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Kps Meeting

Two point function, hydrodynamics and spectral function

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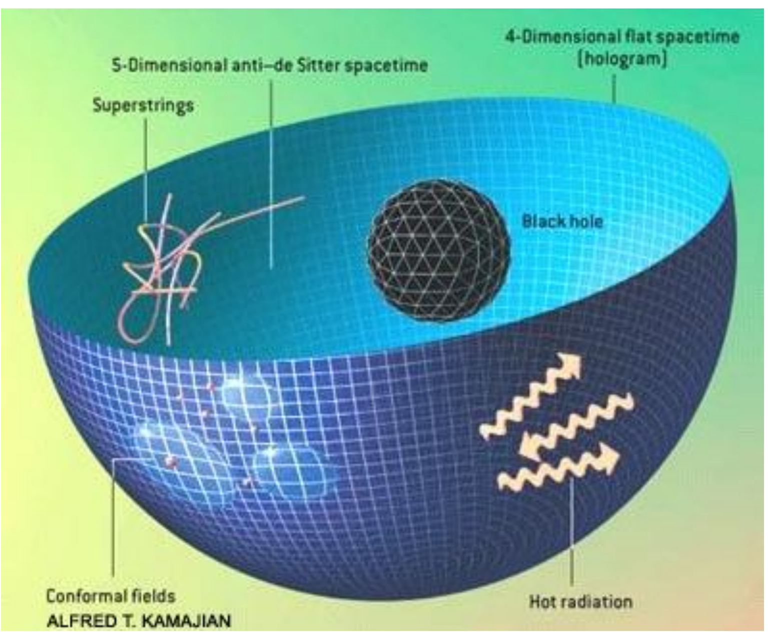
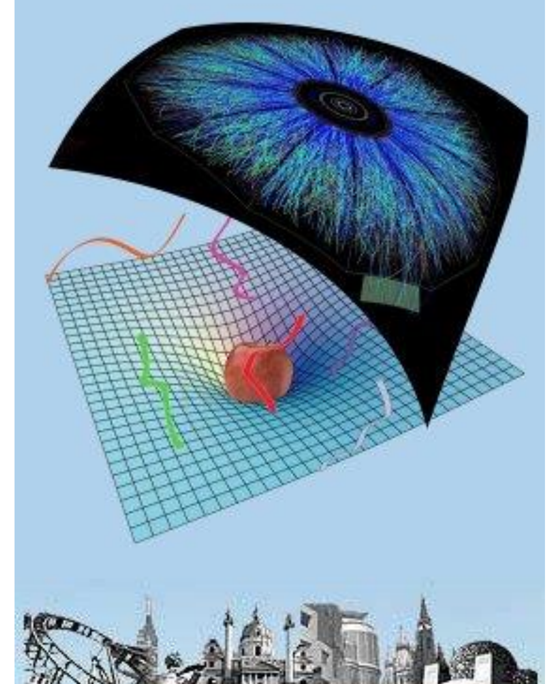
AdS/QCD

- AdS/CFT : AdS_5 X S^5 ~ N=4 SYM in 4 dim.
- AdS/QCD :
  $aAdS_d$ ~ low energy Yang-Mills in d-1 dim.





Phase transition in AdS/QCD

T	Field theory side	Gravity side
T<Tc	Hadron	Thermal AdS
Tc	Confinement/deconfinement phase transition.	Hawking – Page transtition
T>Tc	Quarks and Gluons	AdS B.H.

Assume that confinement and chiral phase transition occurs at same scale, Tc.

T	Field theory side	Gravity side
T<Tc	Vev is not zero	Thermal AdS
Tc	Chiral symmetry breaking/restoration phase transition	Hawking – Page transtition
T>Tc	Vev is zero	AdS B.H.

SU(2)L X SU(2)R flavor sym. -> SU(2)V

Two point function

Microscopic theory

(E > Λ_c) : Yang-Mills

perturbative

Non perturbative

Z_{gauge}

Generating functional

Intermediate energy regime

$$\frac{\delta^2 Z_{gauge}}{\delta\varphi_i \delta\varphi_j} = G_{ij}$$

Two point function

Low energy effective theory

(L > l_s) : Einstein gravity

~~perturbative~~

Non perturbative

Z_{string}

Generating functional

Computable regime

$$\frac{\delta^2 Z_{string}}{\delta\varphi_i \delta\varphi_j} = G_{ij}$$

Two point function

Z_{gauge}

Generating functional

AdS/CFT

$=$

Non perturbative

Z_{string}

Generating functional

Strongly coupled gauge theory

weakly coupled gravity theory

$$\frac{\delta^2 Z_{gauge}}{\delta\varphi_i\delta\varphi_j} = G_{g,ij}$$

$=$

$$\frac{\delta^2 Z_{string}}{\delta\varphi_i\delta\varphi_j} = G_{s,ij}$$

How to compute Green function

Find : $g_{ij}^{(0)}, A_t^{(0)}$ / Perturb : $g_{ij}^{(0)}+\delta g_{ij}$, $A_t^{(0)}+\delta A_t$

Find and solve the equations of motion for $\delta g_{ij}, \delta A_t$

plug it into the original action : Seff

Differentiate Seff with respect to the source field

Correlators

th/0205051

Holography : Bulk(d+1 dim) projected onto the Boundary(d dim)

$$e^{-I_{SUGRA}} \simeq Z_{\text{string}} = Z_{\text{gauge}} = e^{-W}$$

AdS/CFT says,
on-shell partition function of Gravity(Bulk)
= CFT partition function(Boundary)

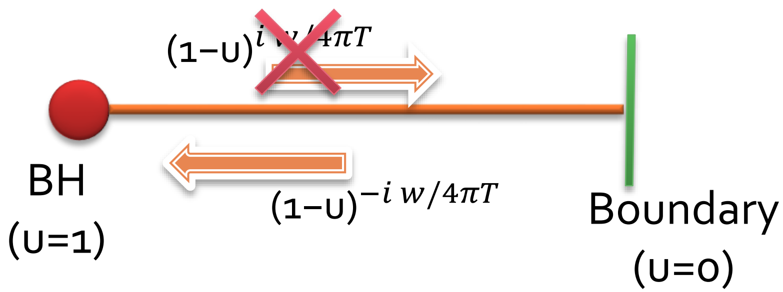
solving Bulk e.o.m. and integrating extra dim of bulk action : Boundary action (On-shell action).

$$S_{bdry}[\phi_0] = \int \frac{d^4k}{(2\pi)^4} \phi_0(-k)\mathcal{G}(k,u)\phi_0(k)\Big|_{u=0}^{u=1}$$

$$G^R_{\mu\nu}(k) = \frac{\delta^2 S}{\delta A^0_\mu(-k)\delta A^0_\nu(k)}$$

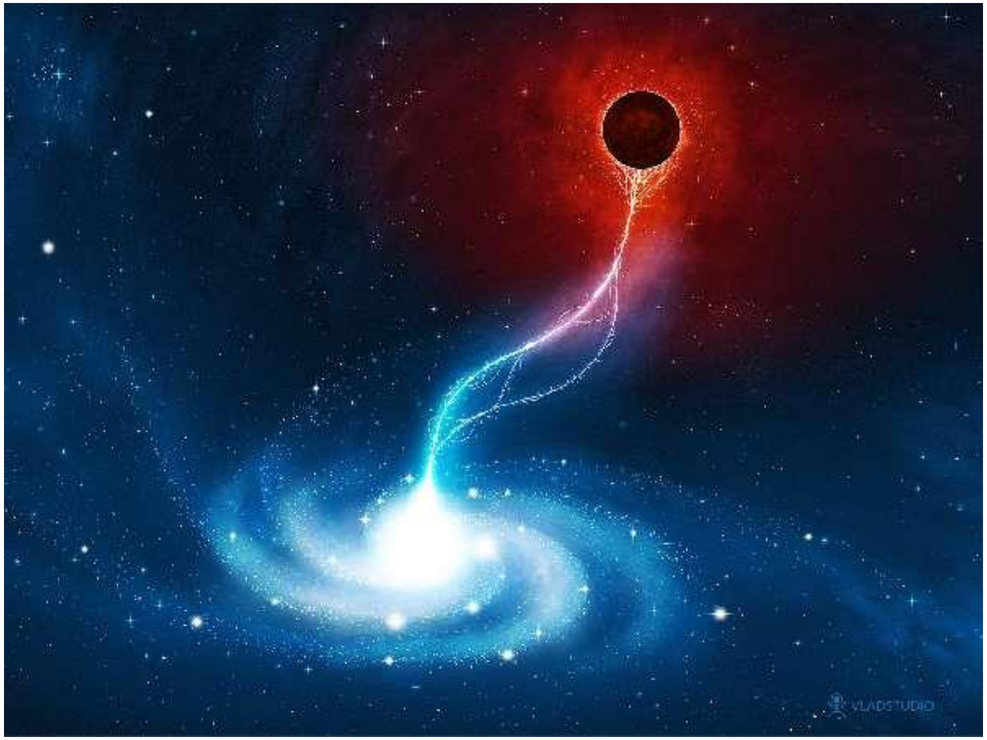
Boundary conditions

- 2^{nd} order linear differential e.o.m.,
  - two integration constants(boundary conditions)



At the horizon : $\phi = (1-u)^{-i\,w/4\pi T}$
At the boundary : $\phi = \phi_0$

Black hole eat everything!



Bottom up approach

Five dimensional action : Einstein + Maxwell + cosmological constant

$$S = \frac{1}{2G_5^2}\int d^5x\sqrt{-g}(\mathcal{R} - 2\Lambda) + \frac{1}{4g_5^2}\int d^5x\sqrt{-g}\; F^2$$

Hadron physics

meson, baryon
π, ρ, ψ etc…

$< \bar{\psi}\psi > \neq 0$
$< TrG^2 > \neq 0$
μ_B, T$\neq$ 0

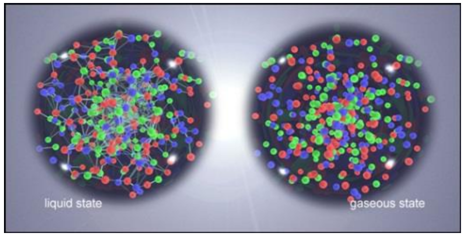
Fluctuation

Non trivial background

hQCD

meson, baryon
δg_{ij}, δA_t, ψ, etc…

$g_{ij} \neq \eta_{ij}$
$A_t \neq 0$



But T$\neq$ 0,There is no hadrons in hQCD but quark-gluon plasma.

Recipe for Green function

EOM for perturbed fields : $\delta g_{ij}, \delta A_t$

$$E''_\alpha(u) + P(\mathfrak{w}, \mathfrak{q}, u)E'_\alpha(u) + Q(\mathfrak{w}, \mathfrak{q}, u)E_\alpha(u) = 0$$

$$S_{bdry}[\phi_0] = \int \frac{d^4k}{(2\pi)^4}\phi_0(-k)\mathcal{G}(k,u)\phi_0(k)\Big|_{u=0}^{u=1}$$

On-shell action and Green function

$$\begin{aligned} G^R_{ij}(K) &= -2\mathcal{G}_{ij}(K, u=0) \quad i=j \\ &= -\mathcal{G}_{ij}(K, u=0) \quad i \neq j \end{aligned}$$

Hydrodynamics

- Hydrodynamics describes the system at large distance and time scale : (small w, k limit).
- Basic equation : conservation of energy momentum

$$\partial_\mu T^{\mu\nu} = 0$$

$$T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu + P g^{\mu\nu} - \sigma^{\mu\nu}$$

$$\sigma^{\mu\nu} = P^{\mu\alpha}P^{\nu\beta}\left[\eta\left(\partial_\alpha u_\beta + \partial_\beta u_\alpha - \frac{2}{3}g_{\alpha\beta}\partial_\lambda u^\lambda\right) + \zeta g_{\alpha\beta}\partial_\lambda u^\lambda\right]$$

$$P^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu$$

Hydrodynamics

- Transport coefficient : viscosity, thermalization time, vorticity etc.

$$\omega = c_s q - i\Gamma q^2 + \frac{\Gamma}{c_s}\left(c_s^2 \tau^{eff} - \frac{\Gamma}{2}\right) q^3 + \mathcal{O}(q^4) \qquad \text{where} \qquad \Gamma = \frac{\eta}{sT}\left(\frac{2}{3} + \frac{\zeta}{2\eta}\right)$$

Hydrodynamics

- Transport coefficient : viscosity, thermalization time, vorticity etc.

$$E''_\alpha(u) + P(\mathfrak{w}, \mathfrak{q}, u)E'_\alpha(u) + Q(\mathfrak{w}, \mathfrak{q}, u)E_\alpha(u) = 0$$

$$E_\alpha(u) = (1-u)^{-i\frac{w}{4\pi T}} F(u, w, q)$$

$$F(u, w, q) = F^{(0)}(u) + w\, F^{(1,0)}(u) + q^2 F^{(0,2)}(u) + \mathrm{O}(w,q)$$

Taylor expansion of the equation of motion in w, q near 0.
The solution of each order in w, q is the $F^{(0)}(u)$, $F^{(i,j)}(u)$.

Hydrodynamics: fundamental d.o.f. = densities of conserved charges

Need to add constitutive relations!

Example: charge diffusion

Conservation law $$\partial_t j^0 + \partial_i j^i = 0$$

Constitutive relation

[Fick's law (1855)] $$j_i = -D\,\partial_i j^0 + O[(\nabla j^0)^2, \nabla^2 j^0]$$

Diffusion equation $$\partial_t j^0 = D\nabla^2 j^0$$

Dispersion relation $$\omega = -i\,D\,q^2 + \cdots$$

Expansion parameters: $\omega \ll T\,, \quad q \ll T$

Transport coefficients in N=4 SYM

in the limit $N_c \to \infty\,, \quad g_{YM}^2 N_c \to \infty$

- Shear viscosity $\eta = \frac{\pi}{8} N_c^2 T^3 \left[1 + O\left(\frac{1}{(g^2 N_c)^{3/2}}, \frac{1}{N_c^2}\right)\right]$
- Bulk viscosity $\zeta = 0$
- Charge diffusion constant $D_R = \frac{1}{2\pi T} + \cdots$
- Thermal conductivity $\frac{\kappa_T\, \mu^2}{\eta\, T} = 8\pi^2 + \cdots$
- Electrical conductivity $\sigma = e^2 \frac{N_c^2\, T}{16\,\pi} + \cdots$

Viscosity/entropy ratio in QCD: current status

Theories with gravity duals in the regime where the dual gravity description is valid

$$\frac{\eta}{s} = \frac{1}{4\pi} \approx 0.08$$

(universal limit)

Kovtun, Son & A.S; Buchel; Buchel & Liu, A.S

QCD: RHIC elliptic flow analysis suggests

$$0 < \frac{\eta}{s} < 0.2$$

QCD: (Indirect) LQCD simulations

$$0.08 < \frac{\eta}{s} < 0.16$$

H.Meyer, 0805.4567 [hep-th]

$$1.2\, T_c < T < 1.7\, T_c$$

Trapped strongly correlated cold alkali atoms $^{6}\mathrm{Li}\,,\ ^{40}K$

$$\left(\frac{\eta}{s}\right)_{\mathrm{min}} \approx 0.5$$

T.Schafer, 0808.0734 [nucl-th]

Liquid Helium-3

$$\left(\frac{\eta}{s}\right)_{\mathrm{min}} \approx 0.7$$

Hydrodynamics, ex1)

Black D3 metric and T_{xy} mode ($\phi = \delta g_{xy}$)

EOM

$$\phi_p'' - \frac{1+u^2}{uf}\phi_p' + \frac{w^2 - q^2 f}{uf^2}\phi_p = 0$$

Eta/s

Solution

$$f_p(z) = (1-u^2)^{-iw/2} + O(w^2, q^2)$$

Greens function

$$G^R_{xy,xy}(\omega, k) = -\frac{\pi^2 N^2 T^4}{4} iw$$

Kubo formular

$$\eta = -\lim_{\omega \to 0} \frac{1}{\omega} \mathrm{Im}\, G^R_{xy,xy}(\omega, 0)$$

$$\eta = \frac{\pi}{8} N^2 T^3$$

Shear viscosity

Hydrodynamics, ex2)

Black D3 metric and R charge current δA_0 mode

EOM

$$A_0''' + \frac{(uf)'}{uf}A_0'' + \frac{w^2 - q^2 f}{uf^2}A_0' = 0 \qquad C = \left.\frac{q^2 A_0 + wqA_3}{iw - q^2}\right|_{u=0}$$

Solution

$$A_0' = C(1-u)^{-iw/2}\left(1 + \frac{iw}{2}\ln\frac{2u^2}{1+u} + q^2\ln\frac{1+u}{2u}\right)$$

Greens function

$$\langle J_0 J_0\rangle_p = \frac{N^2 T}{16\pi}\frac{k^2}{i\omega - Dk^2} \qquad D = \frac{1}{2\pi T}$$

R-charge Diffusion constant

Hydrodynamics, ex3)

Black D3 metric and T_{01} mode, $a_0 = \delta g_{01}$

EOM

$$a_0''' - \frac{2u}{f} a_0'' + \frac{2uf - q^2 f + w^2}{uf^2} a_0' = 0$$

Solution

$$a_0' = C(1-u)^{-iw/2}\left[u - iw\left(1 - u - \frac{u}{2}\ln\frac{1+u}{2}\right) + \frac{q^2}{2}(1-u)\right]$$

Greens function

$$G_{tx,tx}(\omega, k) = \frac{\xi k^2}{i\omega - \mathcal{D}k^2}$$

$$\xi = \frac{\pi}{8} N^2 T^3, \qquad \mathcal{D} = \frac{1}{4\pi T}$$

Bulk viscosity

Momentum diffusion constant

$$\mathcal{D} = \frac{\eta}{\epsilon + P}$$

Spectral function

- Imaginary part of Green function.

$$G = \frac{1}{p - m - i\,\Gamma} = \frac{p - m + i\,\Gamma}{(p-m)^2 + \Gamma^2}$$

$$\chi = Im\,G = \frac{\Gamma}{(p-m)^2 + \Gamma^2}$$

Maximum value $1/\Gamma$ at $p=m$

Correlator (Finite temperature)

N-point function = differentiating N times partition function with respect to the source

Local series solutions : two indicial solutions
(2^{nd} order differential equations)

Near horizon

$$\phi_1 = (1-u)^{-i\mathfrak{w}/2}(1+\cdots)$$
$$\phi_2 = (1-u)^{i\mathfrak{w}/2}(1+\cdots).$$

Near boundary

$$\Phi_1 = u^{\Delta_-}(1+\cdots)$$
$$\Phi_2 = u^{\Delta_+}(1+\cdots)$$

Infalling condition $\quad \phi_1(u) = \mathcal{A}(\mathfrak{w},\mathfrak{q})\Phi_1(u) + \mathcal{B}(\mathfrak{w},\mathfrak{q})\Phi_2(u)$

$$S_{\text{on-shell}} \sim \int d^d x \mathcal{A}^2 \left[\Delta_- u^{\Delta_- - \Delta_+} + (\Delta_+ + \Delta_-)\frac{\mathcal{B}}{\mathcal{A}}\right]_{u\to 0}$$

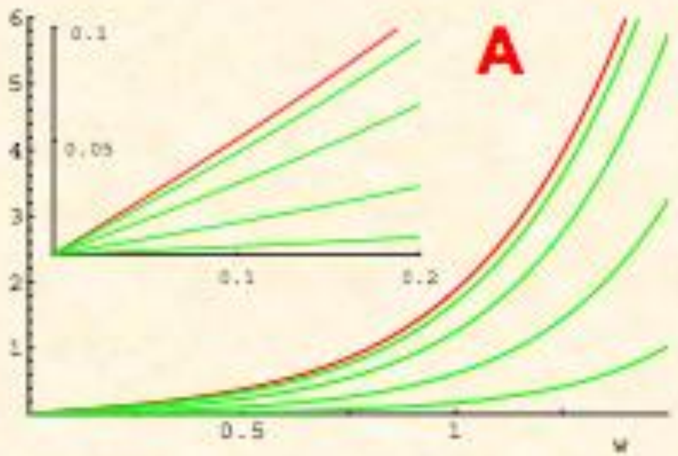
$$\frac{\mathcal{B}}{\mathcal{A}} = \frac{1}{\Phi_2(u)}\left(\frac{\phi_1(u)}{\phi_1(0)} - \Phi_1(u)\right)$$

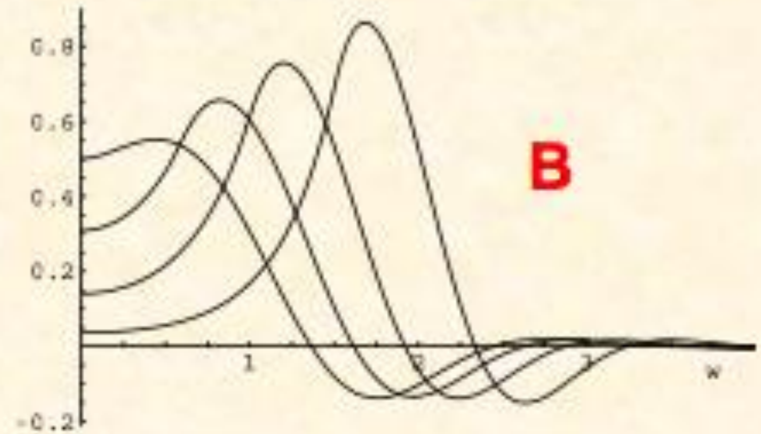
$$\text{Im}\left(\frac{\phi_1(u)}{\phi_1(u=0)}\right) = \text{Im}\left(\mathcal{B}\frac{\Phi_2(u)}{\phi_1(u=0)}\right), \quad \text{where } \mathcal{A} = \phi_1(u=0)$$

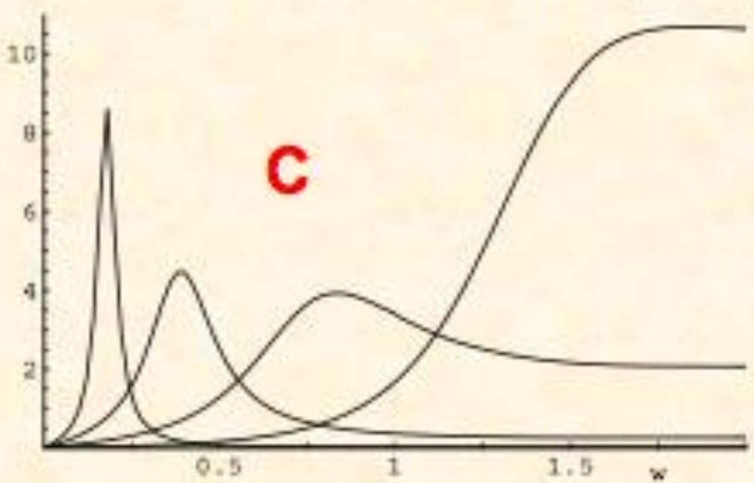
$$\chi = \text{Im}G^{\text{ret}} = \text{Im}\frac{\mathcal{B}}{\mathcal{A}}$$

Spectral function and quasiparticles

$$\chi_{\mu\nu,\alpha\beta}(k) = \int d^4x\, e^{-ikx} \langle \left[T_{\mu\nu}(x) T_{\alpha\beta}(0) \right] \rangle = -2\,\mathrm{Im}\, G^R_{\mu\nu,\alpha\beta}(\omega, q)$$







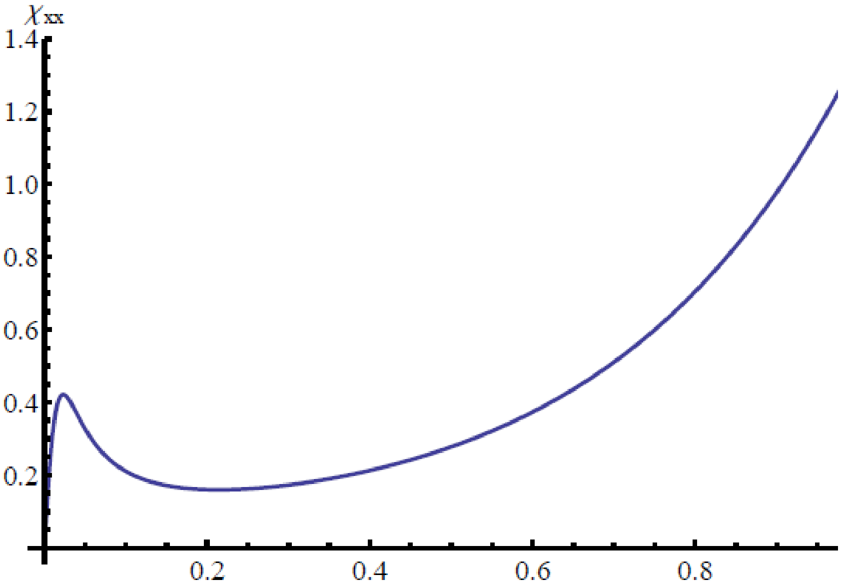
A: scalar channel

B: scalar channel - thermal part

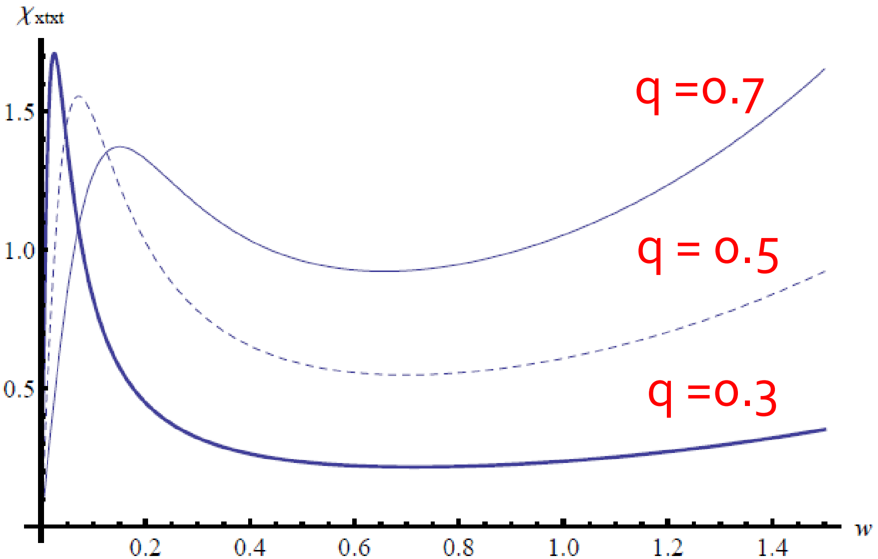
C: sound channel

Spectral function (Im G)

Im G_{xx}



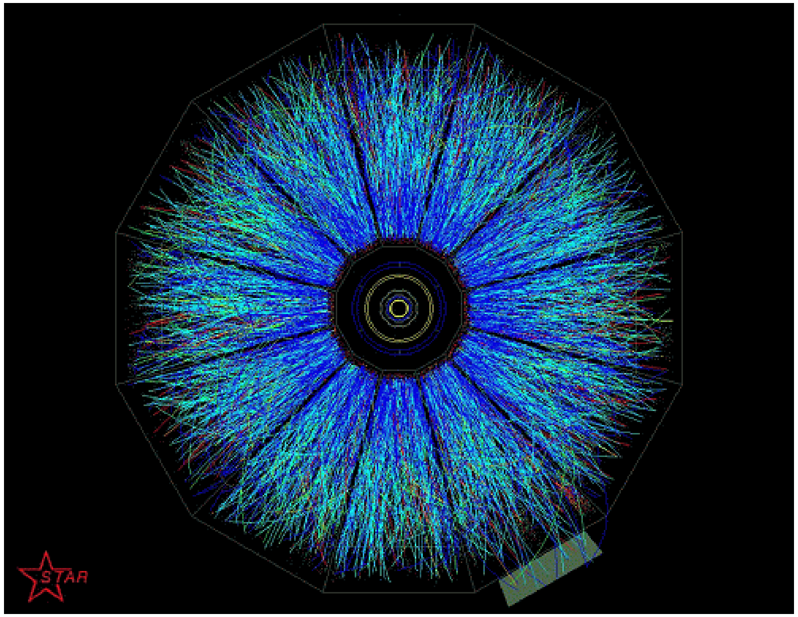
Im G_{xtxt}



$$\chi_{xx}^{hydro} = \frac{2l}{8e^2b^2}\left[\frac{3a}{1+a}\frac{Dk^2w}{D^2k^4+w^2} + \frac{2(1-a/2)^2}{(1+a)^2}bw\right]$$

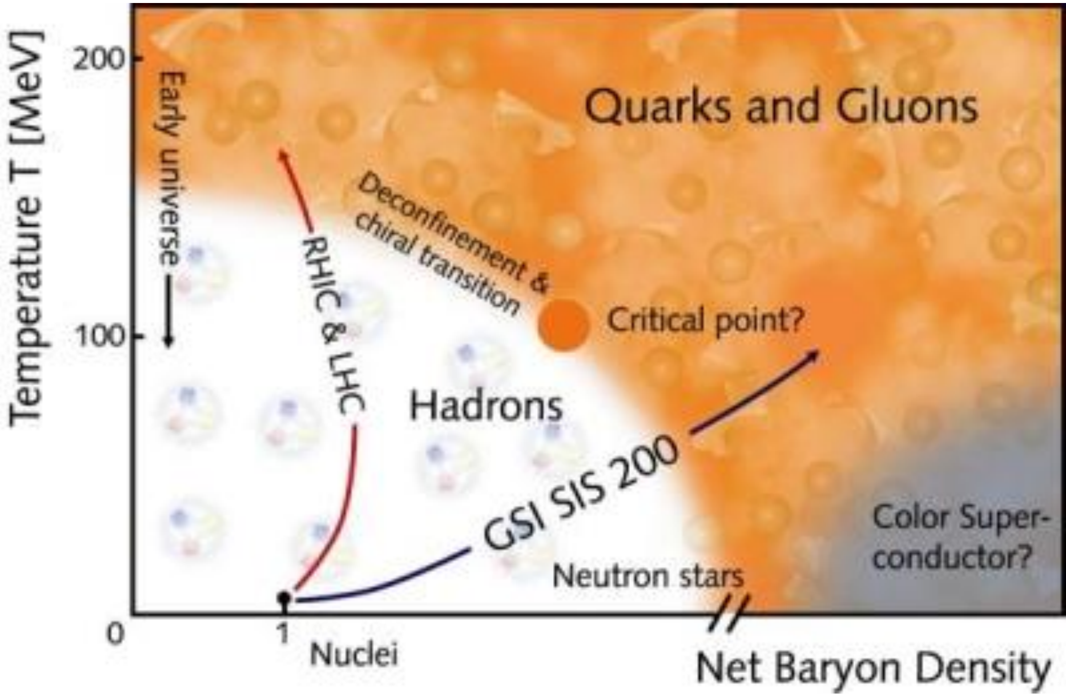
$$= \frac{2l}{8e^2b^2}\left[\frac{3a}{1+a}\frac{D_a\tilde{\mathfrak{q}}^2\tilde{\mathfrak{w}}}{D_a^2\tilde{\mathfrak{q}}^4+\tilde{\mathfrak{w}}^2} + \frac{2(1-a/2)^3}{(1+a)^2}\tilde{\mathfrak{w}}\right], \quad D_a = \frac{1}{2(1+a)}$$

In medium effects of QCD



Particle production
In RHIC

QCD phase diagram



In medium effects of QCD

Theory has parameters

Hadron physics	hQCD
T (temperature) μ (chemical potential)	r_h (black hole size) Q (black hole charge)

The w, k is scale by temperature

$$w = \frac{\omega}{2\pi T}, \qquad q = \frac{k}{2\pi T}$$

By changing μ, we will see the density effect on hydrodynamic quantities and spectral function

RN AdS5

Metric

$$ds^2 = \frac{r^2}{l^2}\left(-f(r)dt^2 + d\vec{x}^2\right) + \frac{l^2}{r^2 f(r)}dr^2$$

$$f(r) = 1 - \frac{ml^2}{r^4} + \frac{q^2 l^2}{r^6}, \quad A_t = -\frac{Q}{r^2} + \mu$$

Temperature

$$T = \frac{r_+^2 f'(r_+)}{4\pi l^2} = \frac{r_+}{\pi l^2}\left(1 - \frac{q^2 l^2}{2r_+^6}\right)$$

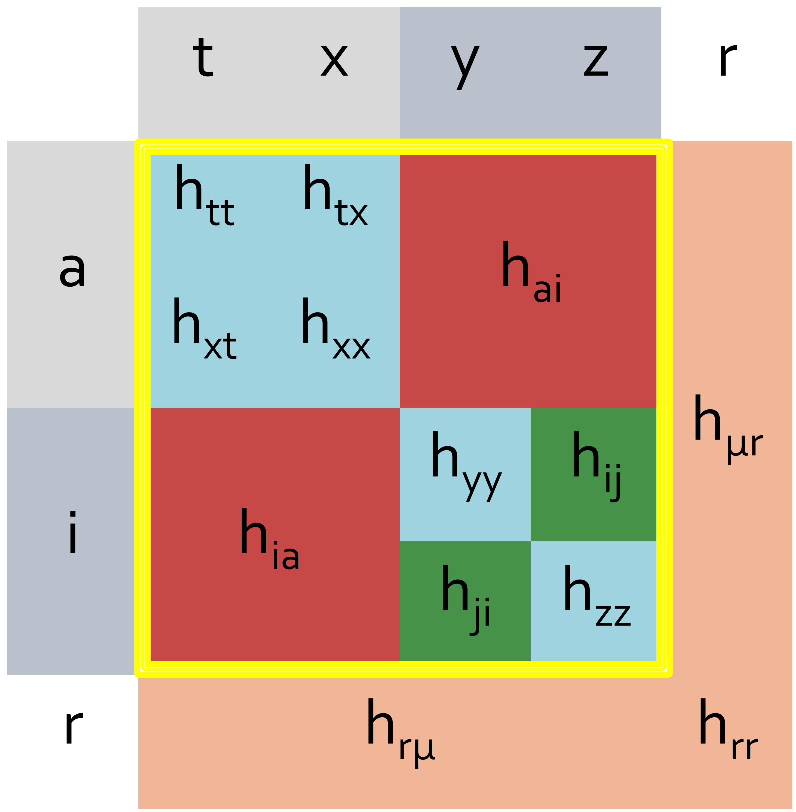
Chemical pot.

$$\mu = \frac{4Qb^2}{l^4} = \frac{1}{2b}\frac{g_5 l}{G_5}\sqrt{\frac{3a}{2}}$$

SO(p-1) classification

Tensor h_{ij}
Vector h_{ai}
Scalar h_{ab}

h_{MN} =



$$x^M = (x^\mu, r) = \left(x^a, x^i, r\right)$$

a=t,x i=y,z

AdSp+2 -> p+1 CFT

SO(p) rotational sym.
---(wave propagating)-->
SO(p-1) rotational sym.

h_{ai} : bi-vector
SO(1,1) X SO(p-1)

Metric and gauge field perturbation : vector mode

1005.0200

$$0 = h_t^{x\prime\prime} - \frac{1}{u}h_t^{x\prime} - \frac{(1-\frac{a}{2})^2}{uf}(\mathfrak{w}\mathfrak{q}h_z^x + \mathfrak{q}^2 h_t^x) - 3auB'$$

$$0 = \mathfrak{q}fh_z^{x\prime} + \mathfrak{w}h_t^{x\prime} - 3a\mathfrak{w}uB$$

$$0 = h_z^{x\prime\prime} + \frac{(f/u)'}{f/u}h_z^{x\prime} + \frac{(1-\frac{a}{2})^2}{uf^2}(\mathfrak{w}^2 h_z^x + \mathfrak{w}\mathfrak{q}h_t^x)$$

$$0 = B'' + \frac{f'}{f}B' + \frac{(1-\frac{a}{2})^2}{uf^2}\left(\mathfrak{w}^2 - \mathfrak{q}^2 f\right)B - \frac{h_t^{x\prime}}{f}.$$

Linearized Einstein eq.

Metric ($g = g_0 + \mathbf{h}$) + gauge field ($A = A_0 + \mathbf{a}$)

With finite density **h** and **a** is coupled via background field g_0 and A_0 .

$$\Psi_\pm = -\frac{f\tilde{\mathfrak{q}}}{\tilde{\mathfrak{w}}^2 - \tilde{\mathfrak{q}}^2 f}Z_1' + \left(-3au\frac{f\tilde{\mathfrak{q}}^2}{\tilde{\mathfrak{w}}^2 - \tilde{\mathfrak{q}}^2 f} + C_\pm\right)B$$

$$\Psi_\pm'' + \frac{f'}{f}\Psi_\pm' + \left(\frac{\tilde{\mathfrak{w}}^2 - \tilde{\mathfrak{q}}^2 f}{uf^2} - \frac{f'}{uf} - \frac{C_\pm}{f}\right)\Psi_\pm = 0$$

decoupling, master variable

$\langle J_x J_x \rangle$ Correlator

Near boundary series solution of master variable

$$\Psi_\pm = \hat{\Psi}_\pm (1 + \cdots + \hat{\Pi}_\pm u + \cdots)$$

From the definition of master variables, get the transformation matrix R to convert original boundary value and their conjugates

$$\begin{pmatrix} \Psi_+ \\ \Psi_- \end{pmatrix} = \mathrm{R} \begin{pmatrix} (Z_1)'(u=0) \\ \hat{B} \end{pmatrix} = \mathrm{R} \begin{pmatrix} (\tilde{\mathfrak{w}}^2 - \tilde{\mathfrak{q}}^2)\hat{Z}_1 \\ \hat{B} \end{pmatrix} \qquad \mathrm{R} = \begin{pmatrix} -\frac{\tilde{\mathfrak{q}}}{\tilde{\mathfrak{w}}^2 - \tilde{\mathfrak{q}}^2} & C_+ \\ -\frac{\tilde{\mathfrak{q}}}{\tilde{\mathfrak{w}}^2 - \tilde{\mathfrak{q}}^2} & C_- \end{pmatrix}$$

$$\begin{pmatrix} \Psi_+ \Pi_+ \\ \Psi_- \Pi_- \end{pmatrix} = \mathrm{R} \begin{pmatrix} \pi_Z \\ \pi_B \end{pmatrix}$$

$$\begin{pmatrix} \pi_Z \\ \pi_B \end{pmatrix} = \mathrm{R}^{-1} \mathrm{Diag}(\Psi_+, \Psi_-) \begin{pmatrix} \Pi_+ \\ \Pi_- \end{pmatrix}$$

$$\mathcal{G}_{x,x} = \frac{l}{4e^2 b^2} \frac{C_+ \hat{\Pi}_+ - C_- \hat{\Pi}_-}{C_+ - C_-}$$

Photoemission rate

th/0607237

$$d\Gamma_\gamma = \frac{d^3k}{(2\pi)^3}\frac{e^2}{2|\boldsymbol{k}|}\,\eta^{\mu\nu}C^<_{\mu\nu}(K)\Big|_{k^0=|\boldsymbol{k}|}$$

# of emitted photon / Vol4
Leading in e^2 expandsion
(e is the EM coupling)

Wightman function of EM currents (not time-ordered)

$$C^<_{\mu\nu}(K) = \int d^4X\, e^{-iK\cdot X}\,\langle J^{\rm EM}_\mu(0)J^{\rm EM}_\nu(X)\rangle$$

Other way to get the Wightman function

$$C^<_{\mu\nu}(K) = n_{\rm b}(k^0)\,\chi_{\mu\nu}(K)$$

$$\chi_{\mu\nu}(K) = -2\,{\rm Im}\,C^{\rm ret}_{\mu\nu}(K)$$

$$n_{\rm b}(k^0) = 1/(e^{\beta k^0}-1)$$

$$\frac{d\Gamma_\gamma}{dk} = \frac{\alpha_{EM}}{\pi}\,k\,\eta^{\mu\nu}C^<_{\mu\nu}(K)|_{\omega=k}$$

Photo emission rate

Photoemission rate

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ph/0111107

3 flavor massless QCD (dashed curve) / SYM (Solid Blue)

SYM computation gives quite good hint of hot QGP

Matching Debye screening mass

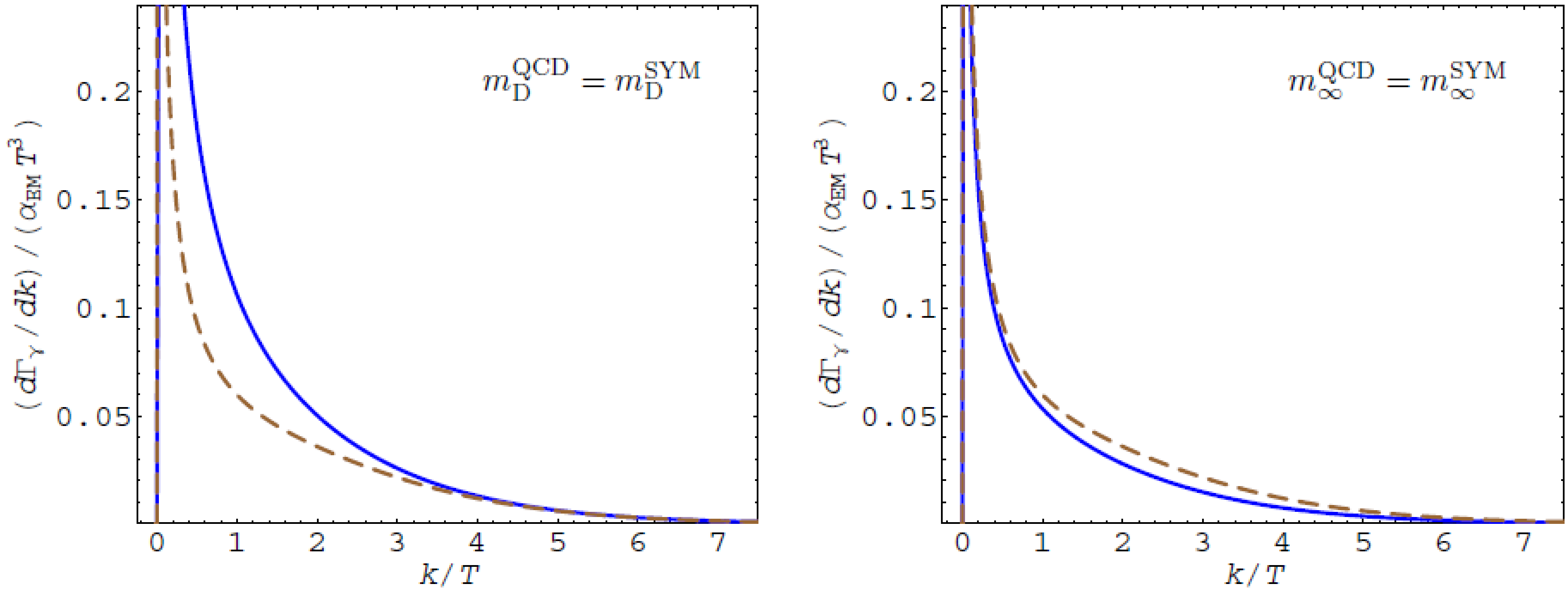
$$m_{\mathrm{D}}^2 = 2m_\infty^2 = 2\lambda T^2$$

$\alpha_{SYM} = .025, \quad \alpha_s = .1$

Matching Asymptotic fermion mass

$$m_\infty^2 = \lambda T^2$$

$\alpha_{SYM} = .011, \quad \alpha_s = .1$



Photoemission rate

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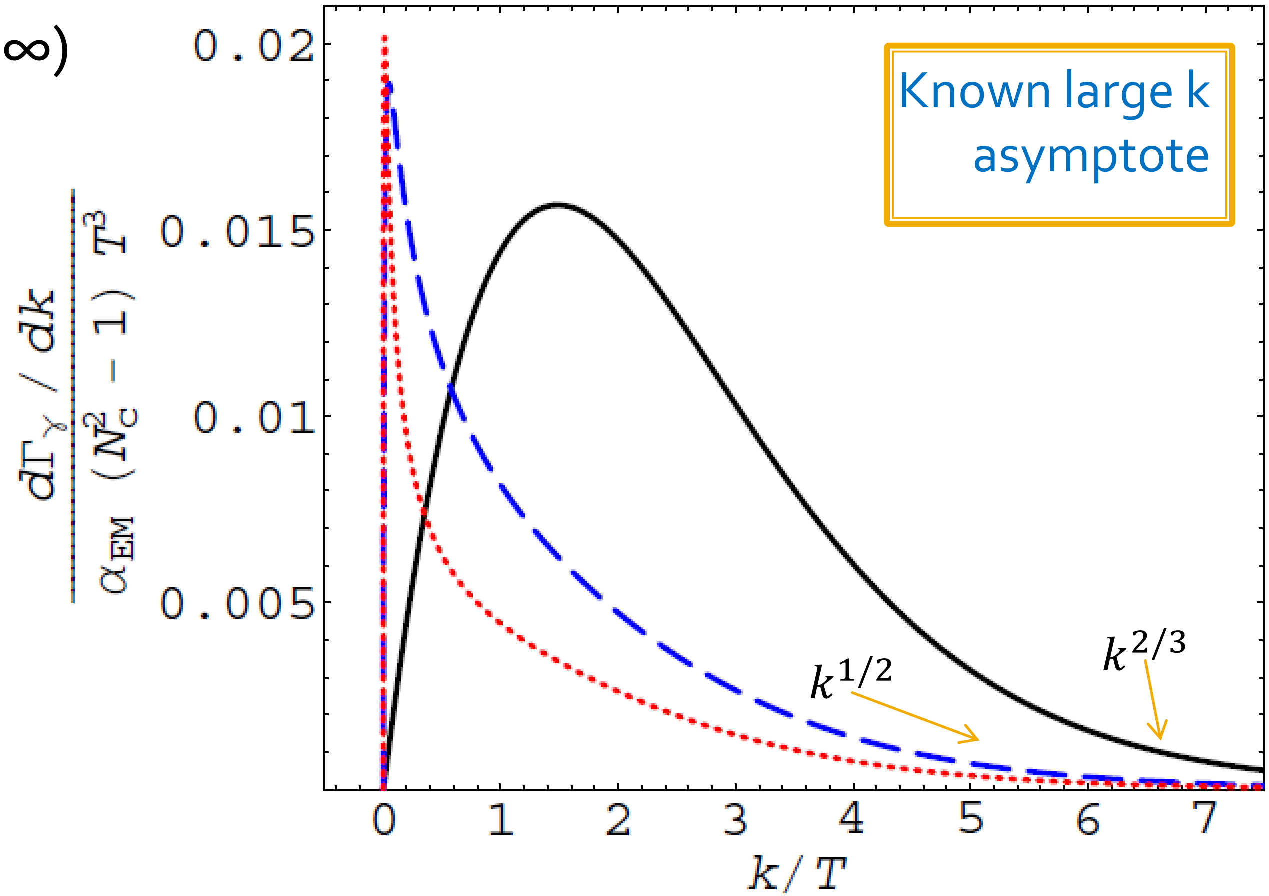
N=4 Super Yang-Mills computation with small t' Hooft coupling

Solid Black (SYM : λ=∞)
Dashed Blue (λ = 0.5)
Dotted Red (λ = 0.2)



Small frequency
Coupling indep.

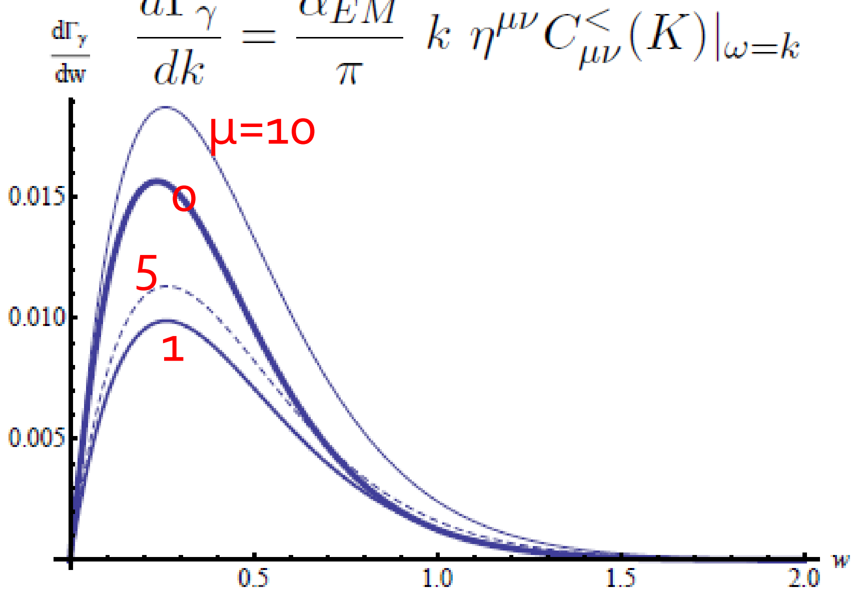
$$\frac{d\Gamma_\gamma}{dk} = \frac{\sigma T}{\pi^2} k$$

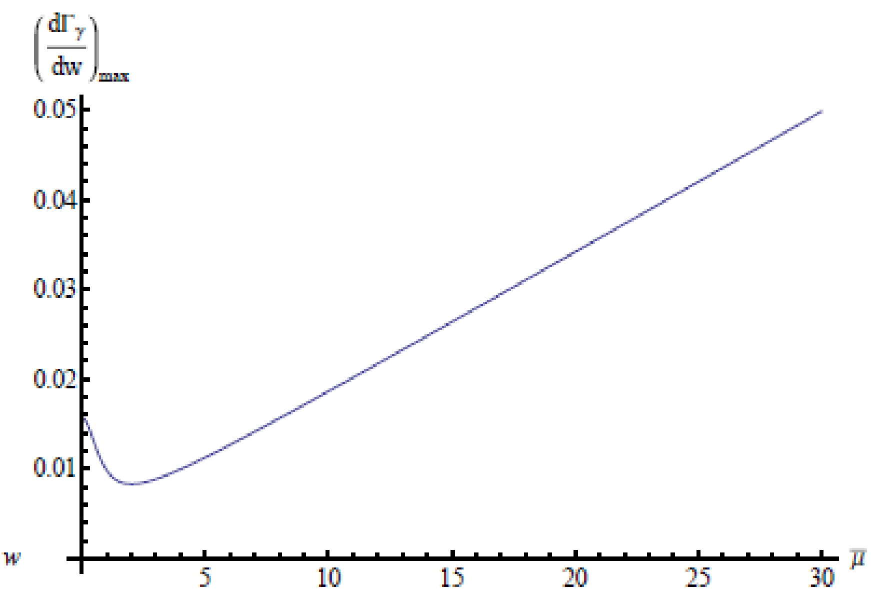
Photo emission rate

Photo emission rate with unit $\alpha_{\rm EM}(N_c^2-1)T^3$

Maximum at $\left(\frac{d\Gamma_\gamma}{dk}\right)_{\rm max} \approx 0.01567\,\alpha_{\rm EM} N_c^2 T^3$

$$\frac{d\Gamma_\gamma}{dk} = \frac{\alpha_{EM}}{\pi}\; k\;\eta^{\mu\nu} C^{<}_{\mu\nu}(K)|_{\omega=k}$$





Maximum rate is decreased first and increased after

Remarks

- Holography : two point function of QGP
- Small w, k limit : Hydrodynamics
  - transport coefficients
- Full w, k regime : Spectral function
  - quasi particle peaks, photoemission
- Finite T and μ, RN AdS is holographic dual.
- Thermal photon production is affected by density effects.