Dense Matter and Holography

I. Zahed

Seoul 10

Outline

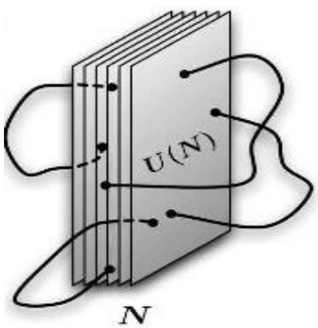
- Holography
- 1,2 Review
- ∞-Nucleon

Holography

Maldacena 97, Witten 98, …

AdS/CFT

N=4 SYM

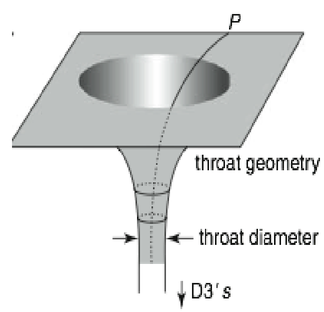


$N_c \times D_3$

Dual

$N_c \gg 1$

$\lambda = g^2 N_c \gg 1$



$AdS_5 \times S^5$

Maldacena 97'

Mesons: 4D

π

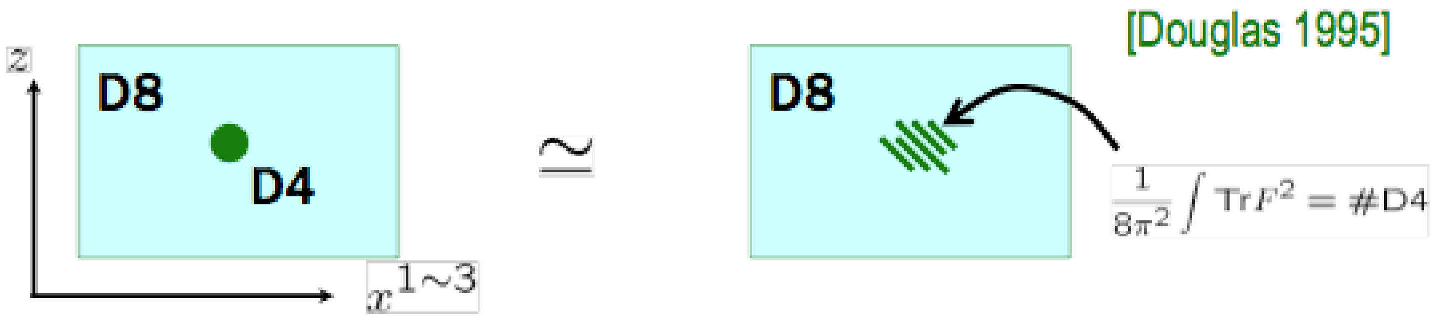
$$\int d^4x \,\mathrm{tr}\left[(\partial_\mu\varphi^{(0)})^2 + \sum_{n=1}^{\infty}\left(\frac{1}{2}(\partial_\mu B_\nu^{(n)} - \partial_\nu B_\mu^{(n)})^2 + \lambda_n M_{KK}^2 (B_\mu^{(n)} - \lambda_n^{-1/2}\partial_\mu\varphi^{(n)})^2\right)\right] + (\text{interaction terms})$$

	ρ	a_1	ρ'	(a_1')	ρ''
exp.(MeV)	776	1230	1459	(1647)	1720
our model	[776]	1189	1607	2023	2435

Below : $M_{\mathrm{KK}} = \frac{m_\rho}{\sqrt{\lambda_1}} \simeq 950 MeV,$

Sakai and Sugimoto 04

Baryon: Instanton



Wrapped D4=Instanton D8=Skyrmion

$$\frac{1}{8\pi^2}\int \mathrm{Tr}\, F^2 = \frac{1}{24\pi^2}\int \mathrm{Tr}\left(U dU^{-1}\right)^3$$

Atiyah+Manton 89!

1,2-Nucleon

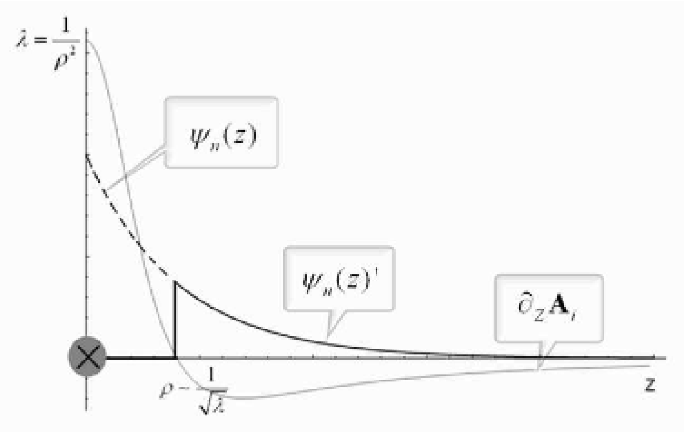
Hong, Rho, Yee and Yi 07; Sakai and Sugimoto 07,09; Kim and Zahed 08,09

Nucleon in 4D

$$\mathbb{F}^R_{MN} = W(t)\mathbb{F}_{MN}W^{-1}(t)$$
$$\mathbb{F}^R_{0M} = -W(t)\omega^a\mathbb{D}_M\left(\frac{\xi^2}{\xi^2+\rho^2}t_a\right)W^{-1}(t)$$

$$\omega^a = -2i\mathrm{tr}\left(t_a W(t)^{-1}\dot{W}(t)\right)$$

$$J_i = \mathbb{I}\omega^i$$
$$I_a t_a = -W J_i t_i W^{-1}$$



$$A_\mu = \mathbb{A}_\mu + C_\mu \,, \qquad C_\mu \equiv v^n_\mu\psi_{2n-1} + a^n_\mu\psi_{2n} + \mathcal{V}_\mu + \mathscr{A}_\mu\psi_0 \,,$$
$$A_Z = \mathbb{A}_Z + C_Z \,, \qquad C_Z \equiv -i\Pi\phi_0 \,,$$

$$\int_{x=Z=0} d\hat{\xi}C\,G^B\mathbb{A}_M$$

Kim and Zahed 08

EM Charge and Radius

$$\int d\vec{x} J^0_{\rm EM}(\vec{x}) = \int d\vec{x} Q_0(x, Z_c) = I_3 + \frac{B}{2}$$

$$\mathcal{Q}_0(x,Z) \equiv \kappa K \mathbb{F}^3{}_{Z0}(x,Z) + \frac{1}{N_c}\kappa K \widehat{\mathbb{F}}_{Z0}(x,Z)$$

Core~1/λ

$$\langle r^2\rangle_{EM} = \int d\vec{x}\, r^2 J^0_{EM}$$
$$= \int d\vec{x}\, r^2\, 2\mathcal{Q}_0(\vec{x}, Z_c) + 6\sum_{n=1}^{\infty} \frac{\alpha_{v^n}\psi_{2n-1}(Z_c)}{m_n^2} \int d\vec{x}\, 2\mathcal{Q}_0(\vec{x}, Z_c)$$

$$= \int dZ \sum_{n=1}^{\infty} \frac{\psi_{2n-1}(Z_c)\psi_{2n-1}(Z)K^{-1/3}(Z)}{m_n^2} = \int dZ \langle Z_c|\Box_{\mathbf{C}}^{-1}|Z\rangle$$

$$\sqrt{\langle r^2\rangle_{EM}} \approx 14.26\left(\frac{1}{2}+I_3\right) M_{\rm KK}^{-2} \approx 0.784\left(\frac{1}{2}+I_3\right) {\rm fm}$$

$$\sqrt{\langle r^2\rangle^{\rm proton}_{EM}} = 0.875\ {\rm fm}\ , \qquad \langle r^2\rangle^{\rm neutron}_{EM} = -0.1161\ {\rm fm}^2$$

Kim and Zahed 08

Cheshire Cat Principle

MIT-bag Brown-Rho-bag Skyrmion

Nadkarni+Nielsen+Zahed 85, Rho …

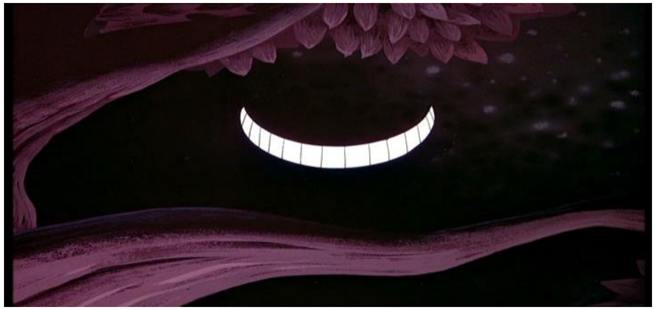
Lewis Caroll Cheshire Cat

Alice in wonderland:

MIT bag

=

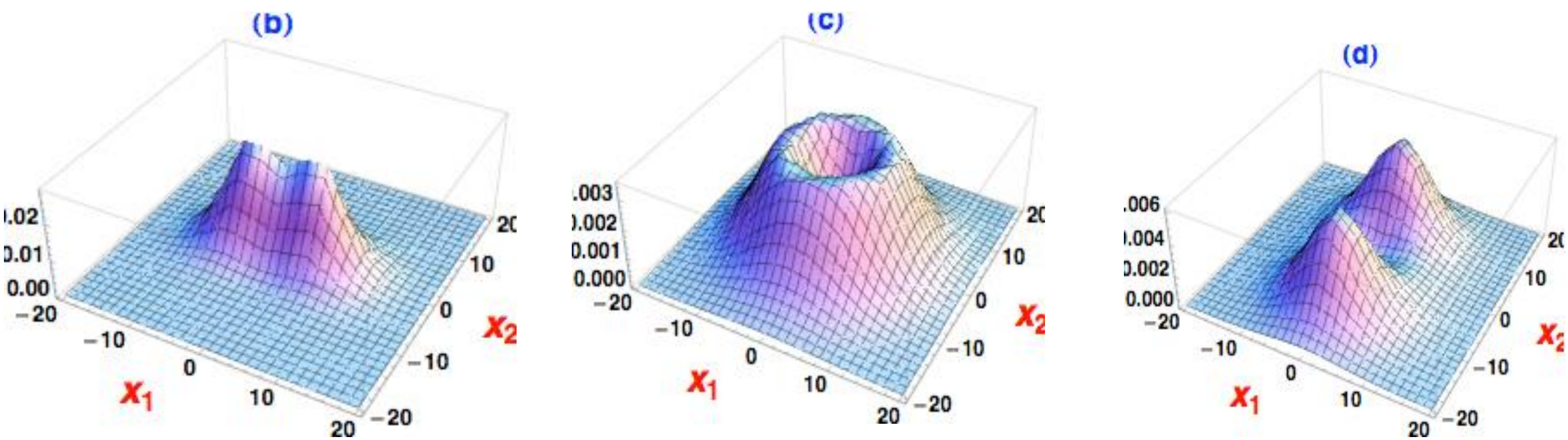
Skyrmion



“I have often seen a cat without a grin but never a grin without a cat”

Combed: $\theta_3 = \pi/2$

$d/\rho =$ 1.7 $\sqrt{2}$ 1



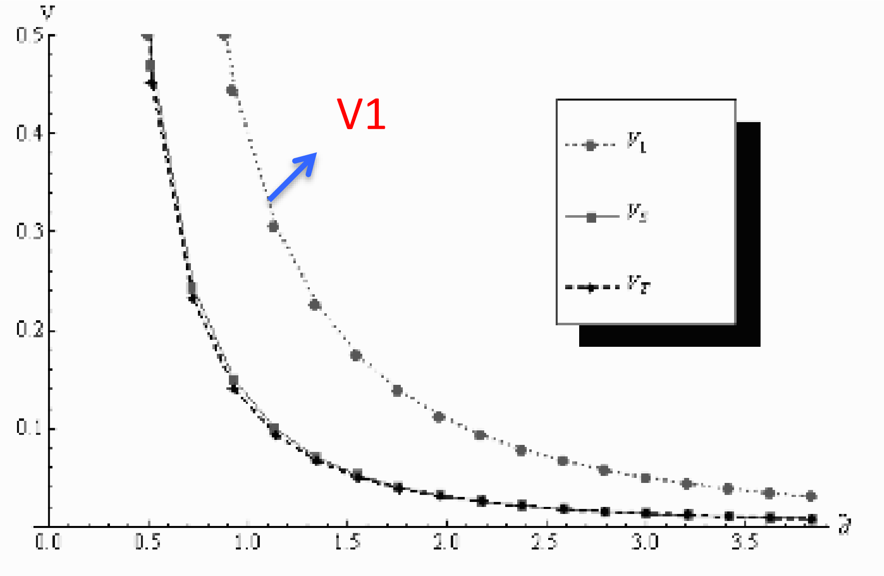
$\rho = 9.64$

Nucleon-Nucleon: Core

$$V = V_1 + V_S\left(4\cos^2|\theta| - 1\right) + V_T^{ab}\left[\left(6\hat{\theta}^a\hat{\theta}^b - 2\delta^{ab}\right)\sin^2|\theta| + 3\epsilon^{abc}\widehat{\theta}^c \sin 2|\theta|\right]$$

$$V_1 \approx \frac{27\pi N_c}{2\lambda}\frac{1}{d^2}$$

4D Coulomb Repulsion !



Nucleon_Nucleon: Cloud

$$V_{NN} = V_1^+ + \vec{\tau}_1 \cdot \vec{\tau}_2 V_1^- + \vec{\sigma}_1 \cdot \vec{\sigma}_2 \left(V_S^+ + \vec{\tau}_1 \cdot \vec{\tau}_2 V_S^- \right) + \left(3(\vec{\sigma}_1 \cdot \hat{d})(\vec{\sigma}_2 \cdot \hat{d}) - \vec{\sigma}_1 \cdot \vec{\sigma}_2 \right) \left(V_T^+ + \vec{\tau}_1 \cdot \vec{\tau}_2 V_T^- \right)$$

$$V^+_{1,\widehat{V}} \approx \sum_n G^2_{1\widehat{V},2n-1} \frac{e^{-m_{2n-1}d}}{4\pi\, d} , \qquad G_{1\widehat{V},2n-1} \equiv \frac{N_c}{2} \psi_{2n-1} \sim \sqrt{\frac{N_c}{\lambda}} ,$$

$$V^-_{S,A} \approx \sum_n G^2_{SA,2n} \frac{e^{-m_{2n}d}}{4\pi d} , \qquad G_{SA,2n} \equiv -\frac{g_A \psi_{2n}}{\sqrt{6}\psi_0} \sim \sqrt{\frac{N_c}{\lambda}} ,$$

$$V^-_{S,V} \approx \sum_n G^2_{SV,2n-1} \frac{e^{-m_{2n-1}d}}{4\pi d} , \qquad G_{SV,2n} \equiv -\frac{g_V \psi_{2n-1}}{\sqrt{6}} \sim \frac{1}{\sqrt{\lambda N_c}}$$

$$V^-_{T,A} \approx \sum_n G^2_{TA,2n} \frac{e^{-m_{2n}d}}{4\pi d} , \qquad G_{TA,2n} \equiv \frac{g_A \psi_{2n}}{\sqrt{12}\psi_0} \sim \sqrt{\frac{N_c}{\lambda}} ,$$

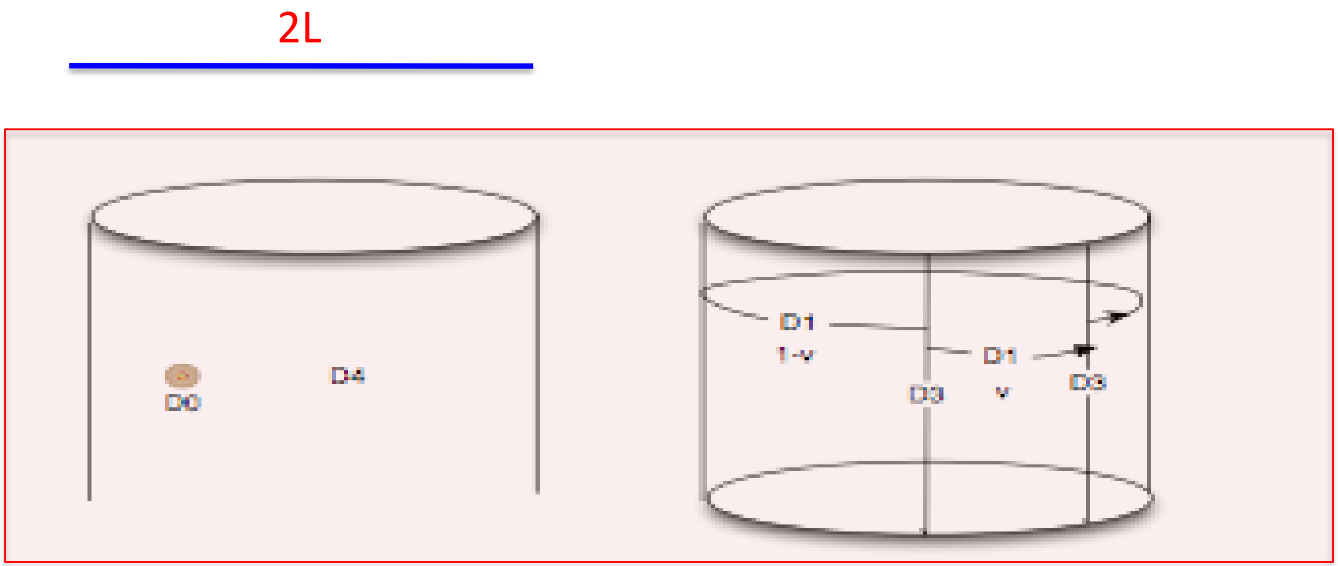
$$V^-_{T,V} \approx \sum_n G^2_{TV,2n-1} \frac{e^{-m_{2n-1}d}}{4\pi d} , \qquad G_{TV,2n} \equiv \frac{g_V \psi_{2n-1}}{\sqrt{12}} \sim \frac{1}{\sqrt{\lambda N_c}} ,$$

$$V^-_{T,\Pi} \approx \frac{1}{16\pi} \left(\frac{g_A}{f_\pi} \right)^2 \frac{1}{d^3} \sim \frac{N_c}{\lambda} .$$

Many-Nucleons

Rho, Sin, Zahed 09

Instanton with Holonomy



$$\langle A_3 \rangle = 0 \rightarrow \frac{2\pi}{2L}\, v\, T_3$$

$$N_f = 2$$

Krann, van Baal 98; Lee, Lu 98 = KvLL

1 Instanton→ 2 Dyons

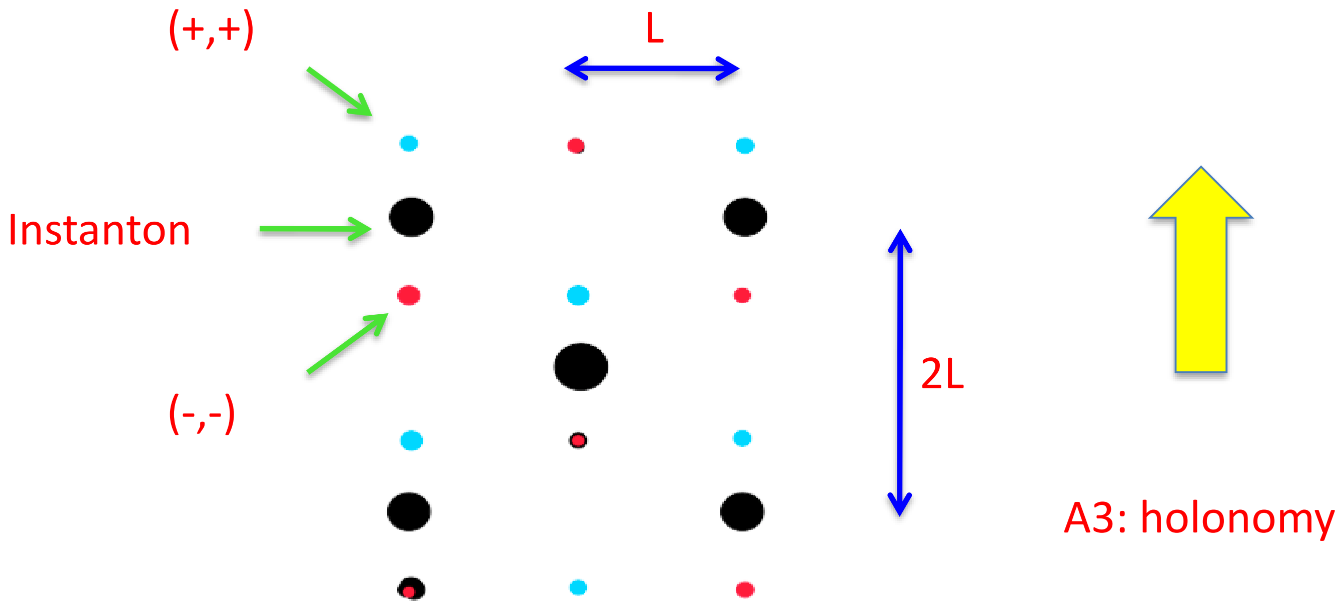
$$+ \equiv (+,+)\,T_3 = (e,g)$$
$$- \equiv (-,-)\,T_3 = (e,g)$$

$$B = B_+ + B_- = v + (1-v)$$
$$M = M_+ + M_- = MB_+ + MB_-$$

$$\langle A_3 \rangle = \frac{2\pi}{2L}\, vT_3$$

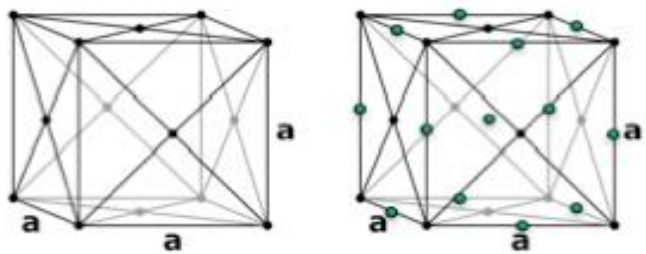
: $0<v<1$!

Array: Instantons→Dyons



I=fcc→D=bcc

Why Salt ?



I-fcc D-bcc

Order $N_c\lambda$: BPS

Order $N_c\lambda^0$: non-BPS with size $\rho \approx 1/\sqrt{\lambda}$
+ and – form bcc with 0<v<1

BCC of ½ Instantons

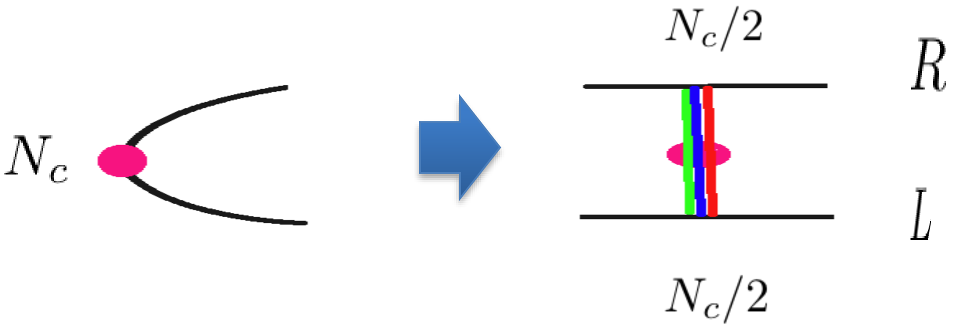
To order N_c/λ ω-core repulsion between v and (1-v) :

$$V_\omega(r) \approx \frac{27\pi}{2}\left(\frac{N_c}{\lambda}\right)\frac{1}{r^2}$$

Repulsion balances v=1-v ie v=1/2

$$n_B = \frac{1/2}{L^3} = \frac{4}{(2L)^3}$$

Holographic Restoration LXR



$$U^R_{1/2}(x) = P\exp\left(\int_0^{+\infty} A_Z(x,Z)\,dZ\right)$$

$$U^L_{1/2}(x) = P\exp\left(\int_{-\infty}^{0} A_Z(x,Z)\,dZ\right)$$

Estimate of fcc→bcc

$2\mathbf{I}/M$ ← bcc ↑

$$R_{+-} = 2\pi\frac{\rho^2}{2L} \equiv L \to L = \sqrt{\pi}\rho \to n_B = \frac{1/2}{(\sqrt{\pi}\rho)^3}$$

↑ KvLL ↓ fcc

$$\mathbf{I} \approx 1/(200\,\mathrm{MeV})$$
$$M \approx 1\,GeV$$

$$\rho \approx 1/2\,\mathrm{fm}$$
$$L \approx 1\,\mathrm{fm}$$
$$n_B \approx 1/2\,\mathrm{fm}^{-3} \approx 3n_{NM}$$

Binding Energy

$$E/N = M - \Delta$$

$$\Delta = (e^2 + g^2)(T^3)^2 \left(\int_{-\rho}^{+\rho} \frac{1}{L^2 + Z^2} \right) M_D = (e^2 + g^2)(T^3)^2 \frac{2\tan^{-1}(\rho/L)}{L} M_D$$

$$\Delta \approx 180\,\mathrm{MeV}$$

Madelung Constant for salt MD=1.748, L=1 fm, $\rho = \sqrt{\pi}/L$

Melting Temperature

Lindemann criterion:

$$\sqrt{\langle x^2 \rangle} \approx (10\%)\, a_{NN}$$

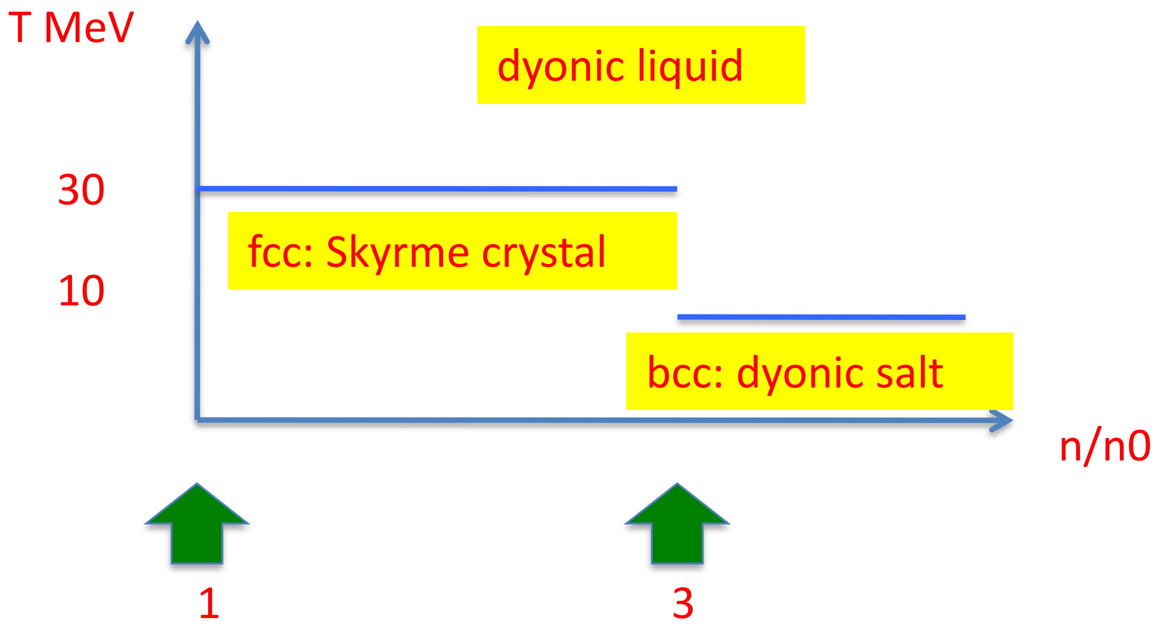
$$M\omega_E^2 \langle x^2 \rangle = k_B T$$

$$\omega_E = c_S\, k_{\max} = c_s \pi / a_{NN}$$

$$c_S^2 / c^2 = (K/(n_0 M c^2))(n_0/n) \approx (0.2 - 0.3)(n_0/n)$$

$$\frac{k_B T}{M c^2} \approx \pi^2 (10\%)^2 \, (0.2 - 0.3) \, (n_0/n)$$

Cold Phase Diagram



Summary

- Baryons are Holograms of Instantons
- 1,2 nucleons BPS (core) VMD (cloud)
- Cheshire Cat found hiding in the 5^{th} Dim
- Nucleons crystalize into salt of ½ Instantons