Dissociation Temperature in QGP and Meson Mass Spectrum

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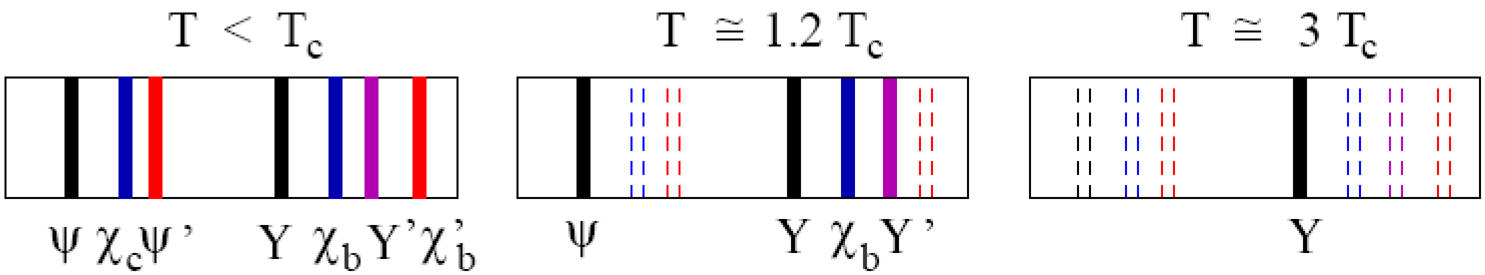
Introduction

- J/ψ **suppression : important signature of QGP**
  - [Matsui & Satz, PLB 178 (1986)]
  - **Heavy quarkonium states with charm(m_c~1.3 GeV)**
- **Why suppressed?**
  - **Deconfined color charges are screened**
  - **r_D decreases with increasing T**
  - **When $r_D < r_b$, no more bound state**
  - **Dissociation(melting) occurs**

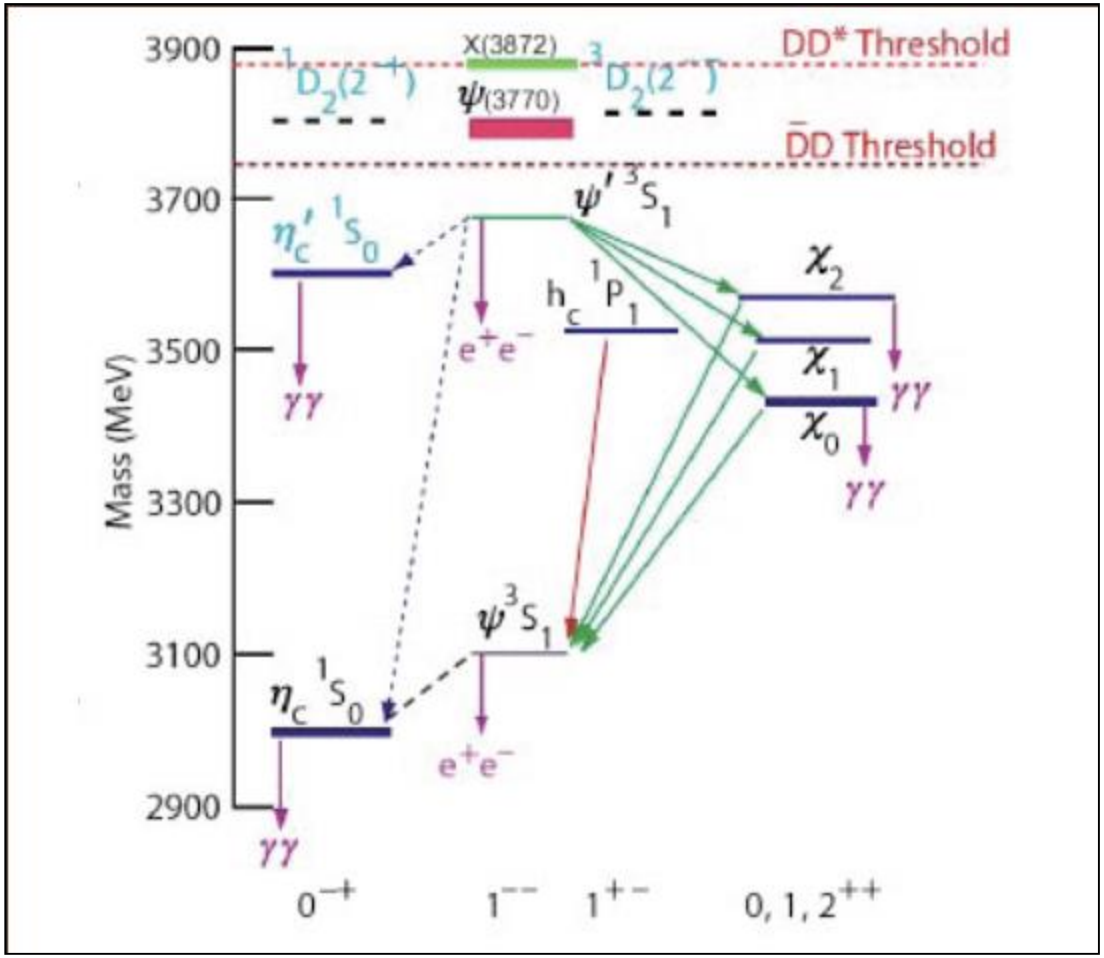
Introduction



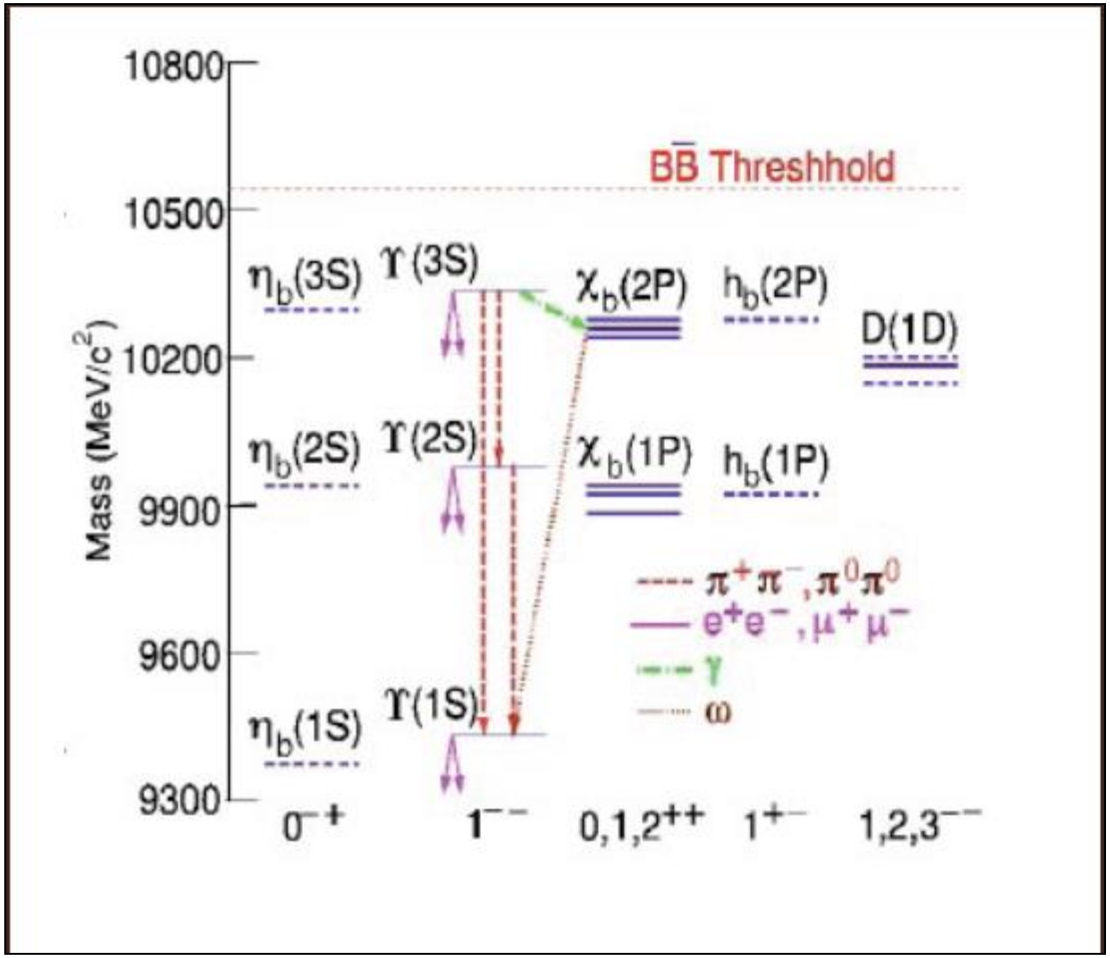
[Satz, NPA 783 (2007)]

Heavy Quarkonium is a good probe for the thermal properties of QGP

Charmonium Family



Bottomonium Family



Quarkonium at T=0

✓ $m_Q >> \Lambda_{QCD}$ and quark velocity v<<1

- non-relativistic approach
- Schrodinger Eq.
- Potential V(r) : interaction between Q and $\bar{Q}$

$$V(r) = -\frac{\alpha_{eff}}{r} + \sigma r$$

[Eichten et al., PRD 21 (1980)]

- Short distance + long distance
- Known as Cornell Potential

Quarkonium at T=0

- In pNRQCD
  - Can derive the $Q\bar{Q}$ potential using scattering amplitude with 1-gluon exchange
  - Verify the Cornell potential

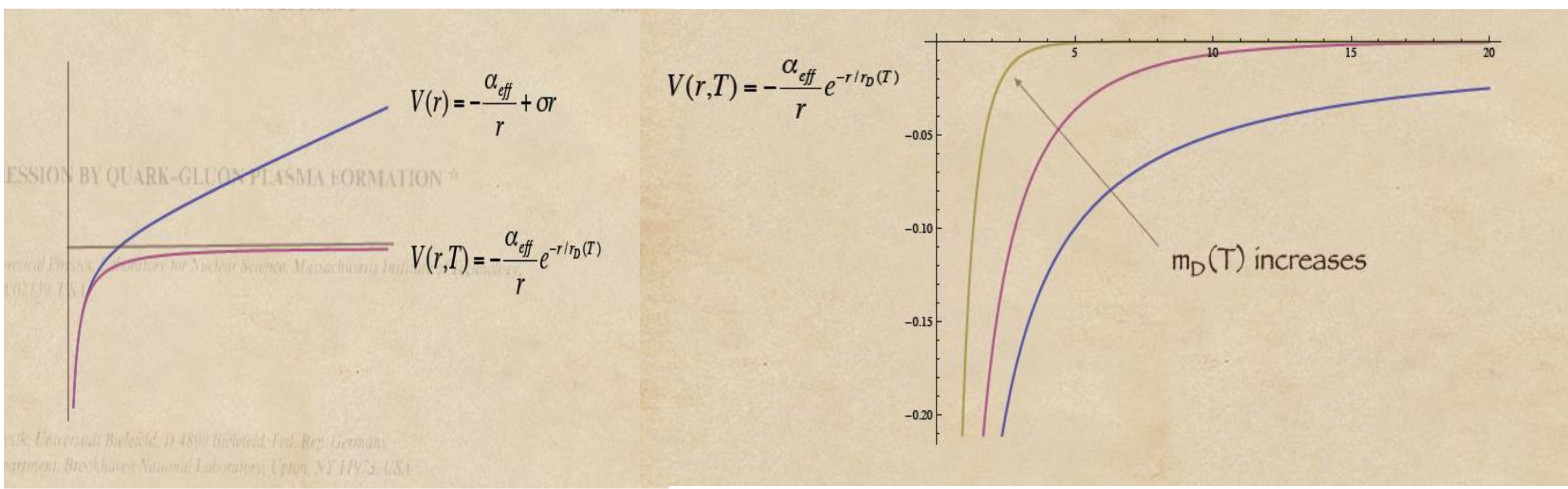
Quarkonium at T≠0

- ✓ **Screening radius decreases as T**

$$-\frac{\alpha_{\rm eff}}{r} \rightarrow -\frac{\alpha_{\rm eff}}{r} e^{-r/r_D(T)}$$

[Matsui & Satz, PLB 178(1986)]

- ➢ **Long range Coulomb → short range Yukawa**



Quarkonium at T≠0

✓ **String Tension Term?**

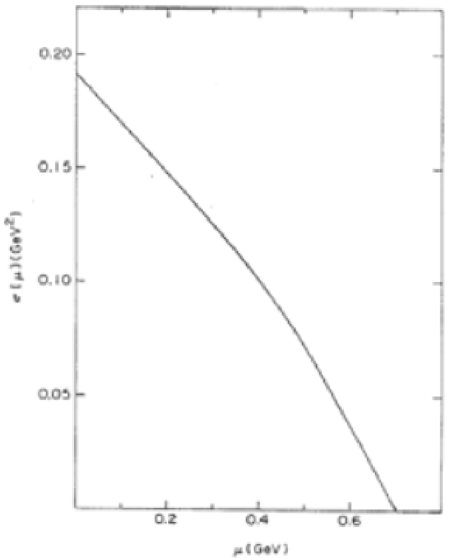
[Karsch, Mehr, Satz(KMS), Z Phys C 37(1988)]

$$\sigma r \rightarrow \sigma r_D(T)(1 - e^{-r/r_D(T)})$$

➢ **T-dependent string tension**

$$\sigma(T) \rightarrow \sigma \frac{(1 - e^{-r/r_D(T)})}{\mu(T) r} \rightarrow \sigma$$

as $\mu = 1/r_D \rightarrow 0$

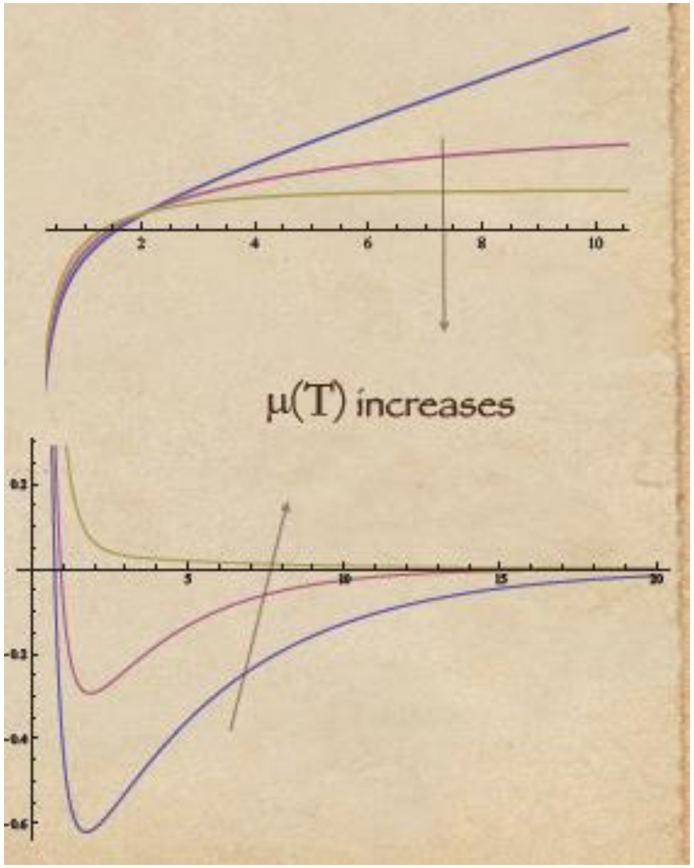


Quarkonium at T≠0

- Screened Cornell Potential

$$V(r) = -\frac{\alpha_{eff}}{r} e^{-\mu(T)r} + \frac{\sigma}{\mu(T)}\left(1 - e^{-\mu(T)r}\right)$$

- Effective binding potential
  - No bound state as μ(T) increases



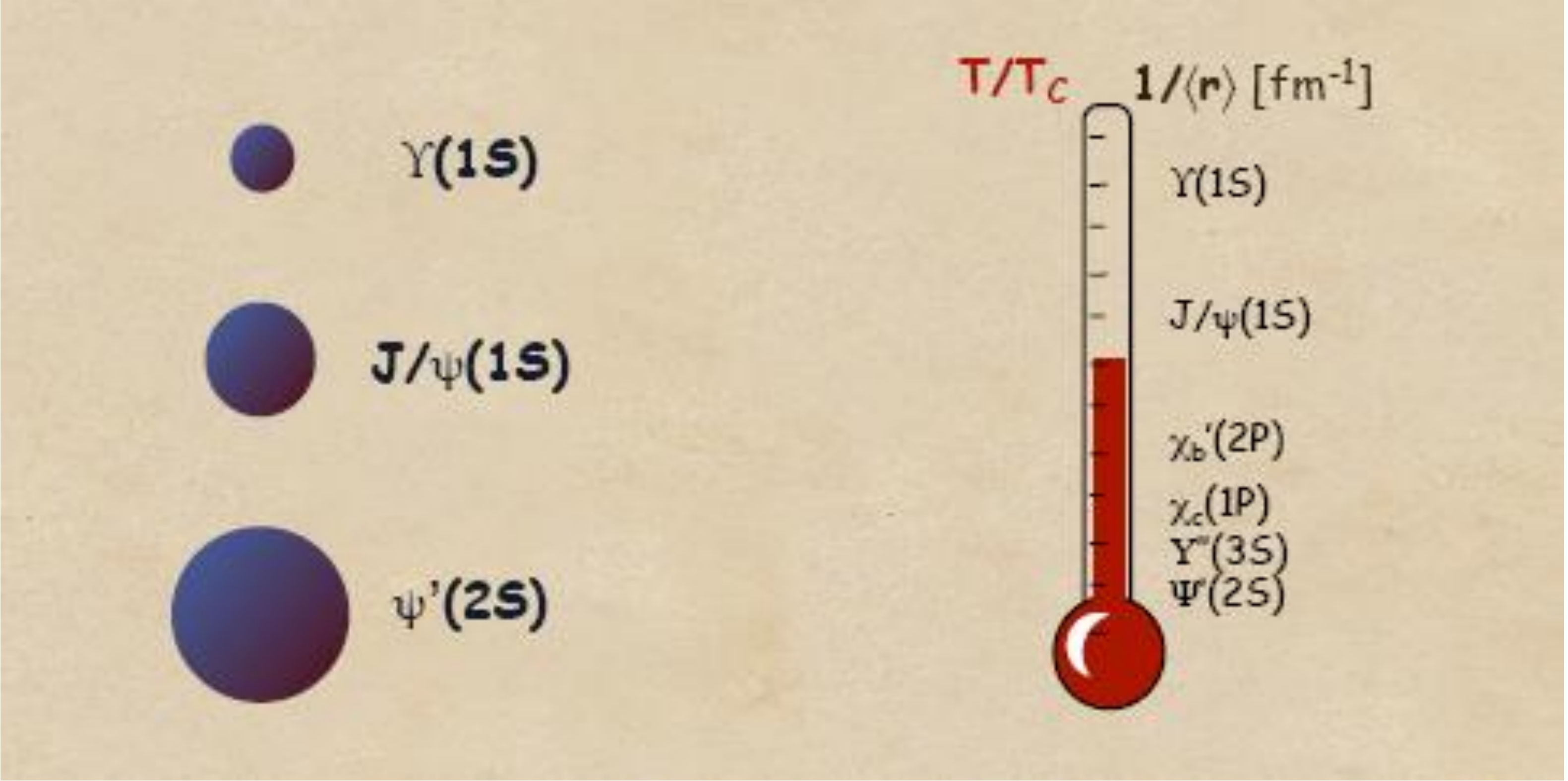
Quarkonium at T=0

- Quarkonium dissociates when $r_D < r_b$
- For J/psi

	T=0	T=200 MeV
α_{eff}	0.52	0.2
r_b	0.41 fm	1.07 fm
r_D	∞	0.59 fm

[Wong, Introduction to HIC (1994)]

QGP Thermometer



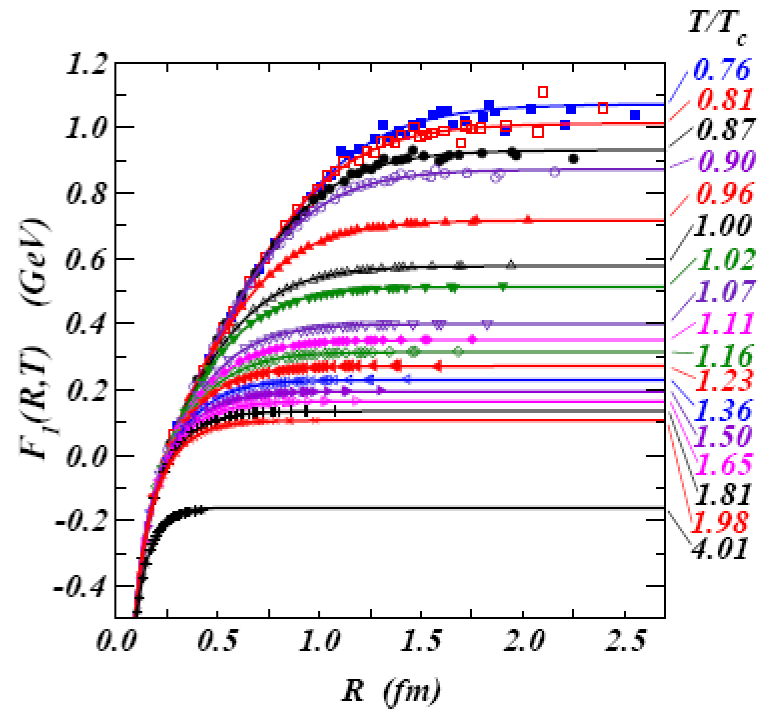
[Mocsy, (2008)]

Lattice QCD Result

- ✓ Gives T-dependent free energy(F)
- ✓ F as V(r)

$q\bar{q}$	T/T_c
J/Ψ	1.10
$\chi_c(1P)$	0.74
$\psi(2S)$	0.1–0.2
$\Upsilon(1S)$	2.31
$\chi_b(1P)$	1.13
$\Upsilon(2S)$	1.10
$\chi_b(2P)$	0.83
$\Upsilon(3S)$	0.75

Digal, Petreczky, Satz, PRD64(2001)



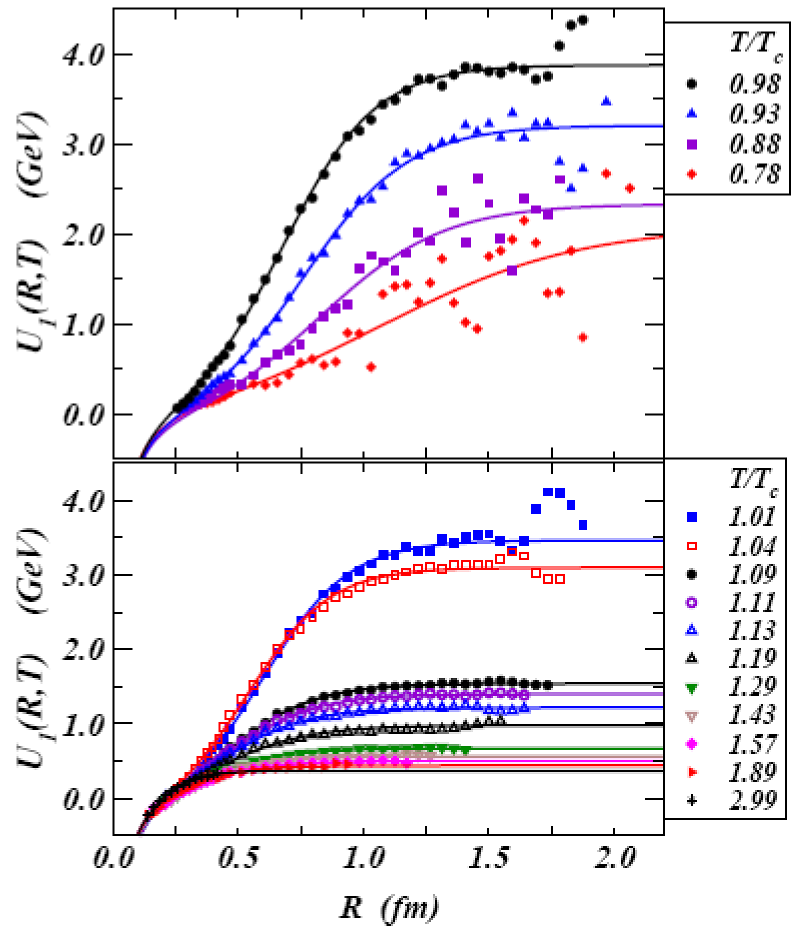
Kaczmarek, Zantow , PRD71(2005)

Lattice QCD Result

- ✓ **However, Entropy plays a role**

$$U_1(R,T) = F_1(R,T) + TS_1(R,T)$$

- ✓ **Internal Energy U(r) as V(r)**



Kaczmarek, Zantow , PRD71(2005)

Lattice QCD Result

- ✓ **U(r) as V(r)**
  - **Same at small r**
  - **Steeper at large r**
  - **Tightly bounded**
  - **Higher Diss. Temp.**

Wong, PRD(2005)

T/T_c	1.1	1.5	2.0	2.5	3.0	3.3
$M[J/\psi, \eta_c]$	2.99	3.13	3.25	3.34	≈3.40	⋯
$E_B[J/\psi, \eta_c]$	0.41	0.27	0.15	0.06	≈0	⋯
$M[\psi(2S)]$	≈3.40	⋯	⋯	⋯	⋯	⋯
$E_B[\psi(2S)]$	≈0	⋯	⋯	⋯	⋯	⋯

T/T_c	1.1	1.5	1.8	2.1	2.7	3.5
$M[\Upsilon, \eta_b]$	9.35	9.47	9.59	9.70	9.81	9.86
$E_B[\Upsilon, \eta_b]$	0.95	0.83	0.71	0.60	0.49	0.44
$M[\Upsilon(2S)]$	10.05	10.18	10.28	⋯	⋯	⋯
$E_B[\Upsilon(2S)]$	0.25	0.12	≈0	⋯	⋯	⋯
$M[\Upsilon(3S)]$	≈10.30	⋯	⋯	⋯	⋯	⋯
$E_B[\Upsilon(3S)]$	≈0	⋯	⋯	⋯	⋯	⋯

TABLE II. Same as in Table I for *P*-wave quarkonia.

T/T_c	1.1	1.3	1.5	2	2.3
$M[\chi_c(1P)]$	3.38	⋯	⋯	⋯	⋯
$E_B[\chi_c(1P)]$	≈0	⋯	⋯	⋯	⋯
$M[\chi_b(1P)]$	9.95	10.05	10.11	10.23	10.30
$E_B[\chi_b(1P)]$	0.35	0.25	0.19	0.07	≈0
$M[\chi_b(2P)]$	10.25	10.30	⋯	⋯	⋯
$E_B[\chi_b(2P)]$	0.05	≈0	⋯	⋯	⋯

Lattice QCD Result

- U(r) as V(r)
  - Large Increase in strength
  - Due to large increase of entropy and internal energy at transition temp.
  - Which is not related to QQ potential
  - Cannot describe the Diss. Temp. suitably
- What is the suitable choice of V?

Kaczmarek, Zantow , PRD71(2005)

Lattice QCD Result

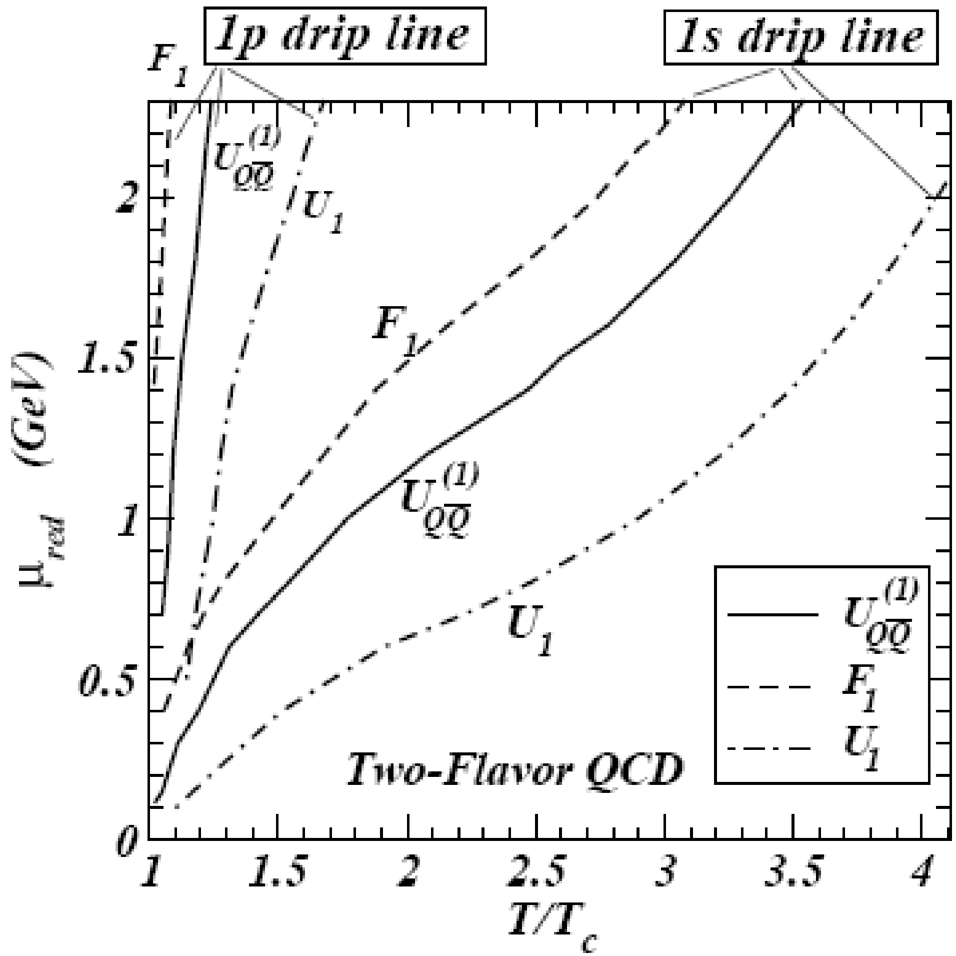
- Wong, J. Phys. G32 (2006)
- Fitted into the following analytic form

$$\{F_1, U_1\}(R,T) = -\frac{4}{3}\frac{\alpha_s(T)}{R} f(R,T) + C(T)[1 - f(R,T)],$$

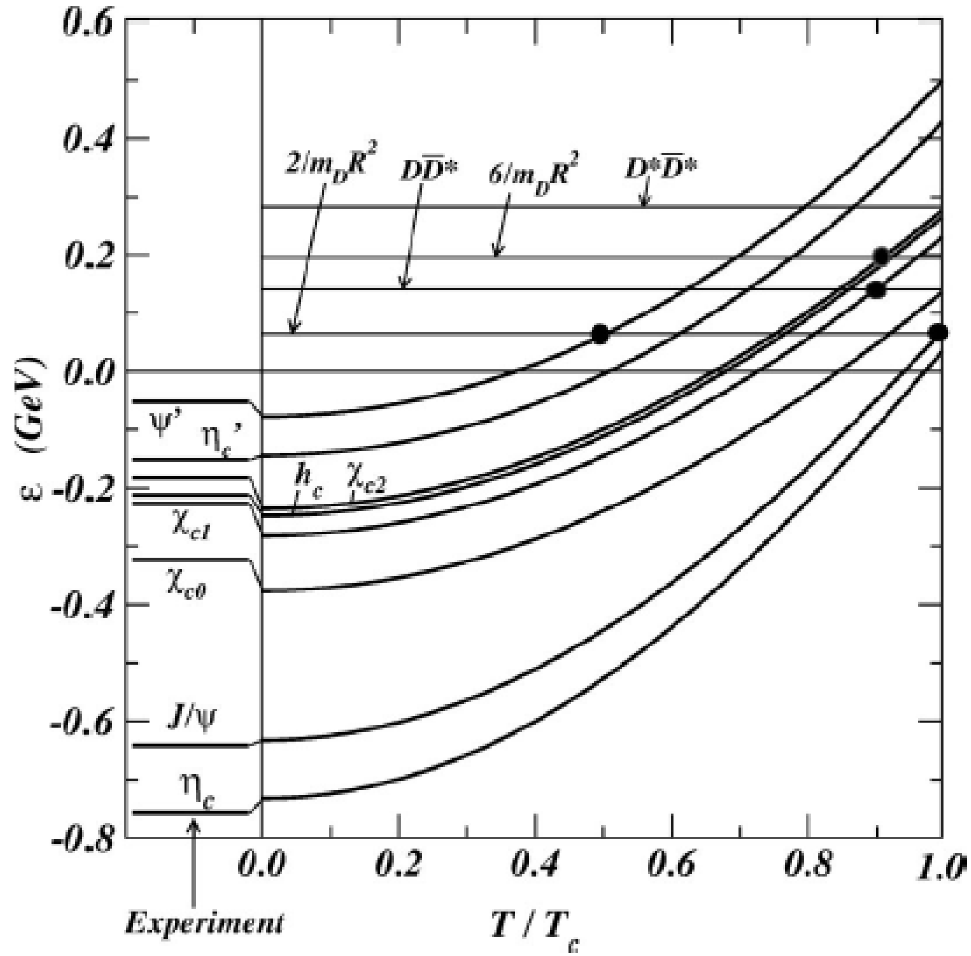
$$f(R,T) = \frac{1}{\exp\{(R - r_0(T))/d(T)\} + 1}.$$

- Some combination between F & U
- fractions of F & U are determined by EOS.

Lattice QCD Result



Lattice QCD Result



Wong, PRC65(2002)

T=0 is not suitably matching

→ Needs some treatment?

→ Rel. approach?

Why Rel.?

Wong, PRC65(2002)

$$V_{spin-spin} + V_{spin-orbit} + V_{tensor}$$
$$= const. \times s_1 \cdot s_2 e^{-d^2 r^2} + const. \times L \cdot S + cont. \times S_{12}$$

- Already tested in e^+e^- system within QED (*Todorov, PRD3(1971)*)
- can be applied to two quark system
- Can treat spin-dep. terms naturally

2-Body Constraint Dynamics

P. van Alstine et al., J. Math. Phys.23, 1997 (1982)

- **Two free spinless particles with the mass-shell constraint**

  → removes relative energy & time

$$H_i^0 \equiv p_i^2 + m_i^2 \approx 0$$

- **Two free spin-half particles with the** ***generalized*** **mass-shell constraint**

  → potential depends on the space-like separation only & P ⊥ p

$$\begin{aligned} H_i &\equiv p_i^2 + M_i^2 \\ &= p_i^2 + m_i^2 + \Phi_i(x, p_1, p_2) \\ &\approx 0 \end{aligned}$$

2-Body Constraint Dynamics

P. van Alstine et al., J. Math. Phys.23, 1997 (1982)

- **Pauli reduction + scale transformation**

16-comp. Dirac Eq. → 4-comp. rel. Schrodinger Eq.

$$H = \frac{\varepsilon_1 H_1 + \varepsilon_2 H_2}{w} = p^2 + \Phi_w(\sigma_1, \sigma_2, p_\perp, A(r), S(r)) = b^2(w)$$

where $b^2(w) = \varepsilon_w^2 - m_w^2$ with $\varepsilon_w = \dfrac{w^2 - m_1^2 - m_2^2}{2w}$ and $m_w = \dfrac{m_1 m_2}{w}$

What’s Good?

- TBDE : 2 fermion basis

  16-component dynamics $\longrightarrow$ 4-component

- Particles interacts through scalar and vector interactions.
- Leads to simple Schrodinger-type equation.
- Spin-dependence is determined naturally.

Formulation

- $\Phi_w =$ central potentials + darwin + SO + SS + Tensor + etc.

$$\Phi_w = 2m_w S + S^2 + 2\varepsilon_w A - A^2 + \Phi_D + \mathbf{L}\cdot(\sigma_1+\sigma_2)\Phi_{SO} + \sigma_1\cdot\sigma_2\Phi_{SS}$$
$$+\sigma_1\cdot\hat{\mathbf{r}}\,\sigma_2\cdot\hat{\mathbf{r}}\,\mathbf{L}\cdot(\sigma_1+\sigma_2)\Phi_{SOT} + (3\sigma_1\cdot\hat{\mathbf{r}}\,\sigma_2\cdot\hat{\mathbf{r}} - \sigma_1\cdot\sigma_2)\Phi_T$$
$$+\mathbf{L}\cdot(\sigma_1-\sigma_2)\Phi_{SOD} + i\mathbf{L}\cdot(\sigma_1\times\sigma_2)\Phi_{SOX}$$

$$\Phi_D = -\frac{2F'(\cosh 2K - 1)}{r} + F'^2 + K'^2 + \frac{2K'\sinh 2K}{r} - \nabla^2 F - \frac{2(\cosh 2K - 1)}{r^2} + m(r),$$

$$\Phi_{SO} = -\frac{F'}{r} - \frac{F'(\cosh 2K - 1)}{r} - \frac{(\cosh 2K - 1)}{r^2} + \frac{K'\sinh 2K}{r},$$

$$\Phi_{SOD} = (l'\cosh 2K - q'\sinh 2K),$$
$$\Phi_{SOX} = (q'\cosh 2K - l'\sinh 2K),$$

(A17)

Φ_{SOD} & Φ_{SOX} =0

when $m_1 = m_2$

$$\Phi_{SS} = k(r) + \frac{2K'\sinh 2K}{3r} - \frac{2F'(\cosh 2K - 1)}{3r} - \frac{2(\cosh 2K - 1)}{3r^2} + \frac{2F'K'}{3} - \frac{\nabla^2 K}{3},$$

$$\Phi_T = \frac{1}{3}\left[n(r) + \frac{3F'\sinh 2K}{r} + \frac{F'(\cosh 2K - 1)}{r} + 2F'K' - \frac{K'\sinh 2K}{r} - \frac{3K'(\cosh 2K - 1)}{r} - \nabla^2 K + \frac{3\sinh 2K}{r^2} + \frac{(\cosh 2K - 1)}{r^2}\right],$$

$$\Phi_{SOT} = -K'\frac{\cosh 2K - 1}{r} + \frac{\sinh 2K}{r^2} - \frac{K'}{r} + \frac{F'\sinh 2K}{r},$$

Formulation

$$k(r) = \frac{1}{3}\nabla^2(K + \mathcal{G}) - \frac{2F'(\mathcal{G}' + K')}{3} - \frac{1}{2}\mathcal{G}'^2$$

$$n(r) = \frac{1}{3}\left[\nabla^2 K - \frac{1}{2}\nabla^2\mathcal{G} + \frac{3(\mathcal{G}' - 2K')}{2r} + F'(\mathcal{G}' - 2K')\right],$$

$$m(r) = -\frac{1}{2}\nabla^2\mathcal{G} + \frac{3}{4}\mathcal{G}'^2 + \mathcal{G}'F' - K'^2, \qquad \text{(A19)}$$

and

$$l'(r) = -\frac{1}{2r}\frac{E_2M_2 - E_1M_1}{E_2M_1 + E_1M_2}(L - \mathcal{G})',$$

$$q'(r) = \frac{1}{2r}\frac{E_1M_2 - E_2M_1}{E_2M_1 + E_1M_2}(L - \mathcal{G})'. \qquad \text{(A20)}$$

$$K' = \frac{\mathcal{G}' + L'}{2},$$

$$\nabla^2 F = \frac{(\nabla^2 L - \nabla^2\mathcal{G})(E_2M_2 + E_1M_1)}{2(E_2M_1 + E_1M_2)} - (L' - \mathcal{G}')^2\frac{(m_1^2 - m_2^2)^2}{2(E_2M_1 + E_1M_2)^2} - \nabla^2\mathcal{G},$$

$$\nabla^2 L = \frac{-L'^2(M_1^2 + M_2^2)}{M_1M_2} + \frac{w}{M_1M_2}\left(\frac{\nabla^2 S(m_w + S) + S'^2}{w - 2A} + \frac{4S'(m_w + S)A' + (2m_wS + S^2)\nabla^2 A}{(w - 2A)^2} + \frac{4(2m_wS + S^2)A'^2}{(w - 2A)^3}\right), \qquad \text{(C3)}$$

$$\nabla^2\mathcal{G} = \frac{\nabla^2 A}{w - 2A} + 2\mathcal{G}'^2. \qquad \nabla^2 K = \frac{\nabla^2\mathcal{G} + \nabla^2 L}{2}.$$

v.

$$F = \frac{1}{2}\log\frac{\mathcal{D}}{\varepsilon_2 m_1 + \varepsilon_1 m_2} - \mathcal{G},$$

$$\mathcal{D} = E_2M_1 + E_1M_2,$$

$$K = \frac{(\mathcal{G} + L)}{2},$$

$$F' = \frac{(L' - \mathcal{G}')(E_2M_2 + E_1M_1)}{2(E_2M_1 + E_1M_2)} - \mathcal{G}',$$

$$E_1 = \frac{\varepsilon_1 - A}{\sqrt{(w - 2A)/w}},$$

$$E_2 = \frac{\varepsilon_2 - A}{\sqrt{(w - 2A)/w}},$$

$$M_1 = \sqrt{m_1^2 + \frac{2m_wS + S^2}{(w - 2A)/w}}, \qquad \text{(C1)}$$

$$M_2 = \sqrt{m_2^2 + \frac{2m_wS + S^2}{(w - 2A)/w}},$$

$$L' = \frac{M_1'}{M_2} = \frac{M_2'}{M_1} = \frac{w}{M_1M_2}\left(\frac{S'(m_w + S)}{w - 2A} + \frac{(2m_wS + S^2)A'}{(w - 2A)^2}\right),$$

$$\mathcal{G}' = \frac{A'}{w - 2A}.$$

$$\cosh 2K = \frac{1}{2}\left(\frac{(\varepsilon_1 + \varepsilon_2)(M_1 + M_2)}{(m_1 + m_2)(E_1 + E_2)} + \frac{(m_1 + m_2)(E_1 + E_2)}{(\varepsilon_1 + \varepsilon_2)(M_1 + M_2)}\right),$$

$$\sinh 2K = \frac{1}{2}\left(\frac{(\varepsilon_1 + \varepsilon_2)(M_1 + M_2)}{(m_1 + m_2)(E_1 + E_2)} - \frac{(m_1 + m_2)(E_1 + E_2)}{(\varepsilon_1 + \varepsilon_2)(M_1 + M_2)}\right),$$

Formulation

- $\Phi_{SOD}=0$ and $\Phi_{SOX}=0$ when $m_1=m_2$
- For singlet state with the same mass,
  - no SO, SOT contribution $\leftarrow$ $\mathbf{L}\cdot(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)=2\mathbf{L}\cdot\mathbf{S}=0$
  - Terms of (Φ_D, Φ_{SS}, Φ_T) altogether vanishes.
  - $H=p^2+2m_wS+S^2+2\varepsilon_wA-A^2$

Formulation

- **For π (S-state),**

$$\left\{-\frac{d^2}{dr^2}+2m_wS+S^2+2\varepsilon_wA-A^2+\Phi_D-3\Phi_{SS}\right\}v_0=b^2v_0.$$

- **For ρ (mixture of S & D-state),**

$$\left\{-\frac{d^2}{dr^2}+2m_wS+S^2+2\varepsilon_wA-A^2+\Phi_D+\Phi_{SS}\right\}u_+ + \frac{2\sqrt{2}}{3}\{3\Phi_T-6\Phi_{SOT}\}u_- = \left\{-\frac{d^2}{dr^2}+\Phi_{++}\right\}u_+ + \Phi_{+-}u_- = b^2u_+, \quad (55)$$

$$\left\{-\frac{d^2}{dr^2}+\frac{6}{r^2}+2m_wS+S^2+2\varepsilon_wA-A^2+\Phi_D-6\Phi_{SO}+\Phi_{SS}-2\Phi_T+2\Phi_{SOT}\right\}u_- + \frac{2\sqrt{2}}{3}\{3\Phi_T\}u_+ = \left\{-\frac{d^2}{dr^2}+\frac{6}{r^2}+\Phi_{--}\right\}u_- + \Phi_{-+}u_+ = b^2u_-. \quad (56)$$

Formulation

**Once we find b², **

Invariant mass $$\mathbf{w} = \sqrt{\mathbf{b}^2 + \mathbf{m}_1^2} + \sqrt{\mathbf{b}^2 + \mathbf{m}_2^2}$$

QCD Potentials

- **Common non-relativistic static quark potential**

$$V(r) = -\frac{\alpha_c}{r} + br$$

  - Dominant Coulomb-like + confinement
  - But asymptotic freedom is missing

- **Richardson(*Phys. Lett. 82B,272(1979)*)**

$$\tilde{V}(q) = -\frac{16\pi}{27}\frac{1}{q^2 \ln(1+q^2/\Lambda^2)}$$

$$\xrightarrow{\text{FT}} \quad V(r) = \frac{8\pi\Lambda^2 r}{27} - \frac{8\pi f(\Lambda r)}{27 r}$$

QCD Potentials

- Richardson potential in coord. space

$$V(r) = \frac{8\pi\Lambda^2 r}{27} - \frac{8\pi f(\Lambda r)}{27r}$$

  - For $r \to 0,\ f(\Lambda r) \to -\frac{1}{\ln(\Lambda r)}$: asymptotic freedom
  - For $r \to \infty,\ f(\Lambda r) \to 1$: confinement

- We will use fitting param.

$$V(r) = \frac{8\pi\Lambda^2 r}{27} - \frac{16\pi}{27r\ln(Ke^2 + B/(\Lambda r)^2)}$$

QCD Potentials

$$\mathbf{V(r)} = \frac{8\pi\Lambda^2 r}{27} - \frac{16\pi}{27r\ln(Ke^2 + B/(\Lambda r)^2)}$$

- **Scalar Pot.**

$$S(r) = \frac{8\pi\Lambda^2 r}{27}$$

- **Vector Pot.**

$$A(r) = -\frac{16\pi}{27r\ln(Ke^2 + B/(\Lambda r)^2)} + \frac{e_1 e_2}{4\pi r}$$

What's Good?

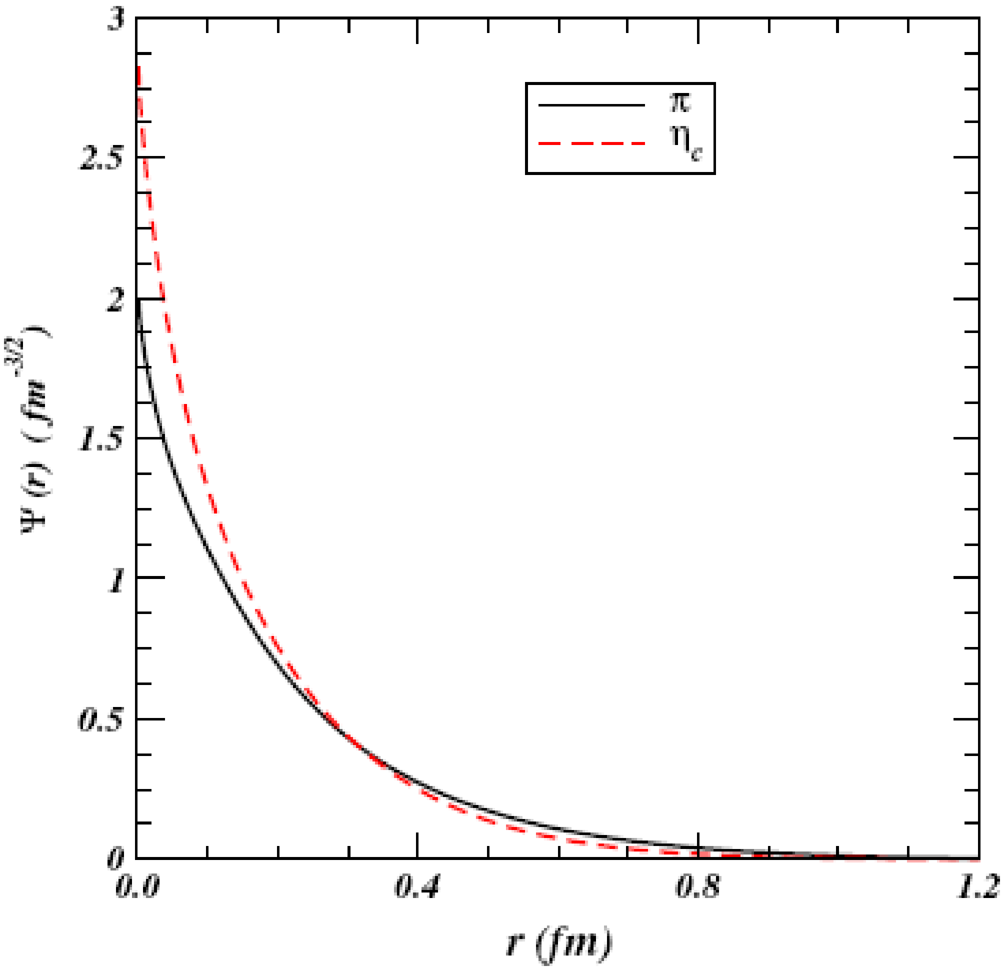
- TBDE : 2 fermion basis

  16-component dynamics $\longrightarrow$ 4-component
- Particles interacts through scalar and vector interactions.
- Yields simple Schrodinger-type equation.
- Spin-dependence is determined naturally.
- **No cutoff parameter**
- **No singularity**

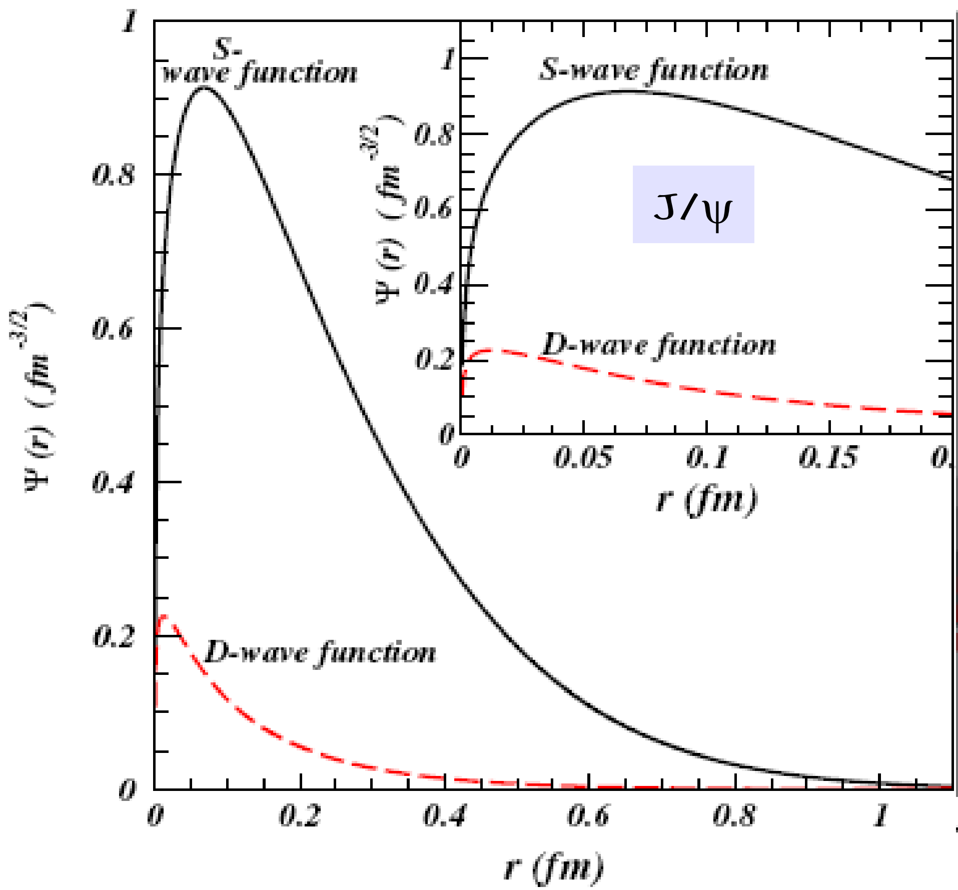
Individual Contribution(π)

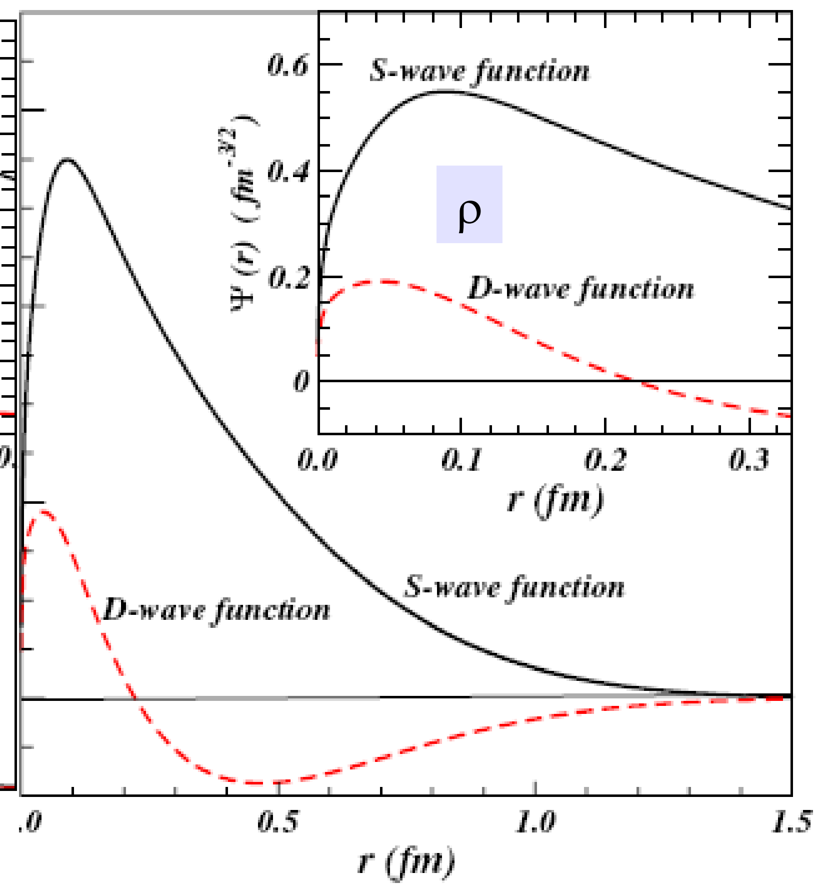
terms	magnitude	terms	magnitude
$\langle d^2/dr^2 \rangle$	0.8508	$\langle \Psi_{SI} \rangle$	-0.3832
$\langle 2m_w S \rangle$	0.0103	$\langle \Psi_D \rangle$	-3.8040
$\langle S^2 \rangle$	0.0942	$\langle \Psi_{SS} \rangle$	-2.8950
$\langle 2\varepsilon_w A \rangle$	-0.0598	$\langle \Psi_{ST} \rangle$	6.2350
$\langle -A^2 \rangle$	-0.4279	$\langle \Psi_{SD} \rangle$	-0.4643
$\langle b^2 \rangle$	0.0033	$\langle \Psi_w \rangle$	-0.8475

Wave Functions(π & η_c)

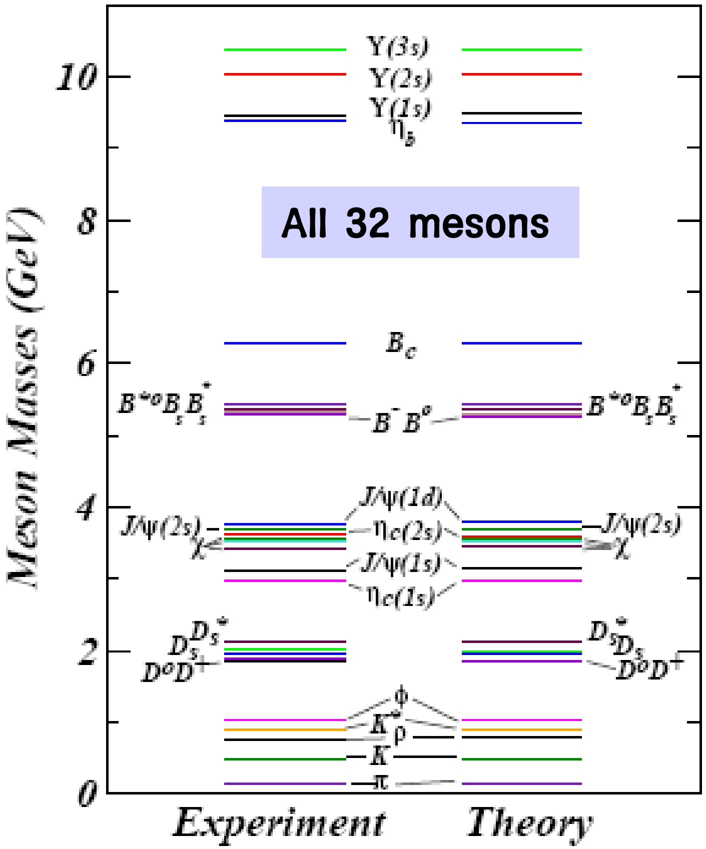


Wave Functions(J/ψ, ρ)





MESON Spectra



L	0.4218 GeV
B	0.05081
K	4.198
m_u	0.0557 GeV
m_d	0.0553 GeV
m_s	0.2499 GeV
m_c	1.476 GeV
m_b	4.844 GeV

MESON Spectra

32 mesons

TABLE II. Selected portions of meson spectrum.

Meson	Exp. (GeV)	Theory (GeV)	Exp. − Theory (GeV)
π: $u\bar{d}1^1S_0$	0.140	0.159	−0.019
ρ: $u\bar{d}1^3S_1$	0.775	0.792	−0.017
K^-: $s\bar{u}1^1S_0$	0.494	0.493	0.001
K^0: $s\bar{d}1^1S_0$	0.498	0.488	0.010
K^*: $s\bar{u}1^3S_1$	0.892	0.903	−0.011
K^*: $s\bar{d}1^3S_1$	0.896	0.901	−0.005
ϕ: $s\bar{s}1^3S_1$	1.019	1.025	−0.006
D^0: $c\bar{u}1^1S_0$	1.865	1.840	0.025
D^+: $c\bar{d}1^1S_0$	1.870	1.845	0.025
D^{*0}: $c\bar{u}1^3S_1$	2.010	1.981	0.029
D^{*+}: $c\bar{d}1^3S_1$	2.007	1.979	0.028
D_s: $c\bar{s}1^1S_0$	1.968	1.965	0.003
D_s^*: $c\bar{s}1^3S_1$	2.112	2.112	0.000
η_c: $c\bar{c}1^1S_0$	2.980	2.978	0.002
$J/\psi(1S)$: $c\bar{c}1^3S_1$	3.097	3.140	−0.043
$\psi(2S)$: $c\bar{c}2^3S_1$	3.686	3.689	−0.003
h_1: $c\bar{c}1^1P_1$	3.526	3.522	0.004
χ_0: $c\bar{c}1^3P_0$	3.415	3.436	−0.021
χ_1: $c\bar{c}1^3P_1$	3.511	3.515	−0.004
χ_2: $c\bar{c}1^3P_2$	3.556	3.541	0.015
η_c: $c\bar{c}2^1S_0$	3.638	3.591	0.047
$\psi(1D)$: $c\bar{c}1^3D_1$	3.773	3.804	−0.031
B^-: $b\bar{u}1^1S_0$	5.279	5.249	0.030
B^0: $b\bar{d}1^1S_0$	5.280	5.248	0.032
B^{*0}: $b\bar{u}1^3S_1$	5.325	5.299	0.026
B_s: $b\bar{s}1^1S_0$	5.366	5.360	0.006
B_s^*: $b\bar{s}1^3S_1$	5.413	5.420	−0.007
B_c^-: $b\bar{c}1^1S_0$	6.276	6.276	0.000
η_b: $b\bar{b}1^1S_0$	9.389	9.345	0.044
$\Upsilon(1S)$: $b\bar{b}1^3S_1$	9.460	9.484	−0.024
$\Upsilon(2S)$: $b\bar{b}2^3S_1$	10.023	10.033	−0.010
$\Upsilon(3S)$: $b\bar{b}3^3S_1$	10.355	10.360	−0.005

Summary

- **Using Dirac's rel. constraints, TBDE successfully leads to the SR-type Eq.**
- **With Coulomb-type + linear pots, non-singular (well-beaved) rel. WF is obtained.**
- **For L=2,**
  - **tensor term cancels the extremely singular S-state pot.**
  - **At small r, S-wave is proportional to D-wave for L=2.**
- **Obtain mass spectrum of mesons**
- **Large $\pi-\rho$ splitting explained**

Future Work

- **Extends this potential to non-zero temperature.**
- **Find the dissociation temperature and cross section of a heavy quarkonium in QGP.**
- **Especially on J/ψ to explain its suppression OR enhancement.**
- **And more…**

Thank you for your attention!

Possible Choice of V(T)

- F & U or their combination from lattice QCD
- Blackhole Potential from hQCD which is analytically expressed and inherits temp-dep. (ongoing) → test the validity of Ads/CFT.

Backup slides

Introduction

✓ **Why heavy Quarkonium?**

- heavy : charm(m_c~1.3 GeV), bottom(m_b~4.7 GeB)
- small size but strong binding
- weak coupling with light mesons
- can survive through the deconfinement transition
- good probe for the thermal properties of QGP

	Mass	radius
π	0.14 GeV	0.06fm
p	0.94 GeV	0.87fm
ψ'	3.68 GeV	0.90fm
χ_c	3.53 GeV	0.72fm
J/ψ	3.1 GeV	0.50fm
Υ	9.5 GeV	0.28fm

Λ_{QCD} ~ 200 MeV

$\alpha_s(M_Q) \ll 1$

Relativistic Application(1)

- Applied to the Binding Energy of chamonium
- Without spin-spin interaction
  - M(exp)=3.067 GeV
  - Compare the result with PRC 65, 034902 (2002)

T/Tc	BE(non-rel.)	BE(rel.)	Rel. Error(%)
0.0	-0.67	-0.61	9.0
0.6	-0.56	-0.52	7.1
0.7	-0.44	-0.41	6.8
0.8	-0.31	-0.30	3.2
0.9	-0.18	-0.17	4.0
1.0	-0.0076	-0.0076	0.0

- At zero temperature, 10% difference at most!

Relativistic Application(2)

- With spin-spin interaction
  - M(S=0) = 3.09693 GeV

    M(S=1) = 2.9788 GeV
  - At T=0, relativistic treatment gives

    BE(S=0) = -0.682 GeV

    BE(S=1) = -0.586 GeV
  - Spin-spin splitting ~100 MeV

Overview of QQ Potential(1)

- **Pure Coulomb :** $A = -\frac{0.2}{r}$ and $S = 0$

  BE=-0.0148 GeV for color-singlet
  =-0.0129 GeV for color-triplet(no convergence)

- **+ Log factor :** $A = -\frac{0.2}{r\frac{1}{2}\ln\left(e^2 + \frac{1}{\Lambda^2 r^2}\right)}$ and $S = 0$

  BE=-0.0124 GeV for color-singlet
  =-0.0122 GeV for color-triplet

- **+ Screening :** $A = -\frac{0.2\, e^{-\mu r}}{r\frac{1}{2}\ln\left(e^2 + \frac{1}{\Lambda^2 r^2}\right)}$ and $S = 0$

  BE=-0.0124 GeV for color-singlet
  No bound state for color-triplet

Overview of QQ Potential(2)

- + String tension(with no spin-spin interaction)

$$A = -\frac{0.2\, e^{-\mu r}}{r \frac{1}{2} \ln\left(e^2 + \frac{1}{\Lambda^2 r^2} \right)} \quad \text{and} \quad S = -br$$

When b=0.17 BE=-0.3547 GeV

When b=0.2 BE=-0.5257 GeV

Too much sensitive to parameters!

QQ Potential

- **Modified Richardson Potential**

$$A=-\frac{4}{3}\alpha_s\frac{e^{-\mu r}}{r\frac{1}{2}\ln\left(e^2+\frac{1}{\Lambda^2 r^2}\right)}$$

and

$$S=-\frac{8\pi}{27}\Lambda^2 r$$

Parameters : m, Λ

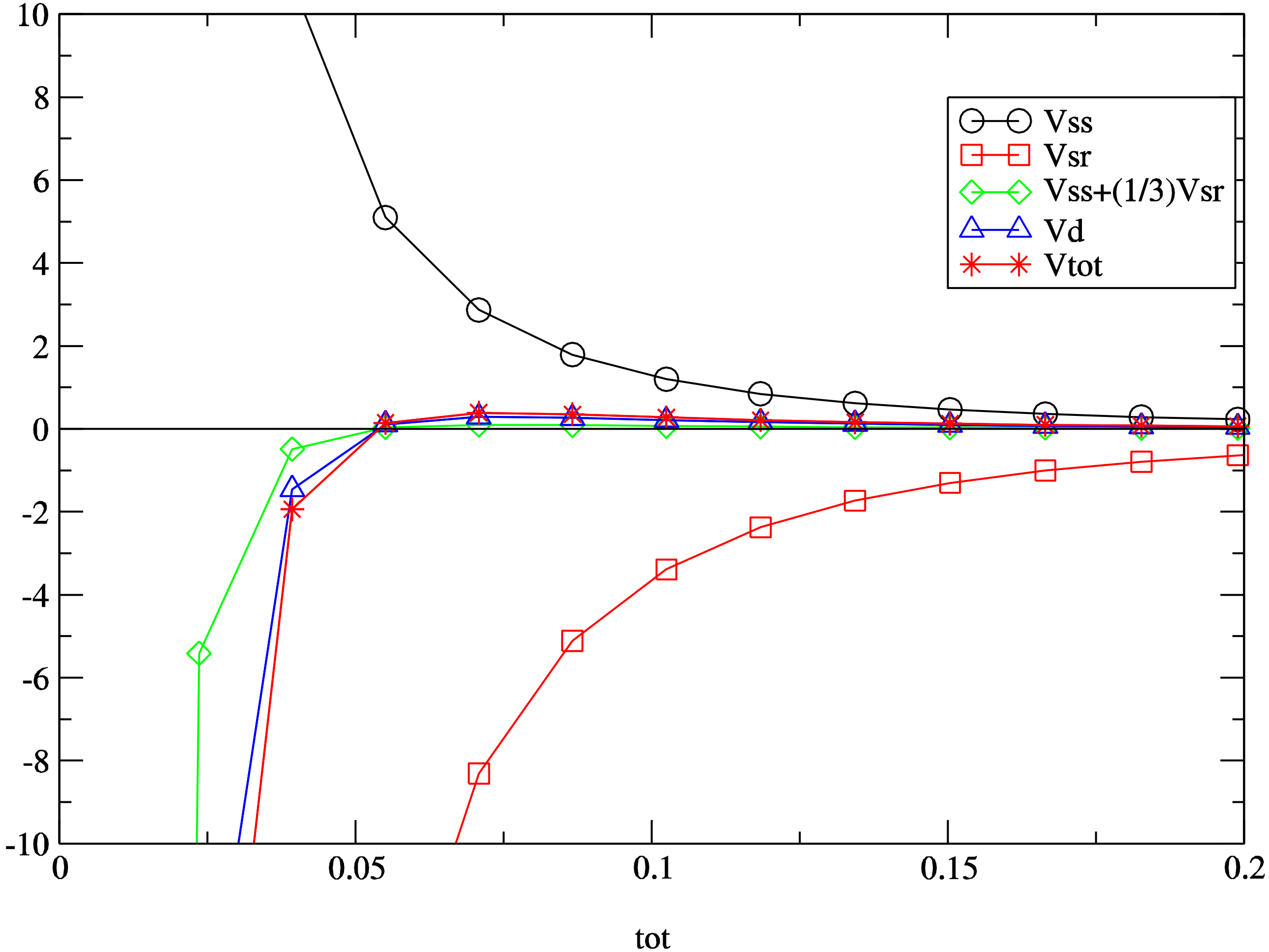
And mass=m(T)

$$\alpha_s=\frac{4\pi}{\left(11-\frac{2}{3}n_f\right)}=\frac{12\pi}{27}$$

A : color-Coulomb interaction with the screening

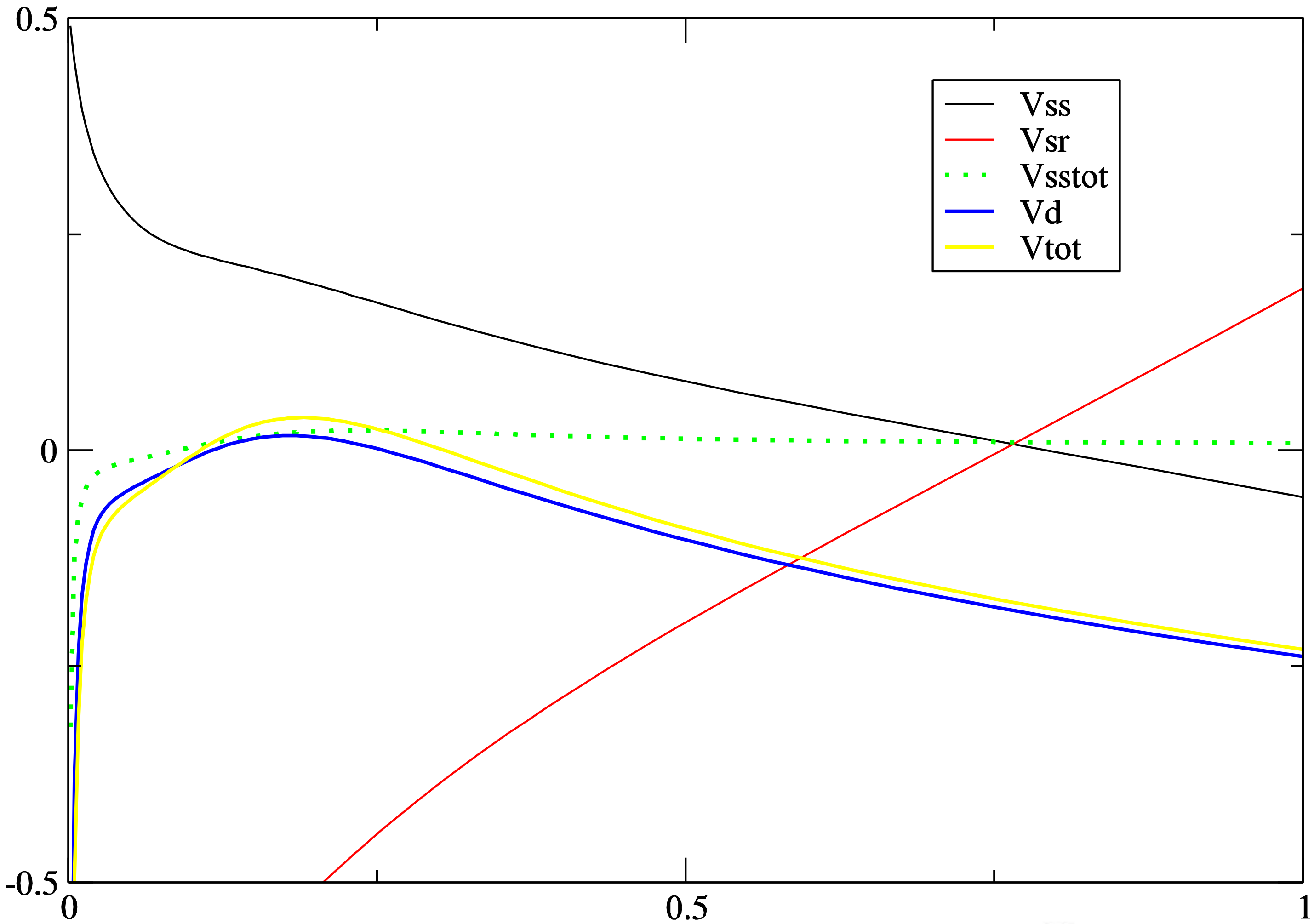
S : linear interaction for confinement

V(J/ψ)

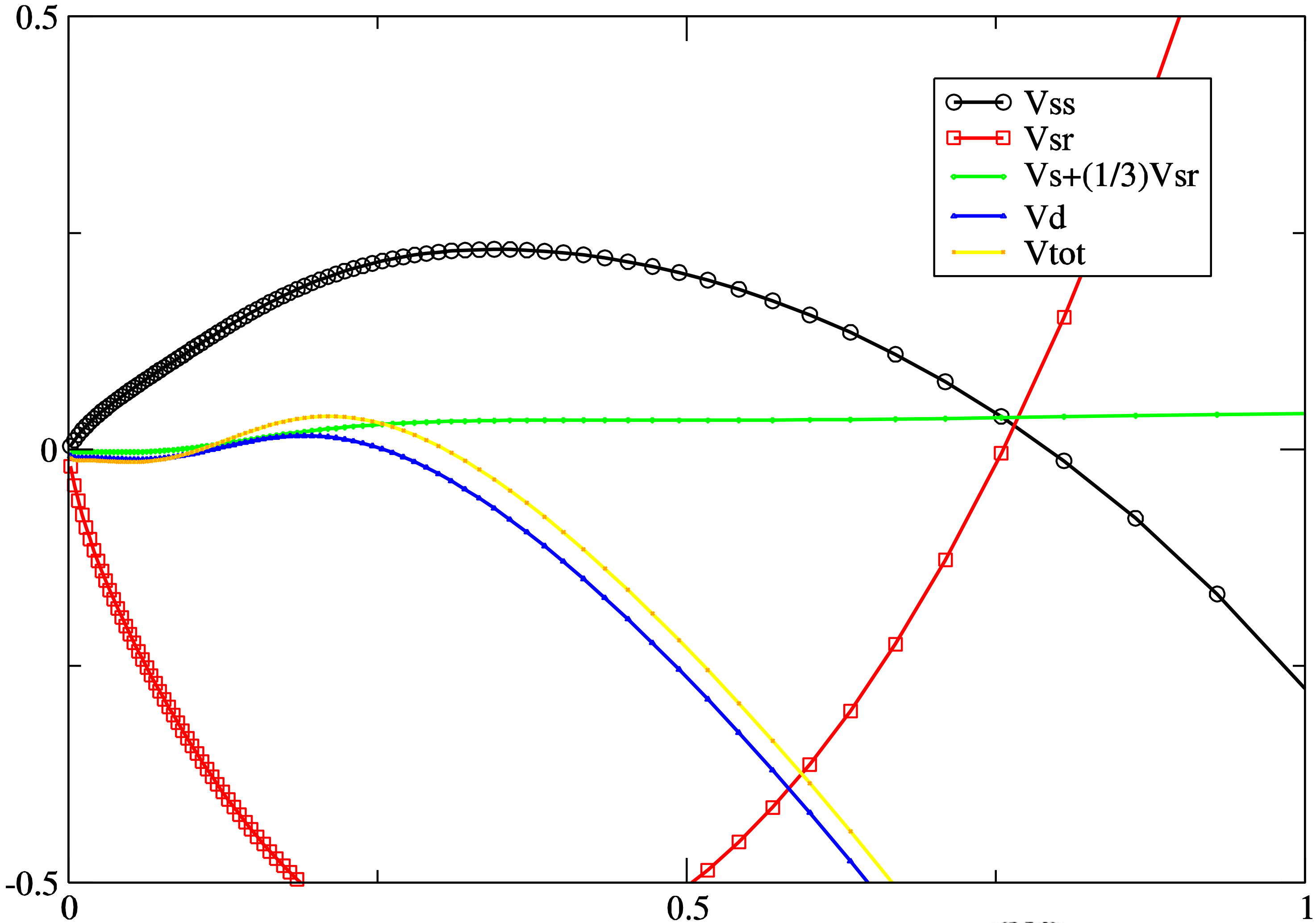


Too Much Attractive!

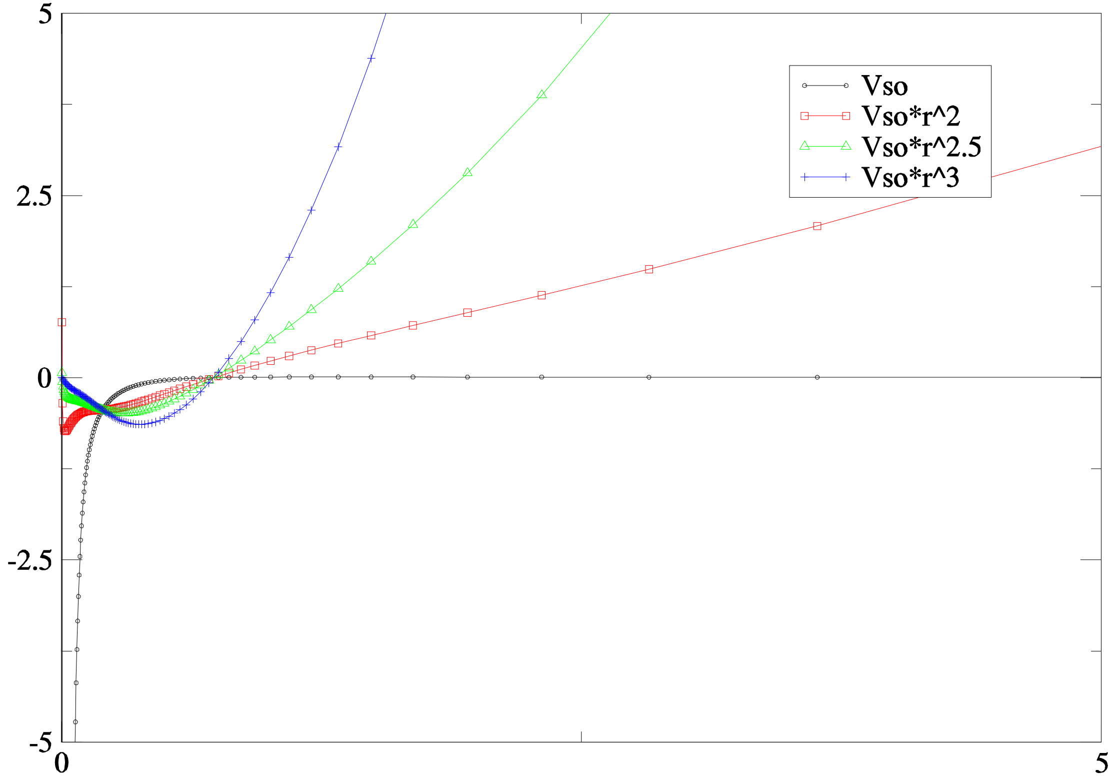
V(J/ψ) *r²



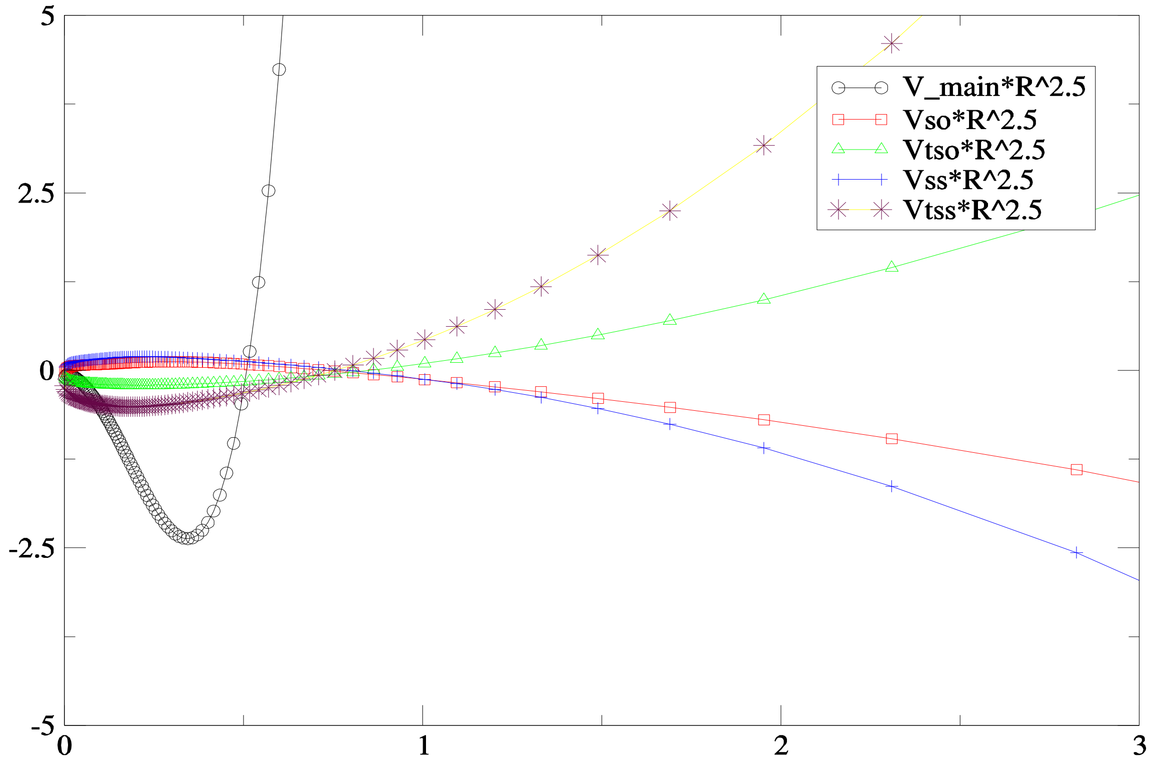
V(J/ψ) *r³



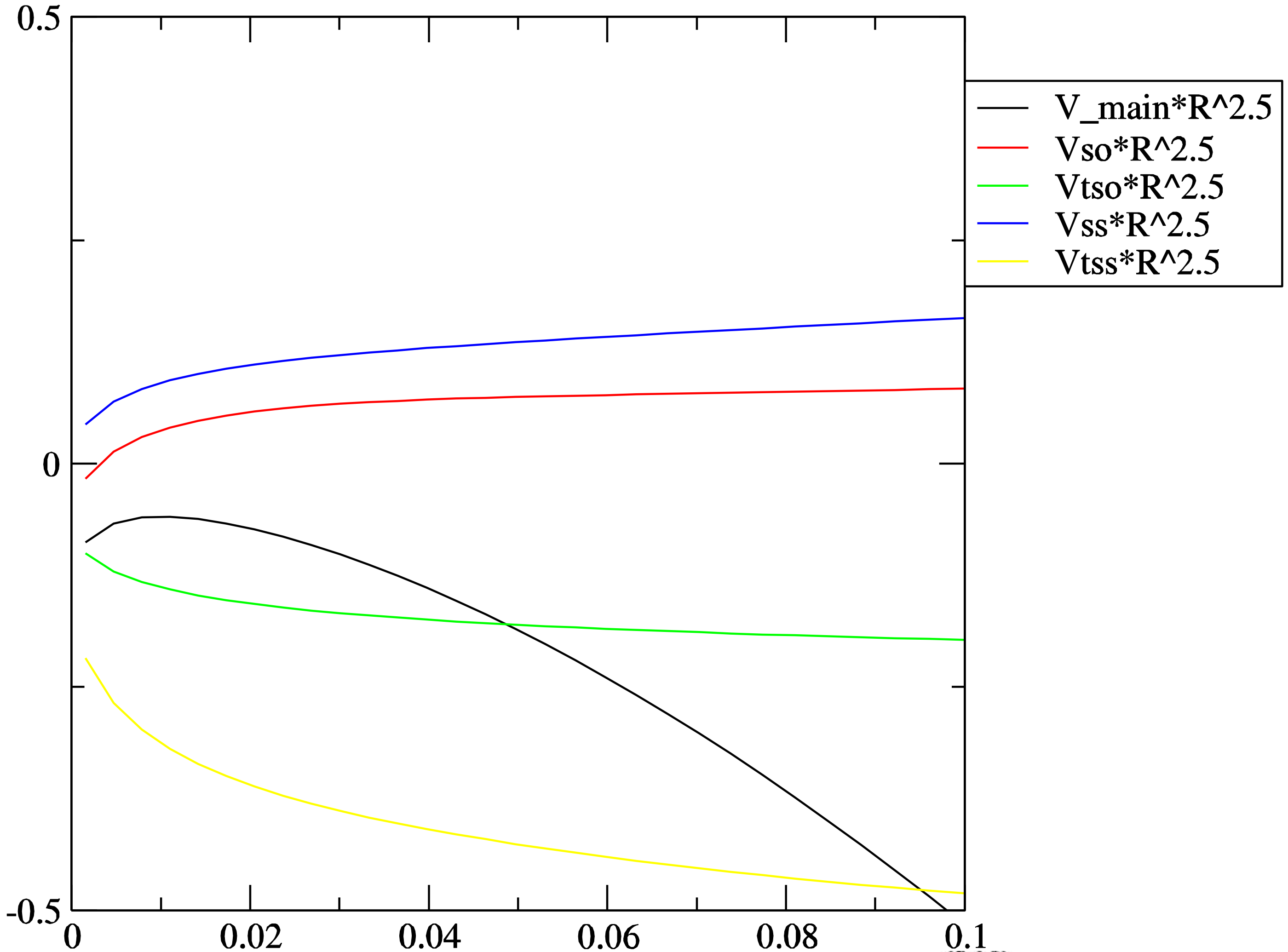
Vso(J/ψ)



$V(J/\psi)*r^{2.5}$



V(J/ψ) at small range

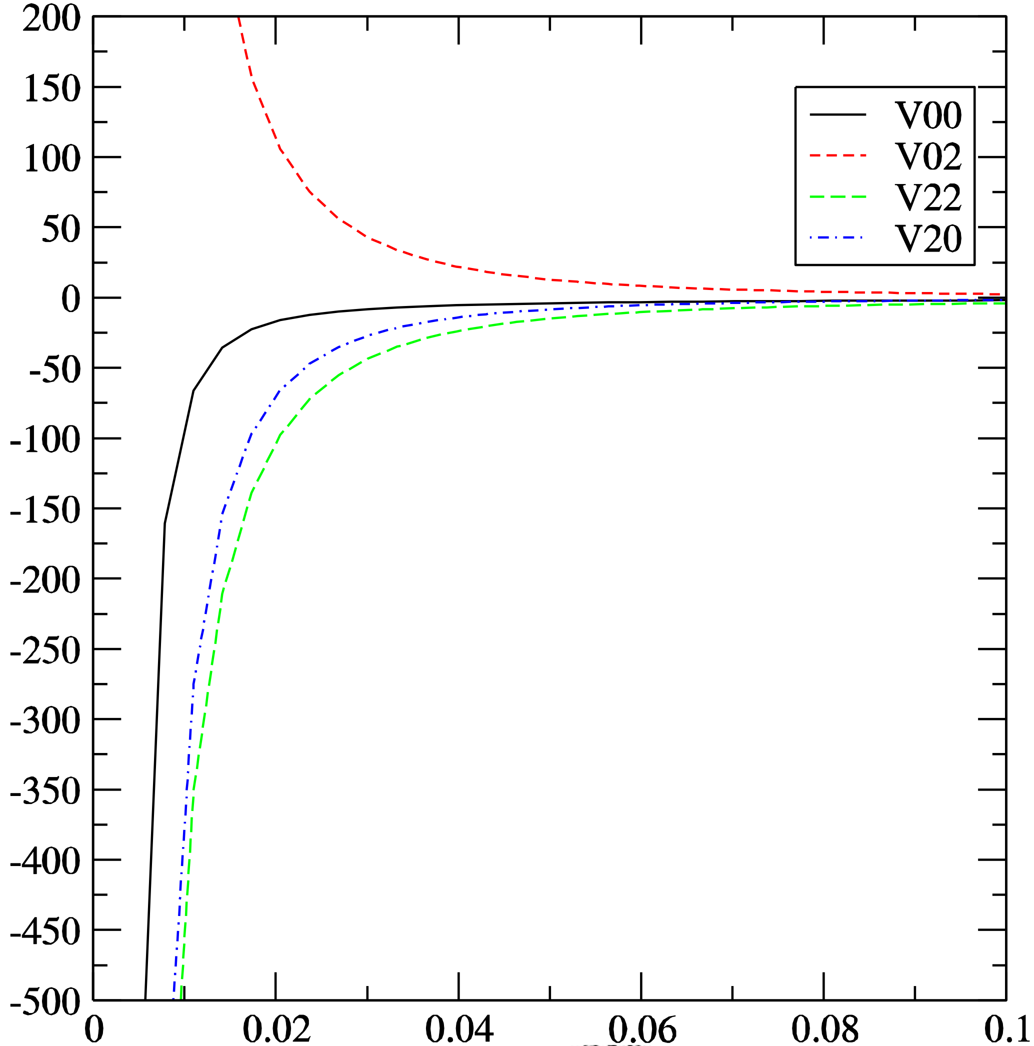


V(J/ψ) with mixing

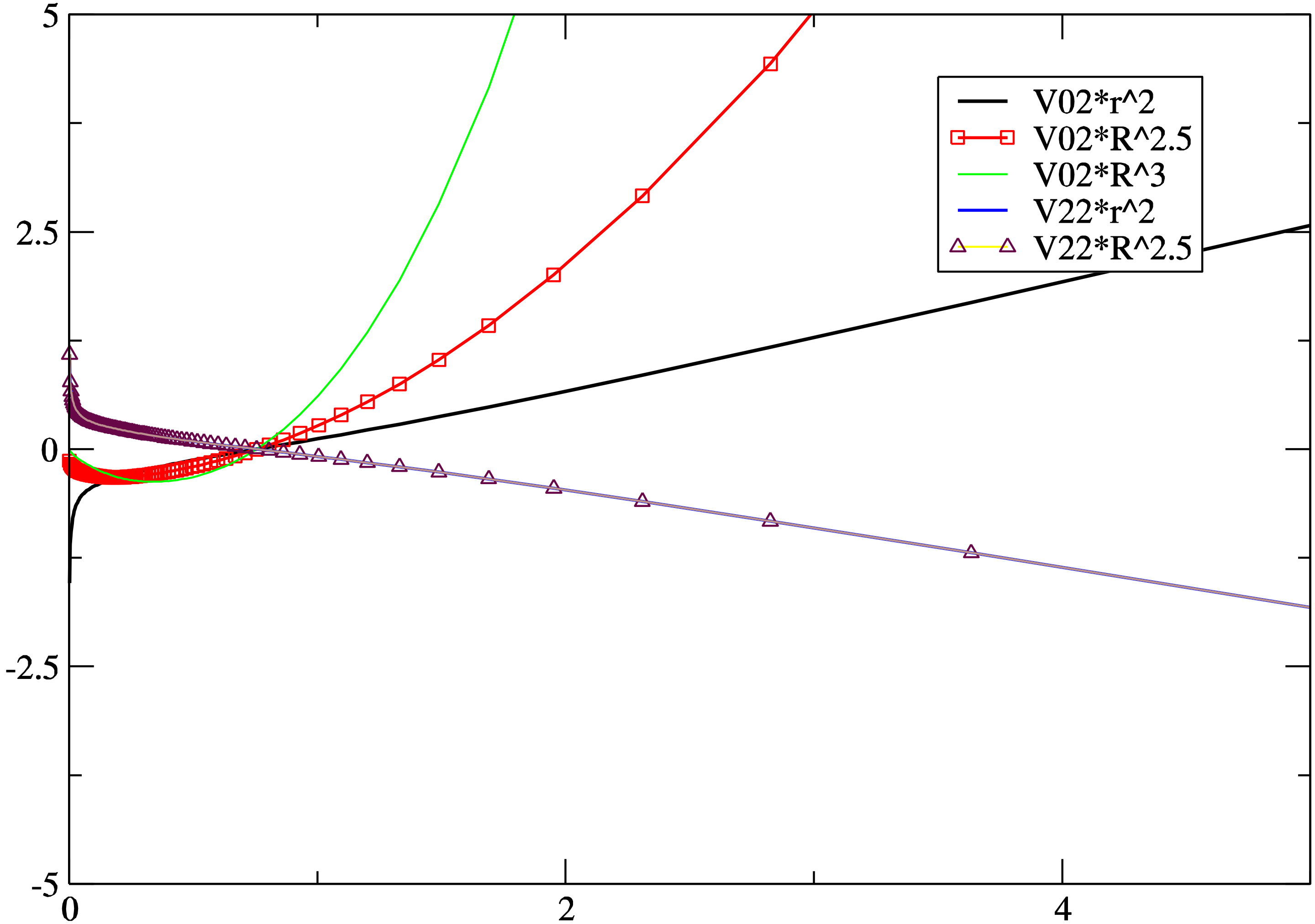
For J/y, S=1 and J=1.

Without mixing(L=0 only), splitting is reversed.

Therefore there has to be mixing between L=0 and L=2 states.



V(J/ψ) with mixing



Work on process

- To solve the S-eq. numerically,
- We introduce basis functions

  $\phi_n(r)=N_n r^l \exp(-n\beta^2 r^2/2)\ Y_{lm}$

  $\phi_n(r)=N_n r^l \exp(-\beta r/n)\ Y_{lm}$

  $\phi_n(r)=N_n r^l \exp(-\beta r/\sqrt{n})\ Y_{lm}$

  …

  - None of the above is orthogonal.
  - We can calculate $<p^2>$ analytically, but all the other terms has to be done numerically.
  - The solution is used as an input again → need an iteration
  - Basis ftns. depend on the choice of β quite sensitively and therefore on the choice of the range of r.