Nuclear Symmetry Energy and Compact Stars

"simple-minded approach"

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Introduction

Nuclear matter (finite or infinite volume)

consituents: nucleon (proton and neutron)

total number: N = $N_n + N_p$

density : $\rho = \rho_n + \rho_p$

Energy of nuclear matter, U

energy per nucleon: E = U / N

energy density : ϵ = U / V = E ρ

energy per nucleon

$$E = E_0\left(\rho_n = \rho_p = \frac{\rho}{2}\right) + E_{symm}$$

$$E_0 = E_0^{free} + E_0^{pot} \quad : \rho$$

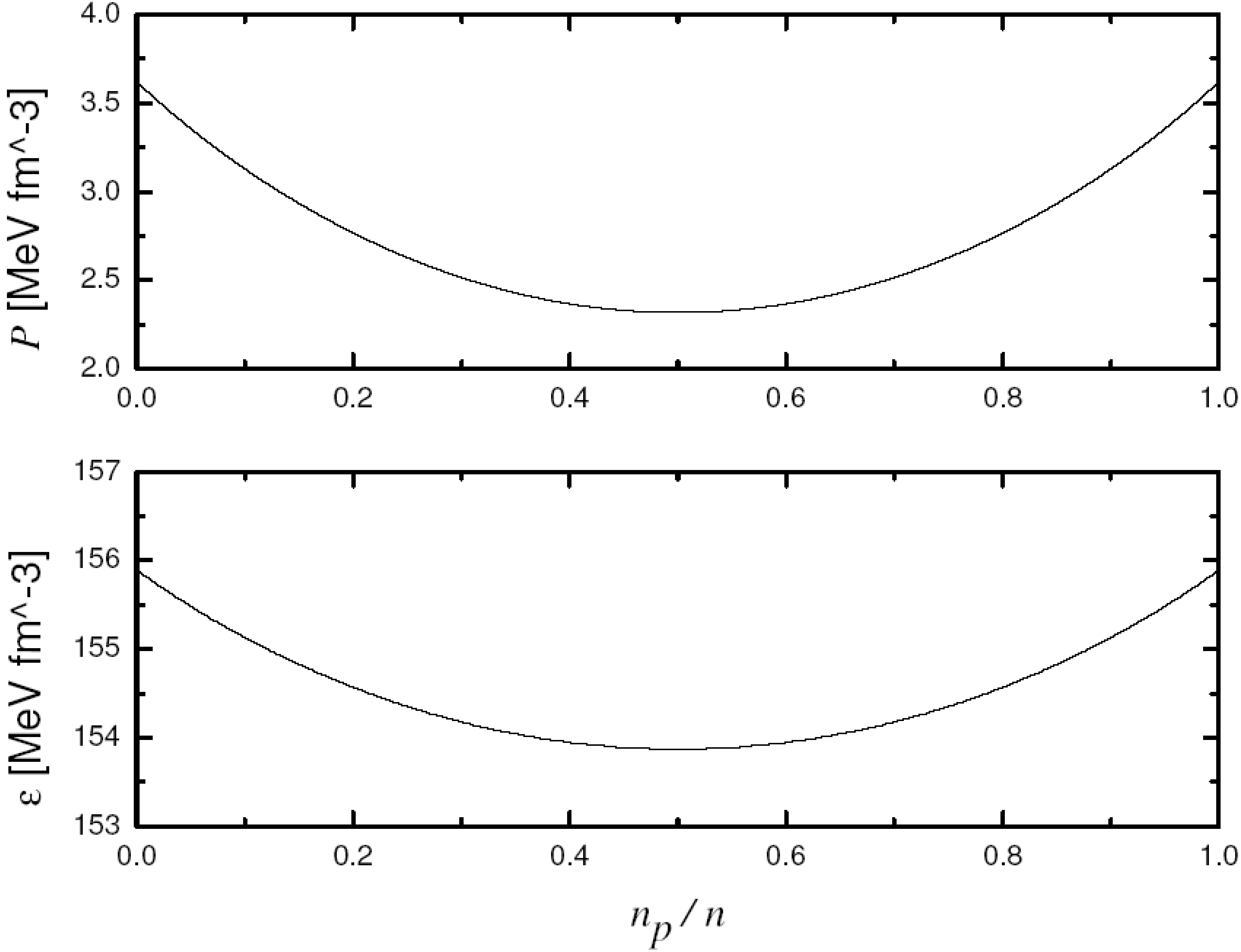
$$E_{symm} = E_{symm}^{free} + E_{symm}^{pot} \quad : \rho_n - \rho_p$$

Free - nucleon matter

$$E_0^{pot} = 0 = E_{symm}^{pot}$$

$$E_{symm}^{free} \neq 0$$

Pauli exclusion principle



4.0
3.5
3.0
2.5
2.0
P [MeV fm^-3]
0.0
0.2
0.4
0.6
0.8
1.0
157
156
155
154
153
ε [MeV fm^-3]
0.0
0.2
0.4
0.6
0.8
1.0
n_p / n

symmetry energy factor, S(n)

$$E_{sym}(n, n_p) = S(n)\left(\frac{n_n - n_p}{n}\right)^2, S(n)$$

$$S_{free}(n) = \left(2^{2/3} - 1\right)\frac{3}{5}E_F^0\left(\frac{n}{n_0}\right)^{2/3}$$

- Model dependent symmetry energy factors
  - Prakash *et al*. (1994)

$$S_F(n) = (2^{2/3} - 1)\frac{3}{5}E_F^0\left[\left(\frac{n}{n_0}\right)^{2/3} - F(n)\right] + S_0F(n),$$

  - B.-A. Li *et al*. (2005)

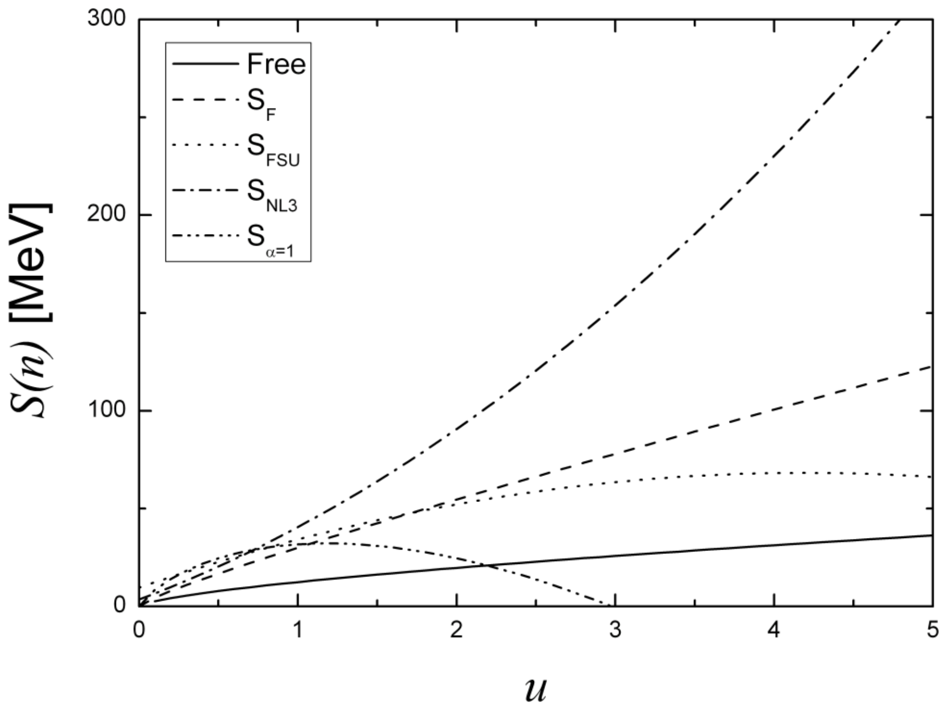
$$S_\alpha = (2^{2/3} - 1)\frac{3}{5}E_F^0\left(\frac{n}{n_0}\right)^{2/3} + A(\alpha)\frac{n}{n_0} + [18.6 - A(\alpha)]\left(\frac{n}{n_0}\right)^{B(\alpha)}$$

  - Piekarewicz and Centelles (2009)

$$S_3(n) \simeq S_0^* + L\rho + \frac{1}{2}K\rho^2,$$

Nuclear Symmetry Energy Factors

(KK and H.K. Lee, arXiv:0909.1398)

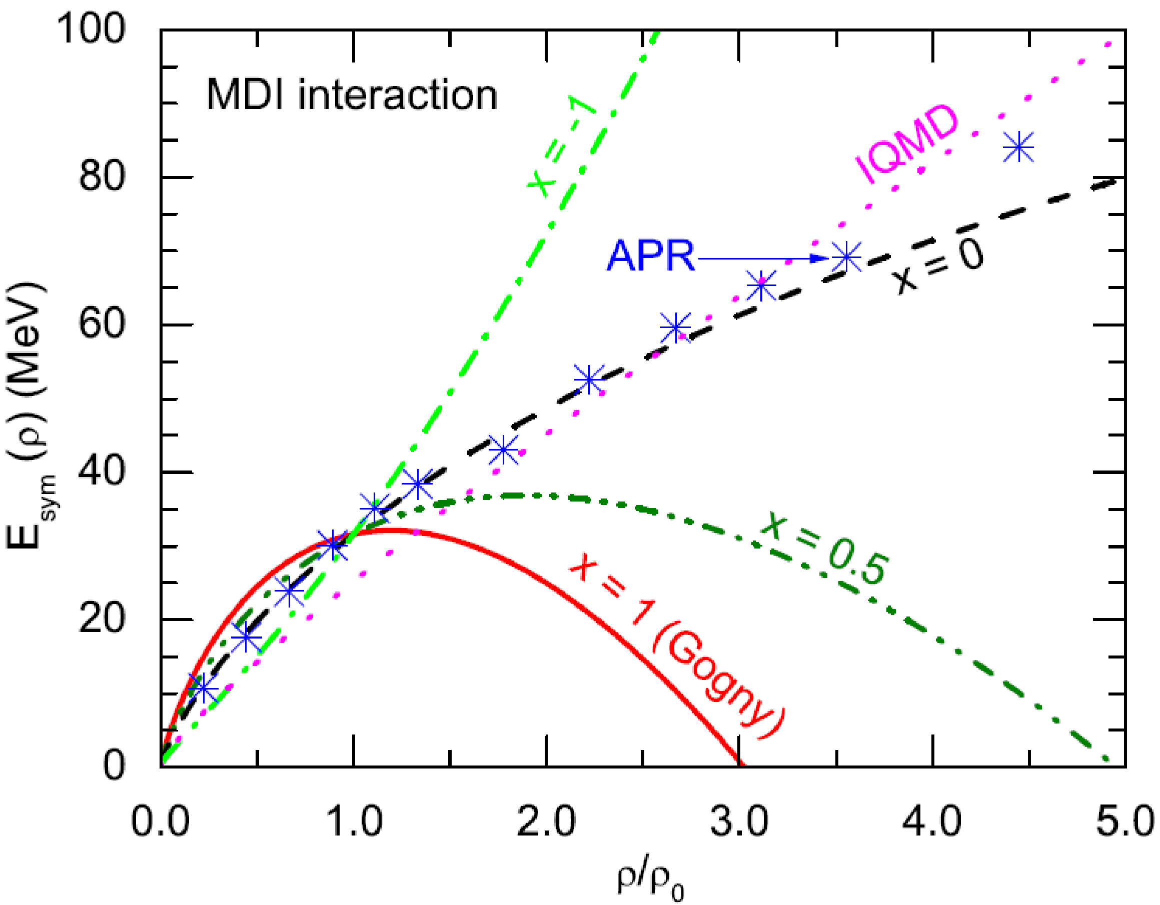


$$\left(u \equiv \frac{n}{n_0} \right)$$

Diverse predictions at u > 1 !

π^-/π^+ ratio in heavy-ion collisions

B-A. Li et al. 0808.0186



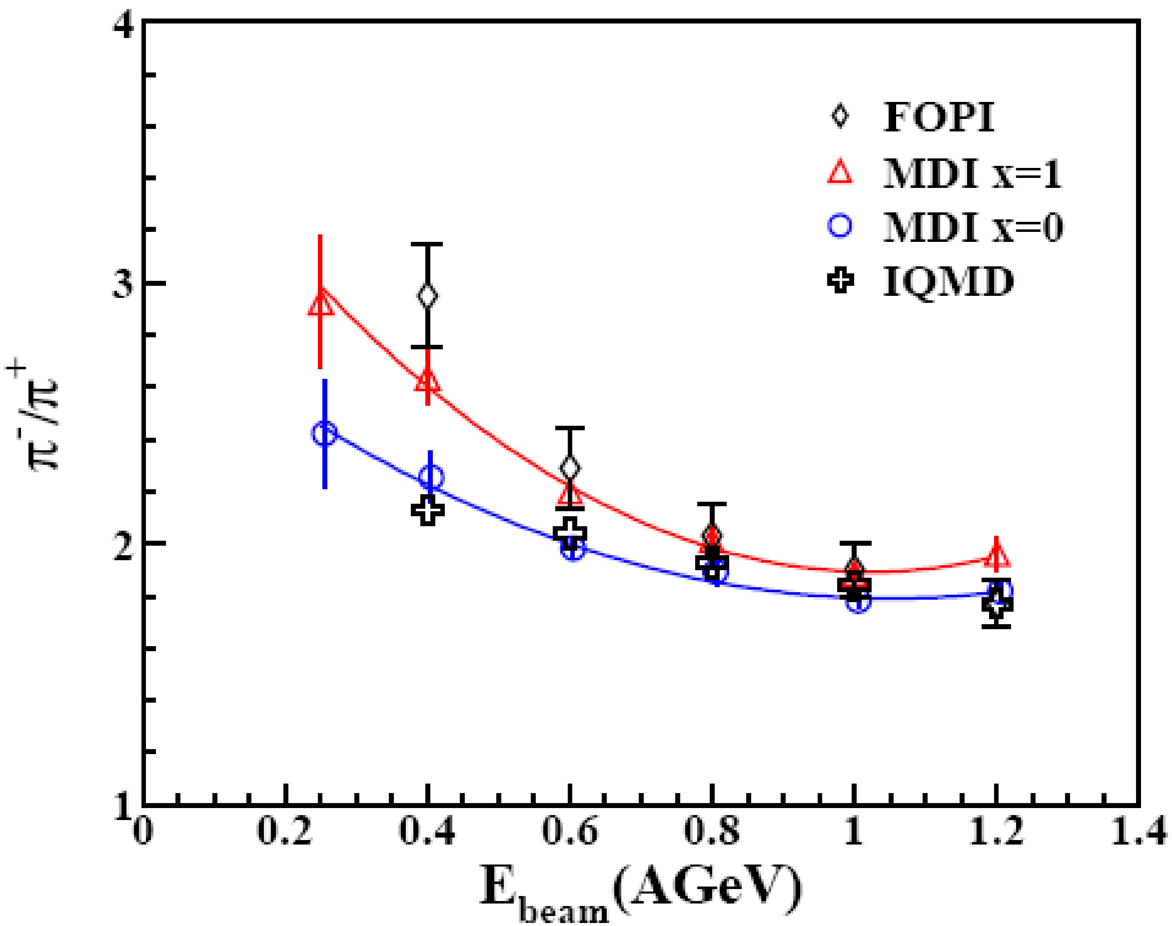
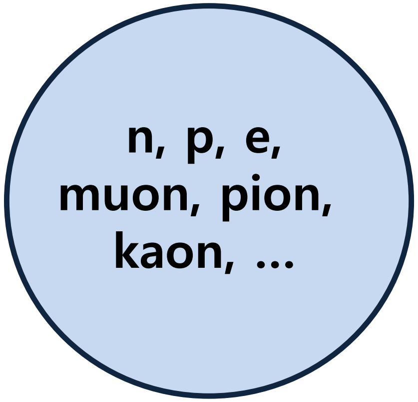


FIG. 4: (Color online)Excitation function of the π^-/π^+ ratio in central Au+Au collisions calculated with the IBUU04 in comparison with the FOPI data and the IQMD prediction.

Neutron star

- Astrophysical Compact Object



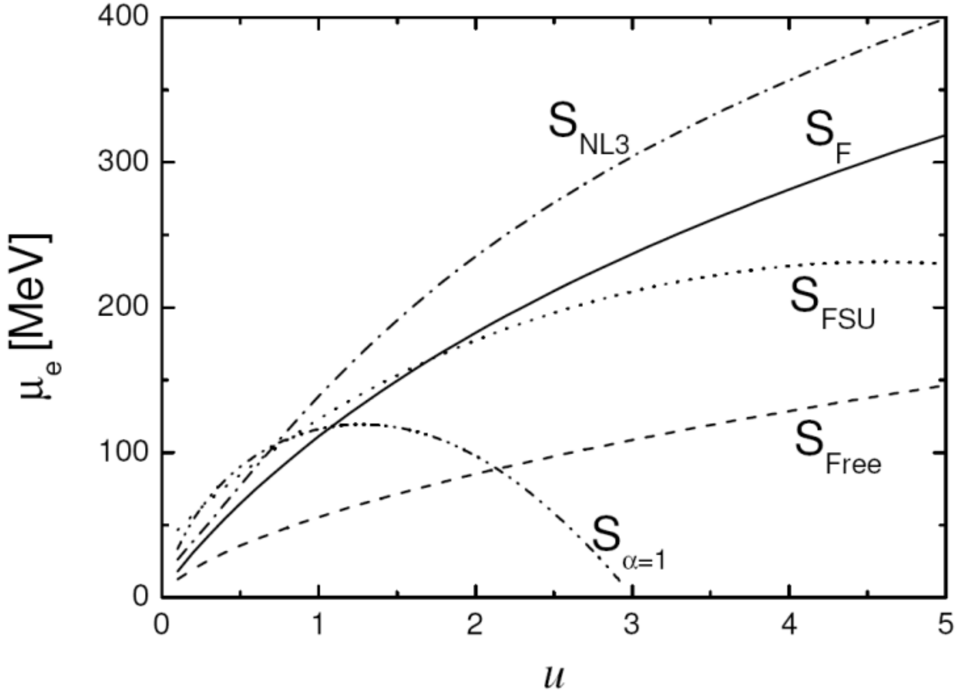
R ~ 10-15 km
M ~ 1-3 M_{sun}

- **Chemical equilibrium (** $\mu_n - \mu_p = \mu_e = \mu_\mu$ **)** and
- **Electrical charge neutrality (** $n_p = n_e + n_\mu$ **)** between particles.
- Pressure ⟺ Gravity: **TOV equation**
- **Equation of State**(EOS) ⟸
  - Free-fermion gas
  - Nuclear Physics (Symmetry Energy)
  - Hadron Physics

Electron Chemical Potential

(KK and H.K. Lee, arXiv:0909.1398)

chemical equilibrium and ***charge neutrality***



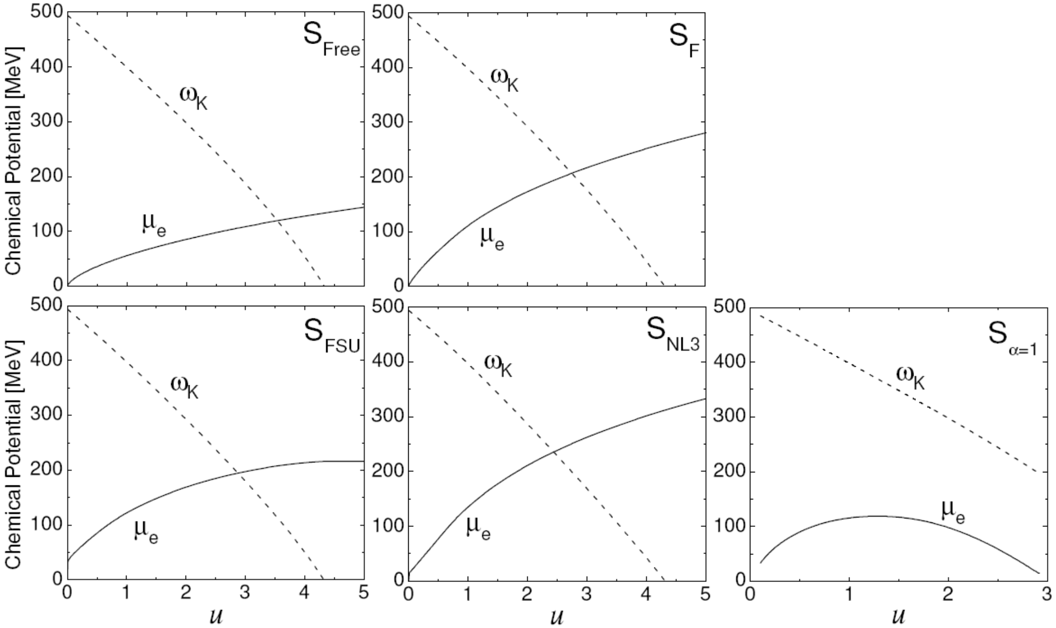
$$\mu_n^{sym} - \mu_p^{sym} = 4\,(1 - 2N_p)\,S(n)$$

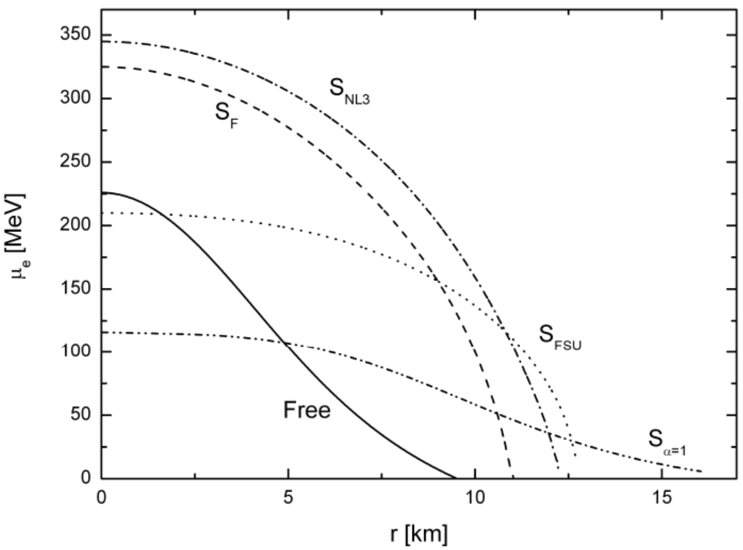
Kaon Condensation Threshold

(KK and H.K. Lee, arXiv:0909.1398)

$$D_{K^-}^{-1} = \omega_K^2 - m_K^2 + \frac{1}{f^2}(n_n/2 + n_p)\omega_K + \frac{\Sigma_{KN}}{f^2}n,$$ (Brown *et al*, 2008)

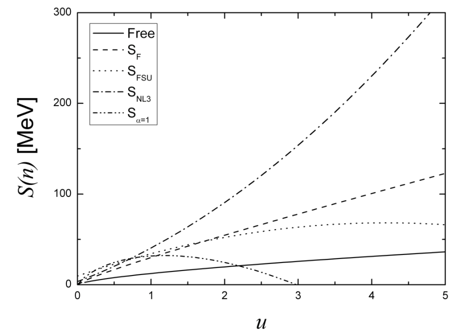
$$f = 93\text{MeV} \quad \Sigma_{KN} = 400\text{MeV}$$





No kaon condensation ?

Equation of States for NS-like Compact Object



- Total energy density and pressure (We put $\hbar = c = 1$)

$$
\begin{aligned}
\epsilon_{tot} &= \epsilon_{nucleon} + \epsilon_{lepton} \\
&= m_N n + nS(n)\left(1 - 2\frac{n_p}{n}\right)^2 + nV(n) + \sum_{i=e,\mu} \frac{m_i}{\lambda_i^3}\chi(x_i), \\
P_{tot} &= P_{nucleon} + P_{lepton} \\
&= n^2(1 - 2\frac{n_p}{n})^2\frac{dS}{dn} + n^2\frac{dV}{dn} + \sum_{i=e,\mu}\frac{m_i}{\lambda_i^3}\phi(x_i),
\end{aligned}
$$

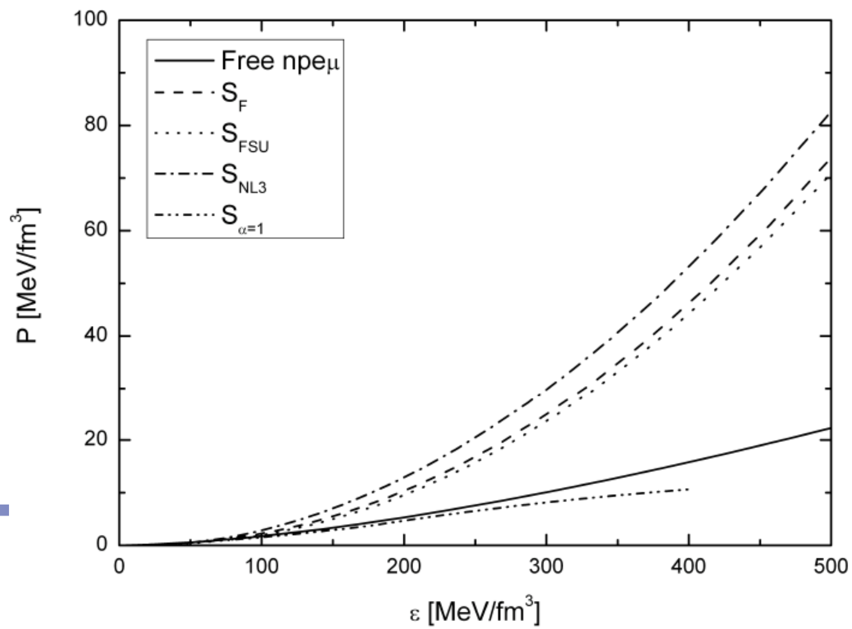
$$\chi(x) = \frac{1}{8\pi^2}\left[x\sqrt{1+x^2}\,(1+2x^2) - \ln\left(x + \sqrt{1+x^2}\right)\right]$$

$$\phi(x) = \frac{1}{8\pi^2}\left[x\sqrt{1+x^2}\left(\frac{2}{3}x^2 - 1\right) + \ln\left(x + \sqrt{1+x^2}\right)\right]$$

$x = \frac{p_F}{m}$: (Dimensionless) Fermi momentum

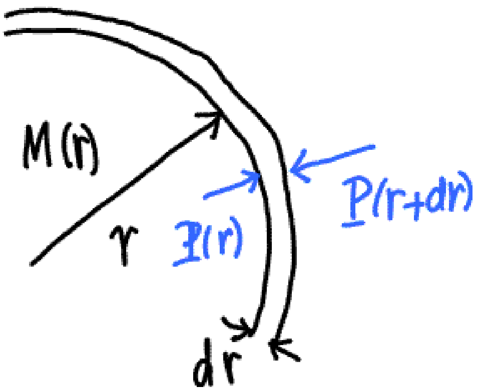
$\lambda = 1/m$: Compton wavelength

(Shapiro & Teukolsky)



TOV(Tolmann-Oppenheimer-Volkov) Equation

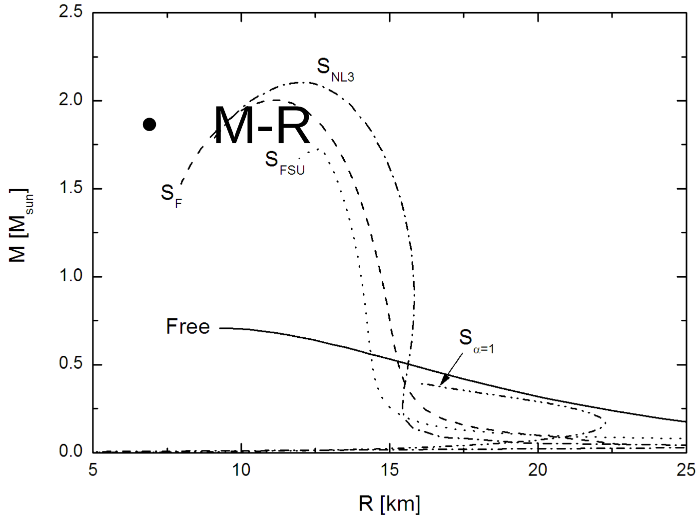
Gravity ⟷ Pressure



$$G\frac{M(r)}{r^2}\epsilon(r)4\pi r^2 dr = -\frac{dP(r)}{dr}4\pi r^2 dr$$

$$\frac{dP}{dr} = -\frac{GM\epsilon}{r^2}\left[1+\frac{P}{\epsilon}\right]\left[1+\frac{4\pi r^3 P}{M}\right]\left[1-\frac{2GM}{r}\right]^{-1}$$

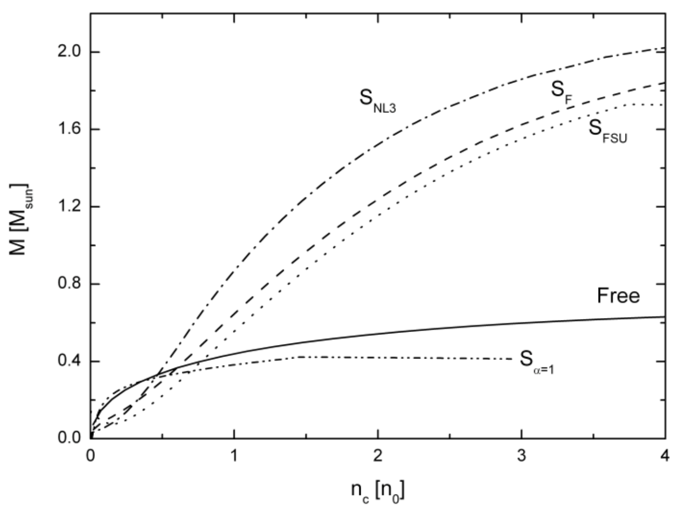
- M-R



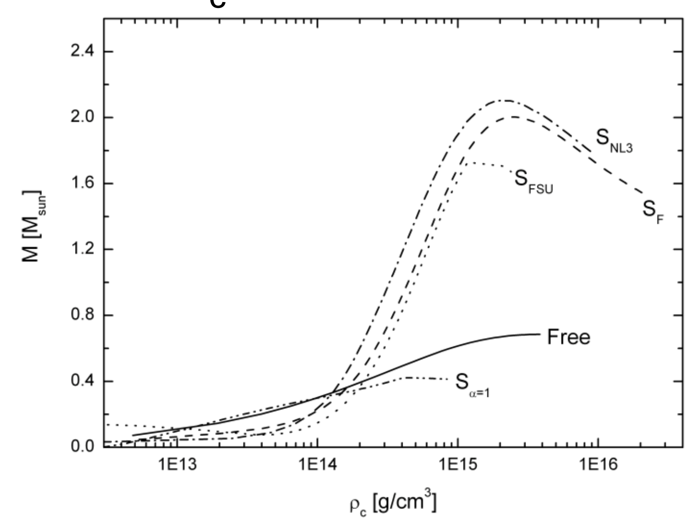
@ Mmax of each models

Free	0.71 M_{sun}, 9.27 km
S_F	2.00 M_{sun}, 11.03 km
S_{FSU}	1.73 M_{sun}, 12.72 km
S_{NL3}	2.10 M_{sun}, 12.22 km
$S_{alpha=1}$	0.39 M_{sun}, 16.08 km

- M-n_c

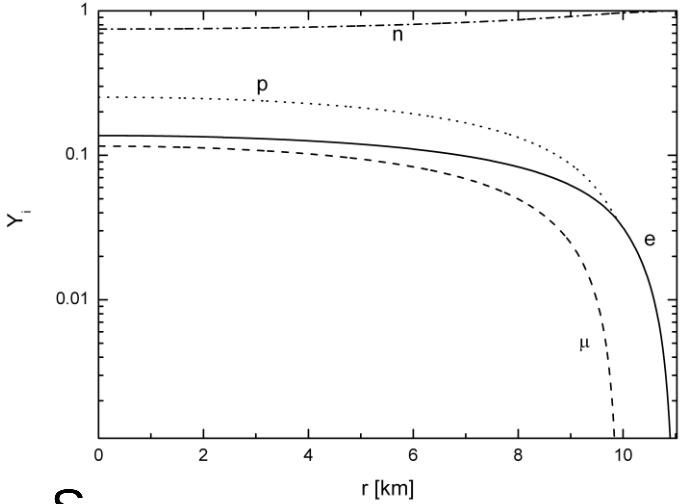


- M-rho_c

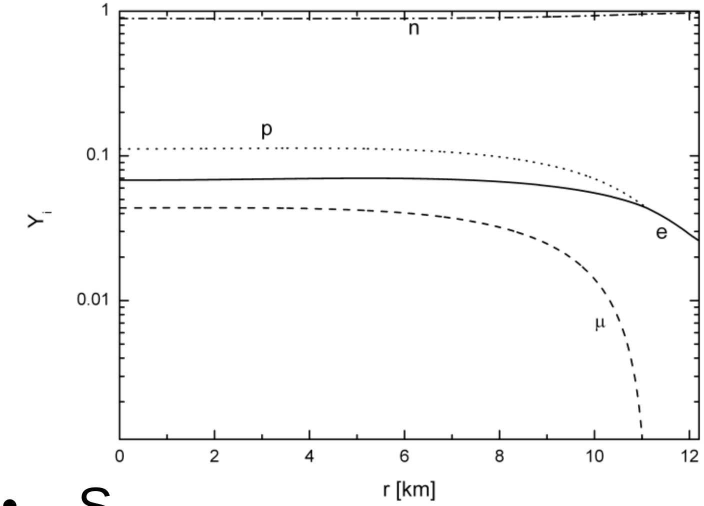


Radial Dependence of Compositions in Compact Objects

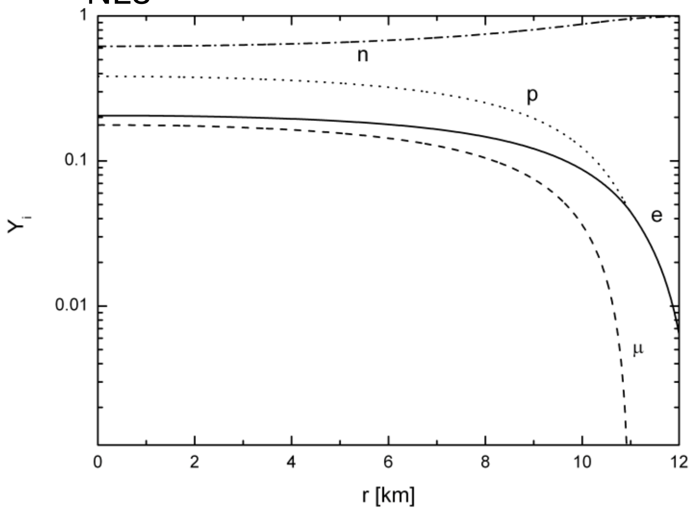
- S_F



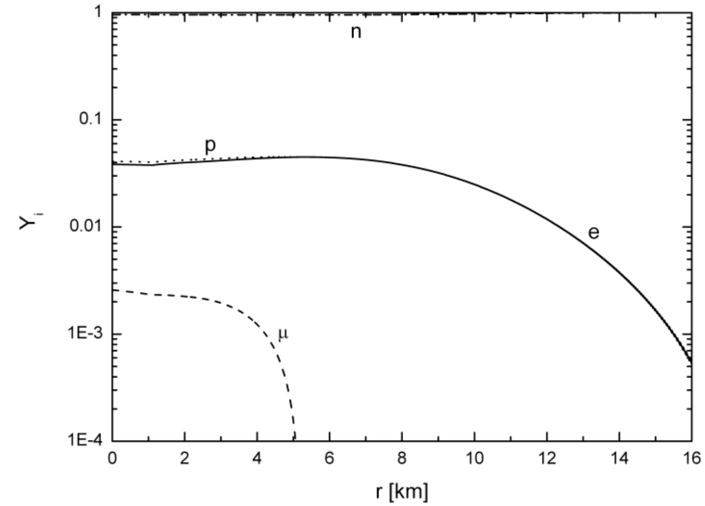
- S_{FSU}



- S_{NL3}

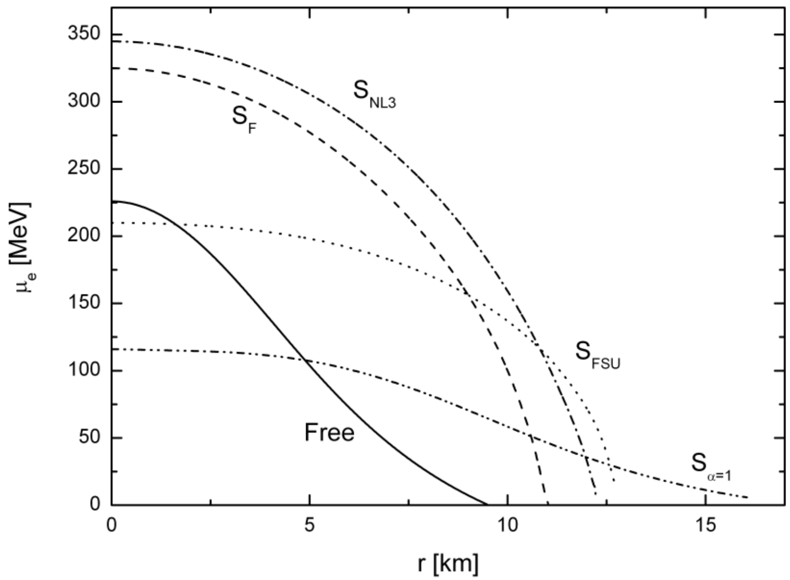


- $S_{alpha=1}$

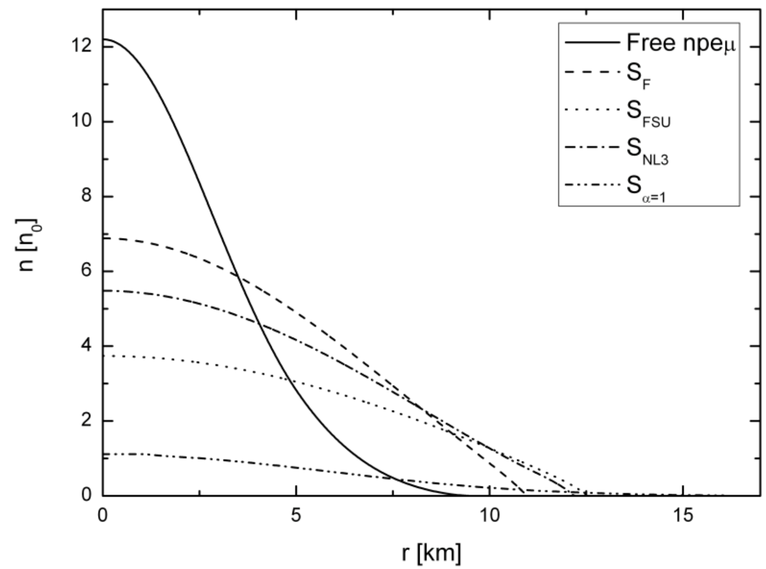


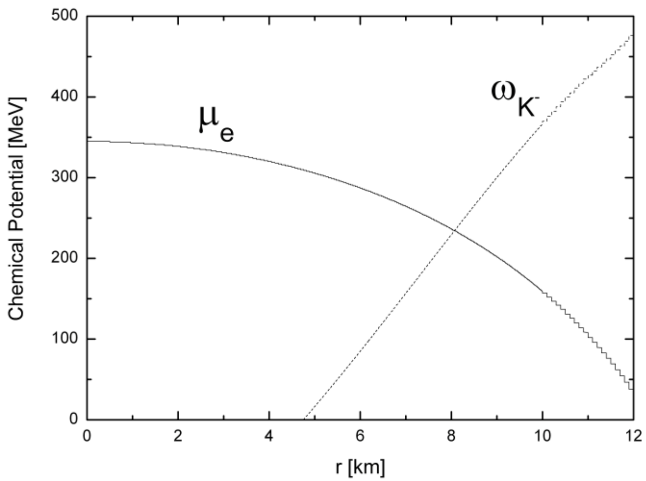
Radial Dependence of Electron Chemical Potential and Nucleon Number Density

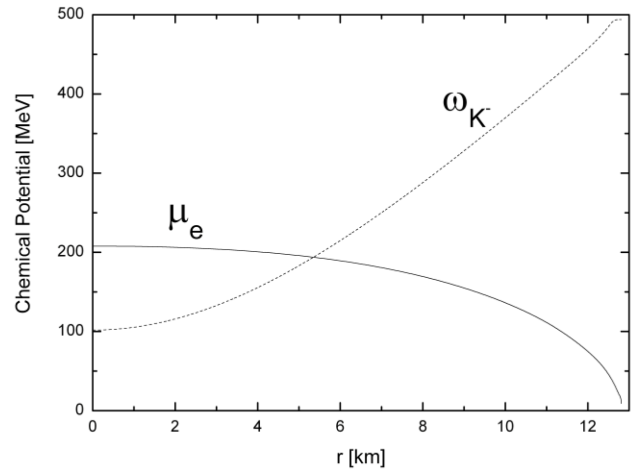
- Electron chemical potential vs. distance from center of compact object @ M_{max} of each models

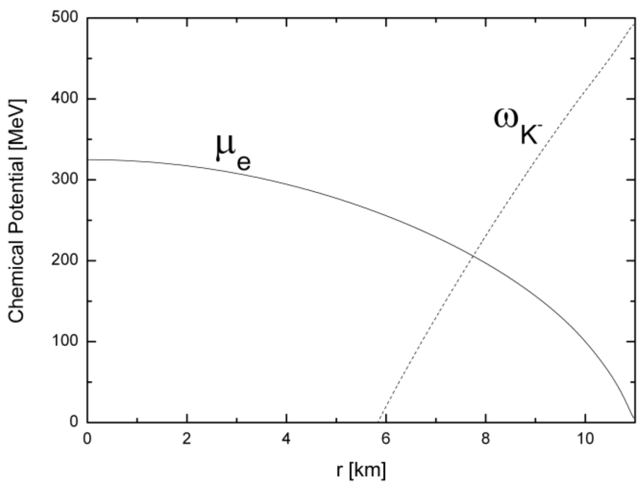


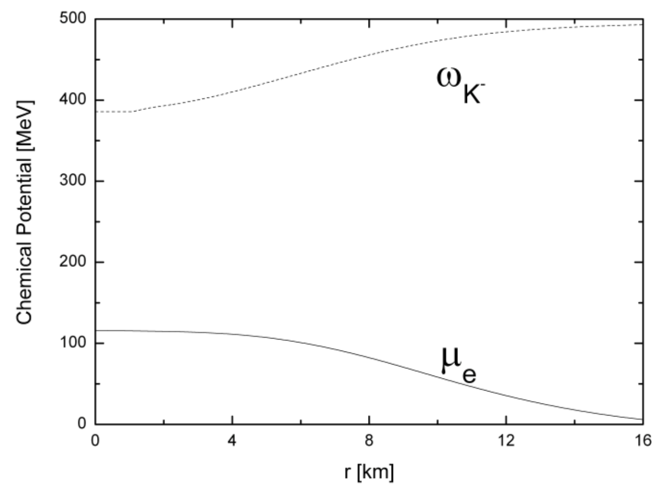
- Nucleon number density vs. distance from center of compact objects @ M_{max} of each models




Chemical Potential [MeV]
r [km]
μ_e
ω_{K^-}

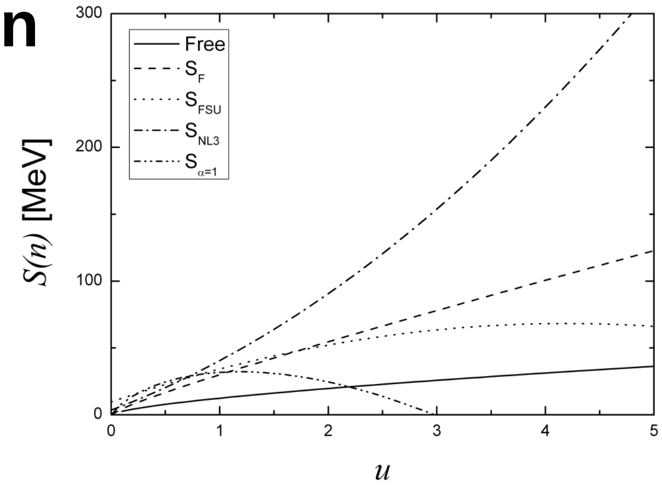

Chemical Potential [MeV]
r [km]
ω_{K^-}
μ_e


Chemical Potential [MeV]
r [km]
μ_e
ω_{K^-}


Chemical Potential [MeV]
r [km]
ω_{K^-}
μ_e

< Nuclear Symmetry Energy at Higher Density >

-Theory: Models for Nuclear Inetraction
Effective Theories for QCD



- Experiments: CBM/FAIR, ALICE/LHC, RHIC…

- Observations: Neutron Stars, Black Holes, .. ,
Gravitaional Waves, ..

Observables ??!!