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A Blast-wave Model with Two Freeze-outs

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- Chemical and thermal analysis, done in a single model

Measurement of hadrons

Particle identification and measurement of momentum gives (p_x, p_y, p_z, E) or (p_T, ϕ, y, E) within certain p_T and y ranges with errors.

How they are presented:

- numbers of particles N_i or ratios $R_{ij}(= N_i / N_j)$
- single particle spectrum

  dN / dp_T , dN / dy , $dN / d\phi$
- two particle correlations, $d^6 N / dp^3_{1,T} dp^3_{2,T}$

Chemical analysis of multiplicities with Statistical Model

Thermal statistical distribution

$$\frac{d^3 N_i}{dp^3} = \frac{d_i}{(2\pi)^3} \int d^3x \int d^3p\, f(x,p) \quad \text{with} \quad f(p) = \frac{1}{e^{(E-\mu_i)/T} \pm 1}$$

Chemical equilibrium is assumed:

- Conservation of baryon number
- Conservation of strangeness number

$$\mu_i = N_q \mu_q + N_q \mu_q$$

By taking ratios, hope to cancel the kinematics, and thus to eliminate the experimental limits(cut-offs)

$$R_{ij} = \frac{N_i}{N_j} = \frac{d_i}{d_j} e^{(\mu_i - \mu_j)/T}$$

❖ Careful treatment of resonance contributions is important.

Thermal analysis of pt spectra with blast-wave model

Information only at freeze-out is needed.
Blast-wave model works well in fitting the slopes of mt or pt spectra of various hadrons simultaneously with a common value of temperature, T.

$$T, \mu_i, \rho, R_0, \eta_{\max}$$

temperature
chemical potential
transverse radius
transverse expansion rapidity
longitudinal rapidity at the surface

However, different normalization constants are needed for different hadrons at SPS or RHIC energies.

❖ Careful treatment of resonance contributions is important.

Blast Wave Model

Cooper-Frye Formula $$E\frac{d^3N}{d^3p} = \frac{g}{(2\pi)^3}\int_{\Sigma_f} p^{\mu} d\sigma_{\mu}(x) f(x,p)$$

$$f(x,p) = \exp(-\frac{p_{\nu}u_{\nu}(x)-\mu}{T})$$

freeze-out hypersurface $d\sigma_{\mu}$

For an ellipsoidally expanding fireball

$$\frac{d^2N_i^{th}}{m_T dm_T dy} = \frac{d_i V}{2\pi}\int_{-\eta_{max}}^{\eta_{max}} d\eta \int_0^{r_{max}(\eta)} r dr m_T \cosh(y-\eta)$$
$$\times \exp(-\frac{m_T\cosh(y-\eta)\cosh\rho - \mu_i}{T}) I_0(\frac{p_T\sinh\rho}{T})$$

$$v_L = z/t$$

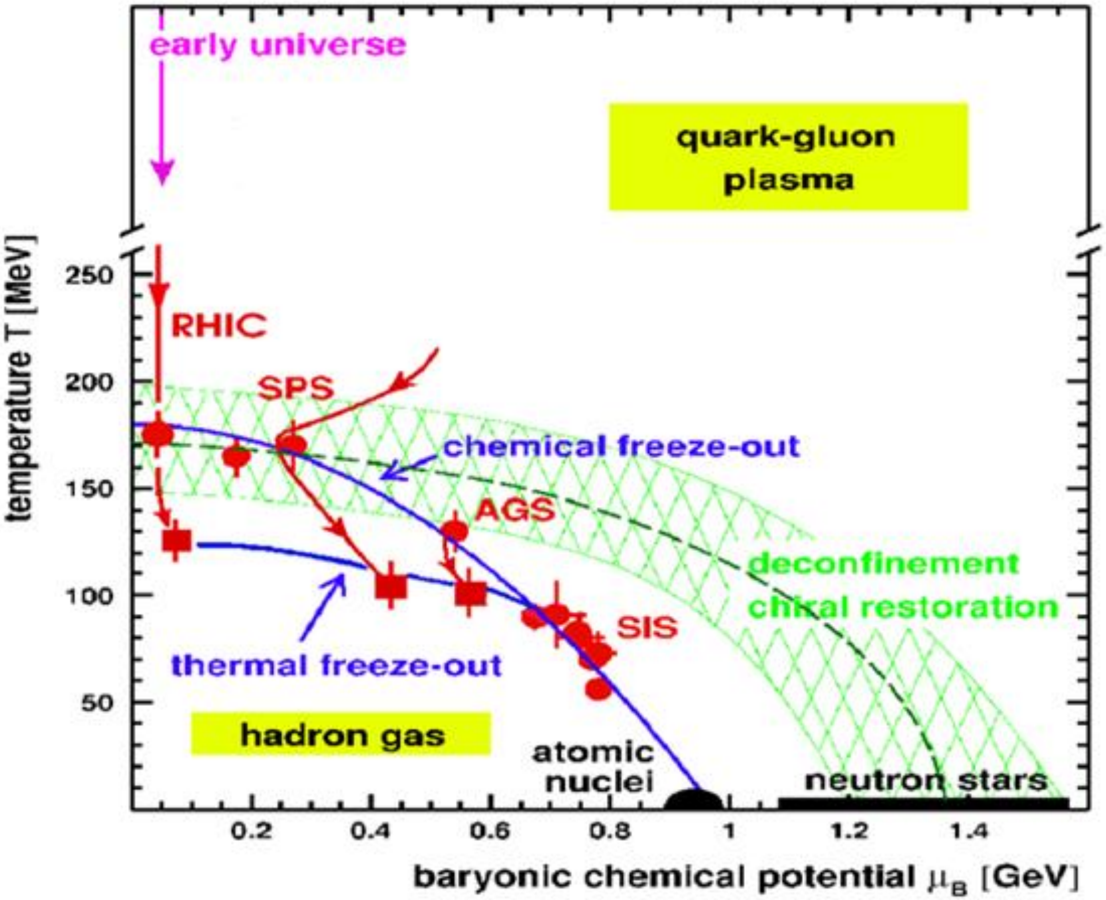
$$\eta = \tanh^{-1} z/t$$

$$r_{max}(\eta) = R_0\sqrt{1-\frac{\eta^2}{\eta_{max}^2}}$$

$$\rho(r) = \rho_0(r/r_{max})^{\alpha}$$

H. Dobler, J. Sollfrank, U. Heinz, P.L. B457,353(1999)

Chemical and thermal analysis of produced hadrons



Chemical analysis of hadron ratios

$$R = e^{-(\mu_i - \mu_j)/T}$$

Thermal analysis of mt spectra of hadrons

$$e^{[-\gamma(E-\beta P_L)-\mu]/T}$$

$$T_{ch} > T_{th}$$

❖ Careful treatment of resonance contributions is important in both analysis.

Interpretation of $T_{ch} > T_{th}$

- At Tch, chemical freeze-out occurs if inelastic collisions, which makes A+B->C+D, are not abundant. Then the numbers of each species, A,B,C, and D are not changing.
- At Tth, thermal freeze-out occurs if elastic collisions are not abundant. Then, the momentum distribution is not changing.
- Earlier chemical freeze-out and later thermal freeze-out.

Models to incorporate the fact that : $T_{ch} > T_{th}$

- Hydrodynamic equation + Hadronic Afterburner (UrQMD)

Nonaka and Bass, Heinz+Bass

- at Tsw, generate hadrons via Monte Carlo method

- Hydrodynamic equation + Partial Chemical Equilibrium (PCE)

Hirano and Tsuda, Teaney, Kolb and Rapp

- below Tch, fix Ni except for short lived resonances (eg. Delta) and solve for μ_i 's (13x13 matrix).

❖ When to compare the hydrodynamic calculation with experiment, one needs one of this scheme. HOT!

A Blast-wave model with two freeze-outs

Suk Choi, K. S. Lee, PRC84, 064905(2011)

- Chemical analysis at Tch
  – Lorentz boosted thermal distribution is used.
- At Tch>T>Tth, number of thermal hadrons of each hadron species fixed.
- Approximation: Short lived hadrons are treated as long lived ones and they decay outside, which causes small error but calculation becomes much simpler and fast.
- At Tth, thermal analysis of mt spectra
- Resonance contribution is carefully considered in both analysis via Sollfrank's program.

Chemical analysis – fitting ratios

$$N_i^{th} = \int\int m_T dm_T dy \frac{d^2 N_i^{th}}{m_T dm_T dy}(T, \mu_i, \eta_{max}, \rho_0, R_0)$$

Chemical Potential $\mu_i = (n_q - n_{\bar{q}})\mu_q + (n_s - n_{\bar{s}})\mu_s$

Chemical equilibrium is assumed up to this point:

- Conservation of baryon number
- Conservation of strangeness number

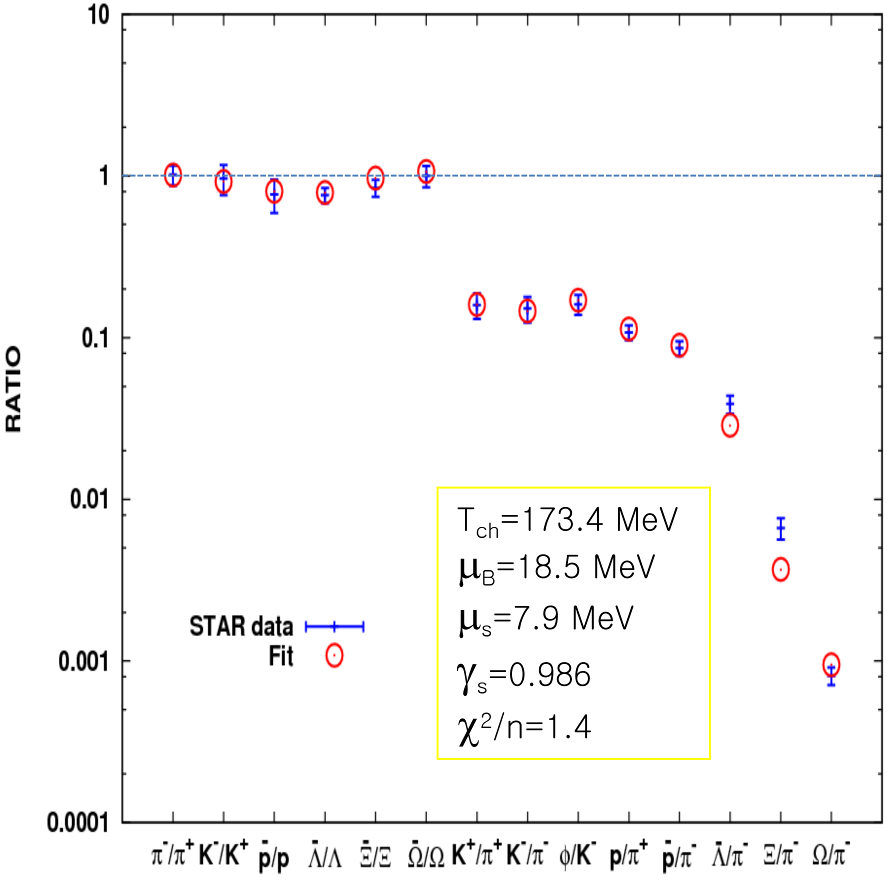
Contribution from decays of higher mass resonances: N_i^{res}

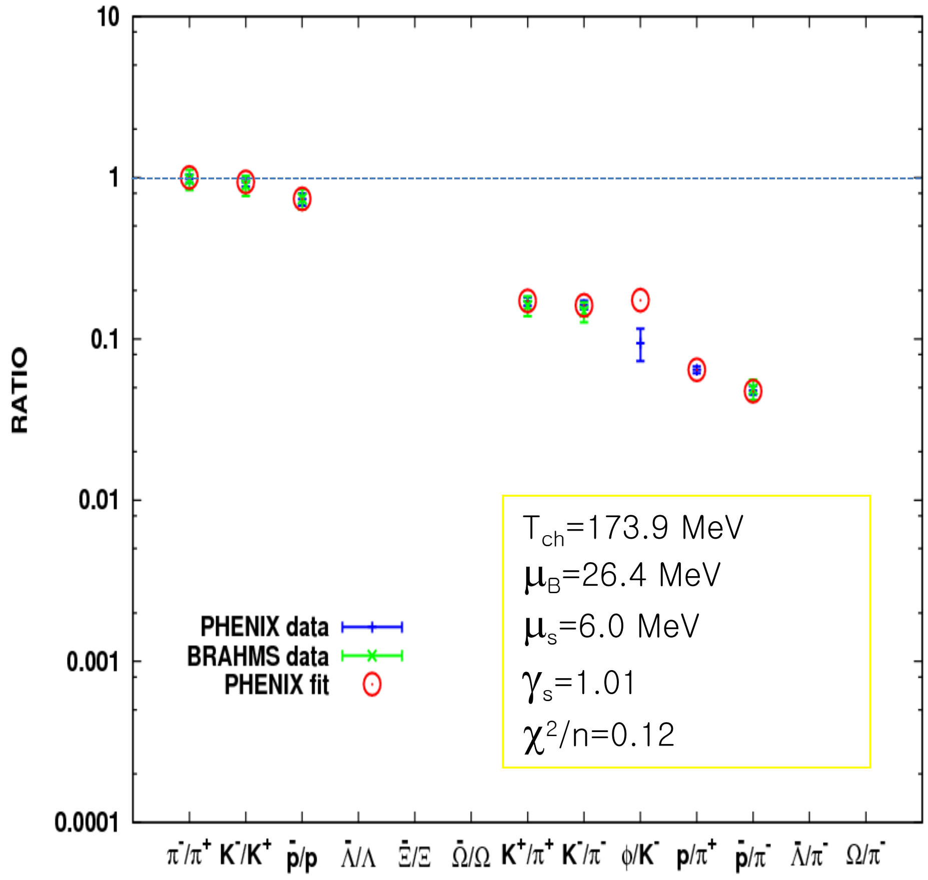
Total Particle Number $N_i = N_i^{th} + N_i^{res}$ or ratios $R_{ij} = \frac{N_i}{N_j}$

Fit for T, μ_q, μ_s

- In the chemical analysis, $\rho, R_0, \eta_{\max}$ are very insensitive parameters.
- Careful treatment of the range of y integration according to the experimental value is important in the chemical analysis.

Results of chemical analysis





Weak decay contribution is properly included.

Thermal analysis - fitting of transverse mass spectra

Transverse Mass Spectrum $$\frac{d^2N_i}{m_T dm_T dy} = \frac{d^2N_i^{th}}{m_T dm_T dy} + (\text{res. contr.})$$

Particle ratios are fixed at Tch.

Conservation of number of each species, Ni determines μ_i

$$\mu_i = \mu_\pi + T\ln\left[R_{i\pi}\frac{\int\int m_T dm_T dy(\frac{d^2N_i'}{m_T dm_T dy})}{\int\int m_T dm_T dy(\frac{d^2N_\pi'}{m_T dm_T dy})}\right]$$

from $R_{i\pi} = N_i^{th}/N_\pi^{th}$ μ_i relative to μ_π

the $'$ denotes that $\exp(\mu_i/T)$ is missing in this equation.

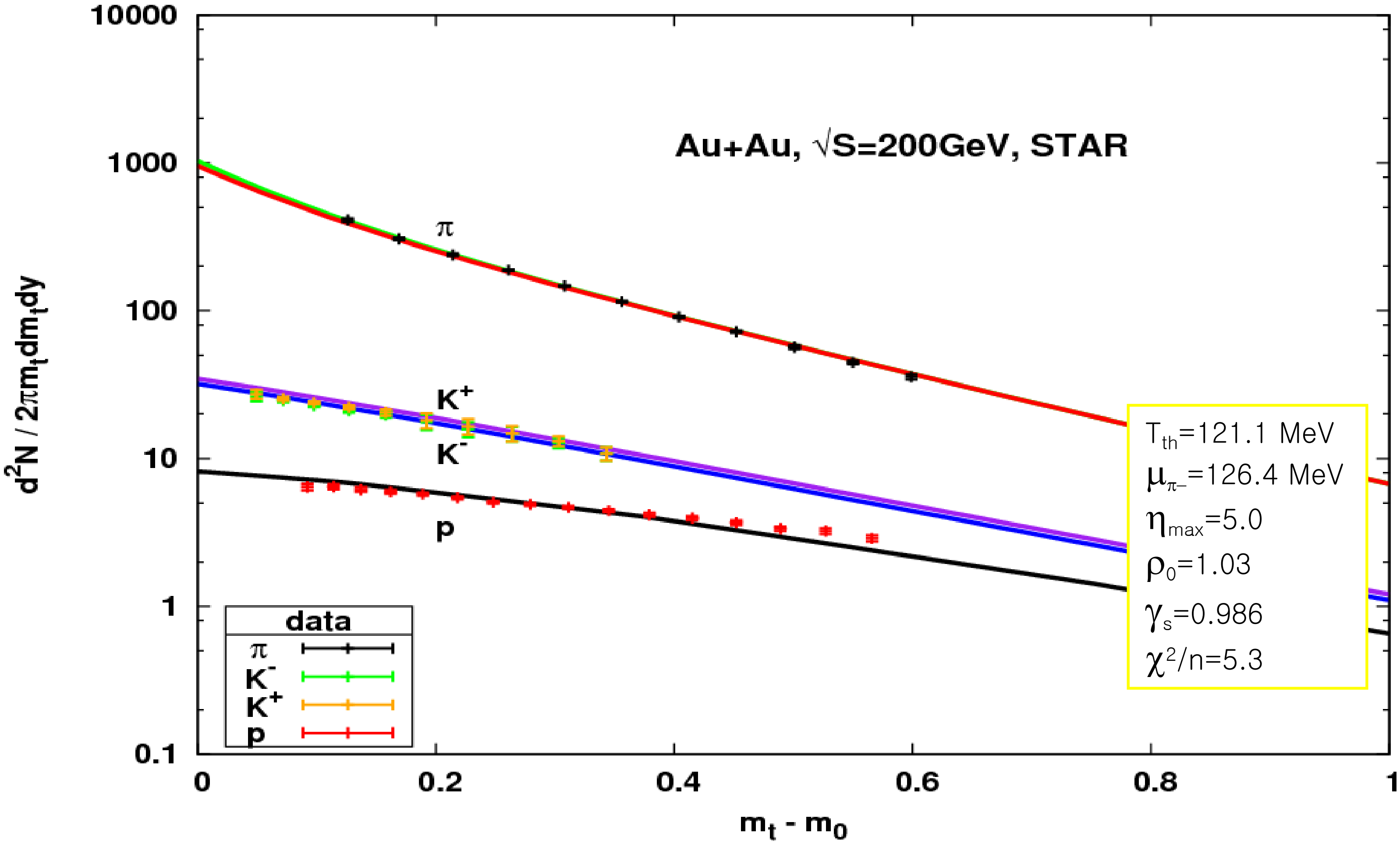
- Careful treatment of the range of y integration is important in the chemical analysis.

- Taking the ratio of number of hadrons with different y-ranges results in a large systematic error. Just publish them with the specification of the y-cutoffs, especially when y-cutoff depends on pT.

- Estimation of number of particles weak-decayed from other hadrons is not simple since the number of the mother particle is not known. Just leave the number as it is.

- In the chemical analysis $\rho, R_0, \eta_{\max}$ are very insensitive parameters.

Results of thermal analysis

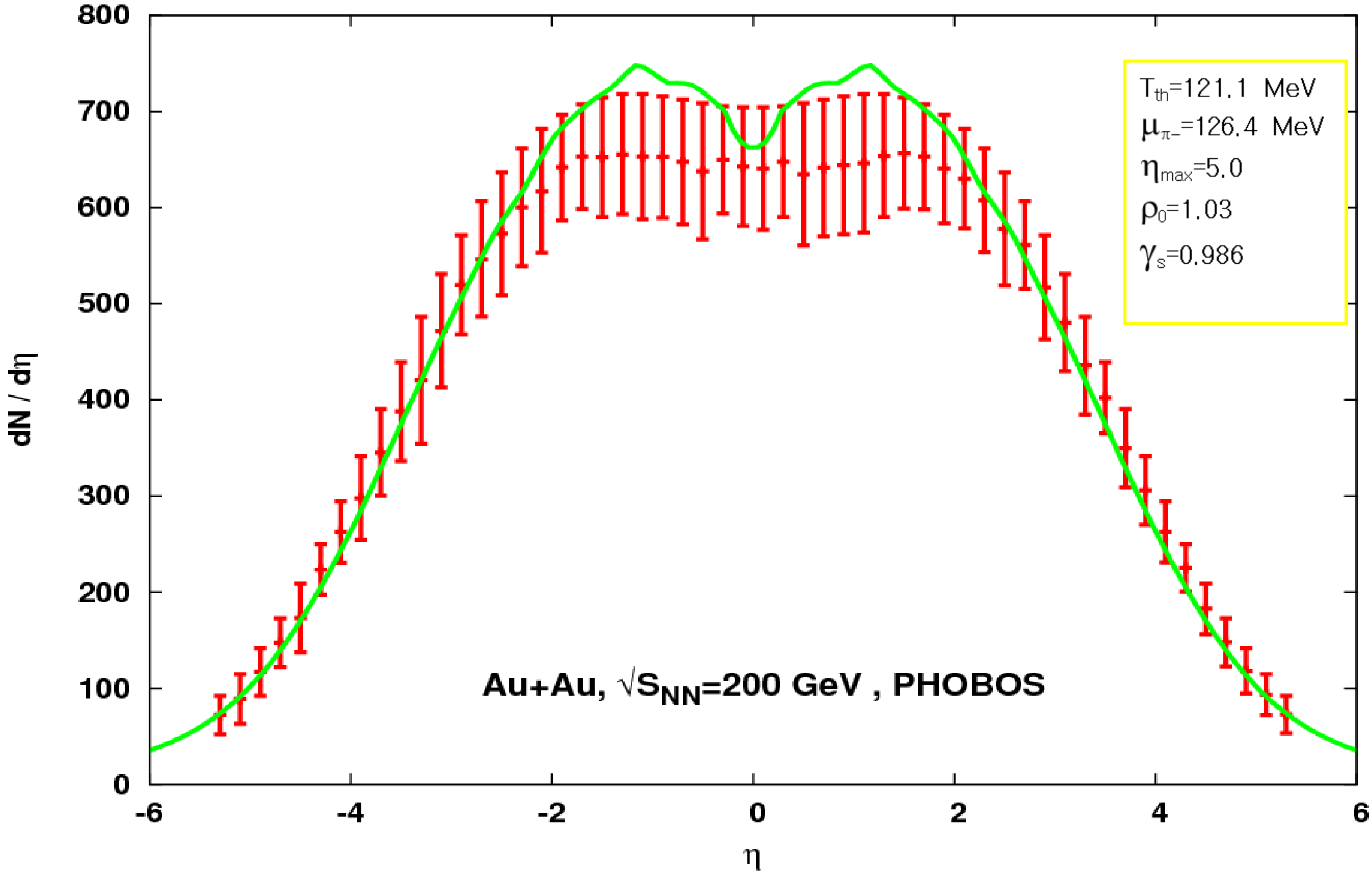


Fit of the width of $dN/d\eta$ distribution with η_{max}

The width of rapidity distribution can be adjusted with values of η_{max} , with all other parameters fixed except for the overall constant, V .

Thus the pseudo-rapidity distribution of charged hadron multiplicity is fitted with (V, η_{max}) as fit parameters.

Pseudo-rapidity distribution of charged hadrons



Conclusion

1. Within an expanding fireball model assuming two freeze-outs, both the yields, the magnitudes and slopes of the p_t spectra, and y-distribution of charged hadrons measured at RHIC are described.

2. Hadron ratios, mt spectra of pions, kaons and protons, and rapidity distribution of total charged hadrons are nicely fitted.

- Resonance contribution is important.
- For mt spectra, we have only one overall constant.
- Wide width of rapidity distribution is also nicely fitted by $\eta_{\max}$.

3. We are waiting for LHC data to analyze.