Equations of state for cold dense matter in D4/D6 model

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based on the work with Y. Kim, Y. Seo and S. J. Sin

JHEP **1106** (2011) 011 [arXiv:1011.0868 [hep-ph]] & arXiv:1108.2751 [hep-ph]

and with Y. Kim, C. -H. Lee and M. -B. Wan

JHEP **1110** (2011) 111 [arXiv:1108.6139 [hep-ph]]

Holographic QCD

- Quantum ChromoDynamics
  : Theory of Strong Interaction
  $\Rightarrow$ Usual tools such as perturbation are not valid

- Holography
  $(d+1)$-dim. (bulk) gravity $\sim$ d-dim. (boundary) field theory
  ex) AdS/CFT J. M. Maldacena, Adv. Theor. Math. Phys. 2, 231 (1998)
  $\mathcal{N}=4$ SYM on 4 dim. $\longleftrightarrow$ Type IIB SUGRA on $AdS_5 \times S^5$

- AdS/CFT dictionary

Gauge theory (boundary)	Gravity (bulk)
operator $\mathcal{O}$	field ϕ
source J	non-normalizable mode ϕ_0
expectation value $\langle\mathcal{O}\rangle$	normalizable mode
$\cdots$	$\cdots$

Holographic QCD

- Toward to QCD ??
  - * $\exists\ \Lambda_{\text{QCD}} \Rightarrow$ Not conformal
  - * No SUSY
  - * Fermions in *fundamental* representation

  $\Rightarrow$ To find an appropriate *dual* geometry to real QCD !!

- Holographic QCD
  - * Bottom-up approach :

    Looking at QCD, gathering field contents

    $\rightarrow$ constructing (DBI) action

    :: Hard-wall model, Soft-wall model, ...
  - * Top-down approach :

    From string theory, setting brane configuration with DBI action

    $\rightarrow$ reproducing QCD-like theory

    :: D3/D7, D4/D6, Sakai-Sugimoto (D4/D8/$\overline{\text{D8}}$), ...

From String to D-brane

- String
  - * one-dimensional object

$$X^{\mu}(\tau,\sigma) = X^{\mu}(\sigma^0,\sigma^1)$$

  - * Nambu-Goto action

$$S_{\mathrm{NG}} = -T\int d\mathcal{A} = -T\int d\tau d\sigma \sqrt{(\dot{X}\cdot X')^2 - \dot{X}^2 X'^2}$$

using an induced metric γ_{ab} on the world-sheet

$$\gamma_{ab} = \eta_{\mu\nu}\frac{\partial X^{\mu}}{\partial\sigma^a}\frac{\partial X^{\nu}}{\partial\sigma^b}$$

in the reparametrization invariant form

$$S_{\mathrm{NG}} = -T\int d^2\sigma\sqrt{-\gamma} \qquad \text{where} \quad \gamma = \det\left(\gamma_{ab}\right)$$

From String to D-brane

- Dp-brane
  - * multi-dimensional object

$$X^\mu(\sigma^0, \sigma^1) \quad \rightarrow \quad X^\mu(\sigma^0, \sigma^1, \cdots, \sigma^p)$$

  - * DBI action

$$S_{\mathrm{D}_p} = -T_p \int d^{p+1}\sigma \sqrt{-\det P[g]}$$

using a pull-back metric $P[g]_{ab}$ on the world-volume

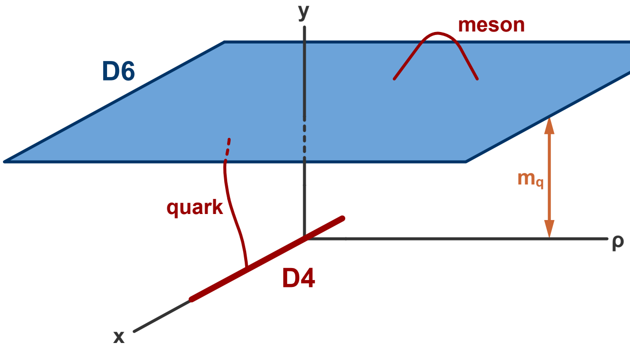
$$P[g]_{ab} = g_{\mu\nu} \frac{\partial X^\mu}{\partial \sigma^a} \frac{\partial X^\nu}{\partial \sigma^b}$$

including a gauge field A_a on the world-volume

$$S_{\mathrm{DBI}} = -T_p \int d^{p+1}\sigma \sqrt{-\det\left(P[g] + 2\pi\alpha' F\right)}$$

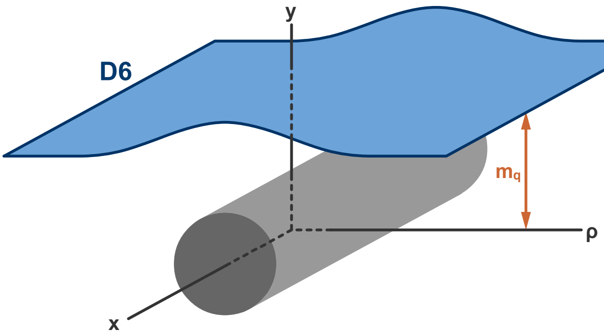
Configuration

- background D4 + probe D6



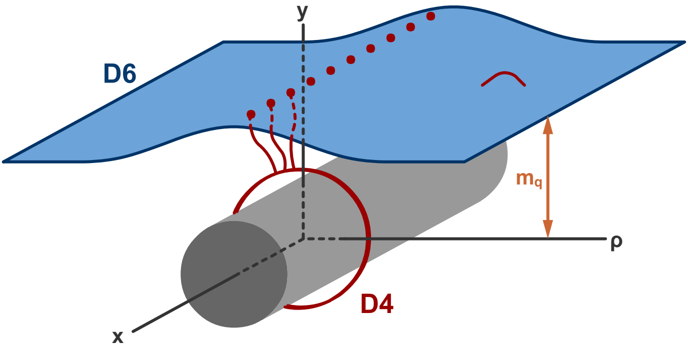
Configuration

- confining D4 background



Configuration

- add compact D4 branes ⇒ Baryon



D4/D6

- background D4 + probe D6

	t	1	2	3	(τ)	ρ	ψ_1	ψ_2	y	ϕ
D4	•	•	•	•	•					
D6	•	•	•	•		•	•	•		

The brane configurations : the background D4 and the probe D6

$$ds^2 = \left(\frac{U}{L}\right)^{3/2}\left(dx_{1,3}^2 + f(U)d\tau^2\right) + \left(\frac{L}{U}\right)^{3/2}\left(\frac{dU^2}{f(U)} + U^2 d\Omega_4^2\right)$$

$$e^{\phi} = g_s\left(\frac{U}{L}\right)^{3/4}\ , \quad F_4 = dC_3 = \frac{2\pi N_c \epsilon_4}{\Omega_4}$$

$$\text{where} \quad f(U) = 1 - (U/U_{\rm KK})^3$$

D4/D6

- background D4 + probe D6

  * DBI action for D6 brane

$$S_{\rm D6} = -\mu_6 \int d^7\sigma\, e^{-\phi}\sqrt{-\det(\mathrm{P}[g] + 2\pi\alpha' F)}$$
$$= -\tau_6 \int dt d\rho\, \rho^2 (1 + 1/\xi^3)^{4/3} \sqrt{(1 + 1/\xi^3)^{4/3}(1 + y'^2) - \tilde{F}^2}$$

  * equation of motion with respect to $\tilde{A}_t$

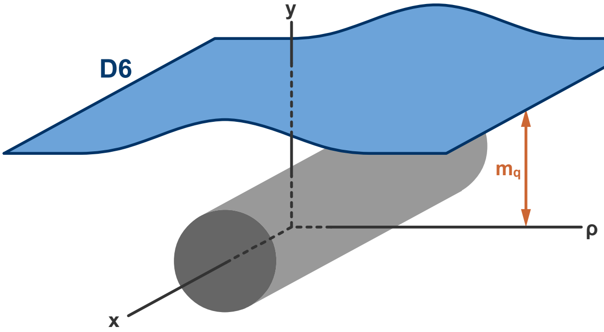
$$\frac{\partial \mathcal{L}_{\rm D6}}{\partial \tilde{F}} = \frac{\rho^2 (1 + 1/\xi^3)^{4/3} \tilde{F}}{\sqrt{(1 + 1/\xi^3)^{4/3}(1 + y'^2) - \tilde{F}^2}} \equiv \tilde{Q}$$

  * through Legendre transformation

$$H_{\rm D6} = \tau_6 \int d\rho \sqrt{(1 + 1/\xi^3)^{4/3}\left[\rho^4 (1 + 1/\xi^3)^{8/3} + \tilde{Q}^2\right](1 + y'^2)}$$

D4/D6

- confining D4 background



$$\lim_{\rho\to\infty} y(\rho) \sim m_q + \frac{c}{\rho} + \cdots$$

Finite density

- background D4 + probe D6 + compact D4

	t	1	2	3	(τ)	ρ	ψ_1	ψ_2	y	ϕ
D4	•	•	•	•	•					
D6	•	•	•	•		•	•	•		
	t	1	2	3	(τ)	U	Ω_4			
cD4	•						•	•	•	•

The brane configurations : the compact D4

* baryon $\sim$ compact D4 transverse to N_c D4 branes
on which N_c fundamental strings are attached

Finite density

- compact D4

  * DBI action for compact D4 brane

$$S_{\rm D4} = -\mu_4 \int d^5\sigma \; e^{-\phi}\sqrt{-\det(\mathrm{P}[g] + 2\pi\alpha' F)}$$
$$= -\tau_4 \int dt d\rho \; \rho^2 (1+1/\xi^3)^{4/3} \sqrt{(1+1/\xi^3)^{4/3}(1+y'^2) - \tilde{F}^2}$$

  * equation of motion with respect to $\tilde{A}_t$

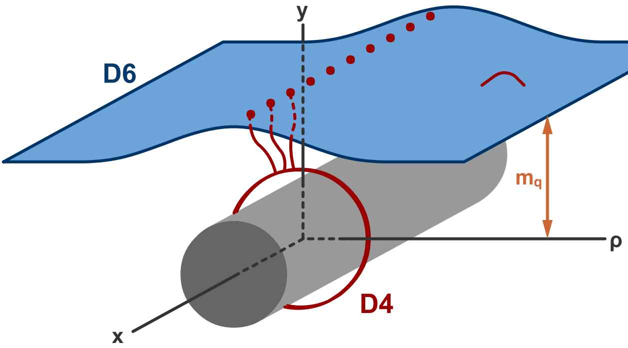
$$\frac{\partial \mathcal{L}_{\rm D4}}{\partial \tilde{F}} = \frac{\sin^3\theta \tilde{F}}{\sqrt{(1+1/\xi^3)^{4/3}(\xi^2+\dot{\xi}^2) - \tilde{F}^2}} \equiv \tilde{D}$$

  * through Legendre transformation

$$H_{\rm D4} = \tau_4 \int d\theta \sqrt{(1+1/\xi^3)^{4/3}(\xi^2+\dot{\xi}^2)} \sqrt{\tilde{D}^2 + \sin^6\theta}$$

Finite density

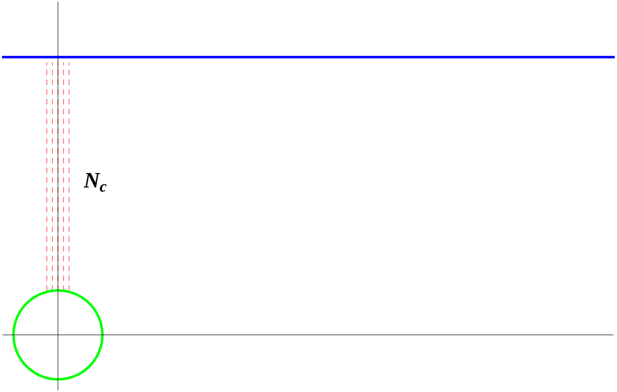
- add compact D4 branes ⇒ Baryon



$$\lim_{\rho\to\infty} A_t(\rho) \sim \mu - \frac{Q}{\rho} + \cdots$$

Force balancing condition

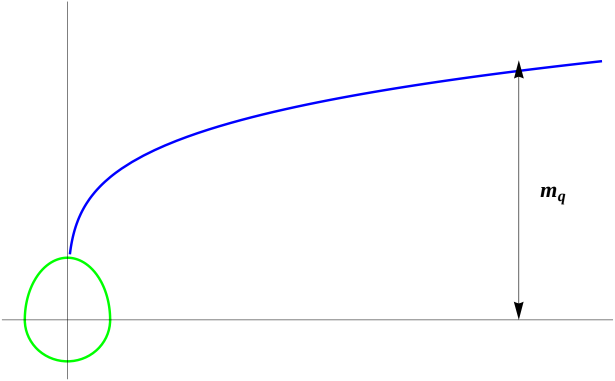
- confining D4 background



$$f \equiv \frac{\partial H}{\partial \xi_c}\bigg|_{\text{fixed other values}}$$

Force balancing condition

- confining D4 background



$$\frac{Q}{N_c} f_{\mathrm{D4}} = f_{\mathrm{D6}}$$

Energy density ϵ and pressure p

- Gibbs relation:

$$\Omega = U - TS - \mu N = -pV$$

- The grand potential is identified with thet on-shell DBI action:

$$\Omega = S_{\mathrm{DBI}}\big|_{\text{on-shell}}$$

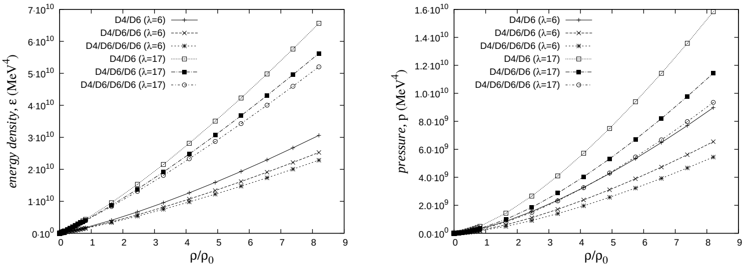
- Finally we can get the thermodynamic quantities:

$$\epsilon = \frac{1}{V_3} H\big|_{\text{on-shell}}$$

$$p = -\frac{1}{V_3} S_{\mathrm{DBI}}\big|_{\text{on-shell}}$$

Holographic EoS

- Equation of States, $\epsilon(\rho)$ and $p(\rho)$, are given from D4/D6 model.

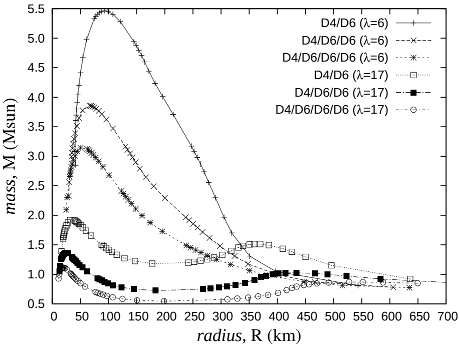


energy density $\epsilon(\rho)$ and pressure $p(\rho)$

Compact star with Holographic EoS

- TOV equation

$$\frac{dP(r)}{dr} = -\frac{G_N\left(\epsilon(r)+P(r)\right)\left(m(r)+4\pi r^3 P(r)\right)}{r\left(r-2G_N m(r)\right)}$$



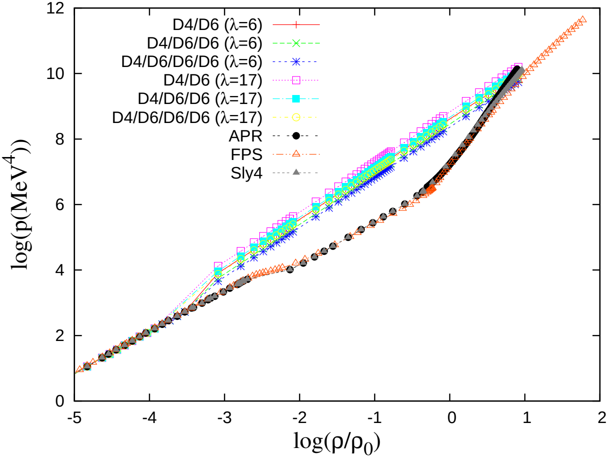
Mass-Radius relation using holographic EoS

Summary

- There is a restriction to deal with strongly coupled system perturbatively.
- AdS/CFT correspondence is a powerful theoretical tool to study strongly-coupled gauge theories.
- Here equation of states are obtained from D4/D6 model as an example.

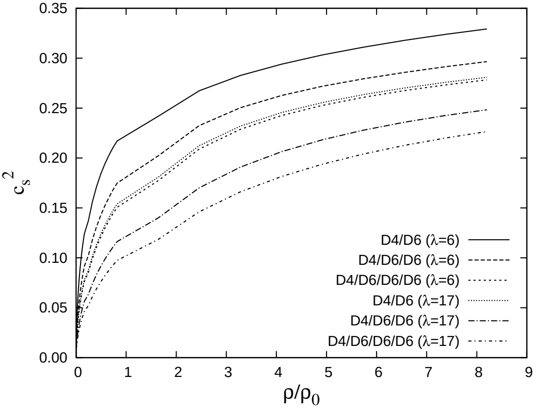
If needed

- Holographic EoS



If needed

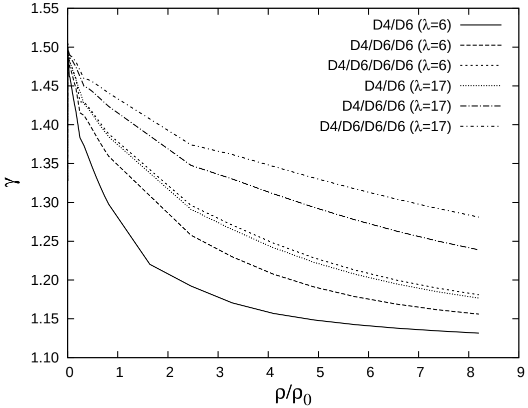
- Holographic EoS



$$c_s^2 \equiv dp/d\epsilon$$

If needed

- Holographic EoS



$$\gamma \equiv c_s^2(\epsilon/p)$$