Classical strongly coupled quark-gluon plasma

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Outline

- Introduction
- Color charges
- Structure factor
- Energy loss
- Conclusion

– This talk is based on

B. Gelman, E. Shuryak, and I. Zahed, Phys. Rev. C 74, 044908 (2006)

B. Gelman, E. Shuryak, and I. Zahed, Phys. Rev. C 74, 044909 (2006)

S. Cho and I. Zahed, Phys. Rev. C 79, 044911 (2009)

S. Cho and I. Zahed, Phys. Rev. C 80, 014906 (2009)

S. Cho and I. Zahed, arXiv : 0909.4725

S. Cho and I. Zahed, arXiv : 0910.1548

S. Cho and I. Zahed, arXiv : 0910.2666

Introduction

− Quark gluon plasma (QGP)

$T >> T_C$: a weakly coupled gas

$T \approx T_C$: a strongly coupled liquid

− Strongly coupled quark-gluon plasma (sQGP)

1) The relativistic hydrodynamics

2) N=4 Super Yang-Mills theory $\frac{\eta_s}{\sigma} = \frac{1}{4\pi}$

3) The model of classical strongly coupled colored plasma

$$\Gamma = \frac{E_V}{E_K} = \frac{(Ze)^2}{a_{WS} k_B T} \left(\frac{N}{V} \frac{4\pi}{3} a_{WS}^3 = 1 \right)$$

− Classical colored plasma

1) A colored Coulomb interaction

$$\frac{g^2}{4\pi}\frac{\vec{Q}\cdot\vec{Q}'}{\left|\vec{r}-\vec{r}'\right|}$$

2) Equations of motion ; Wong equations

$$\frac{d\vec{r}}{dt}=\frac{\vec{p}}{m}$$

$$\frac{d\vec{p}}{dt}=-\frac{g^2}{4\pi}\nabla_{\vec{r}}\frac{\vec{Q}\cdot\vec{Q}'}{\left|\vec{r}-\vec{r}'\right|}$$

$$\frac{dQ^\alpha}{dt}=\frac{g^2}{4\pi}\sum_{\beta}^{N_c^2-1}f^{\alpha\beta\gamma}\frac{Q'^\beta}{\left|\vec{r}-\vec{r}'\right|}Q^\gamma$$

Color Charges

- Canonical variables

$$\vec{\phi} = (\phi_1, \cdots, \phi_{\frac{1}{2}N_c(N_c-1)}) \qquad \vec{\pi} = (\pi_1, \cdots, \pi_{\frac{1}{2}N_c(N_c-1)})$$

$$\{\phi_\alpha, \pi_\beta\} = \delta_{\alpha\beta} \qquad \{Q^\alpha, Q^\beta\} = f^{\alpha\beta\gamma} Q^\gamma$$

- Canonical Poisson bracket

$$\{A, B\}_{PB} = \frac{\partial A}{\partial x_i}\frac{\partial B}{\partial p_i} - \frac{\partial B}{\partial p_i}\frac{\partial A}{\partial x_i} + \frac{\partial A}{\partial \phi_\alpha}\frac{\partial B}{\partial \pi_\alpha} - \frac{\partial B}{\partial \pi_\alpha}\frac{\partial A}{\partial \phi_\alpha}$$

- Color charges for SU(2)

$$Q^1 = \cos\phi_1\sqrt{J^2 - \pi_1^2} \qquad Q^2 = \sin\phi_1\sqrt{J^2 - \pi_1^2} \qquad Q^3 = \pi_1$$

– Quadratic and cubic Casimirs

$$q_2 = \sum_{\alpha}^{N_c^2-1} Q^\alpha Q^\alpha = \frac{1}{3}(J_1^2 + J_1 J_2 + J_2^2)$$

$$q_3 = \sum_{\alpha,\beta,\gamma}^{N_c^2-1} d_{\alpha\beta\gamma} Q^\alpha Q^\beta Q^\gamma = \frac{1}{18}(J_1 - J_2)(J_1 + 2J_2)(2J_1 + J_2)$$

d_{118}	d_{146}	d_{157}	d_{228}	d_{247}	d_{256}	d_{338}	d_{344}	d_{355}	d_{366}	d_{377}	d_{448}	d_{558}	d_{668}	d_{778}	d_{888}
$\frac{1}{\sqrt{3}}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{\sqrt{3}}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{\sqrt{3}}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{\sqrt{3}}$

– The phase space volume element for SU(2), SU(3)

$$dQ = c_R d\pi_1 d\phi_1 J dJ \delta(J^2 - q_2)$$

$$dQ = c_R d\phi_1 d\phi_2 d\phi_3 d\pi_1 d\pi_2 d\pi_3 dJ_1 dJ_2 \frac{\sqrt{3}}{48} J_1 J_2 (J_1 + J_2)$$

$$\times \delta(q_2 - \frac{1}{3}(J_1^2 + J_1 J_2 + J_2^2)) \delta(q_3 - \frac{1}{18}(J_1 - J_2)(J_1 + 2J_2)(2J_1 + J_2))$$

– Integration of color charge(s)

$$\int dQ Q^{\alpha} = 0$$

$$\int dQ = 1$$

$$\int dQ Q^{\alpha} Q^{\beta} = C_2 \delta^{\alpha\beta} = \frac{1}{N_C^2 - 1} q_2 \delta^{\alpha\beta}$$

$$\int dQ Q^{\alpha} Q^{\beta} Q^{\gamma} = \frac{1}{4} d^{\alpha\beta\gamma}$$

$$\int dQ Q^{\alpha} Q^{\beta} Q^{\gamma} Q^{\delta} = A(\sum_n d^{\alpha\beta n} d^{\gamma\delta n} + \sum_n d^{\alpha\gamma n} d^{\beta\delta n} + \sum_n d^{\alpha\delta n} d^{\beta\gamma n}) + B(\delta^{\alpha\beta}\delta^{\gamma\delta} + \delta^{\alpha\gamma}\delta^{\beta\delta} + \delta^{\alpha\delta}\delta^{\beta\gamma})$$

$B = \frac{1}{15} q_2^2$: SU(2) $A + 3B = \frac{3}{80} q_2^2$: SU(3)

Structure factors

– Density structure factor

$$S_{00}(\vec{k}) = \frac{1}{N}\left\langle \left| n_{\vec{k}} \right|^2 \right\rangle \qquad n_{\vec{k}} = \sum_{i=1}^{N} e^{i\vec{k}\cdot\vec{r}_i}$$

– Charge structure factor

$$S_{01}(\vec{k}) = \frac{1}{N}\left\langle \left| \rho_{\vec{k}} \right|^2 \right\rangle \qquad \rho_{\vec{k}} = \sum_{i=1}^{N} \vec{Q}_i e^{i\vec{k}\cdot\vec{r}_i}$$

– The pair and direct correlation function

.

$$-\frac{1}{\beta} c_D(\vec{r}-\vec{r}', \vec{Q}\cdot\vec{Q}') = \frac{\vec{Q}\cdot\vec{Q}'}{\left|\vec{r}-\vec{r}'\right|} + \left.\frac{\delta^2 F_{ex}}{\delta n(\vec{r})\delta n(\vec{r}')}\right|_{n(\vec{r})=n}$$

$$h(\vec{r}-\vec{r}', \vec{Q}\cdot\vec{Q}') = \frac{1}{n^2}\left\langle \sum_{i\neq j} \delta(\vec{r}-\vec{r}_i)\delta(\vec{r}-\vec{r}_j)\delta(\vec{Q}-\vec{Q}_i)\delta(\vec{Q}-\vec{Q}_j) \right\rangle$$

– Ornstein-Zernicke relation for each color partial wave

$$h_l(\vec{k}) = c_D(\vec{k}) + nh_l(\vec{k})c_D(\vec{k})$$

– Static structure factor

$$S_0(\vec{k},\vec{Q}\cdot\vec{Q}') = \delta(\vec{Q}-\vec{Q}') + nh(\vec{k},\vec{Q}\cdot\vec{Q}')$$

$$S_{0l}(\vec{k}) = 1 + nh_l(\vec{k})$$
$$S_{0l}(\vec{k})^{-1} = 1 - nc_{Dl}(\vec{k})$$

− The structure factor

$$S_{00}^{-1}(\vec{k}) = S_{01}^{-1}(\vec{k}) = 1 + 2\left(\frac{\kappa_D}{k}\right)^2 \int_0^1 d\lambda \frac{\lambda}{w_\lambda} \cos\left(\frac{1}{\lambda}\frac{k}{\kappa_D}(w_\lambda - 1)\right)$$

$$+ 2\left(\frac{\kappa_D}{k}\right)^3 \int_0^1 d\lambda \frac{\lambda^2}{w_\lambda} \sin\left(\frac{1}{\lambda}\frac{k}{\kappa_D}(w_\lambda - 1)\right) - 2\int_0^1 \frac{d\lambda}{\lambda}\frac{w_\lambda - 1}{w_\lambda} \mathfrak{I}_0^+\left(w_\lambda - 1, \frac{1}{\lambda}\frac{k}{\kappa_D}\right)$$

$$+ \sum_l (2l+1)\left(2\frac{\kappa_D}{k}\int_0^1 d\lambda \frac{(w_\lambda - 1)^{l+2}}{w_\lambda g_{l+1}(w_\lambda - 1)} j_l\left(\frac{1}{\lambda}\frac{\kappa_D}{k}(w_\lambda - 1)\right)\mathfrak{I}_l^+\left(w_\lambda - 1, \frac{1}{\lambda}\frac{k}{\kappa_D}\right)\right.$$

$$+ (-1)^{l+1}\int_0^1 \frac{d\lambda}{\lambda}\frac{w_\lambda - 1}{w_\lambda}\left(\left(\frac{g_{l+1}(-w_\lambda + 1)}{g_{l+1}(w_\lambda - 1)}\mathfrak{I}_l^+\left(w_\lambda - 1, \frac{1}{\lambda}\frac{k}{\kappa_D}\right)\right)^2\right.$$

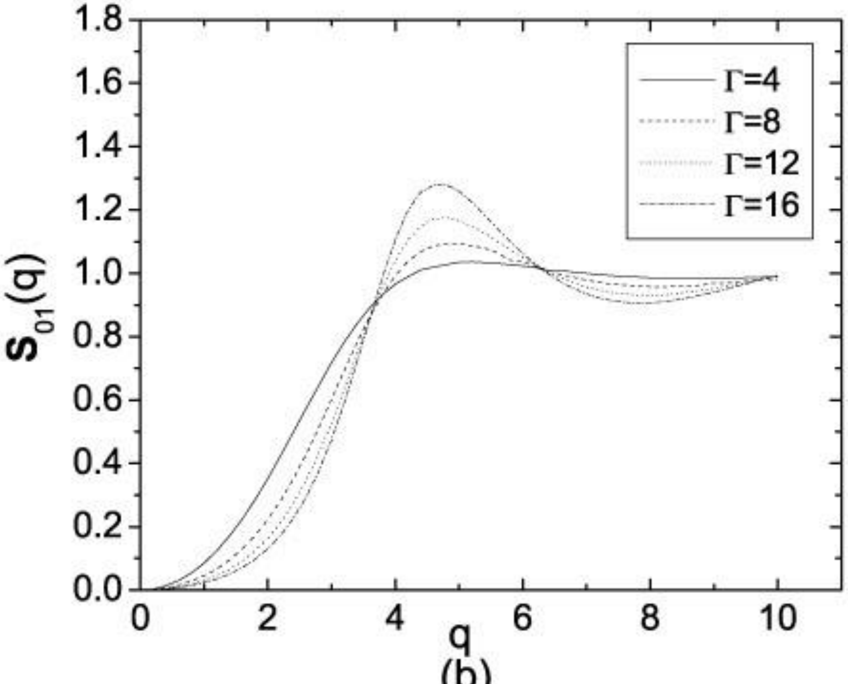
$$\left.\left. + 2\mathfrak{I}_l^+\left(w_\lambda - 1, \frac{1}{\lambda}\frac{k}{\kappa_D}\right)\mathfrak{I}_l^-\left(w_\lambda - 1, \frac{1}{\lambda}\frac{k}{\kappa_D}\right) - 2\mathfrak{I}_l^0\left(w_\lambda - 1, \frac{1}{\lambda}\frac{k}{\kappa_D}\right)\right)\right)$$

With three integrals and $w_\lambda = (1+\lambda^3(3\Gamma)^{2/3})^{1/3}$ $g_{l+1}(s) = e^s s^{l+1} k_l(s)$

$$\Im_l^-(s,\alpha) = \int_0^s d\xi \xi^{-l} g_l(-\xi) j_l(\alpha\xi)$$

$$\Im_l^0(x,\alpha) = \int_x^\infty ds s^{-l} g_l(s) j_l(\alpha s) \Im_l^-(s,\alpha) e^{(2(x-s))}$$

$$\Im_l^+(x,\alpha) = \int_x^\infty ds s^{-l} g_l(s) j_l(\alpha s) e^{(2(x-s))}$$



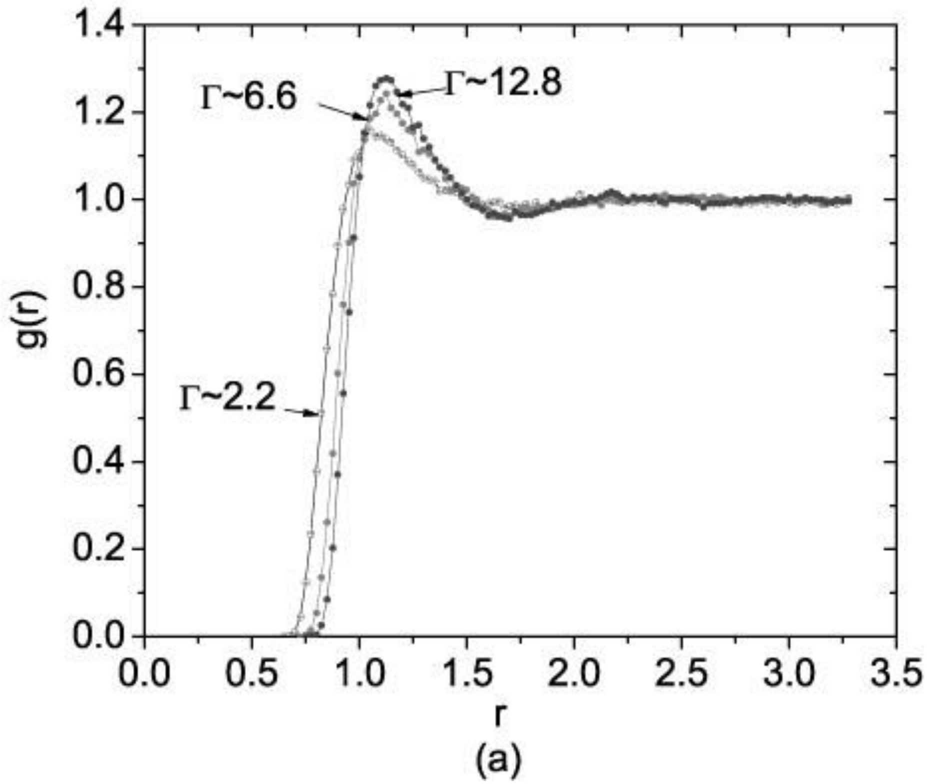
(b)

– Molecular dynamics simulation : SU(2)

$$V(\vec{r},\vec{Q}\cdot\vec{Q}') = \frac{g^2}{\lambda}\left[\frac{1}{9}\left(\frac{\lambda}{r}\right)^9 + \vec{Q}\cdot\vec{Q}'\frac{\lambda}{r}\right]$$

$$\frac{dQ^\alpha}{dt} = \frac{g^2}{4\pi}\sum_{\beta}^{N_c^2-1} f^{\alpha\beta\gamma}\frac{Q'^\beta}{|\vec{r}-\vec{r}'|}Q^\gamma$$

$$\rightarrow \frac{d\vec{Q}}{dt} = \frac{g^2}{4\pi}\frac{\vec{Q}\times\vec{Q}'}{|\vec{r}-\vec{r}'|}$$



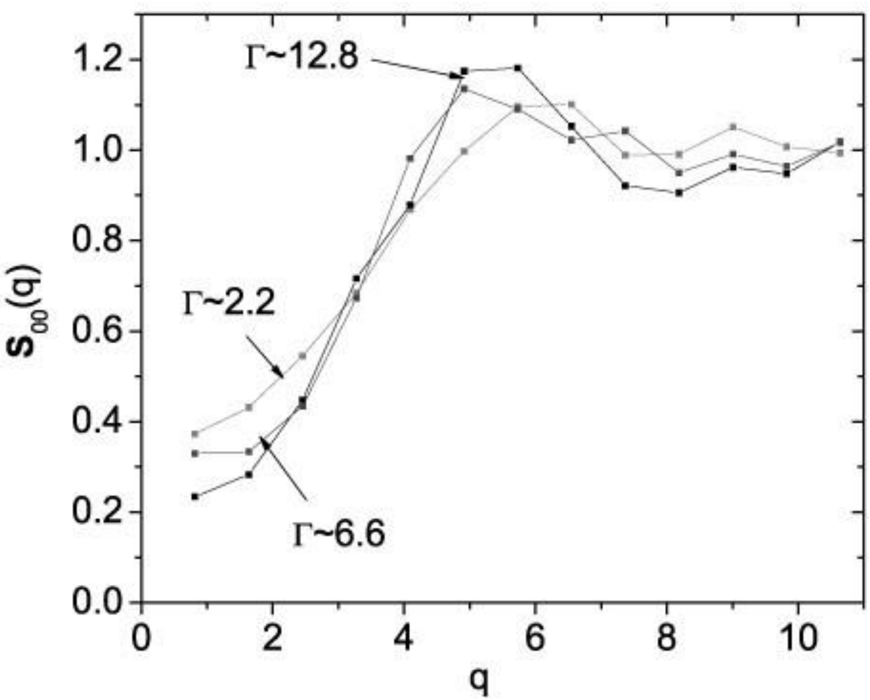
– The radial distribution function

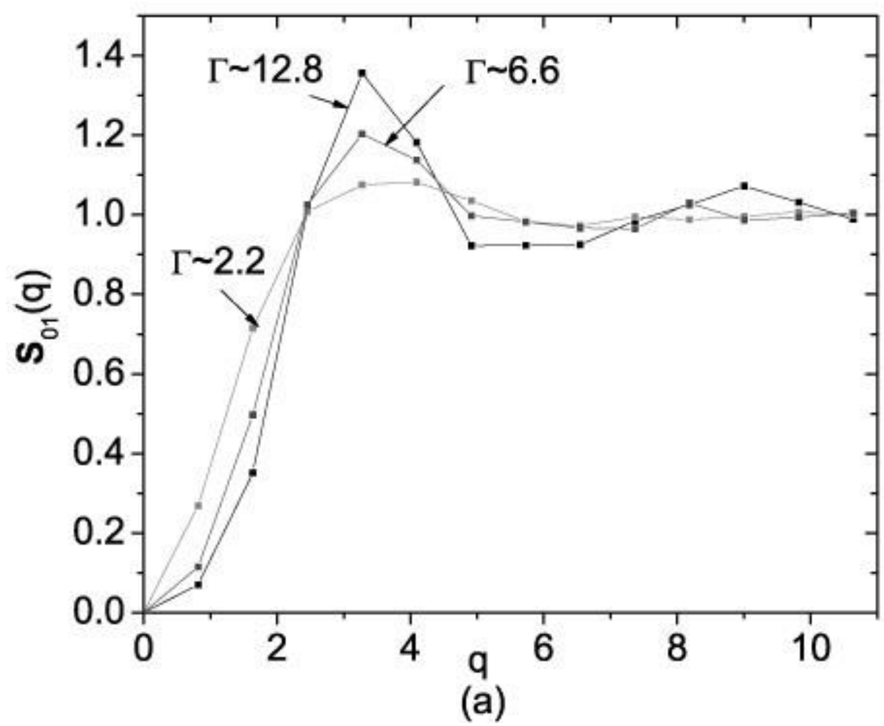
$$g(r) = \frac{1}{nN}\left\langle \sum_{i\neq j}\delta(\vec{r}-\vec{r}_{ij})\right\rangle$$

− Density (l=0) and charge (l=1) structure factors

$$S_{00}(\vec{k}) \approx \frac{k^2}{c_S^2 k^2} = \frac{1}{c_S^2}$$

$$S_{01}(\vec{k}) \approx \frac{k^2}{\kappa_D^2}$$





Energy loss

− The energy loss per unit length

$$-\frac{dK}{dr} = -\frac{q^2}{\pi v^2}\int \frac{dk}{k}\int_{-kv}^{kv}\omega d\omega\, \mathrm{Im}\left[\frac{1}{\varepsilon_L(\omega+i0,k)}\right]$$

−Linear response

$$\left(\frac{1}{\varepsilon_L(\omega,k)}-1\right)\delta^{ab} = -\frac{4\pi}{k^2}\Delta_R^{ab}(\omega,k)$$

$$\Delta_R^{ab}(\omega,k) = -i\int d\vec{r}dt e^{-i\omega\cdot t+i\vec{k}\cdot\vec{r}}\left\langle R(J_0^a(t,r)J_0^b(t',r'))\right\rangle$$

– The fluctuation-dissipation theorem

$$\mathrm{Im}\,\Delta_R^{ab}(\omega,k)=\delta^{ab}\frac{n\omega}{2T}S_1(\omega,k)=-\delta^{ab}\frac{n\omega}{T}\mathrm{Im}\,S_1(z,k)$$

$$\therefore\ \mathrm{Im}\frac{1}{\varepsilon_L(z,k)}=-\frac{\kappa_D^2}{k^2}\frac{1}{nc_{D1}(k)}\mathrm{Im}\frac{1}{\varepsilon_1(z,k)}$$

$$\varepsilon_l(z,k)=1+nc_{Dl}(k)\int d\vec{p}\,\frac{\vec{k}\cdot\vec{p}/m}{z-\vec{k}\cdot\vec{p}/m}f_0(\vec{p})$$

– The final result

$$-\frac{dK}{dr}=-\frac{q^2}{\pi v^2}\int\frac{d\vec{k}}{k^3}\frac{\kappa_D^2 S_{01}(k)}{1-S_{01}(k)}\int_{-kv}^{kv}\omega d\omega\,\mathrm{Im}\left[\frac{1}{\varepsilon_1(\omega+i0,k)}\right]$$

– SU(2) plasmon $(k << \kappa_D)$

$$\varepsilon_1(\omega,k) \approx 1 - \frac{\omega_p^2}{\omega^2}\left(1 - i\frac{\sqrt{\pi}}{2}x^3 e^{-x^2/2}\right) \qquad x = \frac{\omega}{k v_T} = \frac{\omega}{k}\sqrt{m\beta}$$

$$\omega_1^2(k) \approx \omega_p^2\left(1 - i\frac{\sqrt{\pi}}{2}\frac{\kappa_D^3}{k^3}e^{-\kappa_D^2/2k^2}\right)$$

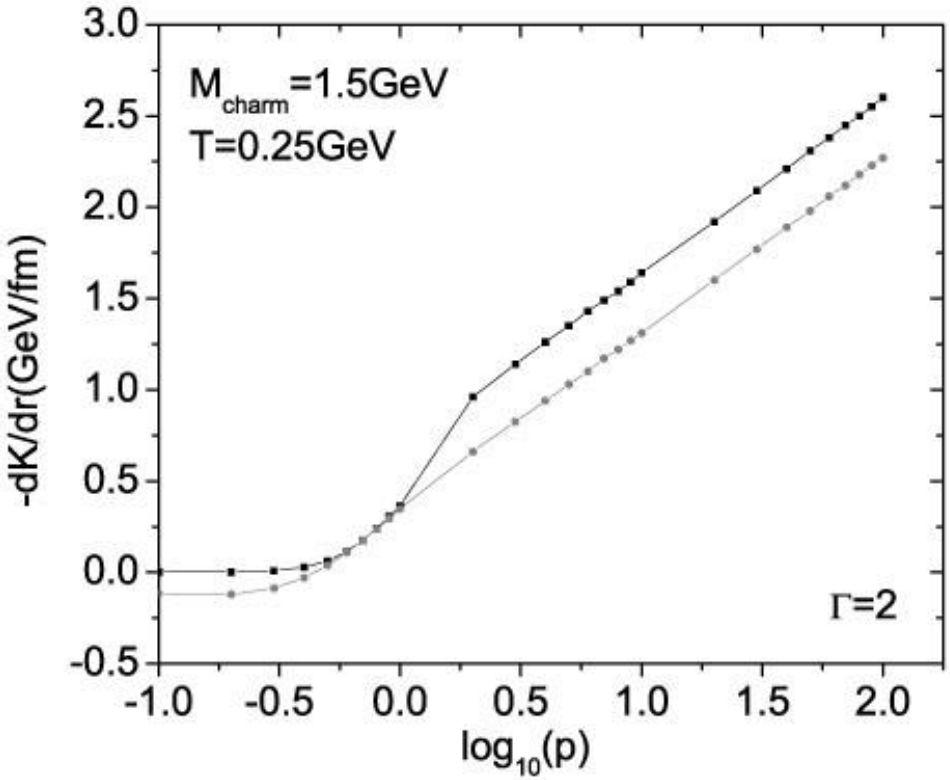
$(k >> \kappa_D)$

$$\varepsilon_1(\omega,k) \approx 1 - inc_{D1}(k)\frac{\sqrt{\pi}}{2}x e^{-x^2/2}$$

$$\sigma_1(\omega,k) = \frac{nc_{D1}(k)}{\sqrt{32\pi}}\frac{\omega^2}{v_T k}e^{-\omega^2/2v_T^2k^2}$$

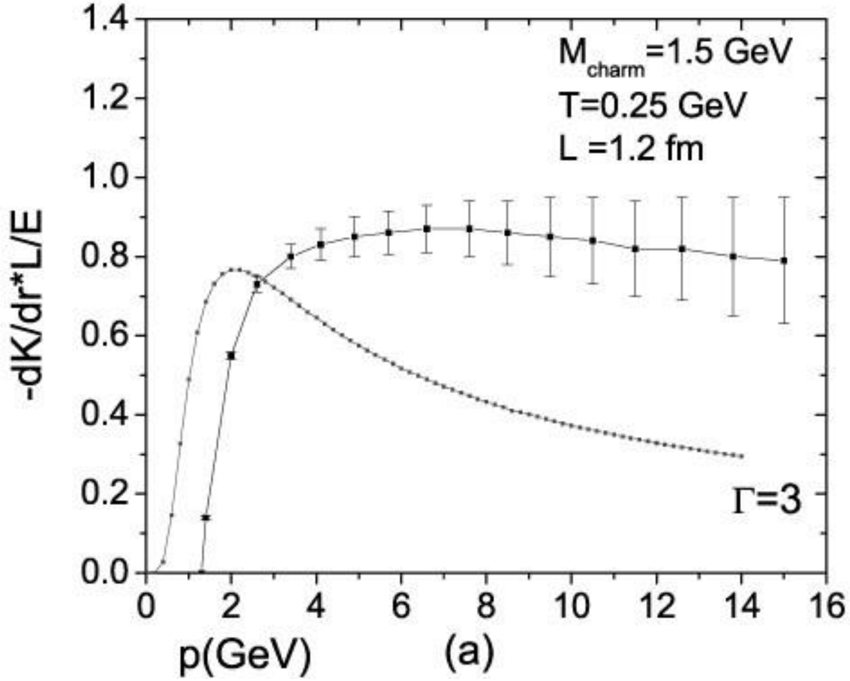
– Energy loss $(k >> \kappa_D)$

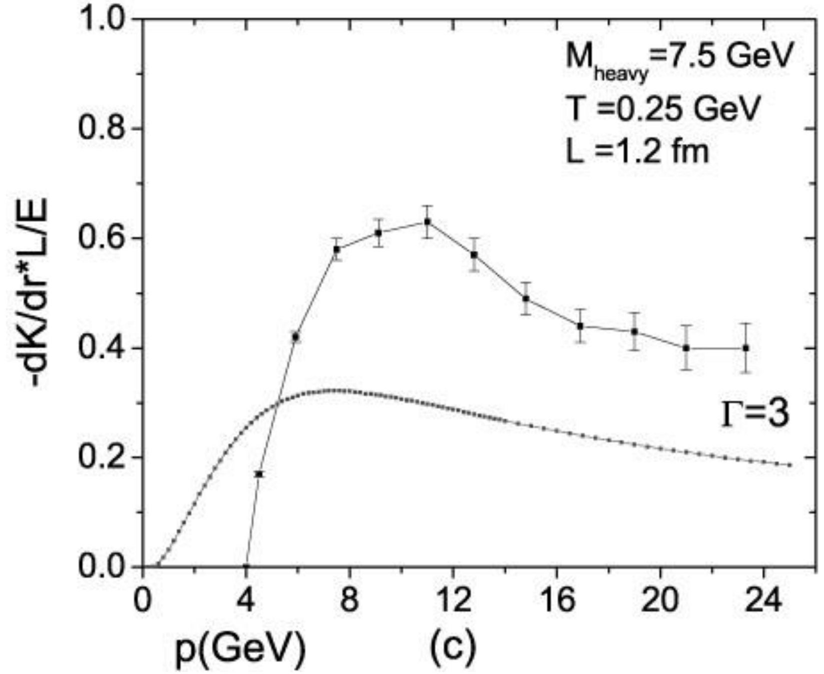
$$-\frac{dK}{dr} \approx \frac{g^2 C_F}{4\pi} \frac{\omega_p^2}{v^2} \left(\sqrt{\frac{2}{\pi}} \int_0^{kv} dx x^2 e^{-x^2/2} \right) \ln\left(\frac{k_{\max}}{\kappa_D} \right)$$



- Energy loss

$$-\frac{dK}{dr} = 3\Gamma^2\left(\frac{C_F}{C_2}\right)\frac{v_T^2}{v^2}\frac{T}{a_{WS}}\int_0^{q_{\max}}\frac{dq}{q}\frac{1}{\sqrt{2\pi_2}}\int_{-v/v_T}^{v/v_T}dx e^{x^2/2}$$
$$\times\left(\left((1-nc_{D1}(q))\frac{e^{x^2/2}}{x}-nc_{D1}(q)\psi(x)\right)+\frac{\pi^2}{2}n^2c_{D1}^2(q)\right)^{-1}$$





Conclusion

- Classical strongly coupled quark-gluon plasma
  1) Color charges
  2) Structure factor
  3) Energy loss

- Outlook
  1) Real SU(3) multicomponent QGP
  2) Quantum effects

– Color charges for SU(3)

$$Q^1 = \cos\phi_1\sqrt{\pi_3{}^2 - \pi_1{}^2} \quad Q^2 = \sin\phi_1\sqrt{\pi_3{}^2 - \pi_1{}^2} \quad Q^3 = \pi_1 \quad Q^8 = \pi_2$$

$$Q^4 = C_{++}\pi_+ A + C_{+-}\pi_- B \quad Q^5 = S_{++}\pi_+ A + S_{+-}\pi_- B$$

$$Q^6 = C_{--}\pi_- A - C_{--}\pi_+ B \quad Q^7 = S_{--}\pi_- A - S_{--}\pi_+ B$$

$$\pi_+ = \sqrt{\pi_3 + \pi_1} \qquad \pi_- = \sqrt{\pi_3 - \pi_1}$$

$$C_{\pm\pm} = \cos(\frac{1}{2}(\pm\phi_1 + \sqrt{3}\phi_2 \pm \phi_3)) \quad S_{\pm\pm} = \sin(\frac{1}{2}(\pm\phi_1 + \sqrt{3}\phi_2 \pm \phi_3))$$

$$A = \frac{1}{2\pi_3}\sqrt{(\frac{J_1 - J_2}{3} + \pi_3 + \frac{\pi_2}{\sqrt{3}})(\frac{J_1 + 2J_2}{3} + \pi_3 + \frac{\pi_2}{\sqrt{3}})(\frac{2J_1 + J_2}{3} - \pi_3 - \frac{\pi_2}{\sqrt{3}})}$$

$$B = \frac{1}{2\pi_3}\sqrt{(\frac{J_1 - J_2}{3} + \pi_3 - \frac{\pi_2}{\sqrt{3}})(\frac{J_1 + 2J_2}{3} - \pi_3 + \frac{\pi_2}{\sqrt{3}})(\frac{2J_1 + J_2}{3} + \pi_3 - \frac{\pi_2}{\sqrt{3}})}$$

…tion of 5 color charges

$$\int dQ Q^\alpha Q^\beta Q^\gamma Q^\delta Q^\varepsilon = \frac{3}{560} q_2 q_3 (\delta^{\alpha\beta} d^{\gamma\delta\varepsilon} + \delta^{\alpha\gamma} d^{\beta\delta\varepsilon} + \delta^{\alpha\delta} d^{\beta\gamma\varepsilon} + \delta^{\alpha\varepsilon} d^{\beta\gamma\delta} + \delta^{\beta\gamma} d^{\alpha\delta\varepsilon} + \delta^{\beta\delta} d^{\alpha\gamma\varepsilon} + \delta^{\beta\varepsilon} d^{\alpha\gamma\delta} + \delta^{\gamma\delta} d^{\alpha\beta\varepsilon} + \delta^{\gamma\varepsilon} d^{\alpha\beta\delta} + \delta^{\delta\varepsilon} d^{\alpha\beta\gamma})$$

– Integration of 6 color charges

$$\begin{aligned}\int dQ Q^\alpha Q^\beta Q^\gamma Q^\delta Q^\varepsilon Q^\zeta = {} & A(d^{\alpha\beta\gamma} d^{\delta\varepsilon\zeta} + d^{\alpha\beta\delta} d^{\gamma\varepsilon\zeta} + d^{\alpha\beta\varepsilon} d^{\gamma\delta\zeta} + d^{\alpha\beta\zeta} d^{\gamma\delta\varepsilon} \\ & + d^{\alpha\gamma\delta} d^{\beta\varepsilon\zeta} + d^{\alpha\gamma\varepsilon} d^{\beta\delta\zeta} + d^{\alpha\gamma\zeta} d^{\beta\delta\varepsilon} + d^{\alpha\delta\varepsilon} d^{\beta\gamma\zeta} + d^{\alpha\delta\zeta} d^{\beta\gamma\varepsilon} + d^{\alpha\varepsilon\zeta} d^{\beta\gamma\delta}) \\ & + B(\delta^{\alpha\beta}\delta^{\gamma\delta}\delta^{\varepsilon\zeta} + \delta^{\alpha\beta}\delta^{\gamma\varepsilon}\delta^{\delta\zeta} + \delta^{\alpha\beta}\delta^{\gamma\zeta}\delta^{\delta\varepsilon} + \delta^{\alpha\gamma}\delta^{\beta\delta}\delta^{\varepsilon\zeta} \\ & + \delta^{\alpha\gamma}\delta^{\beta\varepsilon}\delta^{\delta\zeta} + \delta^{\alpha\gamma}\delta^{\beta\zeta}\delta^{\delta\varepsilon} + \delta^{\alpha\delta}\delta^{\beta\gamma}\delta^{\varepsilon\zeta} + \delta^{\alpha\delta}\delta^{\beta\zeta}\delta^{\gamma\varepsilon} \\ & + \delta^{\alpha\delta}\delta^{\beta\varepsilon}\delta^{\gamma\zeta} + \delta^{\alpha\varepsilon}\delta^{\beta\gamma}\delta^{\delta\zeta} + \delta^{\alpha\varepsilon}\delta^{\beta\zeta}\delta^{\gamma\delta} + \delta^{\alpha\varepsilon}\delta^{\beta\delta}\delta^{\gamma\zeta} \\ & + \delta^{\alpha\zeta}\delta^{\beta\gamma}\delta^{\delta\varepsilon} + \delta^{\alpha\zeta}\delta^{\beta\varepsilon}\delta^{\gamma\delta} + \delta^{\alpha\zeta}\delta^{\beta\delta}\delta^{\gamma\varepsilon})\end{aligned}$$

	A	B
SU(2)		$\frac{1}{105} q_2^3$
SU(3)	$-\frac{9}{8!} q_2^3 + \frac{27}{2}\frac{1}{7!} q_3^2$	$\frac{85}{2}\frac{1}{8!} q_2^3 - \frac{6}{8!} q_3^2$