QCD Phase Structure at High Baryon Density

T. Hatsuda (University of Tokyo)

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(1) Tachibana, Yamamoto, Baym & T.H., Phys. Rev. Lett. 97 (2006) 122001.

(2) Yamamoto, Tachibana, Baym & T.H., Phys. Rev. D 76 (2007) 074001.

(3) Tachibana, Yamamoto & T.H., Phys. Rev. D 78 (2008) 011501.

(4) Maeda, Baym and T.H., in preparation (2009)

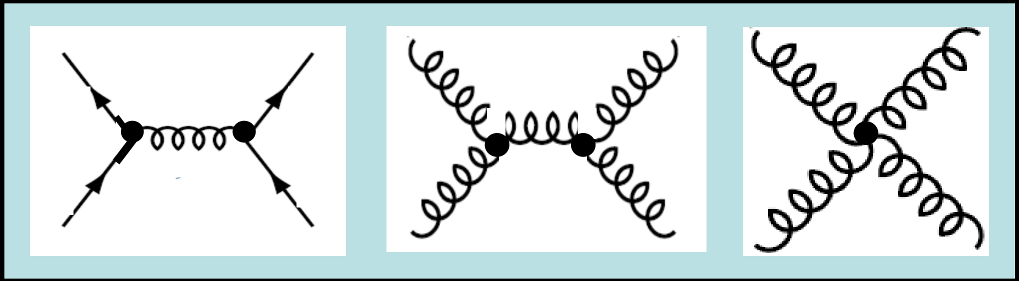
HIM meeting, April. 10, 2009

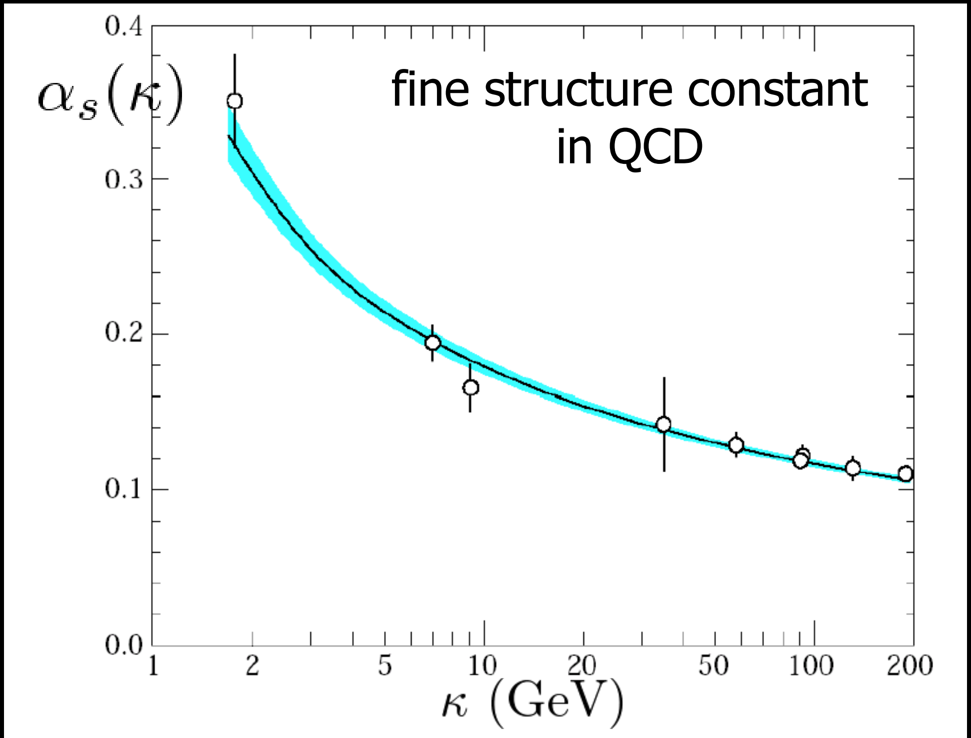
Quantum Chromo Dynamics

$$L = -\frac{1}{4} G^{a}_{\mu\nu} G_{a}^{\mu\nu} + \bar{q}\gamma^{\mu}(i\partial_{\mu} - g t^{a} A^{a}_{\mu})q - m\bar{q}q$$

QCD: SU(3) gauge theory for color charge

Y. Nambu





- SU(3) YM for strong interaction (Nambu ’66)
- Asymptotic freedom (Gross, Wilczek & Politzer ’73)
- Confinement criterion (Wilson ’74)

QCD vacuum and its symmetry

Chiral basis : $q_L = \frac{1}{2}(1-\gamma_5)q, \quad q_R = \frac{1}{2}(1+\gamma_5)q$

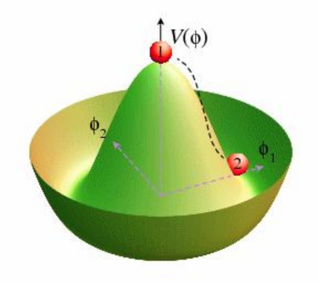
QCD Lagrangian : $\mathcal{L}_{cl} = \mathcal{L}_{cl}(q_L, A) + \mathcal{L}_{cl}(q_R, A)$

classical QCD symmetry (m=0)

$$\mathcal{G} = SU(3)_C \times [SU(N_f)_L \times SU(N_f)_R] \times U(1)_B \times U(1)_A$$

Quantum QCD vacuum (m=0)

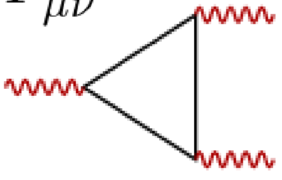
Chiral condensate : spontaneous mass generation



$$\langle \bar{q}_R q_L \rangle \neq 0$$

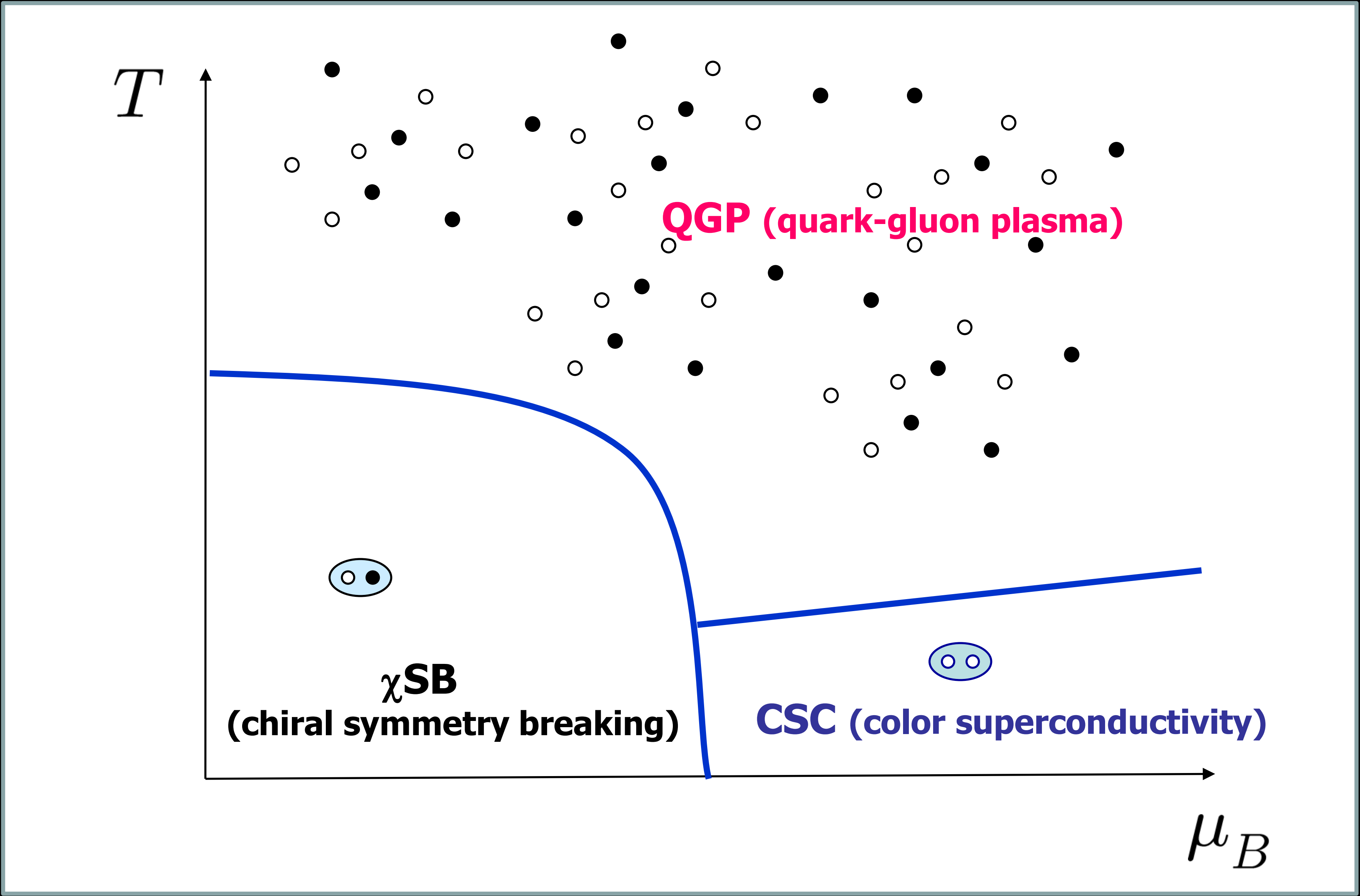
Axial anomaly : quantum violation of $U(1)_A$

$$\partial_\mu J^\mu_A = -2N_f \frac{\alpha_s}{8\pi} F^{\mu\nu}_a \tilde{F}^a_{\mu\nu}$$



$$SU(3)_C \times SU(N_f)_{L+R} \times U(1)_B$$

Phases in QCD

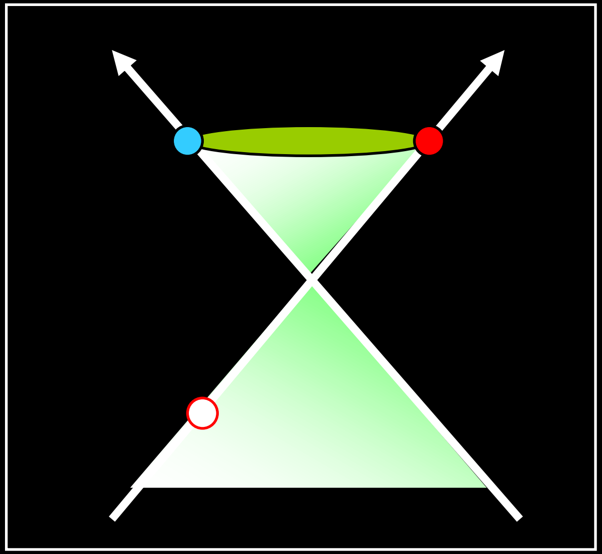


Dirac mass vs. Majorana mass

$$\Psi = (q, q^C)^{\mathrm{t}}$$

$$L_{\mathrm{eff}} = \frac{1}{2}\bar{\Psi}\begin{pmatrix} i\gamma\cdot\partial - \Phi & \bar{\Delta} \\ \Delta & i\gamma\cdot\bar{\partial} - \Phi \end{pmatrix}\Psi$$

Nambu-Gor'kov (cond-mat)
= Hartree-Fock-Bogoliubov (nucl-th)
= Dirac-Majorana (hep-ph)



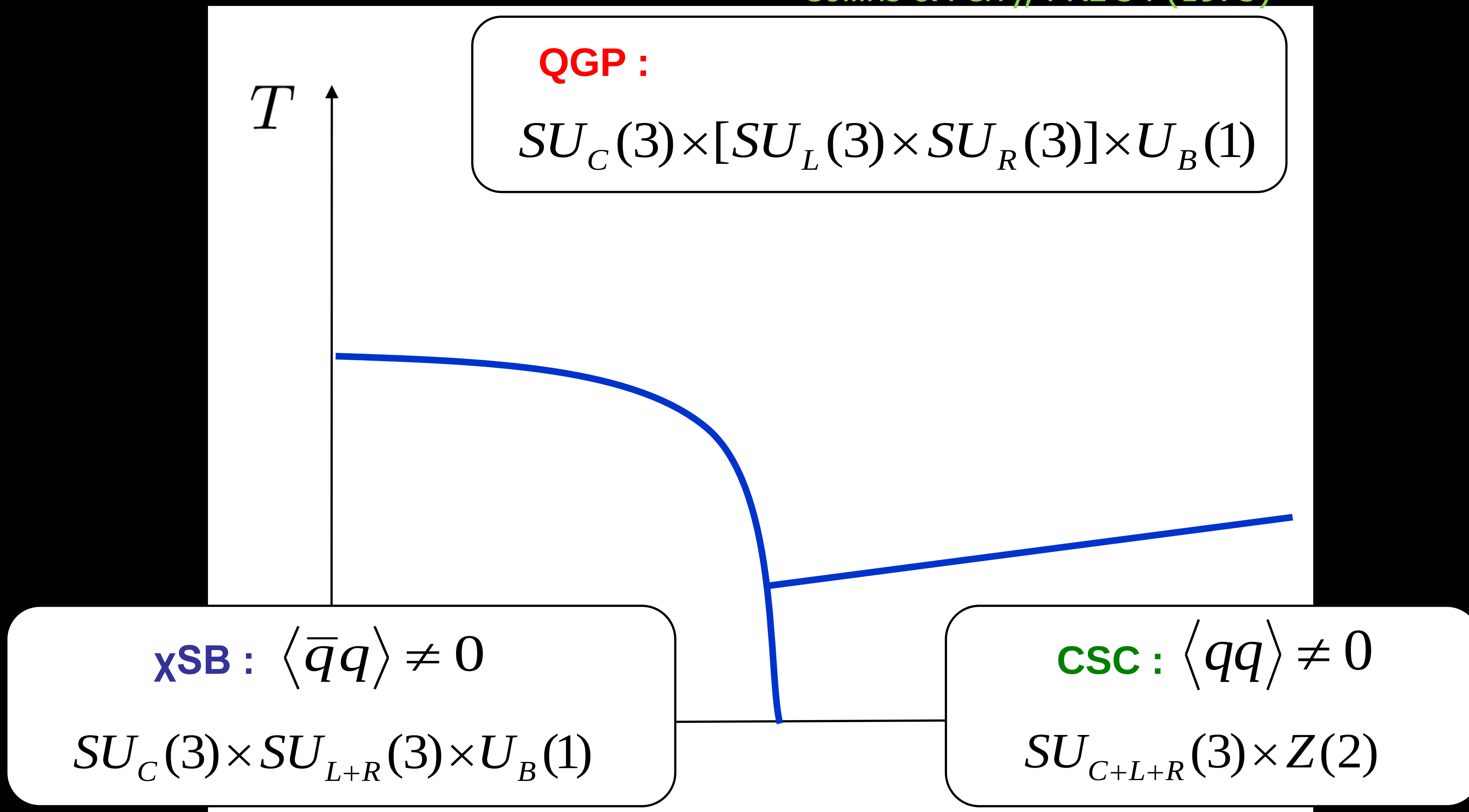
$$\Phi_{ij} \sim \langle \bar{q}_j q_i \rangle, \quad \Delta_{ij}^{ab} \sim \langle q_i^a C q_j^b \rangle$$

Dirac mass Majorana mass

$$SU(3)_C \times \left[SU(3)_L \times SU(3)_R \right] \times U(1)_B$$

Symmetry realization in hot/dense QCD *(for $m_{u,d,s}$=0 case)*

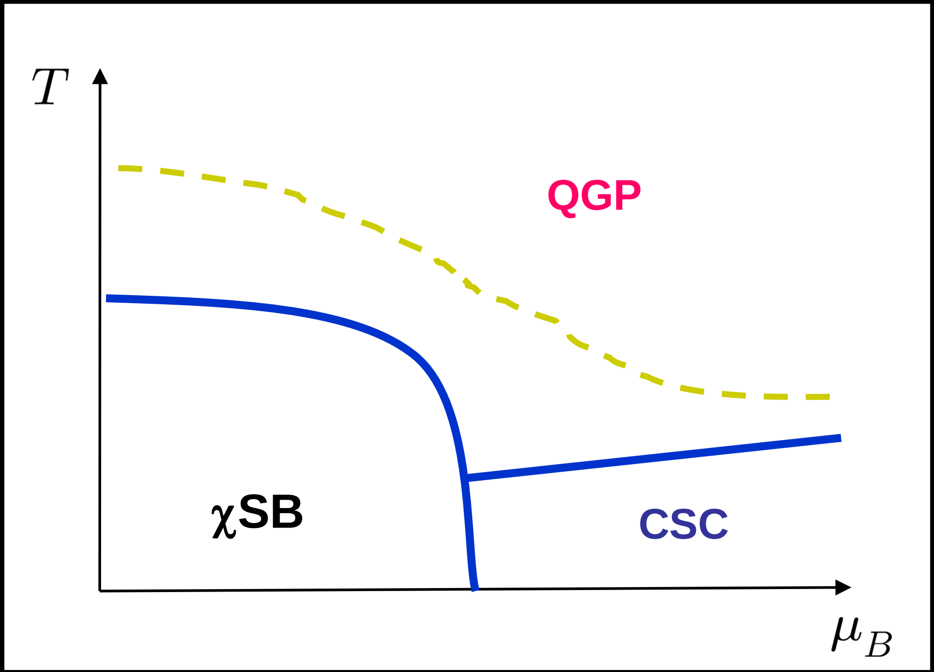
Collins & Perry, PRL 34 (1975)

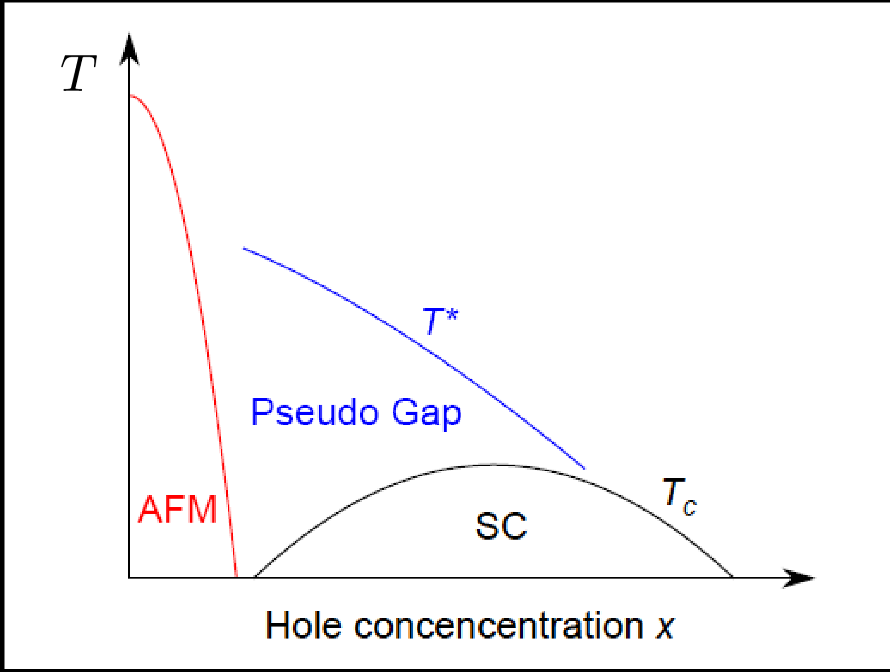


Nambu, PRL 4 (1960)

Alford, Rajagopal & Wilczek, NP B537 (1999)

QCD and high temperature superconductivity (HTS)





Common features in QCD, HTS, and cold atoms

1. Competition between different orders
2. Strong coupling

- Babaev, Int. J. Mod. Phys. A16 ('01)
- Kitazawa, Nemoto, Kunihiro, PTP ('02)
- Abuki, Itakura & Hatsuda, PRD ('02)
- Chen, Stajic, Tan & Levin, Phys. Rep. ('05)
- Baym, Hatsuda, Tachibana & Yamamoto (2008)

PRL 97, 122001 (2006) PHYSICAL REVIEW LETTERS week ending 22 SEPTEMBER 2006

New Critical Point Induced By the Axial Anomaly in Dense QCD

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(Received 10 May 2006; published 18 September 2006)

We study the interplay between chiral and diquark condensates within the framework of the Ginzburg-Landau free energy, and classify possible phase structures of two and three-flavor massless QCD. The QCD axial anomaly acts as an external field applied to the chiral condensate in a color superconductor and leads to a crossover between the broken chiral symmetry and the color superconducting phase, and, in particular, to a new critical point in the QCD phase diagram.

DOI: 10.1103/PhysRevLett.97.122001 PACS numbers: 12.38.−t, 26.60.+c

PRL 99, 130406 (2007) PHYSICAL REVIEW LETTERS week ending 28 SEPTEMBER 2007

Superfluidity and Magnetism in Multicomponent Ultracold Fermions

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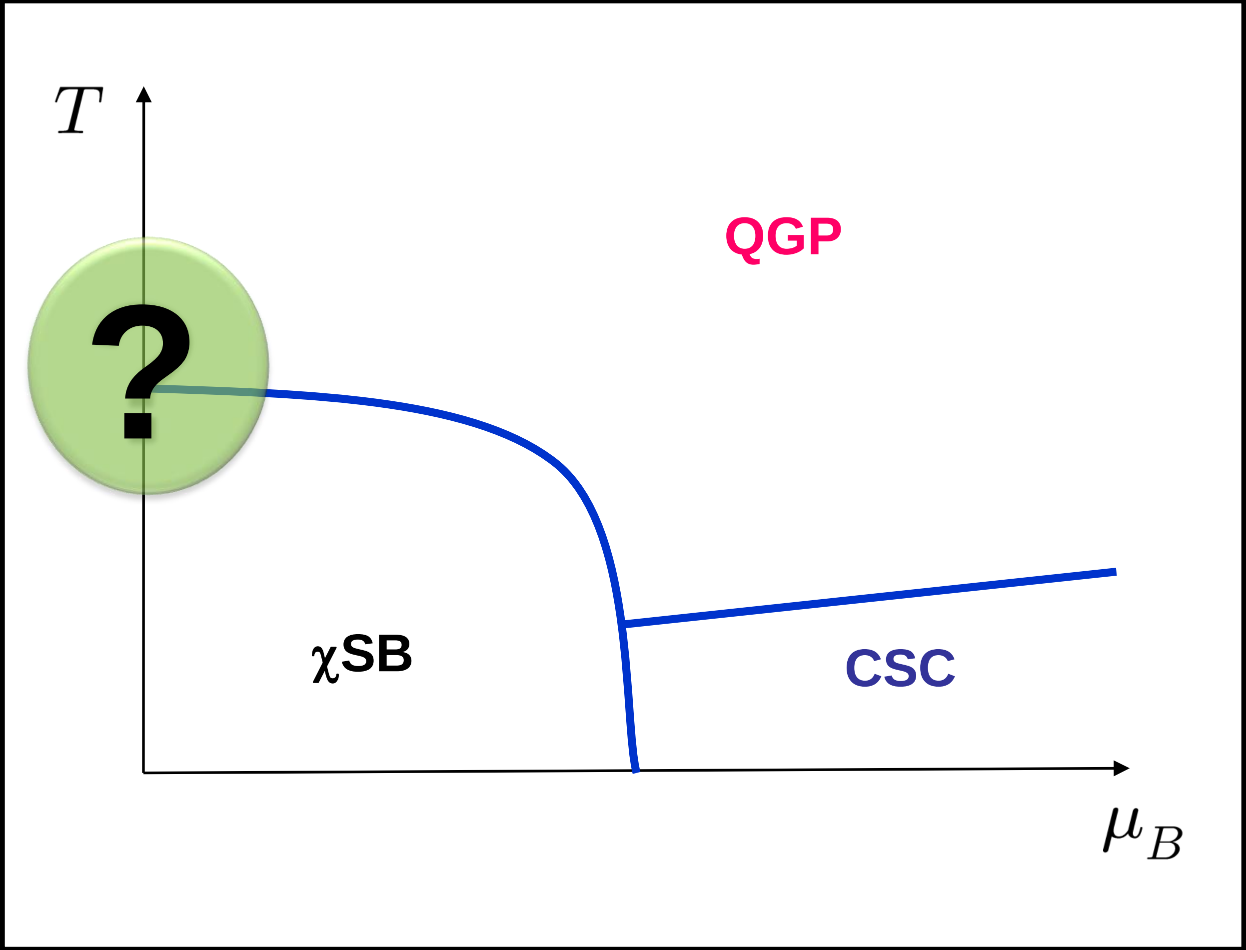
(Received 2 May 2007; published 28 September 2007)

We study the interplay between superfluidity and magnetism in a multicomponent gas of ultracold fermions. Ward-Takahashi identities constrain possible mean-field states describing order parameters for both pairing and magnetization. The structure of global phase diagrams arises from competition among these states as functions of anisotropies in chemical potential, density, or interactions. They exhibit first and second order phase transition as well as multicritical points, metastability regions, and phase separation. We comment on experimental signatures in ultracold atoms.

DOI: 10.1103/PhysRevLett.99.130406 PACS numbers: 05.30.Jp, 03.75.Mn, 03.75.Ss

Chiral Transition at Finite T



How to study QCD phase transition ?

Ginzburg-Landau-Wilson (GLW) approach : model independent, analytic

1. Topological structure of the phase diagram
2. Order of the phase transition
3. Critical properties

Recipe

$$Z = \int [d\sigma] \exp\left(- \int d\mathbf{x}\ \mathcal{L}_{\rm eff}(\sigma(\mathbf{x}); K) \right)$$

$\sigma(\mathbf{x})$: Order parameter field

$$\mathcal{L}_{\rm eff} = \frac{1}{2}(\nabla\sigma)^2 + \sum_n a_n(K)\sigma^n$$

Same symmetry with underlying theory

$K = \{ T, m, \mu, \ldots \}$: External parameters

Ginzburg-Landau = Saddle point approximation

Wilson = Fluctuations in renormalization group method

Caution

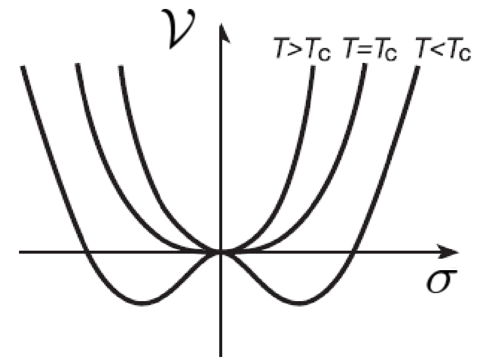
- Valid for continuous or weak 1st order transitions
- Choice of $\sigma(\mathbf{x})$ is an "art"
- Results should be eventually checked by lattice QCD

Some examples of GL potential

- 2nd order phase transition

$$\mathcal{V} = \frac{1}{2}a\sigma^2 + \frac{1}{4}b\sigma^4$$

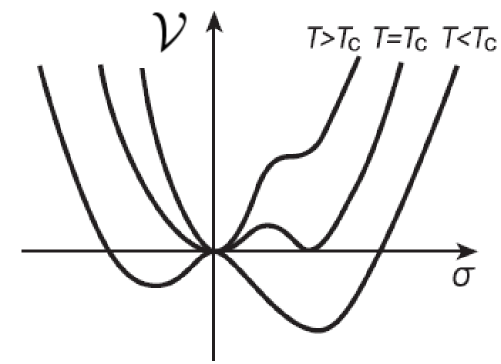
Z(2) Ising model
N_f=2 QCD



- 1st order phase transition

$$\mathcal{V} = \frac{1}{2}a\sigma^2 - \frac{1}{3}c\sigma^3 + \frac{1}{4}b\sigma^4$$

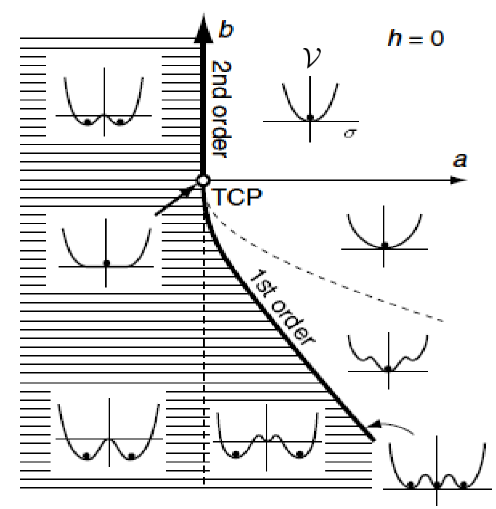
Z(3) Potts model
N_f=3 QCD



- Tri-critical behavior

$$\mathcal{V} = \frac{1}{2}a\sigma^2 + \frac{1}{4}b\sigma^4 + \frac{1}{6}c\sigma^6$$

Meta-magnet
N_f=2+1 QCD



GLW analysis of hot QCD

Pisarski & Wilczek ('84)

Symmetry: $SU(3)_C \times [SU(N_f)_L \times SU(N_f)_R] \times U(1)_B \times \cancel{U(1)}_A$

Chiral field: $\Phi_{ij} \sim \frac{1}{2}\bar{q}^j(1-\gamma_5)q^i = \bar{q}_R{}^j q_L^i$

Chiral transformation: $\Phi \rightarrow e^{-2i\alpha_A} \; V_L \; \Phi \; V_R^\dagger$

$$\mathcal{L}_{\rm eff} = \frac{1}{2}\,\mathrm{tr}\,\partial\Phi^\dagger\partial\Phi + \frac{a}{2}\,\mathrm{tr}\,\Phi^\dagger\Phi + \frac{b_1}{4!}\left(\mathrm{tr}\,\Phi^\dagger\Phi\right)^2 + \frac{b_2}{4!}\,\mathrm{tr}\left(\Phi^\dagger\Phi\right)^2$$

} $SU(N_f)_L x SU(N_f)_R x U(1)_A$

$$-\frac{c}{2}\left(\det\Phi + \det\Phi^\dagger\right)$$

} $SU(N_f)_L x SU(N_f)_R$

$$-\frac{1}{2}\,\mathrm{tr}\,h(\Phi+\Phi^\dagger).$$

} quark mass term

Axial anomaly

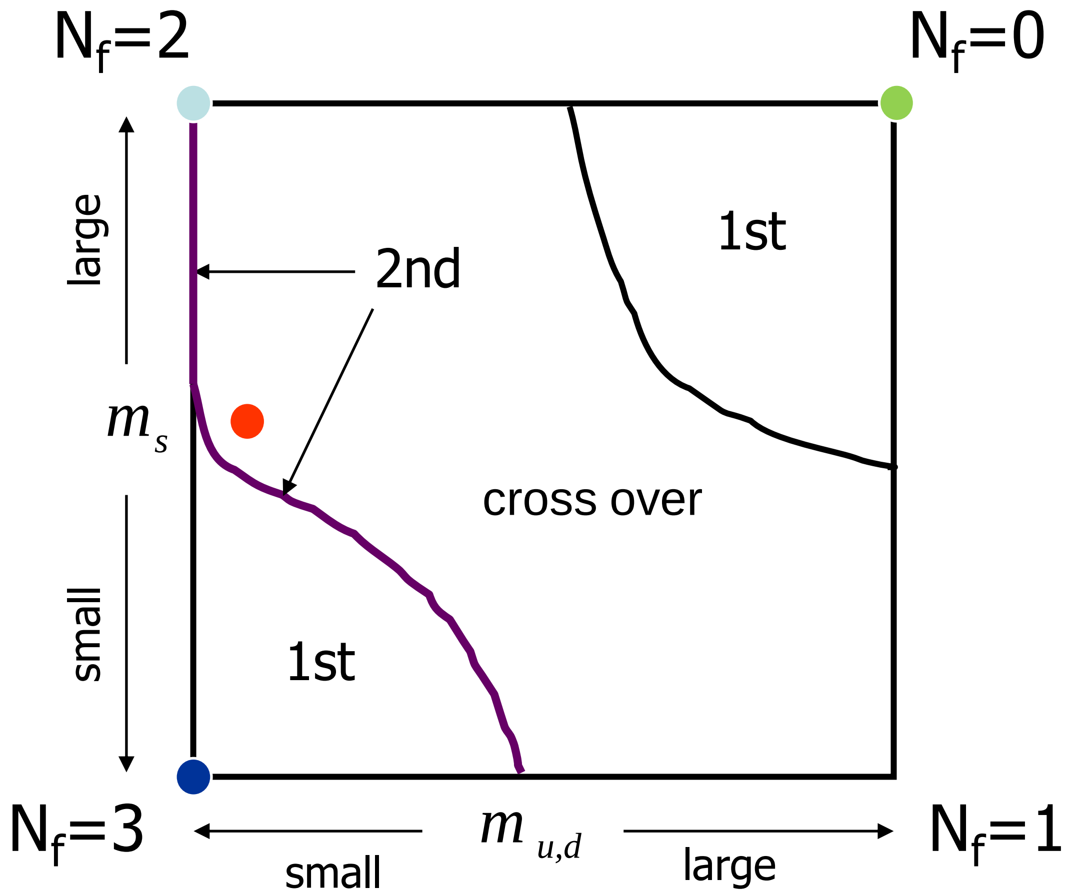
Order of the thermal QCD transition (μ=0)

Svetitsky & Yaffe, NPB210 ('82)

Pisarski and Wilczek, PRD29 ('84)

$$\mathcal{V} = \frac{a}{2}\phi^2 + \frac{b}{4}\phi^4 - h\phi$$

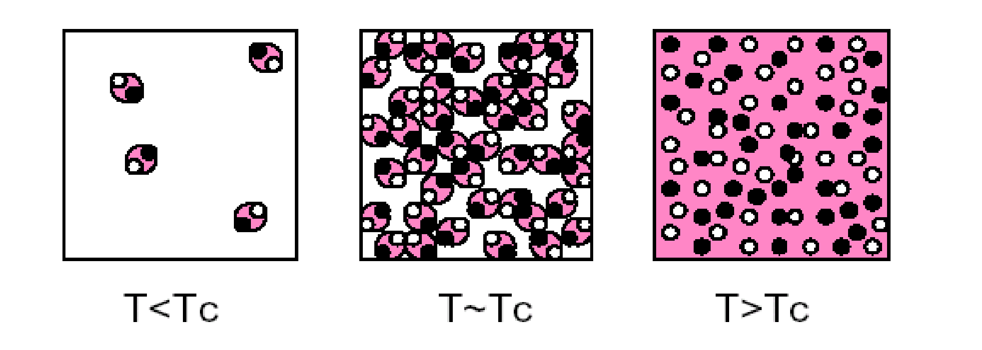
$$\mathcal{V} = \frac{a}{2}L^2 - \frac{c}{3}L^3 + \frac{b}{4}L^4 - hL$$

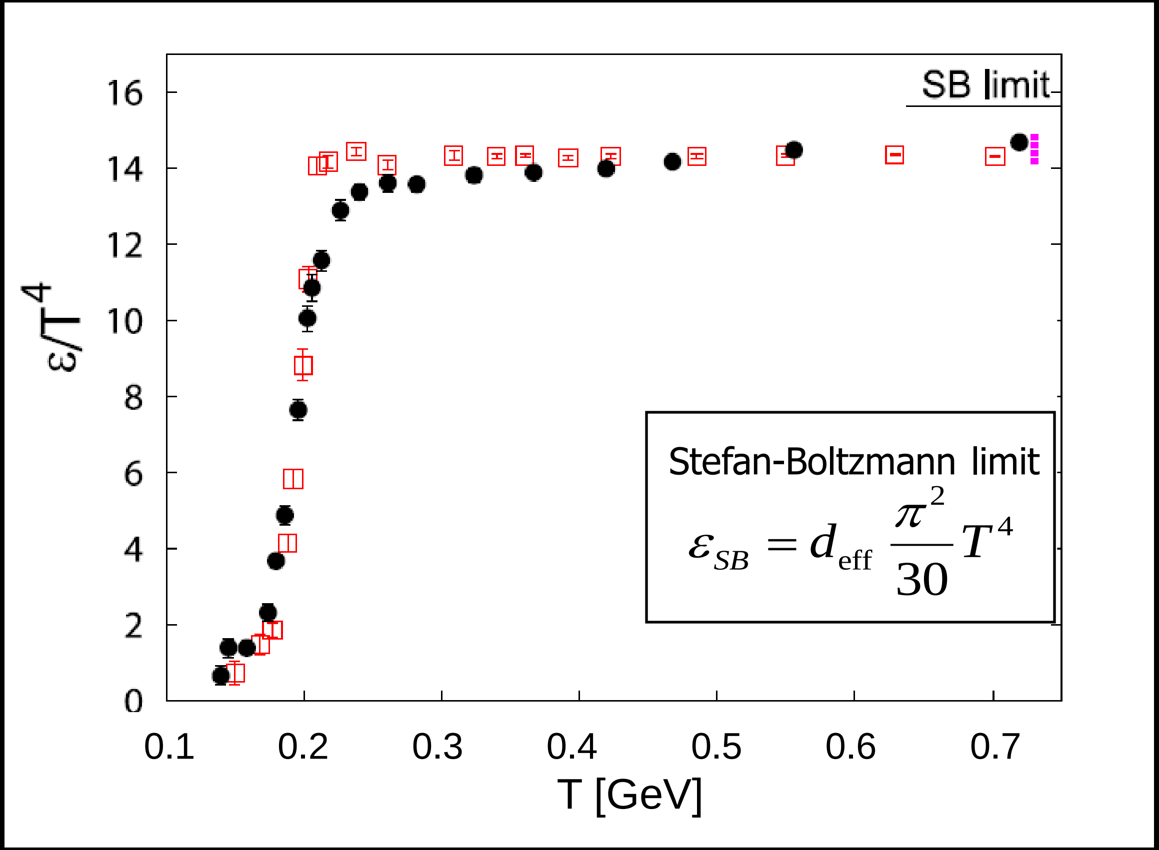


$$\mathcal{V} = \frac{a}{2}\sigma^2 - \frac{c}{3}\sigma^3 + \frac{b}{4}\sigma^4 - h\sigma$$

$$\mathcal{V} = -A\chi + \frac{a}{2}\chi^2 + \frac{b}{4}\chi^4$$

Thermal Transition on the Lattice: (2+1)-flavor, KS fermion, m_π=220 MeV





Lattice QCD: Cheng et al., Phys. Rev. D77 (2008) 014511

Critical Temperature

T_c : 150 – 200 MeV

$\sim 10^{12}$ [K]

Critical Energy Density

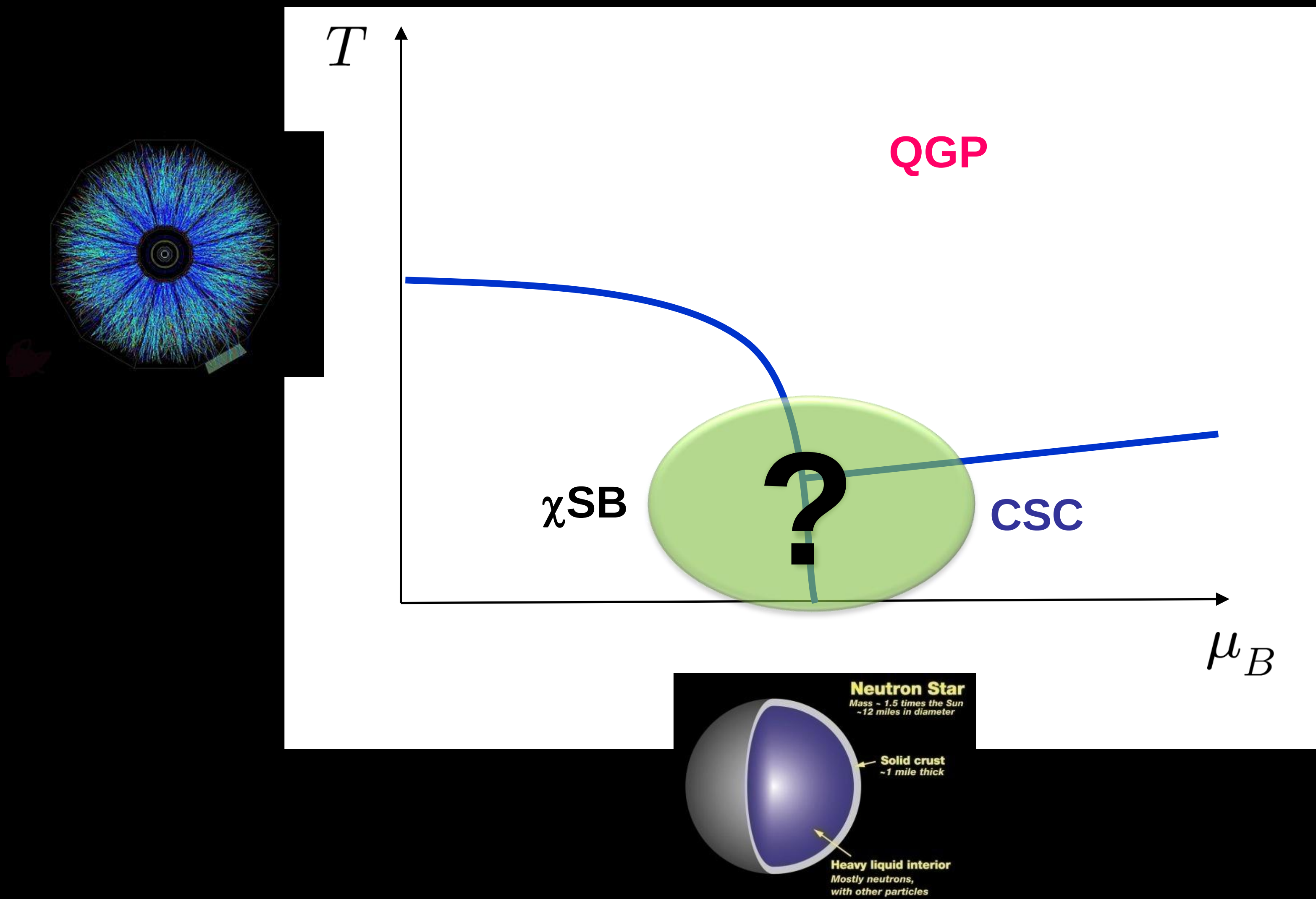
ε_c : ~ 2 GeV/fm^3

~ 10 ε_{nm}

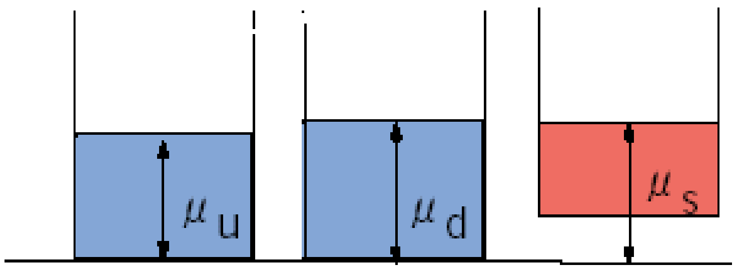
Order of Phase Transition

2nd order	(u,d; m=0)
1st order	(u,d,s; m=0)
crossover	(real world)

Chiral-super interplay at finite μ



Color superconductivity at high density



$$(d_L)_{ia} \sim \epsilon_{ijk}\epsilon_{abc}(q_L)_b^j C (q_L)_c^k$$

$$(d_R)_{ia} \sim \epsilon_{ijk}\epsilon_{abc}(q_R)_b^j C (q_R)_c^k$$

flavor color

major differences from the standard BCS superconductor

1. Relativistic fermi system
   color-magnetic int. dominant

   Son, PRD59 ('99),
   Schafer & Wilczek, PRD60 ('99)
   Pisarski & Rischke, PRD61 ('00)

$$|d| \sim \varepsilon_F \; e^{-c/\sqrt{\alpha_s}}$$

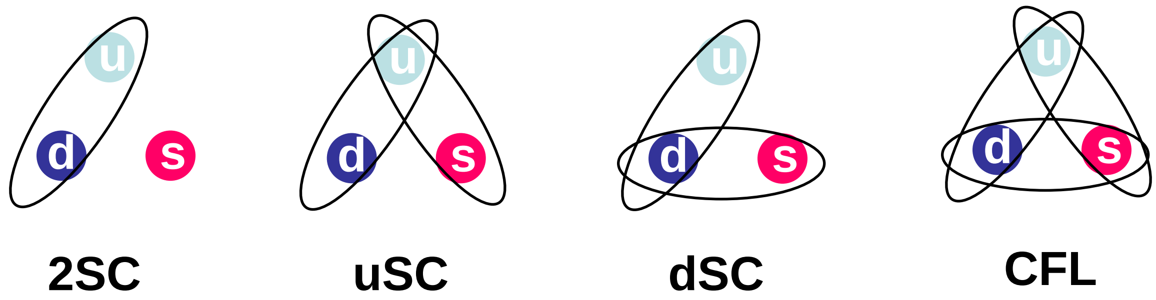
High T_c : $T_c/\varepsilon_F \sim 0.1$
Compact pair : r~ 1-10 fm

2. Color-flavor entanglement

$$d_{ia}$$

Various phases (c.f. Ice, ^{3}He)
2SC, uSC, dSC, CFL etc

Color superconductivity at high density



major differences from the standard BCS superconductor

1. Relativistic fermi system
   color-magnetic int. dominant

   Son, PRD59 ('99),
   Schafer & Wilczek, PRD60 ('99)
   Pisarski & Rischke, PRD61 ('00)

→ $|d| \sim \varepsilon_F \; e^{-c/\sqrt{\alpha_s}}$

- High T_c : $T_c/\varepsilon_F \sim 0.1$
- Compact pair : $r \sim$ 1-10 fm

2. Color-flavor entanglement

$$d_{ia}$$

→ Various phases (c.f. Ice, ^{3}He)
2SC, uSC, dSC, CFL etc

GL analysis for chiral-super interplay in QCD (N_f=3)

Symmetry: $$SU(3)_C \times [SU(3)_L \times SU(3)_R] \times U(1)_B \times U(1)_A$$

Chiral field:

$$\Phi_{ij} \sim (\bar{q}_R)^j_a (q_L)^i_a$$

$$\Phi \rightarrow \mathrm{e}^{-2i\alpha_A} V_L \Phi V_R^\dagger$$

diquark field:

$$(d_L)_{ia} \sim \epsilon_{ijk}\epsilon_{abc}(q_L)^j_b C (q_L)^k_c$$

$$d_L \rightarrow e^{2i\alpha_A} e^{2i\alpha_B} V_L d_L V_C^{\mathrm{T}}$$

$$\mathcal{V}(\Phi, d) = \mathcal{V}_\chi(\Phi) + \mathcal{V}_d(d_L, d_R) + \mathcal{V}_{\chi d}(\Phi, d_L, d_R)$$

- $\mathcal{V}_\chi(\Phi)$: Pisarski & Wilczek, PRD29 ('84)
- $\mathcal{V}_d(d_L, d_R)$:
  - Iida & Baym, PRD63 ('01)
  - Iida, Matsuura, Tachibana & TH, PRD71 ('05)
- $\mathcal{V}_{\chi d}(\Phi, d_L, d_R)$: Yamamoto, TH, Tachibana & Baym, PRL97 ('06)

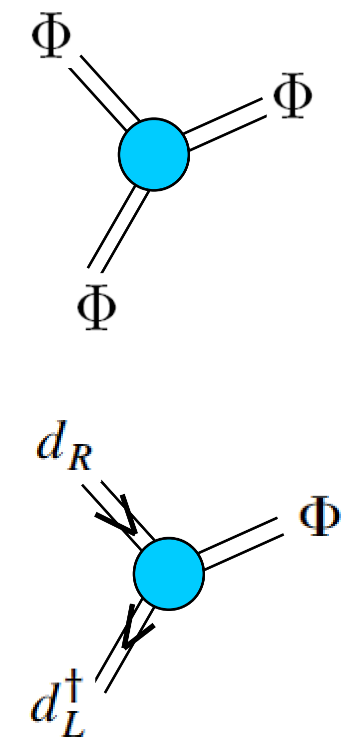
Complete classification of the GL potential (m=0)

$$\mathcal{V}_\chi = \frac{a_0}{2}\mathrm{tr}\,\Phi^\dagger\Phi + \frac{b_1}{4!}\left(\mathrm{tr}\,\Phi^\dagger\Phi\right)^2 + \frac{b_2}{4!}\mathrm{tr}\left(\Phi^\dagger\Phi\right)^2 - \frac{c_0}{2}\left(\det\Phi + \det\Phi^\dagger\right),$$

$$\mathcal{V}_d = \alpha_0\ \mathrm{tr}[d_L d_L^\dagger + d_R d_R^\dagger] + \beta_1\left([\mathrm{tr}(d_L d_L^\dagger)]^2 + [\mathrm{tr}(d_R d_R^\dagger)]^2\right) + \beta_2\left(\mathrm{tr}[(d_L d_L^\dagger)^2] + \mathrm{tr}[(d_R d_R^\dagger)^2]\right) + \beta_3\ \mathrm{tr}[(d_R d_L^\dagger)(d_L d_R^\dagger)] + \beta_4\ \mathrm{tr}(d_L d_L^\dagger)\mathrm{tr}(d_R d_R^\dagger)$$

$$\mathcal{V}_{\chi d} = \gamma_1\ \mathrm{tr}[(d_R d_L^\dagger)\Phi + (d_L d_R^\dagger)\Phi^\dagger] + \lambda_1\ \mathrm{tr}[(d_L d_L^\dagger)\Phi\Phi^\dagger + (d_R d_R^\dagger)\Phi^\dagger\Phi] + \lambda_2\ \mathrm{tr}[d_L d_L^\dagger + d_R d_R^\dagger]\cdot\mathrm{tr}[\Phi^\dagger\Phi] + \lambda_3\left(\det\Phi\cdot\mathrm{tr}[(d_L d_R^\dagger)\Phi^{-1}] + h.c\right)$$

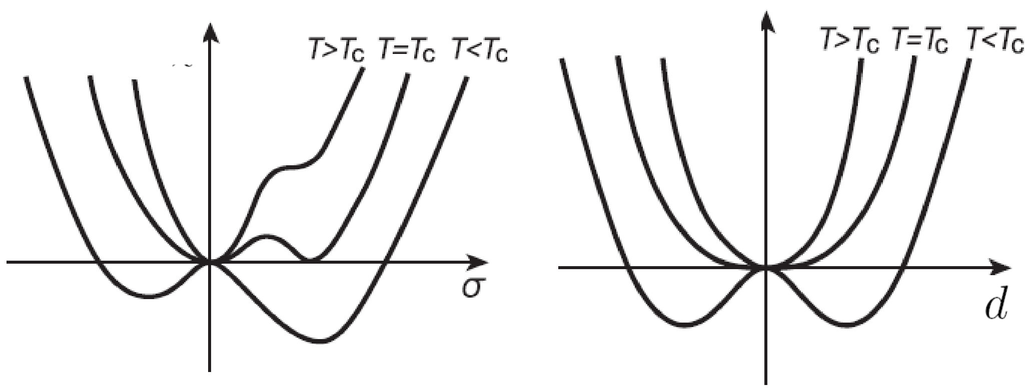
Axial anomaly



Chiral-CFL interplay in N_f=3

$$\Phi = \begin{pmatrix} \sigma & & \\ & \sigma & \\ & & \sigma \end{pmatrix} \qquad d_L = -d_R = \begin{pmatrix} d & & \\ & d & \\ & & d \end{pmatrix}$$

$$\mathcal{V} = \left(\frac{a}{2}\sigma^2 - \frac{c}{3}\sigma^3 + \frac{b}{4}\sigma^4\right) + \left(\frac{\alpha}{2}d^2 + \frac{\beta}{4}d^4\right) - \gamma d^2\sigma + \lambda d^2\sigma^2$$

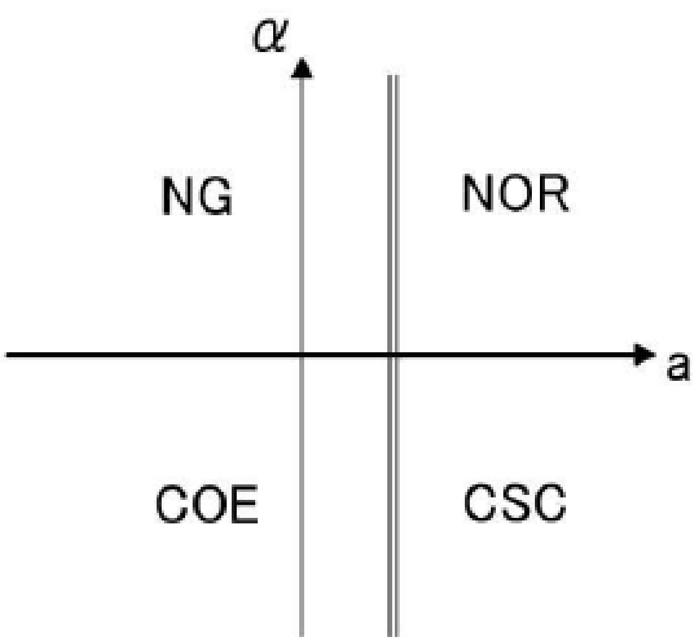


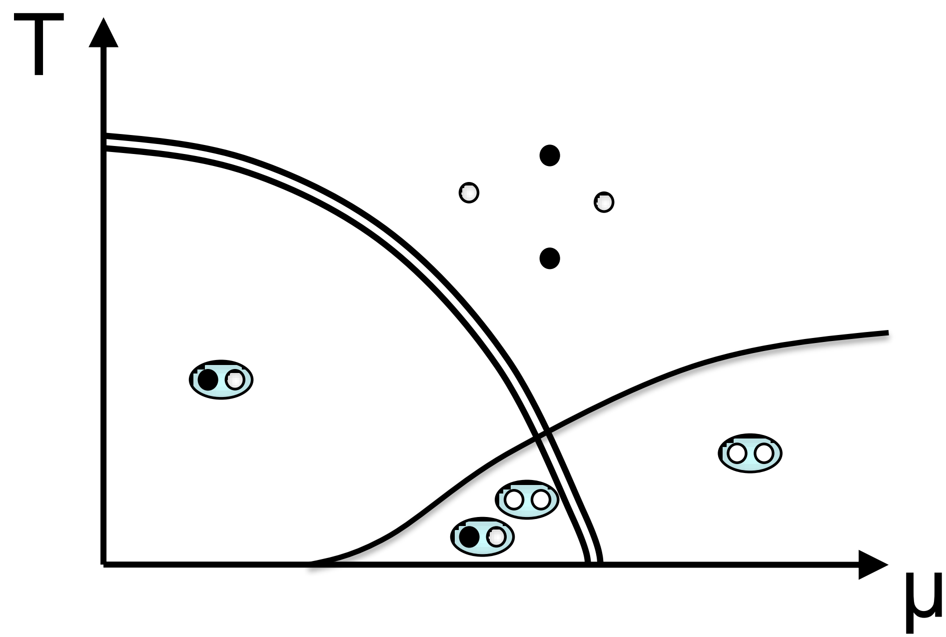
Natural parameter relations

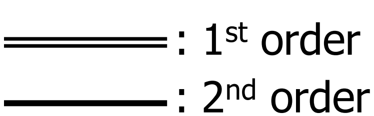
$$\begin{cases} \beta > 0, \ b > 0 \\ \gamma \sim c > 0 \\ 1 \gg \lambda/\beta > 0 \end{cases}$$

phase diagram (without d-σ coupling)

$$\mathcal{V} = \left(\frac{a}{2}\sigma^2 - \frac{c}{3}\sigma^3 + \frac{b}{4}\sigma^4\right) + \left(\frac{\alpha}{2}d^2 + \frac{\beta}{4}d^4\right) - \gamma d^2\sigma$$

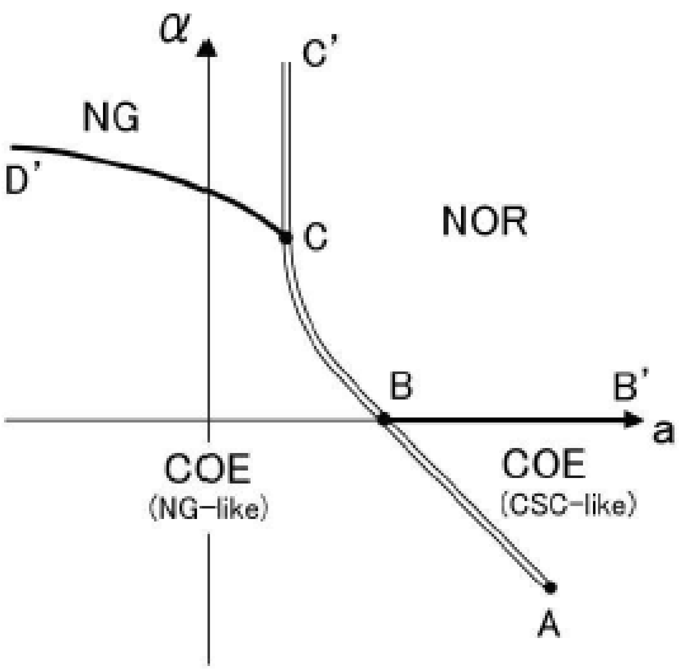


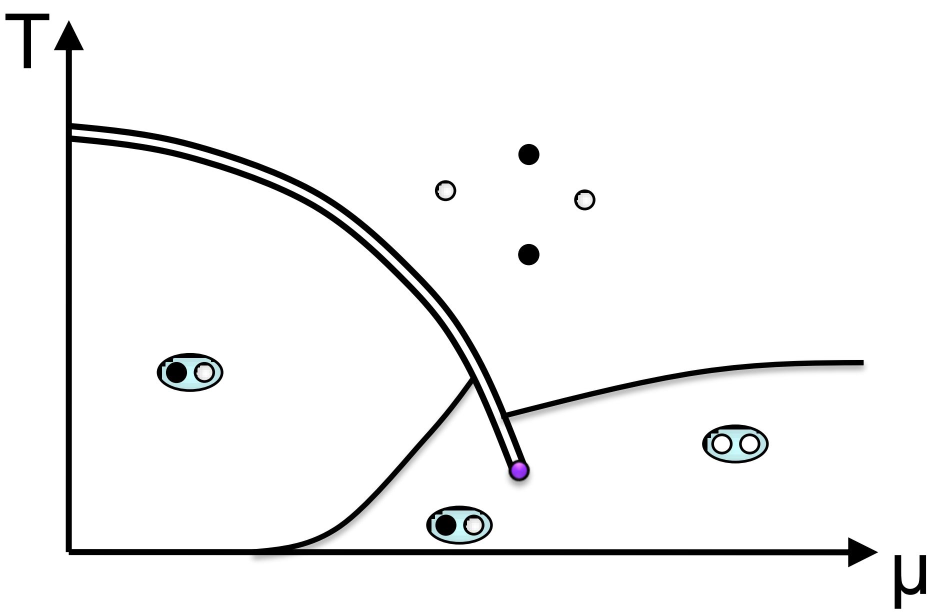




phase diagram (with d-σ coupling)

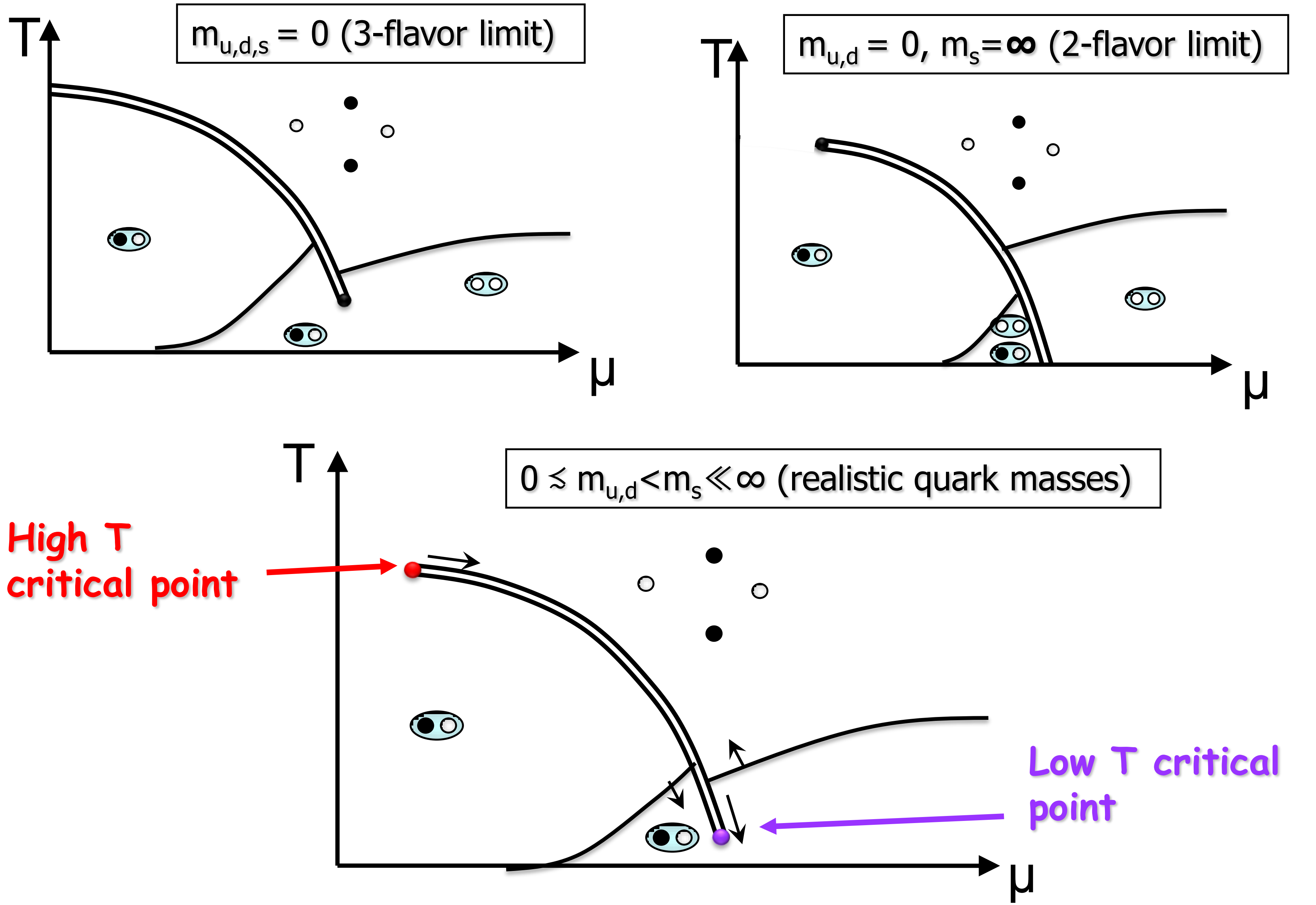
$$\mathcal{V} = \left(\frac{a}{2}\sigma^2 - \frac{c}{3}\sigma^3 + \frac{b}{4}\sigma^4\right) + \left(\frac{\alpha}{2}d^2 + \frac{\beta}{4}d^4\right) - \gamma d^2 \sigma$$





A new critical point driven by the axial anomaly

Realistic phase diagram in N_f=2+1 ?



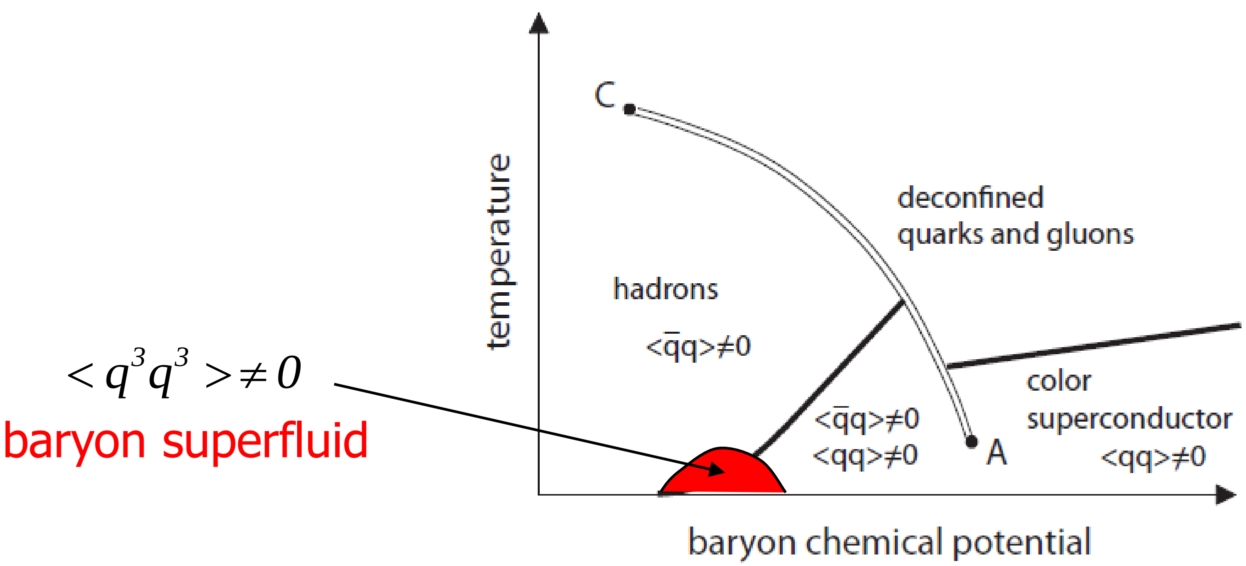
Frequently asked questions

1. Location of the new critical point in physical unit ?

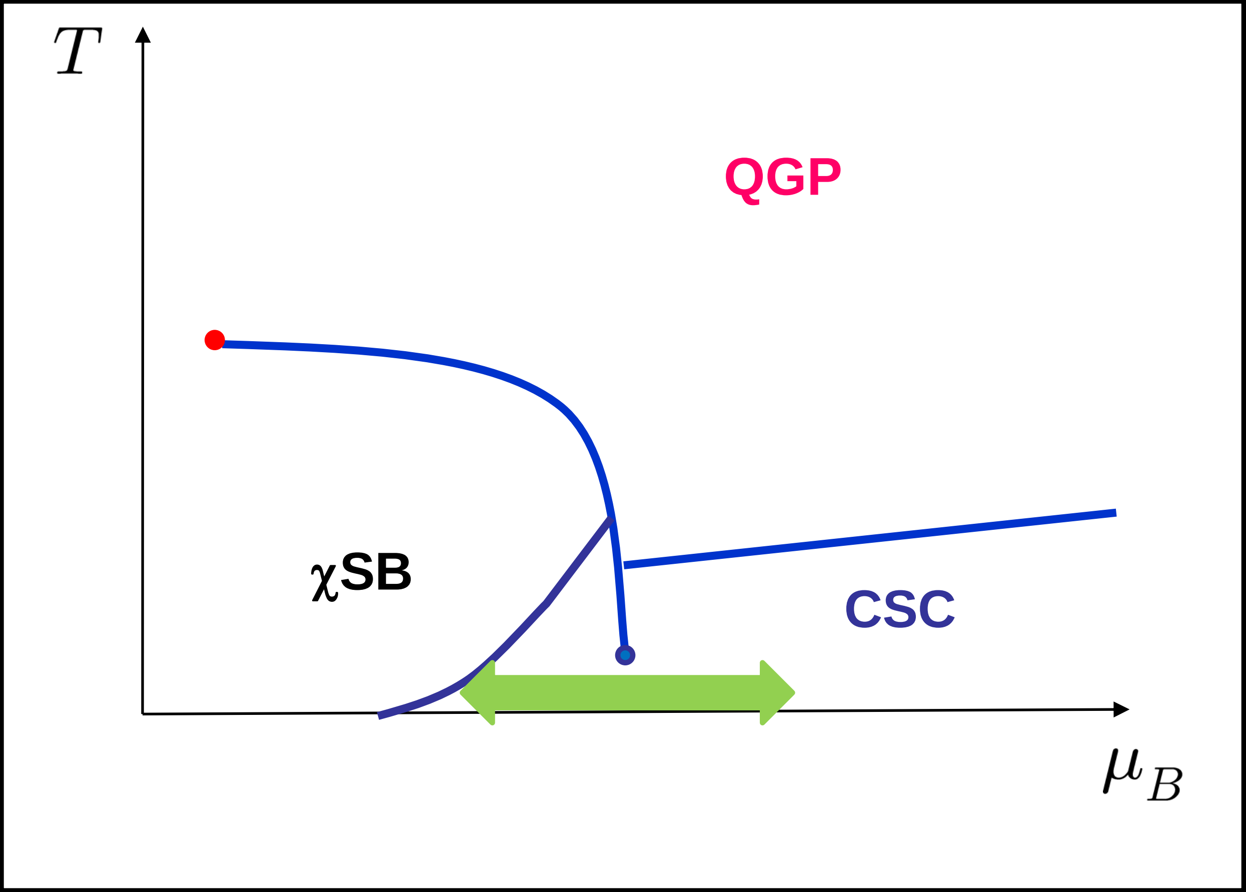
   - No definite answer at present
   - NJL & PNJL model calculation underway

2. Connection to baryon superfluidity at low μ ?

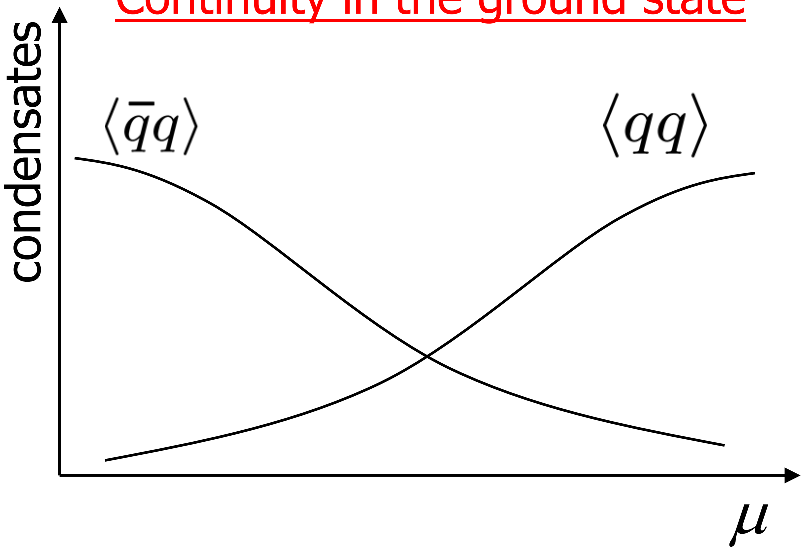
   - Mechanism for the crossover from $<q^2>$ to $<q^3q^3>$
     similar to bose-fermi mixture in cold atoms (Maeda, Baym & TH)



Spectral continuity at finite μ



Continuity in the ground state



Continuity in the excited state??

excitation	Low μ	High μ
NGs	π(8) & H	π′ (8) & H
Vectors	V (9)	gluons (8)
Fermions	baryons (8)	Quarks (9)

Schafer and Wilczek, PRL 82 (1999)

Explicit realization of spectral continuity

○ Generalized Gell-Mann-Oakes-Renner relation :

$$m_{\tilde{\pi}}^2 \simeq \frac{m_q}{f_\pi^2 + f_{\pi'}^2} \left[\alpha \langle \bar{q}q \rangle + \beta \langle qq \rangle^2 \right]$$

Yamamoto, Tachibana, Baym + T.H., PR D76 ('07)

○ Gauge invariant method to show the continuity of vector mesons

In-medium QCD sum rules

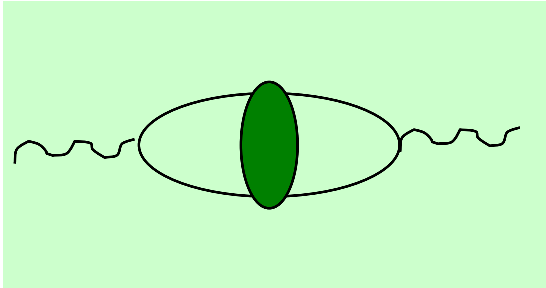
T.H., Tachibana and Yamamoto, PRD78 (2008)

QCD sum rules in the superconducting medium

- Vector current: $J_\mu^{(8)} = \bar{q}\tau^a\gamma_\mu q$, $J_\mu^{(1)} = \bar{q}\tau^0\gamma_\mu q$

- Current correlation function:

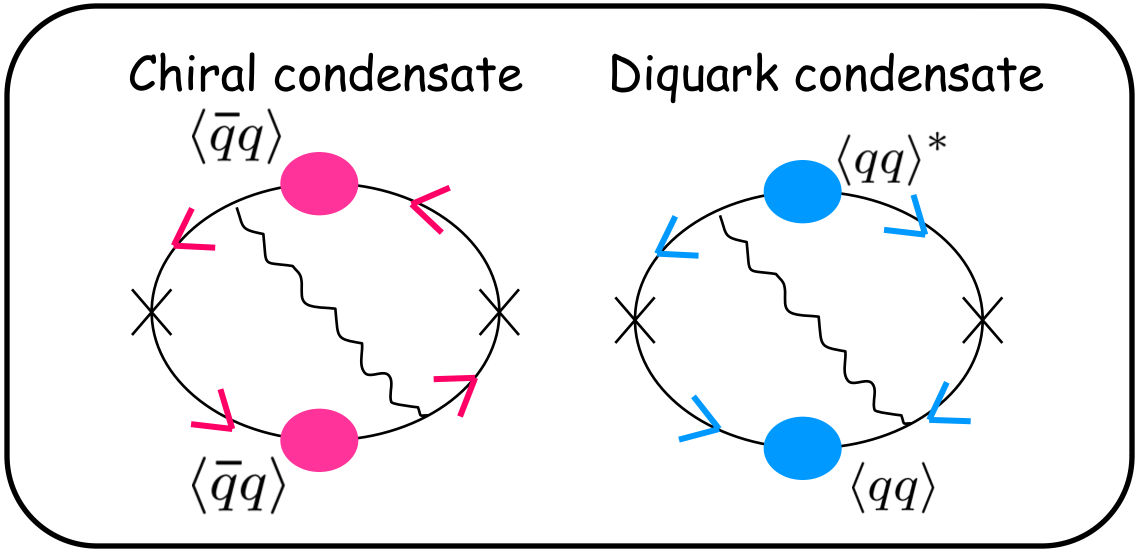
$$\Pi_{\mu\nu}(q) = i\int d^4x e^{iqx}\langle RJ_\mu(x)J_\nu(0)\rangle$$



- Operator Product Expansion (OPE) up to $\mathcal{O}(1/Q^6)$:

4-quark condensate

$$\langle(\bar{q}\Gamma q)(\bar{q}\Gamma q)\rangle$$



Mass formula from Finite Energy Sum Rules

At low density:

$$\left(m_{\rm V}\right)^2 \to \left(\frac{448\pi^3\alpha_s}{27}\langle\bar{q}q\rangle^2\right)^{1/3}$$

At intermediate density:

$$\left(m_{\rm V}^{(8)}\right)^2 \simeq \frac{56\pi^3\alpha_s}{81\mu^4}\left(\langle\bar{q}q\rangle^2 + \frac{15}{7}\langle qq\rangle^2\right)$$

$$\left(m_{\rm V}^{(1)}\right)^2 \simeq \frac{1}{f}\frac{56\pi^3\alpha_s}{81\mu^4}\left(\langle\bar{q}q\rangle^2 - \frac{66}{7}\langle qq\rangle^2\right)$$

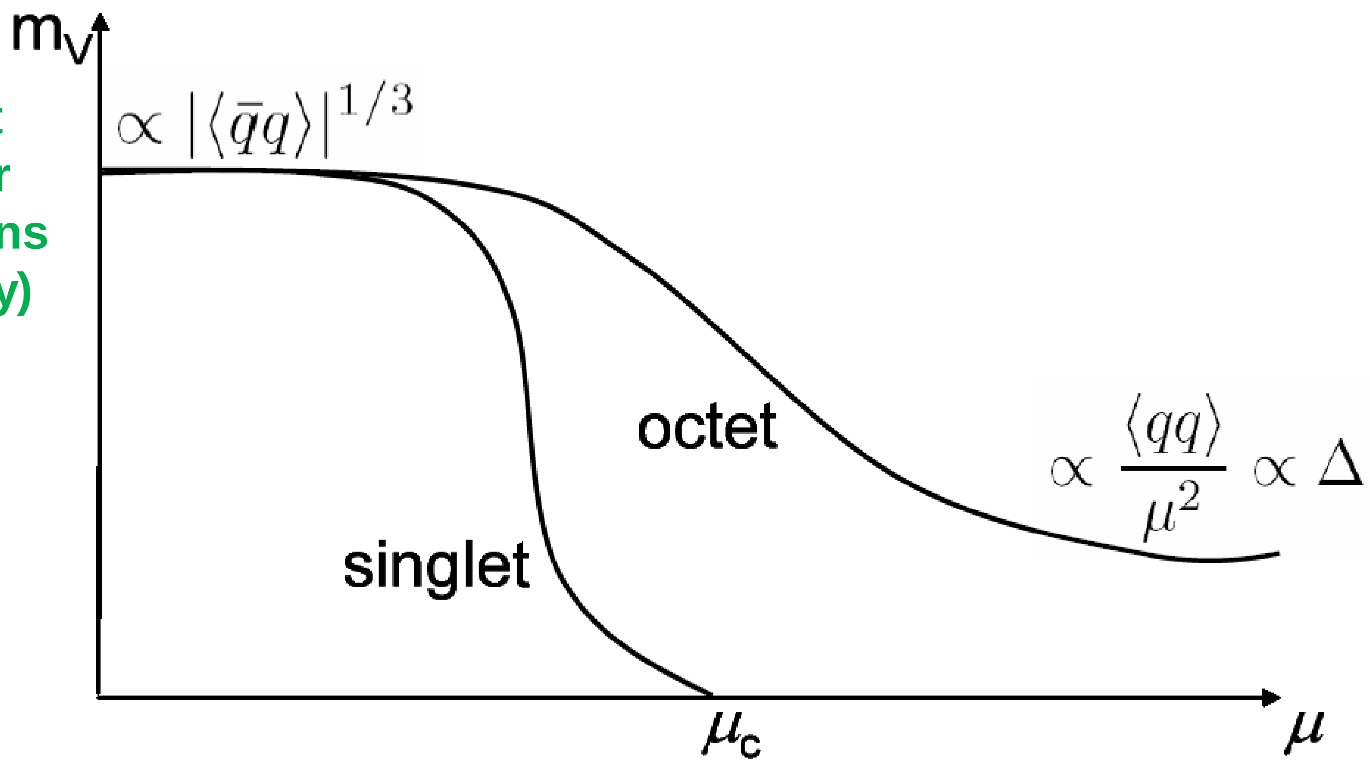
At high density:

$$m_{\rm V}^{(8)} \to \sqrt{\frac{20}{3}}\,\Delta \simeq 2.6\Delta$$

Spectral continuity of vector mesons

T.H., Tachibana and Yamamoto, PRD78 (2008)

Nonet vector mesons (heavy)



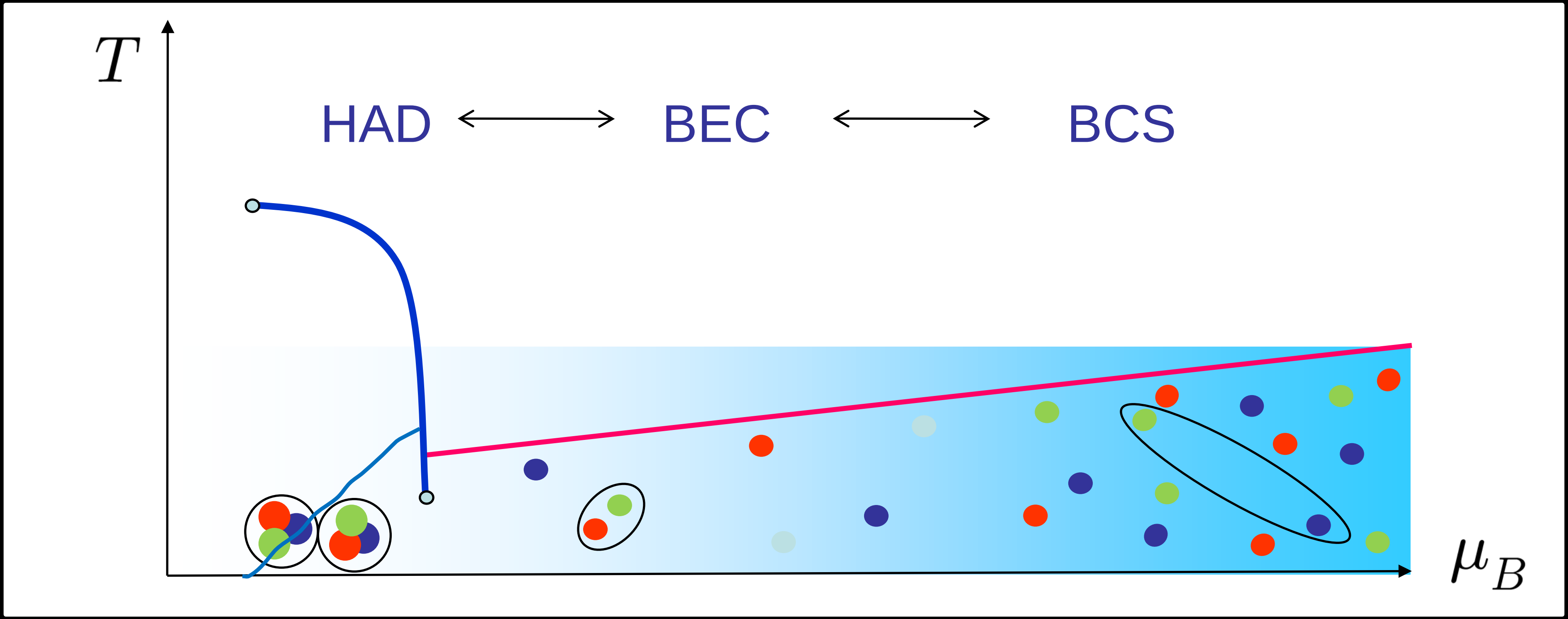
Octet vector mesons (light)

Octet gluons in CFL: m_g=1.362Δ

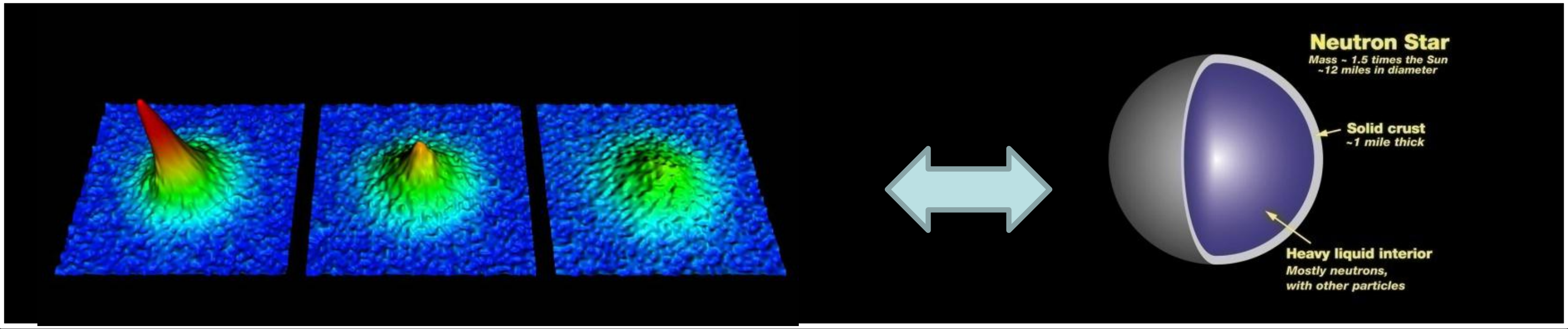
Gusynin & Shovkovy, NPA700 (2002)

Malekzadeh & Rischke, PRD73 (2006)

UCA/QCD correspondence



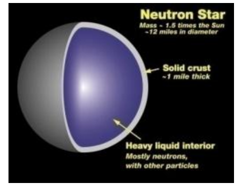
Possibility to simulate BCS-BEC-HAD crossover
using <u>boson-fermion mixture</u> or <u>fermion with three species</u> in cold atoms ?



T[K]

10^{12} Quark-gluon plasma

10^{9} **Superfluid neutron matter**



10^{7} Center of sun

10^{2} Boiling water

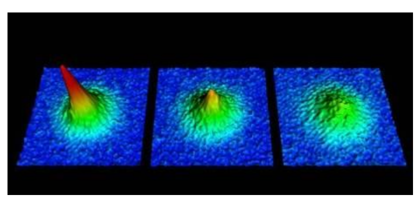
Freezing water

10 Liquid nitrogen

1 Superfluid, superconductor

10^{-3} Superfluid of ^{3}He

10^{-7} **Ultracold atoms**



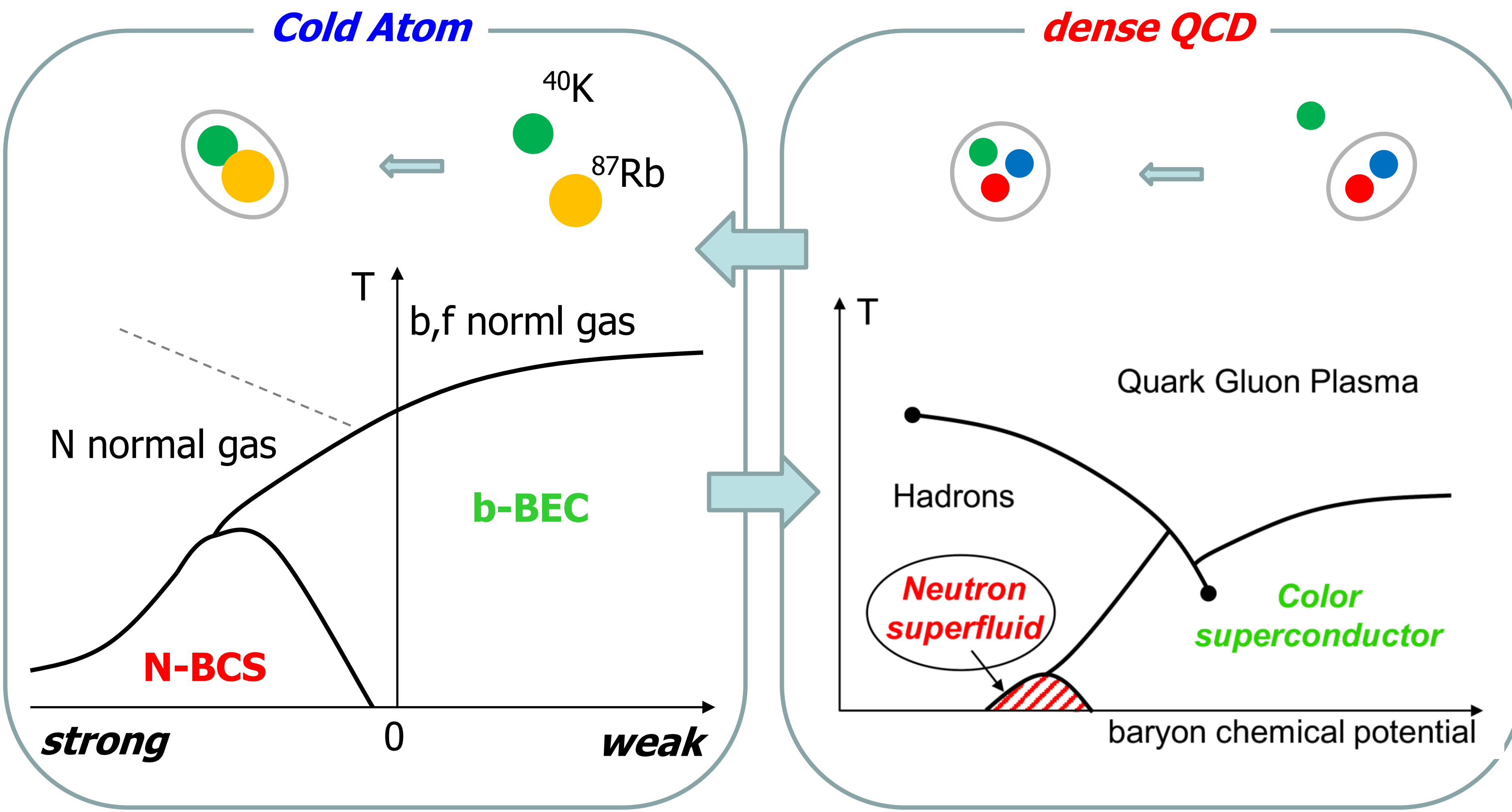
Ultracold Atoms (UcA)

- T ~ 10^{-7} K
- *hyperfine states magnetically controllable*
- density 10^{14} - 10^{15} cm^{-3} (cf. Air ~ $10^{19} cm^{-3}$)

Bose-Fermi mixture in Ultracold Atoms and Dense QCD
-- Induced superfluidity of composite-fermions --

$$a_{\mathrm{NN}}^{\mathrm{Born}} = -\frac{m_{\mathrm{N}}}{2m_{\mathrm{R}}} a_{\mathrm{bf}}$$

Maeda, Baym & Hatsuda ('09)



Phases of attractively interacting boson-fermion mixtures

Kenji Maeda,[1] Gordon Baym,[2], and Tetsuo Hatsuda[1]
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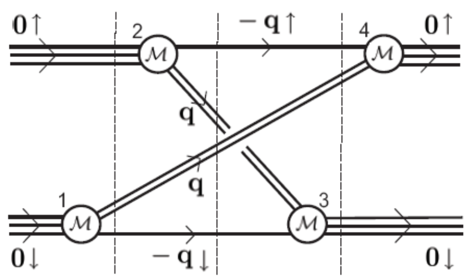
[2]*Department of Physics, University of Illinois, 1110 W. Green St., Urbana, Illinois 61801, USA*

We study a many-body mixture of an equal number of bosons (b) and fermions (f) in two hyperfine states, and having a tunable boson-fermion (b-f) attraction. For weak b-f attraction, the system is a mixture of a Bose condensate and degenerate fermions interacting through density fluctuations, while for strong b-f attraction, the system forms degenerate composite fermions, N=(bf), which are superfluid due to the N-N attaction in the spin singlet channel. We delineate the possible phase structure of the mixture and its symmetry breaking pattern at finite temperature as a function of the b-f coupling strength. The relevance of the results to cold atomic systems and the dense quark matter is also discussed.

$$\mathcal{H} = \frac{1}{2m_{\rm b}}\nabla\phi^*(x)\cdot\nabla\phi(x) - \mu_{\rm b}\phi^*(x)\phi(x) + \frac{1}{2}g_{\rm bb}|\phi(x)|^4 + \frac{1}{2m_{\rm f}}\nabla\psi_\sigma^\dagger(x)\cdot\nabla\psi_\sigma(x) - \mu_{\rm f}\psi_\sigma^\dagger(x)\psi_\sigma(x) + \frac{1}{2}g_{\rm ff}\psi_\sigma^\dagger(x)\psi_\sigma(x)\psi_{-\sigma}^\dagger(x)\psi_{-\sigma}(x) + g_{\rm bf}|\phi(x)|^2\psi_\sigma^\dagger(x)\psi_\sigma(x)\ .$$



$$a_{\rm NN}^{\rm Born} = \frac{m_{\rm N}}{4\pi}{\rm T_N}(\mathbf{0},\mathbf{0}) = -\frac{m_{\rm N}}{2m_{\rm R}}a_{\rm bf}\ .$$

$$T_{\rm c}(\text{N-BCS}) = \frac{\gamma}{\pi}\left(\frac{2}{e}\right)^{7/3}\varepsilon_{\rm N}\exp\left(\frac{\pi}{2k_{\rm F}a_{\rm NN}}\right)\ .$$

Hierarchical spontaneous symmetry breaking

Y. Nambu, *Masses as a problem and as a clue*, May 2004

The BCS mechanism is most relevant to the mass problem because introduces an energy (mass) gap for fermions, and the Goldstone and Higgs modes as low-lying bosonic states. An interesting feature of the SSB is the possibility of hierarchical SSB or "tumbling". Namely an SSB can be a cause for another SSB at lower energy scale.

. . . [examples are]

1. the chain crystal–phonon–superconductivity. . . . Its NG mode is the phonon which then induces the Cooper pairing of electrons to cause superconductivity.

2. the chain QCD–chiral SSB of quarks and hadrons–π and σ mesons–nuclei formation and nucleon pairing–nuclear π and σ modes–nuclear collective modes.

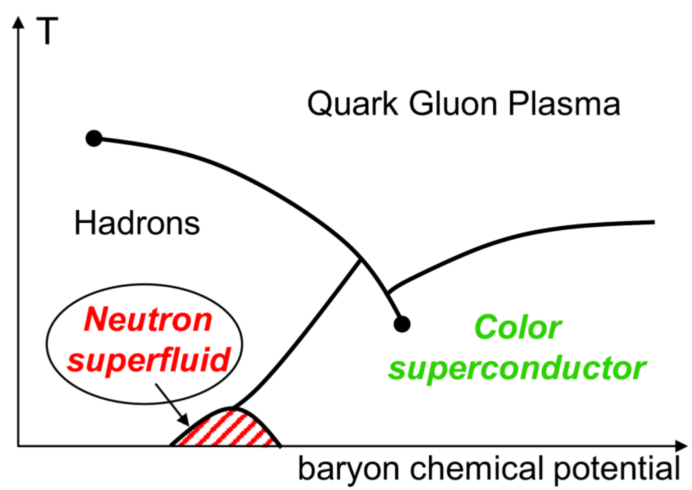
Summary and Future

1. QCD phase structure

- Three major phases in QCD: χSB, QGP and CSC
- Axial anomaly plays crucial roles everywhere
- Close similarity with high Tc supercond. & multi-comp. cold atoms

2. Chiral-super interplay driven by axial anomaly

- A new critical point at low T and high μ
- Continuity of χSB phase and CSC phase



3. Spectral continuity in high density QCD

- Pions are pions.
- Vector mesons are gluons.

4. Future

- Real location of the new critical point ?
- How to detect critical lines and points in lab. experiment ?
- Tabletop simulations of high density QCD using cold atoms ?