Symmetry energy of dense matter in holographic QCD

Yunseok Seo

CQUeST

December 10, 2010

Based on

 ${\sf arXive:} 1011.0868: \ {\sf Youngman} \ {\sf Kim, \, YS, \, Ik \, Jae \, Shin, \, Sang-Jin \, Sin}$

JHEP 0804:010, 2008: YS, Sang-Jin Sin

JHEP 1003:074, 2010: Youngman Kim, YS, Sang-Jin Sin



Table of Contents

- Motivation
- AdS/CFT correspondence
- (A)symmetric dense matter
- Symmetry energy
- Conclusion and Discussion

 Nuclear symmetry energy is important for understanding not only the structure of nuclei but also many critical issue in astrophysics and cosmology.

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- One of main research project in KoRIA(Korea Rare Isotope Accelerator).

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- One of main research project in KoRIA(Korea Rare Isotope Accelerator).
- QCD phenomena at finite density is hard to understand.
- AdS/CFT may give clues to understand non-perturbative phenomema.

String theory

Open Strings

Closed Strings

String theory

Open Strings Closed Strings
massless excitation massless excitation

Gauge Filed A_{μ}

massless excitation Graviton $G_{\mu
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String theory

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O P

D-branes

Closed Strings

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Curved spacetime

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D-branes

low energy limit $\mathcal{N}=4$, D=4 SYM

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low energy limit 10d Supergravity

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Large N limit Super conformal Theory Closed Strings

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Near horizon limit $AdS_5 \times S^5$

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D-branes

Curved spacetime

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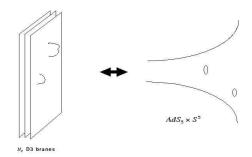
low energy limit 10d Supergravity

Large *N* limit
Super conformal Theory

Near horizon limit $AdS_5 \times S^5$

• There is Open-Closed string duality





- Weak coupling limit ($\lambda << 1$): $\mathcal{N}=$ 4, D= 4, $SU(N_C)$ SYM
- Strong coupling limit ($\lambda >> 1$): Classical gravity in $AdS_5 \times S^5$
- From calculating classical gravity, we can obtain some quantities in gauge theory with strong coupling.



AdS/CFT dictionaryGauge Theory(boundary) | Gravity(bulk)

AdS/CFT dictionary

Gauge Theory(boundary)

Operator $\mathcal O$

Gravity(bulk)

Field ϕ

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mass of field m_ϕ

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u}$

Conformal dimension Δ_{ϕ}

Global symmetry

. . .

Gravity(bulk)

Field ϕ

Graviton $g_{\mu\nu}$

mass of field m_ϕ

Gauge symmetry

•

- But it is too far from realistic QCD !!
 - SUSY, zero temperature, no fundamental field ...

- Toward to QCD
 - Break SUSY
 - D4 brane background with circle compactification

$$ds^{2} = \left(\frac{U}{R}\right)^{3/2} \left(-dt^{2} + d\vec{x}^{2} + dx_{4}^{2}\right) + \left(\frac{R}{U}\right)^{3/2} \left(dU^{2} + r^{2}d\Omega_{4}^{2}\right)$$

$$e^{\phi} = g_s \left(\frac{U}{R}\right)^{3/4}, \quad F_4 = \frac{2\pi N_c}{\Omega_4} \epsilon_4, \quad R^3 = \pi g_s N_c I_s^3$$

By imposing anti-periodic condition to fermion along x_4 direction, we can break supersymmetry

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- Finite temperature
 - Shcwarzschild type solution

$$ds^{2} = \left(\frac{U}{R}\right)^{3/2} \left(-f(U)dt^{2} + d\vec{x}^{2} + dx_{4}^{2}\right) + \left(\frac{R}{U}\right)^{3/2} \left(\frac{dU^{2}}{f(U)} + U^{2}d\Omega_{4}^{2}\right)$$
$$f(U) = 1 - \left(\frac{U_{h}}{U}\right)^{3}, \ T \sim U_{h}$$

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$$\begin{split} ds^2 &= \left(\frac{U}{R}\right)^{3/2} \left(-dt^2 + d\vec{x}^2 + dx_4^2\right) + \left(\frac{R}{U}\right)^{3/2} \left(dU^2 + r^2 d\Omega_4^2\right) \\ e^\phi &= g_s \left(\frac{U}{R}\right)^{3/4}, \quad F_4 = \frac{2\pi N_c}{\Omega_4} \epsilon_4, \quad R^3 = \pi g_s N_c I_s^3 \end{split}$$

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- Confining phase
 - Double Wick rotation

$$t \leftrightarrow ix_4, x_4 \leftrightarrow i\tau$$

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$$f(U) = 1 - \left(\frac{U_{KK}}{U}\right)^3$$



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 - Flavor degrees of freedom
 - Add probe branes

$$y(\rho) \sim m_q + \frac{c}{\rho^\#} + \cdots$$

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 - Source of gauge field
 - endpoint of fundamental strings
 - baryon vertex (spherical D-brane with N_c fundamental strings)

- Basic Setup
 - Confining phase

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 - \bullet Confining phase \leftrightarrow D4 background with double Wick rotation

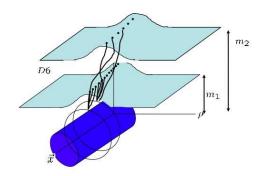
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- Basic Setup
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- DBI action
 - D4 brane(baryon vertex)

$$S_{D4} = -\mu_4 \int e^{-\phi} \sqrt{\det(g + 2\pi \alpha' F)} + \mu_4 \int A_{(1)} \wedge G_{(4)}$$

D6 brane(flavor brane)

$$\mathcal{S}_{D6} = \int dt \mathcal{L}_{D6} = -\mu_6 \int \mathrm{e}^{-\phi} \sqrt{\det(g + 2\pi lpha' F)}$$

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- Legendre transformed 'Hamiltonian'
 - D4 brane(baryon vertex)

$$\mathcal{H}_{D4} = au_4 \int d heta \sqrt{\omega_+^{4/3} (\xi^2 + \xi'^2)} \sqrt{D(heta)^2 + \sin^6 heta}$$

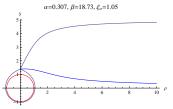
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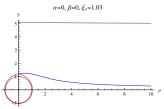
$$\mathcal{H}_{D6}= au_{6}\int d
ho\sqrt{\omega_{+}^{4/3}\left(ilde{Q}^{2}+
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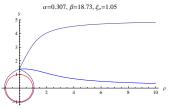
• Solve equation of motion for DBI action with force balance condition

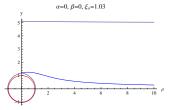
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Solve equation of motion for DBI action with force balance condition

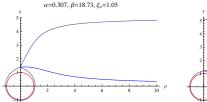


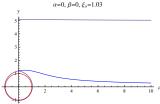


Total free energy

$$E_{tot} = \frac{Q}{N_C} \mathcal{H}_{D4} + \mathcal{H}_{D6}((1-\alpha)Q) + \mathcal{H}_{D6}(\alpha Q)$$

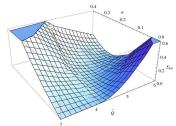
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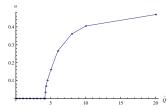


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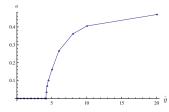
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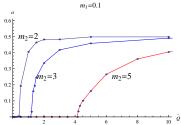
• Transition from nuclear matter to strange matter



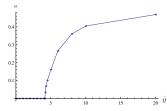
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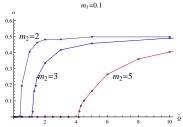
• Different mass ratio



Transition from nuclear matter to strange matter



Different mass ratio



• $m_2/m_1=1$, physical system is lpha=0.5 for any density ightarrow symmetric matter

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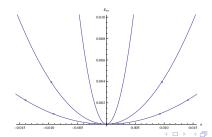
$$E(\rho, \tilde{\alpha}) \sim E(\rho, 0) + S_2(\rho)\tilde{\alpha}^2$$

 $\tilde{\alpha} \equiv (N - Z)/A = 1 - 2\alpha$
 $S_2 \sim \frac{\partial^2 E_{tot}}{\partial \alpha^2}$

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$$E(\rho,\alpha) = E(\rho,0) + \frac{\partial E}{\partial \tilde{\alpha}} \Big|_{\tilde{\alpha}=0} \tilde{\alpha} + \frac{1}{2} \frac{\partial^2 E}{\partial \tilde{\alpha}^2} \Big|_{\tilde{\alpha}=0} \tilde{\alpha}^2 \cdots$$
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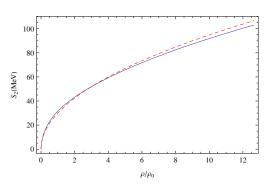
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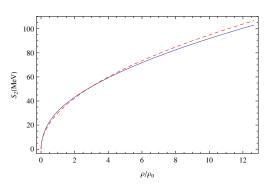
$$E_1 = 0$$

$$E_2 = \frac{2\tau_6}{N_B} \int d\rho \frac{\sqrt{1 + \dot{y}^2} \tilde{Q}^2 \omega_+^{10/3} \rho^4}{(\tilde{Q}^2 + 4\omega_+^{8/3} \rho^4)^{3/2}}$$

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• Analytic calculation($m_q >> 1$, $\tilde{Q} << 1$)

$$\textit{E}_{2} = \left(\Gamma\left(\frac{5}{4}\right)\right)^{2}\sqrt{\frac{\lambda\rho_{0}}{2\textit{M}_{\textit{KK}}}}\sqrt{\frac{\rho}{\rho_{0}}}$$



Physical quantities

$$M_q = \frac{m_{\infty} \lambda M_{KK}}{9\pi}$$

$$\rho \equiv \frac{N_b}{V_3} = \frac{Q}{N_c V_3}$$

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 - Off-diagonal fluctuation spectrum corresponds to mass of 'Kaon'
 - We can study density dependence of mass of Kaon



Thank you !!!