Scalar D mesons in nuclear matter

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Procedure

1. Motivation

2. QCD sum rule
in vacuum & in nuclear matter

3. Application to Scalar D mesons

4. Conclusions

1. Motivation

- Charm-strange scalar meson $D_{s0}^{+}(2317)$ was discovered in 2003
- Its mass is too lower than the expected values in quark models and other theoretical predictions.
- It has been interpreted as the isosinglet state (conventional scalar meson), a four-quark state, a mixed state of both, a DK molecule, …
- Later charm scalar meson $D_0^*(2400)$ with broad width was discovered in 2004
- Its mass is similar to or larger than that of $D^+_{s0}(2317)$, even though $D_s(0^-)$ is 100 MeV higher than $D(0^-)$.

2. QCD sum rule

1. OPE (operator product expansion)

2. Phenomenological side

3. Dispersion relation

4. Borel transformation

2. 1. OPE (operator product expansion)

$$\lim_{x\to 0} A(x)B(0) = \sum_n C_n(x)O_n(0)$$

$C_n(x)$: Wilson Coefficien t (short range effect)

$O_n(0)$: Nonsingula r operator with dimension n

(long range effect)

As an example,

The polarization function of charm scalar current is

$$\Pi = i\int d^4x e^{iq\cdot x}\left\langle T\left[j(x), j^+(0)\right]\right\rangle$$

$$= i\int d^4x e^{iq\cdot x}\left\langle T\left[\bar{q}(x)c(x), \bar{c}(0)q(0)\right]\right\rangle$$

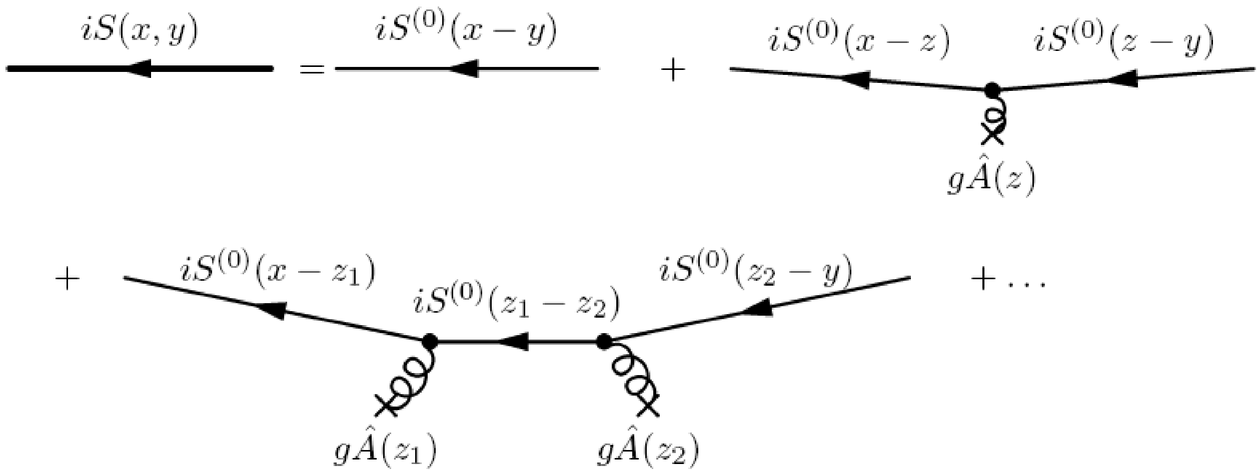
$$= i\int d^4x e^{iq\cdot x}\left\langle :Tr\left[S_q(-x)S_c(x)\right]: + :\bar{q}(x)S_c(x)q(0): \right.$$

$$\left. :\bar{c}(0)S_q(-x)c(x): + :\bar{q}(x)q(0)\bar{c}(0)c(x):\right\rangle$$

$$\equiv \Pi^{(0)} + \Pi^{(1)} \cancel{+ \Pi^{(2)} + \Pi^{(3)}}$$ may be ignored

, where $q = u, d, s$

Perturbative quark propagator in a weak gluonic background field
(The gluons are supposed to emerge from the ground state)



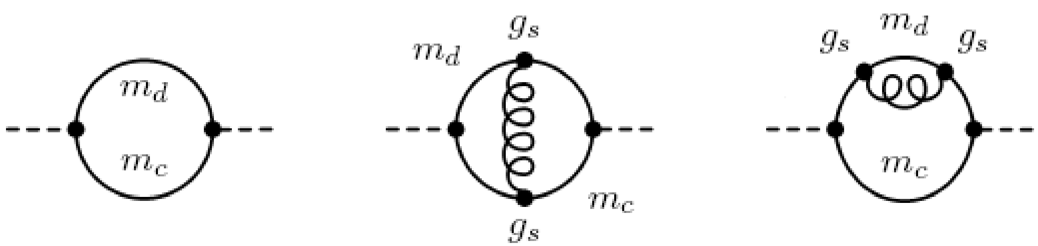
$$S^{(0)}(p) = \frac{\hat{p}+m}{p^2-m^2} \, ,$$

$$S^{(1)}(p) = i\frac{g}{2}\mathscr{G}_{\mu\nu}(0)S^{(0)}(p)\gamma^\mu S^{(0)}(p)\gamma^\nu S^{(0)}(p) \, ,$$

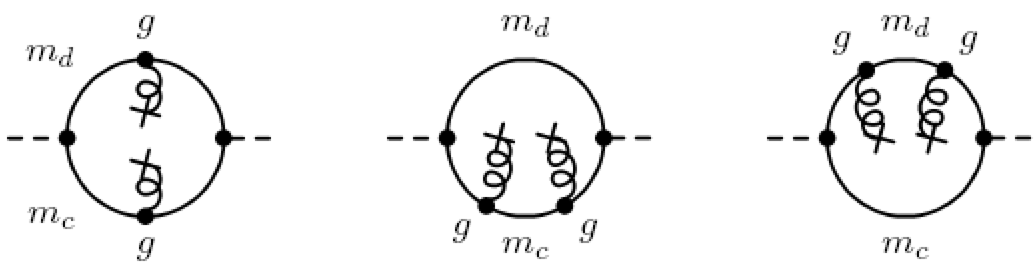
$$S^{(2)}(p) = \left(i\frac{g}{2}\right)^2 \mathscr{G}_{\mu\nu}(0)\mathscr{G}_{\kappa\lambda}(0)T^{\mu\nu\kappa\lambda}(p) \, ,$$

$$\begin{aligned} T^{\mu\nu\kappa\lambda}(p) :=& S^{(0)}(p)\gamma^\mu S^{(0)}(p)\gamma^\nu S^{(0)}(p)\gamma^\kappa S^{(0)}(p)\gamma^\lambda S^{(0)}(p) \\ &+ S^{(0)}(p)\gamma^\mu S^{(0)}(p)\gamma^\kappa S^{(0)}(p)\gamma^\nu S^{(0)}(p)\gamma^\lambda S^{(0)}(p) \\ &+ S^{(0)}(p)\gamma^\mu S^{(0)}(p)\gamma^\kappa S^{(0)}(p)\gamma^\lambda S^{(0)}(p)\gamma^\nu S^{(0)}(p) \, . \end{aligned}$$

Considering $\Pi^{(0)} \sim :Tr\left[S_q(-x)S_c(x)\right]:$



Perturbative part
I : dimension 0



Gluon condensates
<G^2> : dimension 4

and more condensates
with higher dimension
or more strong couplings

in Fock Schwinger Gauge $x^{\mu} A_{\mu}(x) = 0$

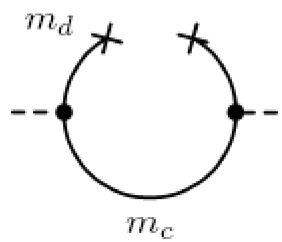
$$A_{\mu}(x) = \sum_{n} \frac{x^{\nu}}{n!(n+2)} x^{\alpha 1} \ldots x^{\alpha n} (D_{\alpha 1} \ldots D_{\alpha n} F_{\nu\mu})_{x=0}$$

$$\Psi(x) = \sum_{n} \frac{1}{n!} x^{\alpha 1} \ldots x^{\alpha n} (D_{\alpha 1} \ldots D_{\alpha n} \Psi)_{x=0}$$

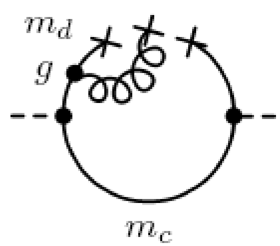
and

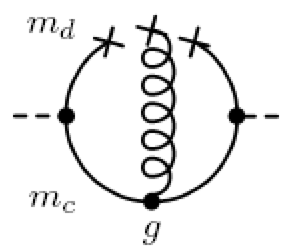
$$\Pi^{(1)} = -\sum_{n} \frac{(-i)^{n}}{n!} \left\langle : \left(\overline{q} D_{\alpha 1} \ldots D_{\alpha n} \right) \partial^{\alpha 1} \ldots \partial^{\alpha n} S_{c}(q)\, q : \right\rangle$$

$\Pi^{(1)}$ is graphicall y



Quark condensate
<qq> : dimension 3





Quark-gluon mixed condensates
<qGq> : dimension 5

and more condensates
with higher dimension
or more strong couplings

OPE up to dimension 5 in vacuum

$$\Pi\,(q^2)$$
$$= c_0(q^2) - \langle\bar{q}q\rangle\frac{m_c}{q^2-m_c^2} + \langle\bar{q}g\sigma\mathscr{G}q\rangle\frac{1}{2}\left(\frac{m_c^3}{(q^2-m_c^2)^3} + \frac{m_c}{(q^2-m_c^2)^2}\right)$$
$$- \langle\frac{\alpha_s}{\pi}G^2\rangle\frac{1}{12}\frac{1}{q^2-m_c^2}$$

, where C_0 (q^2) is perturbative part

OPE up to dimension 5 in nuclear matter

in the rest frame of the matter, $v = (1, \vec{0})$,

and considering scalar meson at $q = (q_0, \vec{0})$,

$\Pi(q_0) = \Pi^e(q_0^2) + q_0 \Pi^o(q_0^2)$,

where

$$
\begin{aligned}
\Pi^e
&= c_0(q_0^2) - \langle \bar{q}q \rangle \frac{m_c}{q_0^2 - m_c^2} + \langle \bar{q} g \sigma \mathcal{G} q \rangle \frac{1}{2} \left(\frac{m_c^3}{(q_0^2 - m_c^2)^3} + \frac{m_c}{(q_0^2 - m_c^2)^2} \right) \\
&- \langle \frac{\alpha_s}{\pi} G^2 \rangle \frac{1}{12} \frac{1}{q_0^2 - m_c^2} \\
&+ \langle \frac{\alpha_s}{\pi} \left(\frac{(vG)^2}{v^2} - \frac{G^2}{4} \right) \rangle \left(\frac{7}{18} + \frac{1}{3} \ln \frac{\mu^2}{m_c^2} + \frac{2}{3} \ln \left(- \frac{m_c^2}{q_0^2 - m_c^2} \right) \right) \\
&\times \left(\frac{m_c^2}{(q_0^2 - m_c^2)^2} + \frac{1}{q_0^2 - m_c^2} \right) \\
&+ \langle q^\dagger i D_0 q \rangle 2 \left(\frac{m_c^2}{(q_0^2 - m_c^2)^2} + \frac{1}{q_0^2 - m_c^2} \right) \\
&+ \left[\frac{1}{3} \langle \bar{q} D_0^2 q \rangle - \frac{1}{24} \langle \bar{q} g \sigma \mathcal{G} q \rangle \right] 12 \left(\frac{m_c^3}{(q_0^2 - m_c^2)^3} + \frac{m_c}{(q_0^2 - m_c^2)^2} \right)
\end{aligned}
$$

$$
\begin{aligned}
\Pi^o
&= -\langle q^\dagger q \rangle \frac{1}{q_0^2 - m_c^2} + \langle q^\dagger D_0^2 q \rangle 4 \left(\frac{m_c^2}{(q_0^2 - m_c^2)^3} + \frac{1}{(q_0^2 - m_c^2)^2} \right) \\
&- \langle q^\dagger g \sigma \mathcal{G} q \rangle \frac{1}{(q_0^2 - m_c^2)^2}
\end{aligned}
$$

List of employed condensate parameters

condensate	vacuum value $\langle \cdots \rangle_{vac}$	density dependent part $\langle \cdots \rangle_{med}$
$\langle \overline{q} q \rangle$	$(-0.245\ \mathrm{GeV})^3$	$45/11\, n$
$\langle \frac{\alpha_s}{\pi} G^2 \rangle$	$(0.33\ \mathrm{GeV})^4$	$-0.65\ \mathrm{GeV}\, n$
$\langle \overline{q} g \sigma \mathscr{G} q \rangle$	$0.8\ \mathrm{GeV}^2 \times (-0.245\ \mathrm{GeV})^3$	$3\, n\ \mathrm{GeV}^2$
$\langle q^\dagger q \rangle$	0	$1.5\, n$
$\langle \frac{\alpha_s}{\pi} \left(\frac{(vG)^2}{v^2} - \frac{G^2}{4} \right) \rangle$	0	$-0.05\ \mathrm{GeV}\, n$
$\langle q^\dagger i D_0 q \rangle$	0	$0.18\ \mathrm{GeV}\, n$
$\langle \overline{q} \left[D_0^2 - \frac{1}{8} g \sigma \mathscr{G} \right] q \rangle$	0	$-0.3\ \mathrm{GeV}^2\, n$
$\langle q^\dagger D_0^2 q \rangle$	0	$-0.0035\ \mathrm{GeV}^2\, n$
$\langle q^\dagger g \sigma \mathscr{G} q \rangle$	0	$0.33\ \mathrm{GeV}^2\, n$

2.2. Phenomenological side

$$\Pi(q^2) = i\int d^4x e^{iq\cdot x} < o\,|\,T[\,j(x),\, j^+(0)]\,|\,o >$$

$$= i\int d^4x e^{iq\cdot x}\theta(x_0) < o\,|\, j(x)\, j^+(0)\,|\,o > +\theta(-x_0) < o\,|\, j^+(0)\, j(x)\,|\,o >$$

Inserting identity operator between tw o current operators

$$I = \sum_{\lambda}\int \frac{dp^3}{(2\pi)^3 2\omega_{\vec{p}}}\,|\,\lambda_{\vec{p}} >< \lambda_{\vec{p}}\,|,$$

and using translati on operators $< o\,|\, j(x)\,|\,\lambda_{\vec{p}} >= e^{-ip\cdot x} < o\,|\, j(0)\,|\,\lambda_{\vec{p}} >$,

and the definition of step function $\theta(x_0) = \frac{1}{2\pi i}\int dw \frac{e^{iwx_0}}{w - i\varepsilon}$,

$$\Pi(q) = \sum_{\lambda}\left\{ \frac{\left|< o\,|\, j(0)\,|\,\lambda_{\vec{q}} >\right|^2}{2E_{\vec{q}}\left(E_{\vec{q}} - q_0 - i\varepsilon\right)} + \frac{\left|< o\,|\, j^+(0)\,|\,\lambda_{-\vec{q}} >\right|^2}{2E_{-\vec{q}}\left(E_{-\vec{q}} + q_0 - i\varepsilon\right)} \right\} .$$

Its imaginary part is extracted by using $\dfrac{1}{x^2 - i\varepsilon} = \dfrac{P}{x^2} + i\pi\delta(x^2)$,

$$\mathrm{Im}\,\Pi = \sum_{\lambda}\left\{\left(\frac{1}{2} + \frac{q_0}{2E_{\vec{q}}}\right)\left|< o\,|\,j\,|\,\lambda_{\vec{q}} >\right|^2 \delta(q_0^2 - E_{\vec{q}}^2) + \left(\frac{1}{2} - \frac{q_0}{2E_{-\vec{q}}}\right)\left|< o\,|\,j^+\,|\,\lambda_{-\vec{q}} >\right|^2 \delta(q_0^2 - E_{-\vec{q}}^2)\right\}$$

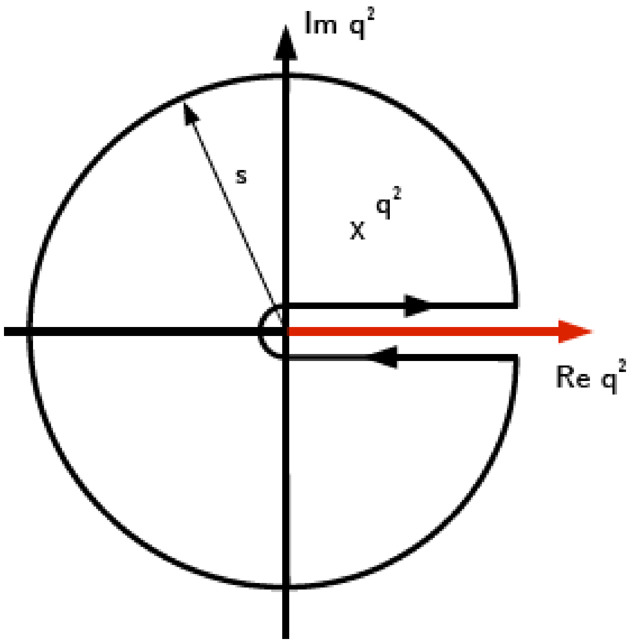
If $\left|< o\,|\,j\,|\,\lambda_{\vec{q}} >\right|^2 = \left|< o\,|\,j^+\,|\,\lambda_{-\vec{q}} >\right|^2$,

$$\mathrm{Im}\,\Pi = \sum_{\lambda}\left|< o\,|\,j\,|\,\lambda_{\vec{q}} >\right|^2 \delta(q_0^2 - E_{\vec{q}}^2) \rightarrow \text{spectral function !!}$$

$$\approx F\delta(q^2 - m_h^2) + \mathrm{Im}\,\Pi\,\theta(q^2 - s_0)$$

, which is so called 'Pole + Continuum Ansatz'

2.3. Dispersion relation (in vacuum)

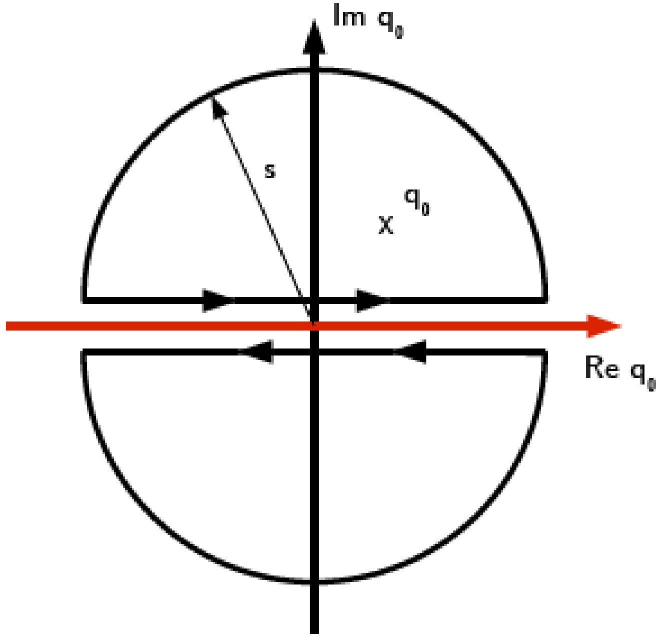


$$\Pi(q^2) = \frac{1}{\pi}\int_0^{\infty} \frac{\Delta\Pi^{vac}(s)}{s-q^2} ds + \sum_{n=0}^{N} a_n q_0^n$$

L.H.S .is a function of QCD parameters such as g, m_q, <qq>, <G²>…, obtained from OPE.

R.H.S. is a function of physical parameters such as m_λ and s_0

Dispersion relation (in nuclear matter)



$$\Pi(q_0, \vec{q}\,) = \frac{1}{\pi}\int_{-\infty}^{+\infty} \frac{\Delta\Pi(s, \vec{q}\,)}{s - q_0} ds + \sum_{n=0}^{N} a_n q_0^n$$

2.4. Borel transformation

Borel transformation $\hat{B} \equiv \lim_{\substack{Q^2 \to \infty \\ n \to \infty \\ Q^2/n = M^2}} \frac{1}{(n-1)!}(Q^2)^n \left(-\frac{d}{dQ^2}\right)^n$, where $Q^2 = -q^2$

1) improves OPE side by suppressing high - dimensional condensate terms

$$\hat{B}\left(\frac{1}{Q^2}\right)^N = \frac{1}{(N-1)!}\left(\frac{1}{M^2}\right)^N$$

2) improves the phenomenological side by suppressing continuum part

$$\hat{B}\oint_C \frac{\mathrm{Im}\,\Pi(s)}{s+Q^2}\,ds = \oint_C \mathrm{Im}\,\Pi(s) e^{-s/M^2}\,ds$$

3. Application to scalar D mesons

1. Charm scalar meson D_0* in vacuum
2. Charm scalar meson D_0* in nuclear matter
3. Charm-strange scalar meson D_{s0} in vacuum & in nuclear matter

3.1. Charm scalar meson D_0* in vacuum

From the dispersion relation

$$\widetilde{F}e^{-\widetilde{m}^2/M^2}+\frac{1}{\pi}\int_{s_0}^{\infty}e^{-s/M^2}\mathrm{Im}\Pi_{per}(s)ds=\frac{1}{\pi}\int_{m_c^2}^{\infty}dse^{-s/M^2}\mathrm{Im}\Pi_{per}(s)$$
$$+e^{-m_c^2/M^2}\left(\ m_c\langle\bar{d}d\rangle-\frac{1}{2}\left(\frac{m_c^3}{2M^4}-\frac{m_c}{M^2}\right)\langle\bar{d}g\sigma\mathscr{G}d\rangle+\frac{1}{12}\langle\frac{\alpha_s}{\pi}G^2\rangle\right)\ ,$$

$$\boxed{\begin{aligned}\widetilde{F}e^{-\widetilde{m}^2/M^2}&=\frac{1}{\pi}\int_{m_c^2}^{s_0}dse^{-s/M^2}\mathrm{Im}\Pi_{per}(s)\\&+e^{-m_c^2/M^2}\left(\ m_c\langle\bar{d}d\rangle-\frac{1}{2}\left(\frac{m_c^3}{2M^4}-\frac{m_c}{M^2}\right)\langle\bar{d}g\sigma\mathscr{G}d\rangle+\frac{1}{12}\langle\frac{\alpha_s}{\pi}G^2\rangle\right)\end{aligned}}$$

the mass of scalar D meson is

$$\widetilde{m}^2=-\frac{\partial\left(\widetilde{F}e^{-\widetilde{m}^2/M^2}\right)/\partial\left(1/M^2\right)}{\widetilde{F}e^{-\widetilde{m}^2/M^2}}$$

Borel window

The range of M^2 satisfying below two conditions

1) continuum/total < 0.3

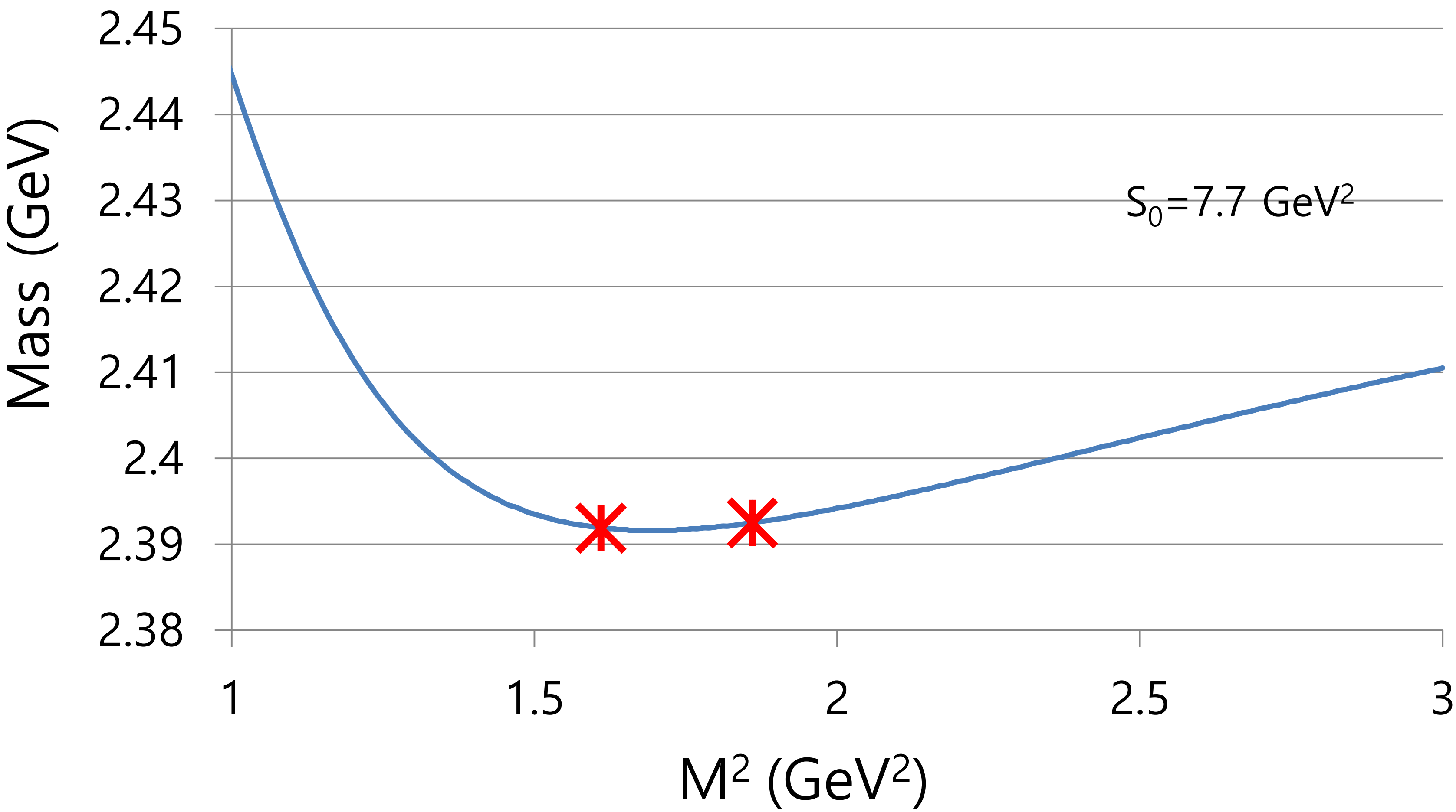
$$\frac{1}{\pi}\int_{s_0}^{\infty} e^{-s/M^2}\mathrm{Im}\Pi_{per}(s)ds \quad \widetilde{F}e^{-\tilde{m}^2/M^2} \quad < 0.3$$

2) Power correction/total < 0.3

$$e^{-m_c^2/M^2}\left(m_c\langle\bar{d}d\rangle - \frac{1}{2}\left(\frac{m_c^3}{2M^4} - \frac{m_c}{M^2}\right)\langle\bar{d}g\sigma\mathscr{G}d\rangle + \frac{1}{12}\langle\frac{\alpha_s}{\pi}G^2\rangle\right) \quad \widetilde{F}e^{-\tilde{m}^2/M^2} \quad < 0.3$$

Borel curve for $D_0^*(2400)$ in vacuum

(adjust s_0 to obtain flattest curve within Borel window)



3.2. Charm scalar meson D_0^* in nuclear matter

- Two dispersion relations for π^e and π^o exist

$$
\begin{aligned}
m_+F_+e^{-m_+^2/M^2} + m_-F_-e^{-m_-^2/M^2} &= \frac{1}{2\pi}\left[\int_{m_c^2}^{s_0^+} + \int_{m_c^2}^{s_0^-}\right] dse^{-s/M^2}\mathrm{Im}\Pi_{per}(s) \\
&+ e^{-m_c^2/M^2}\Bigg(m_c\langle\bar{d}d\rangle - \frac{1}{2}\left(\frac{m_c^3}{2M^4} - \frac{m_c}{M^2}\right)\langle\bar{d}g\sigma\mathcal{G}d\rangle + \frac{1}{12}\langle\frac{\alpha_s}{\pi}G^2\rangle \\
&+ \left[\left(\frac{7}{18} + \frac{1}{3}\ln\frac{\mu^2 m_c^2}{M^4} - \frac{2\gamma_E}{3}\right)\left(\frac{m_c^2}{M^2} - 1\right) - \frac{2}{3}\frac{m_c^2}{M^2}\right]\langle\frac{\alpha_s}{\pi}\left(\frac{(vG)^2}{v^2} - \frac{G^2}{4}\right)\rangle \\
&+2\left(\frac{m_c^2}{M^2} - 1\right)\langle d^\dagger iD_0 d\rangle - 4\left(\frac{m_c^3}{2M^4} - \frac{m_c}{M^2}\right)\left[\langle\bar{d}D_0^2 d\rangle - \frac{1}{8}\langle\bar{d}g\sigma\mathcal{G}d\rangle\right]\Bigg)
\end{aligned}
\equiv f(s_0^+, s_0^-)
$$

$$
\begin{aligned}
F_+e^{-m_+^2/M^2} - F_-e^{-m_-^2/M^2} &= \frac{1}{2\pi}\int_{s_0^-}^{s_0^+}\frac{ds}{\sqrt{s}}\, e^{-s/M^2}\mathrm{Im}\Pi_{per}(s) \\
&e^{-m_c^2/M^2}\left(\langle d^\dagger d\rangle - 4\left(\frac{m_c^2}{2M^4} - \frac{1}{M^2}\right)\langle d^\dagger D_0^2 d\rangle - \frac{1}{M^2}\langle d^\dagger g\sigma\mathcal{G}d\rangle\right)
\end{aligned}
\equiv g(s_0^+, s_0^-)
$$

After proper recombinat ion of two functions,

$$m_+^2 = -\frac{\dfrac{d}{d(1/M^2)} f(s_0^+, s_0^-) + m_- \dfrac{d}{d(1/M^2)} g(s_0^+, s_0^-)}{f(s_0^+, s_0^-) + m_- g(s_0^+, s_0^-)}$$

$$m_-^2 = -\frac{\dfrac{d}{d(1/M^2)} f(s_0^+, s_0^-) - m_+ \dfrac{d}{d(1/M^2)} g(s_0^+, s_0^-)}{f(s_0^+, s_0^-) - m_+ g(s_0^+, s_0^-)}.$$

That is,

$$m_+ = m_+(m_-, s_0^+, s_0^-)$$

$$m_- = m_-(m_+, s_0^+, s_0^-) \ ,$$

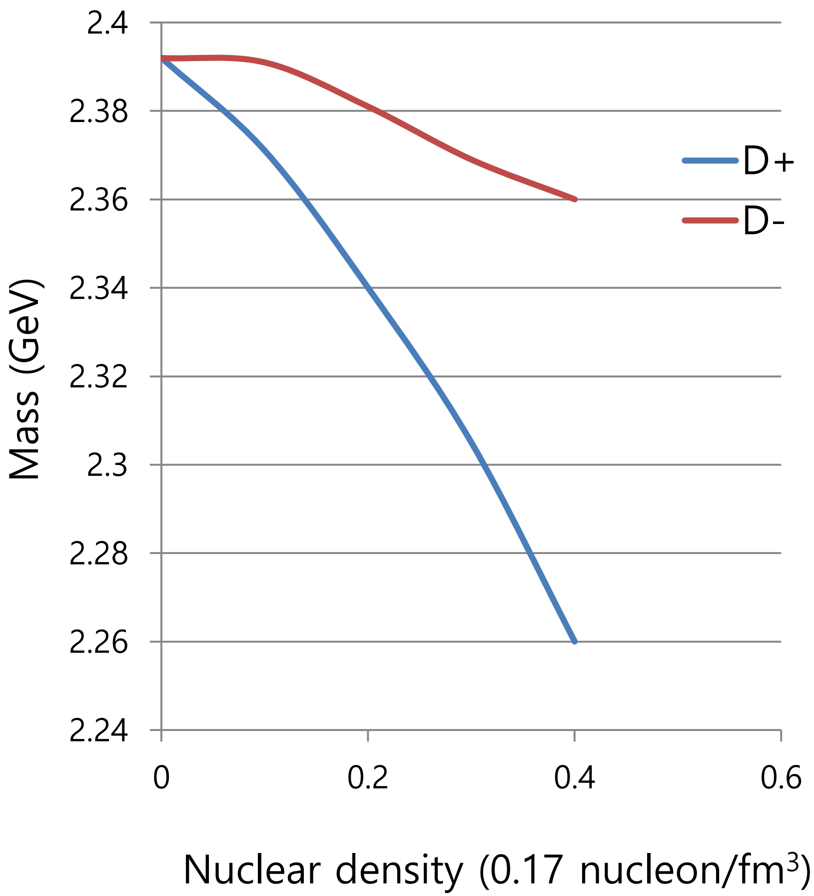
and they are solved by iterative method.

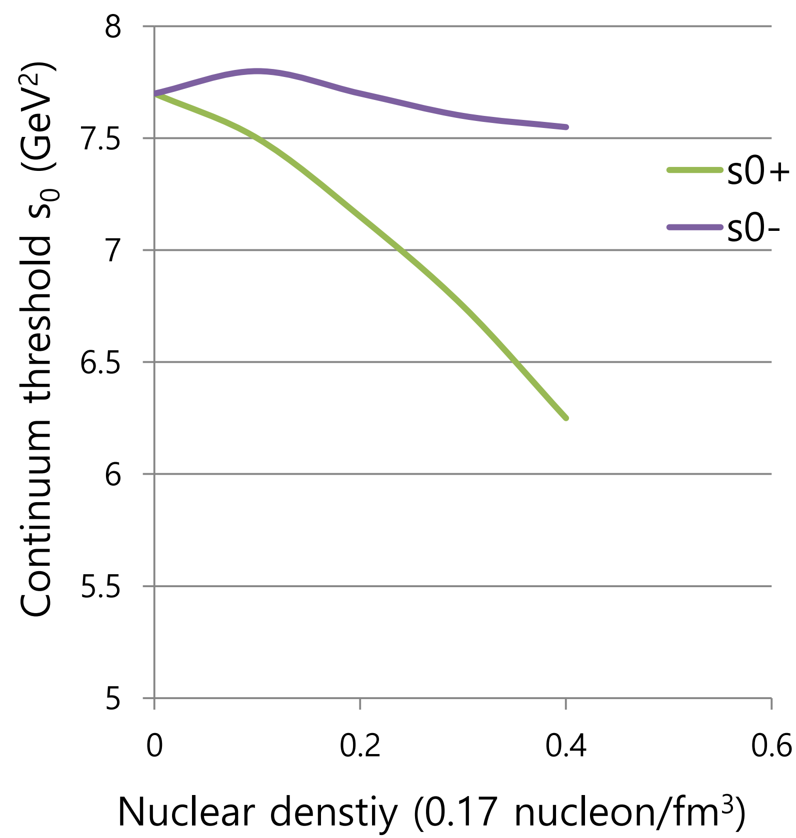
For example, in the case of 0.1 times nuclear saturation density,

iteration No.	D+ s0	D+ mass	D- s0	D- mass
0			7.7	2.392
1	7.55	2.373		
2			7.8	2.391
3	7.5	2.371		
4			7.8	2.391

Mass shift of $D_0^{*\pm}(2400)$ in nuclear matter

(The conditions for Borel window are continuum/total<0.5 & power correction/total <0.5)





Physical interpretation of the result

The quark component of D_0^{*+} is $\bar{d}c$, and that of D_0^{*-} is $\bar{c}d$.

Nuclear matter is rich in u and d quarks.

As a result, $\bar{d}c\left(D_0^{*+}\right)$ is more attractive than $\bar{c}d\left(D_0^{*-}\right)$.

That is the reason why the mass of D_0^{*+} decreases more than that of D_0^{*-} in nuclear matter.

3.3. Charm-strange scalar meson D_{s0} in vacuum & in nuclear matter

The mass of strange quark is ignored for simplicity .

Condensate parameters are changed as belows.

$$\langle \bar{d}d \rangle \to \langle \bar{s}s \rangle = 0.8 \langle \bar{d}d \rangle_{vacuum} + y \langle \bar{d}d \rangle_{matter}$$

$$\langle \bar{d}g\sigma F d \rangle \to \langle \bar{s}g\sigma F s \rangle = 0.8\,\mathrm{GeV}^2 \langle \bar{s}s \rangle$$

$$\langle d^+ d \rangle \to \langle s^+ s \rangle = 0$$

$$\langle d^+ iD_0 d \rangle \to \langle s^+ iD_0 s \rangle = 0.018\,\mathrm{GeV}\; n$$

$$\left\langle \bar{d}\left[D_0^2 - \frac{1}{8} g\sigma F \right] d \right\rangle \to \left\langle \bar{s}\left[D_0^2 - \frac{1}{8} g\sigma F \right] s \right\rangle = y \left\langle \bar{d}\left[D_0^2 - \frac{1}{8} g\sigma F \right] d \right\rangle$$

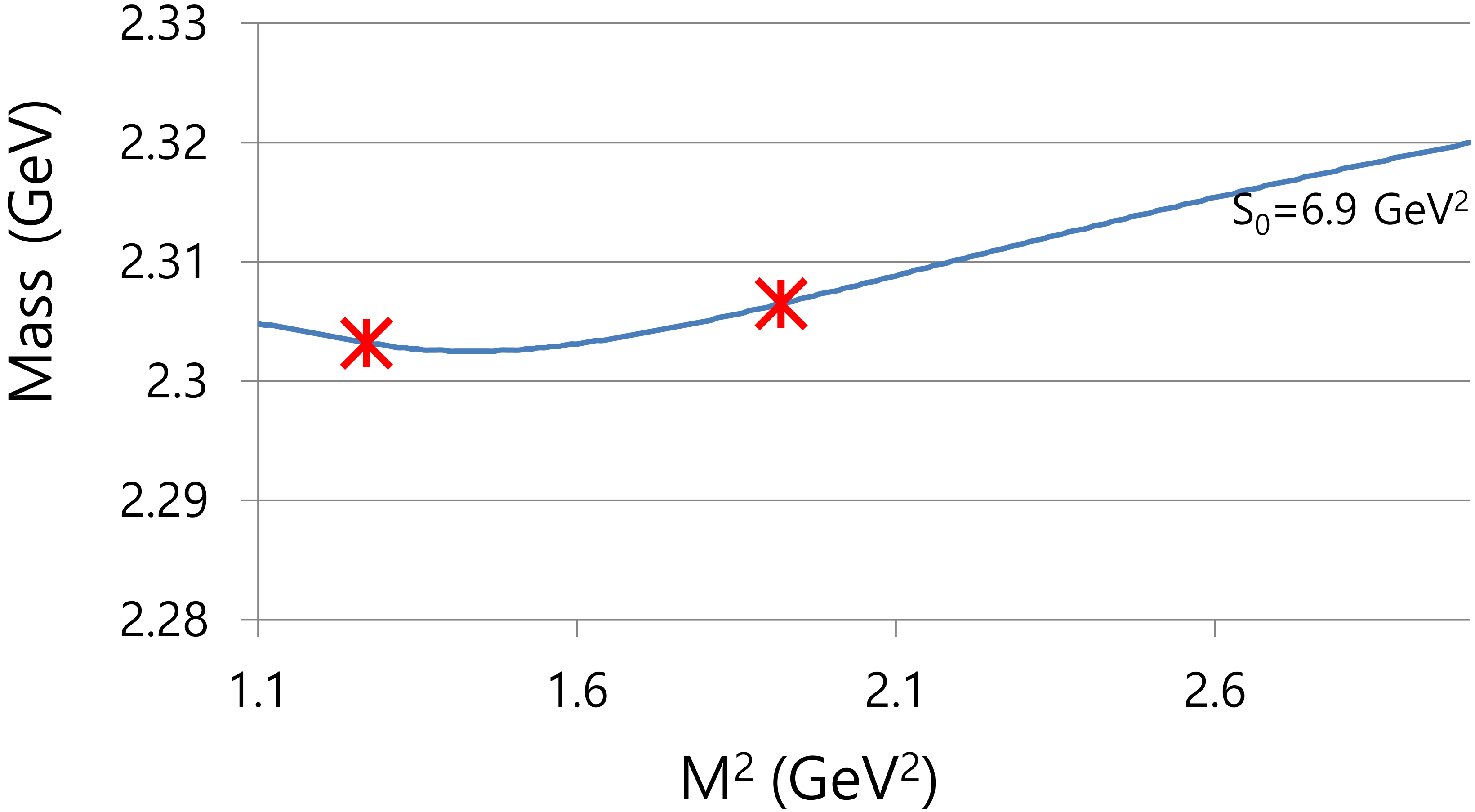
$$\langle d^+ D_0^2 d \rangle \to \langle s^+ D_0^2 s \rangle = y \langle d^+ D_0^2 d \rangle$$

$$\langle d^+ g\sigma F d \rangle \to \langle s^+ g\sigma F s \rangle = y \langle d^+ g\sigma F d \rangle$$

and $y = 0.36$ from lattice calculatio ns.

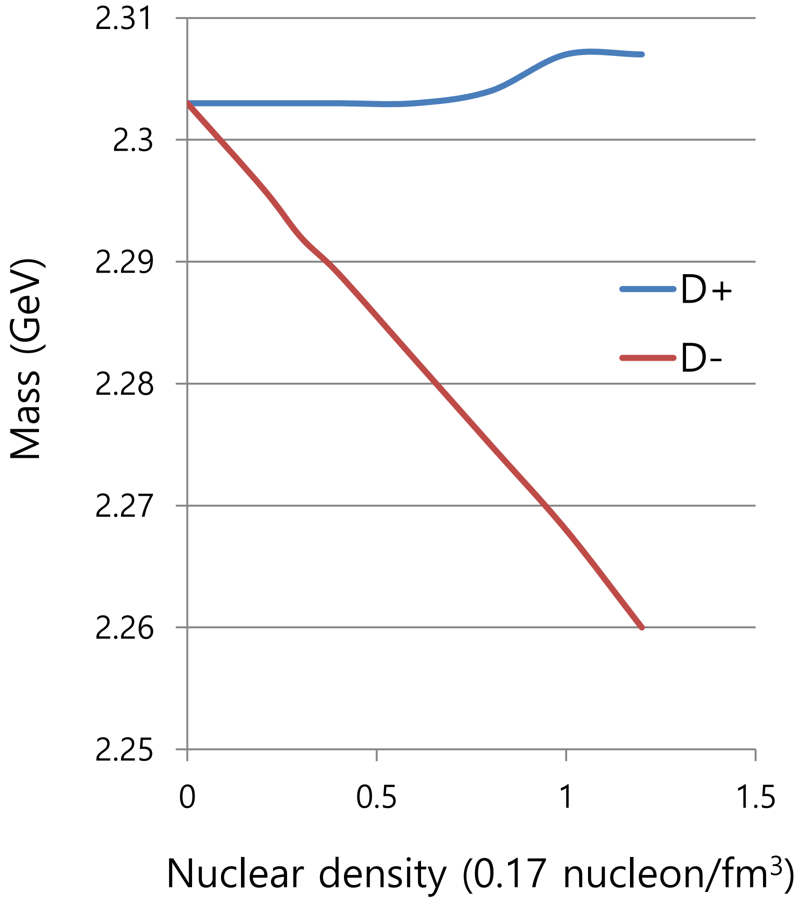
Borel curves for D_{s0} (2317) in vacuum

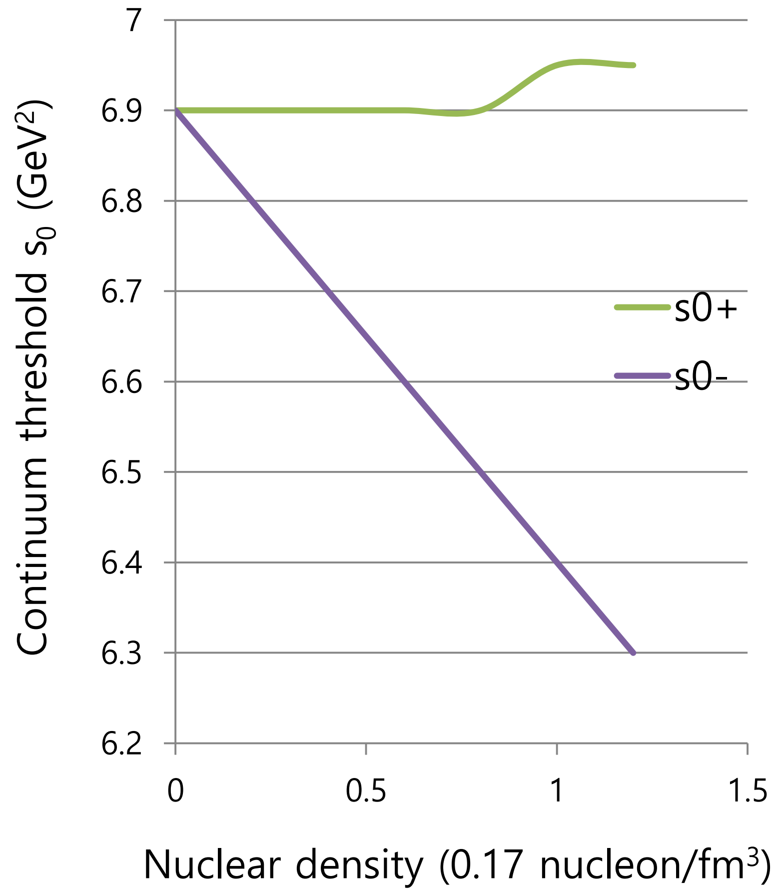
(The conditions for Borel window are continuum/total<0.5 & power correction/total <0.5)



Mass shift of $D_{s0}^{\pm}(2317)$ in nuclear matter

(The conditions for Borel window are continuum/total<0.5 & power correction/total <0.5)





4. Conclusion

1. We successfully reproduced the mass of charm scalar meson D_0^* in vacuum and that of charm-strange scalar meson D_{s0} by using QCD sum rule
2. Based on this success, their mass shifts in the nuclear matter are estimated by considering the change of condensate parameters and adding new condensate parameters which do not exist in vacuum state.
3. The mass of D_0^{*+} decreases in the nuclear matter more than that of D_0^{*-}, as naively expected.
4. But the mass of D_{s0}^+ increases a little in the nuclear matter, while that of D_{s0}^- decreases.
5. The quark component of these scalar particles are still in question, whether they are conventional mesons or quark-quark states or their combinations or others.
6. We expect that the behavior of their masses in nuclear matter serves to determine the exact quark component of those scalar particles.

4.1. Future work

- QCD sum rule for pseudoscalar D meson, which is the chiral partner of charm scalar meson – the mass difference between scalar and pseudoscalar meson is expected to decrease in nuclear matter due to the partial restoration of chiral symmetry.
- QCD sum rule for charm scalar meson as four quark state – The comparison of its mass shift in nuclear matter with that of conventional meson will reveal the exact component of charm scalar meson.