)\, l^+(p_l)\, \nu_l(p_\nu) \,:\, K^+_{l3}, \\
K^0(p_K) &\to \pi^-(p_\pi)\, l^+(p_l)\, \nu_l(p_\nu) \,:\, K^0_{l3},
\end{aligned}
$$

Decay amplitude defined by

$$T_{K\to l\nu\pi} = \frac{G_F}{\sqrt{2}} \sin\theta_c \left[w^\mu(p_l, p_\nu) F\mu(p_K, p_\pi)\right]$$

Cabbibo-Maskawa-Kobayashi (CMK) matrix element ~ $\sin\theta_c$

SiNam and HChK, PRD 75, 094011 (2007)

Mesonic properties

Vector and scalar form factors for decay (K_{l3})

Weak current and Flavor changing matrix element

$$
\begin{aligned}
w^\mu(p_l,p_\nu) &= \bar{u}(p_\nu)\gamma^\mu(1-\gamma_5)v(p_l),\\
F_\mu(p_K,p_\pi) &= c\langle K(p_K)|j^{su}_\mu|\pi(p_\pi)\rangle = c\langle K(p_K)|\bar{\psi}\gamma_\mu\lambda^{4-i5}\psi|\pi(p_\pi)\rangle\\
&= (p_K+p_\pi)_\mu f_{l+}(t) + (p_K-p_\pi)_\mu f_{l-}(t),
\end{aligned}
$$

where t indicates the momentum transfer by W-boson

Decay form factor in terms of scalar and vector form factors

$$
F_\mu(p_K,p_\pi) = f_{l+}(t)(p_K+p_\pi)_\mu - \frac{(m_\pi^2-m_K^2)(p_K-p_\pi)_\mu}{(p_K-p_\pi)^2}\left[f_{l+}(t)-f_{l0}(t)\right]
$$

where the scalar form factor is defined by

$$
f_{l0}(t) = f_{l+}(t) + \left[\frac{t}{m_K^2-m_\pi^2}\right] f_{l-}(t)
$$

SiNam and HChK, PRD 75, 094011 (2007)

Mesonic properties

Low energy constants related to K_{l3}

i	$L_i^r(M_\rho) \times 10^3$	source	Γ_i
1	0.4 ± 0.3	$K_{e4}, \pi\pi \to \pi\pi$	$3/32$
2	1.35 ± 0.3	$K_{e4}, \pi\pi \to \pi\pi$	$3/16$
3	-3.5 ± 1.1	$K_{e4}, \pi\pi \to \pi\pi$	0
4	-0.3 ± 0.5	Zweig rule	$1/8$
5	1.4 ± 0.5	$F_K : F_\pi$	$3/8$
6	-0.2 ± 0.3	Zweig rule	$11/144$
7	-0.4 ± 0.2	Gell-Mann–Okubo,L_5, L_8	0
8	0.9 ± 0.3	$M_{K^0} - M_{K^+}, L_5,$ $(2m_s - m_u - m_d) : (m_d - m_u)$	$5/48$
9	6.9 ± 0.7	$\langle r^2 \rangle_V^\pi$	$1/4$
10	-5.5 ± 0.7	$\pi \to e\nu\gamma$	$-1/4$
11			$-1/8$
12			$5/24$

SiNam and HChK, PRD 75, 094011 (2007)

Mesonic properties

Low energy constants related to K_{l3}

Low energy constant L_5 in the large N_c limit

$$L_9 = \frac{F_\pi^2 \langle r^2 \rangle^{K\pi}}{12}$$

Low energy constant L_9 in the large N_c limit ~ Callam-Treiman Theorem

$$\frac{F_K}{F_\pi} = 1 + \frac{4}{F_\pi^2}\left(m_K^2 - m_\pi^2\right) L_5$$

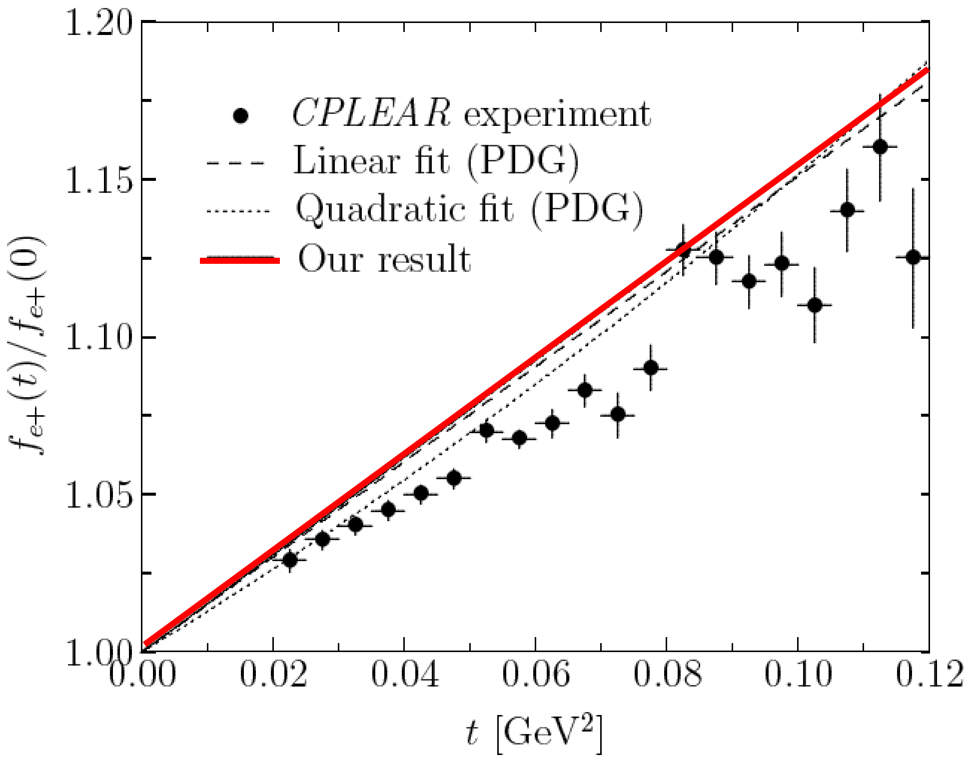
$L_5 = 1.6\sim2.0\times10^{-3}$ (exp)

$L_9 = 6.7\sim7.4\times10^{-3}$ (exp)

SiNam and HChK, PRD 75, 094011 (2007)

Mesonic properties

Ratio of $f_+(t)/f_+(0)$ for K_{l3}



Almost linear

Well consistent with data

λ_+ = 0.0303 (0.0296 by PDG)

SiNam and HChK, PRD 75, 094011 (2007)

Mesonic properties

Test of the low energy theorems

Ademollo-Gatto theorem

$$f_{e0}(m_K^2 - m_\pi^2) = \frac{F_K}{F_\pi}$$ → F_K/F_π =1.08 (1.22 exp)

$$\frac{F_K}{F_\pi} = 1 + \frac{4}{F_\pi^2}\left(m_K^2 - m_\pi^2\right) L_5$$ → $L_5 = 7.67\times10^{-4}$ (1.45×10^{-3}) !!!

$$L_9 = \frac{F_\pi^2 \langle r^2 \rangle^{K\pi}}{12}$$ → $L_9 = 6.78\times10^{-3}$ ($6.9 \sim 7.4\times10^{-3}$)

SiNam and HChK, PRD 75, 094011 (2007)

Instanton at finite density (μ)

❖ Considerable successes in the application of the instanton vacuum

❖ Extension of the instanton model to a system at finite ρ and T

❖ QCD phase structure: nontrivial QCD vacuum: instanton at finite density

· Simple extension to the quark matter:

· Breakdown of the Lorentz invariance at finite temperature

⟶ Future work!

· Instanton at finite temperature: Caloron with Polyakov line

· Very phenomenological approach such as the pNJL

· Using periodic conditions (Matsubara sum)

Focus on the basic QCD properties at finite quark-chemical potential, μ

Instanton at finite density (μ)

Modified Dirac equation with μ

$$[i\partial\!\!\!/ - i\mu\!\!\!/ + A\!\!\!/_{I\bar{I}}]\,\Psi^{(n)}_{I\bar{I}} = \lambda_n \Psi^{(n)}_{I\bar{I}} \qquad \mu_\mu \;=\; (0,0,0,\mu_q)$$

Instanton solution (singular gauge)

$$A^\alpha_\mu(x) = \frac{2\bar{\eta}^{\alpha\nu}_\mu\,\bar{\rho}^2 x_\nu}{x^2(x^2+\bar{\rho}^2)}$$

Assumption:

No modification from μ

Zero-mode equation

$$[i\partial\!\!\!/ - i\mu\!\!\!/ + A\!\!\!/_{I\bar{I}}]\,\Psi^{(0)}_{I\bar{I}} = 0.$$

$$S_{I\bar{I}} = \frac{1}{i\partial\!\!\!/ - i\mu\!\!\!/ + A\!\!\!/_{I\bar{I}}} \approx S_0 - \frac{\Psi^{(0)\dagger}_{I\bar{I}}\Psi^{(0)}_{I\bar{I}}}{im_q}$$

$$S_0 = (i\partial\!\!\!/ - i\mu\!\!\!/)^{-1}$$

Instanton at finite density (μ)

Quark propagator with the Fourier transformed zero-mode solution,

$$S = \frac{1}{i\not{\partial} - i\not{\mu} + iM(i\partial, \mu)} \qquad M(p,\mu) = M_0(p+i\mu)^2\psi^2(p,\mu)$$

$$\psi_4(p,\mu) = \frac{\bar{\rho}^2}{8|\vec{p}|}\Big\{(|\vec{p}| - \mu_q - ip_4)\left[(2p_4 + i\mu_q)F^a_-(p,\mu) + i(|\vec{p}| - \mu_q - ip_4)F^b_-(p,\mu)\right]$$
$$+(|\vec{p}| + \mu_q + ip_4)\left[(2p_4 + i\mu_q)F^a_+(p,\mu) - i(|\vec{p}| + \mu_q + ip_4)F^b_+(p,\mu)\right]\Big\},$$
$$\vec{\psi}(p,\mu) = \frac{\bar{\rho}^2\hat{p}}{8|\vec{p}|}\Big\{(2|\vec{p}| - \mu_q)(|\vec{p}| - \mu_q - ip_4)F^a_-(p,\mu) + (2|\vec{p}| + \mu_q)(|\vec{p}| + \mu_q + ip_4)F^a_+(p,\mu)$$
$$+\left[2(|\vec{p}| - \mu_q)(|\vec{p}| - \mu_q - ip_4) - \frac{1}{|\vec{p}|}(\mu_q + ip_4)\left[p_4^2 + (|\vec{p}| - \mu_q)^2\right]\right]F^b_-(p,\mu)$$
$$+\left[2(|\vec{p}| + \mu_q)(|\vec{p}| + \mu + ip_4) + \frac{1}{|\vec{p}|}(\mu_q + ip_4)\left[p_4^2 + (|\vec{p}| + \mu_q)^2\right]\right]F^b_+(p,\mu)\Big\}, \qquad (23)$$

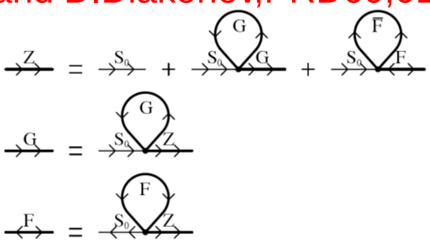
$$F^a_\pm = \frac{I_1(z_\pm)K_0(z_\pm) - I_0(z_\pm)K_1(z_\pm)}{z_\pm}, \quad F^b_\pm = \frac{I_1(z_\pm)K_1(z_\pm)}{z_\pm^2}$$
$$z_\pm = \frac{\bar{\rho}}{2}\sqrt{p_4^2 + (|\boldsymbol{p}| \pm \mu)^2}.$$

Instanton at finite density (μ)

Schwinger-Dyson-Gorkov equation (N_f=2, N_c=3)

G.W.Carter and D.Diakonov,PRD60,016004 (1999)

$$\begin{aligned} Z(p) &= 1 - G(p)A(p,\mu)M_0, \\ G(p) &= Z(p)\psi^2(p)M_0, \\ F(p) &= 2Z(-p)\psi_\mu(p,\mu)\psi^\mu(-p,\mu)\Delta \end{aligned}$$



$$\begin{aligned} A(p,\mu) &= (p+i\mu)^2\psi^2(p,\mu), \\ B(p,\mu) &= (p^2+\mu^2)\psi_\mu(p,\mu)\psi^\mu(-p,\mu) + (p+i\mu)_\mu\psi^\mu(p,\mu)(p-i\mu)_\nu\psi^\nu(-p,\mu) \\ &- (p+i\mu)_\mu\psi^\mu(-p,\mu)(p-i\mu)_\nu\psi^\nu(p,\mu). \end{aligned}$$

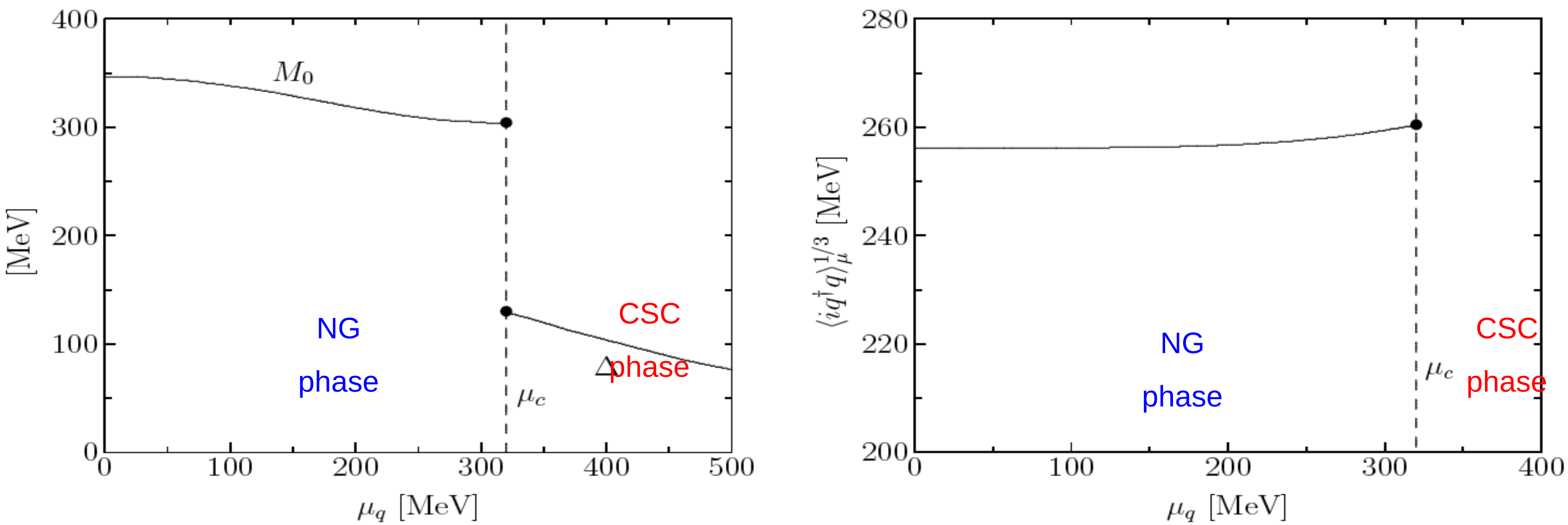
$$g(\mu) = \frac{\lambda M_0}{N_c^2-1}\int\frac{d^4p}{(2\pi)^4}\frac{\alpha(p,\mu)}{1+\alpha(p,\mu)M_0^2}, \quad f(\mu) = \frac{2\lambda\Delta}{N_c^2-1}\int\frac{d^4p}{(2\pi)^4}\frac{\beta(p,\mu)}{1+4\beta(p,\mu)\Delta^2},$$

$$\alpha(p,\mu) = A(p,\mu)\psi^2(p,\mu), \quad \beta(p,\mu) = B(p,\mu)\psi_\mu(p,\mu)\psi^\mu(-p,\mu)$$

$$M_0 = \left(2N_c - \frac{2}{N_c}\right)g(\mu), \quad \Delta = \left(1+\frac{1}{N_c}\right)f(\mu) \quad \left.\frac{f(\mu)}{g(\mu)}\right|_{\mu=\mu_c} = \left[\frac{N_c(N_c-1)}{2}\right]^{\frac{1}{2}}$$

M_0, Δ, and $<iq^+q>$ (N_f=2)

1st order phase transition occurred at μ ~ 320 MeV



Metastable state (mixing of σ and Δ) ignored for simplicity here

chiral condensate with μ

$$\langle iq^\dagger q\rangle_\mu = 4N_c \int \frac{d^4k}{(2\pi)^4} \left[\frac{M(k,\mu)}{(k+i\mu)^2 + M^2(k,\mu)} \right]$$

Magnetic susceptibility at finite μ

QCD magnetic susceptibility with externally induced EM field

$$\langle iq^\dagger\sigma_{\mu\nu}q\rangle_F = ie_q F_{\mu\nu}\langle iq^\dagger q\rangle_0 \chi$$

Only for the NG phase!!

Magnetic phase transition of the QCD vacuum

Effective chiral action with tensor source field *T*

$$\mathcal{S}_{\text{eff}}[T,\mu] = -\text{Sp}_{cf\gamma}\ln\left[i\not{D} - i\not{\mu} + iM(iD,\mu) + \sigma\cdot T\right]$$

Evaluation of the matrix element

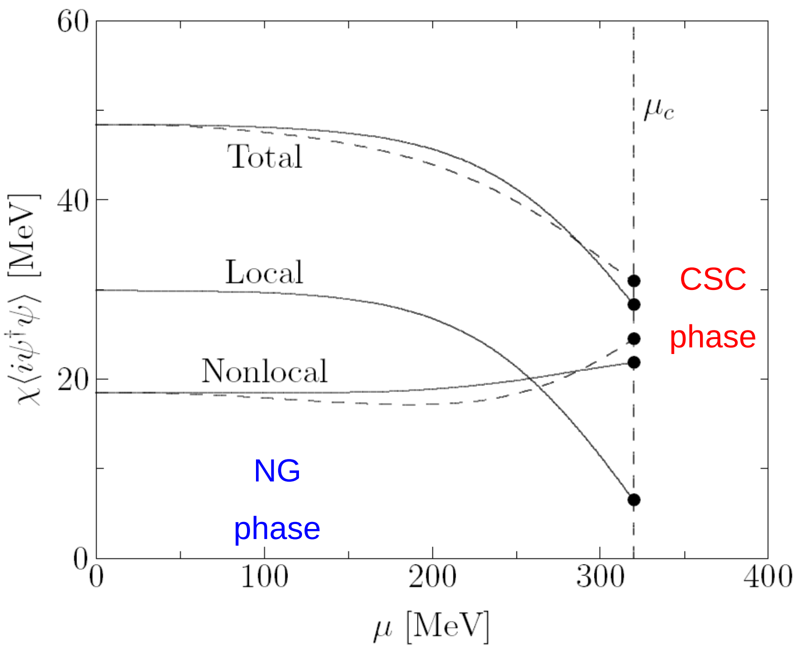
$$\frac{\delta}{\delta T_{\mu\nu}}\mathcal{S}_{\text{eff}}[T,\mu]\Big|_{T=0} = \langle 0|\psi^\dagger\sigma_{\mu\nu}\psi|0\rangle_F = \text{Tr}_{cf\gamma}[S\sigma_{\mu\nu}$$

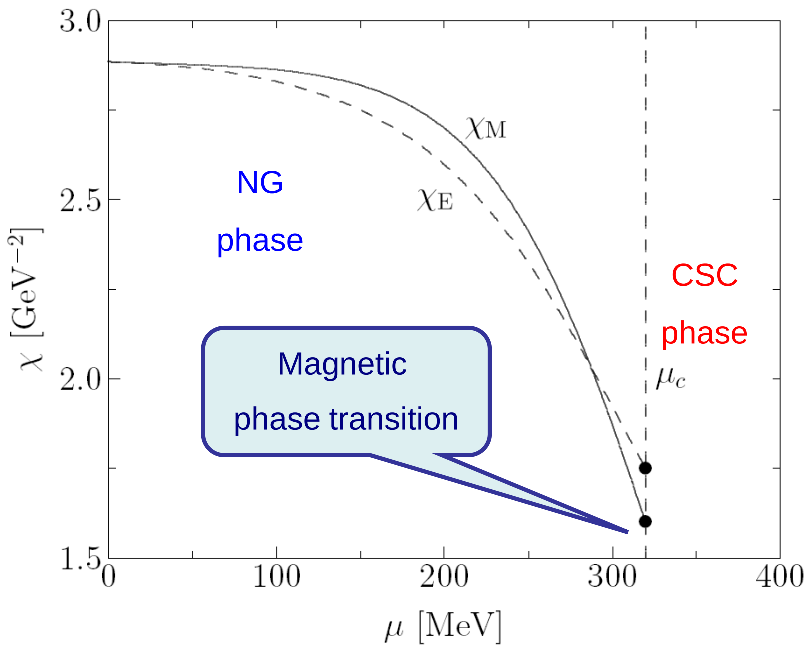
SiNam. HYRyu, MMusakhanov and HChK, arXiv:0804.0056 [hep-ph]

Magnetic susceptibility at finite μ

$$\chi_{\mathrm{M}}\langle iq^\dagger q\rangle = 4N_c \int \frac{d^4p}{(2\pi)^4}\left[\frac{M(\bar{p}) - \bar{p}^2\tilde{M}'(\bar{p})}{[\bar{p}^2 + M^2(\bar{p})]^2}\right]$$

$$\chi_{\mathrm{E}}\langle i\psi^\dagger\psi\rangle = \chi_{\mathrm{M}}\langle i\psi^\dagger\psi\rangle + \underbrace{4N_c \int \frac{d^4p}{(2\pi)^4}\left[\frac{\mu^2\tilde{M}'(\bar{p})}{[\bar{p}^2 + M^2(\bar{p})]^2}\right]}_{\text{Breakdown of Lorentz invariance}}$$





1st order magnetic phase transition at μ_c

Pion weak decay constant at finite μ

Pion-to-vacuum transition matrix element

$$\langle 0|A^a_\mu(x)|\pi^b(P)\rangle = i\sqrt{2}F_\pi\delta^{ab}P_\mu e^{-iP\cdot x}$$

Separation for the time and space components

$$\langle 0|\boldsymbol{A}^a(x)|\pi^b(P)\rangle = i\sqrt{2}F^s_\pi\delta^{ab}\boldsymbol{P}e^{-iP\cdot x}, \quad \langle 0|A^a_4(x)|\pi^b(P)\rangle = i\sqrt{2}F^t_\pi\delta^{ab}P_4 e^{-iP\cdot x}$$

Effective chiral action with μ

$$\mathcal{S}_{\rm eff}[\pi,\mu] = -\mathrm{Sp}\ln\left[i\bar{\not\partial} + i\sqrt{M(i\bar{\partial})}U_5\sqrt{M(i\bar{\partial})}\right]$$

Renormalized auxiliary pion field corresponding to the physical one

$$\pi^a_{\rm phy} = \frac{1}{C_r}\pi^a$$

SiNam and HChK, PLB 666, 324 (2008)

Pion weak decay constant at finite μ

Effective chiral action with axial-vector source

$$\mathcal{S}_{\rm eff}[\pi,\mu,J^a_{5\mu}] \;=\; -{\rm Sp}\ln\left[i\bar{\not\partial}+\gamma_5\gamma^\mu\frac{\tau^a}{2}J^a_{5\mu}+\sqrt{M(i\bar{\partial},J^a_{5\mu})}U_5\sqrt{M(i\bar{\partial},J^a_{5\mu})}\right]$$

Using the LSZ (Lehmann-Symanzik-Zimmermann) reduction formula

$$\begin{aligned} i\sqrt{2}\delta^{ab}F_\pi(q^2,\mu)q_\mu \;&=\; \mathcal{K}_\pi\int d^4x\langle 0|T\left[A^a_\mu(x)\pi^b_{\rm phy}(0)\right]|0\rangle\, e^{iq\cdot x} \\ &=\; \frac{\mathcal{K}_\pi}{C_r(\mu)}\int d^4x\langle 0|T\left[A^a_\mu(x)\pi^b(0)\right]|0\rangle\, e^{iq\cdot x}\end{aligned}$$

$$\langle 0|T[A^a_\mu(x)\pi^b(0)]|0\rangle = \frac{\delta^2\ln\mathcal{Z}_{\rm eff}[\pi,\mu,J^a_{5\mu}]}{\delta J^a_{5\mu}(x)\,\delta J^b_5(0)} = \int d^4z\frac{\delta^2\mathcal{S}_{\rm eff}[\pi,\mu,J^a_{5\mu}]}{\delta J^a_{5\mu}(x)\,\delta\pi^b(z)}\mathcal{K}^{-1}_\pi(z)$$

Pion weak decay constant at finite μ

Analytic expression for the pion weak decay constant

$$F_\pi(\mu)P_\mu = \frac{4N_c}{C_r F_0}\int\frac{d^4k}{(2\pi)^4}\Bigg[\underbrace{\frac{\sqrt{M(\bar{k})M(\bar{k}-P)}[(P_\mu-\bar{k}_\mu)M(\bar{k})+\bar{k}_\mu M(\bar{k}-P)]}{[\bar{k}^2+M^2(\bar{k})][(\bar{k}-P)^2+M^2(\bar{k}-P)]}}_{\text{local cont.}}$$

$$-\underbrace{\frac{M(\bar{k})\sqrt{M(\bar{k})}\sqrt{M(\bar{k}-P)}_\mu - M(\bar{k})\sqrt{M(\bar{k})}_\mu\sqrt{M(\bar{k}-P)}}{\bar{k}^2+M^2(\bar{k})}}_{\text{nonlocal cont.}}\Bigg],$$

$$\bar{k} = (\vec{k}, k_4 + i\mu)$$

Pion weak decay constant at finite μ

Local contributions

$$F^s_{\pi,\mathrm{L}}(\mu) \;=\; \frac{4N_c}{C_r F_0}\int \frac{d^4k}{(2\pi)^4}\frac{1}{(k^2+\mathcal{M}^2)^2}\left[\mathcal{M}^2 - \frac{1}{2}k^2\mathcal{M}\tilde{\mathcal{M}}' - 5\mu^2\, k_4^2\tilde{\mathcal{M}}'^2\right],$$

$$F^t_{\pi,\mathrm{L}}(\mu) \;=\; \frac{4N_c}{C_r F_0}\int \frac{d^4k}{(2\pi)^4}\frac{1}{(k^2+\mathcal{M}^2)^2}\left[\mathcal{M}^2 - \frac{1}{2}k^2\mathcal{M}\tilde{\mathcal{M}}' - \mu^2\, k_4^2\tilde{\mathcal{M}}'^2\right],$$

Nonlocal contributions

$$F^s_{\pi,\mathrm{NL}}(\mu) \;=\; -\frac{4N_c}{C_r F_0}\int \frac{d^4k}{(2\pi)^4}\frac{1}{k^2+\mathcal{M}^2}\left[\mathcal{M}\tilde{\mathcal{M}}' + \frac{1}{2}k^2\mathcal{M}\tilde{\mathcal{M}}'' - \frac{1}{2}k^2\tilde{\mathcal{M}}'^2 - 4\mu^2 k_4^2\tilde{\mathcal{M}}'\tilde{\mathcal{M}}''\right]$$

$$F^t_{\pi,\mathrm{NL}}(\mu) \;=\; F^s_{\pi,\mathrm{NL}}(\mu).$$

When the density switched off,

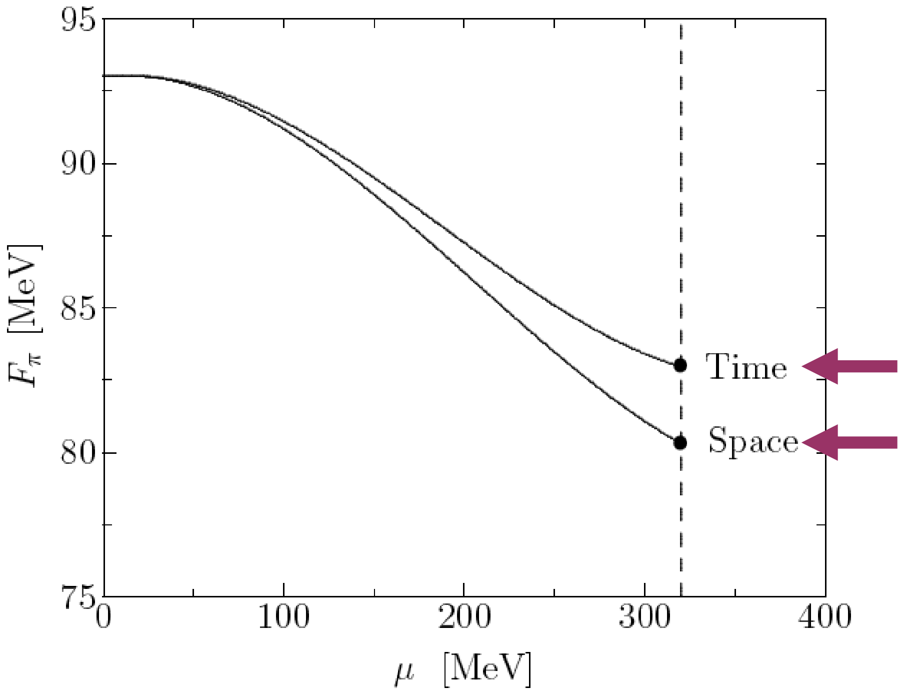
$$F_\pi(0) = \frac{4N_c}{C_r F_0}\int \frac{d^4k}{(2\pi)^4}\left[\frac{\mathcal{M}^2 - \frac{1}{2}k^2\mathcal{M}\tilde{\mathcal{M}}'}{(k^2+\mathcal{M}^2)^2} - \frac{\mathcal{M}\tilde{\mathcal{M}}' + \frac{1}{2}k^2\mathcal{M}\tilde{\mathcal{M}}'' - \frac{1}{2}k^2\tilde{\mathcal{M}}'^2}{k^2+\mathcal{M}^2}\right]$$

Pion weak decay constant at finite μ

Time component > Space component

$$F_\pi^s(\mu) \approx F_\pi^{\rm exp} + \mu^2 \left[\frac{N_c}{F_\pi^{\rm exp}} \int \frac{d^4k}{(2\pi)^4} \left(\frac{8k_4^2 \tilde{\mathcal{M}}' \tilde{\mathcal{M}}''}{k^2 + \mathcal{M}^2} - \frac{10 k_4^2 \tilde{\mathcal{M}}'^2}{[k^2 + \mathcal{M}^2]^2} \right) \right]$$

$$F_\pi^t(\mu) \approx F_\pi^s(\mu) + \mu^2 \left[\frac{N_c}{F_\pi^{\rm exp}} \int \frac{d^4k}{(2\pi)^4} \frac{8k_4^2 \tilde{\mathcal{M}}'^2}{[k^2 + \mathcal{M}^2]^2} \right],$$



Pion weak decay constant at finite μ

Comparison with the in-medium ChPT

K.Kirchbach and A.Wirzba, NPA616, 648 (1997)

$$F_\pi^s(\rho_0) = \left[1 + \frac{2c_3\rho_0}{(F_\pi^{\exp})^2}\right]\left[1 - \frac{\Sigma_{\pi N}\,\rho_0}{(F_\pi^{\exp})^2 m_\pi^2}\right]^{-1},$$

$$F_\pi^t(\rho_0) = \left[1 + \frac{2(c_2+c_3)\rho_0}{(F_\pi^{\exp})^2}\right]\left[1 - \frac{\Sigma_{\pi N}\,\rho_0}{(F_\pi^{\exp})^2 m_\pi^2}\right]^{-1}$$

$$c_3 < 0 \text{ and } c_2 > 0$$

$F^s / F^t < 0.5$

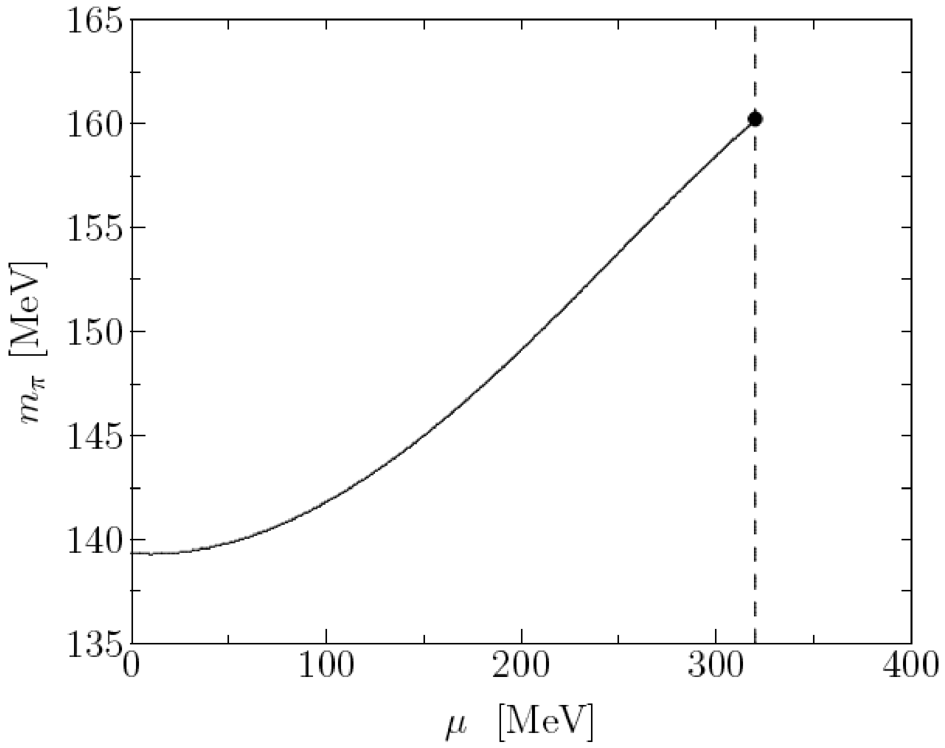
Critical *p*-wave contribution

Comparison with the QCD sum rules

m_π^* (MeV)	D_5 (GeV^2)	f_t/f_π	f_s/f_π
139	-0.019	0.79 (0.77)	0.78 (0.57)
159	-0.025	0.69 (0.63)	0.68 (0.37)

(H.c.Kim and M.Oka, NPA720, 386 (2003))

Pion weak decay constant at finite μ



GOR relation satisfied
within the present framework

$$(m_\pi^* F_\pi^t)^2 = 2m_q \langle i q^\dagger q \rangle^*$$

Changes of the pion properties with μ

	F_π^s	F_π^t	m_π
$\mu = 0$	93 MeV	93 MeV	139.33 MeV
$\mu = \mu_c \approx 320$ MeV	80.29 MeV	82.96 MeV	160.14 MeV
Modification	16% ↓	13% ↓	15% ↑

Pion EM form factor at finite μ

$$\langle \pi^+(p_f)|j_\mu^{\rm EM}(0)|\pi^+(p_i)\rangle = (p_f+p_i)_\mu F_\pi(Q^2)$$

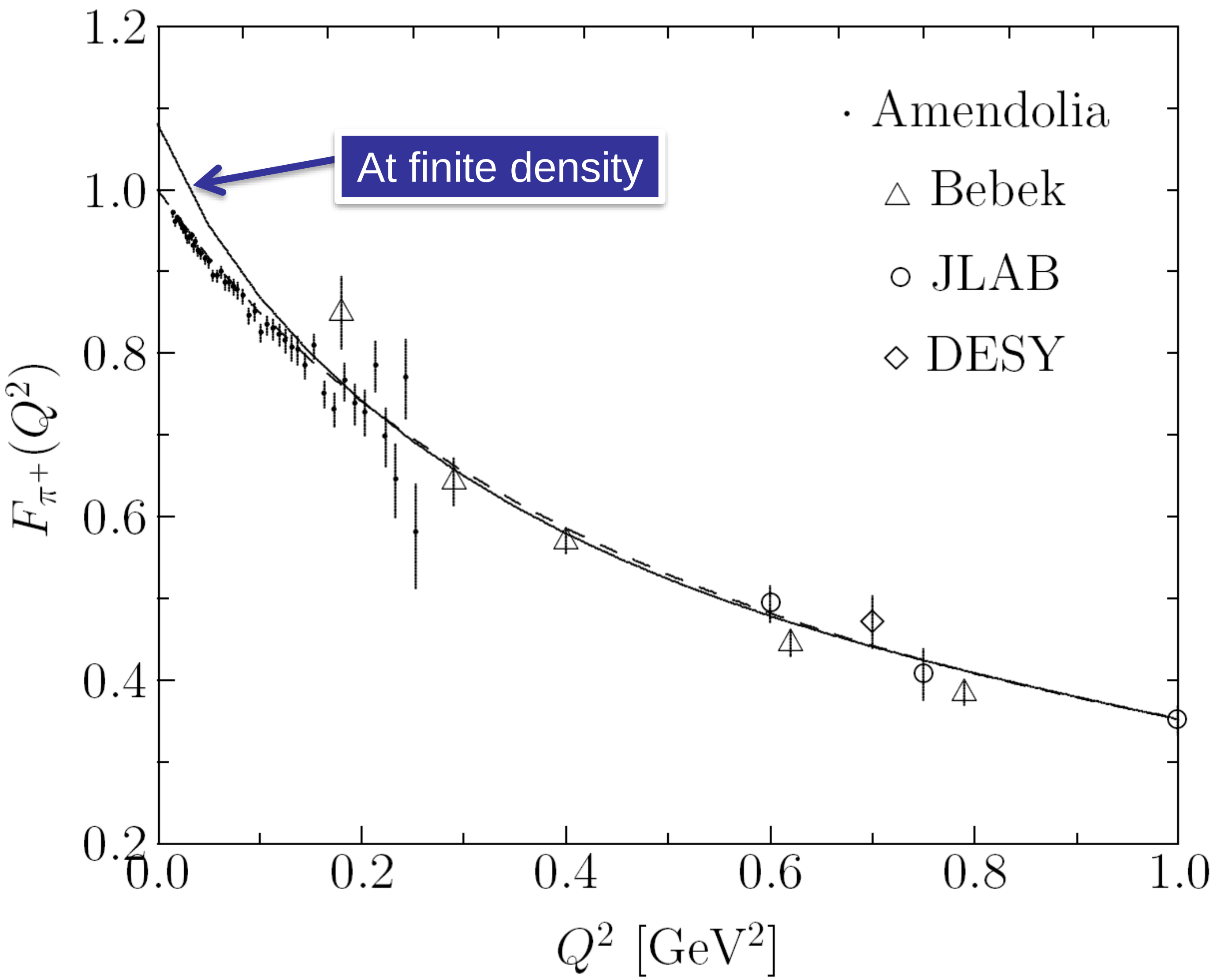
$$j_\mu^{\rm EM}(x) = iq^\dagger(x)\hat{Q}q(x) = i\frac{2}{3}u^\dagger(x)\gamma_\mu u(x) - i\frac{1}{3}d^\dagger(x)\gamma_\mu d(x)$$

$$F_\pi^{*\,\rm local} = \sum_{\rm flavor}\frac{8e_qN_c}{(p_i\cdot q+2m_\pi^2)}\int_{-\infty}^{\infty}\frac{d^4k}{(2\pi)^4}\left[\frac{\sqrt{\mathcal{M}_b\mathcal{M}_c}(\mathcal{M}_ck_{bd}+\mathcal{M}_bk_{cd})}{2(k_b^2+\mathcal{M}_b^2)(k_c^2+\mathcal{M}_c^2)}\right.$$
$$\left.+\frac{\mathcal{M}_a\sqrt{\mathcal{M}_b\mathcal{M}_c}(k_{ab}k_{cd}+k_{ac}k_{bd}-k_{bc}k_{ad}+\mathcal{M}_a\mathcal{M}_ck_{bd}+\mathcal{M}_a\mathcal{M}_bk_{cd}-\mathcal{M}_c\mathcal{M}_ck_{ad})}{(k_a^2+\mathcal{M}_a^2)(k_b^2+\mathcal{M}_b^2)(k_c^2+\mathcal{M}_c^2)}\right],$$

$$F_\pi^{*\,\rm nonlocal} = \sum_{\rm flavor}\frac{8e_qN_c}{(2p_i\cdot q+M_\pi^2)}\int_{-\infty}^{\infty}\frac{d^4k}{(2\pi)^4}\left[\frac{\sqrt{\mathcal{M}_b\mathcal{M}_c}(\sqrt{\mathcal{M}_c}\hat{\mathcal{M}}_{bd}-\sqrt{\mathcal{M}_b}\hat{\mathcal{M}}_{cd})(k_{bc}-\mathcal{M}_b\mathcal{M}_c)}{(k_b^2+\mathcal{M}_b^2)(k_c^2+\mathcal{M}_c^2)}\right.$$
$$+\frac{\mathcal{M}_a\sqrt{\mathcal{M}_b\mathcal{M}_c}(\sqrt{\mathcal{M}_b}\hat{\mathcal{M}}_{cd}-\sqrt{\mathcal{M}_c}\hat{\mathcal{M}}_{bd})(\mathcal{M}_ck_{ab}+\mathcal{M}_bk_{ac}-\mathcal{M}_ak_{bc}+\mathcal{M}_a\mathcal{M}_b\mathcal{M}_c)}{(k_a^2+\mathcal{M}_a^2)(k_b^2+\mathcal{M}_b^2)(k_c^2+\mathcal{M}_c^2)}$$
$$+\frac{\sqrt{\mathcal{M}_a\mathcal{M}_c}\left[\sqrt{\mathcal{M}_b}\hat{\mathcal{M}}_{ad}-\sqrt{\mathcal{M}_a}\hat{\mathcal{M}}_{bd}\right](k_{ac}+\mathcal{M}_a\mathcal{M}_c)}{2(k_a^2+\mathcal{M}_a^2)(k_c^2+\mathcal{M}_c^2)}$$
$$\left.+\frac{\sqrt{\mathcal{M}_a\mathcal{M}_b}\left[\sqrt{\mathcal{M}_c}\hat{\mathcal{M}}_{ad}-\sqrt{\mathcal{M}_a}\hat{\mathcal{M}}_{cd}\right](k_{ab}+\mathcal{M}_a\mathcal{M}_b)}{2(k_a^2+\mathcal{M}_a^2)(k_b^2+\mathcal{M}_b^2)}\right],$$

$$\hat{\mathcal{M}}_{\alpha\beta} = \frac{\partial\sqrt{\mathcal{M}_\alpha}}{\partial k_\alpha^\mu}k_{\beta\mu}$$

Pion EM form factor at finite μ



Pion EM form factor at finite μ

Charge radius of the pion

$$\langle r^2\rangle^* = -6\frac{\partial F_\pi^*(Q^2)}{\partial Q^2}\bigg|_{Q^2=0}$$

$$\langle r^2(\mu_q)\rangle^* = \langle r^2(0)\rangle \mathcal{C}^*(\mu_q)\left[\frac{m_\rho}{m_\rho^*(\mu_q)}\right]^2 \approx 0.45\,\mathrm{fm}^2 \times \mathcal{C}^*(\mu_q)\left[\frac{m_\rho}{m_\rho^*(\mu_q)}\right]^2$$

Pion form factor with VMD

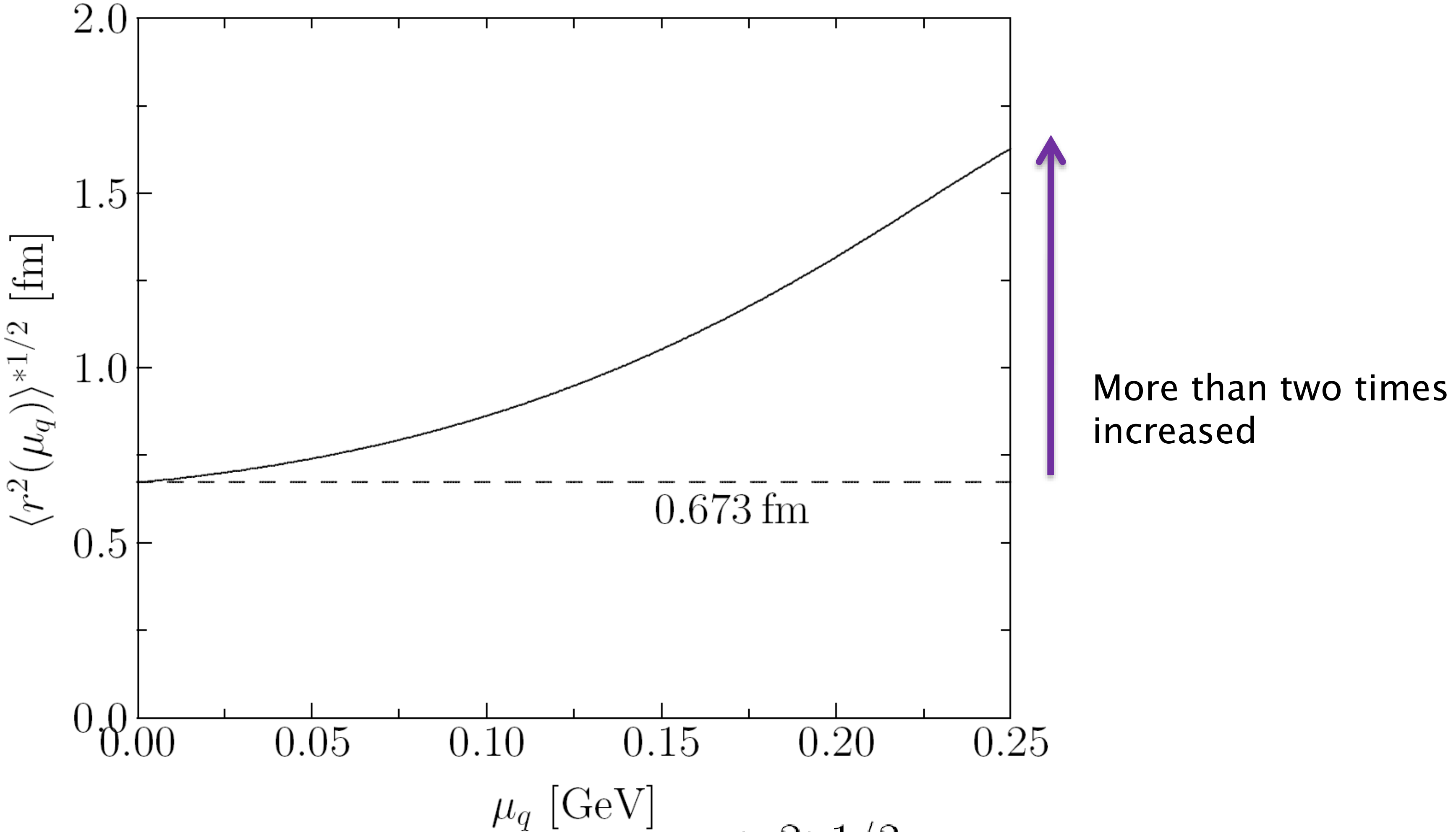
$$F_\pi^*(Q^2) \approx \frac{\mathcal{C}^*\, m_\rho^{*2}}{m_\rho^{*2} + Q^2 + i\Gamma_\rho^* m_\rho^*}, \qquad \mathcal{C} = \frac{f_{\rho\pi\pi}}{f_\rho}$$

$$m_\rho^*(\mu_q) = m_\rho\left[\frac{0.45\,\mathrm{fm}^2 \times \mathcal{C}^*(\mu_q)}{\langle r^2(\mu_q)\rangle^*}\right]^{1/2}$$

: Modification of the rho meson mass

Pion EM form factor at finite μ

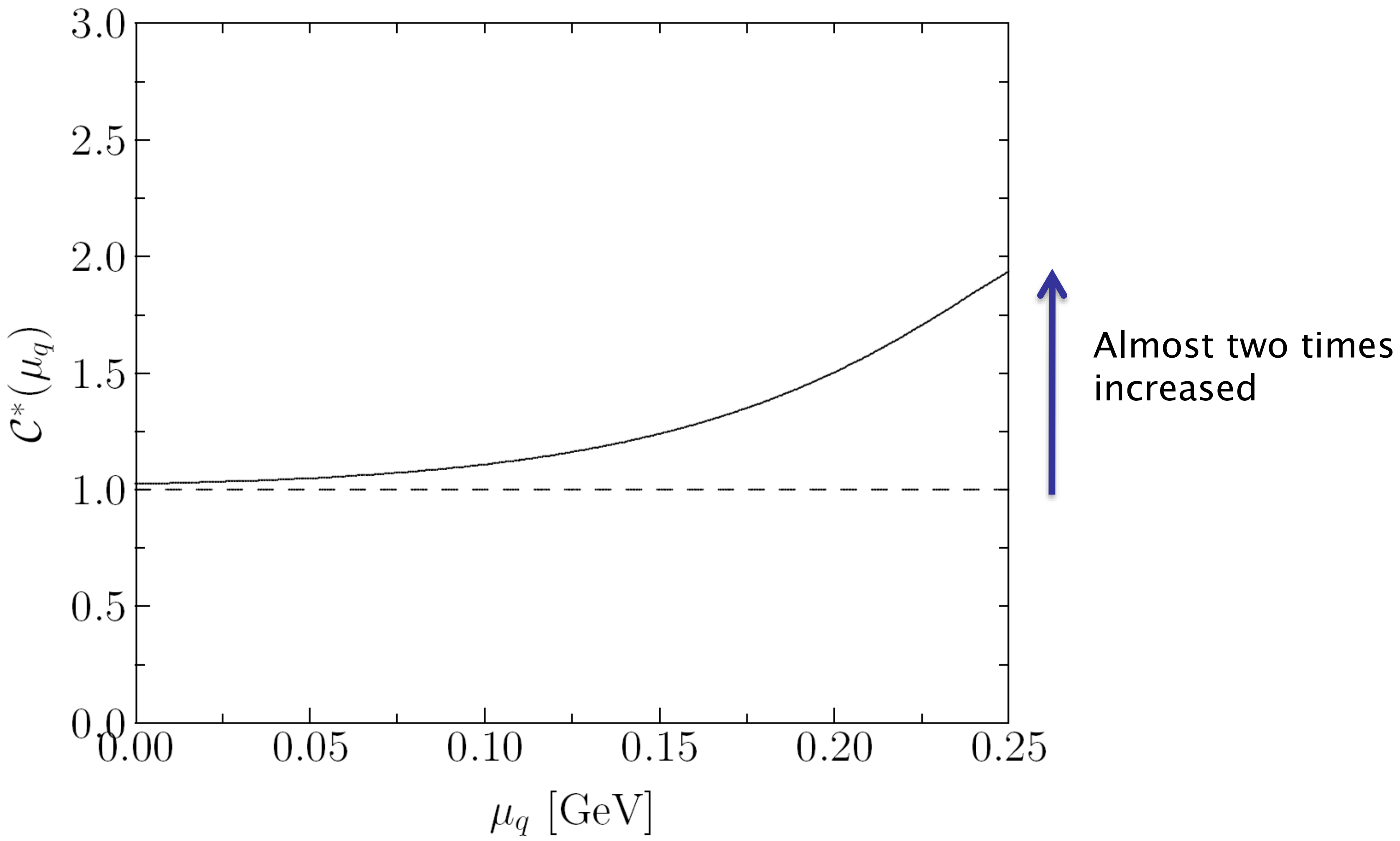
Charge radius of the pion



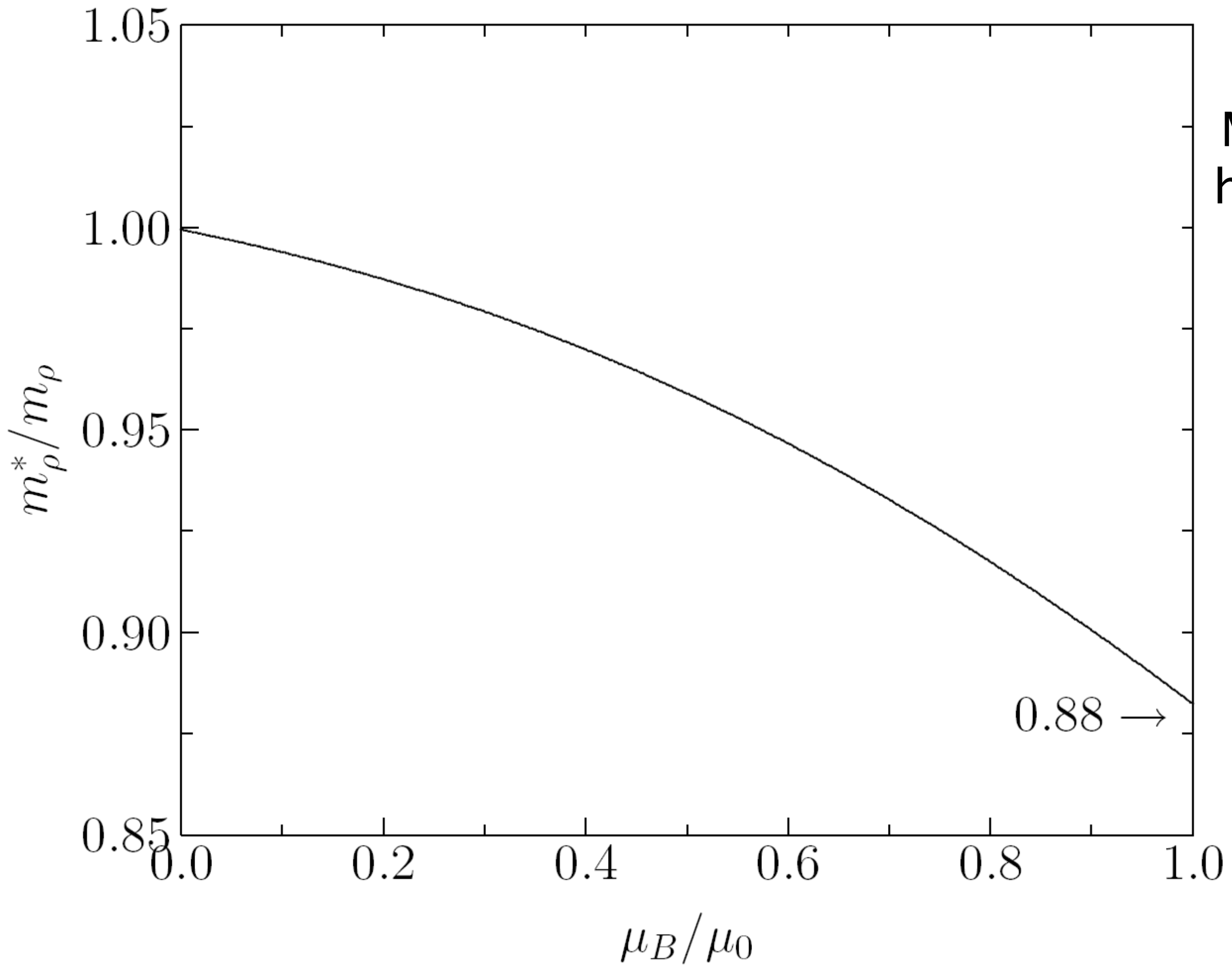
$$\langle r^2\rangle^{1/2} = 0.672 \pm 0.008\,\mathrm{fm}$$

Pion EM form factor at finite μ

Coupling constant C*



ρ meson mass dropping at finite μ



Modification of the Width has not been considered!

$$\alpha \approx 0.12$$

$$\frac{m^*_\rho}{m_\rho} \approx 1 - \alpha\,\frac{\mu_B}{\mu_0}$$

Slightly smaller than
Hatsuda & Lee, PRC 46, 34 (1992)

Summary and outlook

The nonlocal chiral quark model from the instanton vacuum is shown to be very successful in describing low-energy properties of mesons and (baryons).

We extended the model to study the QCD vacuum and pion properties at finite μ :

• QCD magnetic susceptibility: 1st-order magnetic phase transition

• Pion weak decay constant at finite density: *p*-wave contribution?

• Pion EM form factors and ρ meson mass dropping (not complete!)

Perspectives

Systematic studies for nonperturbative hadron properties with μ

Several works (effective chiral Lagrangian, LEC..) under progress

Extension to the finite *T* (Dyon with nontrivial holonomy, Caloron)

Though this be madness,
yet there is method in it.

Hamlet Act 2, Scene 2

Thank you very much!