η' at finite temperature

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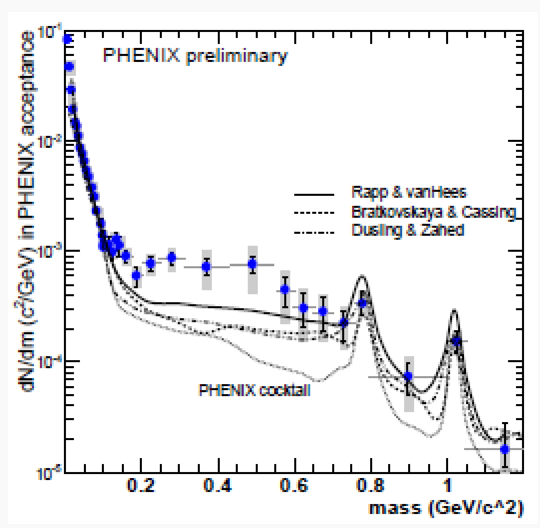
Quark condensate and the η' mass

1. Quark condensate and the η' meson

1. Some introduction
2. Casher Banks formula
3. Lee-Hatsuda formula
4. Witten – Veneziano formula
5. At finite temperature and density

Some introduction

- QCD symmetry: $SU(N)_L x\ SU(N)_R \rightarrow SU(N)_V$ and $U(1)_A$ is always broken by Anomaly
  QCD $\rightarrow$ confinement
- Kapusta, Kharzeev, McLerran PRD 96 : Effective $U(1)_A$ restoration in medium in SPS data
- Rapp, Wambach, van Hees: SPS dilepton data
- RHIC dilepton puzzle



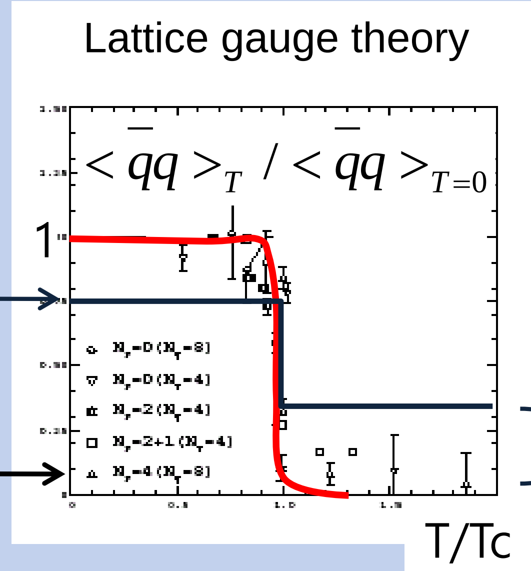
- Csorgo, Vertesi, Sziklai : Additional contribution from η' mass reduction
- Experimental and theoretical works on η' in nuclear medium

Quark condensate – Chiral order parameter

Finite temperature

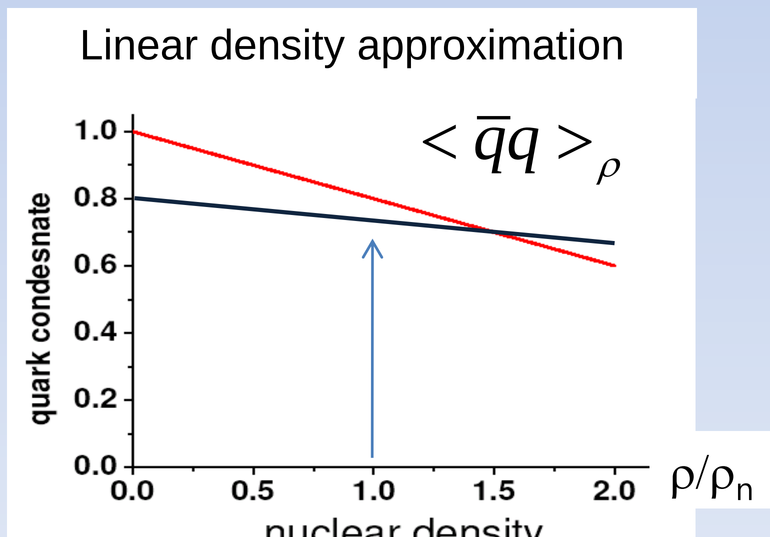
$$< \bar{s}s >_T / < \bar{q}q >_{T=0} = 0.8$$

$$\langle \bar{c}c \rangle = -\frac{1}{12 m_c} \left\langle \frac{\alpha}{\pi} G^2 \right\rangle$$



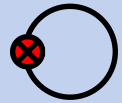
$$< \bar{s}s >_T \propto m_s \exp(-m_s / T)$$

Finite density



Casher Banks formula - Chiral symmetry breaking (m→0)

- *Quark condensate*



$$\langle \bar{q}(0) q(0) \rangle = -\lim_{x\to 0} \mathrm{Tr}\left[S(x,0)\right] = -\int dA e^{-S_{QCD}} \mathrm{Tr}\left[\left\langle 0 \right| \frac{1}{\not{D}+m} \left| 0 \right\rangle\right]$$

→ Phenomenologically need constituent quark mass

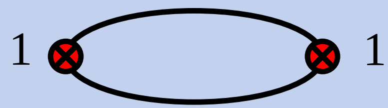
→ Casher Banks formula in the chiral limit

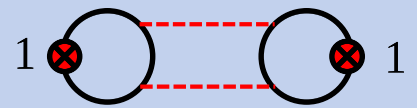
$$\langle \bar{q}(0) q(0) \rangle = -\mathrm{Tr}\left[\left\langle 0 \right| \frac{1}{\not{D}+m} \left| 0 \right\rangle\right] = \int d\lambda \, \psi_\lambda^{+}(0) \psi_\lambda(0) \frac{m}{m^2+\lambda^2} \xrightarrow{m\to 0} \pi \rho(\lambda = 0)$$

• *Other order parameters: $\sigma - \pi$ correlator (mass difference)*

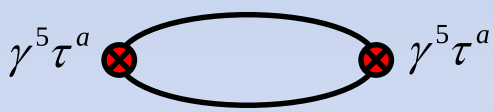
$$\frac{1}{V}\int d^4x \left[\langle \bar{q}(x)q(x), \bar{q}(0)q(0)\rangle - \langle \bar{q}(x)\tau^a i\gamma^5 q(x), \bar{q}(0)\tau^a i\gamma^5 q(0)\rangle \right]$$

$$= -\mathrm{Tr}\left[S(x,0)\; S(0,x)\right] + \langle \mathrm{Tr}\left[S(x,x)\right] \times \mathrm{Tr}\left[S(0,0)\right]\rangle$$





$$+\mathrm{Tr}\left[\tau^a i\gamma^5 S(x,0)\; \tau^a i\gamma^5 S(0,x)\right]$$



→ Phenomenologically constituent quark mass terms survives

→ Generalized Casher Banks formula in the chiral limit

$$= \mathrm{c}\left(\pi\rho(\lambda = 0)\right)^2$$

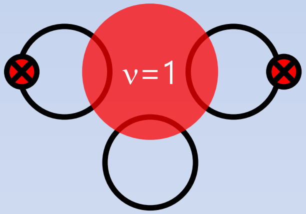
- *$\eta' - \pi$ correlator (mass difference)*

$$\frac{1}{V}\int d^4x e^{ikx}\left[\left\langle \bar{q}(x) i\gamma^5 q(x), \bar{q}(0) i\gamma^5 q(0)\right\rangle - \left\langle \bar{q}(x)\tau^a i\gamma^5 q(x), \bar{q}(0)\tau^a i\gamma^5 q(0)\right\rangle\right]$$

$$= \quad -\mathrm{Tr}\left[i\gamma^5 S(x,0) i\gamma^5 S(0,x)\right] \qquad + \qquad \left\langle \mathrm{Tr}\left[i\gamma^5 S(x,x)\right]\times \mathrm{Tr}\left[i\gamma^5 S(0,0)\right]\right\rangle$$

$$+\mathrm{Tr}\left[\tau^a i\gamma^5 S(x,0)\tau^a i\gamma^5 S(0,x)\right]$$

$$+\frac{1}{V}\int d^4x \left\langle \bar{u}_0(x) d_0(0)\, \bar{d}_0(0) u_0(x) \int d^4y\, \bar{s}_0(y) m_s s_0(y) + \text{permutations}\right\rangle_{\nu\neq 0}$$



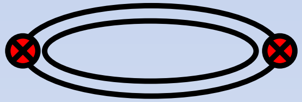
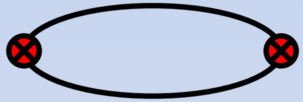
$$< \ \left(\pi\rho(\lambda=0)\right)^2 \qquad + \left[\text{const for SU(2)}\right] \text{x} \prod_{q>3} m_q \langle \bar{q} q\rangle$$

T. Cohen (96) — Lee, Hatsuda (96)

→ U(1) $_A$ symmetry will effectively be restored up to quark mass terms in SU(3)

η' mass? Witten-Veneziano formula - I

- Correlation function $$P(k) = -i\int dx e^{ikx}\langle G\tilde{G}(x), G\tilde{G}(0)\rangle$$
- Contributions from glue only $$P_0(k=0) \neq 0$$ from low energy theorem
- When massless quarks are added $$P(k) = -i\int dx e^{ikx}\langle \partial^{\mu} j_{\mu}^{5}(x), \partial^{\mu} j_{\mu}^{5}(0)\rangle \propto k^{\mu}k^{\nu}P_{\mu\nu} \xrightarrow{k=0} 0$$
- Large Nc argument $$P(k) = \sum_{glueballs} \frac{\langle 0 | G\tilde{G} | glueball\rangle^2}{k^2 - m_n^2} + \sum_{mesons} \frac{\langle 0 | G\tilde{G} | meson\rangle^2}{k^2 - m_n^2}$$

$G\tilde{G}$  $G\tilde{G}$ $G\tilde{G}$  $G\tilde{G}$

N_c^2 N_c

- Need η' meson $$+\frac{\langle 0 | G\tilde{G} | \eta'\rangle^2}{k^2 - m_{\eta'}^2} \quad \text{with} \quad m_{\eta'}^2 \approx O\left(\frac{1}{N_c}\right)$$

$$\rightarrow \quad P(k=0) = P_0(0) - \frac{\langle 0 | G\tilde{G} | \eta'\rangle_c^2}{m_{\eta'}^2}$$

Witten-Veneziano formula – II

- η' meson

$$\frac{\left\langle 0 | G\tilde{G} | \eta' \right\rangle^2}{m_{\eta'}^2} = P_0(0)$$

$$\frac{\left(\frac{4\pi}{\alpha}\right)^2 \left(\frac{N}{2N_F}\sqrt{N_F}\, m_{\eta'}^2 f_{\eta'}\right)^2}{m_{\eta'}^2} = \left(\frac{4\pi}{3\alpha}\right)^2 \frac{8}{11} \left\langle \frac{\alpha}{\pi} G^2 \right\rangle$$

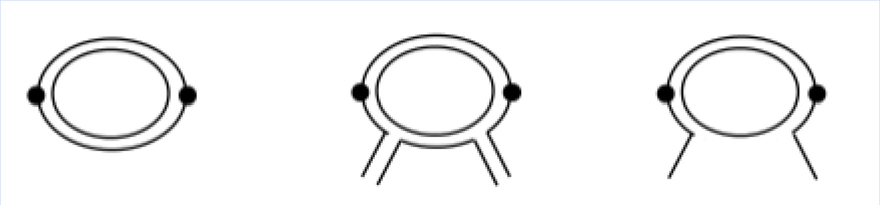
Lee, Zahed (01)

$$m_{\eta'}^2 f_{\eta'}{}^2 = N_F \left(\frac{2}{3N}\right)^2 \frac{8}{11} \left\langle \frac{\alpha}{\pi} G^2 \right\rangle \rightarrow \quad m_{\eta'} = 432 \text{ MeV}$$

$$m_{\eta'}(958) - m_\eta(547) = 411\,\text{MeV}$$

Witten-Veneziano formula in medium (Kwon, Morita, Wolf, Lee in prep)

- *Large Nc counting* $$P(k) = i\int dx e^{ikx}\left\langle G\tilde{G}(x), G\tilde{G}(0)\right\rangle_m$$



N_c^2 N_c^2 N_c

- *LET (Novikov, Shifman, Vainshtein, Zhakarov) at finite temperature for S(k): Ellis, Kapusta, Tang (98)*

$$\frac{d}{d\left(-1/4g_0^2\right)}\left\langle Op\right\rangle = -i\int dx e^{ikx}\left\langle Op(x), g_0^2 GG(0)\right\rangle$$

$$\left\langle Op\right\rangle_T = const\left[M_0 \exp\left(-\frac{8\pi^2}{bg_0^2}\right)\right]^d + c'T^d = \left\langle Op\right\rangle_{T0} + c'T^d$$

$$\frac{d}{d\left(-1/4g_0^2\right)}\left\langle Op\right\rangle_T = \frac{32\pi^2}{b}\left(d - T\frac{\partial}{\partial T}\right)\left\langle Op\right\rangle_T = \frac{32\pi^2}{b}\left(d - T\frac{\partial}{\partial T}\right)\left\langle Op\right\rangle_{T0}$$

- *LET at finite temperature for P(k) : Lee, Zahed (01)*

$$P(k=0)=\left(\frac{4\pi}{3\alpha}\right)^2\frac{2}{11}\left[d-T\frac{\partial}{\partial T}\right]\left\langle\frac{\alpha}{\pi}G^2\right\rangle$$

- $\left\langle 0\,|\,G\tilde{G}\,|\,\eta'\right\rangle$

$$P(k)=\int d^4x e^{ikx}\left[\left\langle G\tilde{G}(x),G\tilde{G}(0)\right\rangle\right] = \frac{k^4\left\langle 0\,|\,G\tilde{G}\,|\,0\right\rangle^2}{k^2-m_{\eta'}^2}+\ldots$$

$$=k^\mu k^\nu\int d^4x e^{ikx}\left(\frac{2\pi N}{\alpha N_F}\right)^2\left[\left\langle\bar{q}(x)i\gamma_\mu\gamma^5 q(x),\bar{q}(0)i\gamma_\nu\gamma^5 q(0)\right\rangle-\left\langle\bar{q}(x)i\gamma_\mu q(x),\bar{q}(0)i\gamma_\nu q(0)\right\rangle\right]$$

$$\xrightarrow{\text{chiral sym restored phase}} 0$$

Therefore, $\left\langle 0\,|\,G\tilde{G}\,|\,\eta'\right\rangle\to 0$

- *W-V formula at finite temperature:*

$$\frac{\langle 0 | G\tilde{G} | \eta' \rangle^2}{m_{\eta'}^2} = P_0(0) \longrightarrow \left(\frac{4\pi}{3\alpha}\right)^2 \frac{2}{11}\left(d - T\frac{\partial}{\partial T}\right)\left\langle \frac{\alpha}{\pi} G^2 \right\rangle$$

$$\downarrow$$

$$\frac{\langle \bar{q} q \rangle^2}{m_{\eta'}^2}$$

Smoothes out temperature change because if

$$\left\langle \frac{\alpha}{\pi} G^2 \right\rangle = -a g^4 T^4 + \left\langle \frac{\alpha}{\pi} G^2 \right\rangle_0$$

$$\left(\frac{4\pi}{3\alpha}\right)^2 \frac{2}{11}\left(d - T\frac{\partial}{\partial T}\right)\left\langle \frac{\alpha}{\pi} G^2 \right\rangle \approx \left(\frac{4\pi}{3\alpha}\right)^2 \frac{2}{11}(d)\left\langle \frac{\alpha}{\pi} G^2 \right\rangle_0$$

Therefore , : $m_{\eta'} \propto \langle \bar{q} q \rangle$ → *Observable consequences ?*

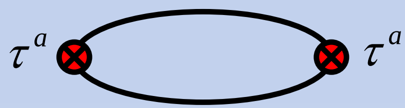
Summary

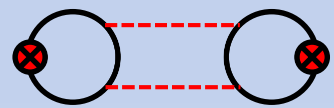
1. η' mass is related to quark condensate and thus should reduce in medium
   →
   a) Could serve as signature of chiral symmetry restoration
   b) Dilepton in Heavy Ion collision
   c) Measurements from nuclear targets

- *Other order parameters:* $\sigma - \pi$ *correlator (mass difference)*

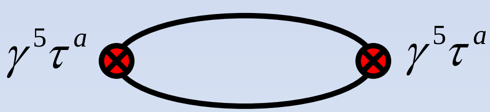
$$\frac{1}{V}\int d^4x \left[\langle \bar q(x) q(x), \bar q(0) q(0)\rangle - \langle \bar q(x)\tau^a i\gamma^5 q(x), \bar q(0)\tau^a i\gamma^5 q(0)\rangle \right]$$

$$= -\mathrm{Tr}\left[\left\langle x\left|\frac{1}{\not D+m}\right|0\right\rangle\left\langle 0\left|\frac{1}{\not D+m}\right|x\right\rangle\right] \qquad + \left\langle \mathrm{Tr}\left[\left\langle x\left|\frac{1}{\not D+m}\right|x\right\rangle\right]\times \mathrm{Tr}\left[\left\langle 0 \,\frac{1}{\not D+m}\right|0\right\rangle\right]\right\rangle$$





$$+\,\mathrm{Tr}\left[\tau^a i\gamma^5\left\langle x\left|\frac{1}{\not D+m}\right|0\right\rangle\tau^a i\gamma^5\left\langle 0\left|\frac{1}{\not D+m}\right|x\right\rangle\right]$$



$$= \mathrm{c}\left(\pi\rho(\lambda=0)\right)^2$$