Quark Number Susceptibility in Deformed AdS

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Quark Number Susceptibility

Ref. Larry Mclerran PRD 36 (1987) 3291

If the effective mass of baryons in the quark-gluon plasma is small compared to what it is in the hadron gas, then the baryon number concentrates in the region of quark-gluon plasma.

The baryon number density near zero chemical potential

$$\rho_B = \mu \left(\frac{\partial}{\partial \mu} \rho_B \right)_{\mu=0} \equiv \mu \kappa_B .$$

$$\kappa_B = \beta \int d^3x \langle \rho_B(x) \rho_B(0) \rangle$$

In large N_c limit, $\rho_q = N_c \rho_B$

Quark Number Susceptibility

Ref. Cheng et al. PRD 79, 074505 (2009)

<The pressure>

$$\frac{p}{T^4} \equiv \frac{1}{VT^3} \ln Z(V, T, \mu_B, \mu_S, \mu_Q)$$ = Grand potential

$$\mu_u = \frac{1}{3}\mu_B + \frac{2}{3}\mu_Q \qquad \mu_d = \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q, \qquad \mu_s = \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q - \mu_S.$$

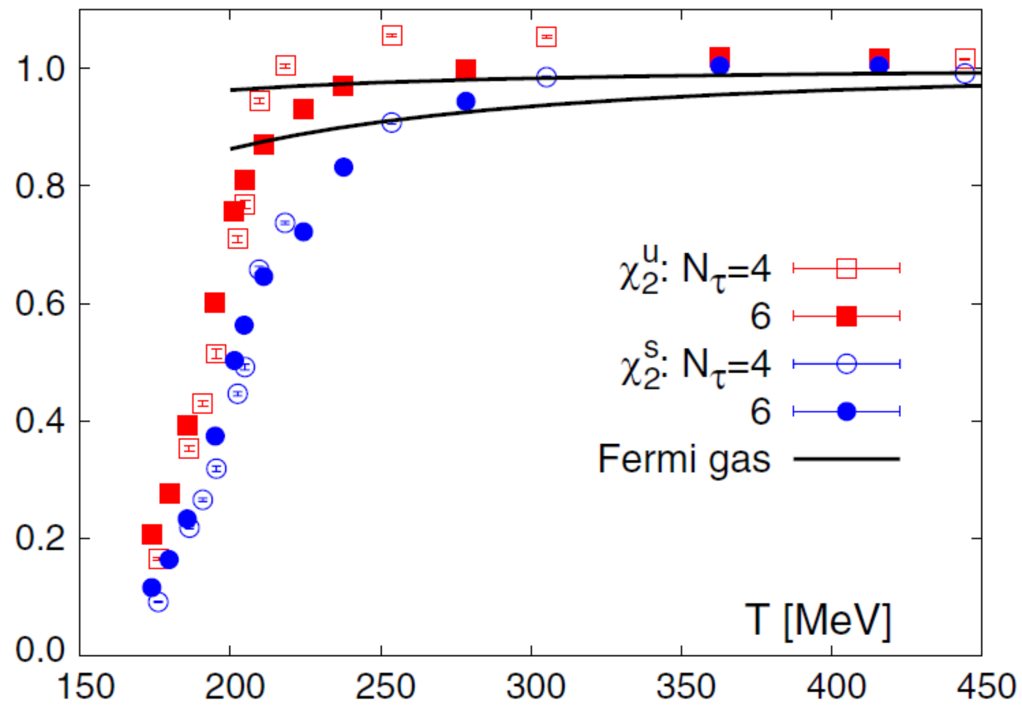
<The definition of susceptibility for various charge>

$$\chi_{ijk}^{BQS} = \frac{\partial^{i+j+k} p/T^4}{\partial \hat{\mu}_B^i \partial \hat{\mu}_Q^j \partial \hat{\mu}_S^k}\Bigg|_{\mu=0} \qquad \hat{\mu}_X \equiv \mu_X/T$$

Under conditions met in RHIC and LHC the net baryon number is small and QCD at vanishing chemical potential provides a good approximation.

Quark Number Susceptibility

Lattice Results



AdS/QCD at finite temperature

Geometry

Confinement phase ⇕ **Thermal AdS** → **Deconfinement phase** ⇕ **AdS Black Hole**

$$ds^2 = \frac{1}{z^2}\left(d\tau^2 + dz^2 + d\vec{x}_3^2\right)$$

The periodicity

$$\beta' = \pi z_h \sqrt{f(\epsilon)}$$

$$ds^2 = \frac{1}{z^2}\left(f(z)d\tau^2 + \frac{dz^2}{f(z)} + d\vec{x}_3^2\right)$$

The Hawking temperature

$$T = 1/(\pi z_h)$$

↓ The black hole horizon

The Euclidean gravitational action

$$S_{grav} = -\frac{1}{2\kappa^2}\int d^5x\sqrt{g}\,(\mathrm{R}+12)$$

Hydrodynamics

The long-distance, low-frequency behavior of any interacting theory at finite temperature must be described by hydrodynamics.

<The retarded thermal Green's function>

$$G^R_{\mu\nu}(\omega,\boldsymbol{q}) = -i\int d^4x\, e^{-iq\cdot x}\,\theta(t)\langle[j_\mu(x),\, j_\nu(0)]\rangle$$

where, $q = (\omega, \boldsymbol{q}),\ x = (t, \boldsymbol{x}).$

- This function determines the response of the system on a small external source coupled to the current.
- When omega and q are small, the external perturbation varies slowly in space and time, and a macroscopic hydrodynamic description for its evolution is possible.

Hydrodynamics in AdS/QCD

<The gravity/gauge theory duality>

$$\langle e^{\int_{\partial M} \phi_0 \hat{O}} \rangle = e^{-S_{\rm cl}[\phi_0]}$$

<The method to obtain the retarded Green's function>

i) From classical action,

$$S_{\rm cl} = \frac{1}{2} \int du\, d^4x\, A(u) (\partial_u \phi)^2 + \cdots$$

ii) With incoming-wave boundary condition at horizon,

$$\phi(u,q) = f_q(u) \phi_0(q)$$ $f_q(u)$ =1 at u=0 by definition of mode eq.

iii) The retarded thermal Green's function is

$$G^R(q) = A(u) f_{-q}(u) \partial_u f_q(u)|_{u \to 0} \,.$$

Hydrodynamics in AdS/QCD

Ref. G.Policastro, D.T.Son and O.Starinets JHEP 09 (2002) 043

How do we calculate the retarded Green's function in AdS?

$$G^R(\omega, \boldsymbol{q}) = -i \int d^4x\, e^{-iq\cdot x}\, \theta(t)\langle[\hat{\mathcal{O}}(x),\, \hat{\mathcal{O}}(0)]\rangle$$

<The metric and action>

$$ds^2 = \frac{1}{u^2}(-f(u)dt^2 + dx^2 dy^2 + dz^2) + \frac{1}{4u^2 f(u)}du^2 \qquad u = z/z_h$$

$$S = -\frac{1}{4g_{SG}^2}\int d^5x \sqrt{-g}\, F^a_{\mu\nu}F^{\mu\nu\, a}$$

<The equation of motion for gauge field>

$$\frac{1}{\sqrt{-g}}\partial_\nu[\sqrt{-g}g^{\mu\rho}g^{\nu\sigma}(\partial_\rho A_\sigma - \partial_\sigma A_\rho)] = 0$$

Hydrodynamics in AdS/QCD

<The Fourier decomposition>

$$A_i = \int \frac{d^4q}{(2\pi)^4} e^{-i\omega t + i\boldsymbol{q}\cdot\boldsymbol{x}} A_i(q,u)$$

We choose, $q = (\omega, 0, 0, q)$

dimensionless quantities, $\mathfrak{w} = \frac{\omega}{2\pi T}\,, \qquad \mathfrak{q} = \frac{q}{2\pi T}$

The e.o.m can be written as,

$$\mathfrak{w} A_t' + \mathfrak{q} f\, A_z' = 0\,,$$

$$A_t'' - \frac{1}{uf}\left(\mathfrak{q}^2 A_t + \mathfrak{w}\mathfrak{q} A_z\right) = 0\,,$$

$$A_z'' + \frac{f'}{f} A_z' + \frac{1}{uf^2}\left(\mathfrak{w}^2 A_z + \mathfrak{w}\mathfrak{q} A_t\right) = 0\,,$$

$$A_\alpha'' + \frac{f'}{f} A_\alpha' + \frac{1}{uf}\left(\frac{\mathfrak{w}^2}{f} - \mathfrak{q}^2\right) A_\alpha = 0\,,$$

Hydrodynamics in AdS/QCD

The equation for time component

$$A_t''' + \frac{(uf)'}{uf} A_t'' + \frac{\mathfrak{w}^2 - \mathfrak{q}^2 f(u)}{uf^2} A_t' = 0$$

We substitute, $A_t' = (1-u)^{\nu} F(u)$

then, we obtain with incoming-wave condition, $\nu_- = -i\mathfrak{w}/2$

The equation for F(u)

$$F'' + \left(\frac{1-3u^2}{uf} + \frac{i\mathfrak{w}}{1-u}\right) F' + \frac{i\mathfrak{w}(1+2u)}{2uf} F + \frac{\mathfrak{w}^2[4-u(1+u)^2]}{4uf^2} F - \frac{\mathfrak{q}^2}{uf} F = 0 .$$

$$\Rightarrow \quad F(u) = F_0 + \mathfrak{w} F_1 + \mathfrak{q}^2 G_1 + \mathfrak{w}^2 F_2 + \mathfrak{w}\mathfrak{q}^2 H_1 + \mathfrak{q}^4 G_2 + \cdots .$$

Deformed AdS Space

Ref. Y.Kim, T.Misumi, and I.j.Shin [arxiv 0911.3205]

5D action for scalar field

$$S = \int d^5x\sqrt{g}\,[-\frac{1}{2\kappa^2}R + \mathrm{Tr}(\partial_M X\partial^M X - V(X))]$$

Scalar field solution for

- Thermal AdS : $X = X_0(z) = (\mathbf{1}_{N_f\times N_f}/2)(m_q z + \sigma z^3)$
- AdS BH : $X = X_0(z) = (\mathbf{1}_{N_f\times N_f}/2)m_q z$

σ=0 is assumed in AdS BH since this background corresponds to high temperature deconfined phase.

Deformed AdS Space

The deformed AdS Black Hole

<The back-reacted metric>

$$ds^2 = e^{-2K(z)}(f(z)dt^2 - d\vec{x}^2) - dz^2/(z^2 f(z))$$

where, $$K(z) = \ln z + \frac{1}{4}\left(\frac{z}{z_Q}\right)^2 \qquad z_Q^2 \equiv 6/(\kappa^2 N_f m_q^2)$$

$$f(z) = 1 - 2\left(\frac{z}{z_Q}\right)^4 \left[1 - \left(1 - \left(\frac{z}{z_Q}\right)^2\right) e^{(z/z_Q)^2}\right]$$

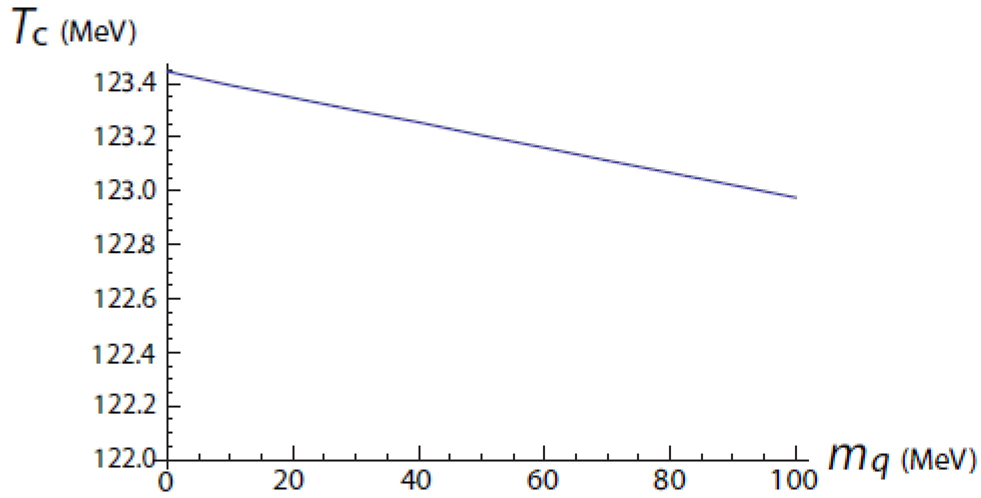
<The Hawking temperature for black-hole>

$$\Rightarrow \quad T = \frac{1}{\pi z_H}\left(\frac{z_H}{z_h}\right)^4 e^{\frac{3}{4}\left(\frac{z_H}{z_Q}\right)^2}$$

where, $$z_H = z_Q\sqrt{1 + \mathrm{ProductLog}\left[\frac{(z_h/z_Q)^4 - 2}{2e}\right]}$$

Deformed AdS Space

<Change of critical temperature by quark mass>



➡ *If we consider only the effect of back-reacted quark mass, the change of critical temperature is small.*
(Here the change of scale is not considered.)

Deformed AdS Space

< A dimensionless coordinate>

$u \equiv (z/z_H)^2$: u=1 is horizon and u=0 is boundary.

The metric rewritten as

$$(\mathrm{d}s)^2 = \mathrm{e}^{-2K(u)}\Big(- f(u)(\mathrm{d}t)^2 + (\mathrm{d}\vec{x})^2\Big) + \frac{(\mathrm{d}u)^2}{4u^2 f(u)}$$

where, $K(u) = \frac{1}{2}\log(z_H^2 u) + \frac{1}{4}\left(\frac{z_H}{z_Q}\right)^2 u$

$$f(u) = 1 - 2\left(\frac{z_Q}{z_h}\right)^4 \left\{1 - \left(1 - \left(\frac{z_H}{z_Q}\right)^2 u\right) \mathrm{e}^{\left(\frac{z_H}{z_Q}\right)^2 u}\right\}$$

Quark Number Susceptibility

<The 5D action for U(1) gauge field>

$$S = -\frac{1}{4g_5^2}\int \mathrm{d}^5x\sqrt{-g}F_{mn}F^{mn}$$ with gauge choice, $A_u(x) = 0$

↳ 5D gauge coupling constant

The Fourier decomposition

$$A_\mu(t,z,u) = \int \frac{\mathrm{d}^4k}{(2\pi)^4}\, \mathrm{e}^{-i\omega t + ikz} A_\mu(\omega,k,u)$$

<The equation of motion for A_t>

$$0 = \left(\mathrm{e}^{-\frac{1}{2}\left(\frac{z_H}{z_Q}\right)^2 u} A_t'(u)\right)' - \frac{z_H^2}{4uf(u)}\left(k^2 A_t(u) + \omega k A_z(u)\right)$$

$$0 = \omega A_t'(u) + kf(u)A_z'(u),$$

Quark Number Susceptibility

<The equation of motion for time component>

$$0 = \left(uf(u)\ \mathrm{e}^{-\left(\frac{z_H}{z_Q}\right)^2 u} A_t''(u)\right)'$$

$$-\left\{\frac{1}{2}\left(\frac{z_H}{z_Q}\right)^2 \mathrm{e}^{-\frac{1}{2}\left(\frac{z_H}{z_Q}\right)^2 u}\left(uf(u)\ \mathrm{e}^{-\frac{1}{2}\left(\frac{z_H}{z_Q}\right)^2 u}\right)' - \frac{z_H^2}{4}\ \mathrm{e}^{-\frac{1}{2}\left(\frac{z_H}{z_Q}\right)^2 u}\left(\frac{\omega^2}{f(u)} - k^2\right)\right\} A_t'(u)$$

We substitue, $A_t'(u) = \mathrm{e}^{\frac{1}{2}\left(\frac{z_H}{z_Q}\right)^2 u} X(u)$

$$\Rightarrow \quad 0 = \left(uf(u)X'(u)\right)' + \frac{z_H^2}{4}\ \mathrm{e}^{\frac{1}{2}\left(\frac{z_H}{z_Q}\right)^2 u}\left(\frac{\omega^2}{f(u)} - k^2\right) X(u).$$

A solution as,

$$X(u) = (1-u)^{\nu} F(u) \qquad \nu = -i\frac{\omega}{4\pi T}$$

Quark Number Susceptibility

The small parameter expansion of F(u)

$$F(u) = F_0(u) + \omega F_\omega(u) + k^2 F_{k^2}(u) + \mathcal{O}(\omega^2, \omega k^2)$$

$$0 = \Big(uf(u)F_0'(u)\Big)'$$

$$0 = \Big(uf(u)F_{k^2}'(u)\Big)' - \frac{z_H^2}{4}\, \mathrm{e}^{\frac{1}{2}\left(\frac{z_H}{z_Q}\right)^2 u} F_0(u)$$

The solution is obtained as following,

$$F_0(u) = C.$$

$$F_{k^2}'(u) = C\frac{z_H^2}{4uf(u)}\int_1^u \mathrm{d}u'\; \mathrm{e}^{\frac{1}{2}\left(\frac{z_H}{z_Q}\right)^2 u'} = C\frac{z_H^2}{4uf(u)} 2\left(\frac{z_Q}{z_H}\right)^2 \left(\mathrm{e}^{\frac{1}{2}\left(\frac{z_H}{z_Q}\right)^2 u} - \mathrm{e}^{\frac{1}{2}\left(\frac{z_H}{z_Q}\right)^2}\right)$$

Quark Number Susceptibility

The solution of A_t

$$A_t(u) = 2C\left(\frac{z_Q}{z_H}\right)^2\left(\mathrm{e}^{\frac{1}{2}\left(\frac{z_H}{z_Q}\right)^2 u} - \mathrm{e}^{\frac{1}{2}\left(\frac{z_H}{z_Q}\right)^2}\right)$$

$$= \frac{A_t^0}{1-\mathrm{e}^{\frac{1}{2}\left(\frac{z_H}{z_Q}\right)^2}}\left(\mathrm{e}^{\frac{1}{2}\left(\frac{z_H}{z_Q}\right)^2 u} - \mathrm{e}^{\frac{1}{2}\left(\frac{z_H}{z_Q}\right)^2}\right)$$

<The quark susceptibility from the on-shell action>

$$S = \frac{1}{g_5^2 z_H^2}\int \frac{\mathrm{d}^4 k}{(2\pi)^4}\ \mathrm{e}^{-\frac{1}{2}\left(\frac{z_H}{z_Q}\right)^2 u} A_t(-k)A_t'(k)\Big|_{u=1}^{u=0}.$$

$$\chi_q = \frac{1}{g_5^2 z_Q^2\left(\mathrm{e}^{\frac{1}{2}\left(\frac{z_H}{z_Q}\right)^2} - 1\right)}$$

Quark Number Susceptibility

Thermodynamic analysis

<The grand potential>

$$\Omega = T S_{\text{on-shell}}$$

The 5D bulk action on-shell action in Euclidean spacetime

Solution of time-component of gauge field, $A_t'(u) = -d\ \mathrm{e}^{\frac{1}{2}\left(\frac{z_H}{z_Q}\right)^2 u}$

$$\Omega = \frac{d^2 V_3}{g_5^2 z_H^2} \int_1^0 \mathrm{d}u\ \mathrm{e}^{\frac{1}{2}\left(\frac{z_H}{z_Q}\right)^2 u} = \frac{2 d^2 V_3 z_Q^2}{g_5^2 z_H^4} \left(1 - \mathrm{e}^{\frac{1}{2}\left(\frac{z_H}{z_Q}\right)^2}\right),$$

when chemical potential is defined as,

$$\mu = \int_1^0 \mathrm{d}u\ A_t'(u) = -\frac{2 d z_Q^2}{z_H^2} \left(1 - \mathrm{e}^{\frac{1}{2}\left(\frac{z_H}{z_Q}\right)^2}\right).$$

Quark Number Susceptibility

<The grand potential>

$$\Omega = T \frac{V_3 \mu^2}{2 g_5^2 z_Q^2 \left(1 - e^{\frac{1}{2}\left(\frac{z_H}{z_Q}\right)^2}\right)}$$

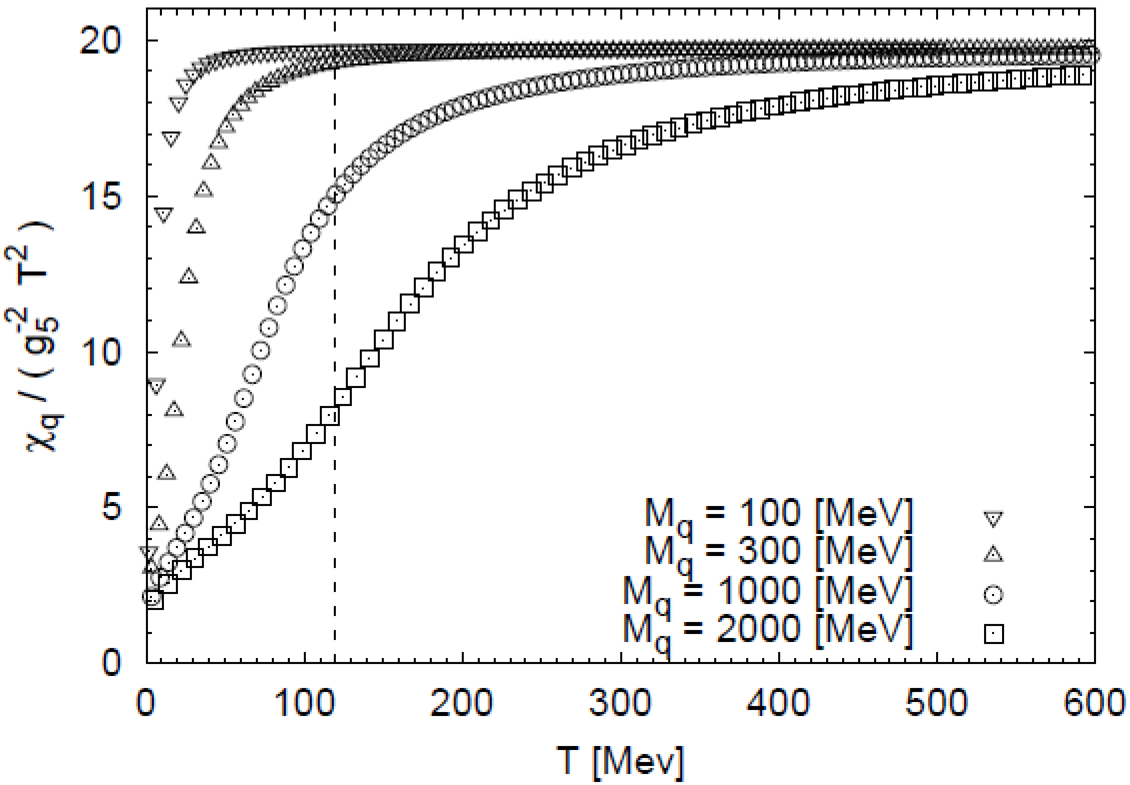
<The quark number susceptibility>

$$\chi_q = \frac{1}{T\, V_3} \frac{\partial^2 \Omega}{\partial \mu^2}$$

$$\Rightarrow \quad \boxed{\chi_q = \frac{1}{g_5^2 z_Q^2 \left(\mathrm{e}^{\frac{1}{2}\left(\frac{z_H}{z_Q}\right)^2} - 1\right)}}$$

Quark Number Susceptibility

<The quark number susceptibility in deformed AdS>



Summary and Outlook

- The quark number susceptibility has a feature that it is suppressed by quark mass.

- Using hydrodynamic and thermodynamic approaches to AdS/QCD, we can obtain same result in quark number susceptibility.

- Using D3/D7 model, in top-down approach we can obtain quark number susceptibility.

Thank you!