HIM-WCU workshop
Hanyang U.
Oct. 31, 2009

Half-Skyrmion Matter
&
Quarkionic Matter

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Contents

1. Skyrme Model?

Skyrmion Model

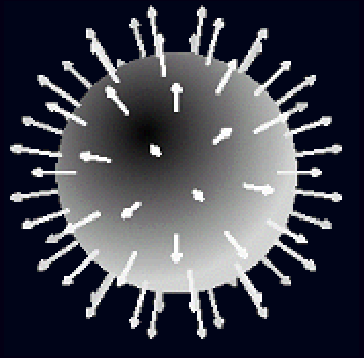
1960, T. H. R. Skyrme

$$\mathcal{L} = \frac{f_\pi^2}{4}\mathrm{Tr}(\partial_\mu U^\dagger \partial^\mu U) + \frac{1}{32e^2}\mathrm{Tr}[U^\dagger \partial_\mu U, U^\dagger \partial_\nu U]^2$$

$U(\vec{x})$: mapping from R^3-{∞}=S^3 to $SU(2)=S^3$

→ topological soliton

R ~ 1 fm
M ~ 1.5 GeV

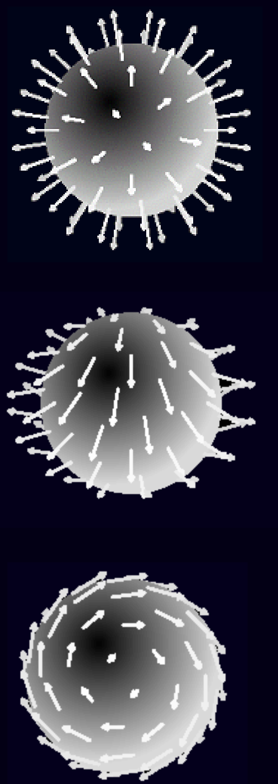


→ BARYON ?

Hadronic World
(Skyrmionist's Viewpoint)

1960 Skyrme

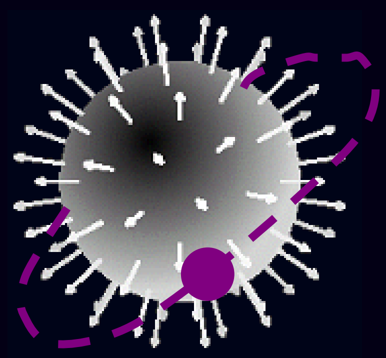
pion



SU(2) Collective Coord. Quantization → N, Δ

SU(3) collective Coordinate Quatization → N, Σ, Ξ, Δ, Σ*, Ξ*, Ω

Hedgehog Soliton

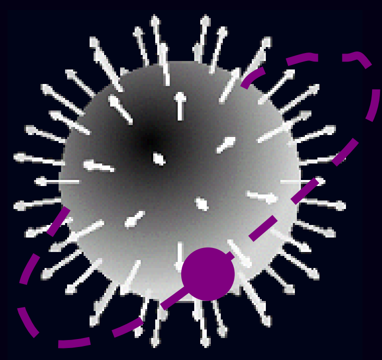


bound kaon

SU(2) collective
Coordinate
Quatization

N, Σ, Ξ,
Δ, Σ*, Ξ*, Ω

Hedgehog Soliton



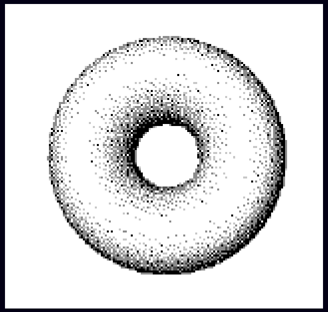
bound D,D*

SU(2) collective
Coordinate
Quatization

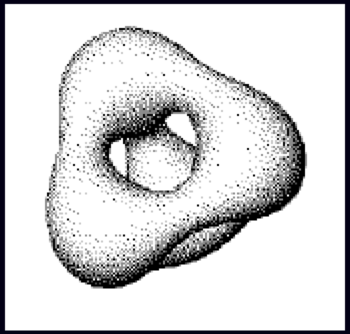
$N, \Sigma, \Xi_c,$

$\Delta, \Sigma_c^*, \Xi_c^*, \Omega_c$

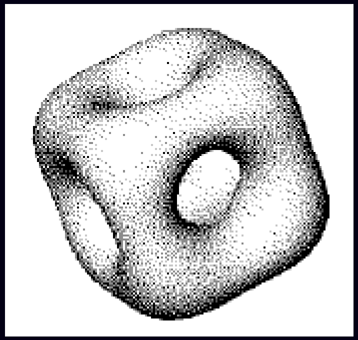
Multi-Skyrmion system



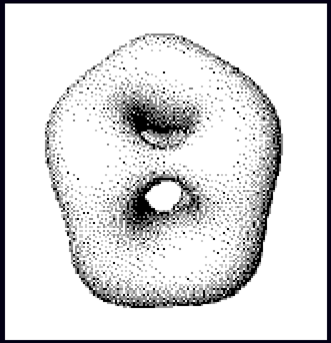
B=2 Torus



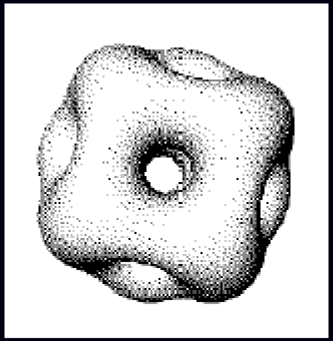
B=3 Tetrahedron



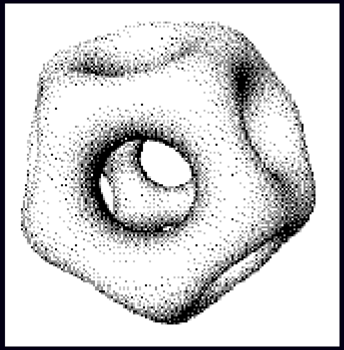
B=4 Cube



B=5 with D_{d2} sym



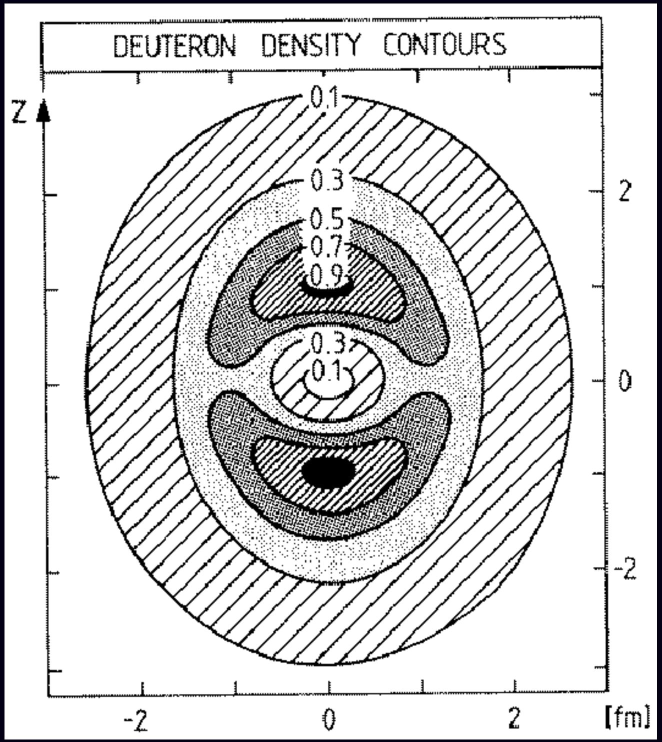
B=6 with D_{d4} sym



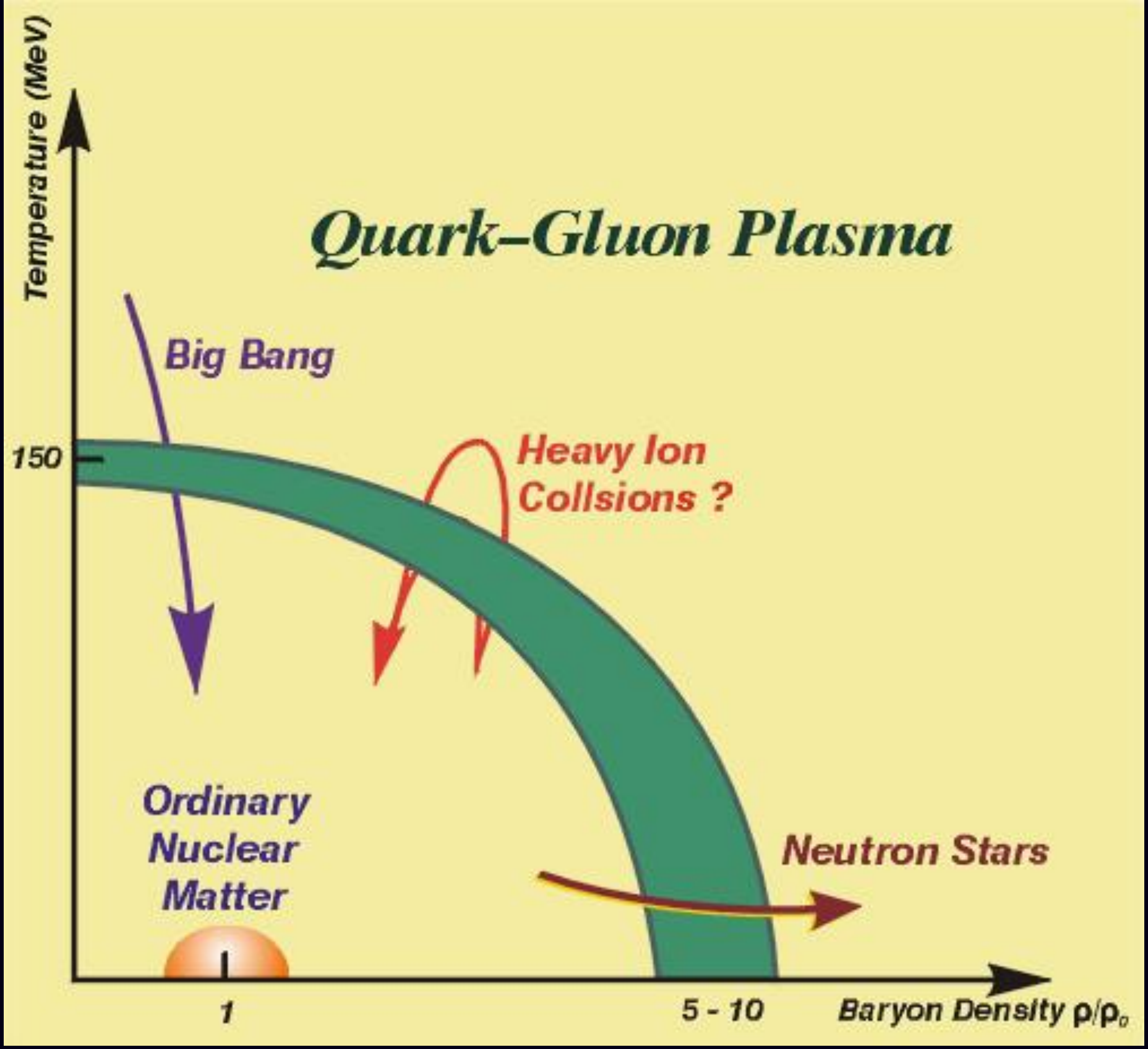
B=7 Dodecahedron

Toroidal B=2 skyrmion

1988, Braaten & Carson,
1995, Leese, Manton & Schroers

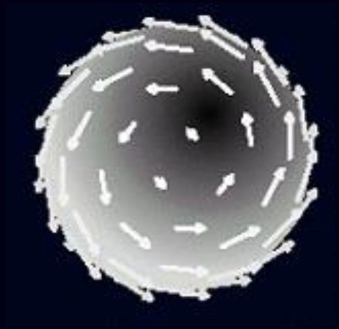


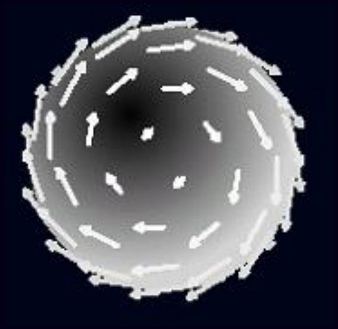
2. Dense Skyrmion Matter



2. Dense Skyrmion matter

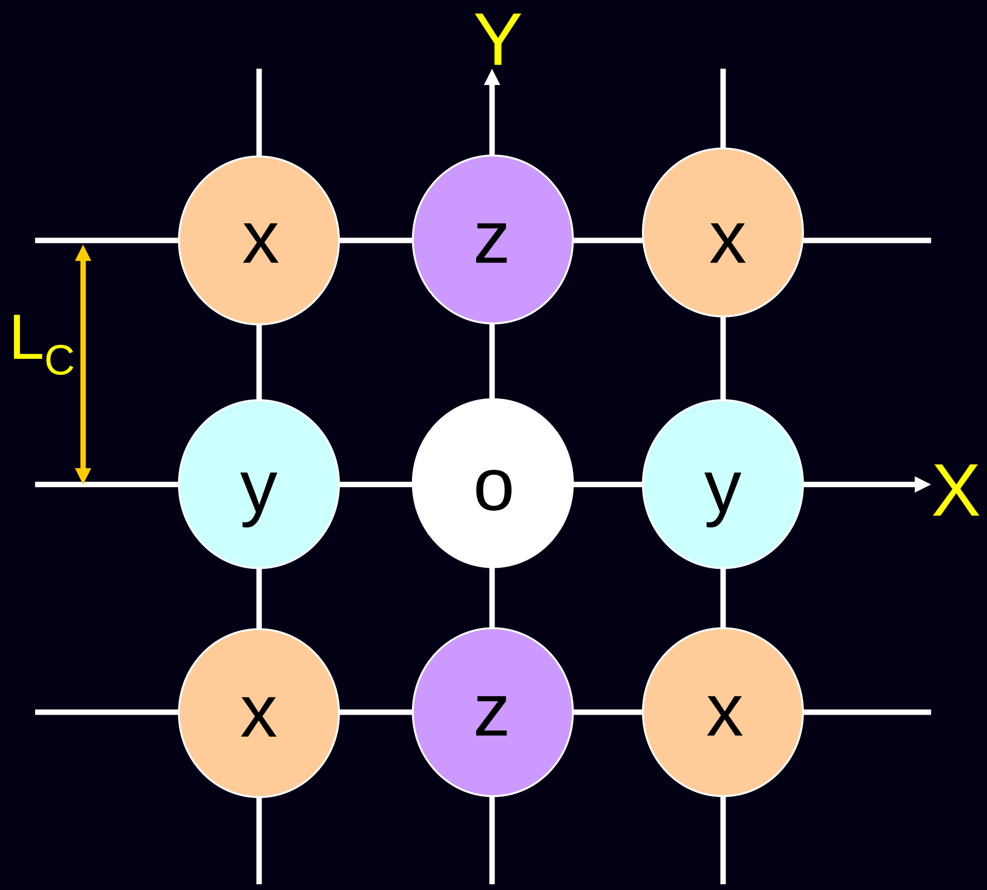
Two skyrmions in the most attractive configuration.





Simple Cubic Skyrmion Crystal

1985, I. Klebanov

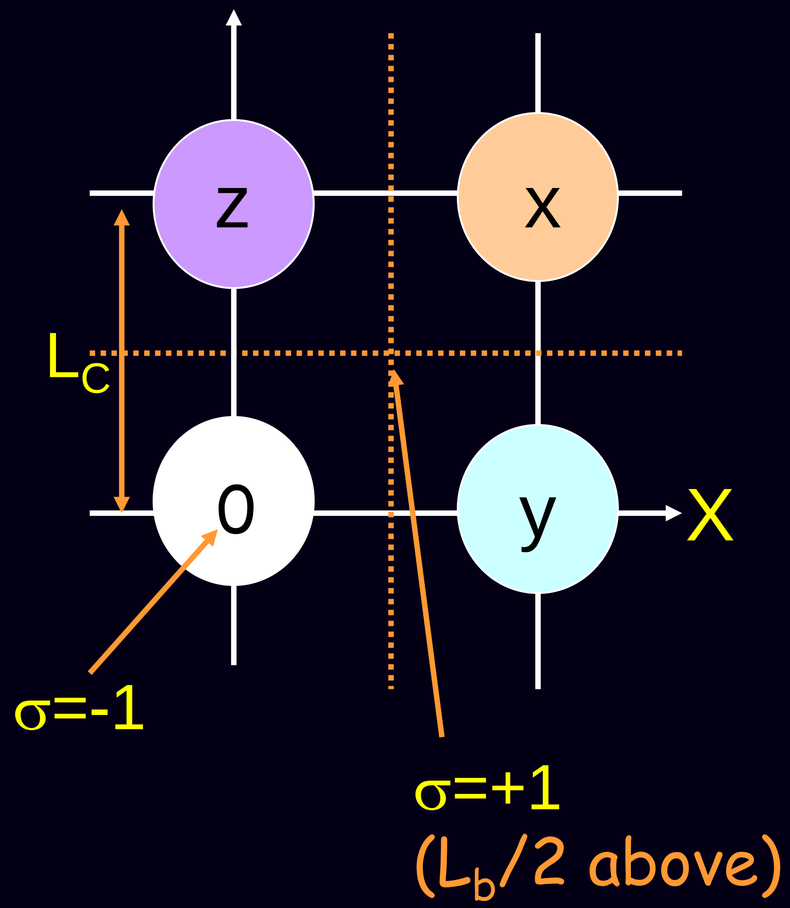


$U(x+L_C,y,z) = \tau_y\, U(x,y,z)\, \tau_y$

$(E/B)_{min}=1.078$ at $L_C=5.56$

Half-Skyrmion Crystal

1987, A. S. Goldhaber & N. S. Manton



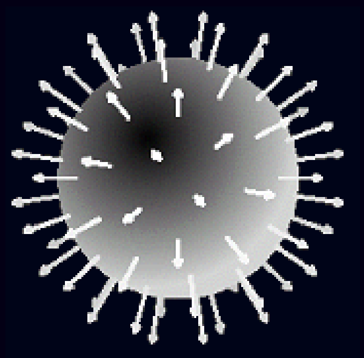
$U(x+L_C,y,z) = \tau_y U(x,y,z) \tau_y$

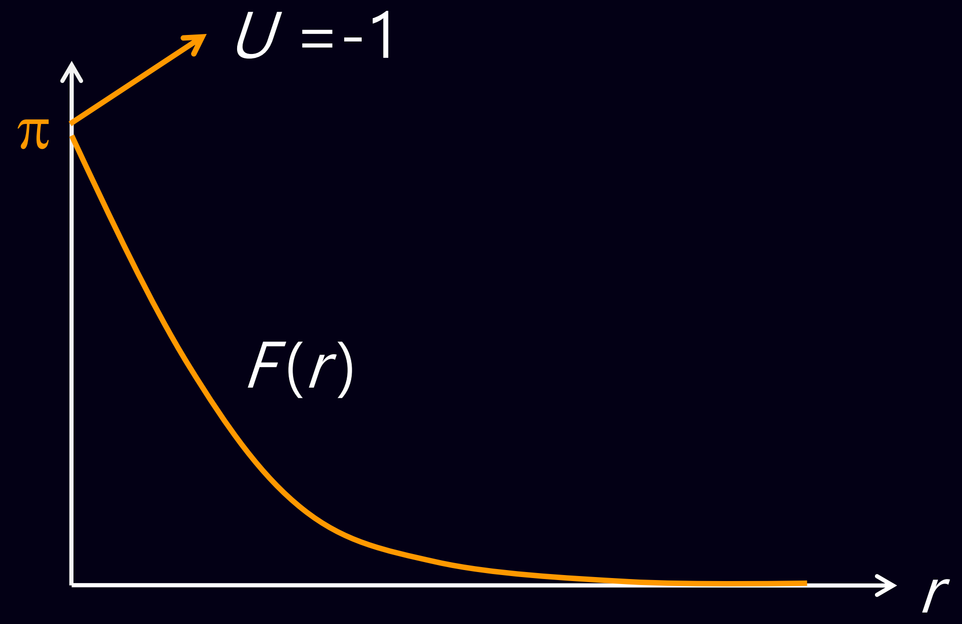
+ additional Symmetry w.r.t. σ → -σ

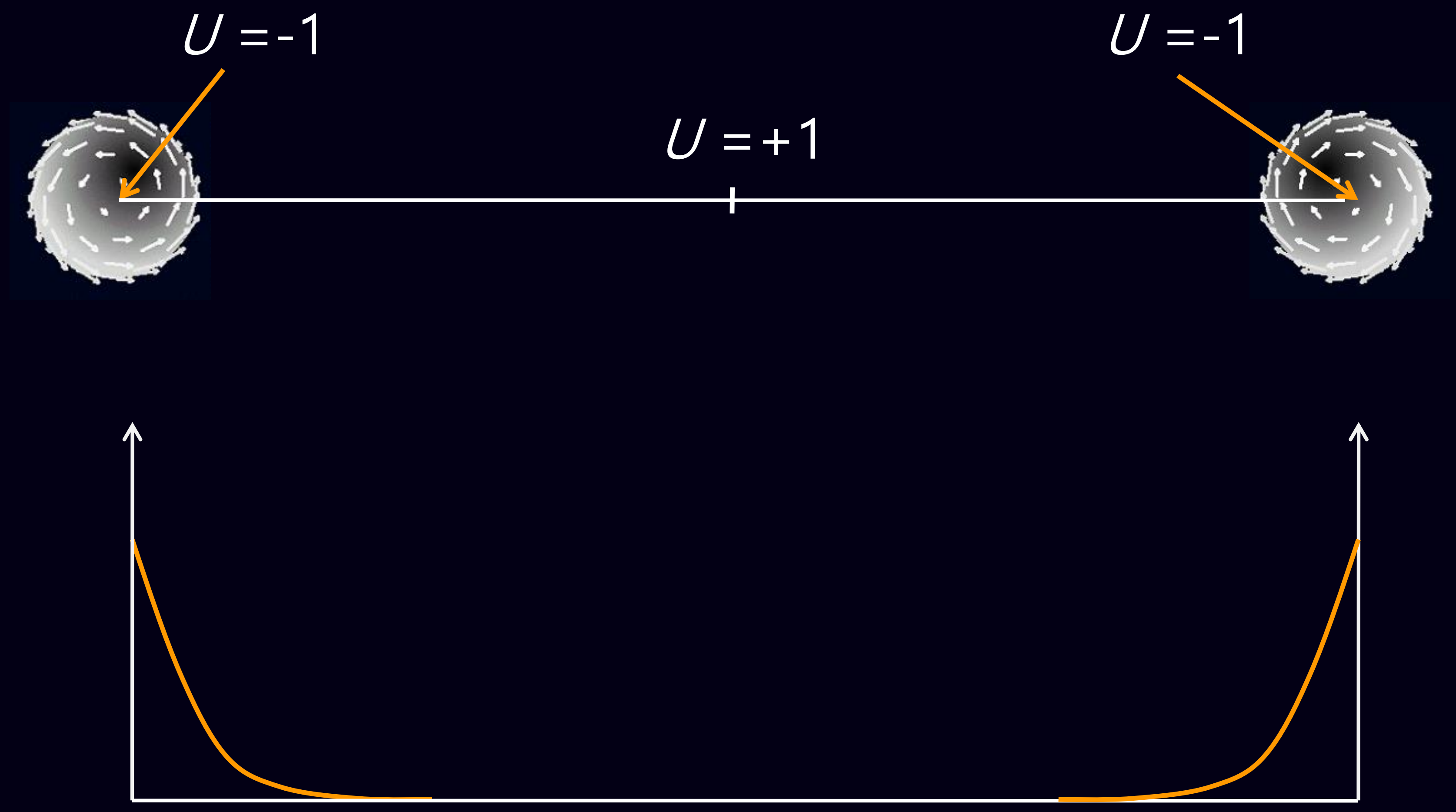
$(E/B)_{min}=1.076$ at $L_C=5.56$

Half-skyrmion?

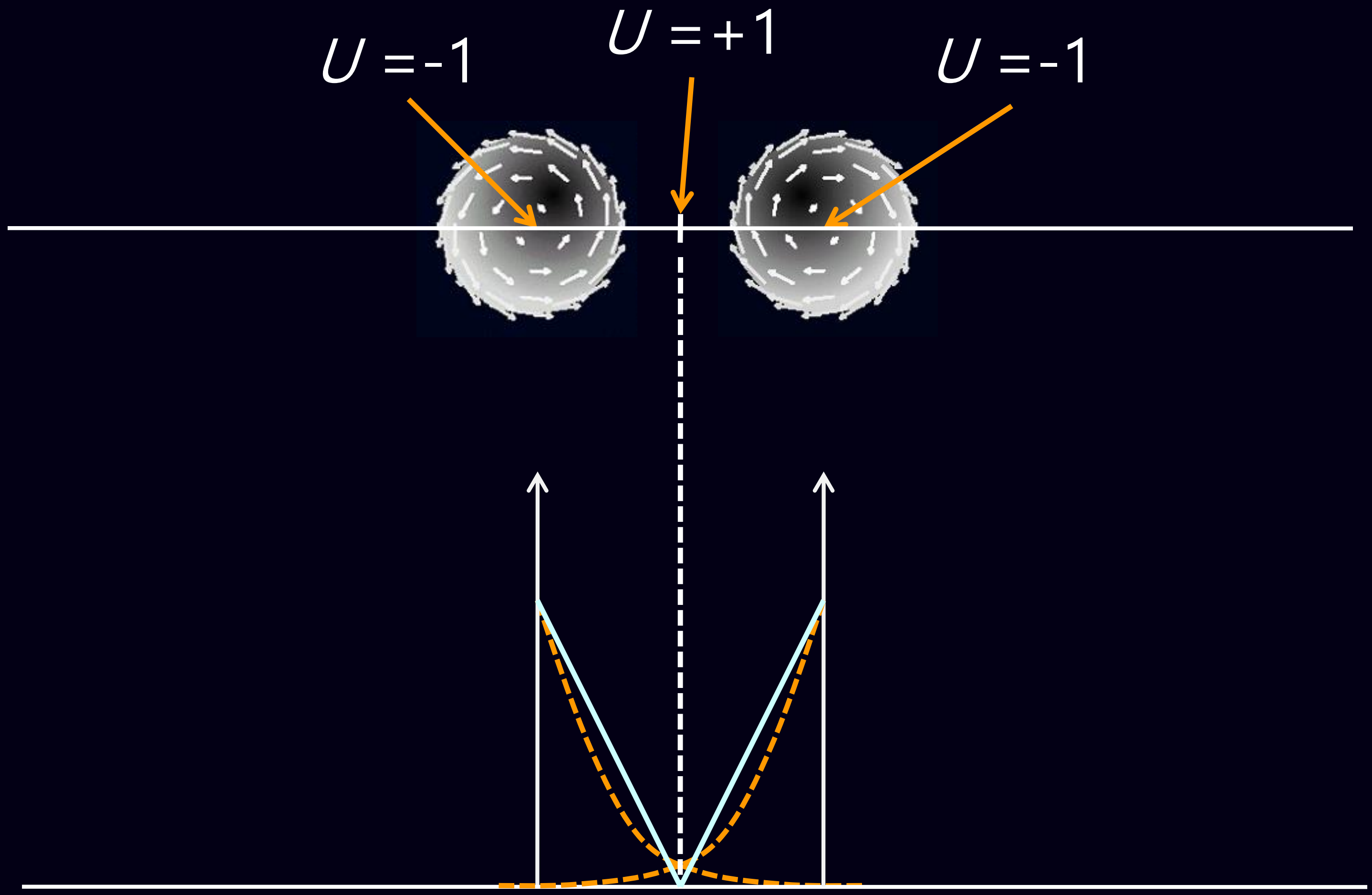
$U = \exp(i\,F(r)\vec{\tau}\cdot\hat{r})$





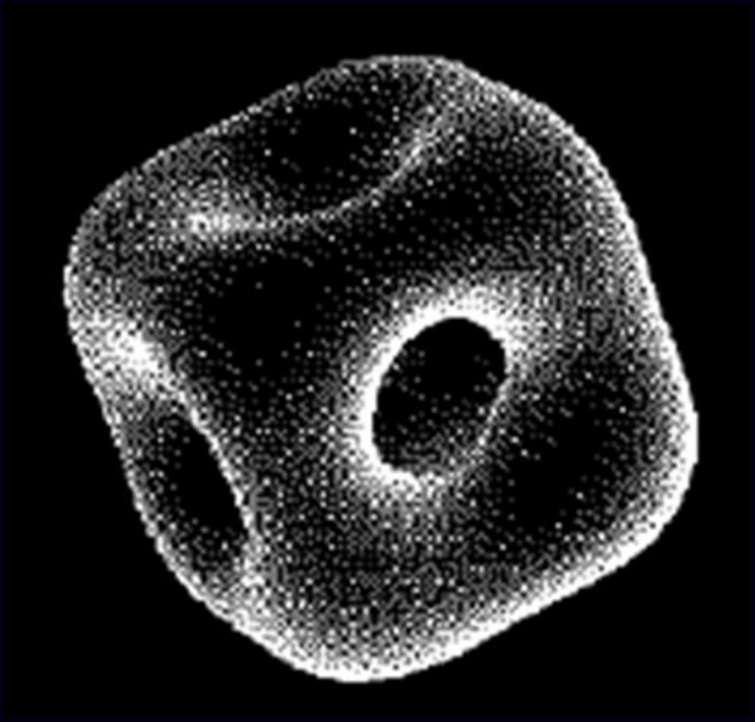


2. Dense Skyrmion matter



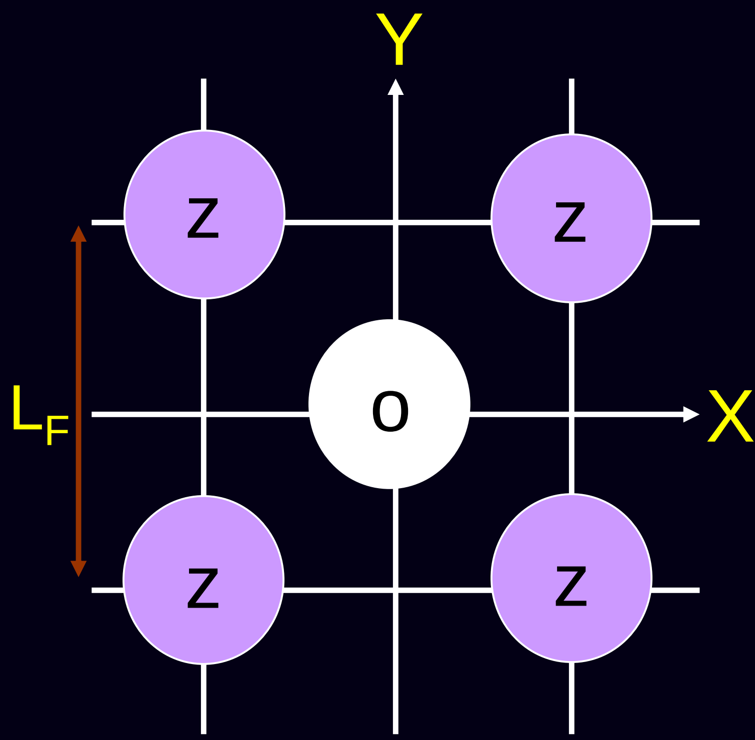
2. Dense Skyrmion matter

B=4 skyrmion

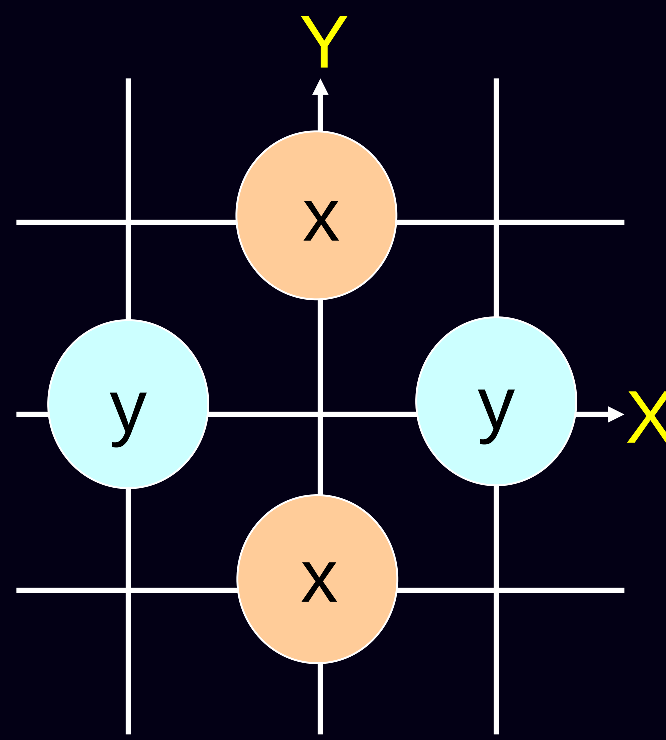


FCC Skyrmion Crystal

1989, L. Castellejo *et al.* & M. Kulger *et al.*

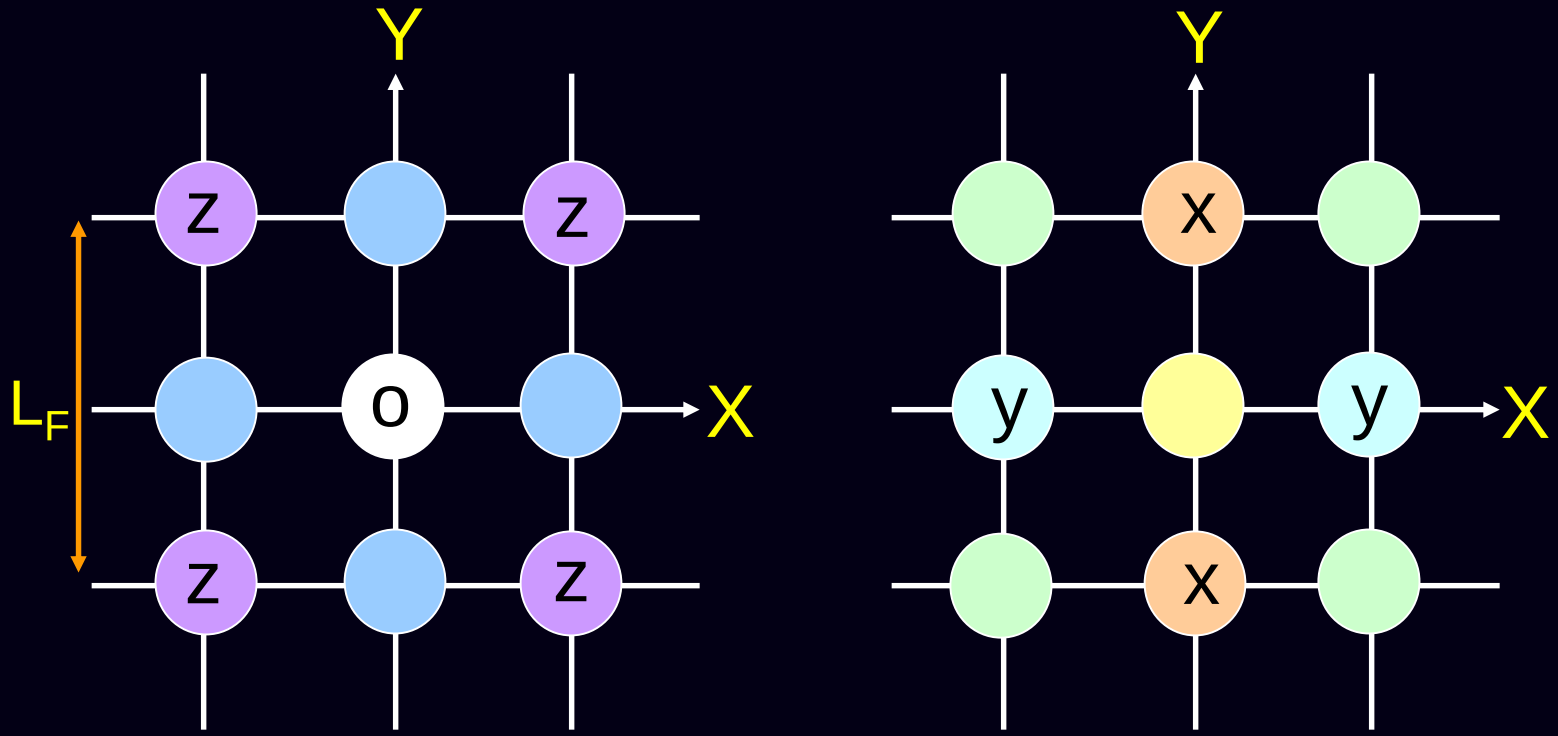


z=0 plane



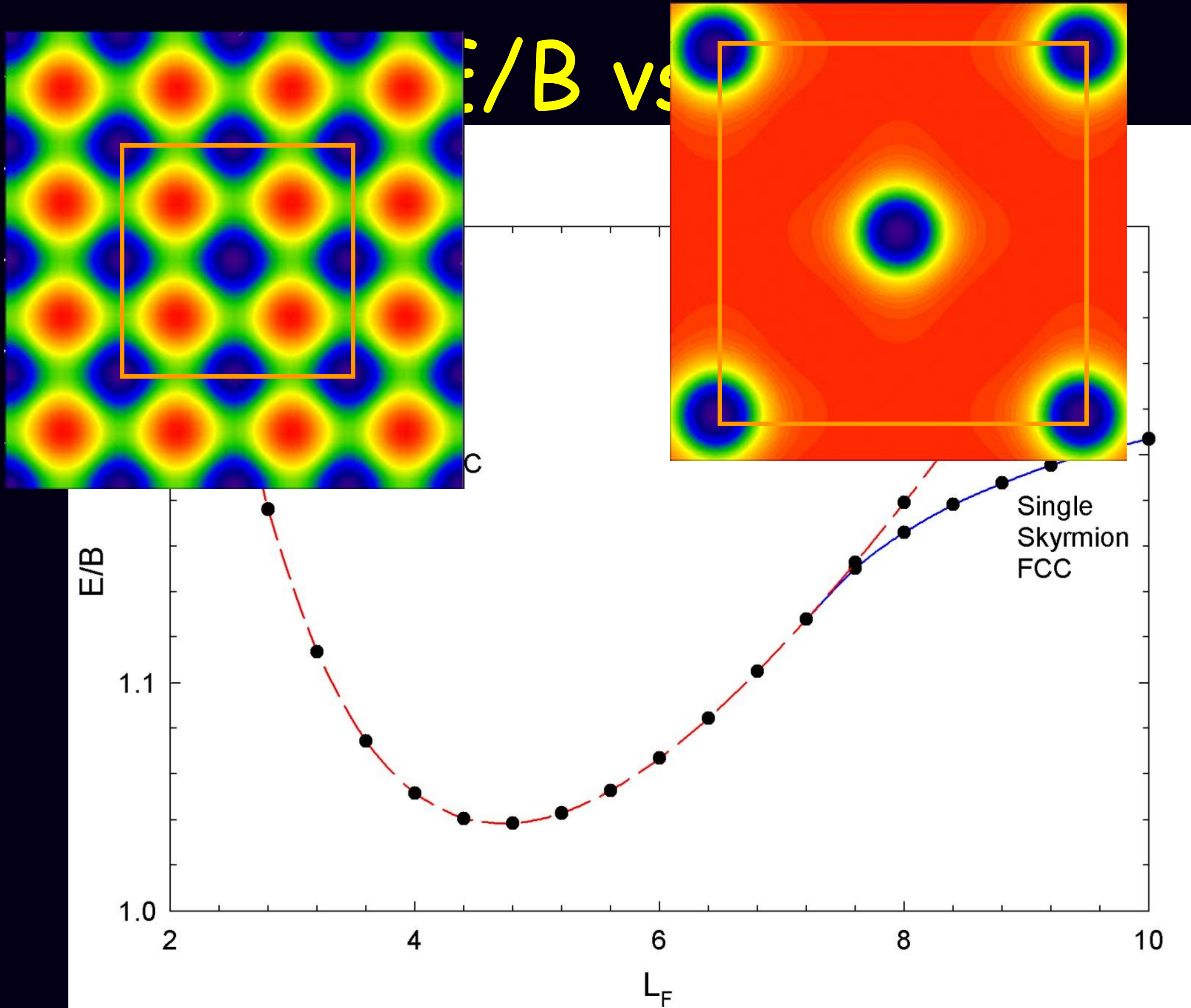
z=L_F/2 plane

Half-Skyrmion CC



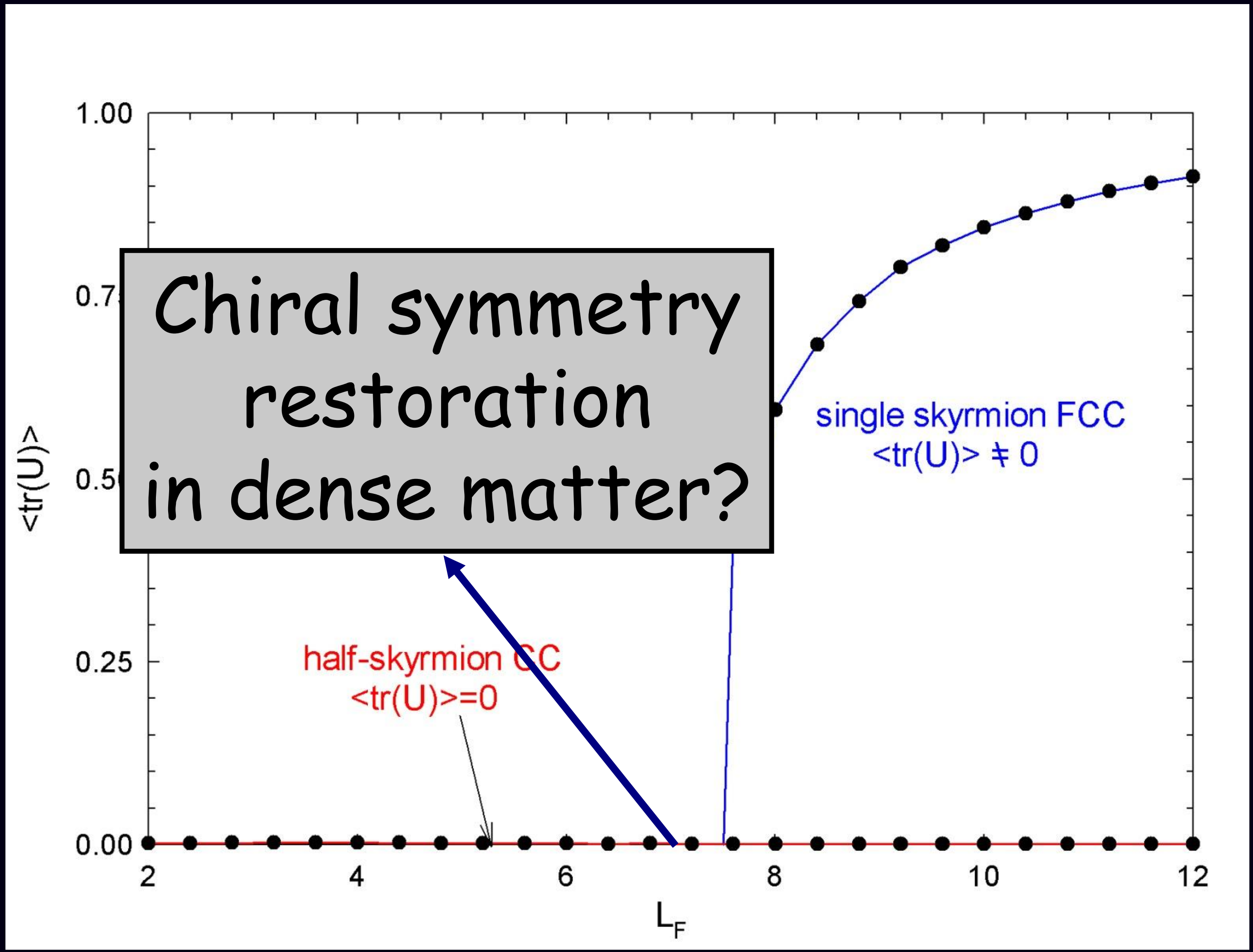
$(E/B)_{min}$=1.038 at L_f=4.72

E/B vs



2. Dense Skyrmion matter

$\langle U \rangle = \langle \sigma \rangle$



Chiral symmetry restoration in dense matter?

2. Dense Skyrmion matter

$$\mathcal{L} = \frac{f_\pi^2}{4}\mathrm{Tr}(\partial_\mu U^\dagger \partial^\mu U) + \frac{1}{32e^2}\mathrm{Tr}[U^\dagger \partial_\mu U, U^\dagger \partial_\nu U]^2$$

$$U(\vec{x}, t) = \sqrt{U_\pi} U_0 \sqrt{U_\pi}$$

fluctuating pions

classical dense matter config.

Pion in the dense baryonic matter?

2. Dense Skyrmion matter

Skyrme Model($m_\pi \neq 0$)

$$\mathcal{L} = \frac{f_\pi^2}{4}\mathrm{Tr}(\partial_\mu U^\dagger \partial^\mu U) + \frac{1}{32e^2}\mathrm{Tr}[U^\dagger \partial_\mu U, U^\dagger \partial_\nu U]^2$$
$$+ \frac{f_\pi^2 m_\pi^2}{4}\mathrm{Tr}(U + U^\dagger - 2)$$

Pion fluctuation
in ρ_B=0 space
Vacuum : U=1

$$U_\pi = \exp(i\vec{\tau}\cdot\vec{\varphi}/f_\pi)$$

$$\mathcal{L} = \frac{1}{2}\partial_\mu \varphi_a \partial^\mu \varphi_a + \frac{1}{2}m_\pi^2 \varphi_a \varphi_a + \cdots$$

π dynamics(ρ≠0)

$$\mathcal{L} = \frac{f_\pi^2}{4}\mathrm{Tr}(\partial_\mu U^\dagger \partial^\mu U) + \frac{1}{32e^2}\mathrm{Tr}[U^\dagger\partial_\mu U, U^\dagger \partial_\nu U]^2 + \frac{f_\pi^2 m_\pi^2}{4}\mathrm{Tr}(U + U^\dagger - 2)$$

Skyrmion matter

$$U = \sqrt{U_\pi}U_0\sqrt{U_\pi}$$

Pion fluctuations on top of the skyrmion matter

$$\mathcal{L} = \tfrac{1}{2}\partial_\mu \varphi_a \partial^\mu \varphi_a + \tfrac{1}{2} m_\pi^2 \varphi_a \varphi_a$$

+ π-skyrmion matter interactions

π dynamics($\rho \neq 0$)

$$\mathcal{L} = \frac{1}{2} G^{ab}(\vec{x}) \partial_\mu \varphi_a \partial^\mu \varphi_b + \frac{1}{2} m_\pi^2 \sigma(\vec{x}) \varphi_a \varphi_a + \varepsilon_{abc} \partial_i \varphi_a \varphi_b V_i^c(\vec{x})$$

$$m_\pi^{*2}(\vec{x}) \sim m_\pi^2 \sigma(\vec{x})$$

π dynamics(ρ≠0)

$$\mathcal{L} = \frac{1}{2}G^{ab}(\vec{x})\partial_\mu\varphi_a\partial^\mu\varphi_b + \frac{1}{2}m_\pi^2\sigma(\vec{x})\varphi_a\varphi_a$$
$$+\varepsilon_{abc}\partial_i\varphi_a\varphi_b V_i^c(\vec{x})$$

$$f_\pi^*(\vec{x})$$

$$U_\pi = \exp(i\vec{\tau}\cdot\vec{\varphi}/f_\pi)$$

$$\exp(i\vec{\tau}\cdot\vec{\varphi}^*/f_\pi^*)$$

$$\mathcal{L} = \frac{1}{2}\partial_\mu\varphi_a^*\partial^\mu\varphi_a^* + \cdots$$

π dynamics(ρ≠0)

$$\mathcal{L} = \frac{1}{2}\langle G_{ab}\rangle \partial_\mu \varphi_a \partial^\mu \varphi_b + \frac{1}{2} m_\pi^2 \langle \sigma \rangle \varphi_a \varphi_a$$

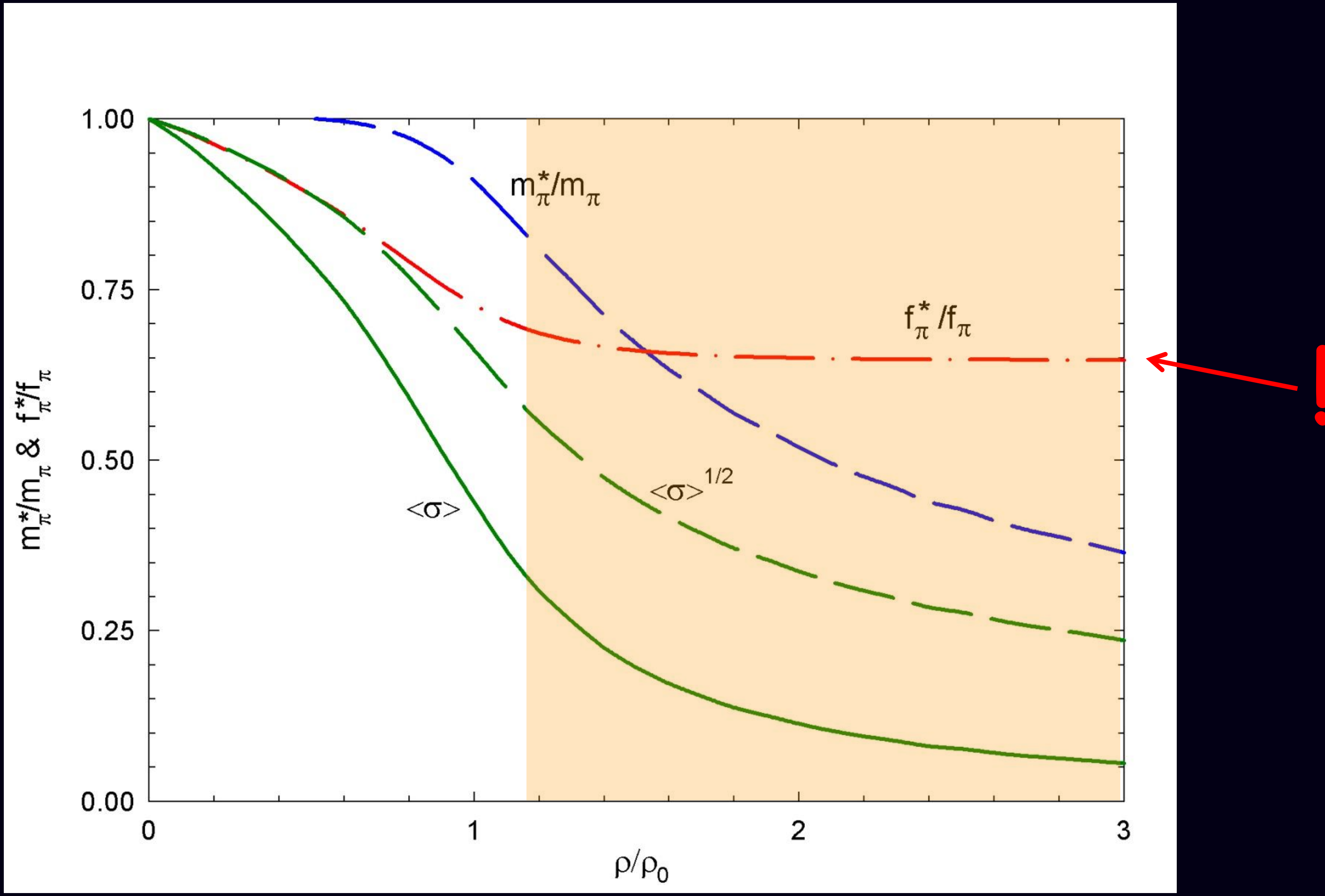
In-Medium pion decay constant ?

$$\frac{f_\pi^{*2}}{f_\pi^2} \sim \langle G_{ab} \rangle$$

In-Medium pion mass ?

$$\frac{m_\pi^{*2}}{m_\pi^2} \sim \frac{\langle \sigma \rangle}{\langle G_{ab} \rangle}$$

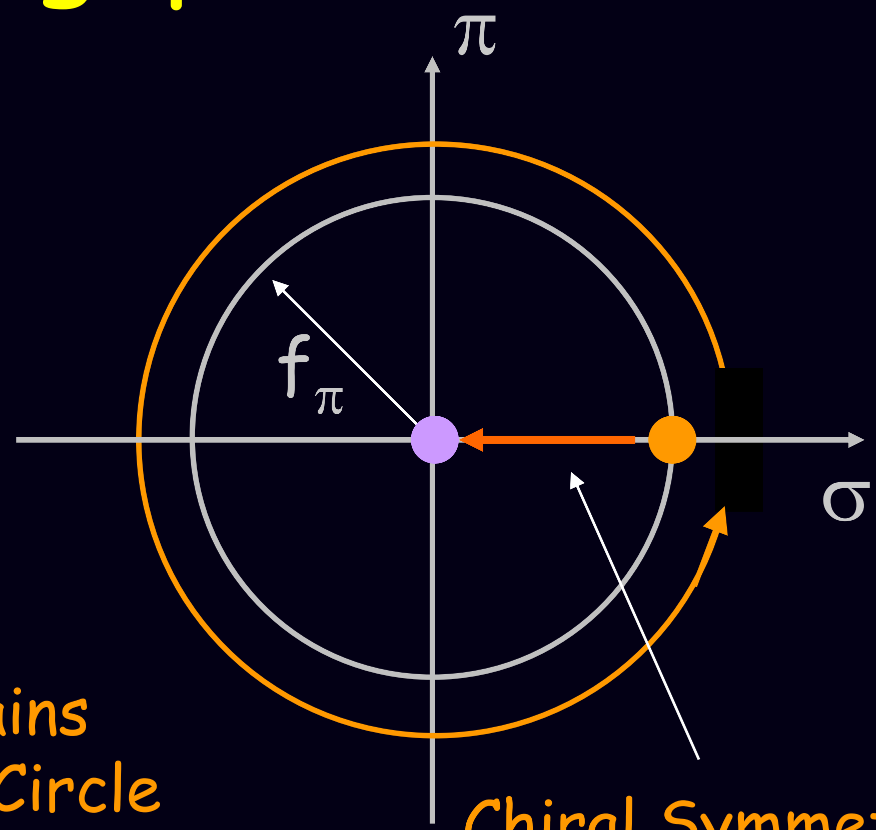
pion effective mass



H.-J. Lee, B.-Y. Park, D.-P. Min, M. Rho, V. Vento, Nucl. Phys. A (2003)

2. Dense Skyrmion matter

Pseudogap?



Chiral Symmetry Restoration

U still remains
on the Chiral Circle
But <U>=0

Zarembo, hep-ph/0104305

Skyrme Lagrangian

$$\mathcal{L} = \frac{f_\pi^2}{4}\mathrm{Tr}(\partial_\mu U^\dagger \partial^\mu U) + \frac{1}{32e^2}\mathrm{Tr}[U^\dagger \partial_\mu U, U^\dagger \partial_\nu U]^2$$
$$+ \frac{f_\pi^2 m_\pi^2}{4}\mathrm{Tr}(U + U^\dagger - 2)$$

Trace Anomaly
of QCD

Skyrme Lagrangian

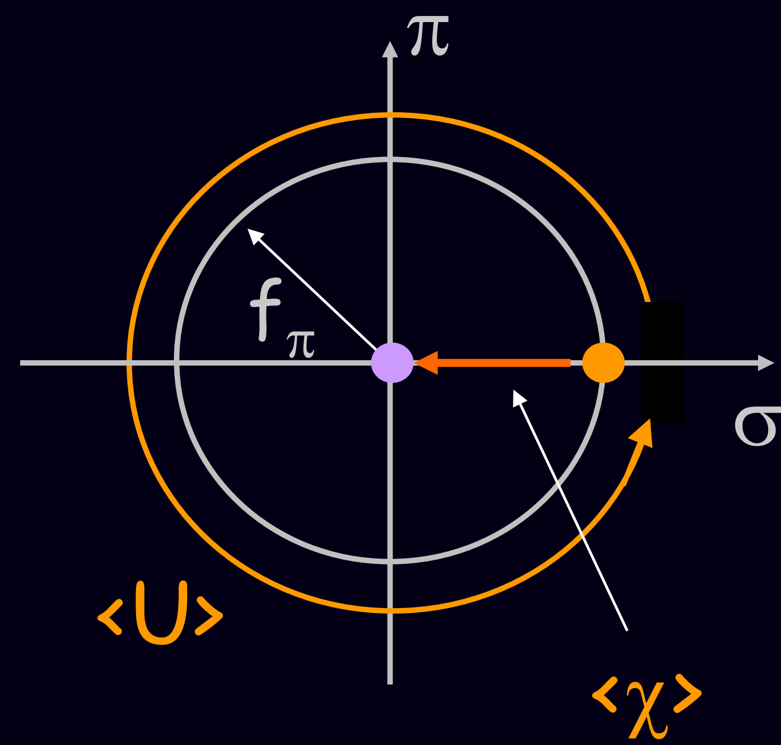
Ellis & Lanik, PLB(1985)

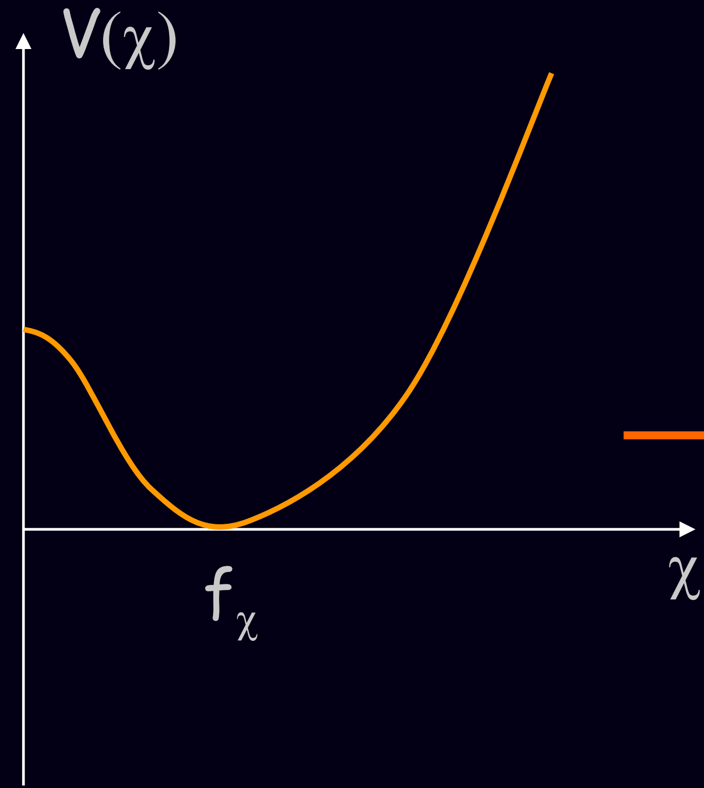
Brown-Rho scaling, PRL(1992)

$$\mathcal{L} = \frac{f_\pi^2}{4}\left(\frac{\chi}{f_\chi}\right)^2 \mathrm{Tr}(\partial_\mu U \partial^\mu U) + \frac{1}{32e^2}\mathrm{Tr}[U^\dagger \partial_\mu U, U^\dagger \partial_\nu U]^2$$
$$+\frac{f_\pi^2 m_\pi^2}{4}\left(\frac{\chi}{f_\chi}\right)^3 \mathrm{Tr}(U^\dagger + U - 2)$$
$$+\frac{1}{2}\partial_\mu \chi \partial^\mu \chi - \frac{1}{4} m_\chi^2 f_\chi^2 \left((\chi^4/f_\chi)^4 (\ln(\chi/f_\chi) - \tfrac{1}{4}) + \tfrac{1}{4}\right)$$

m_χ ~ 720 MeV, f_χ~240 MeV

$U \rightsquigarrow \chi U$





Vacuum ($\rho=0$)

$U=1$

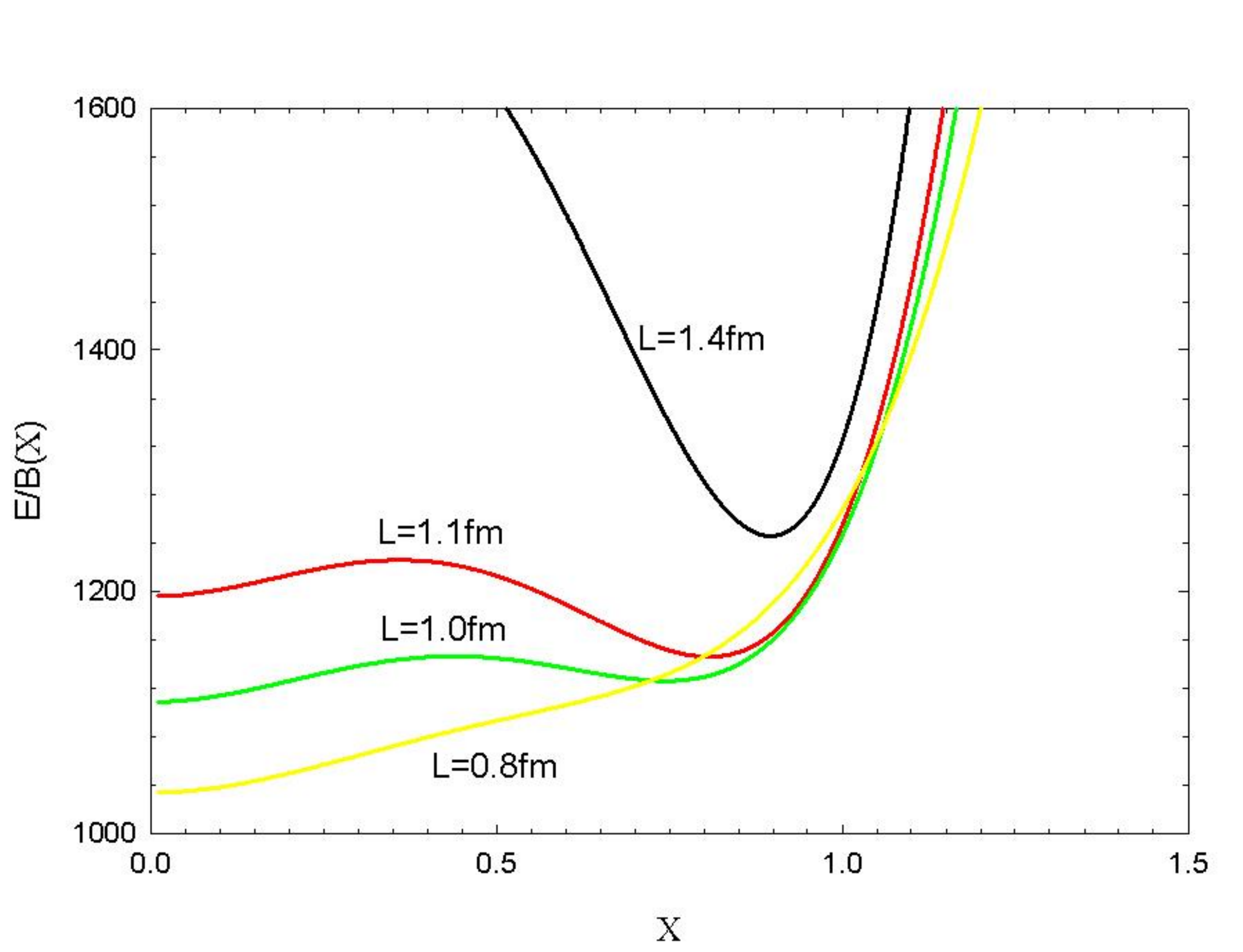
$\chi=f_\chi$

Naive Estimation

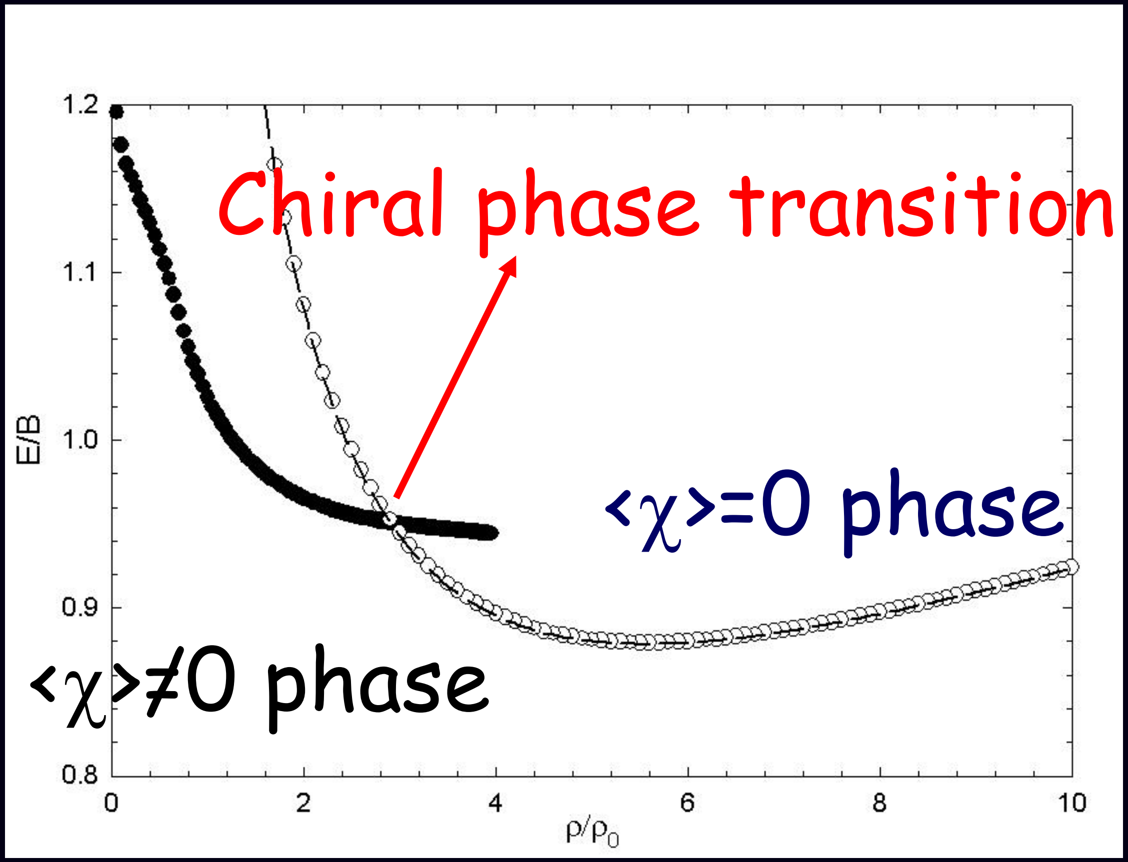
$$\mathcal{L} = \frac{f_\pi^2}{4}\left(\frac{\chi}{f_\chi}\right)^2 \mathrm{Tr}(\partial_\mu U \partial^\mu U) + \frac{1}{32e^2}\mathrm{Tr}[U^\dagger \partial_\mu U, U^\dagger \partial_\nu U]^2$$
$$+\frac{f_\pi^2 m_\pi^2}{4}\left(\frac{\chi}{f_\chi}\right)^3 \mathrm{Tr}(U^\dagger + U - 2)$$
$$+\tfrac{1}{2}\partial_\mu \chi \partial^\mu \chi - \tfrac{1}{4} m_\chi^2 f_\chi^2 \left((\chi^4/f_\chi)^4(\ln(\chi/f_\chi) - \tfrac{1}{4}) + \tfrac{1}{4}\right)$$

$$E/B = M_2(L)\chi^2 + M_4(L) + M_m(L)\chi^3 + V(\chi)L^3$$

Naive Estimation

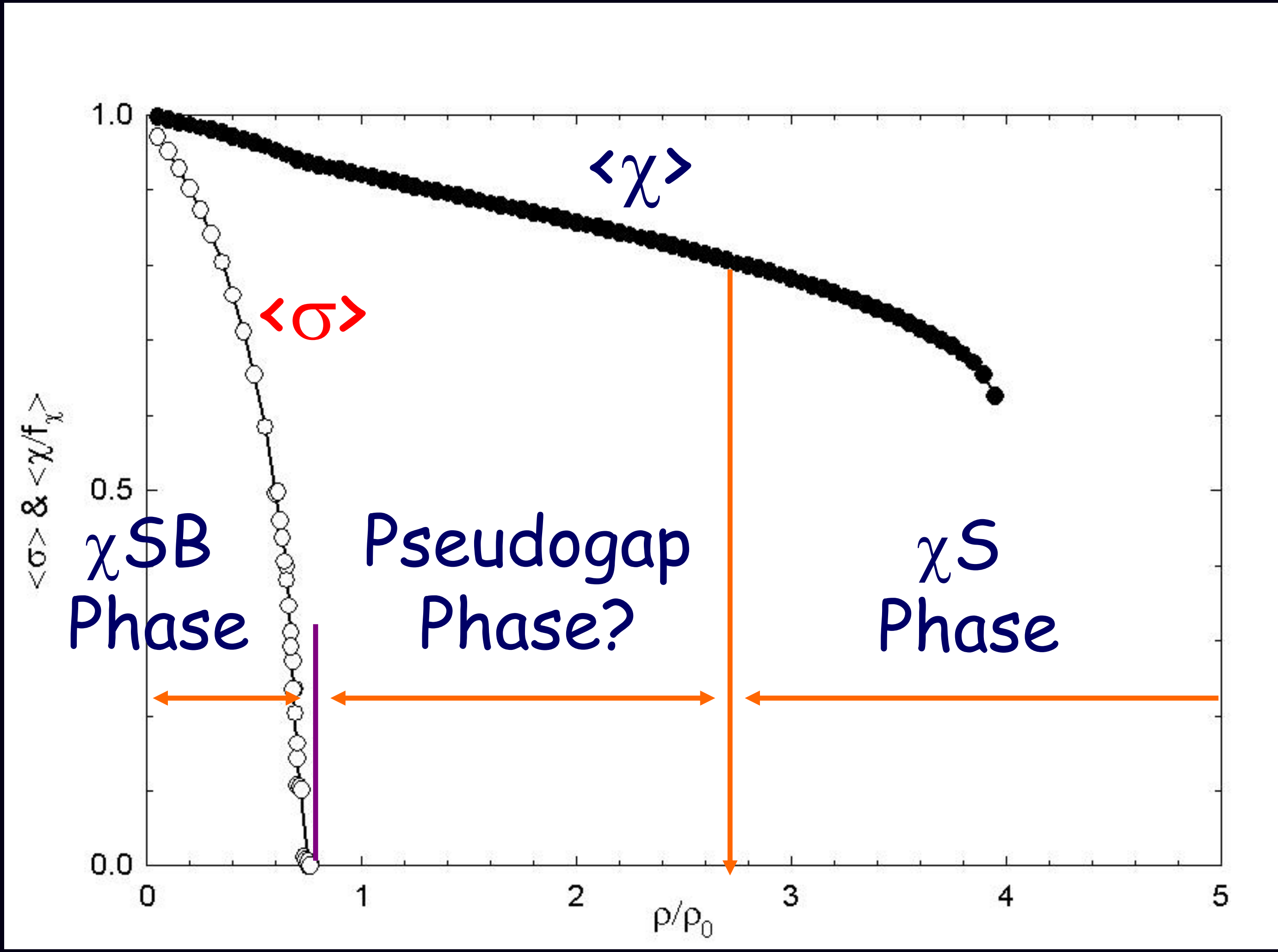


E/B


Chiral phase transition
<χ>=0 phase
<χ>≠0 phase
E/B
1.2
1.1
1.0
0.9
0.8
0
2
4
6
8
10
ρ/ρ0

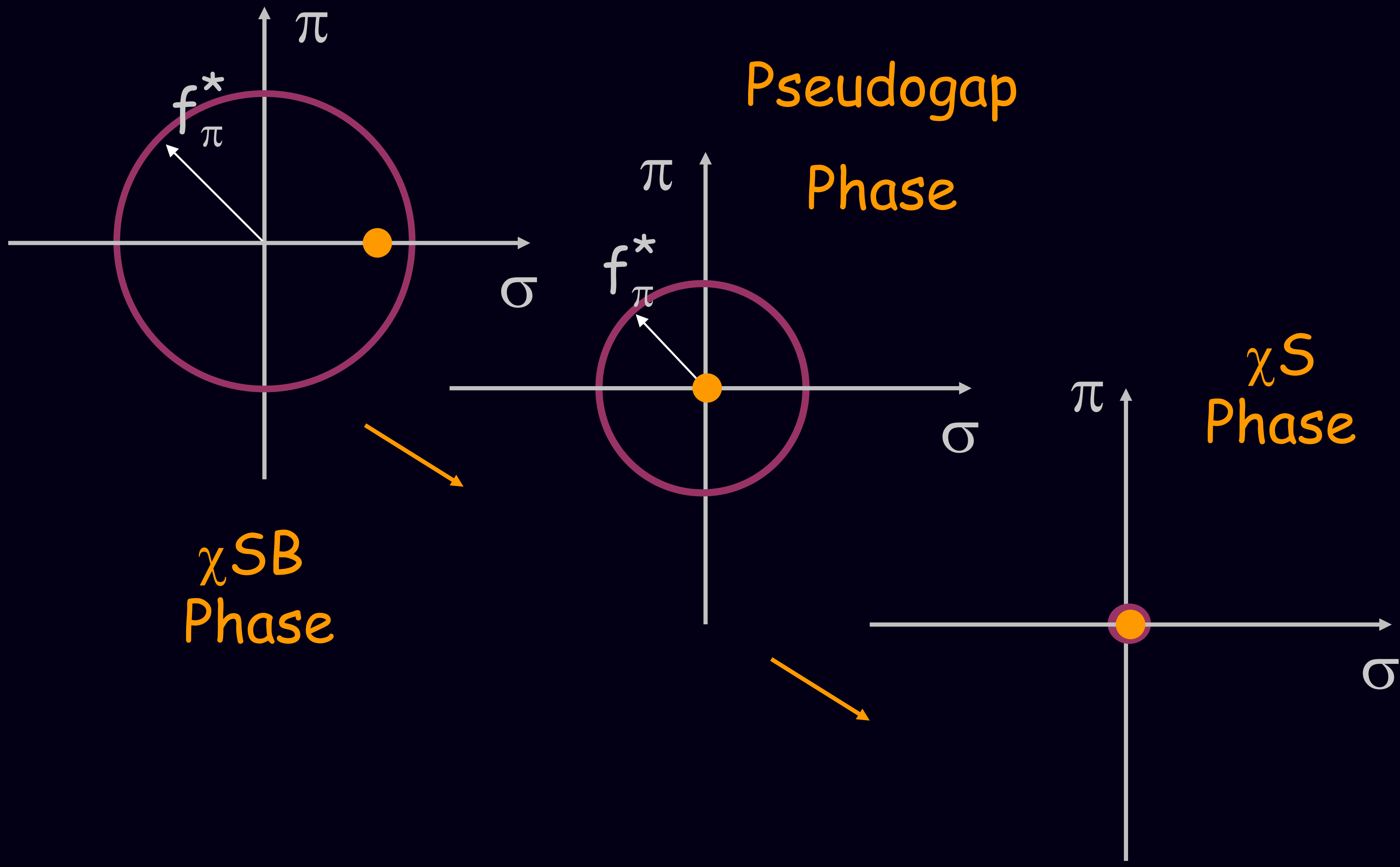
$\langle \chi \rangle$ & $\langle U \rangle$

$m_\pi=0$



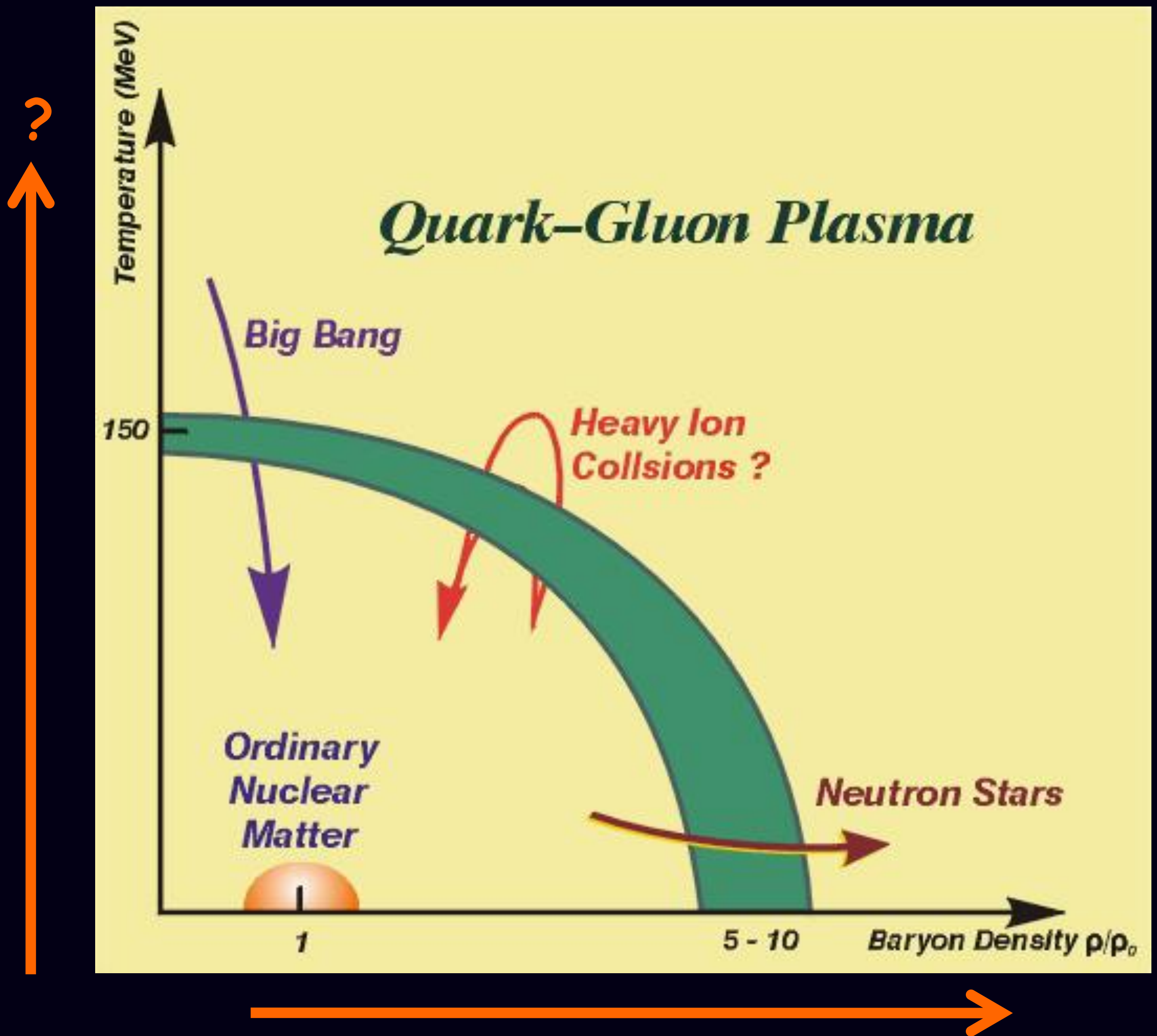
2. Dense Skyrmion matter

Pseudogap still remains?



2. Dense Skyrmion matter

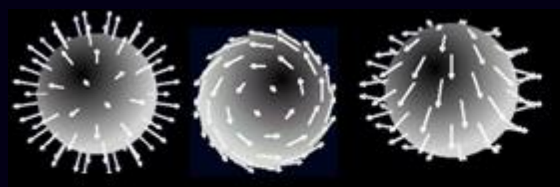
3. Hot & Dense Skyrmion Matter



2. Hot & Dense Skyrmion matter

Skyrmion Soup

Main ingredients : pions



Condiments :

dilaton

vector mesons

Heat

B.-Y. **Park**, H.-J. Lee, V. Vento,
Phys. Rev. D80 (2009)

Skyrmion from Instanton

1989, M. F. Atiyah & N. S. Manton

$$U(\vec{x}) = C\mathcal{S}\left\{\mathcal{P}\exp\left[\int_{-\infty}^{\infty} -A_4(\vec{x},t)dt\right]\right\}C^\dagger$$

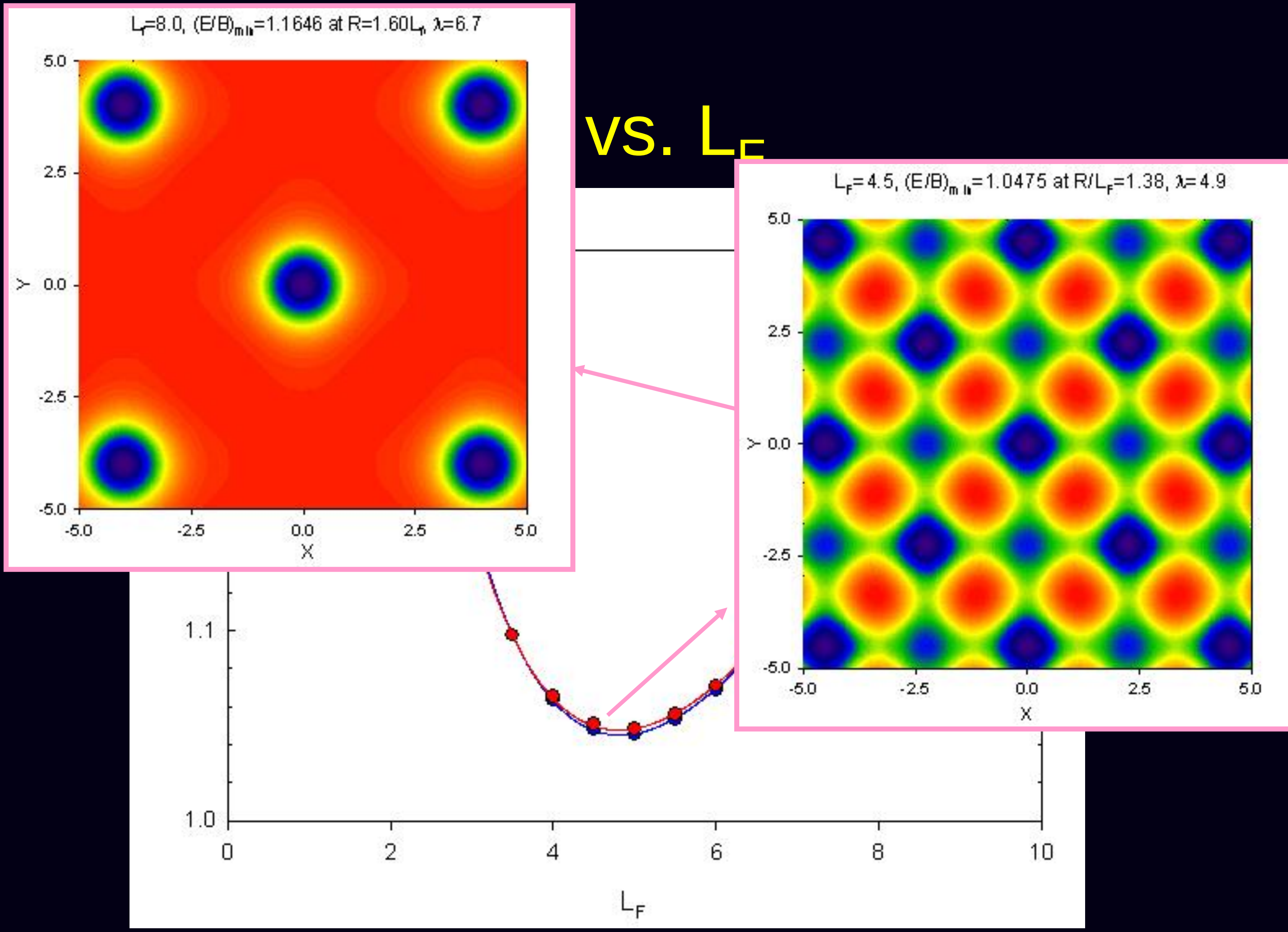
time component of SU(2) gauge potential for the instanton field of charge N

time-ordering

constant matrix to make U approaches 1 at infinity

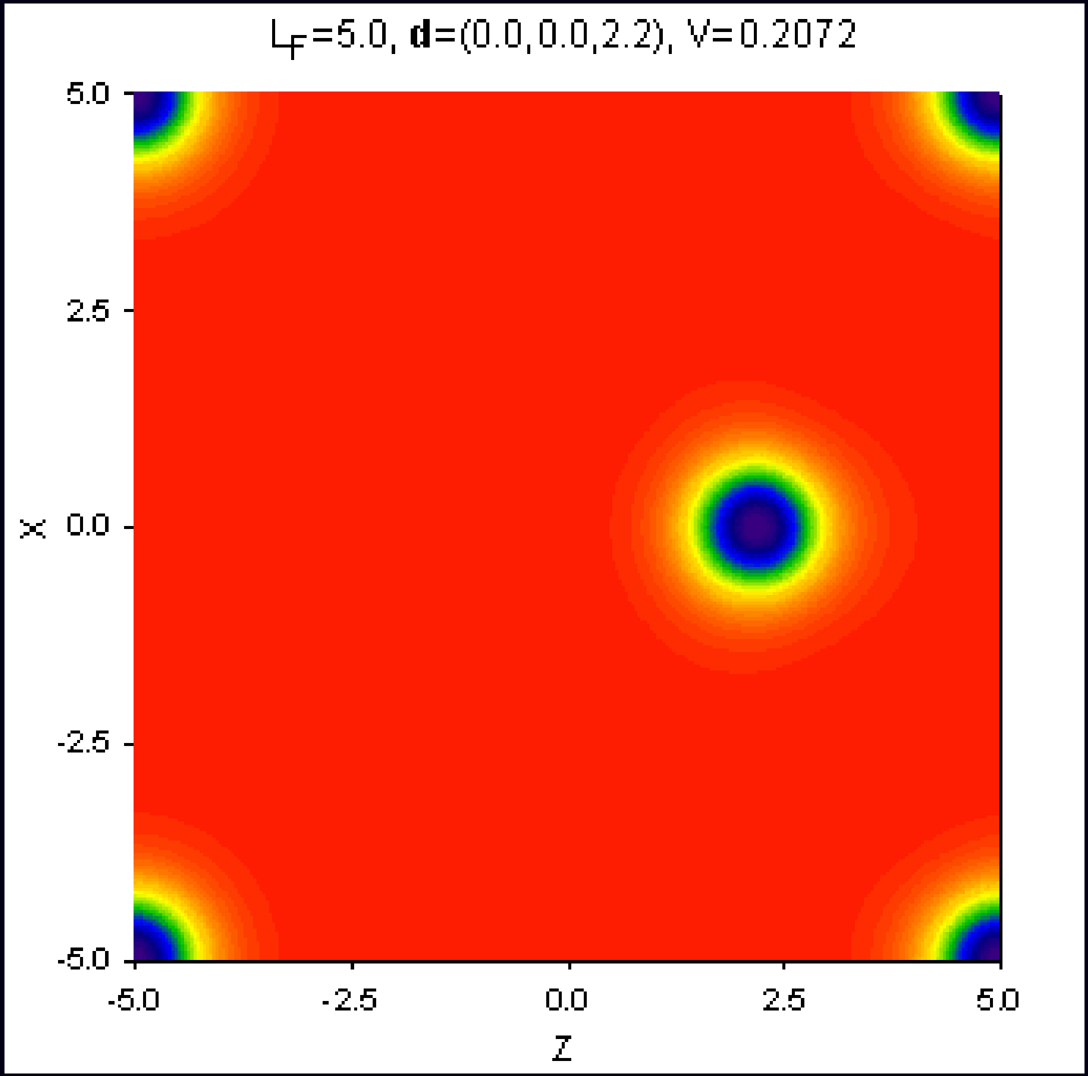
constant rotation matrix

vs. L_F



2. Hot & Dense Skyrmion matter

B.-Y. Park, D.-P. Min, V. Vento & M. Rho,
Nucl. Phys. A707(2002) 381



2. Hot & Dense Skyrmion matter

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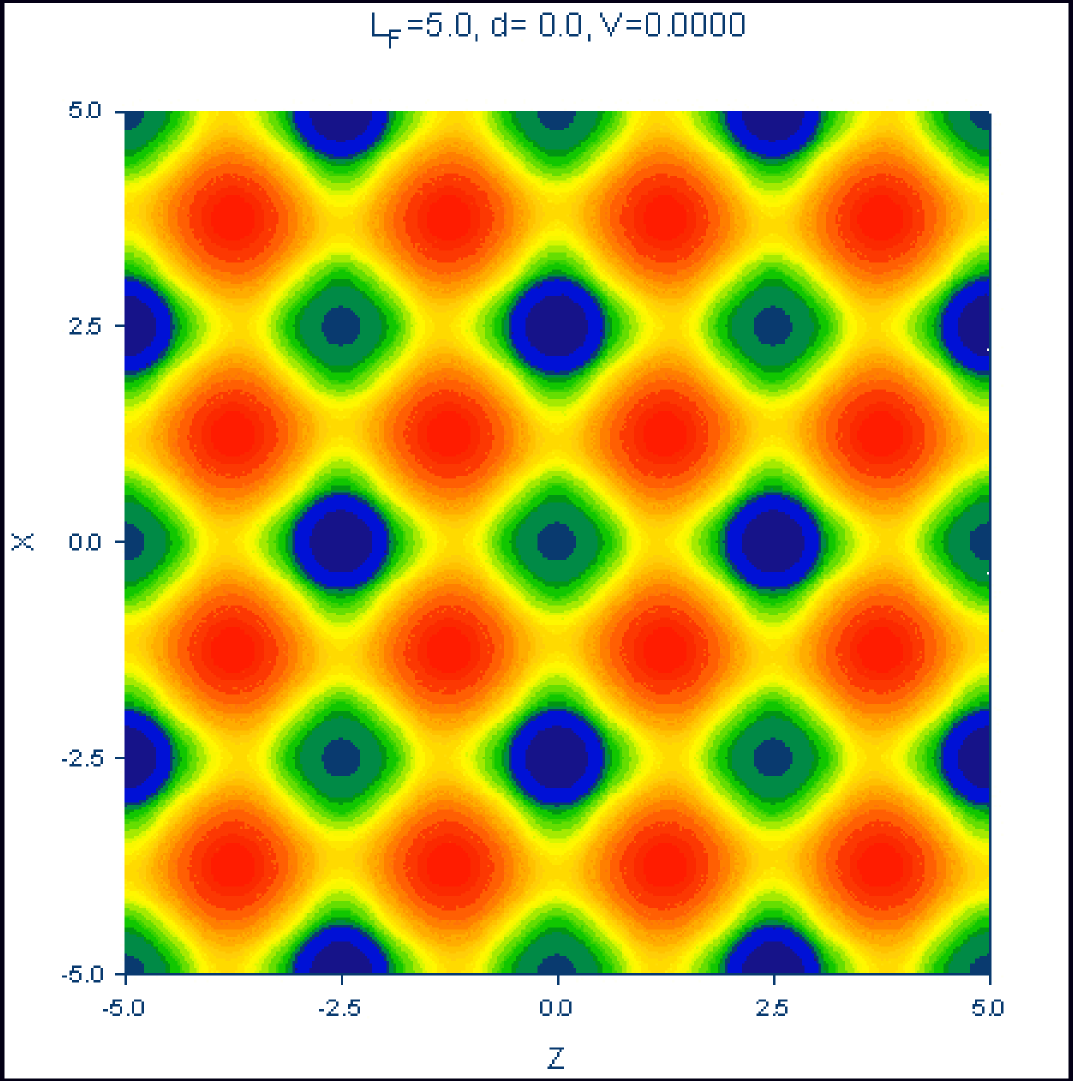
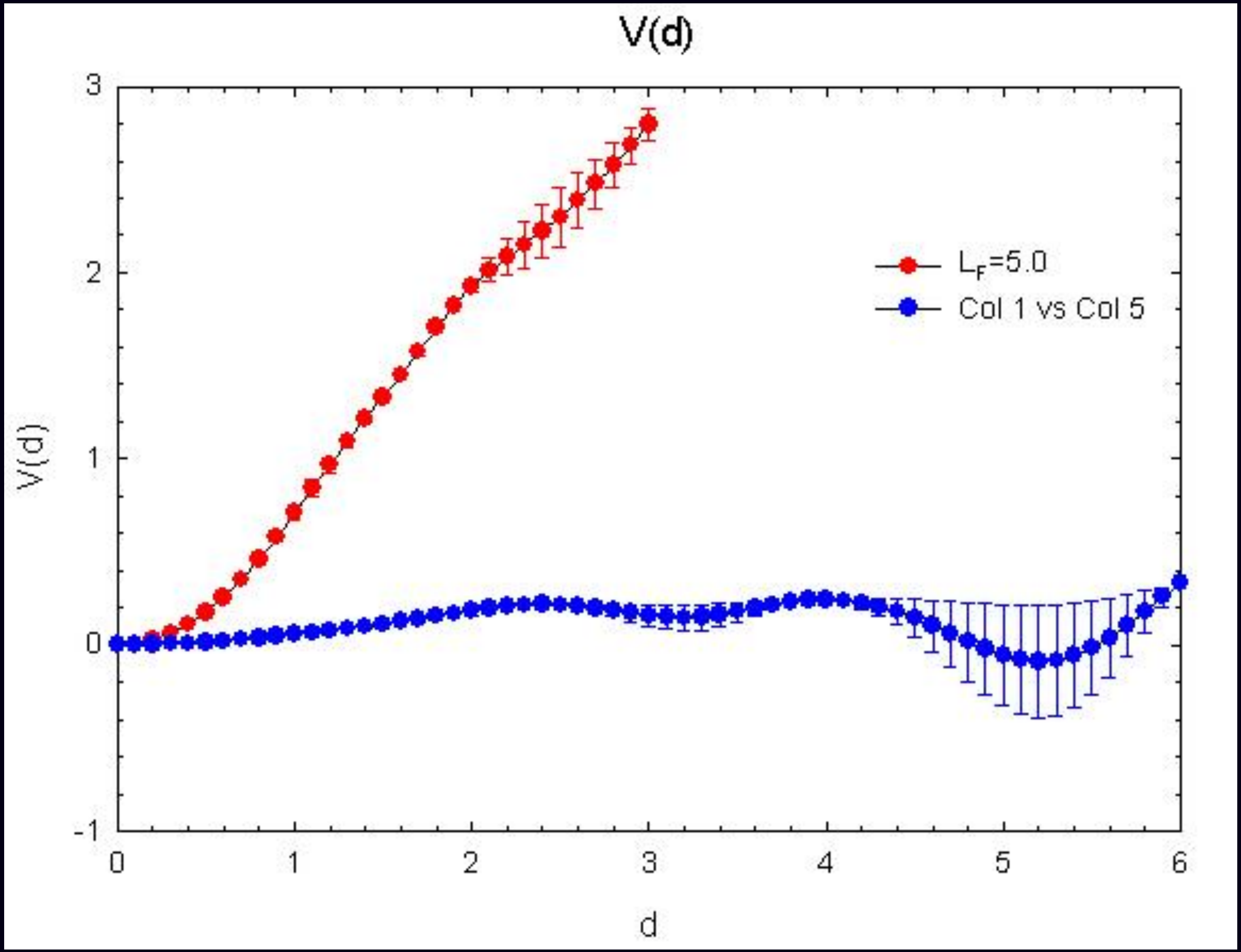


2. Hot & Dense Skyrmion matter



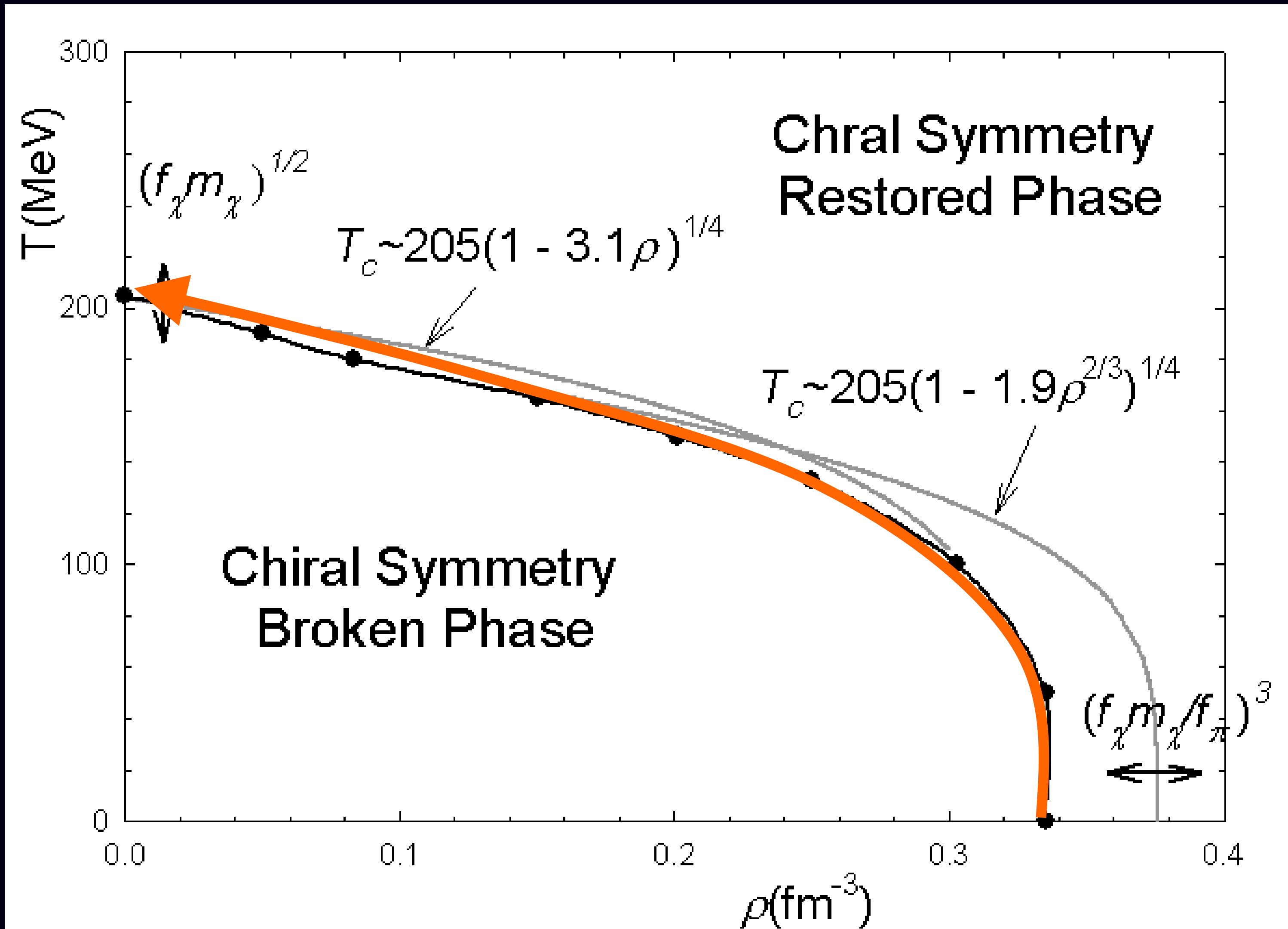
2. Hot & Dense Skyrmion matter

Pion Gas!

$$P = \frac{\pi^2}{30} T^4 \longrightarrow E/B = 3PV$$

$$\mathcal{L} = \frac{f_\pi^2}{4} \left(\frac{\chi}{f_\chi} \right)^2 \mathrm{Tr}(\partial_\mu U \partial^\mu U)$$

$$E/B = (M_2(\rho) + 3PV)\chi^2 + M_4(\rho) + V(\chi)/\rho^{1/3}$$



2. Hot & Dense Skyrmion matter

Summary

Skyrme Model can be applied to study

(1) dense hadronic matter

(2) hadron properties in dense matter

Thank You!