# QCD sum rules for  $\rho$ -meson<br>at finite density or temperature

### Youngshin Kwon

Heavy Ion Meeting at KNU October 9, 2010



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#### contents based on :

<span id="page-0-0"></span>YK., Procura & Weise [ Phys. Rev. C78, 055203 (2008) ] YK, Sasaki & Weise [ Phys. Rev. C81, 065203 (2010) ]

### **Outline**

- ◎ Introduction & motivation
- $\circ$  QCD sum rules for  $\rho$ -meson, in vacuum and in medium

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- $\circ$  Finite energy sum rules at finite density
- $\circ$  Finite energy sum rules at finite temperature
- <span id="page-1-0"></span><sup>®</sup> Summary and outlook

8

**Goal**: reliable framework of in-medium QCD sum rules for vector mesons  $\Rightarrow$  constraints for the in-medium spectral properties

### **Motivation**

- Spontaneous chiral symmetry breaking:
	- **•** quark condensate:  $\langle \bar{q}q \rangle \neq 0$
	- **Goldstone bosons:**  $\pi$ ,  $K$ , etc. pion decay constant:  $f_{\pi} \approx 92.4$  MeV
	- $\cdot$  mass splitting of chiral partners  $(e.g. \rho(770)-a_1(1250))$
- 0.0 0.5 1.0 1.5 2.0 2.5 3.0  $_{0}$ 2 F 4 6þ  $s$  [GeV<sup>2</sup>] Spectral function Ρ *a*<sup>1</sup>
- $\circ$  Chiral symmetry restoration in nuclear medium:
	- ► degenerate chiral partners ⇒ modifications of hadron spectrum

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Restoration scenarios in medium

- Pole mass shift:
	- $\triangleright$  masses of parity partners degenerate in medium.
	- F moving toward each other or going to zero (Brown-Rho).<br>Για το κατά (Sam (Sam (Sam (Sam (Sam (Sam)).

Brown & Rho [ PRL 66, 2720 (1991) ]

**⊘** Width broadening:  $\circ$ 

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- $\blacktriangleright$  in-medium spectral functions are broadly distributed.  $\dots$ 9 · (2π+<sub>1)</sub>π να τ erc<br><sub>ho [ ]</sub><br>spe
- $\blacktriangleright$  the continuum merges the broadened spectral distributions.



by perturbative continuation  $\ell$ 

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#### Dilepton spectroscopy

- $\circ$  Dilepton production in RHIC ( $γ^*$  →  $l^+l^-$ ):
	- $\blacktriangleright$  EM probe with pure information of the hot and/or dense region
	- ► dilepton emission ⇔ in-medium vector-meson spectroscopy



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## <span id="page-5-0"></span>General review of QCD sum rules (in vacuum)

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[Introduction](#page-1-0) **International COD sum rules [In-medium FESR](#page-19-0)** [Summary](#page-30-0) Summary

General review of QCD sum rules

 $\circ$  Current correlation function:

$$
\Pi^{\mu\nu}(q) = i \int d^4x \ e^{iq \cdot x} \langle \mathcal{T} j^{\mu}(x) j^{\nu}(0) \rangle
$$

 $\triangleright$  isovector vector- and axialvector-currents:

$$
j_{\rho}^{\mu} = \frac{1}{2} \left( \bar{u} \gamma^{\mu} u - \bar{d} \gamma^{\mu} d \right), \qquad j_{A}^{\mu} = \frac{1}{2} \left( \bar{u} \gamma^{\mu} \gamma_5 u - \bar{d} \gamma^{\mu} \gamma_5 d \right)
$$

- $\blacktriangleright$  invariant correlator:  $\Pi(q^2) = \frac{1}{3} g_{\mu\nu} \Pi^{\mu\nu}(q)$
- ⊚ Operator product expansion (quark & gluon d.o.f.) at large  $Q^2 = -q^2$ :

$$
\frac{12\pi^2}{Q^2}\Pi(Q^2) = -c_0 \ln\left(\frac{Q^2}{\mu^2}\right) + \frac{c_1}{Q^2} + \frac{c_2}{Q^4} + \cdots
$$

◎ Spectral representation (hadronic d.o.f.) at resonance region:

$$
\Pi(q^2) = \Pi(0) + q^2 \Pi'(0) + \frac{q^4}{\pi} \int ds \frac{\operatorname{Im} \Pi(s)}{s^2(s - q^2 - i\epsilon)}
$$

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◎ Borel transformation:

$$
12\pi^2 \Pi(0) + \int_0^\infty ds \, R(s) \, e^{-s/M^2} = c_0 M^2 + c_1 + \frac{c_2}{M^2} + \frac{c_3}{2M^4} + \cdots
$$

- $\blacktriangleright$  dimensionless spectral function: *R*(*s*) ≡  $-\frac{12π}{s}$  Im Π(*s*)
- } Coefficients *cn*:

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 $\circledcirc$  Borel transformation:

$$
12\pi^2 \Pi(0) + \int_0^\infty ds \, R(s) \, e^{-s/M^2} = c_0 M^2 + c_1 + \frac{c_2}{M^2} + \frac{c_3}{2M^4} + \cdots
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- $\blacktriangleright$  dimensionless spectral function: *R*(*s*) ≡  $-\frac{12π}{s}$  Im Π(*s*)
- $\circledcirc$  Coefficients  $c_n$ :

$$
c_0 = \frac{3}{2} \left( 1 + \frac{\alpha_s}{\pi} \right) + \cdots, \qquad c_1 \propto m_q^2 \text{ : negligibly small}
$$
\n
$$
c_2 = \frac{\pi^2}{2} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle \pm 6\pi^2 \left( m_u \langle \bar{u}u \rangle + m_d \langle \bar{d}d \rangle \right)
$$
\n
$$
c_3 \propto \mp \langle (\bar{q}q)^2 \rangle \text{ uncertain value}
$$

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Borel transformation:

$$
12\pi^2 \Pi(0) + \int_0^\infty ds \, R(s) \, e^{-s/M^2} = c_0 M^2 + c_1 + \frac{c_2}{M^2} + \frac{c_3}{2M^4} + \cdots
$$

- $\triangleright$  dimensionless spectral function: *R*(*s*) ≡  $-\frac{12π}{s}$  Im Π(*s*)
- **Expand for**  $s_0 \ll M^2$  and compare term by term
- ◎ Coefficients  $c_n$ :

$$
c_0 = \frac{3}{2} \left( 1 + \frac{\alpha_s}{\pi} \right) + \cdots, \qquad c_1 \propto m_q^2 : \text{negligibly small}
$$
\n
$$
c_2 = \frac{\pi^2}{2} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle \pm 6\pi^2 \left( m_u \langle \bar{u}u \rangle + m_d \langle \bar{d}d \rangle \right)
$$
\n
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c_3 \propto \mp \langle (\bar{q}q)^2 \rangle \text{ uncertain value}
$$

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[Introduction](#page-1-0) **International COD sum rules [In-medium FESR](#page-19-0)** [Summary](#page-30-0) Summary

Finite energy sum rules

 $\circledcirc$  Hierarchy of finite energy sum rules for moments of  $R(s)$ :

$$
0^{th} \text{ moment}: \int_0^{s_0} ds R(s) = s_0 c_0 + c_1 - 12\pi^2 \Pi(0)
$$
  

$$
1^{st} \text{ moment}: \int_0^{s_0} ds s R(s) = \frac{s_0^2}{2} c_0 - c_2
$$

 $\circ$  Spectral distribution (resonance + continuum):



$$
R(s) = R_{\rho}(s)\theta(s_0 - s) + R_c(s)\theta(s - s_0)
$$

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$$
\sqrt{s_0} \simeq 4\pi f_\pi
$$

[Introduction](#page-1-0) **International COD sum rules [In-medium FESR](#page-19-0)** [Summary](#page-30-0) Summary

Finite energy sum rules

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$$
\sqrt{s_0} \simeq 4\pi f_\pi
$$

Consistency with current algebra

 $\circ$  Identification of  $\sqrt{s_0}$  with  $\Lambda_{\text{CSB}} \simeq 4\pi f_\pi$ :



### $\triangleright$  KSRF relation

Kawarabayashi & Suzuki [ PRL 16, 255 (1966) ] Riazuddin & Fayyazuddin [ PR 147, 1071 (1966) ]

 $\blacktriangleright$  Weinberg sum rules

Weinberg [ PRL 18, 507 (1967) ]

$$
\Rightarrow m_{a_1} = \sqrt{2} m_\rho = 4\pi f_\pi
$$

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**MOMENTS** of **SPECTRAL FUNCTIONS** (contd.)

**MOMENTS** of **SPECTRAL FUNCTIONS** (contd.)

Introduction **International COD sum rules In-medium FESR** [Summary](#page-30-0) Summary **CCD sum rules CCD sum rules In-medium FESR** 

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**QCD SUM RULES** for



 $\circledcirc$  Identification of  $\sqrt{s_0}$  with  $\Lambda_{\text{CSB}} \simeq 4\pi f_\pi$ :

### $\triangleright$  KSRF relation

**Aswarabayasin & Suzuki [ PR 147, 1071 (1966) ]**<br> **Riazuddin & Fayyazuddin [ PR 147, 1071 (1966)** ] Kawarabayashi & Suzuki [ PRL 16, 255 (1966) ]

► Weinberg sum rules

Weinberg [ PRL 18, 507 (1967) ]



Other number:

\n
$$
\int_0^{s_0} ds \, R_{\rho}(s) = \frac{3}{2} s_0
$$
\n
$$
\Rightarrow m_{\rho}^2 = 2g^2 f_{\pi}^2
$$
\nSubstituting the values of the following matrices:

\n
$$
\int_0^{s_0} ds \, sR_{\rho}(s) = \frac{3}{2} s_0
$$
\n
$$
\Rightarrow g = 2\pi
$$

**disk moment:**  $(s) = \frac{1}{4}s_0$  $\mathcal{C}^{s_0}$  $\int$  ds  $\int^{s_0}$  $\int_0^{s_0} ds \, sR_\rho(s) = \frac{3}{4}$  $\frac{3}{4} s_0^2$  $\Rightarrow$   $g = 2\pi$ 

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#### <span id="page-14-0"></span>Vacuum sum rule analysis

 $\circ$  Input:  $R_{\rho}(s)$  from chiral effective field theory + vector mesons (VMD)





<span id="page-15-0"></span>Vacuum sum rule analysis

 $\circ$  Input:  $R_{\rho}(s)$  from chiral effective field theory + vector mesons (VMD)



[Introduction](#page-1-0) **International COD sum rules [In-medium FESR](#page-19-0)** [Summary](#page-30-0) Summary

within an error band determined by the uncertainties of the uncertainties of the input summarized in Table 4.1 and Eq. (4.21). This test turns out to be successful. The detailed analysis of  $\mathcal{A}$  to of uncertainties performed with Eq. (4.32) for the first moment is shown in Fig. 4.3.

using the physical value f<sup>π</sup> = 92.[4 Me](#page-15-0)V of the p[ion d](#page-16-0)ecay const[ant.](#page-18-0) [T](#page-19-0)[he](#page-5-0) [post](#page-6-0)ulate,

<span id="page-16-0"></span>s<sup>0</sup> = 1.14±0.01 GeV is w[ithin](#page-17-0) [2%](#page-15-0) o[f th](#page-17-0)[e em](#page-18-0)[pir](#page-5-0)[ical](#page-6-0) 4πf<sup>π</sup> ' 1.[16](#page-18-0) [GeV](#page-19-0)

Sensitivity to threshold modeling

◎ replace the Heaviside step function with a ramp function:

$$
R(s) = R_{\rho}(s)\theta(s_2 - s) + R_c(s)W(s)
$$

with the weight function *W*(*s*)



© No dependence on details of the threshold modeling

[Introduction](#page-1-0) **International COD sum rules In-medium FESR** Summary Summary [between](#page-19-0) resonance and conti[nuum re](#page-30-0)gion, as follows: as follows: as follows:  $\sum_{n=1}^{\infty}$ tion (11). A test can be performed replacing the step f[u](#page-19-0)[n](#page-26-0)[c](#page-21-0)[t](#page-22-0)[i](#page-23-0)[o](#page-29-0)n by a ramp function [t](#page-28-0)ransition to  $\mathcal{A}$  smooth transition to  $\mathcal{A}$ 

with the weight function  $\mathcal{L}_\text{c}$ 

respect to variations in the slope (s<sup>2</sup> <sup>−</sup> <sup>s</sup>1)−<sup>1</sup> of the ramp function W(s), thus confirming that the step function

function has <sup>s</sup><sup>2</sup> <sup>−</sup> <sup>s</sup><sup>1</sup> " 1 GeV<sup>2</sup> (see Fig.1). It can be

R(s)= Rρ(s) Θ(s<sup>2</sup> − s)+ Rc(s) W(s) , (27)

for s<sup>1</sup> ≤ x ≤ s<sup>2</sup>

 $S$ et intervals (s $\mathbb{R}$ ) are the determi[ne](#page-18-0)[d](#page-19-0) [so](#page-5-0) as  $\mathbb{R}$ ) are then determined so as

Sensitivity to threshold modeling where the weight function,  $\mathcal{L}_{\text{S}}$ 

"

 $\circledast$  replace the Heaviside step function with a ramp function: 0 for x ≤ s<sup>1</sup> of  $\alpha$  the the narrow (less than 1 %) under narrow (less than 1 %) under narrow (less than 1 %) under narrow (

$$
R(s) = R_{\rho}(s)\,\theta(s_2 - s) + R_c(s)\,W(s)
$$

with the weight function  $W(s)$  $\mathcal{L} = \mathcal{L} \mathcal{L} \mathcal{L}$ 

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© No dependence on details of the threshold modeling<br>○ No dependence on details of the threshold modeling − energy extensive possible po<br>December 2009 energy extensive possible possible possible possible possible possible possible possible possib

Sets of intervals and intervals  $\bigcirc$ 

The analysis is performed at the baryon density of normal nuclear<br>Eti  $\infty$ 

The step function behavior is recovered for W(x)in the

<span id="page-17-0"></span>in comparison in Fig.4.

### <span id="page-18-0"></span>Finite energy sum rules at finite density

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### Medium-modification of the sum rules

- The existence of nuclear matter causes breaking Lorentz invariance:
	- $\triangleright$  two invariant correlator: longitudinal and transverse parts.
	- choosing a preferred reference frame of the medium ( $q = 0$ ), longitudinal and transverse correlators coincide.

$$
\Pi^{L}(\omega, q = 0) = \Pi^{T}(\omega, q = 0) \equiv \Pi(\omega, q = 0)
$$

- New operators with spin appear due to the broken Lorentz invariance:
	- $\cdot$  the first moment of in-medium FESR involves twist-2 operator (e.g.  $\langle \bar{q}\gamma_{\nu}D_{\mu}q\rangle$ ) to be considered.
- Medium-dependence in the OPE side contributes only to the condensates:
	- $\triangleright$  non-perturbative contributions in OPE appear to be clearly separated into the condensates.
	- $\blacktriangleright$  medium effects are non-perturbative.

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### Density-dependence of OPE

Hatsuda & Lee [ Phys. Rev. C 46, R34 (1992) ]

 $\circledcirc$  Expectation value: vacuum  $\rightarrow$  ground state of nuclear matter

$$
\langle 0 | O | 0 \rangle \equiv \langle O \rangle_0 \rightarrow \langle O \rangle_{\rho_N} = \langle N | O | N \rangle
$$

 $\circledcirc$  In-medium coefficients:  $c_n \to c_n + \delta c_n$ 



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[Introduction](#page-1-0) **[In-medium FESR](#page-19-0)** [Summary](#page-30-0) COD sum rules **In-medium FESR** Summary Summary

### Spectral functions at finite density

 $\circ$  *ρ*-meson spectral functions in nuclear medium ( $\rho_N = \rho_0 = 0.17$  fm<sup>−3</sup>):



- $\triangleright$  KKW: SU(3) chiral dynamics with vector meson dominance Klingl, Kaiser & Weise [ Nucl. Phys. A624, 527 (1997) ]
- <sup>I</sup> RW: particle-hole excitations (∆(1232)-*h* and *N* ∗ (1520)-*h*)) Rapp & Wambach [ Adv. Nucl. Phys. 25, 1 (2000) ]

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[Introduction](#page-1-0) **[In-medium FESR](#page-19-0)** [Summary](#page-30-0) COD sum rules **In-medium FESR** Summary Summary

Results for  $\rho$ -meson at finite density

0.95 1.00 1.05 1.10 0.6 0.7 0.8  $\theta$ . 1.0 1.1  $1.2<sub>1</sub>$ 1.3  $1.4<sub>F</sub>$  $s_0^{1/2}$  [GeV]  $1.2$ <br>  $1.1$ <br>  $1.0$ l.h.s <sup>H</sup>RW<sup>L</sup>  $l.h.s$   $(KKN)$  $\mathcal{S}$  $\rho_N = \rho_0$ 

In vacuum: 
$$
\sqrt{s_0} \approx 1.14 \,\text{GeV} \approx 4\pi f_\pi
$$

$$
\bar{m}^2 \equiv \frac{\int_0^{s_0} \mathrm{d}s \ s R(s)}{\int_0^{s_0} \mathrm{d}s \ R(s)}
$$



In-medium RW spectrum:  $\sqrt{s_0^*} \approx 1.09 \pm 0.01 \,\text{GeV}$  $\sqrt{\frac{s_0^*}{s_0}} \simeq \frac{\bar{m}^*}{\bar{m}}$  $\frac{m}{\bar{m}} \approx 0.96$ 

Kwon, Procura & Weise [ PRC 78, 055203 (2008) ]

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## <span id="page-23-0"></span>Finite energy sum rules at finite temperature

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Temperature-dependence of OPE

Hatsuda, Koike & Lee [ Nucl. Phys. B 394, 221 (1993) ]

 $\circ$  Thermal expectation value:

$$
\langle O \rangle_0 \to \langle O \rangle_T = \frac{\text{Tr}\, O\, \exp(-H/T)}{\text{Tr}\, \exp(-H/T)}
$$

 $\circledcirc$  In-medium coefficients:  $c_n \to c_n + \delta c_n$ 



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[Introduction](#page-1-0) **[In-medium FESR](#page-19-0)** [Summary](#page-30-0) COD sum rules **In-medium FESR** Summary Summary

Vector & axialvector mixing with temperature

 $\circ$  Mixing of vector and axialvector:

$$
R_V(s, T) = R_V(s, 0) (1 - \epsilon(T)) + R_A(s, 0) \epsilon(T)
$$
  
\n
$$
R_A(s, T) = R_A(s, 0) (1 - \epsilon(T)) + R_V(s, 0) \epsilon(T)
$$
  
\nEletsky & lefte [ PRO 47, 3083 (1993). PRO 51, 2371 (1995)]

If the mixing parameter  $\epsilon(T)$  is given by the thermal pion loop:

$$
\epsilon(T) = \frac{2}{f_{\pi}^2} \int \frac{d^3k}{\omega(2k)^3} \frac{1}{e^{\omega/T} - 1} \xrightarrow{m_{\pi} \to 0} \frac{T^2}{6 f_{\pi}^2}
$$
  
where  $\omega^2 = k^2 + m_{\pi}^2$ .

At critical temperature where  $\epsilon \simeq \frac{1}{2}$ ,  $R_V$  and  $R_A$  become identical.

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### Mixing of finite-width spectrum

◎ Spectral functions with finite decay width:





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Mixing of finite-width spectrum

 $\circ$  Sum rule result for vector channel:



**Average**  $\rho$ **-meson mass:** 

$$
\bar{m}_{\rho}^{2} = \frac{\int_{0}^{s_{0}} \mathrm{d}s \ s R_{\rho}(s)}{\int_{0}^{s_{0}} \mathrm{d}s \ R_{\rho}(s)}
$$

 $\blacktriangleright$  Comparison with ChPT:

$$
f_{\pi}(T) = f_{\pi}\left(1 - \frac{1}{2}\epsilon(T)\right)
$$

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Simple test beyond *V*-*A* mixing

} Dropping pole mass in addition to the *V*-*A* mixing:



The simplest ansatz (zero width):

$$
R_{\rho}(s,0) = F_{\rho}^{2} \delta\left(s - m_{\rho}^{2}\right)
$$

$$
R_{a}(s,0) = F_{a}^{2} \delta\left(s - m_{a}^{2}\right)
$$

$$
R_{\rho}(s,T) = R_{\rho}(s,0) \left(1 - \epsilon\right) + R_{a}(s,0) \epsilon
$$

Brown-Rho scaling hypothesis:

$$
m_{\rho}^2 \to m_{\rho}^2 \left(1 - \frac{1}{2} \epsilon(T)\right)^2
$$

 $\Rightarrow$  better agreement :  $\sqrt{s_0} = 4\pi f_\pi(T)$ 

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### About four-quark condensates

- ◎ Sum rules for 0th and 1st moments: RHS quantities are accurately determined (pQCD and leading condensates)
- ◎ Sum rules for 2nd moment: involving four-quark condensates

$$
\int_0^{s_0} ds s^2 R(s) = \frac{s_0^3}{3} + c_3
$$
  

$$
c_3 = -6\pi^3 \alpha_s \left[ \langle (\bar{u}\gamma_\mu \gamma_5 \lambda^a u - \bar{d}\gamma_\mu \gamma_5 \lambda^a d)^2 \rangle + \frac{2}{9} \langle (\bar{u}\gamma_\mu \lambda^a u + \bar{d}\gamma_\mu \lambda^a d) \sum_{q=u,d,s} \bar{q} \gamma^\mu \lambda^q q \rangle \right]
$$

 $\circ$  Ground state saturation ( $\kappa = 1$ )

$$
\langle (\bar{q}\gamma_\mu\gamma_5 \lambda^a q)^2 \rangle = -\langle (\bar{q}\gamma_\mu \lambda^a q)^2 \rangle = \frac{16}{9} \kappa \, \langle \bar{q}q \rangle^2
$$

valid approximation?  $\Rightarrow$  Always  $\kappa > 3$  and large uncertainties.

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### **Summary**

- The sum rules for the lowest two moments of the  $\rho$ -meson spectral function involve perturbative contributions and only leading condensates as small corrections: accuracy both in vacuum and in medium
- $\circ$  Chiral gap scale:  $4\pi f_\pi$  meaningful both in vacuum and in-medium.
- ◎ For broad spectral distributions, "mass shift" vs. "broadening" discussion must be specified in terms of first moment.
- $\circ$  Brown-Rho scaling as a statement involving the lowest two moments in the window of low-mass enhancement.
- <span id="page-30-0"></span>Further step: extension to nonvanishing three-momentum.

### <span id="page-31-0"></span>Thank you for your attention!

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