Equations of state and compact stars in gauge/gravity duality

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Outline

- Short note for string theory
- Short note for AdS/CFT
- Motivation
- Gravity degrees of freedom in gauge/gravity correspondence
- Simple compact stars and TOV equation
- The simplest holographic model without matter
- Adding the neutral scalar condensation to the model
- Summary and discussion

Short note for string theory

- Quantum field theories and the gravity theories including super gravity theories are point particle theories.
- Point particle theories have UV infinity problem.
- In field theory side, we have a method extracting physical quantities, renormalization.
- In gravity theory side, we don't have any way regarding quantum effect(Some Sugra theories, Loop quatum gravity, Horava gravity,…..).
- But the point particle theory cannot avoid the UV divergence problem, because the size of particles is zero.
- The only way to avoid is eliminating all dimensionful couplings. We have many successful examples in QFT.
- We cannot do such a thing in gravity theory, because gravity theories have the famous dimensionful coupling Planck's constant.

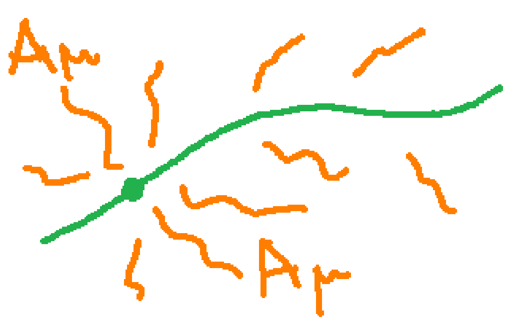
Short note for string theory

- So we may take some theory which have the extended objects(string, membrane and so on…..) as fundamental particles.
- An example of point particle theory

$$S \sim m\int dt\sqrt{\dot{x}^\mu \dot{x}_\mu} + q\int dt\,\frac{dx^\mu}{dt}A_\mu(t)$$

or

$$m\int ds + q\int A$$



Short note for string theory

- String theory

$$S_s \sim \int d\sigma^1 d\sigma^2 \sqrt{-\det h_{ab}} + \int B$$

$$h_{ab} : \frac{\partial X^\mu}{\partial \sigma^a} \frac{\partial X^\nu}{\partial \sigma^b} \eta_{\mu\nu}$$

$$X^\mu(\sigma^1, \sigma^2)$$

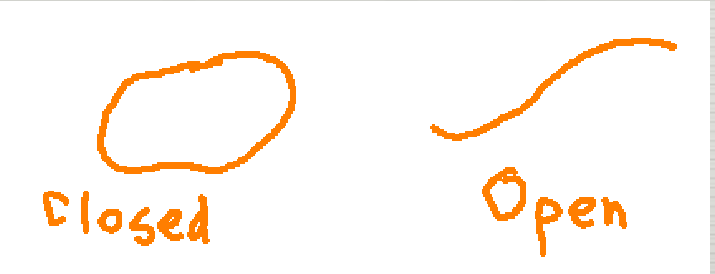
$$B = \frac{1}{2} B_{\mu\nu}(X(\sigma^a))\, dx^\mu \wedge dx^\nu$$

Short note for string theory

- Better Action ; Polyakov Action

$$S_{pol} \sim \int d^2\sigma \sqrt{-h}\, h^{ab}\, \partial_a X^M \partial_b X^N G_{MN}$$

- Quantization



Short note for string theory

- Closed string -> periodic boundary condition
- Result

tachyon ϕ $m^2<0$ TT

Φ dilaton

B_{MN} Antisymmetric 2 form fied

$G_{\mu\nu}$ Graviton

$m^2=0$

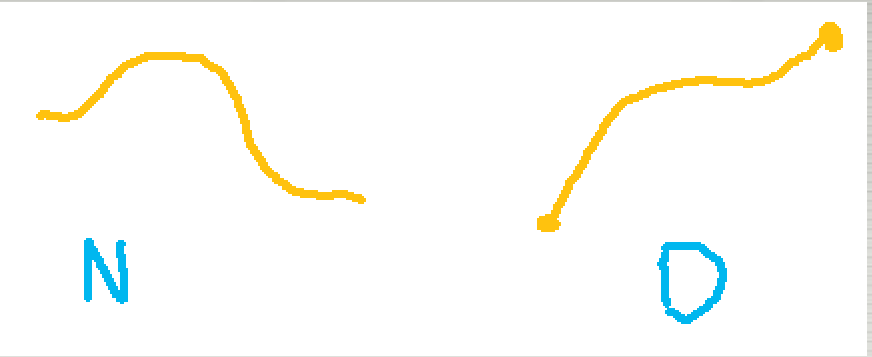
Massive ptls (mass unit planck scale)

∞ tower

- D=26

Short note for string theory

- Open string ;
  -Neumann boundary condition
  -Derichlet Boundary condition



- Result(N. B.C)

tachyon $m^2<0$ T T

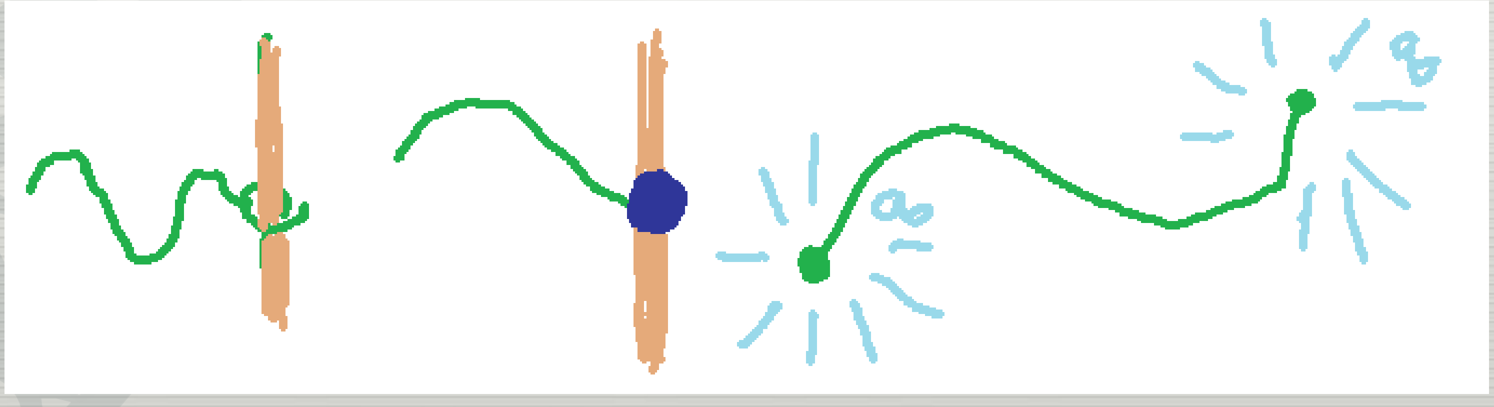
U(1) gauge field $A_\mu (m^2=0)$

Massive ptls

$D=26$

Short note for string theory

- D . B.C
  -From the wave mechanics, we can see that the physical meaning of such a B.C is related to existence of other object.
- If there are some object at the end points of open strings, open string ends can have generalized charges.



Short note for string theory

- D . B.C -> Some extended object -> “D-brane”
- D(-1) brane world point instantonlike object
- D 0 brane world line particle like object
- D1 brane world sheet string like object
- D2 brame world volume membrane like object
- D3 brane 3 dimensional object

- It is well known that these object are really dynamical objects.

Short note for string theory

- What is effective action for D-p brane

$$S = \int d^{p+1}\xi\, e^{\Phi} \sqrt{-\det\left(G_{ab}+B_{ab}+F_{ab}\right)} + \int_{M_{p+1}} e^{B+F} \sum C_{(p)}$$

- Ramond fields: Generalized gauge fields sauced by D branes

$$D_p \text{ brane} \sim C_{(p+1)} : \frac{1}{(p+1)!} C_{\mu_1 \cdots \mu_{p+1}} dx^{\mu_1} \cdots dx^{\mu_{p+1}}$$

Short note for string theory

- In order to eliminate tachyons, one may introduce super symmetry.

$$S \sim S_{pol} + \int d\tau\, d\sigma \sqrt{h}\; i\, \bar{\psi}^{M} \rho^{a}\, \partial_{a} \psi_{M}$$

- Results
  - No tachyon
  - Consistent theories
  --type I (D=10)
  --type IIA (D=10)
  --type IIB (D=10)
  --Heterotic E8 X E8 (D= 10+26)
  --Heterotic SO(32) (D= 10+ 26)

Short note for string theory

- One example- type IIB string theory

$m^2 = 0$

Bosonic part

Φ Dilaton

B_{MN} Anti-symmetric tensor

G_{MN} Metric

C Ramon scalar

$C^{(2)}$ 〃 2form ~ D1 brane

$C^{(4)}$ 〃 4form ~ D3 brane

+ ∞ Massive ptls

Short note for string theory

- Low energy effective field theory.
  --→ Type IIB super gravity theory

$$S=\frac{1}{2\kappa^2}\int\Big[e^{-2\phi}\left(*R+4d\phi\wedge*d\phi-\frac{1}{2}H_3\wedge*H_3\right)$$
$$-\frac{1}{2}F_1\wedge*F_1-\frac{1}{2}\tilde{F}_3\wedge*\tilde{F}_3-\frac{1}{4}\tilde{F}_5\wedge*\tilde{F}_5$$
$$-\frac{1}{2}C_4^+\wedge H_3\wedge F_3$$
$$+\text{Ferlmons}\Big]$$

$$F_1=dC_0,\quad H_3=dB,\quad F_3=dC_2$$
$$F_5=dC_4^+,\quad \tilde{F}_3=F_3-C_0\wedge H_3$$
$$\tilde{F}_5=F_5-\frac{1}{2}C_2\wedge H_3+\frac{1}{2}B\wedge F_3$$
$$*\tilde{F}_5=\tilde{F}_5$$

Summary of string theory

- One supersymmetric black 3-brane solution

$$ds^2 = H(r)^{-1/2}\left(-dx^{0\,2} + dx^{1\,2} + \cdots + dx^{3\,2}\right) + H(r)^{1/2}\left(dr^2 + r^2 d\Omega_5\right)$$

$$F_5 = dx^0 \wedge dx^1 \wedge \cdots \wedge dx^3 \wedge dH^{-1}(r)$$

$$H(r) = 1 + (\text{constant})\,\frac{g_s N \alpha'^2}{r^4} \qquad \kappa \sim g_s \sqrt{\alpha}$$

- Actually this blackhole-like 3 brane solution is same with D3 brane on end point of open strings. (Polchinski)

Short note for string theory

- So this is main difference from usual supergravity. In old supergravity, this black brane solution has nothing to do with fundamental particle.s It is just a soliton solution in the supergravity.

$$ds^2 = H(r)^{-1/2}\left(-dx^{0\,2} + dx^{1\,2} + \cdots + dx^{3\,2}\right) + H(r)^{1/2}\left(dr^2 + r^2 d\Omega_5\right)$$

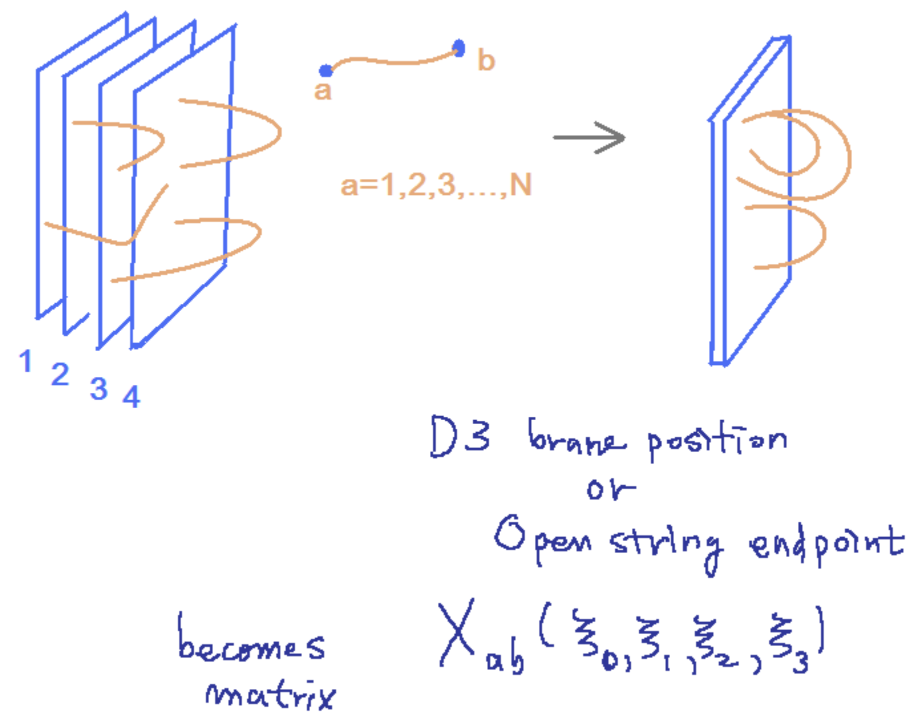
$$F_5 = dx^0 \wedge dx^1 \wedge \cdots \wedge dx^3 \wedge dH^{-1}(r)$$

$$H(r) = 1 + (\text{constant})\,\frac{g_s N \alpha'^2}{r^4} \qquad \kappa \sim g_s \sqrt{\alpha}$$

- However, the supergravity theory as a low energy effective theory for string theory has exact relation between the black brane soliton solution and fundamental string.

Short note for AdS/CFT

- Now we use the fact in the previous short note. Let's consider N D3-brane. The D-brane has two description, D-brane in string theory and Black brane in supergravity.
- D-brane in stringy picture



Short note for AdS/CFT

- D3 brane action becomes Non-abelian with some presciptions.

$$S_{D3} = \int d^4\xi \, \mathrm{Tr}\sqrt{-\det\left(G_{ab}+B_{ab}+F_{ab}\right)} + \int \mathrm{Tr}\, e^{B+F} \sum_{i} C^{(i)}$$

$$+ \left(\text{Fermionic action}\right)$$

$$\left.\begin{aligned}
G_{ab} &: \frac{\partial X^M(\xi)}{\partial \xi^a}\frac{\partial X^N}{\partial \xi^b} G_{MN}(X) \\
B_{ab} &= \frac{\partial X^M}{\partial \xi^a}\frac{\partial X^N}{\partial \xi^b} B_{MN}(X) \\
F_{ab}&(\xi_0,\cdots,\xi_3) : \\
C^{(i)}&
\end{aligned}\right\} \begin{array}{l} N\times N \\ \text{Matrix} \end{array}$$

- The perturbation theory is welll defined, when

g_s : string coupling

N : trace of Chan-Paton factors

$$g_s N \ll 1$$

Short note for AdS/CFT

- The low enegy limit
- The higher derivative terms can be ignored.
- The Sugra fields are decoupled from Dbrane.
- Then the action governing D3 brane turns out to be U(N) Super Yang-Mill theory with 4 super-charges.

$$\mathcal{L}_{\substack{SYM \\ \mathcal{N}=4}} = \frac{1}{g_{YM}^2} \mathrm{tr}\Big[-\frac{1}{2} F_{\mu\nu}^2 - i\bar{\lambda}^a \not{D} \lambda_a - (D_\mu X^i)^2 + C_i^{ab} \lambda_a [X^i, \lambda_a] + c.c. + \frac{1}{2}[X^i, X^j]^2 \Big] + \frac{\theta}{16\pi^2} \mathrm{tr}\, \epsilon^{\mu\nu\lambda\sigma} F_{\mu\nu} F_{\lambda\sigma}$$

$$g_{YM}^2 \propto g_{string}$$

Short note for AdS/CFT

- Super gravity point of view

$$ds^2 = H(r)^{-1/2}\left(-dx^{0^2} + dx^{1^2} + \cdots + dx^{3^2}\right) + H(r)^{1/2}\left(dr^2 + r^2 d\Omega_5\right)$$

$$F_5 = dx^0 \wedge dx^1 \wedge \cdots \wedge dx^3 \wedge dH^{-1}(r)$$

$$H(r) = 1 + (\text{constant})\,\frac{g_s N \alpha'^2}{r^4} \qquad \kappa \sim g_s\sqrt{\alpha}$$

- Near horizon limit(r -> 0)

$$H(r) \to 4\pi \frac{g_s N \alpha'^2}{r^4}$$

Black brane sol $\to AdS_5 \times S^5$ with radius $(4\pi g N)^{1/4}$ in string unit

- The SUGRA is valid when two following quantities are large.

$$L/\alpha'^{1/2} = (4\pi g N)^{1/4} \qquad L/L_{\text{Planck}} \propto N^{1/4}$$

Short note for AdS/CFT

There are two limit for same object N D3 brane

$gN \propto g_{YM}^2 N \propto \lambda$ small $\rightarrow$ D brane description is better.

$gN \gg 1$ $N \gg 1$ $\rightarrow$ SUGRA is better

Low-energy limit

type IIB string in $AdS_5 \times S^5$

$\mathcal{N}=4$ SYM with $U(\infty)$

This is the AdS/CFT.

Short note for AdS/CFT

- This super conformal YM is very strange theory which is very far from real YM theory QCD. In order to make similar theory to real QCD, we need to deform the conjecture.

top-down approach

$$AdS_5 \times S_5 \quad + (\text{Probe brane})$$

$$\downarrow \qquad \downarrow$$

$$AdS_5 \times (X)$$

$$(X) \times (X)$$

Short note for AdS/CFT

- Or we may think phenomenological model

Bottom up approach

AdS_5 + matter field

$\boxed{AdS_5}$ + matter field

Short note for AdS/CFT

- An example ; EKSS(J. Erlich, E. Katz, Dam T. Son and M. A. Stephanov, 05'), model

TABLE I: Operators/fields of the model

4D: $\mathcal{O}(x)$	5D: $\phi(x,z)$	p	Δ	$(m_5)^2$
$\bar{q}_L\gamma^\mu t^a q_L$	$A^a_{L\mu}$	1	3	0
$\bar{q}_R\gamma^\mu t^a q_R$	$A^a_{R\mu}$	1	3	0
$\bar{q}^\alpha_R q^\beta_L$	$(2/z)X^{\alpha\beta}$	0	3	-3

$$ds^2 = \frac{1}{z^2}(-dz^2 + dx^\mu dx_\mu), \qquad 0 < z \le z_m.$$

$$S = \int d^5x \sqrt{g}\, \mathrm{Tr}\left\{|DX|^2 + 3|X|^2 - \frac{1}{4g_5^2}(F_L^2 + F_R^2)\right\}$$

TABLE II: Results of the model for QCD observables. Model A is a fit of the three model parameters to m_π, f_π and m_ρ (see asterisks). Model B is a fit to all seven observables.

Observable	Measured (MeV)	Model A (MeV)	Model B (MeV)
m_π	139.6±0.0004 [8]	139.6*	141
m_ρ	775.8±0.5 [8]	775.8*	832
m_{a_1}	1230±40 [8]	1363	1220
f_π	92.4±0.35 [8]	92.4*	84.0
$F_\rho^{1/2}$	345±8 [15]	329	353
$F_{a_1}^{1/2}$	433±13 [6, 16]	486	440
$g_{\rho\pi\pi}$	6.03±0.07 [8]	4.48	5.29

Motivation

- QCD is still mysterious region in physics.
- In general, strong coupling behavior, density and temperature make field theory difficult. We have no way to control these situations.
- AdS/CFT gave us some possibilities for controlling the situations.
- At least, for some super conformal field theories, we can overcome these difficulties.
- Although it is not clear whether this way can produce the quantitative explanation for real QCD or not, we may extend the correspondence due to a likelihood of qualitative explanations.

Motivation

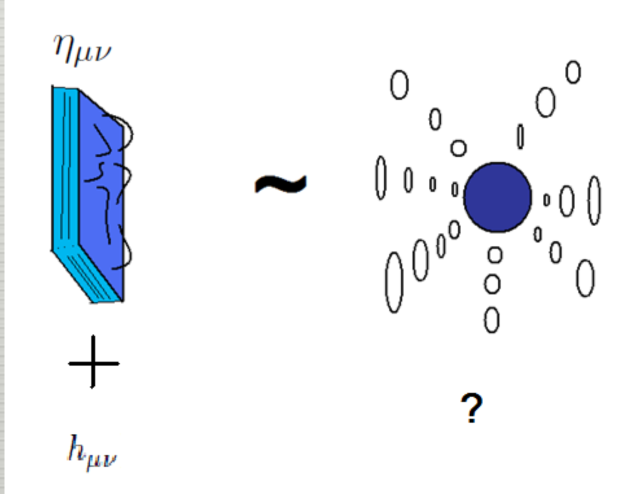
- The compact star(neutron star, white dwarfs, quark stars, strange stars,…) is another interesting research topic, because they can tell us the equation of state for some region in the phase space of QCD.
- There are many interesting observations from neutron stars.
- Since these objects are self bound objects with nuclear force and gravity. there are two obstacles, the density and the gravity.
- First one is already mentioned. It is well known that we can avoid this problem by introducing bulk gauge field.
- For the second one, we have to introduce the gravity degrees of freedom in the boundary of AdS space. This is the source for boundary energy-momentum tensor.
- So it is also interesting to realize similar configuration in holographic QCD. This is our motivation.

Gravity degrees of freedom in gauge/gravity correspondence

- Randall-Sundrum model was already considered for the gravity degrees of freedom in our brane. The gravity strength is represented by warping factor.
- The first work investigating gravity wave functions is given by Gherghetta et al, 05'. In the model, they considered a mass term for IR brane and obtained composite graviton.
- Another work is given by Kiritsis and Nitti 06'. They considered massless 4d gravitons in the Asymptotically AdS5 geometries.
- Now I want to describe their idea.

Gravity degrees of freedom in gauge/gravity correspondence

- The original AdS/CFT correspondence was about correspondence between two possible descriptions of a same object. One description is a theory decoupled from bulk degrees of freedom, i,e. a field theory on a flat spacetime. The other one is the string theory on a curved geometry, AdS X Sphere.
- If we take decoupling limit, there is no way to introduce gravity degrees of freedom in the field theory system. Then the field theory should be defined in a flat space time.



Gravity degrees of freedom in gauge/gravity correspondence

- So it is difficult to realize compact stars in usual AdS/CFT correspondence. Because the compact star is a self bound configuration with not only nuclear interaction but also gravity interaction.

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = < T_{\mu\nu} >$$

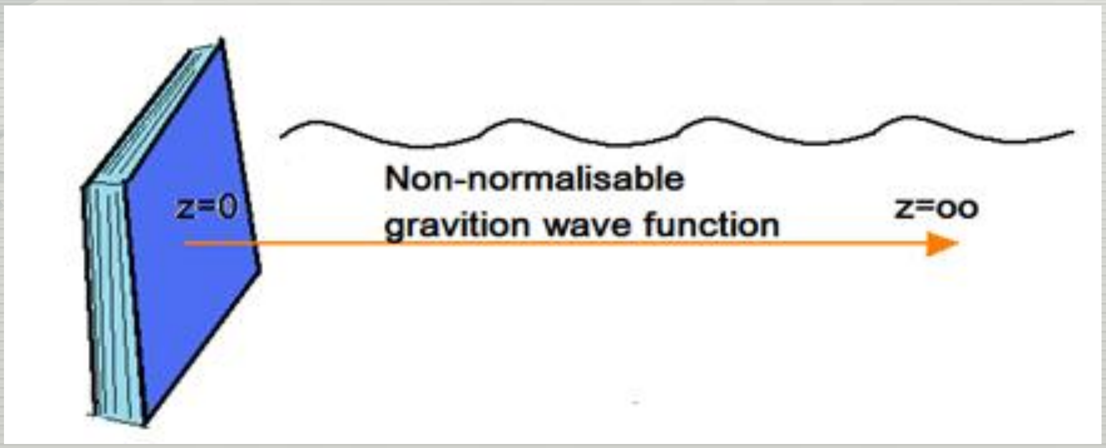
- In string theory point or view, this configuration is a bound state of open strings(Field theory degrees of freedom) and closed strings degrees of freedom(Gravity degrees of freedom).
- We need closed string degrees of freedom for compact star, thus we have to take an extension of usual AdS/CFT into account and we should be very careful of the decoupled limit. It is not easy problem.

Gravity degrees of freedom in gauge/gravity correspondence

- If we introduce the gravity fluctuation in the AdS space with Poincare coordinates.

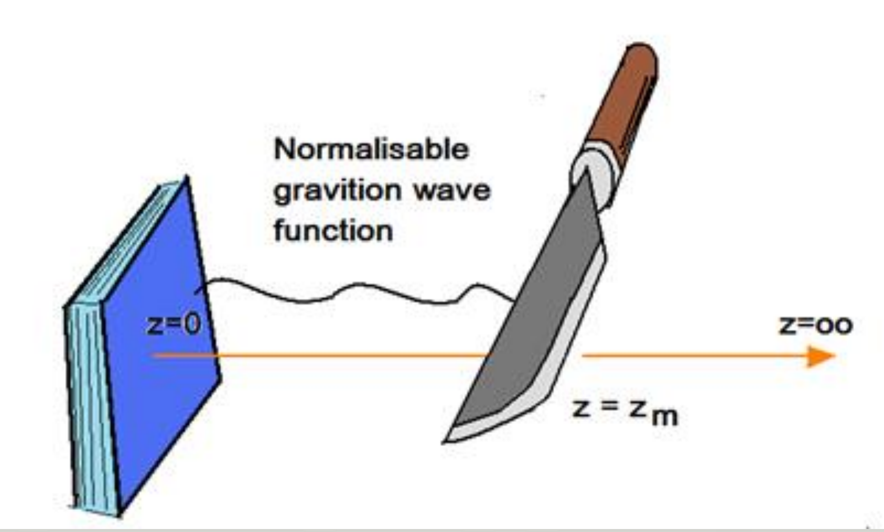
$$a^2(y)\left[(1+2\phi)\,dy^2 + 2A_\mu dy dx^\mu + (\eta_{\mu\nu} + h_{\mu\nu})\,dx^\mu dx^\nu\right]$$

- Solving Einstein equation, this gravity wave function is not a normalizable function., where y is from 0 to infinity.



Gravity degrees of freedom in gauge/gravity correspondence

- One idea for normalizable wave function is introducing cutoff and suitable boundary condition for gravity wave function.
- This make the existence of graviton on the boundary.



- So we can have gauge+gravity/gravity correspondence.

Simple compact stars and TOV equation

- Now we know how to construct existence of gravity d.o.f on the boundary of AdS space.
- Now we want to describe the compact stars with this construction.
- For this, we summarize how the usual 4 dimensional Einstein gravity consider for simplest perfect fluid stars.

$$G_{\mu\nu} = T_{\mu\nu}$$

- Where the energy momentum tensor is perfect fluid energy momentum tensor.

$$\begin{pmatrix} \rho & & & \\ & P & & \\ & & P & \\ & & & P \end{pmatrix}$$

Simple compact stars and TOV equation

- The metric is given by

$$g_{tt}(r)dt^2 + g_{rr}(r)dr^2 + g_{\theta\theta}(r)\left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

- Where

$$g_{tt} = -e^{2f(r)}\,, \quad g_{rr} = \frac{1}{\left(1 - \frac{2m(r)}{r}\right)}\,, \quad g_{\theta\theta} = r^2$$

- Then Einstein equation gives

$$f'(r) = \frac{m(r) + 4\pi r^3 P(r)}{r\,(r - 2m(r))}$$

$$m(r) = 4\pi \int_0^r \rho(r')r'^2 dr'$$

Simple compact stars and TOV equation

- If consider hydrostatic equation, we can obtain TOV

$$P'(r) = -(P(r)+\rho(r))\frac{m(r)+4\pi r^3 P(r)}{r(r-2m(r))}$$

- Thus we have to solve following tree equations

$$f'(r) = \frac{m(r)+4\pi r^3 P(r)}{r(r-2m(r))}$$

$$m(r) = 4\pi\int_0^r \rho(r')r'^2 dr'$$

$$P'(r) = -(P(r)+\rho(r))\frac{m(r)+4\pi r^3 P(r)}{r(r-2m(r))}$$

- We have only tree equations for four unknown ftns.

Simple compact stars and TOV equation

- Usual study needs one more equation, for example ,the equation of state.

$$P(\rho)$$

- If we know equation of state, we can solve the four equations.
- Then we can see full function of mass, pressure and energy density.
- From these, we can read the radius and the mass of a compact star.
- This computation can be related to observation.
- The EOS is from the microscopic nature of the nuclear matter, the radius and mass are from the observation.
- Thus the compact stars are good object to understand microscopic physics from observation.
- In this usual case, we have to assume or obtain EOS from QCD. But we don't know yet.

The simplest holographic model without matter

- If we believe possibility for existence of gravity degrees of freedom, we may consider holographic model in a curved spacetime.
- This is a deformation from the gauge/gravity correspondence.
- In order to obtain first intuition, we had better take a simplest toy model. So we are going to consider the Einstein-Hilbert action with negative cosmological constant in 5 dimension.

$$R_{IJ} - \frac{1}{2} g_{IJ} R = \frac{6}{L^2} g_{IJ}$$

- The general behavior already studied by Skenderis et al(2000).

The simplest holographic model without matter

- The summary of their work is as follows.
- If we take the Fefferman-Graham coordinates,

$$ds^2 = \frac{L^2}{z^2}\left(dz^2 + g_{\mu\nu}(x,z)dx^\mu dx^\nu\right)$$

The general behavior of metric is

$$g_{\mu\nu}(x,z) = h_{\mu\nu}(x)z^4 \log z^2 + g^{(0)}_{\mu\nu}(x) + z^2 g^{(2)}_{\mu\nu}(x) + z^4 g^{(4)}_{\mu\nu}(x) + \sum_{n=5}^{\infty} z^n g^{(n)}_{\mu\nu}(x)$$

- The expectation value of energy-momentum tensor is

$$\langle T_{ij}\rangle = \frac{4}{16\pi G_N}[g_{(4)ij} - \frac{1}{8}g_{(0)ij}[(\mathrm{Tr}\, g_{(2)})^2 - \mathrm{Tr}\, g^2_{(2)}] - \frac{1}{2}(g^2_{(2)})_{ij} + \frac{1}{4}g_{(2)ij}\mathrm{Tr}\, g_{(2)}]$$

The simplest holographic model without matter

- Again, the bulk metric is

$$g_{\mu\nu}(x,z) = h_{\mu\nu}(x) z^4 \log z^2 + g^{(0)}_{\mu\nu}(x) + z^2 g^{(2)}_{\mu\nu}(x) + z^4 g^{(4)}_{\mu\nu}(x) + \sum_{n=5}^{\infty} z^n g^{(n)}_{\mu\nu}(x)$$

- The first term is related to conformal anomaly and scheme dependent term, when we consider holographic renormalization. For simplicity, we assume that this term vanishes.
- The second term is interpreted as the metric of boundary system. Usually, this is taken as flat metric. In our case we will put nontrivial non-flat metric into this boundary metric.

The simplest holographic model without matter

- Since we will take star object into account, we assume that our metric has rotational symmetry. Then our ansatz for bulk metric is

$$g_{\mu\nu}(x,z)dx^{\mu}dx^{\nu} = g_{tt}(r,z)dt^2 + g_{rr}(r,z)dr^2 + g_{\theta\theta}(r,z)\left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

- From this ansatz, we may solve the bulk equation.

$$R_{IJ} - \frac{1}{2}g_{IJ}R = \frac{6}{L^2}g_{IJ}$$

The simplest holographic model without matter

- Now we take a star metric as the boundary metric.

$$g_{tt}^{(0)} = -e^{2f(r)}\ , \quad g_{rr}^{(0)} = \frac{1}{\left(1-\frac{2m(r)}{r}\right)}\ , \quad g_{\theta\theta}^{(0)} = r^2$$

$$g_{\mu\nu}(x,z) = h_{\mu\nu}(x) z^4 \log z^2 + g_{\mu\nu}^{(0)}(x) + z^2 g_{\mu\nu}^{(2)}(x) + z^4 g_{\mu\nu}^{(4)}(x) + \sum_{n=5}^{\infty} z^n g_{\mu\nu}^{(n)}(x)\ ,$$

- Then we can solve the bulk Einstein equation order by order in z.
- Up to zeroth order equation, g^(2) is given in terms of g^(0).

The simplest holographic model without matter

- Up to second order, the metric must satisfy following constraint equation.

$$
\begin{aligned}
0 = & -3m(r)^2 + r\left(-4r + 6m(r) + rm'(r)\right)m'(r) + 2r^2\left(r - 2m(r)\right)m''(r) - 2r\left(2r^2\right. \\
& + \left(-5r + 3m(r)\right)m(r) + \left(-3r + 4m(r) + rm'(r)\right)m'(r) + 2r^2\left(r - 2m(r)\right)m''(r)\left.\right)f'(r) \\
& + r^2\left(9r^2 + \left(-32r + 29m(r)\right)m(r) + r\left(-4r + 6m(r) + rm'(r)\right)m'(r)\right. \\
& \left. + 2r^2\left(r - 2m(r)\right)m''(r)\right)f'(r)^2 + 2r^3\left(r - 2m(r)\right)\left(-3r + 5m(r) + rm'(r)\right)f'(r)^3 \\
& + r^4\left(r - 2m(r)\right)^2 f'(r)^4 - \left(4r^2\left(r - 2m(r)\right)\left(-r + m(r) + rm'(r)\right)\right. \\
& \left. - 4r^3\left(r - 2m(r)\right)\left(-m(r) + rm'(r)\right)f'(r) + 2r^4\left(r - 2m(r)\right)^2 f'(r)^2\right)f''(r) \\
& + r^4\left(r - 2m(r)\right)^2 f''(r)^2 - 2r^3\left(r - 2m(r)\right)^2\left(-1 + rf'(r)\right)f'''(r) \qquad \text{(A.1)}
\end{aligned}
$$

- In other words, If the boundary metric satisfies this equation, then the full bulk metric is solution of the bulk Einstein equation up to second order.

The simplest holographic model without matter

- We take one more assumption. If the boundary metric is a perfect fluid star, we have to consider following constraints.

$$f'(r) = \frac{m(r) + 4\pi r^3 P(r)}{r\,(r - 2m(r))}$$

$$m(r) = 4\pi \int_0^r \rho(r') r'^2 dr'$$

$$P'(r) = -(P(r) + \rho(r)) \frac{m(r) + 4\pi r^3 P(r)}{r\,(r - 2m(r))}$$

$$g_{tt}^{(0)} = -e^{2f(r)}\ , \ \ g_{rr}^{(0)} = \frac{1}{\left(1 - \frac{2m(r)}{r}\right)}\ , \ \ g_{\theta\theta}^{(0)} = r^2$$

- These come from the boundary Einstein equation and TOV equation.
- Then previous complicate constraint becomes simple.

$$\rho'(r) = \frac{(P(r) + \rho(r))\,(3m(r) - 4\pi r^3 \rho(r))}{r\,(r - 3m(r) - 4\pi r^3 P(r))}$$

The simplest holographic model without matter

- If we solve the following equations

$$m(r) = 4\pi \int_0^r \rho(r')r'^2 dr'$$

$$P'(r) = -(P(r)+\rho(r))\frac{m(r)+4\pi r^3 P(r)}{r(r-2m(r))}$$

$$\rho'(r) = \frac{(P(r)+\rho(r))\left(3m(r)-4\pi r^3\rho(r)\right)}{r(r-3m(r)-4\pi r^3 P(r))}$$

with initial condition P(0),rho(0) and m(0)=0, then we can obtain pressure and energy density as functions of the radial coordinate.

The simplest holographic model without matter

- Indeed, these equation can be solved, the solution is well known uniform density solution.

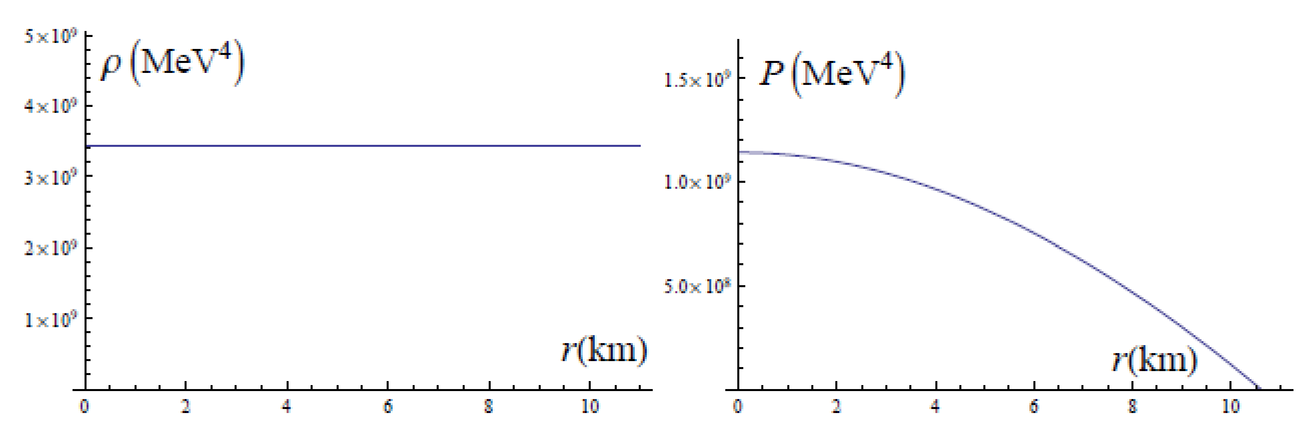


Figure 1: Energy density and pressure for a uniform density star

- This is a simplest toy model, so we expect that more realistic model can give more realistic equation of state.

The simplest holographic model without matter

- Main reason for this determination comes from our rotation symmetry and the bulk Einstein equation. The bulk Einstein equation has more component than the boundary Einstein equation. Such a extra degrees of equation can make following constraint equation.

$$
\begin{aligned}
0 = & -3m(r)^2 + r\left(-4r + 6m(r) + rm'(r)\right)m'(r) + 2r^2\left(r - 2m(r)\right)m''(r) - 2r\left(2r^2\right. \\
& + \left(-5r + 3m(r)\right)m(r) + \left(-3r + 4m(r) + rm'(r)\right)m'(r) + 2r^2\left(r - 2m(r)\right)m''(r)\Big)f'(r) \\
& + r^2\left(9r^2 + \left(-32r + 29m(r)\right)m(r) + r\left(-4r + 6m(r) + rm'(r)\right)m'(r)\right. \\
& \left. + 2r^2\left(r - 2m(r)\right)m''(r)\right)f'(r)^2 + 2r^3\left(r - 2m(r)\right)\left(-3r + 5m(r) + rm'(r)\right)f'(r)^3 \\
& + r^4\left(r - 2m(r)\right)^2 f'(r)^4 - \Big(4r^2\left(r - 2m(r)\right)\left(-r + m(r) + rm'(r)\right) \\
& - 4r^3\left(r - 2m(r)\right)\left(-m(r) + rm'(r)\right)f'(r) + 2r^4\left(r - 2m(r)\right)^2 f'(r)^2\Big)f''(r) \\
& + r^4\left(r - 2m(r)\right)^2 f''(r)^2 - 2r^3\left(r - 2m(r)\right)^2\left(-1 + rf'(r)\right)f'''(r) \qquad \text{(A.1)}
\end{aligned}
$$

Adding neutral scalar to the model

- Now we move to more realistic configuration.
- First one we can think is giving a simplest matter to the system.
- So we have to add a neutral scalar bulk field as follows.

$$R_{IJ} - \frac{1}{2}g_{IJ}(R + \frac{6}{L^2}) = T_{IJ}$$
$$T_{IJ} = \frac{1}{2}\partial_I\phi\partial_J\phi - \frac{1}{4}g_{IJ}((\partial\phi)^2 + m_\phi^2\phi^2)$$

- Eom for matter fields

$$(\nabla^2 - m_\phi^2)\phi = 0$$

$$m_\phi^2 = -3/L^2$$

Adding neutral scalar and gauge field to the model

- The ansatz for the matter field is as follows.

$$\phi(r,z) = \sum_{n=0}^{\infty} \phi_n(r) z^n$$

- The metric ansatz is same with the previous case.

$$g_{\mu\nu}(x,z)dx^{\mu}dx^{\nu} = g_{tt}(r,z)dt^2 + g_{rr}(r,z)dr^2 + g_{\theta\theta}(r,z)\left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

- We can solve the Einstein equation and the equations of motion for matter field near boundary(z=0) of the AdS5 order by order in z.

Adding neutral scalar and gauge field to the model

- For the matter fields, we can obtain the result

$$\phi_0(r) = \phi_2(r) = \phi_4(r) = 0,$$

- Again a complicate equation for metric

$$\begin{aligned}
0 = & \frac{1}{2} r^6 \phi_1(r)^4 + 2r^3 \left(m(r) - r\left(r - 2m(r)\right) f'(r)\right) \phi_1(r)^2 \\
& - 2r^4 \left(r - 2m(r)\right) \left(2 + r f'(r)\right) \phi_1(r) \phi_1'(r) - 3r^5 \left(r - 2m(r)\right) \phi_1'(r)^2 \\
& - 3m(r)^2 + r\left(-4r + 6m(r) + r m'(r)\right) m'(r) + 2r^2 \left(r - 2m(r)\right) m''(r) - 2r \left(2r^2\right. \\
& \left. + \left(-5r + 3m(r)\right) m(r) + \left(-3r + 4m(r) + r m'(r)\right) m'(r) + 2r^2 \left(r - 2m(r)\right) m''(r)\right) f'(r) \\
& + r^2 \left(9r^2 + \left(-32r + 29m(r)\right) m(r) + r\left(-4r + 6m(r) + r m'(r)\right) m'(r)\right. \\
& \left. + 2r^2 \left(r - 2m(r)\right) m''(r)\right) f'(r)^2 + 2r^3 \left(r - 2m(r)\right) \left(-3r + 5m(r) + r m'(r)\right) f'(r)^3 \\
& + r^4 \left(r - 2m(r)\right)^2 f'(r)^4 - \left(4r^2 \left(r - 2m(r)\right) \left(-r + m(r) + r m'(r)\right)\right. \\
& \left. - 4r^3 \left(r - 2m(r)\right) \left(-m(r) + r m'(r)\right) f'(r) + 2r^4 \left(r - 2m(r)\right)^2 f'(r)^2\right) f''(r) \\
& + r^4 \left(r - 2m(r)\right)^2 f''(r)^2 - 2r^3 \left(r - 2m(r)\right)^2 \left(-1 + r f'(r)\right) f'''(r)
\end{aligned} \tag{A.3}$$

Adding neutral scalar and gauge field to the model

- And for the scalar field

$$
\begin{aligned}
0 = {} & r^2\phi_1(r)^3 + \left(2m'(r) + \left(-2r + 3m(r) + rm'(r) - r\left(r - 2m(r)\right) f'(r)\right) f'(r)\right. \\
& \left. -r\left(r - 2m(r)\right) f''(r)\right) \phi_1(r) + 3\left(-2r + 3m(r) + rm'(r) - r\left(r - 2m(r)\right) f'(r)\right) \phi_1'(r) \\
& - 3r\left(r - 2m(r)\right) \phi_1''(r) \qquad \text{(A.2)}
\end{aligned}
$$

- If we give perfect fluid metric condition and TOV equation to these solution

$$
\begin{aligned}
f'(r) &= \frac{m(r) + 4\pi r^3 P(r)}{r\,(r - 2m(r))} \\
m(r) &= 4\pi \int_0^r \rho(r') r'^2 dr' \\
P'(r) &= -(P(r) + \rho(r)) \frac{m(r) + 4\pi r^3 P(r)}{r\,(r - 2m(r))} \quad ,
\end{aligned}
$$

Adding neutral scalar and gauge field to the model

- The two equations become much simpler

$$\phi_1''(r) = \frac{1}{3r\left(r-2m(r)\right)}\left(r^2\phi_1(r)^3 - 4\pi r^2\phi_1(r)(3P(r)-\rho(r))\right.$$
$$\left. - 6\left(r - m(r) + 2\pi r^3\left(P(r)-\rho(r)\right)\right)\phi_1'(r)\right) \tag{3.5}$$

$$\rho'(r) = \frac{1}{32\pi r(-r+3m(r)+4\pi r^3 P(r))}\left(-r^2\left(-r\phi_1(r)^4 + 8\phi_1(r)\phi_1'(r) + 6r\phi_1'(r)^2\right.\right.$$
$$\left. + 16\pi r P(r)\phi_1(r)\left(\phi_1(r) + r\phi_1'(r)\right)\right) + 32\pi\left(-3m(r) + 4\pi r^3 P(r)\right)\rho(r)$$
$$\left. + 12m(r)\left(-8\pi P(r) + r\phi_1'(r)\left(\phi_1(r) + r\phi_1'(r)\right)\right) + 128\pi^2 r^3\rho(r)^2\right)\,, \tag{3.6}$$

Adding neutral scalar and gauge field to the model

- Thus the equations we have to solve are

$$\phi_1''(r) = \frac{1}{3r\left(r-2m(r)\right)}\Big(r^2\phi_1(r)^3 - 4\pi r^2\phi_1(r)(3P(r)-\rho(r)) \\ - 6\left(r - m(r) + 2\pi r^3\left(P(r)-\rho(r)\right)\right)\phi_1'(r)\Big) \quad (3.5)$$

$$\rho'(r) = \frac{1}{32\pi r(-r+3m(r)+4\pi r^3 P(r))}\Big(-r^2\left(-r\phi_1(r)^4 + 8\phi_1(r)\phi_1'(r) + 6r\phi_1'(r)^2\right. \\ +16\pi r P(r)\phi_1(r)\left(\phi_1(r)+r\phi_1'(r)\right)\Big) + 32\pi\left(-3m(r)+4\pi r^3 P(r)\right)\rho(r) \\ +12m(r)\left(-8\pi P(r) + r\phi_1'(r)\left(\phi_1(r)+r\phi_1'(r)\right)\right) + 128\pi^2 r^3 \rho(r)^2\Big)\,, \quad (3.6)$$

$$m(r) = 4\pi\int_0^r \rho(r')r'^2 dr'$$

$$P'(r) = -(P(r)+\rho(r))\frac{m(r)+4\pi r^3 P(r)}{r\left(r-2m(r)\right)}$$

Adding neutral scalar and gauge field to the model

- By regularity condition of the star metric, the relevant parameter for this solution are P(0), rho(0), phi_1(0)

$$\phi_1''(r) = \frac{1}{3r\left(r-2m(r)\right)}\left(r^2\phi_1(r)^3 - 4\pi r^2\phi_1(r)(3P(r)-\rho(r))\right. \\ \left. - 6\left(r - m(r) + 2\pi r^3\left(P(r)-\rho(r)\right)\right)\phi_1'(r)\right) \quad (3.5)$$

$$\rho'(r) = \frac{1}{32\pi r(-r+3m(r)+4\pi r^3 P(r))}\left(-r^2\left(-r\phi_1(r)^4 + 8\phi_1(r)\phi_1'(r) + 6r\phi_1'(r)^2\right.\right. \\ \left. +16\pi r P(r)\phi_1(r)\left(\phi_1(r) + r\phi_1'(r)\right)\right) + 32\pi\left(-3m(r)+4\pi r^3 P(r)\right)\rho(r) \\ \left. +12m(r)\left(-8\pi P(r) + r\phi_1'(r)\left(\phi_1(r)+r\phi_1'(r)\right)\right) + 128\pi^2 r^3\rho(r)^2\right)\ , \quad (3.6)$$

$$m(r) = 4\pi\int_0^r \rho(r')r'^2 dr'$$

$$P'(r) = -(P(r)+\rho(r))\frac{m(r)+4\pi r^3 P(r)}{r\left(r-2m(r)\right)}$$

- If we give thess parameters, we can obtain holographic star solutions.

Adding neutral scalar and gauge field to the model

Now we add one more assumption at surfaces of stars to eliminate the parameter $\phi_1(0)$ whose physical meaning is not clear. At the surface of a star, the pressure $P(R)$ vanishes with very small energy density $\rho(R)$, where R is the radius of the star. So we may take $P(R) = \rho(R) = 0$ as a first approximation. Then our parameter space becomes two dimensional space parameterized by (P_c, ρ_c) giving clear physical meaning as center values of the pressure and the energy density. [1]. In this two dimensional space, we have obtained the resulting mass and radius shown in table.1. For every point in the parameter space, we can generate equation of state. We have shown the energy densities, the pressures and the equations of state for one case in Fig.2.

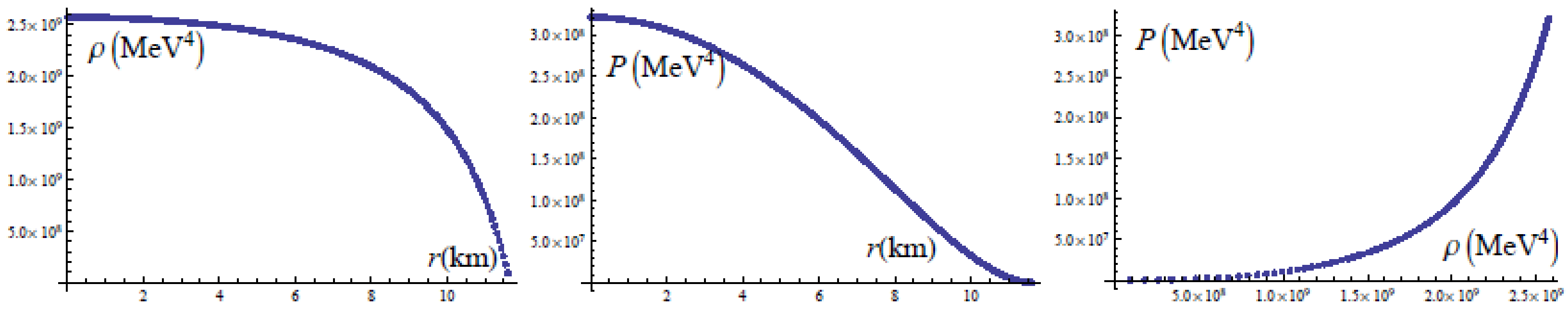


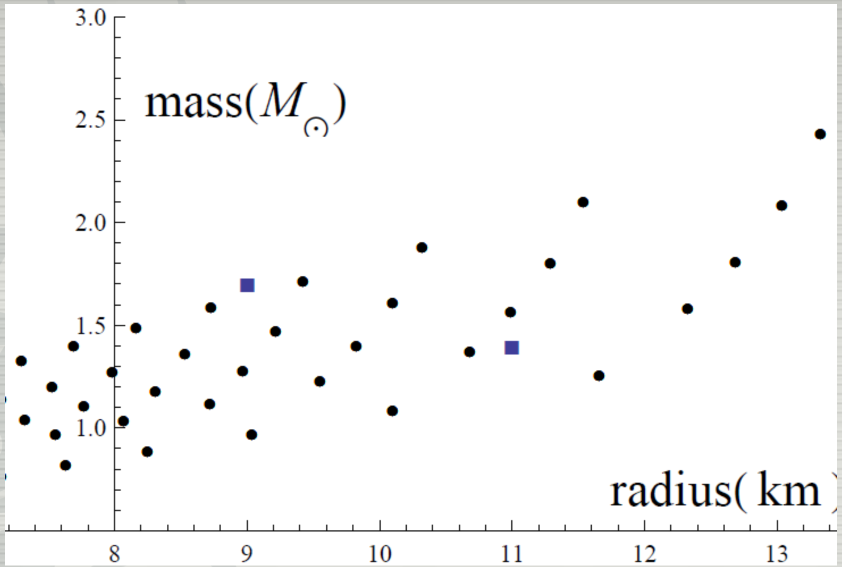
Figure 2: Energy density, pressure and equation of state for $\rho_c = 2.5713 \times 10^9 MeV^4$, $P_c = 3.2141 \times 10^8 MeV^4$. This configuration gives the radius 11.66 km and the mass 1.26 $M_\odot$

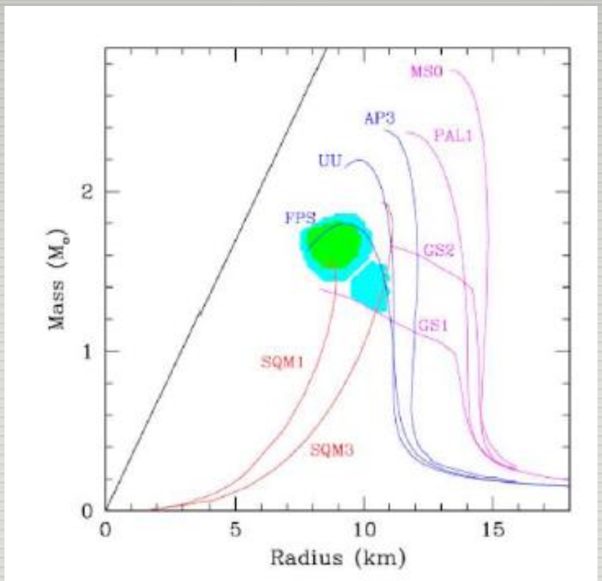
Adding neutral scalar and gauge field to the model

$\rho_c(8.5704 \times 10^8 MeV^4)$	3	4	5	6	7	8	9	10
$P_c(8.5704 \times 10^8 MeV^4)$	$\frac{1}{3} \times 3$	$\frac{1}{3} \times 4$	$\frac{1}{3} \times 5$	$\frac{1}{3} \times 6$	$\frac{1}{3} \times 7$	$\frac{1}{3} \times 8$	$\frac{1}{3} \times 9$	$\frac{1}{3} \times 10$
Mass$(M_\odot)$	2.43	2.11	1.89	1.72	1.59	1.49	1.40	1.33
Radius(km)	13.33	11.54	10.32	9.42	8.73	8.16	7.70	7.30
$\rho_c(8.5704 \times 10^8 MeV^4)$	3	4	5	6	7	8	9	10
$P_c(8.5704 \times 10^8 MeV^4)$	$\frac{1}{4} \times 3$	$\frac{1}{4} \times 4$	$\frac{1}{4} \times 5$	$\frac{1}{4} \times 6$	$\frac{1}{4} \times 7$	$\frac{1}{4} \times 8$	$\frac{1}{4} \times 9$	$\frac{1}{4} \times 10$
Mass$(M_\odot)$	2.09	1.81	1.62	1.48	1.37	1.28	1.20	1.14
Radius(km)	13.04	11.29	10.10	9.22	8.53	7.98	7.53	7.14
$\rho_c(8.5704 \times 10^8 MeV^4)$	3	4	5	6	7	8	9	10
$P_c(8.5704 \times 10^8 MeV^4)$	$\frac{1}{5} \times 3$	$\frac{1}{5} \times 4$	$\frac{1}{5} \times 5$	$\frac{1}{5} \times 6$	$\frac{1}{5} \times 7$	$\frac{1}{5} \times 8$	$\frac{1}{5} \times 9$	$\frac{1}{5} \times 10$
Mass$(M_\odot)$	1.81	1.57	1.40	1.28	1.19	1.11	1.05	0.99
Radius(km)	12.69	10.99	9.83	8.97	8.31	7.77	7.32	6.95
$\rho_c(8.5704 \times 10^8 MeV^4)$	3	4	5	6	7	8	9	10
$P_c(8.5704 \times 10^8 MeV^4)$	$\frac{1}{6} \times 3$	$\frac{1}{6} \times 4$	$\frac{1}{6} \times 5$	$\frac{1}{6} \times 6$	$\frac{1}{6} \times 7$	$\frac{1}{6} \times 8$	$\frac{1}{6} \times 9$	$\frac{1}{6} \times 10$
Mass$(M_\odot)$	1.59	1.38	1.23	1.12	1.04	0.97	0.92	0.87
Radius(km)	12.33	10.68	9.55	8.72	8.07	7.55	7.12	6.75
$\rho_c(8.5704 \times 10^8 MeV^4)$	3	4	5	6	7	8	9	10
$P_c(8.5704 \times 10^8 MeV^4)$	$\frac{1}{8} \times 3$	$\frac{1}{8} \times 4$	$\frac{1}{8} \times 5$	$\frac{1}{8} \times 6$	$\frac{1}{8} \times 7$	$\frac{1}{8} \times 8$	$\frac{1}{8} \times 9$	$\frac{1}{8} \times 10$
Mass$(M_\odot)$	1.26	1.09	0.97	0.89	0.82	0.77	0.73	0.69
Radius(km)	11.66	10.10	9.03	8.25	7.64	7.14	6.73	6.39

Table 1: Mass and Radius

Adding neutral scalar and gauge field to the model





tightly constrained pairs of values

$M = 1.7\ M_\odot$ and $R = 9$ km.

$M = 1.4\ M_\odot$ and $R = 11$ km

2 sigma error

Adding neutral scalar and gauge field to the model

- By holography, one can obtained equation of state depending on some paremeters.
- The back reactin was included in our construction. But we are still trying to undet stand this point.
- And we can make a situation whose surface energy density does not vanish.

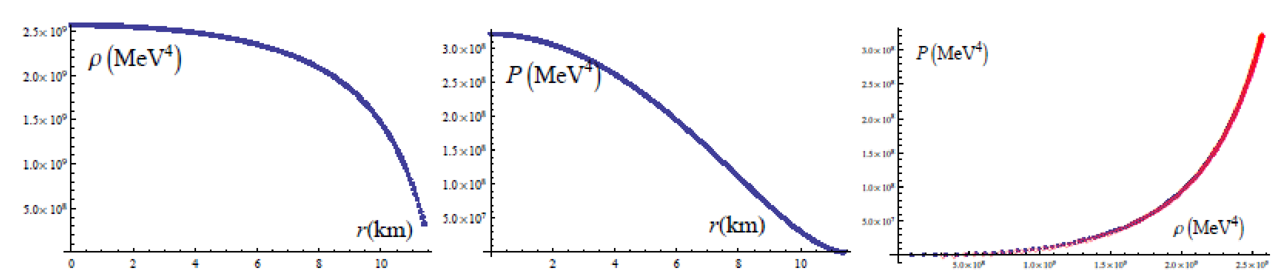


Figure 3: Energy density, pressure and equation of state for $\rho_c = 2.5713 \times 10^9 MeV^4$ and $P_c = 3.2141 \times 10^8 MeV^4$ with surface energy density $3.26 \times 10^8 MeV^4$. This configuration gives the radius 11.45 km and the mass 1.26 $M_\odot$(slightly smaller than Fig.2). In the third figure, we compare this equation of state to the equation of state of Fig.2. The empty diamonds are non-vanishing surface density configuration and the dots correspond to Fig.2.

- In this case, we can interpret the scalar field parameter as a parameter related to surface energy density.

Summary and discussion

- In order to consider compact stars, we have deformed boundary metric which is not flat.
- To embed the compact star geometry, we have taken rotational symmetry on the metric.
- Since the bulk Einstein equation gives more degrees of freedom than the boundary Einstein equation, our system gives one complicate constraint.
- By taking perfect fluid star as the boundary metric, the constraint becomes very simple we can solve.
- The simplest solution is well-known uniform density star.
- We think that this simple structure is from the simplicity of the model.
- Thus we introduced other matter fields.
- Another toy model gave more realistic configuration.
- As a next step, we will study full bulk geometry by solving partial differential equation with boundary metric, then we can understand which IR condition is relevant for this configuration.
- Then we can compare the IR condition to the already known cutoff and boundary conditions.