Hydro + UrQMD model with the QCD Critical Point

Nagoya University
Chiho NONAKA

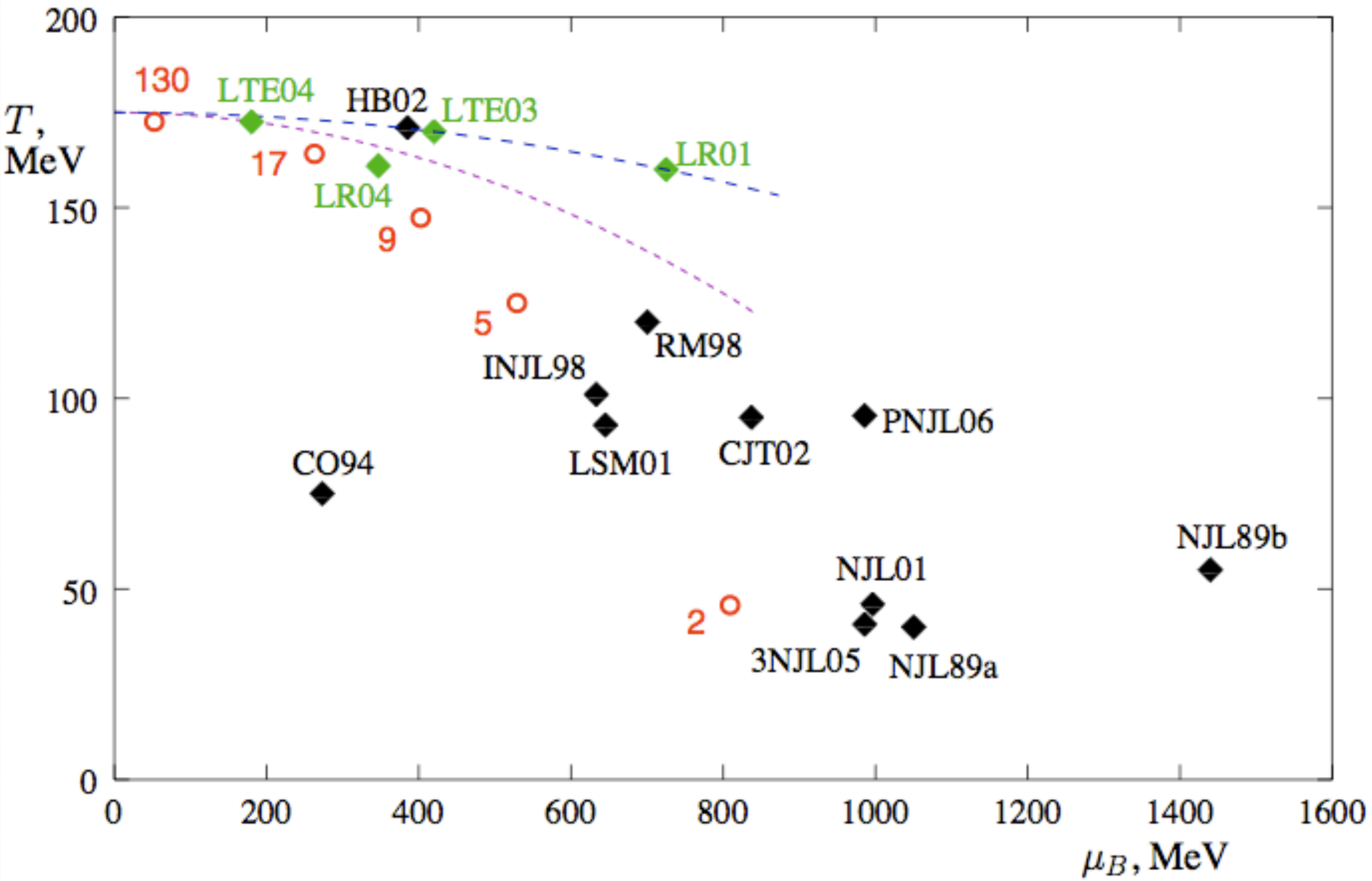
In collaboration with Asakawa, Bass, and Mueller

September 25, 2009@HIM

Where is the QCD Critical Point?

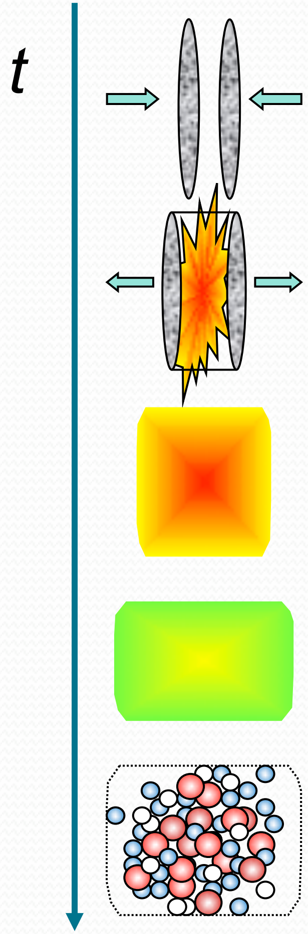
- Lattice QCD, Effective theories….

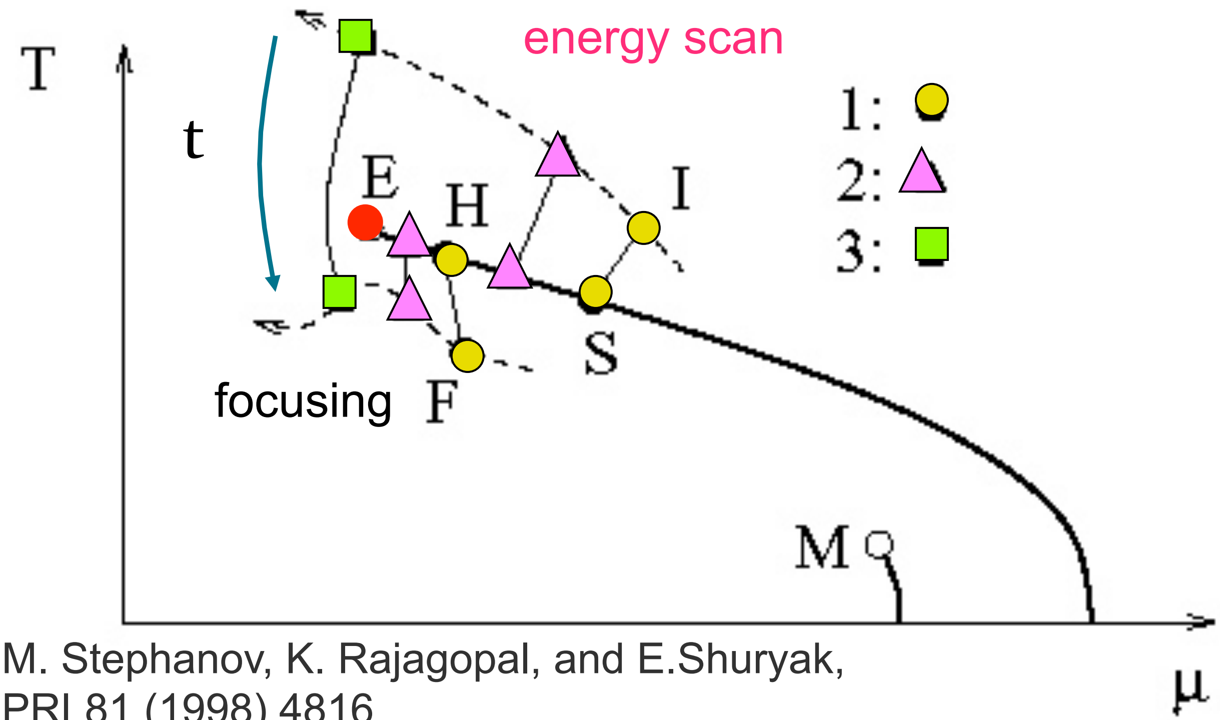
Stephanov,hep-lat/0701002



Chiho Nonaka

QCP Search in Heavy Ion Collisions





M. Stephanov, K. Rajagopal, and E.Shuryak, PRL81 (1998) 4816

- QCP search in heavy ion collisions
  - -Energy scan
  - -Experiments and phenomenology

Chiho Nonaka

Towards Quantitative Analyses

- Realistic dynamical model
  - 3D Hydro + UrQMD (hadron base event generator)
- Equation of state with QCD critical point
- Physical observables

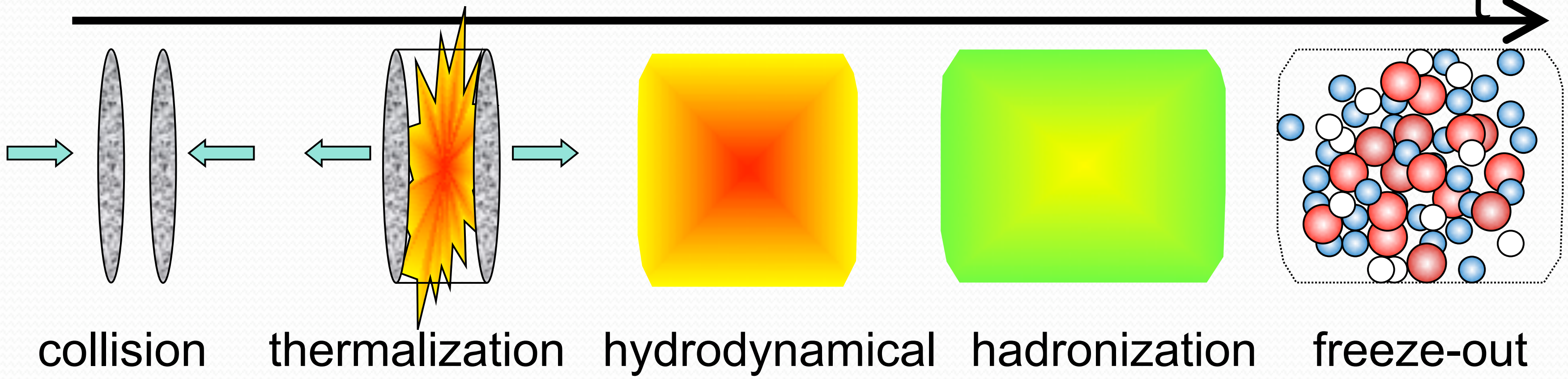
Signals of QCP should survive after freezeout process.

  - Fluctuations
  - Hadron ratios

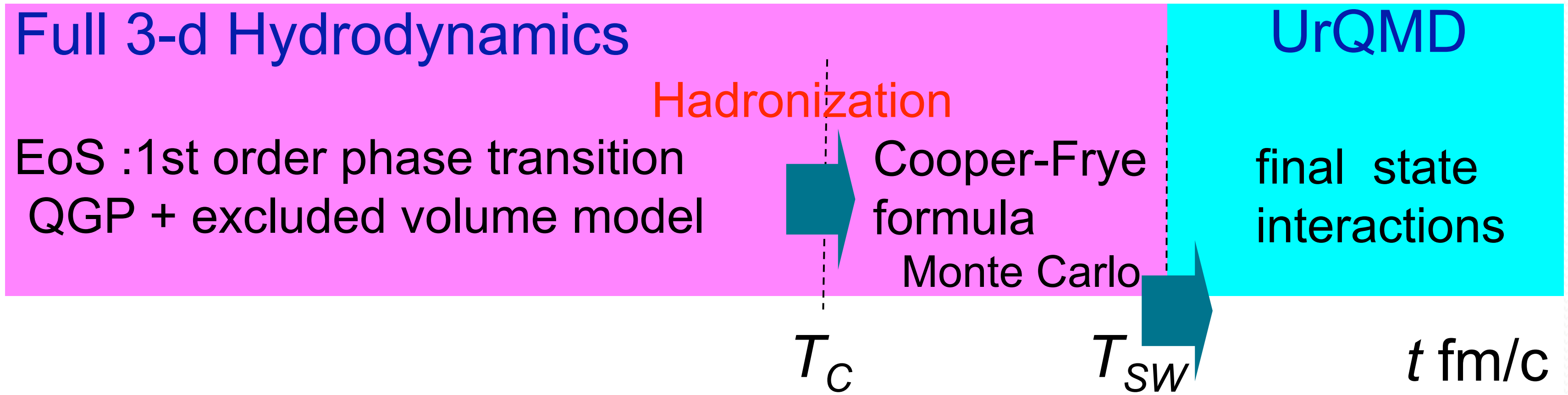
3D Hydro+UrQMD Model

- Schematic sketch

Nonaka and Bass PRC75:014902(2007)



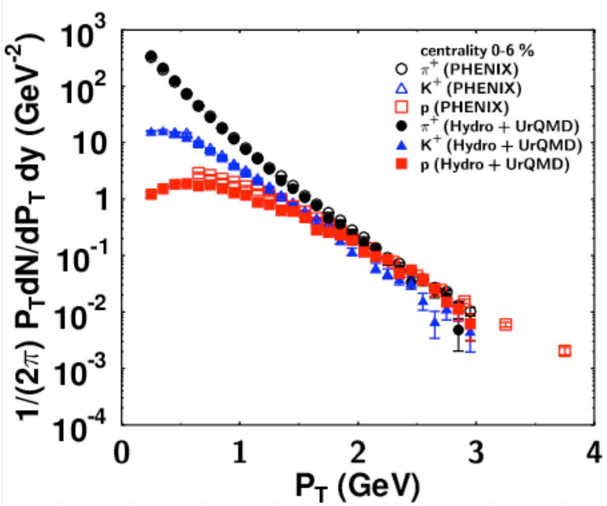
- 3D Hydro + UrQMD

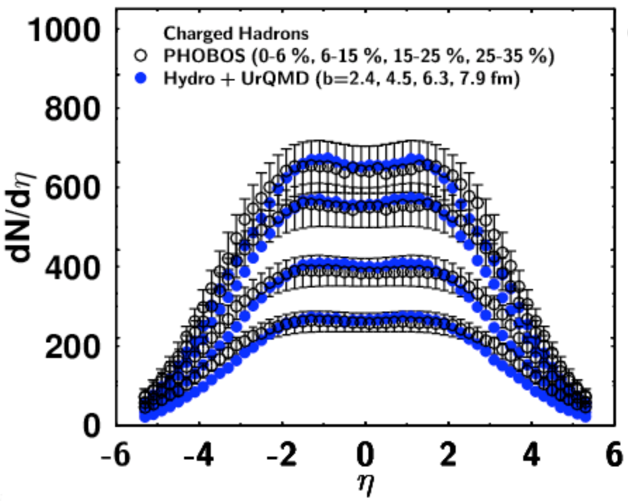


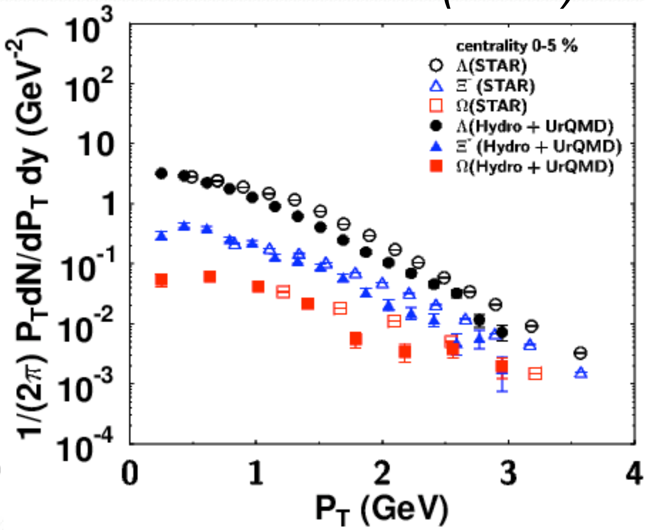
T_C:critical temperature > T_{SW}: Hydro ➔ UrQMD

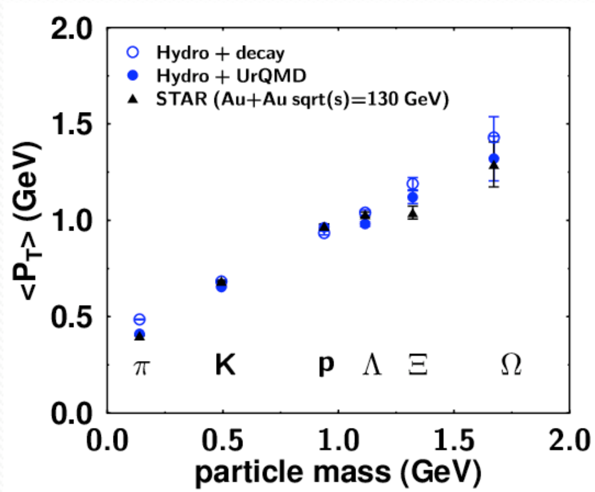
Highlights of 3D Hydro+UrQMD

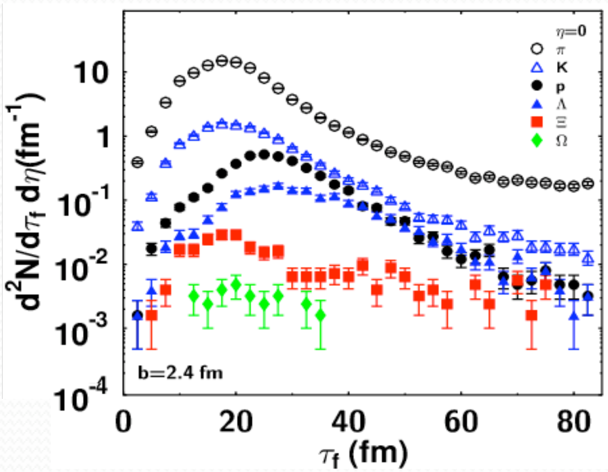
Nonaka and Bass PRC75:014902(2007)

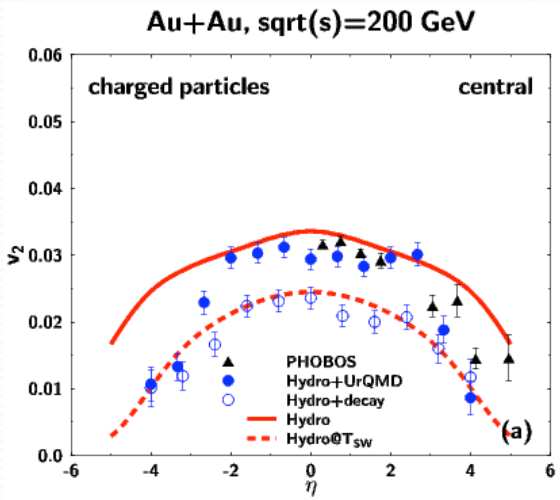










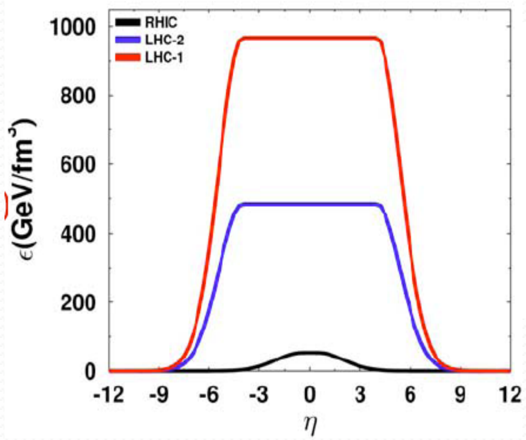


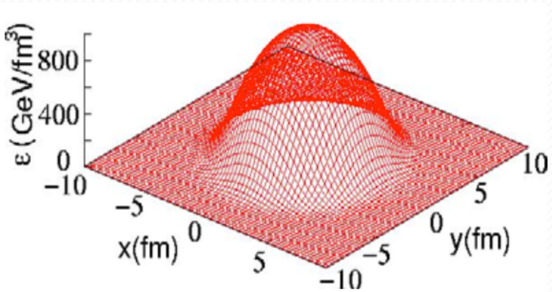
Chiho Nonaka

Initial Parameters for LHC

- Initial Conditions
  - Energy density
    $\varepsilon(x,y,\eta) = \varepsilon_{max} W(x,y;b) H(\eta)$
  - Baryon free
    preliminary results
  - Flow
    $v_L = \eta$ (Bjorken's solution) $v_T = 0$

•longitudinal direction •transverse plane

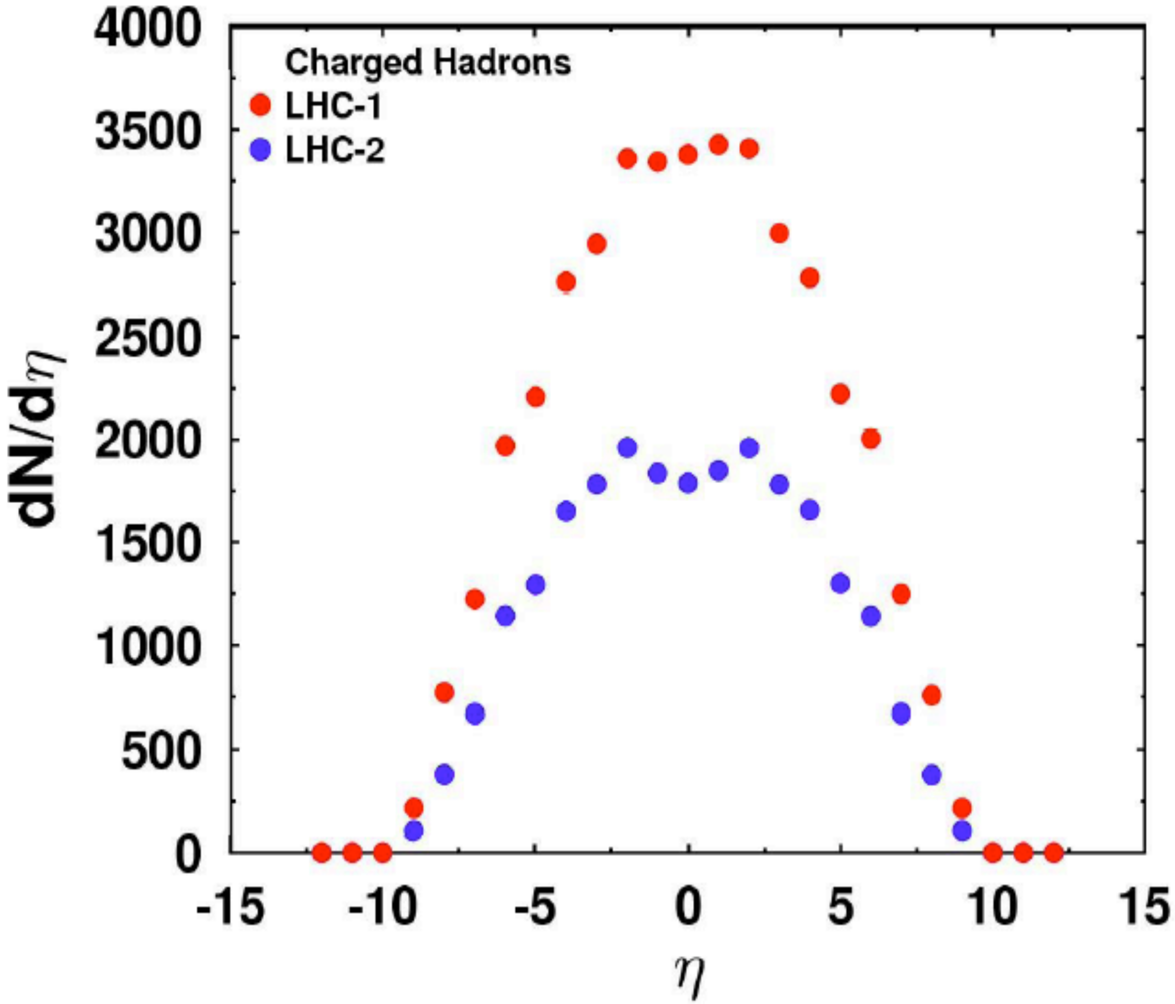




- Equation of State
  1st order phase transition,
  T_c=160 MeV
- Switching temperature
  T_{SW}=150 [MeV]

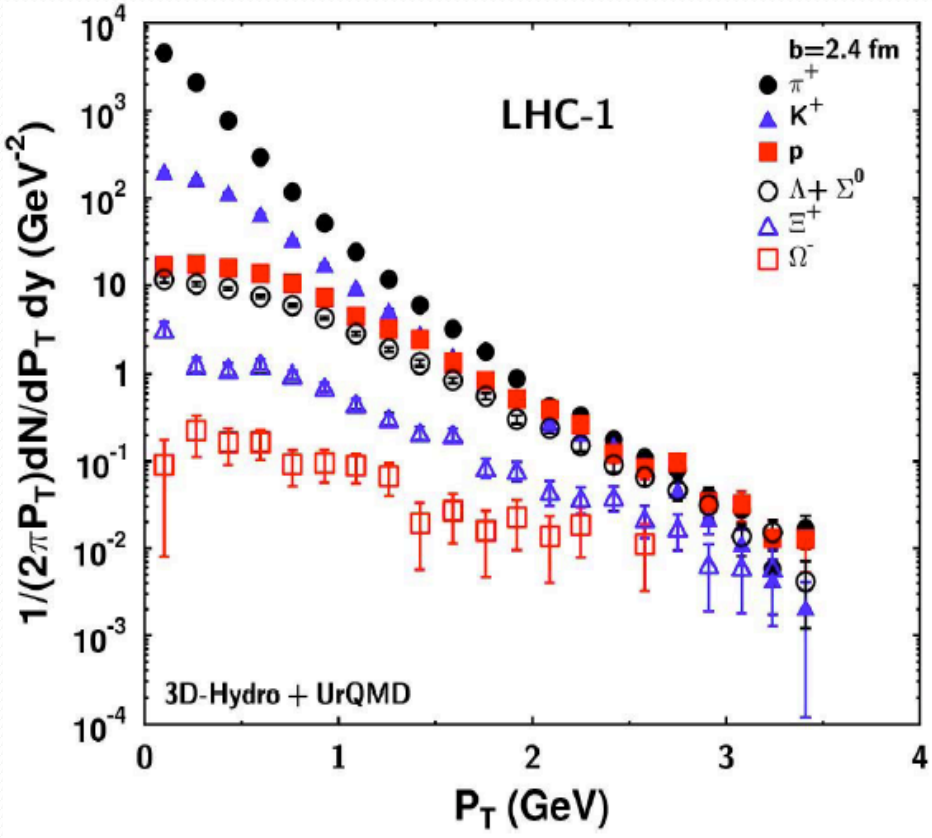
	RHIC	LHC1	LHC2
τ_0(fm)	0.6	0.2	0.2
ε_{max}(GeV/fm^3)	55	1000	500
η_0, σ_η	0.5, 1.5	1.0, 6.0	1.0, 6.0

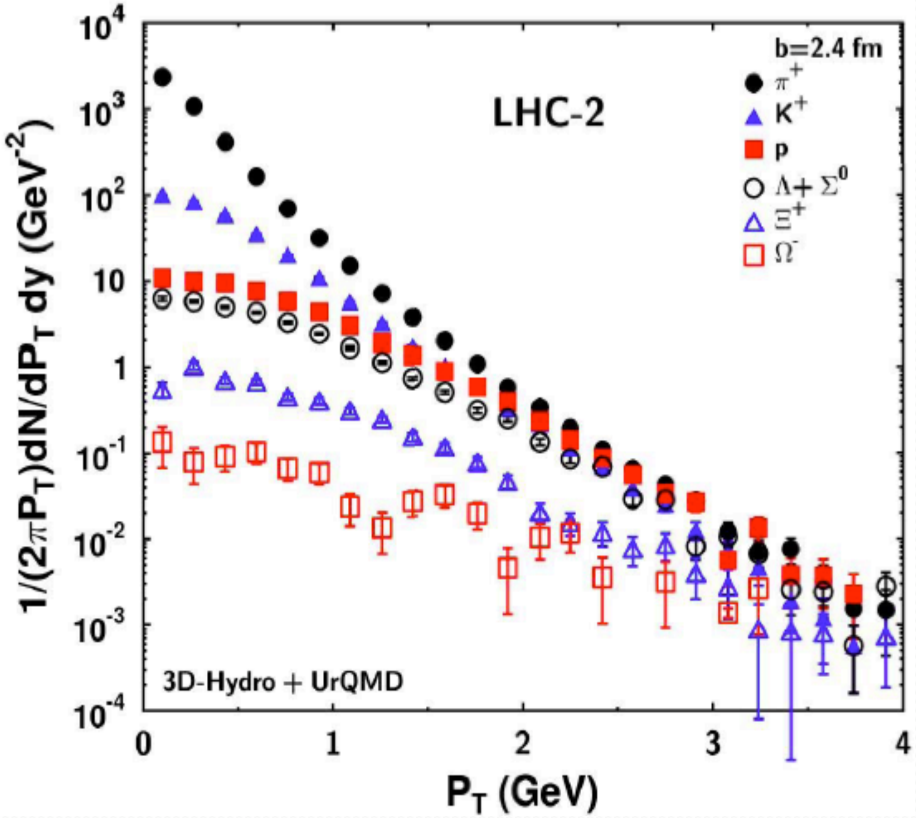
Multiplicities



dN/dy at y_{CM}	LHC-1	LHC-2
π^+	1715	904
K^+	228	123
p	57	34
$\Lambda^0+\Sigma^0$	33	19
Ξ^+	4.3	2.5
Ω^-	0.85	0.52

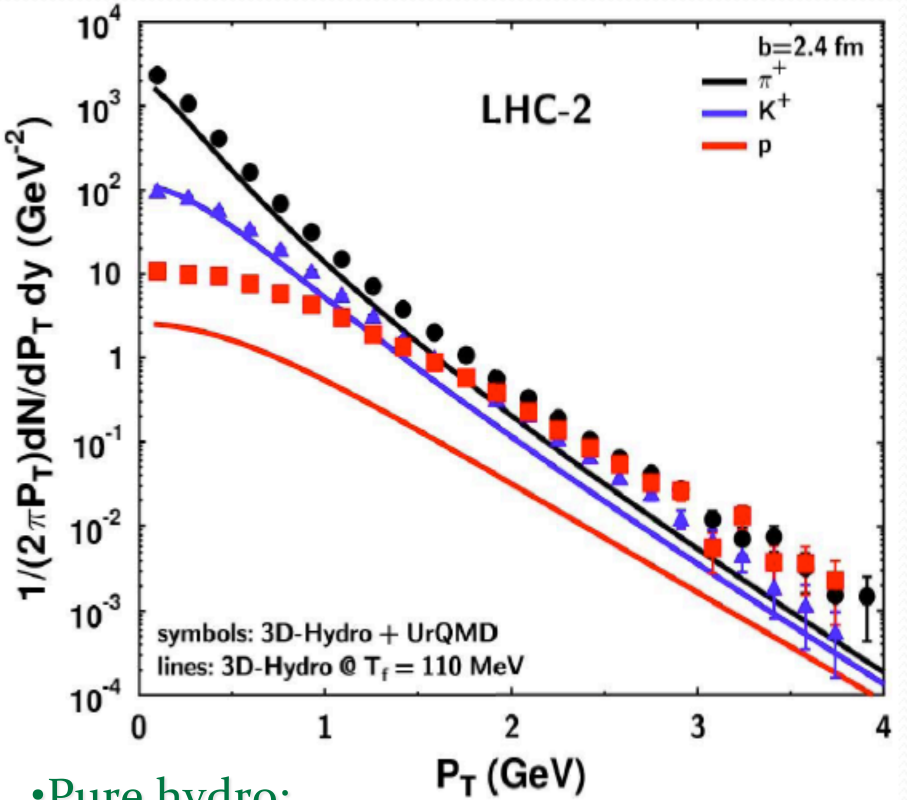
P_T Spectra



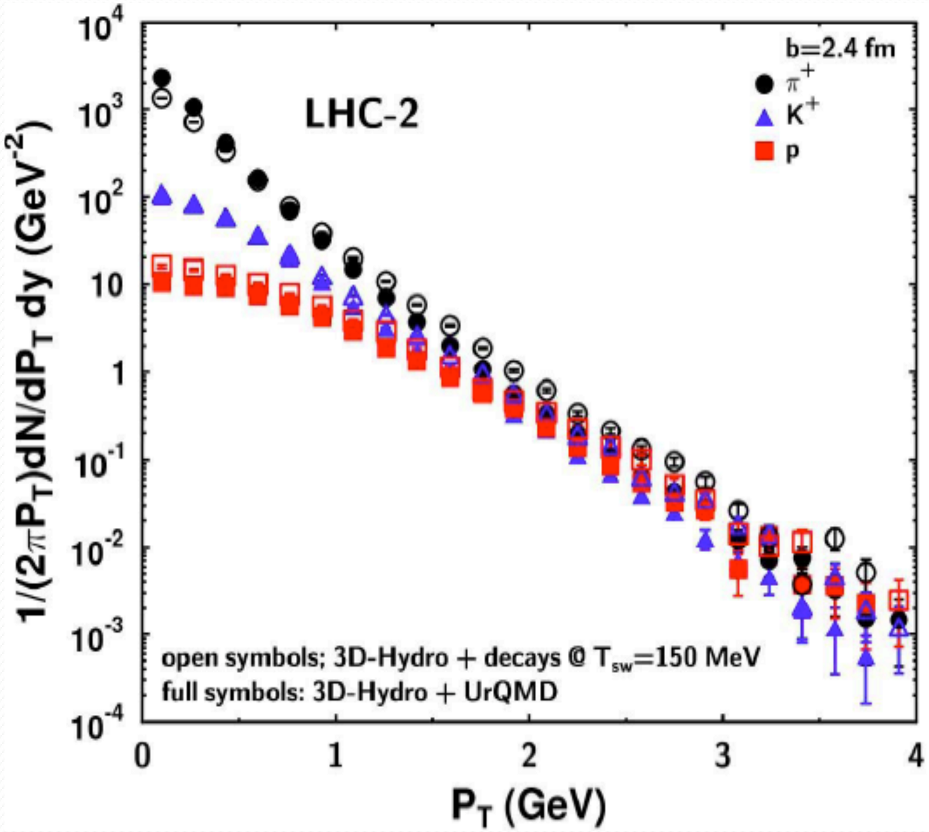


Chiho Nonaka

Reaction Dynamics



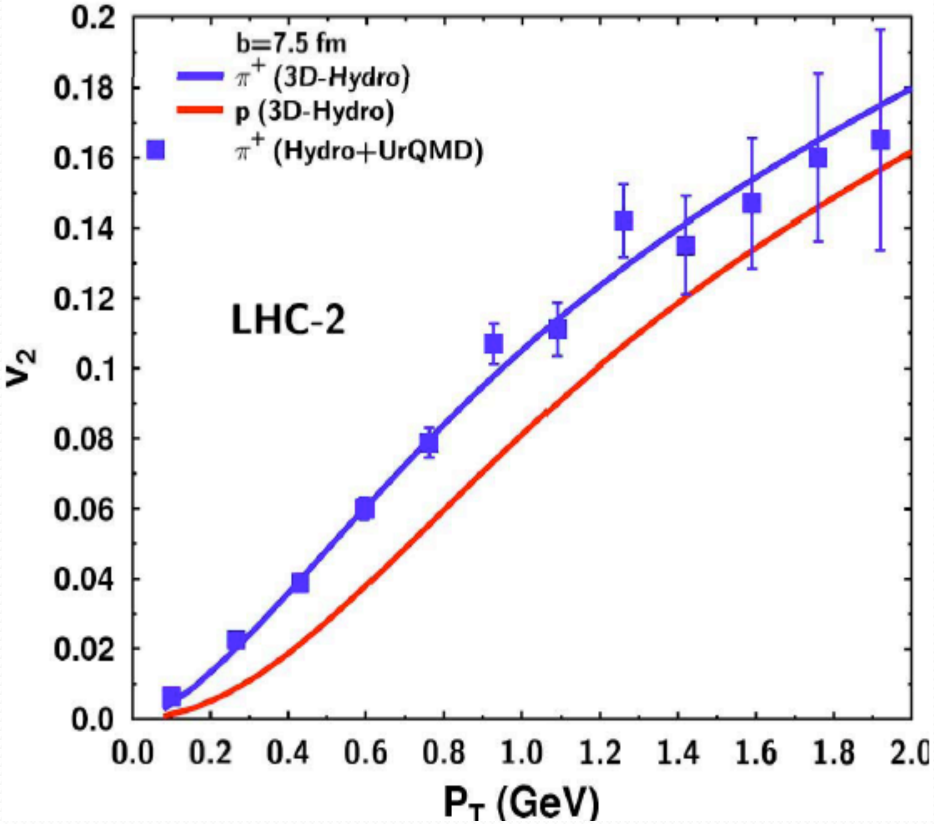
- pion wind:
  P_T of pions ➔ P_T of protons



- Pure hydro:
  invalid freezeout process
- Viscosity effect
  ← final state interactions

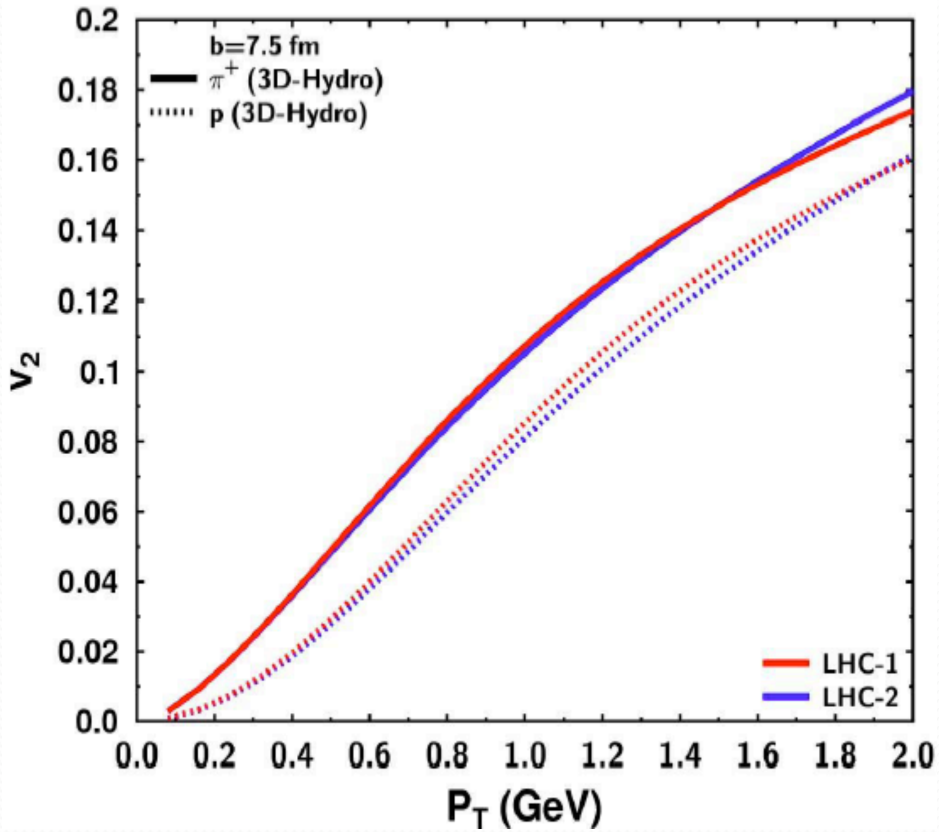
Chiho Nonaka

Elliptic Flow at LHC



•Elliptic flow in hadron phase does not build-up

•No significant difference in initial energy difference



Chiho Nonaka

Summary for LHC

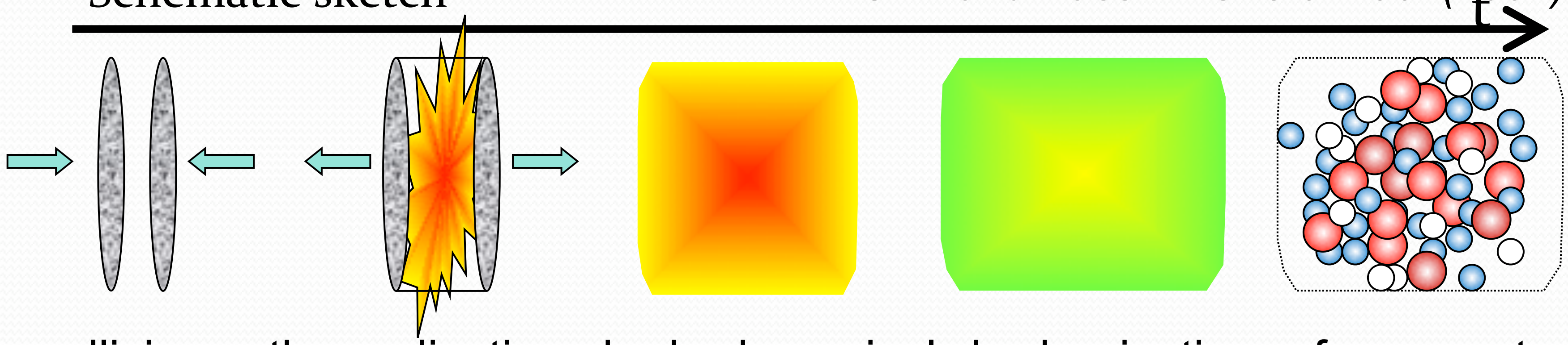
- Baryon free calculation
- Single particle spectra
  - P_T spectra

    Viscosity effect from final state interactions, Pion wind
- Elliptic flow
  - Insensitive to initial conditions
- Work in progress
  - With baryon current

3D Hydro+UrQMD Model

- Schematic sketch

C.N. and Bass PRC75:014902(2007)



- 3D Hydro + UrQMD

Full 3-d Hydrodynamics

EoS with QCD critical point

Hadronization

Cooper-Frye formula

Monte Carlo

UrQMD

final state interactions

Asakawa, Bass, Mueller, C.N. PRL101:122302(2008),
C.N., Asakawa, Phys.Rev.C71:044904(2005)

T_C T_{SW} t fm/c

T_C:critical temperature > T_{SW}: Hydro ➔ UrQMD

Chiho Nonaka

EOS with QCD Critical Point

C.N. and Asakawa, PRC71,044904(2005)

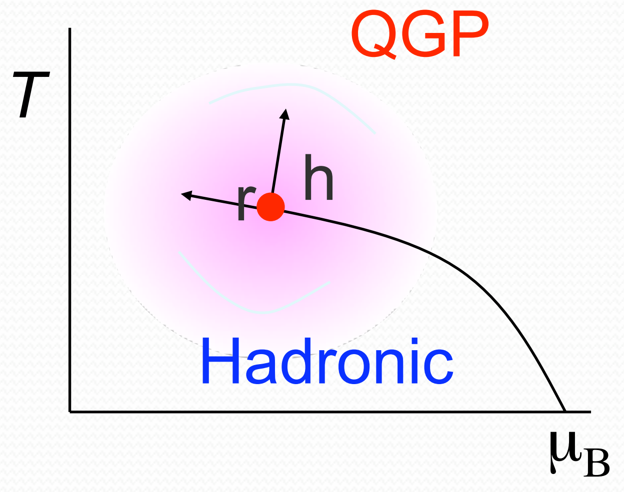
- Singular part near QCD critical point + Non-singular part
  - Singular part

3d Ising Model	⟷	QCD
$r = \frac{T - T_C}{T_C}$	Same Universality Class	T, μ_B
h: extermal magnetic field	$(r, h) \longleftrightarrow (T, \mu_B)$	

- Mapping: (r, h) 3-d Ising Model

↓

(T, μ_B) QCD



Chiho Nonaka

EOS of 3-d Ising Model

- Parametric Representation of EOS

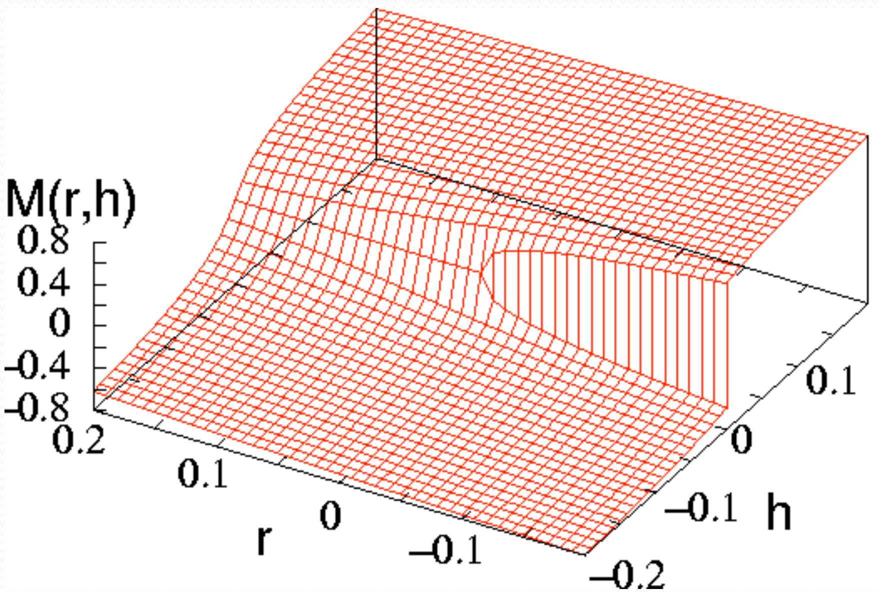
$$\begin{cases} M = M_0 R^\beta \theta \\ h = h_0 R^\beta \tilde{h}(\theta) = h R_0^{\beta\delta}(\theta - 0.762201\theta^3 + 0.00804\theta^5) \\ r = R(1-\theta^2) \qquad (R \geq 0, \ \ -1.154 \leq \theta \leq 1.154) \end{cases}$$

$$r = \frac{T - T_C}{T_C}$$

h : external magnetic field

$\beta = 0.326$

$\delta = 4.8$



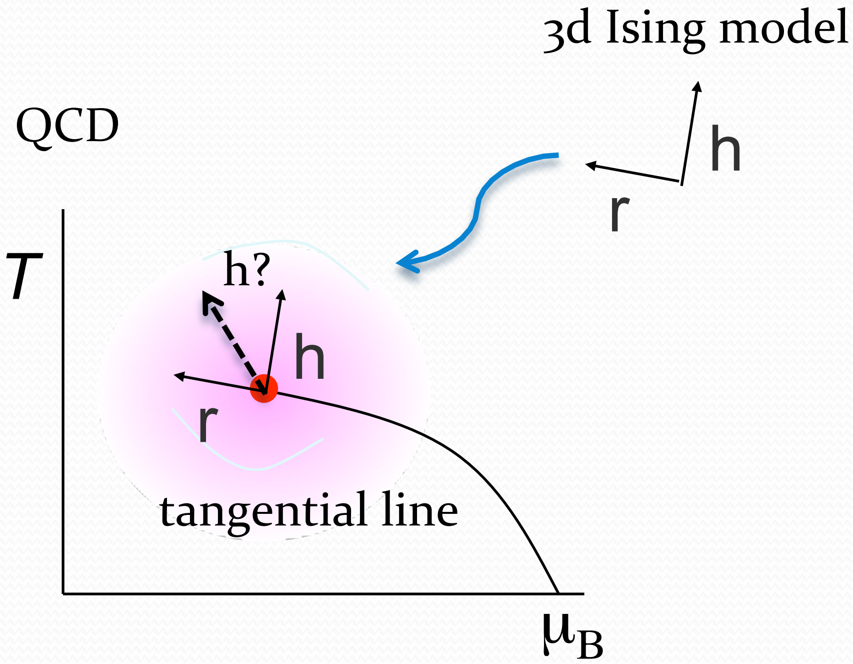
Guida and Zinn-Justin NPB486(97)626

Chiho Nonaka

Mapping : 3D Ising Model → QCD

No Universality

- h axis ?



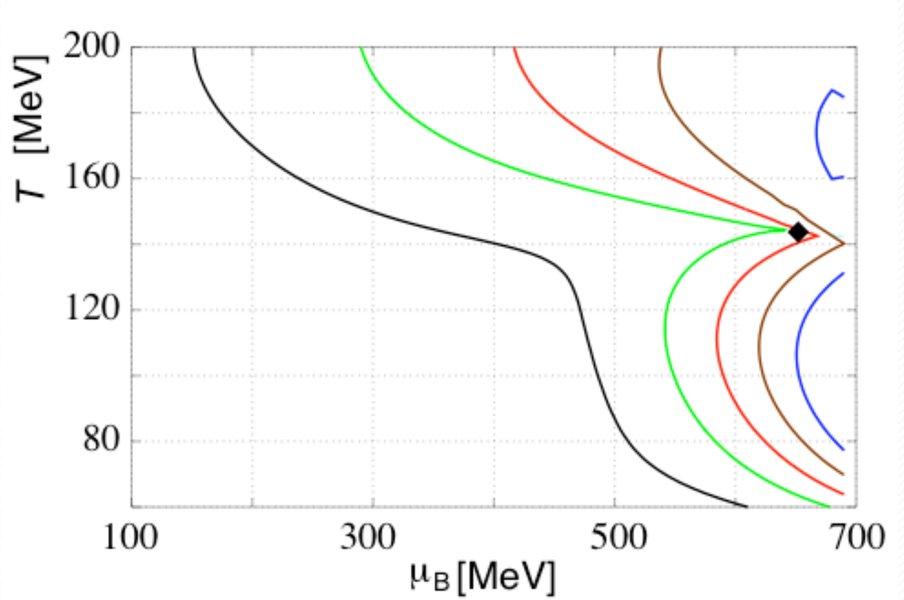
- Critical Region?
  - Lattice QCD
  - Effective theory
  - Experiments

inputs in our model

Focusing Effect

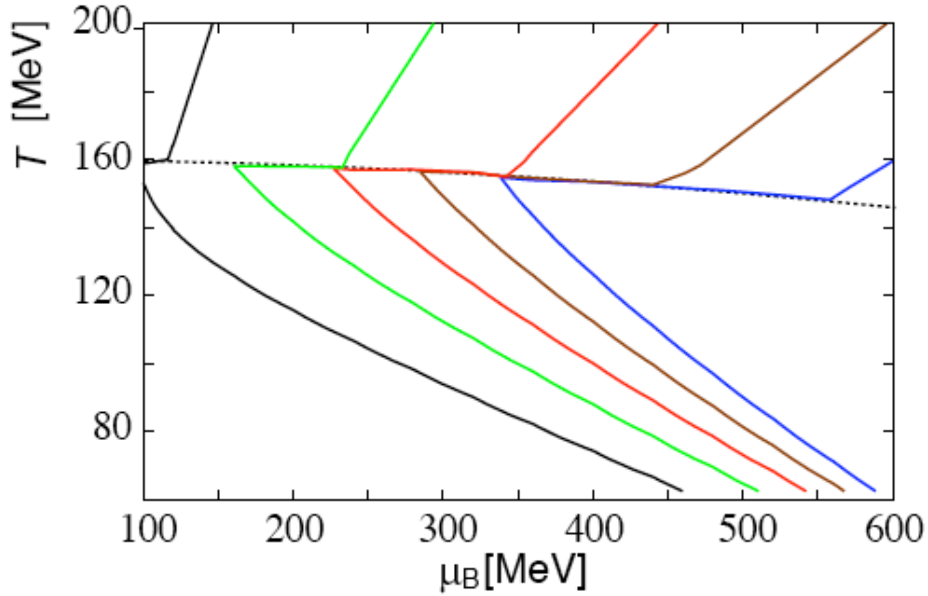
- Isentropic Trajectories on QCD phase diagram

With QCD critical point



Focused

Bag Model +
Excluded Volume Approximation
(No Critical Point)

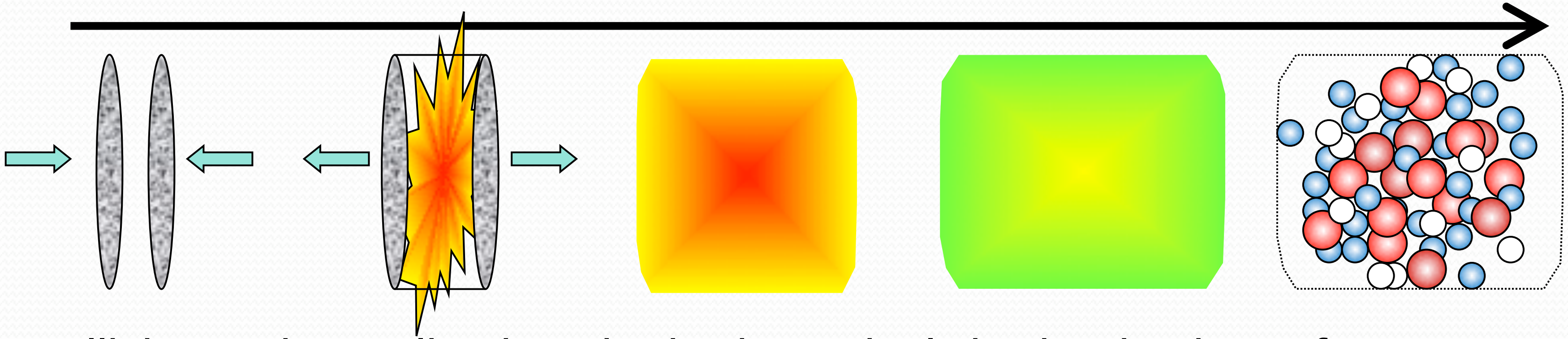


Not Focused

Chiho Nonaka

Signal of QCP

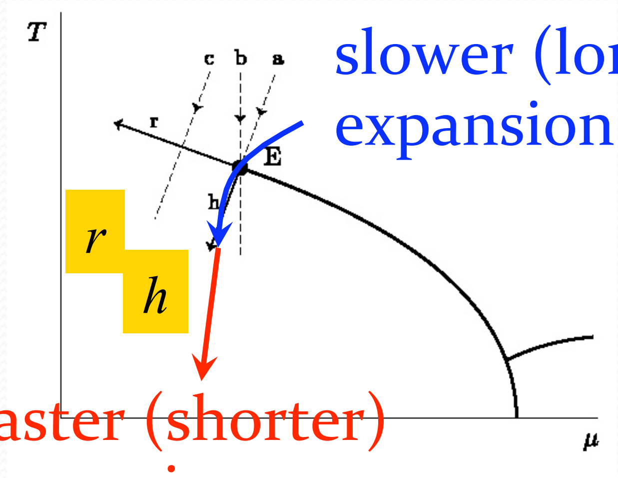
- Signal of QCD critical point should survive even after freezeout process.



- Fluctuations: conserved values ex. charge, baryon number
- Hadron ratios: fixed at chemical freezeout temperature T_{ch}

Slowing out of Equilibrium

- **Berdnikov and Rajagopal's Schematic Argument**



slower (longer) expansion

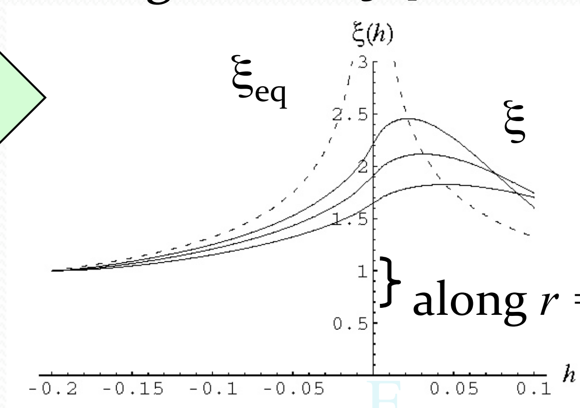
r

h

faster (shorter) expansion

B. Berdnikov and K. Rajagopal,
Phys. Rev. D61 (2000) 105017

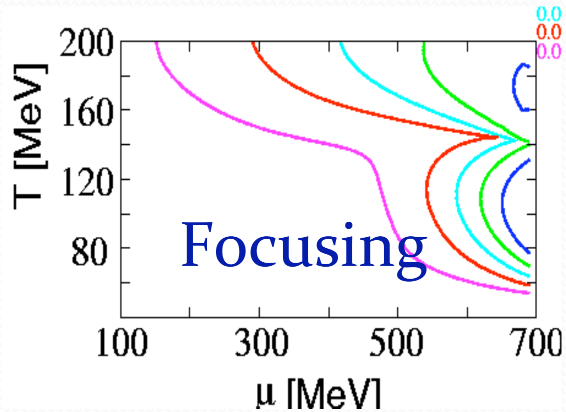
Correlation length longer than ξ_{eq}



} along r = *const.* line

h

- **Effect of Focusing on ξ?**

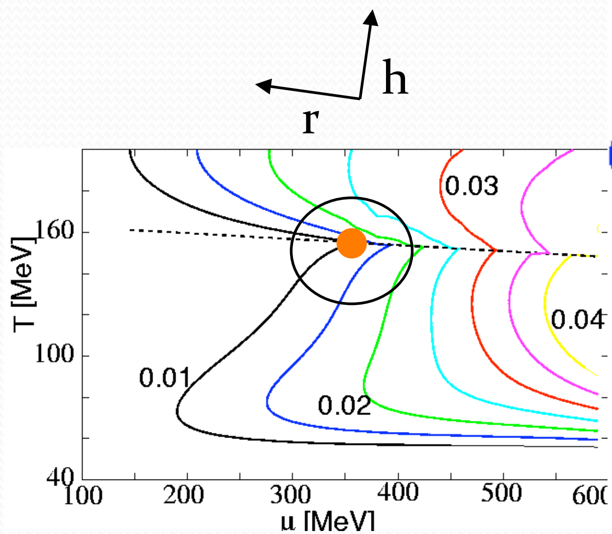


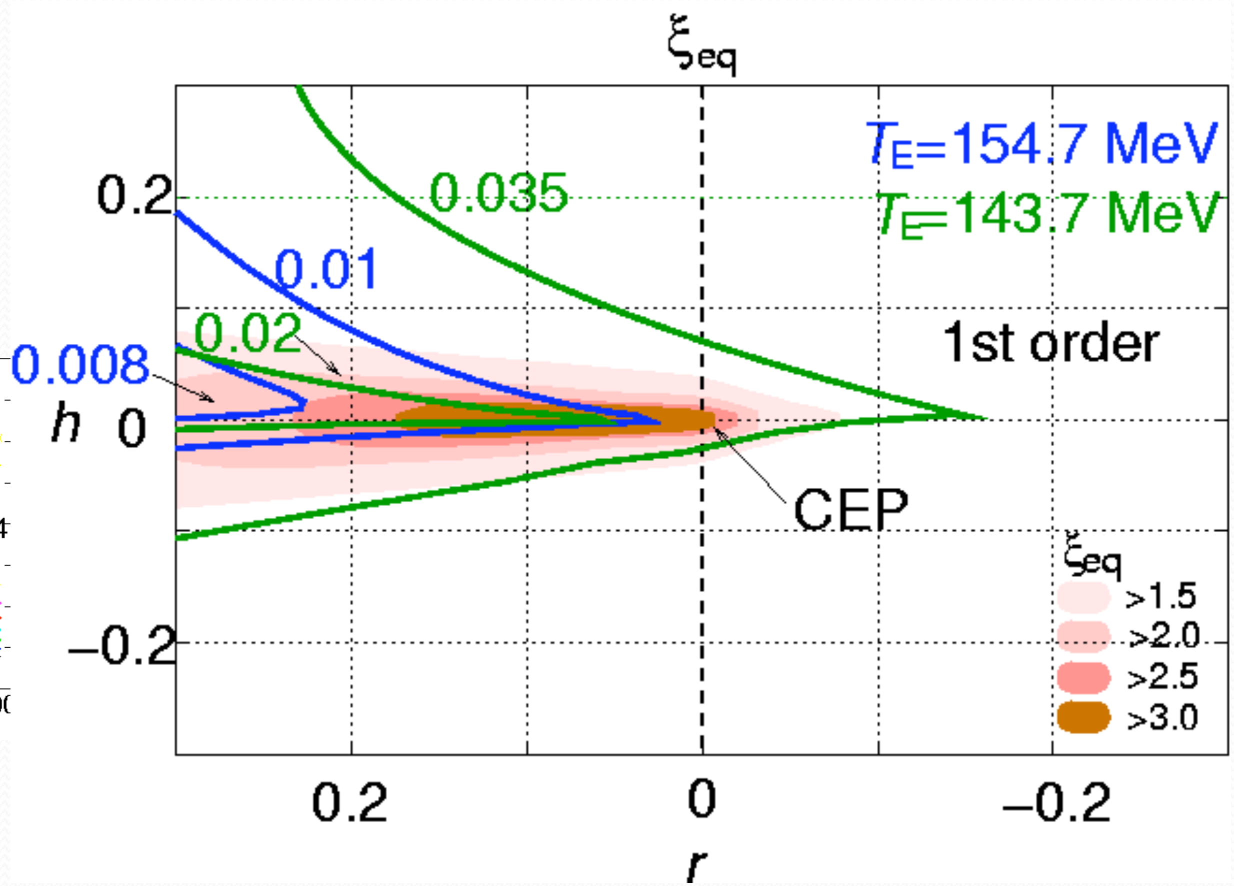
Time evolution : Bjorken's scaling solution along n_B/s
$\tau_0 = 1$ fm, T_o = 200 MeV

Correlation Length

Widom's scaling low

$$\xi_{\rm eq}^2(r,M) = f^2 M^{-2\nu/\beta} g\left(\frac{|r|}{|M|^{1/\beta}}\right)$$





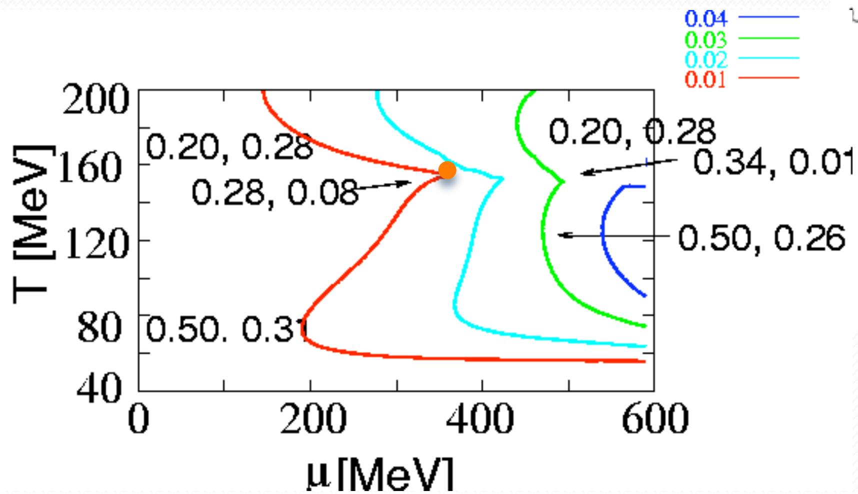
Chiho Nonaka

Fluctuation in 1d Hydro.

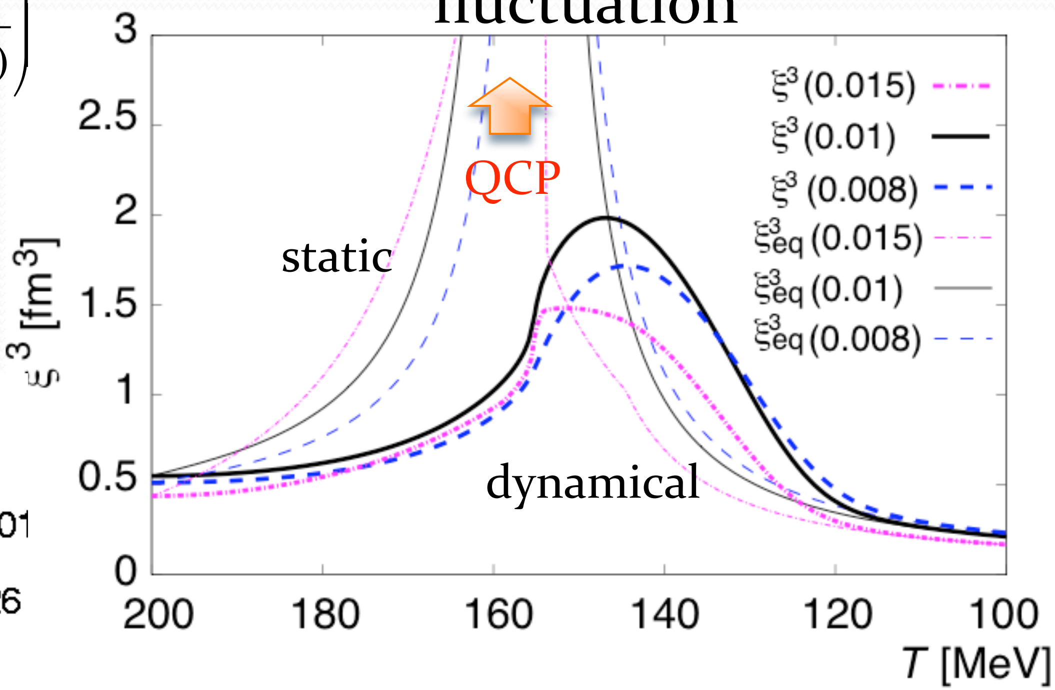
- $$\frac{d}{d\tau} m_\sigma(\tau) = -\Gamma\left[m_\sigma(\tau)\right]\left(m_\sigma(\tau) - \frac{1}{\xi_{eq}(\tau)}\right)$$

  $$\Gamma(m_\sigma) = \frac{a}{\xi_0}(m_\sigma \xi_0)^z, \quad m_\sigma(\tau) = 1/\xi(\tau)$$

  $z = 3.0$ Model H
  (Halperin RMP49(77)435)



fluctuation



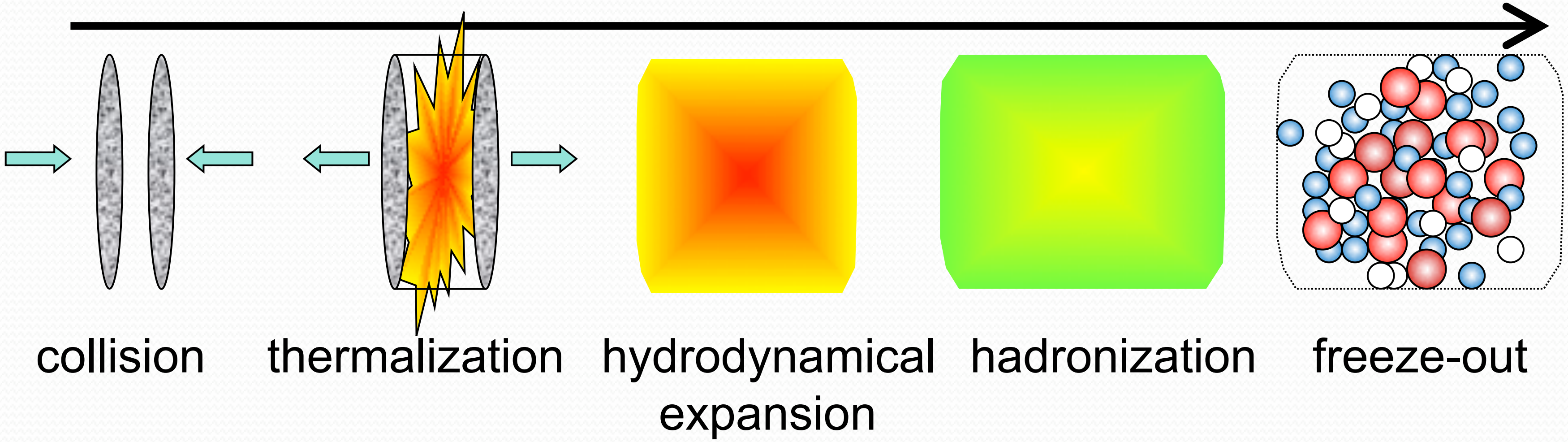
- critical slowing down
- evolution rate

fluctuation : clear signal?

Chiho Nonaka

Signal of QCP

- Signal of QCD critical point should survive even after freezeout process.

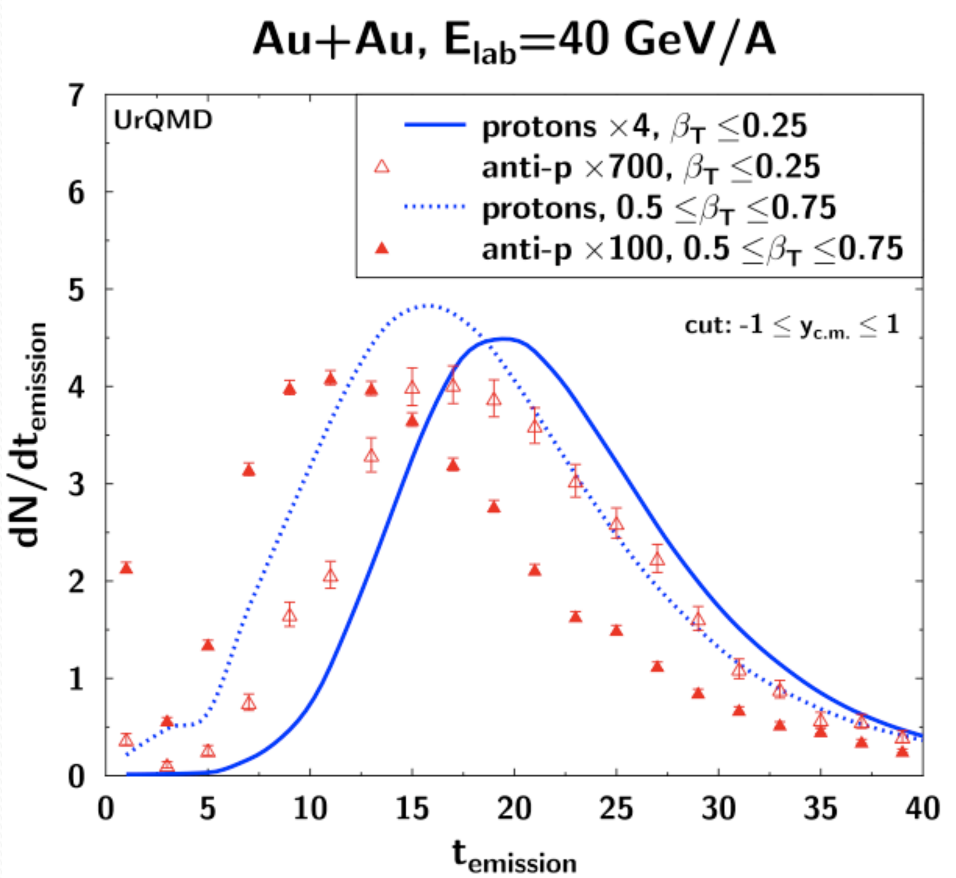


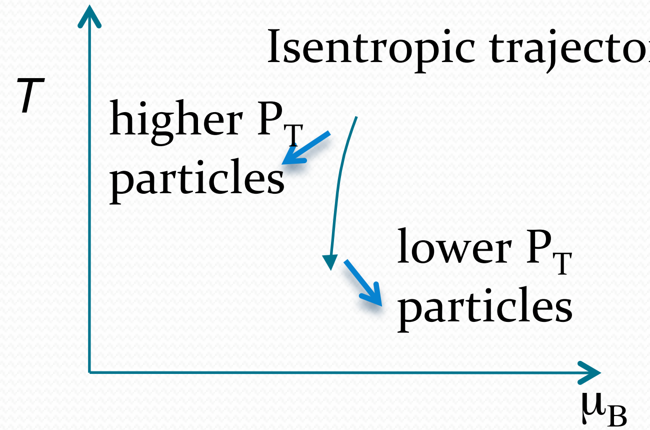
- Fluctuations: conserved values ex. charge, baryon number
- Hadron ratios: fixed at chemical freezeout temperature T_{ch}

Key: T_{ch} depends on transverse momentum

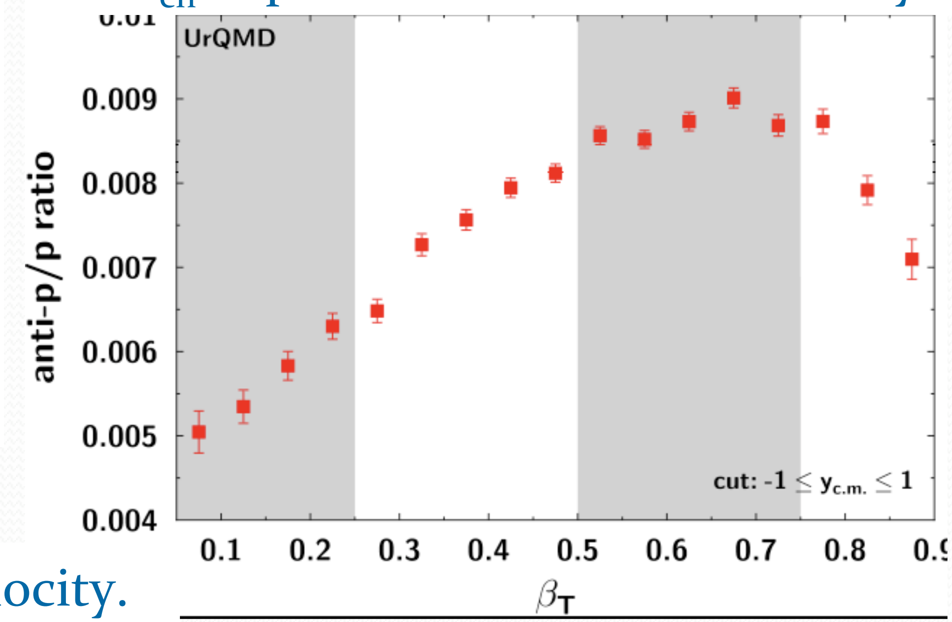
Hadron Production on n_B/s const. line

UrQMD : no QCD critical point





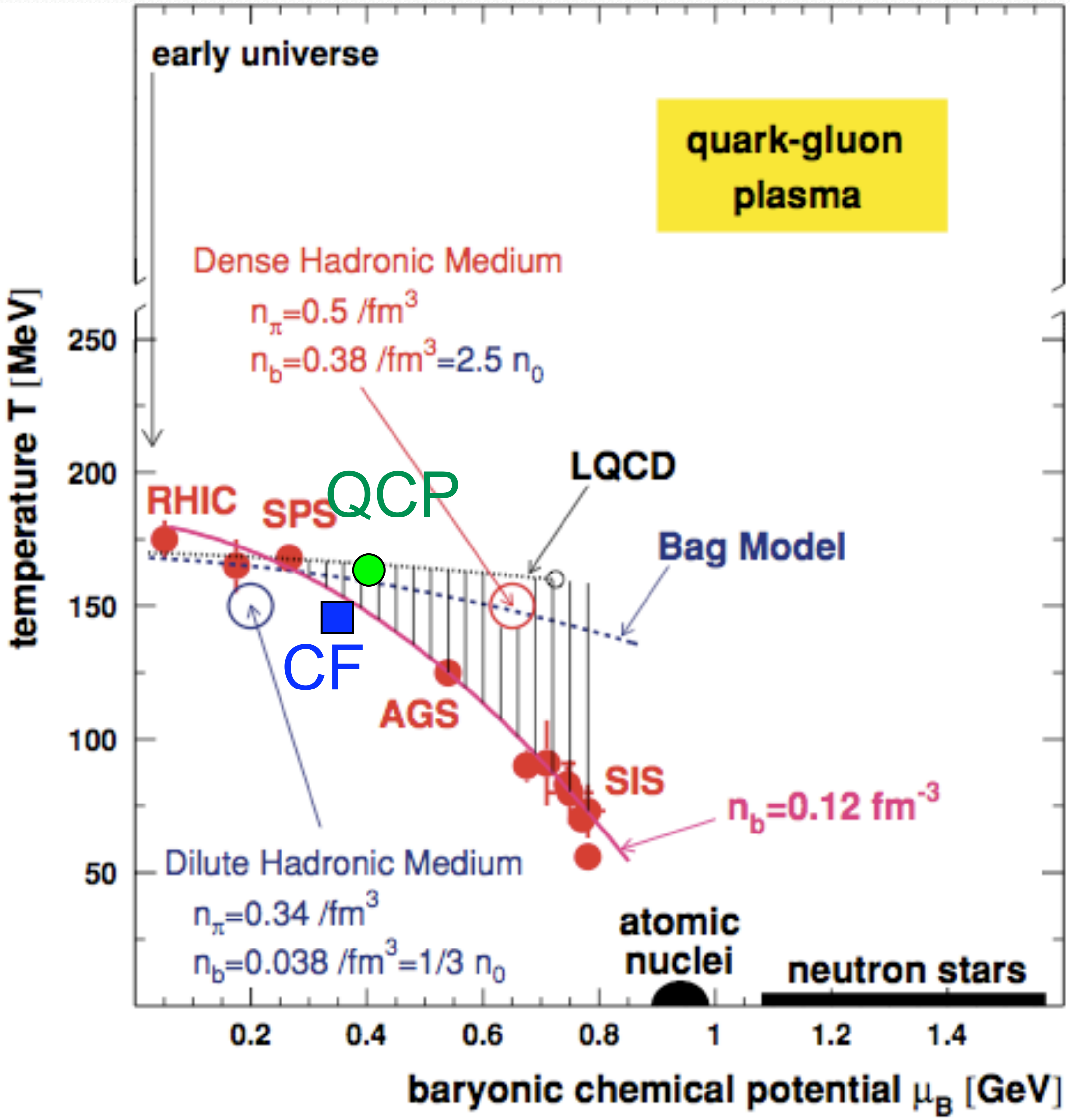
• T_{ch} depends on transverse velocity.



• Hadron ratios depend on transverse velocity.

Chiho Nonaka

Demonstration

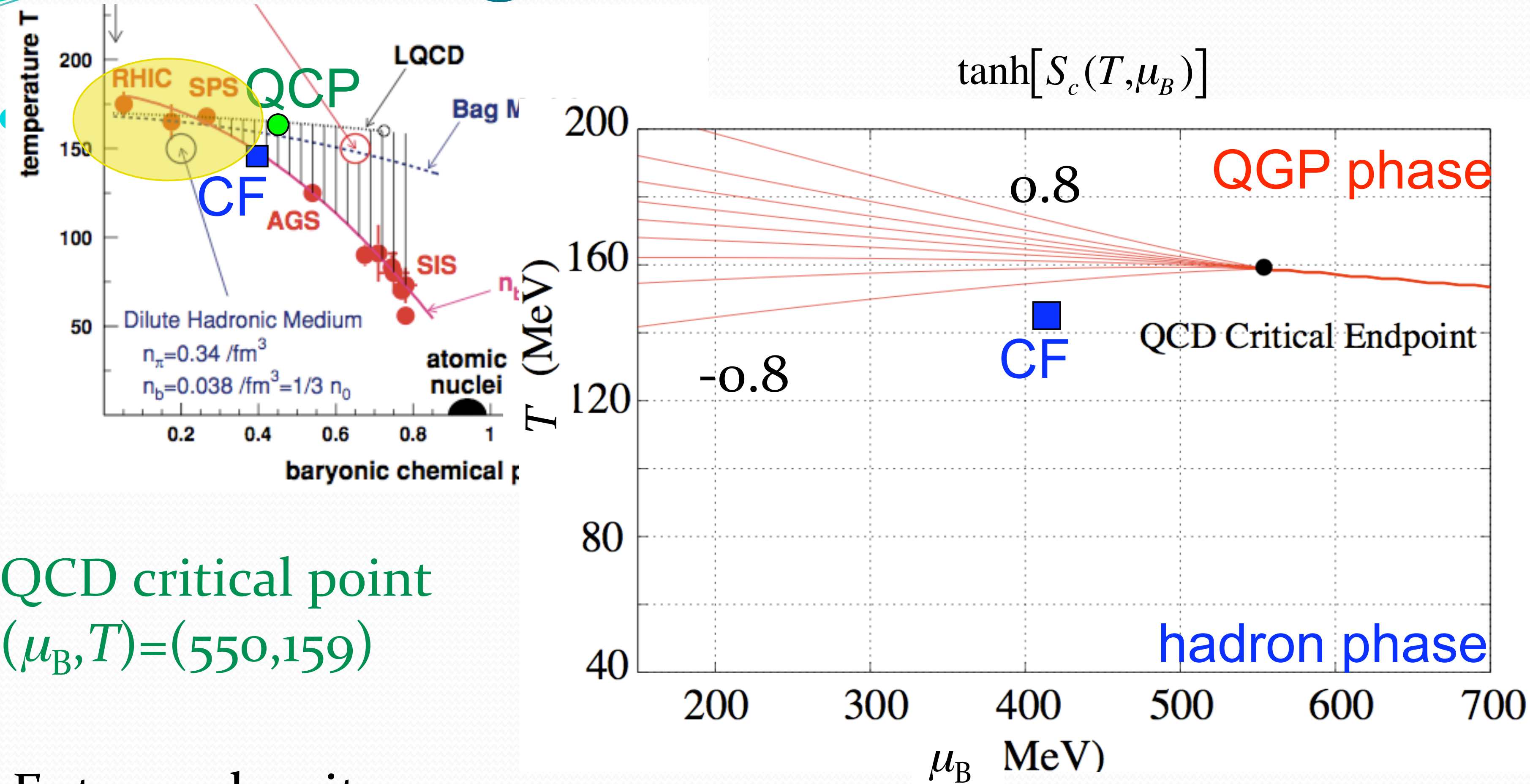


Search of the QCD critical point from experiments

SPS

- Location of QCP
  $(\mu_B, T)=(550,159)$
- Critical Region parameter
- Chemical freezeout point
  $(\mu_B, T)=(406,145)$
  from statistical model

Critical Region



QCD critical point

$(\mu_B, T)=(550,159)$

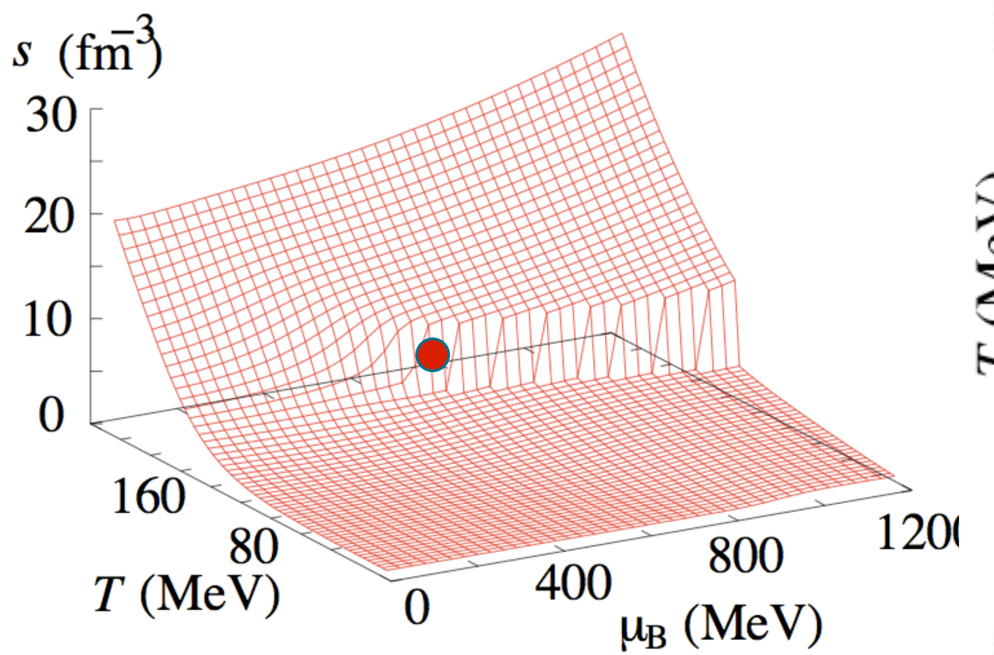
- Entropy density

$$S_{\text{real}}(T,\mu_B) = \frac{1}{2}\{1-\tanh[S_c(T,\mu_B)]\}S_H(T,\mu_B) + \frac{1}{2}\{1+\tanh[S_c(T,\mu_B)]\}S_Q(T,\mu_B)$$

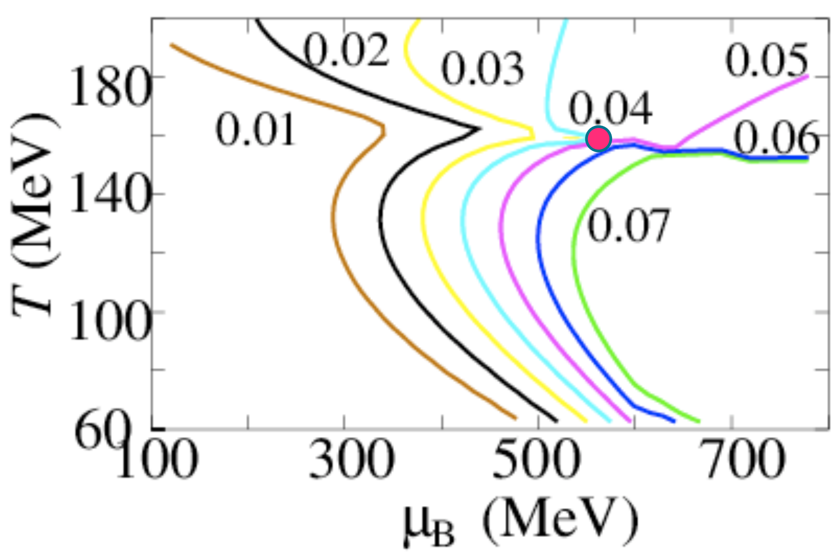
Equation of State

- QCD critical point $(\mu_B, T)=(550,159)$

entropy density

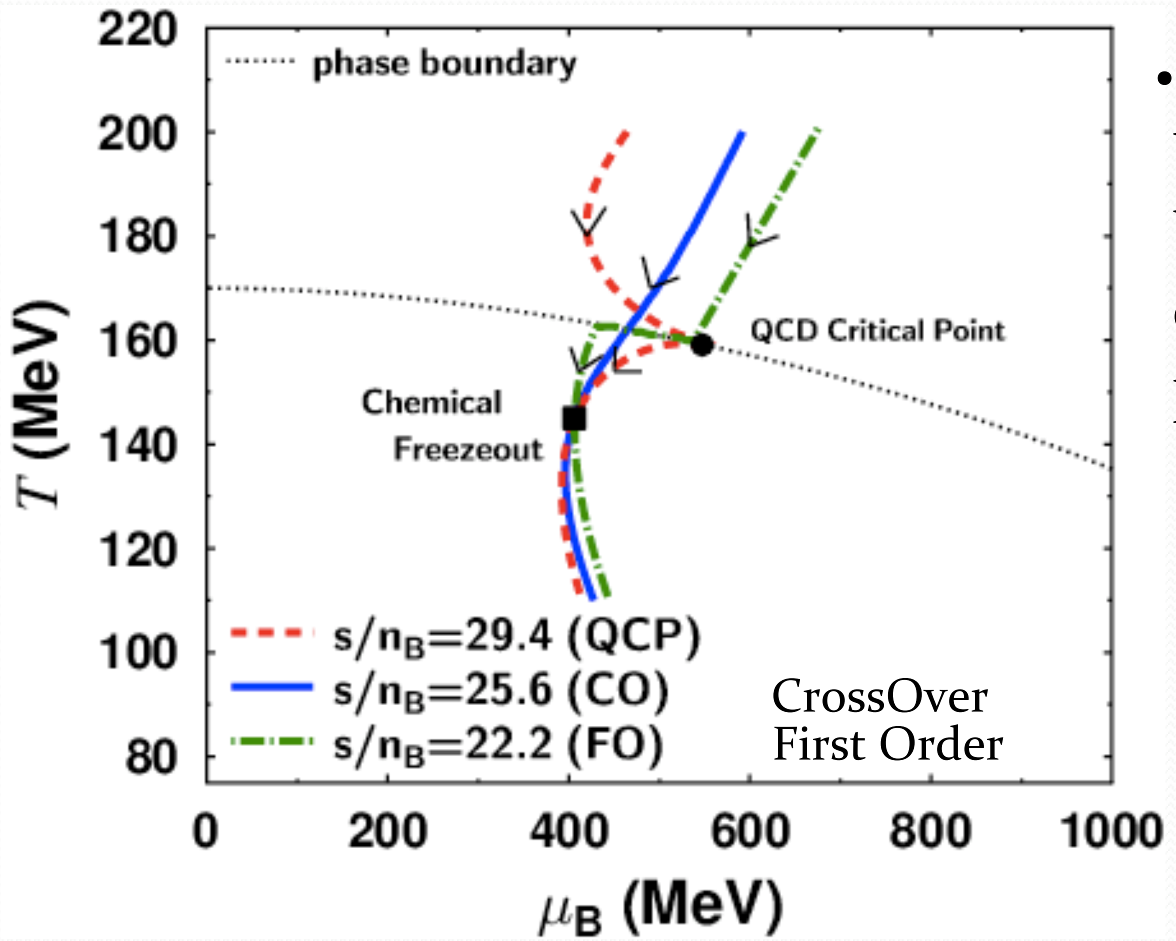


trajectories



Chiho Nonaka

Isentropic Trajectories



- Hadronization occurs between the phase boundary and chemical freezeout point

- FO, CO ↗
- QCP →

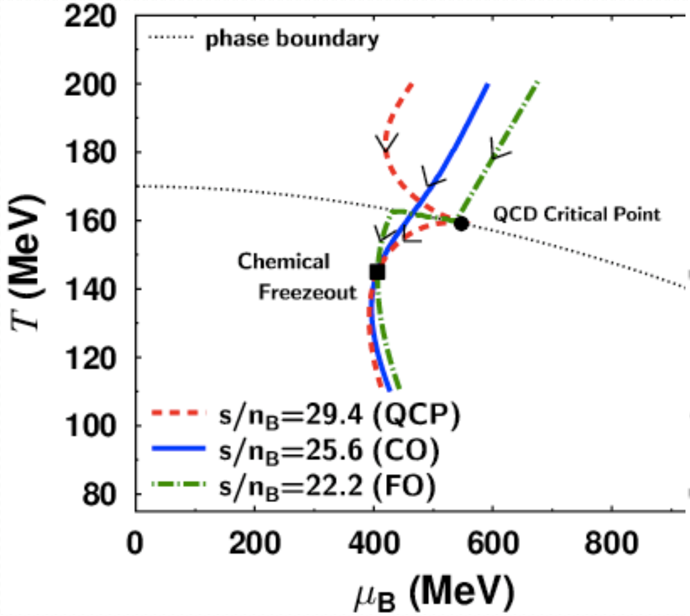
$\frac{\mu_B}{T}$

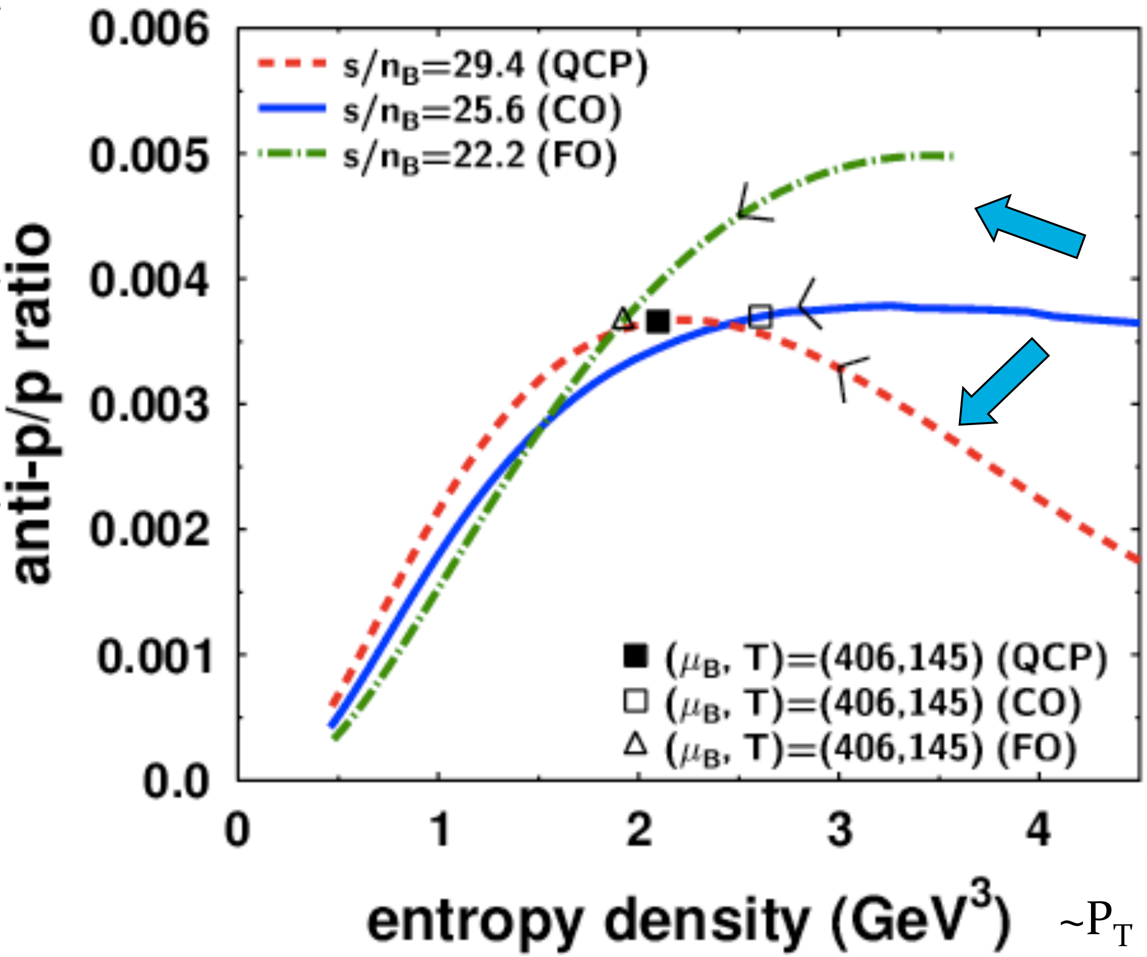
➡ $\bar{p}/p$ ratio

$$\bar{p}/p \sim \exp\left(-\frac{2\mu_B}{T}\right)$$

Chiho Nonaka

Signature of QCP





~P_T

$$\bar{p}/p \sim \exp\left(-\frac{2\mu_B}{T}\right)$$

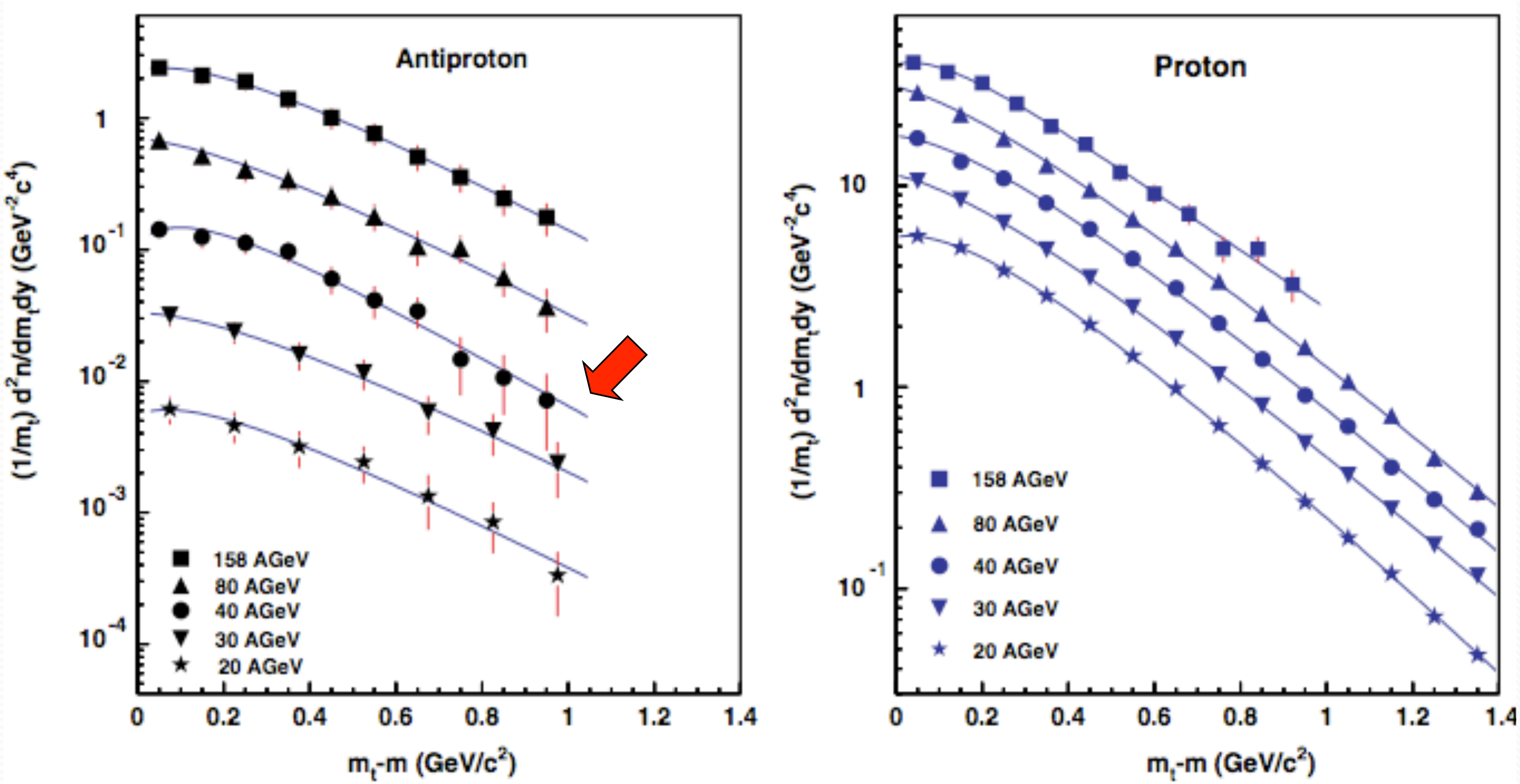
- decreases (FO,CO)
- increases (QCP)

with QCP
steeper $\bar{p}$ spectra at high P_T

Chiho Nonaka

QCD Critical Point?

NA49



steeper $\bar{p}$ spectra at high P_T

NA49, PRC73,044910(2006)

3D Hydro+UrQMD with QCP

- Initial Conditions
  - Energy density

    $\varepsilon(x,y,\eta) = \varepsilon_{\max} W(x,y;b) H(\eta)$

  - Baryon number density

    $n_B(x,y,\eta) = n_{B\max} W(x,y;b) H(\eta)$

  - Parameters

    τ_0=0.6 fm/c

    η_0=0.5 σ_η=1.5

  - Flow

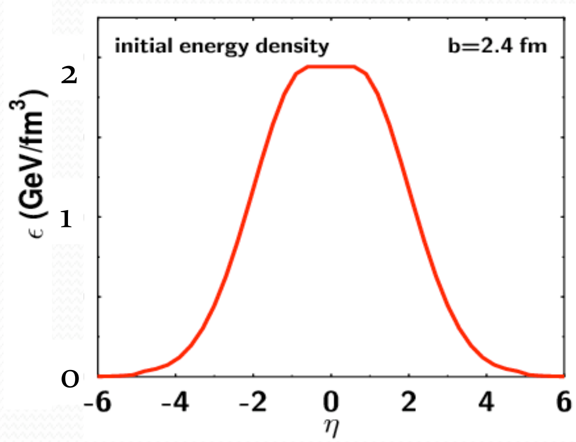
    v_L=η (Bjorken's solution); v_T=0
- EOS: QCP, Bag Model
- Switching temperature

  T_{SW}=150 [MeV]

QCP: $T_E = 159$ MeV, $\mu_E = 550$ MeV

T_c=170 MeV at μ_B=0 MeV

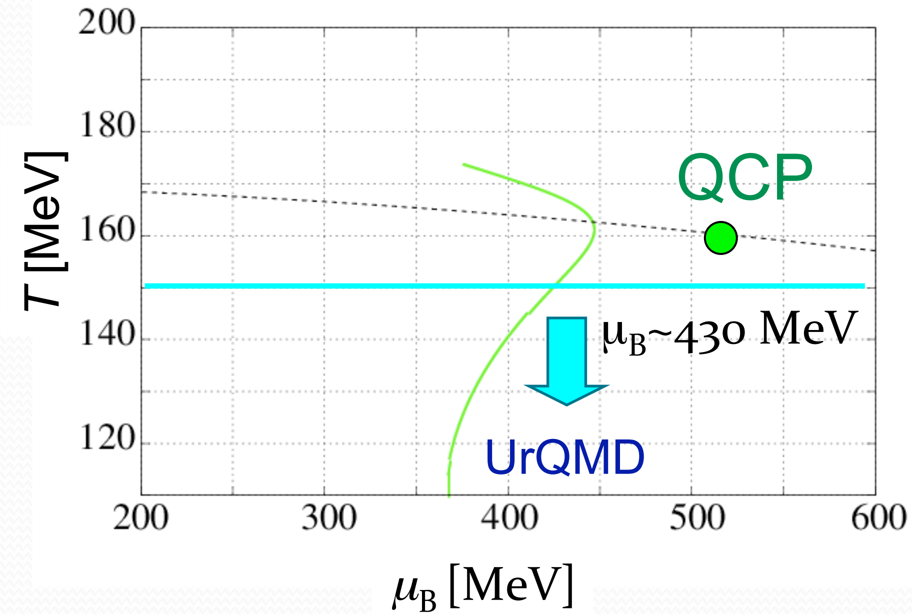
•longitudinal direction: $H(\eta)$



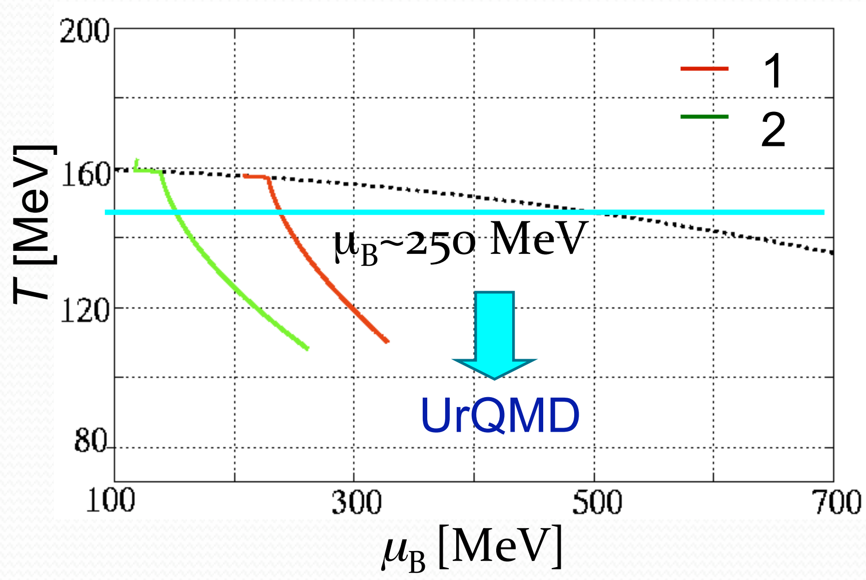
Chiho Nonaka

3D Hydro + UrQMD

Hydro with QCD critical point



Hydro with Bag Model

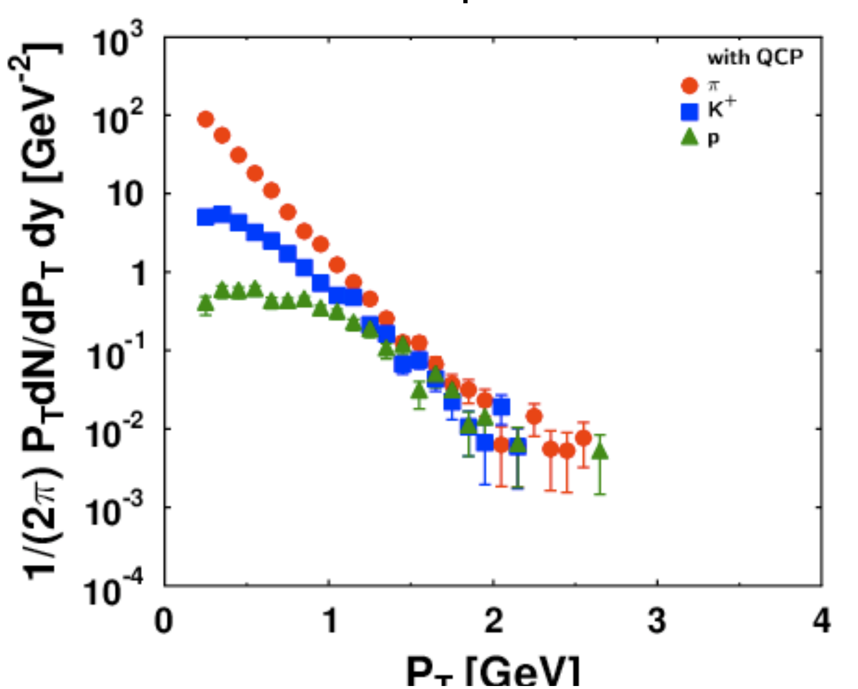


Switching temperature: 150 MeV

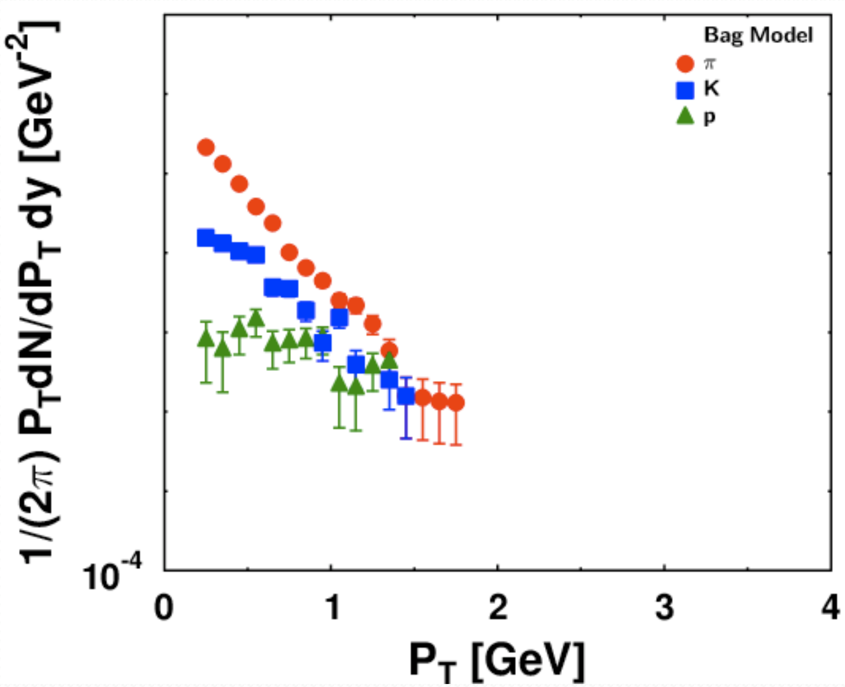
Chiho Nonaka

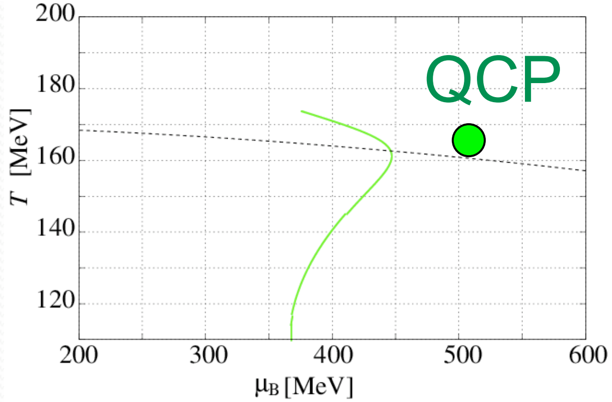
P_T Spectra

QCD critical point



Bag Model





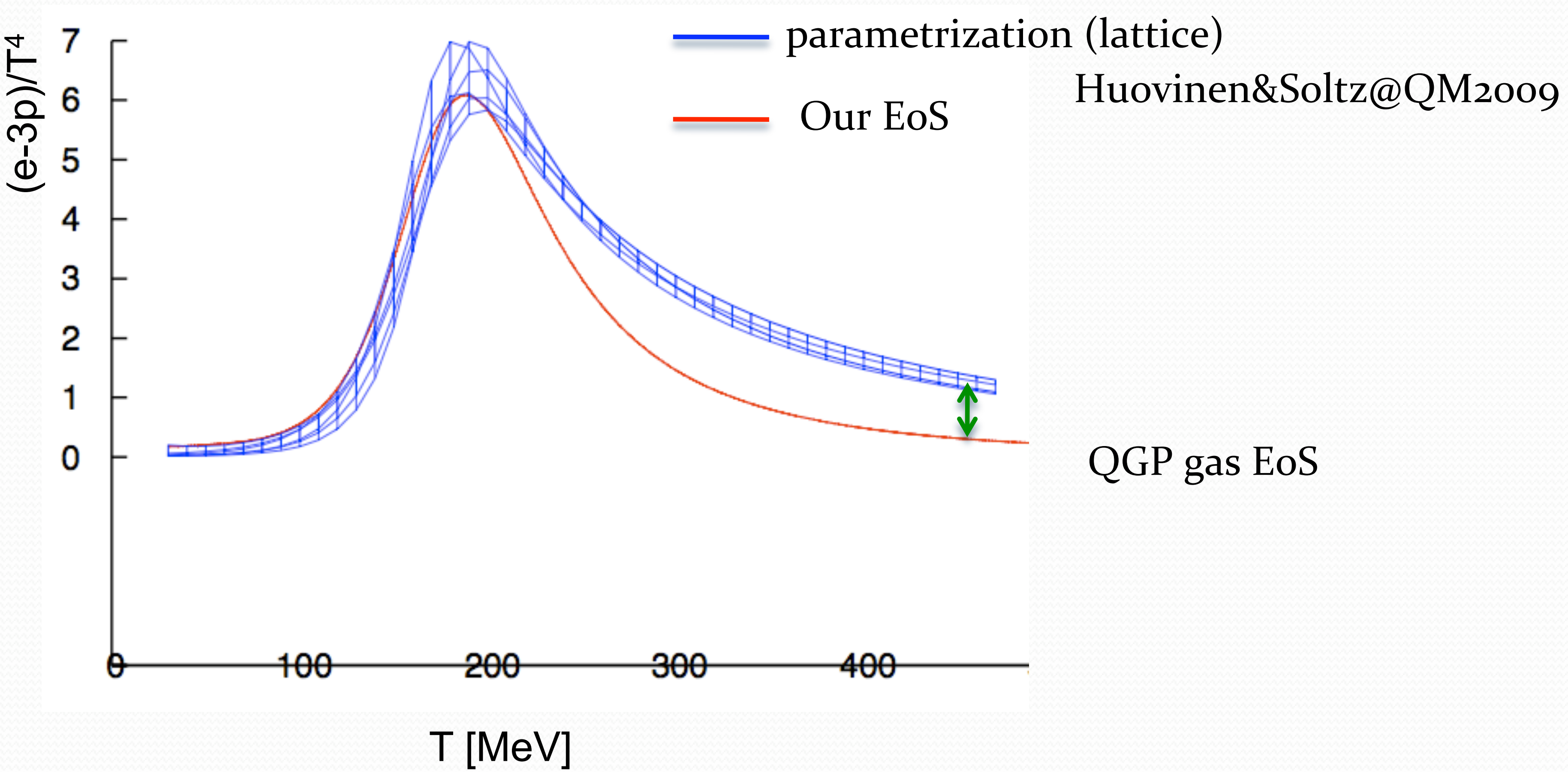
Because of focusing effect

At T_{SW} $\langle \mu_B \rangle_{QCP} > \langle \mu_B \rangle_{BG} \Rightarrow \frac{p}{\pi}_{QCP} > \frac{p}{\pi}_{BG}$

Chiho Nonaka

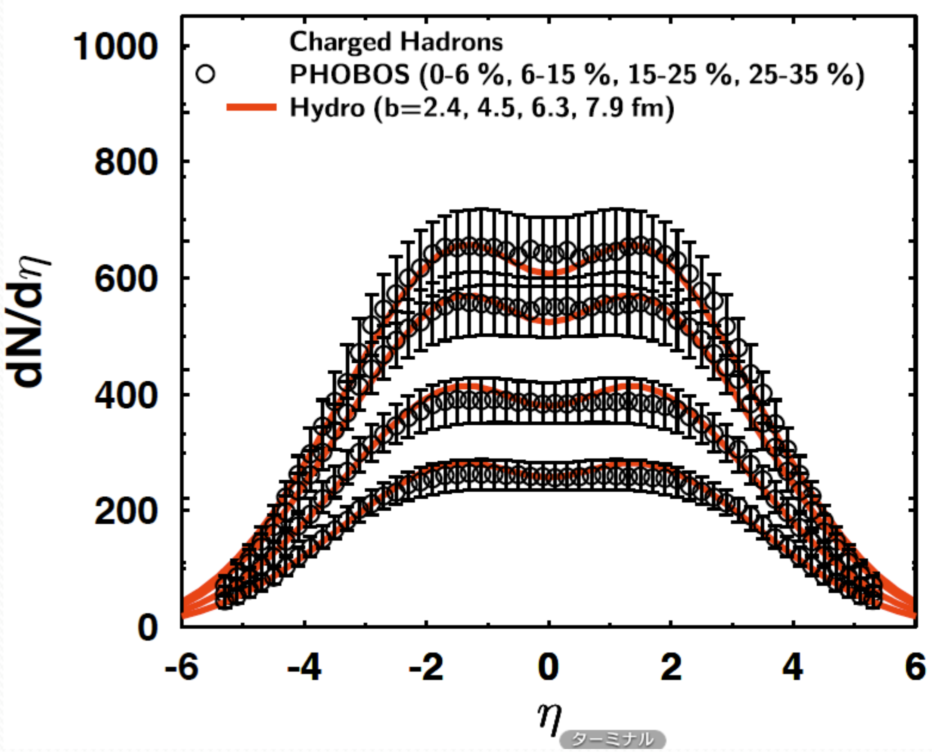
At RHIC

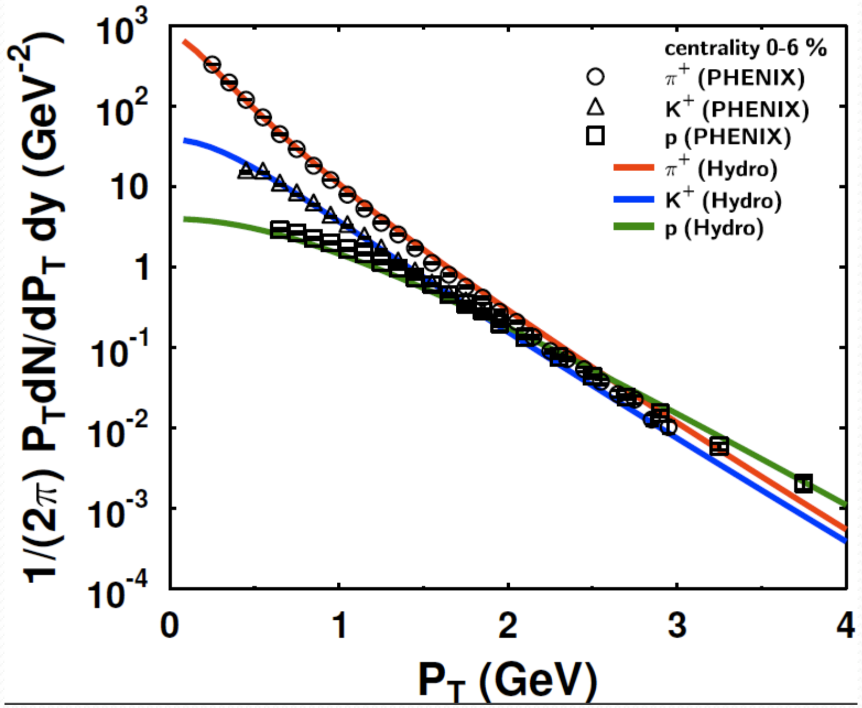
- Smooth connection to lattice data at μ=0



Chiho Nonaka

At RHIC





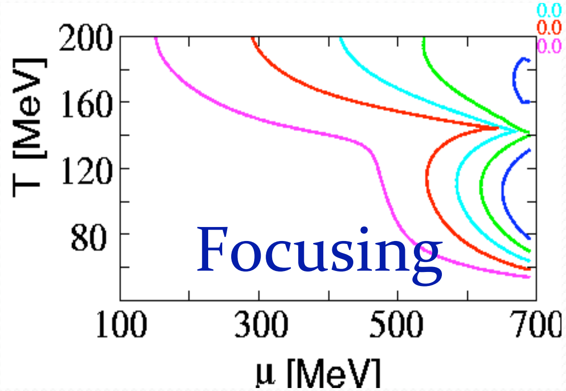
Chiho Nonaka

Summary

Signal of QCD critical point:
$\bar{p}/p$ ratio as a function of transverse momentum

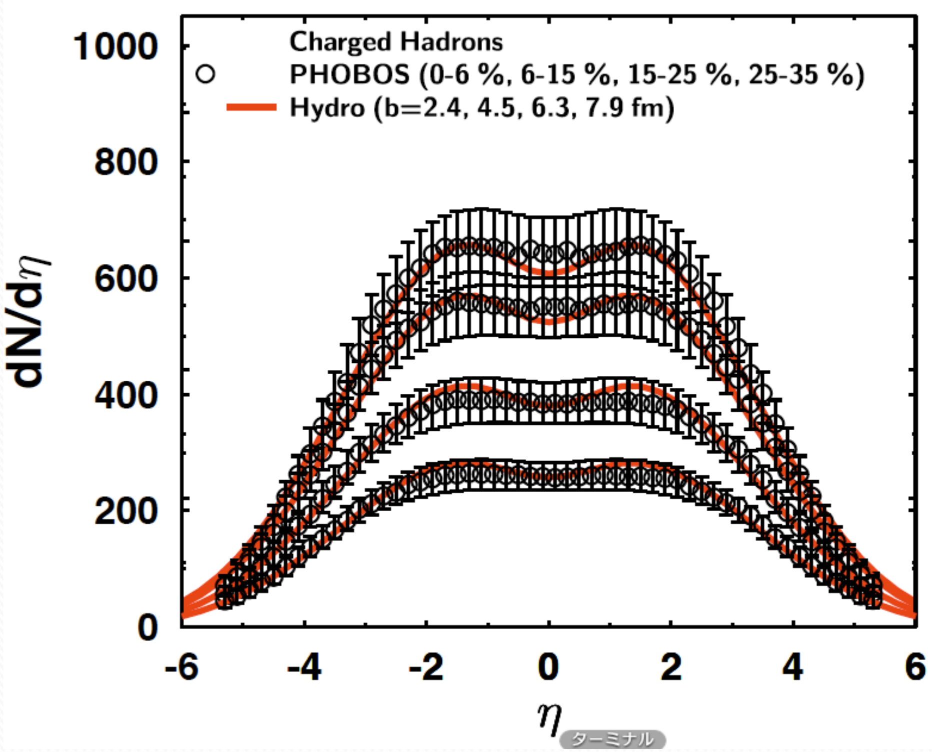
- Towards quantitative analyses
  - EoS with QCP

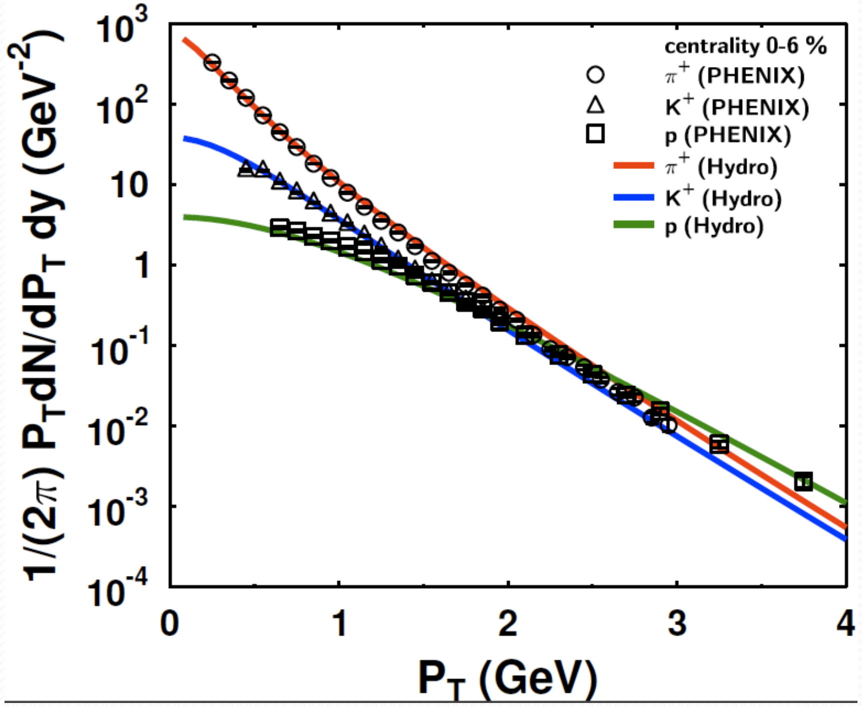
    Focusing effects in isentropic trajectories



- 3D Hydro + UrQMD model
  - P_T spectra, hadron ratios

At RHIC





ε_{max} GeV/fm^3	n_{Bmax} fm^{-3}
68.0	**0.15**

Chiho Nonaka

Back Up

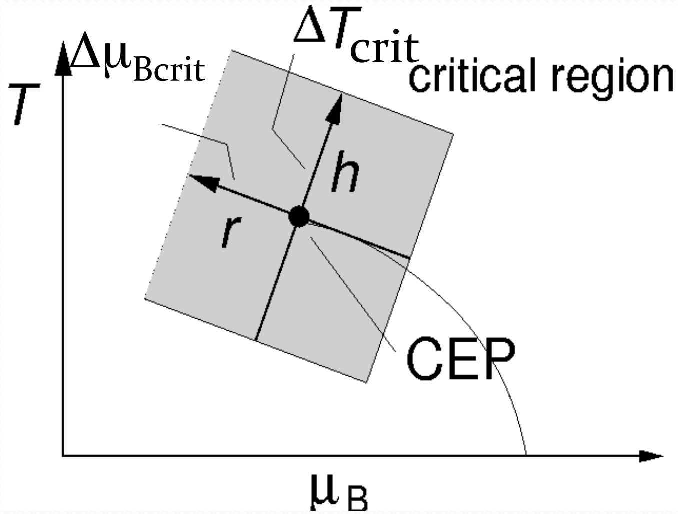
Chiho Nonaka

Singular Part + Non-singular Part

- Entropy density

$$S_{\rm real}(T,\mu_B)=\frac{1}{2}\{1-\tanh[S_c(T,\mu_B)]\}S_H(T,\mu_B)+\frac{1}{2}\{1+\tanh[S_c(T,\mu_B)]\}S_Q(T,\mu_B)$$

- $S_H(T,\mu_B)$ Hadron Phase (excluded volume model)
  $S_Q(T,\mu_B)$ QGP phase



- Dimensionless parameter: Sc

$$S_c(T,\mu_B)=s_c\sqrt{(\Delta T_{\rm crit})^2+(\Delta\mu_{\rm crit})^2}\times D$$

Critical domain

- Choice of parameters $\Delta T_{\rm crit},\Delta\mu_{\rm crit},D$

Thermodynamical inequalities

$$\left(\frac{\partial S}{\partial T}\right)\bigg|_{V,N}\geq 0,\quad \left(\frac{\partial P}{\partial V}\right)\bigg|_{T,N}\geq 0,\quad \left(\frac{\partial \mu}{\partial N}\right)\bigg|_{T,V}\geq 0$$

Critical exponent near CEP keeps correctly.

Chiho Nonaka

Singular Part of EOS

- Gibbs Free Energy

$$G(h,r) = F(M,r) - Mh$$

Guida and Zinn-Justin NPB486(97)626

Free energy: $F(M,r) = h_0 M_0 R^{2-\alpha} g(\theta)$ ← $h = \left(\frac{\partial F}{\partial M}\right)\Big|_r$ $\alpha = 0.11$

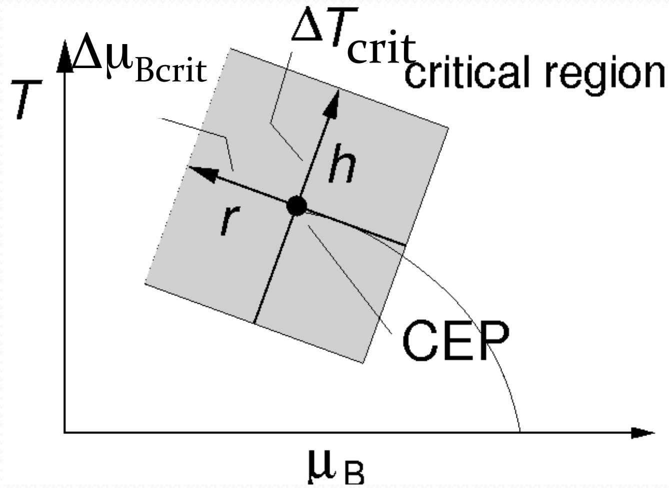
- Entropy Density for Singular Part

$$s_c = -\frac{\partial G}{\partial T}\Big|_\mu = -\frac{\partial G}{\partial h}\Big|_r \frac{\partial h}{\partial T} - \frac{\partial G}{\partial r}\Big|_h \frac{\partial r}{\partial T}$$

$$\begin{cases} \frac{\partial G}{\partial h}\Big|_r = -M \\ \frac{\partial G}{\partial r}\Big|_h = \frac{\partial F}{\partial r}\Big|_h - \frac{\partial M}{\partial r}\Big|_h h \end{cases}$$

mapping

$(r,h) \leftrightarrow (T,\mu_B)$



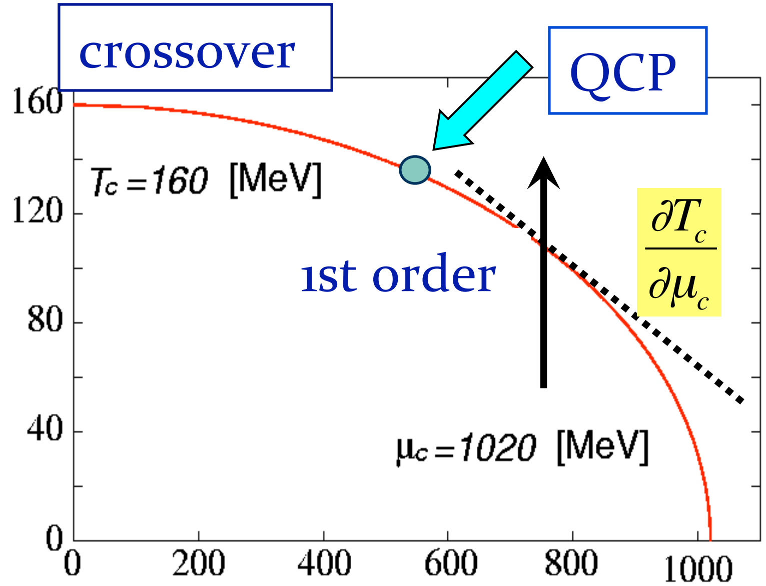
Thermodynamical Quantities

$$S_{\text{real}}(T,\mu_B) = \frac{1}{2}\{1-\tanh[S_c(T,\mu_B)]\}S_H(T,\mu_B) + \frac{1}{2}\{1+\tanh[S_c(T,\mu_B)]\}S_Q(T,\mu_B)$$

- Baryon number density

$$n_B(T,\mu_B) = \frac{\partial P}{\partial \mu_B} = \int_0^T \frac{\partial s}{\partial \mu_B}(T',\mu_B)dT' + n_B(0,\mu_B)$$
$$+\left|\frac{\partial T_C(\mu_B)}{\partial \mu_C}\right|\left(S_Q\left(T_C,\mu_B(T_C)\right) - S_H\left(T_C,\mu_B(T_C)\right)\right)$$

1st order



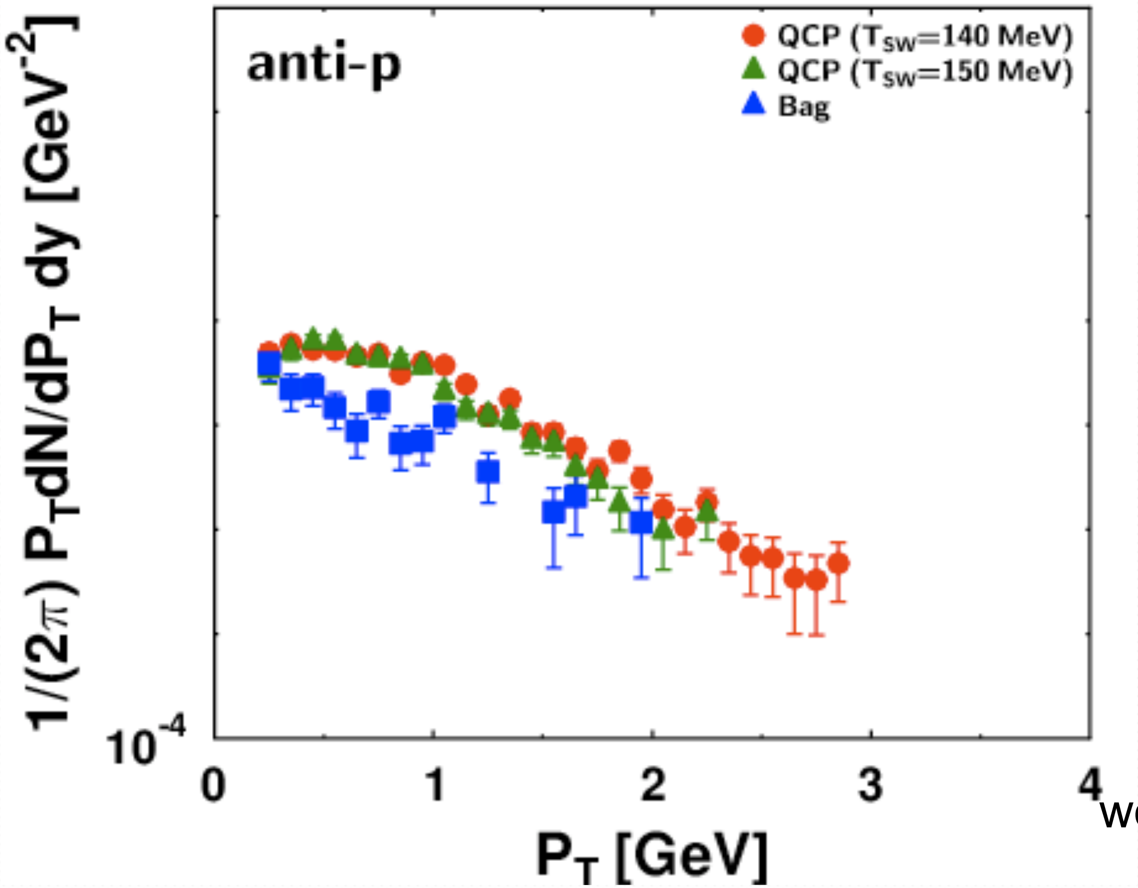
- Pressure

$$P(T,\mu_B) = \int_0^T S_{\text{real}}(T',\mu_B)dT' + P(0,\mu_B)$$

- Energy Density

$$\varepsilon = TS_{\text{real}} - P - \mu_B n_B$$

$\bar{p}$ Spectra



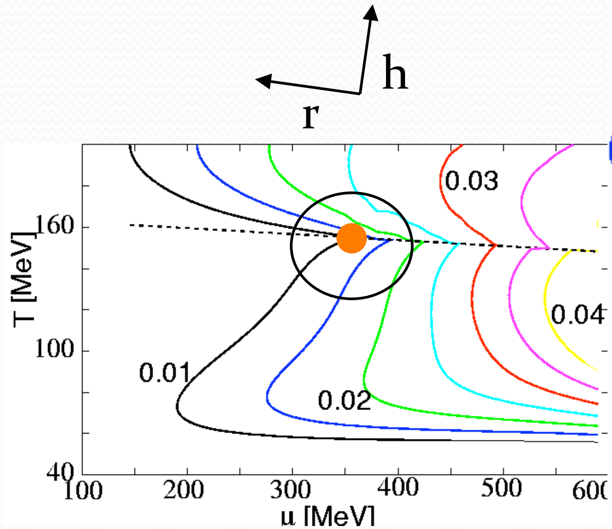
work in progress

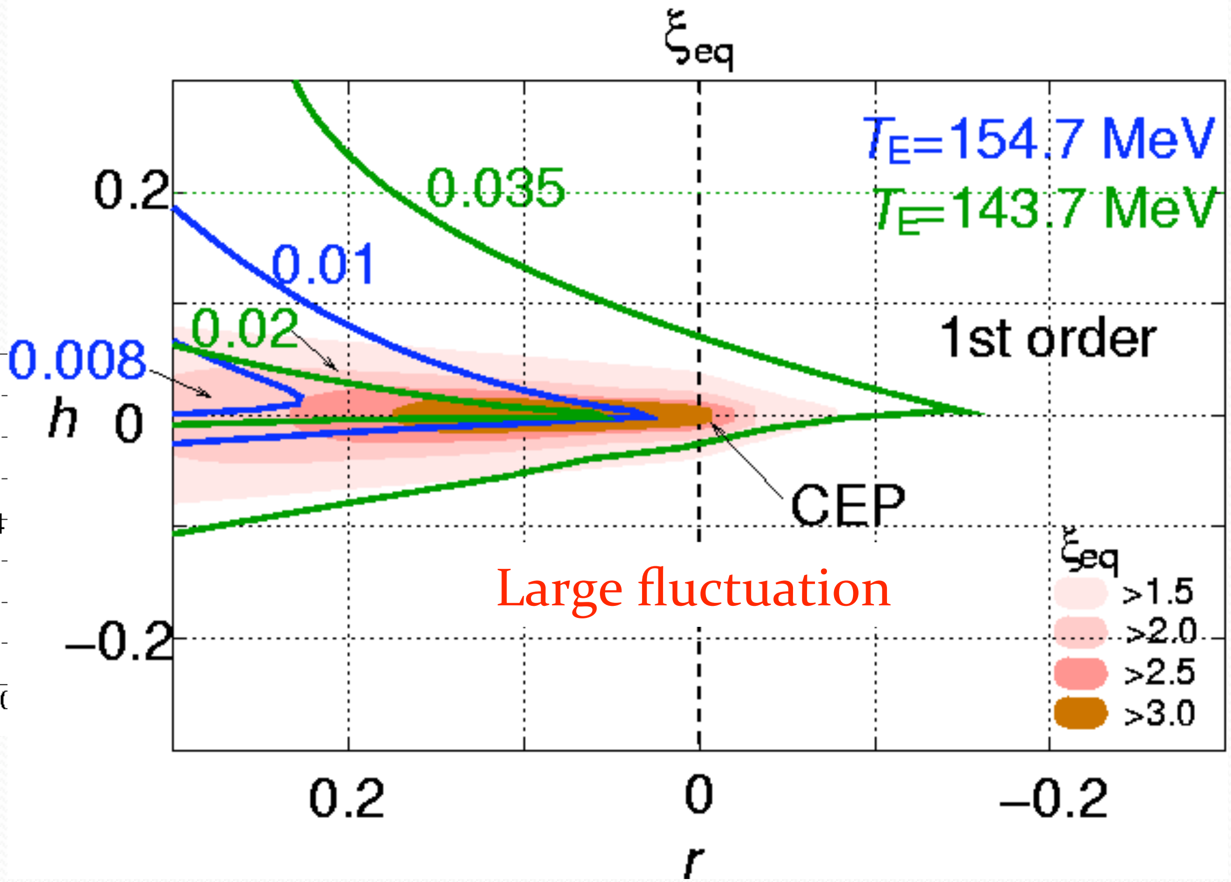
- Narrow collision energy window is needed. ← NA49: anti-p spectra
- Careful choice for initial condition of hydro calculation

Correlation Length

Widom's scaling low

$$\xi_{\rm eq}^2(r,M) = f^2 M^{-2\nu/\beta} g\left(\frac{|r|}{|M|^{1/\beta}}\right)$$





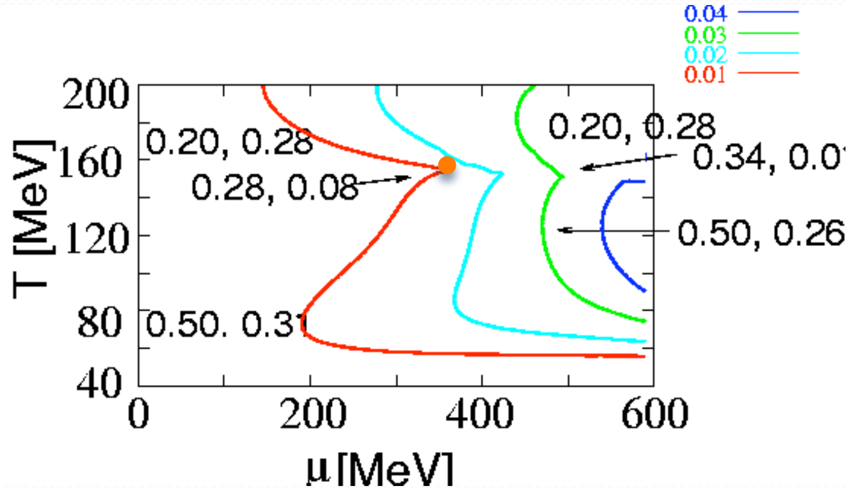
Chiho Nonaka

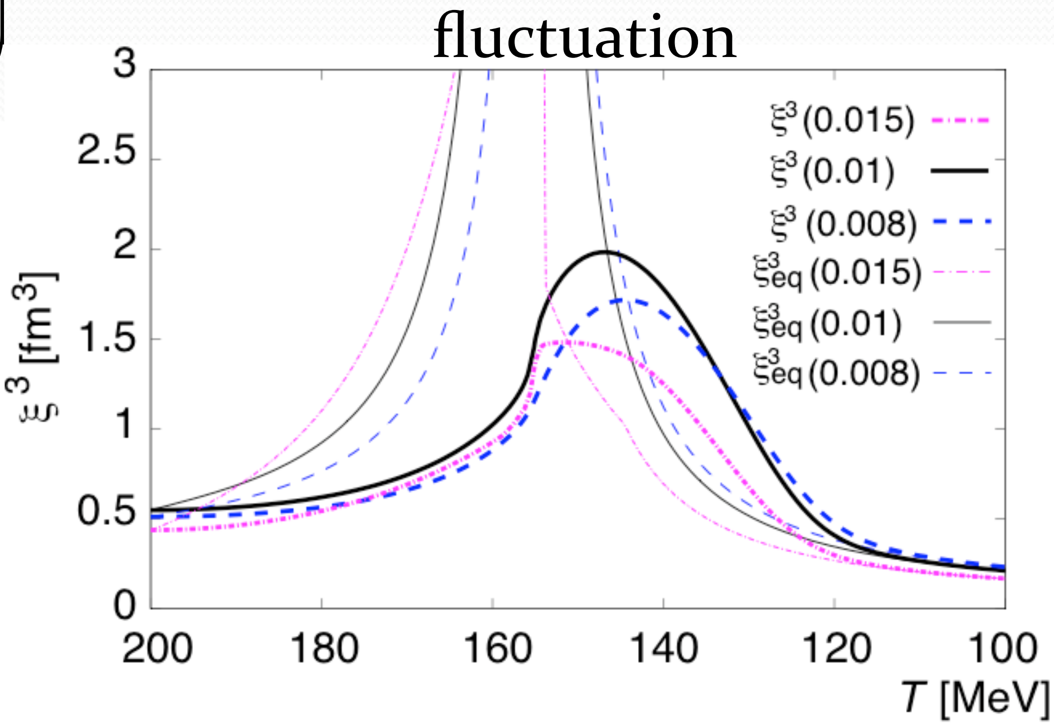
Fluctuation in 1d evolution

- $$\frac{d}{d\tau} m_\sigma(\tau) = -\Gamma\left[m_\sigma(\tau)\right]\left(m_\sigma(\tau) - \frac{1}{\xi_{eq}(\tau)}\right)$$

$$\Gamma(m_\sigma) = \frac{a}{\xi_0}(m_\sigma \xi_0)^z, \quad m_\sigma(\tau) = 1/\xi(\tau)$$

$z = 3.0$ Model H (Halperin RMP49(77)435)

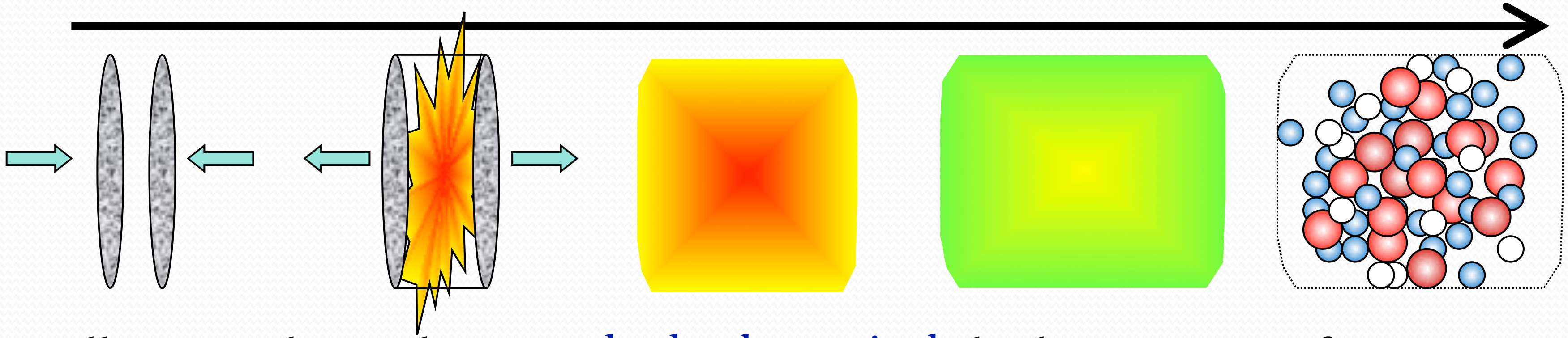




- Critical slowing down
- Evolution rate

Chiho Nonaka

Recent Status of Dynamical Models



collision thermalization hydrodynamical expansion hadronization freeze-out

initial conditions
- parametrization
  Glauber type

- Color Glass Condensate
- Flux Tube
- Initial fluctuation

equation of states
- 1^{st} order phase transition

- Lattice QCD

freezeout process
- Sudden freezeout
- Chemical equilibrium

- Partial chemical equilibrium
- Viscosity effect of hadron phase
- Final state interactions

Viscous Hydrodynamics

Chiho Nonaka

Focusing Effect

3d Ising Model ⟷ QCD

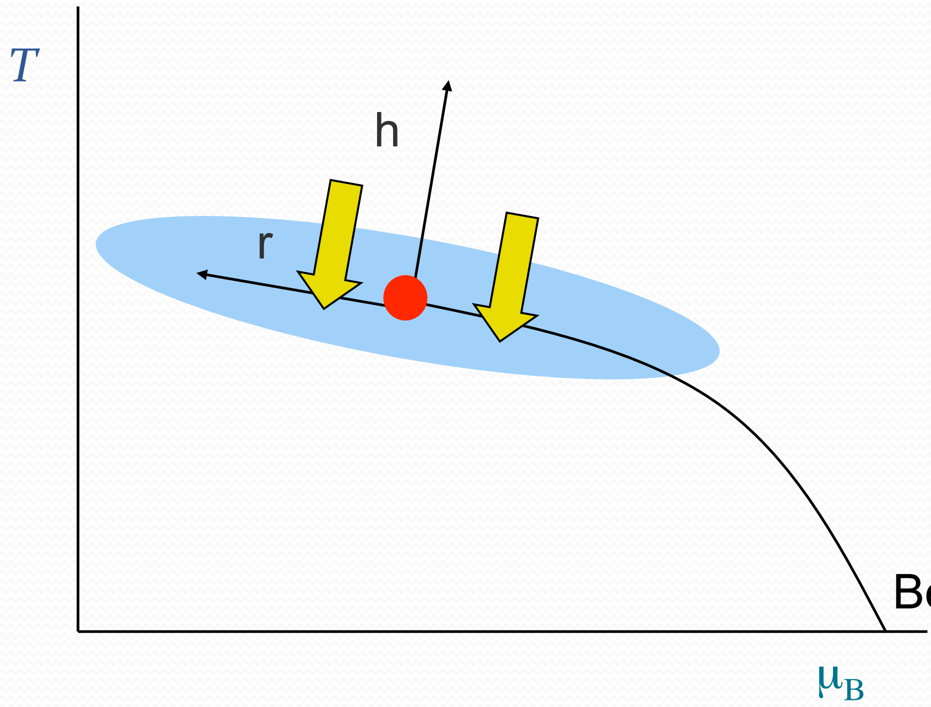
Same Universality Class

$r = \frac{T - T_C}{T_C}$

h : extermal magnetic field

$(r, h) \longleftrightarrow (T, \mu_B)$

$T, \ \mu_B$



•Critical exponent

$$M = \begin{cases} |r|^{\beta} & h = +0 \\ -|r|^{\beta} & h = -0 \end{cases} \qquad M = \mathrm{sgn}(h)|h|^{1/\delta} \ r = 0$$

$$\beta = 0.36, \delta = 4.86$$

Guida and Zinn-Justin, NPB486(97)626

•Critical slowing down

dynamical critical exponent along *h* is dominant.

Berdnikov and Rajagopal,PRC61,105017(99)

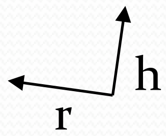
Chiho Nonaka

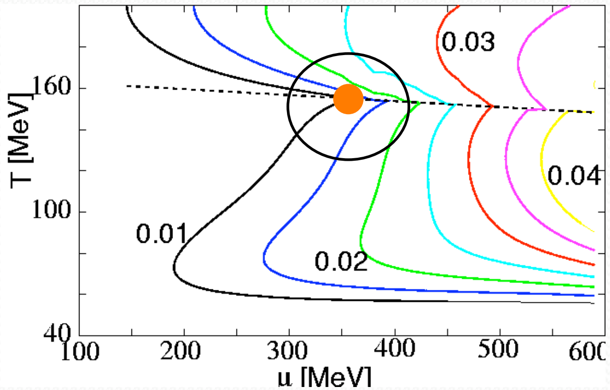
Correlation Length (I)

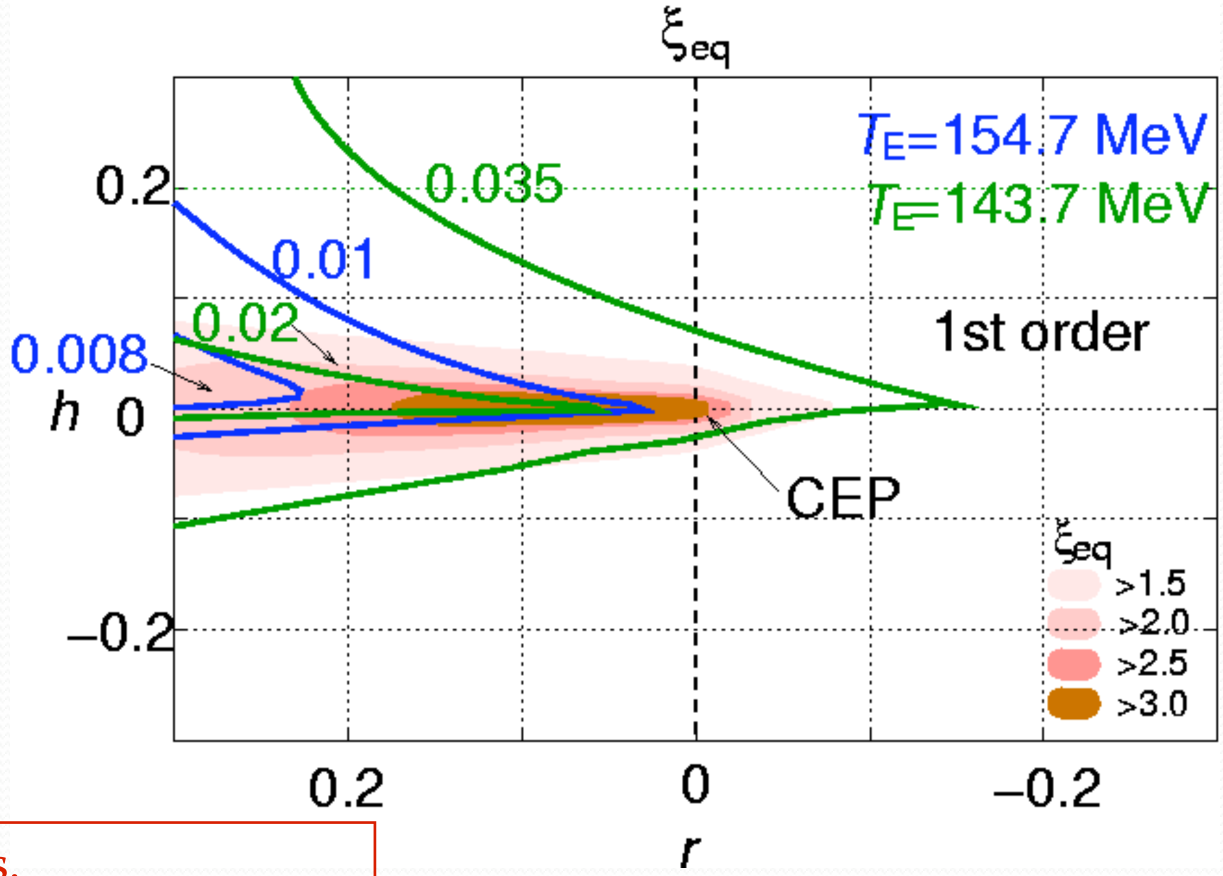
- ξ eq

Widom's scaling low

$$\xi_{\rm eq}^2(r,M) = f^2 M^{-2\nu/\beta} g\left(\frac{|r|}{|M|^{1/\beta}}\right)$$







- Max. ξ_{eq} depends on n_B/s.
- Trajectories pass through the region where ξ_{eq} is large. (focusing)
- n_B/s ←— non-critical component of the EOS

Chiho Nonaka

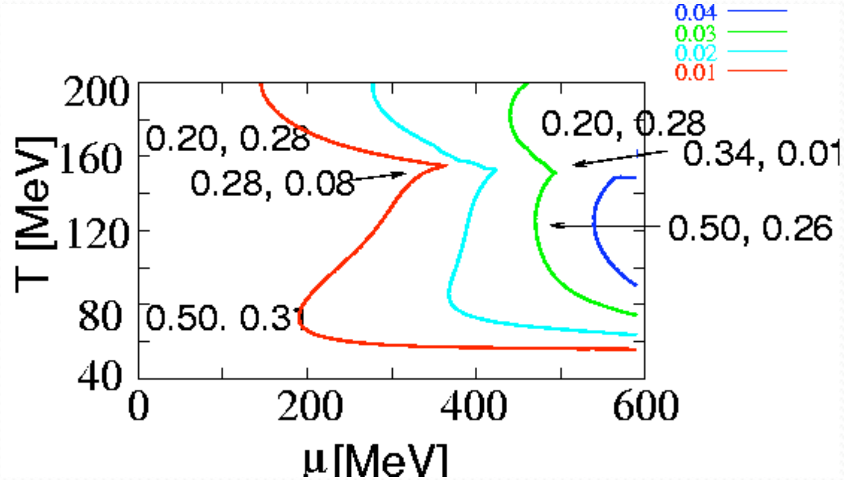
Correlation Length (II)

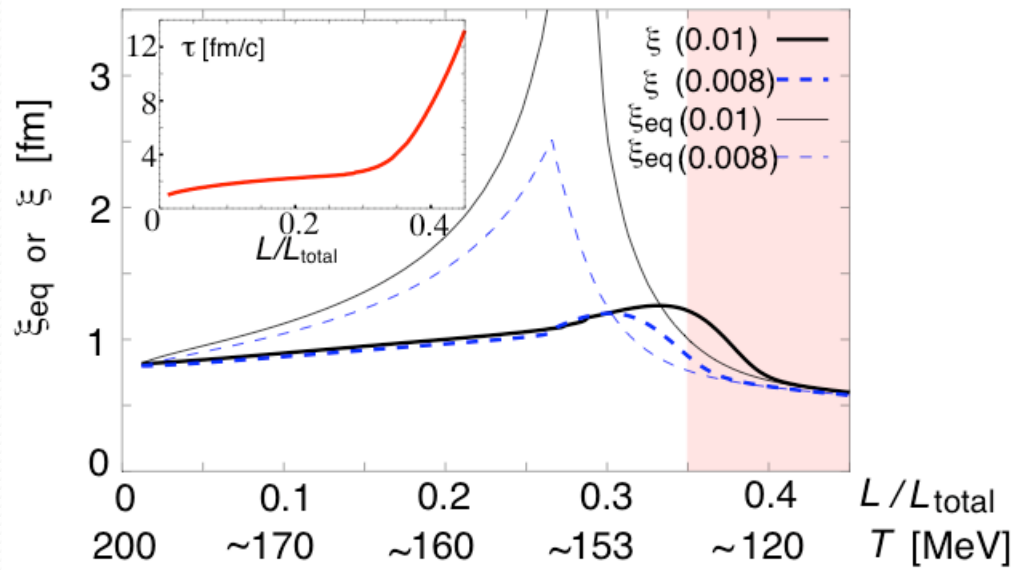
- ξ : time evolution (1-d)

$$\frac{d}{d\tau} m_\sigma(\tau) = -\Gamma[m_\sigma(\tau)]\left(m_\sigma(\tau) - \frac{1}{\xi_{eq}(\tau)}\right)$$

$$\Gamma(m_\sigma) = \frac{a}{\xi_0}(m_\sigma \xi_0)^z, \quad m_\sigma(\tau) = 1/\xi(\tau)$$

$z = 3.0$ Model H (Halperin RMP49(77)435)





- ξ is larger than ξ_{eq} at T_f. ⟵ Critical slowing down
- Differences among ξs on n_B/s are small.
- In 3-d, the difference between ξ_{eq} and ξ becomes large due to transverse expansion.