Nuclear Symmetry Energy from QCD Sum Rule

Heavy Ion Meeting 2012-04,
April 13, 2012

Kie Sang JEONG

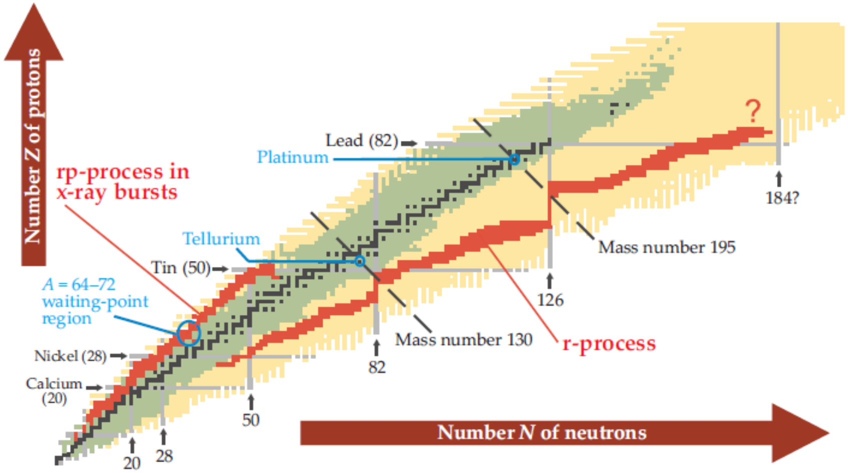
Su Houng LEE

(Theoretical Nuclear and Hadron Physics Group)

Yonsei University

Motivation 1 – KoRIA plan

- Rare Isotope Accelerator Plan

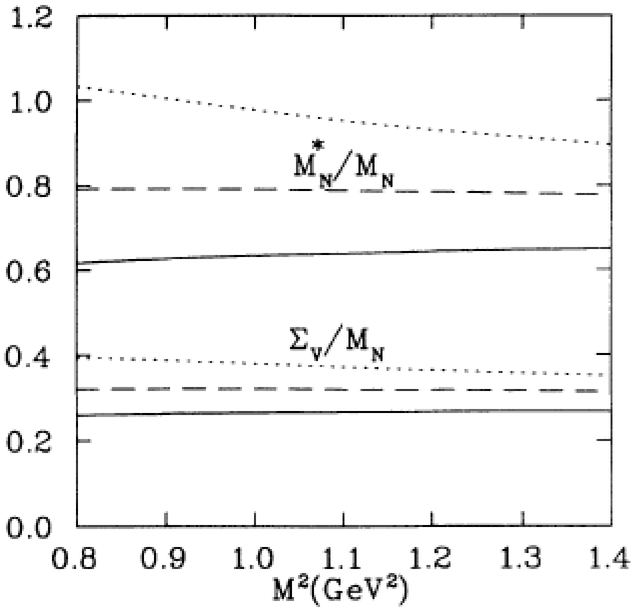


(Quoted from Physics Today November 2008)

Nuclear Symmetry Energy plays key role in Rare Isotope and Neutron Star study

Motivation 2 – RMFT vs QCD SR

- Dirac phenomenology of nucleon scattering on nuclear target suggests nucleon potential to consist of strong vector repulsion and scalar attraction
- This tendency also comes naturally in RMFT



Physical Review C 49, 464 (1993)

- For symmetric nuclear matter, it is confirmed that this result can be justified with QCD, by Thomas Cohen et al. (1992)

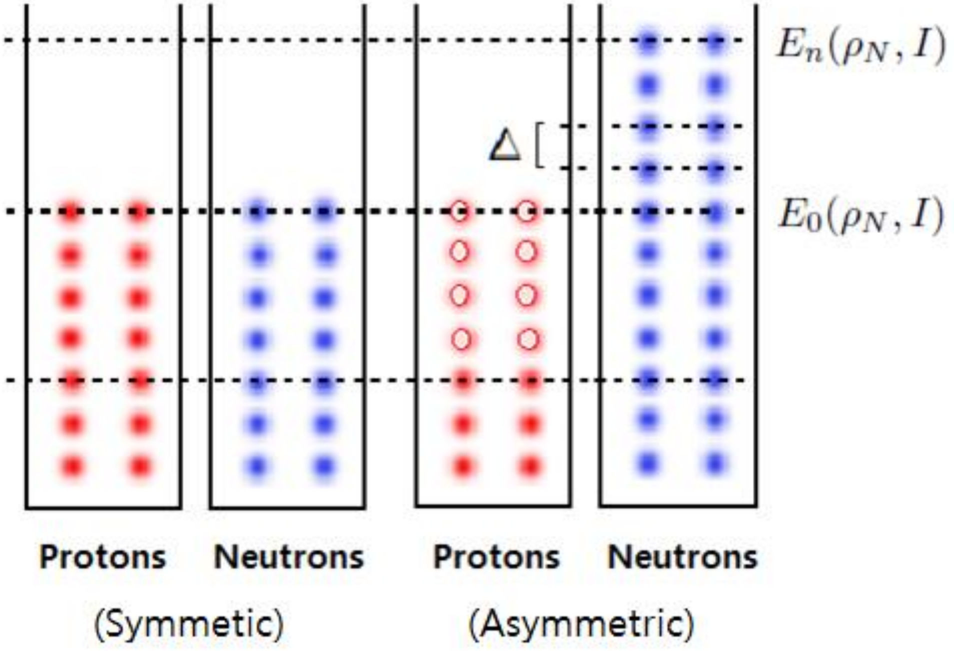
- Motivated by these, we applied QCD Sum Rule to asymmetric nuclear matter

Early attempt for finite nuclei

- Liquid drop model

$$m_{tot} = N m_n + Z m_p - E_B/c^2$$

$$E_B = a_V A - a_S A^{\frac{2}{3}} - a_C (Z(Z-1))A^{-\frac{1}{3}} - \boxed{a_A I^2 A} + \delta(A, Z) \qquad I = (N-Z)/A$$



- Total shifted energy

$$a_A I^2 A = \boxed{\left(\frac{1}{2} I A\right)} \cdot (E_n(\rho, I) - E_0(\rho, I))$$

Total shifted state number

$$= \left(\frac{1}{2} I A\right) \cdot \frac{1}{2}(E_n(\rho, I) - E_p(\rho, I))$$

Nuclear Symmetry Energy

$$a_A = \frac{1}{4I}(E_n(\rho, I) - E_p(\rho, I))$$

This simple concept can be generalized to infinite matter case

For infinite nuclear matter

- Energy per a nucleon

Nuclear Symmetry Energy

$$\begin{aligned} E &= \frac{N}{A}\overline{E}_n + \frac{Z}{A}\overline{E}_p \\ &= \frac{1}{2}(\overline{E}_n + \overline{E}_p) + \frac{1}{2}I(\overline{E}_n - \overline{E}_p) = E(\rho) + \boxed{E_{sym}(\rho)} I^2 + 0(I^4) \end{aligned}$$

- Single nucleon energy

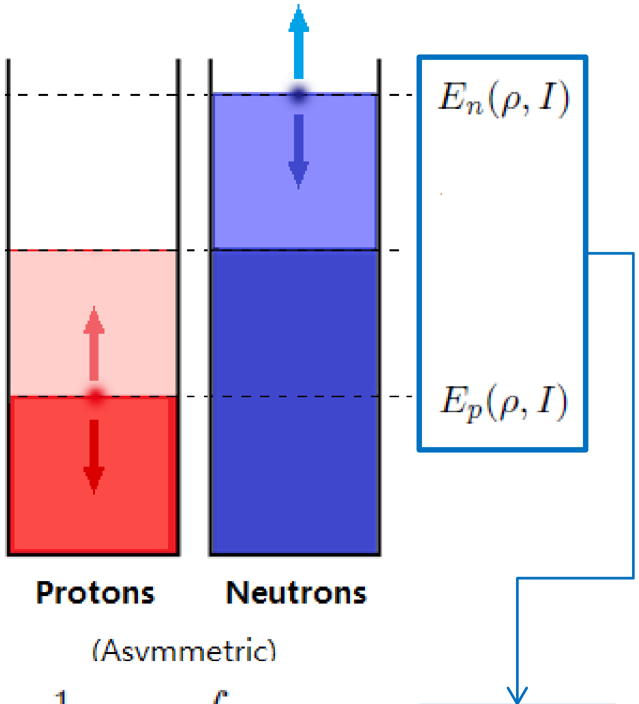
$$\begin{aligned} E_n &= m_0 + a\rho_p + b\rho_n + \cdots \\ &= m_0 + \frac{1}{2}\rho(a+b) + \frac{1}{2}I\rho(b-a) + \cdots \\ E_p &= m_0 + \frac{1}{2}\rho(a+b) - \frac{1}{2}I\rho(b-a) + \cdots \end{aligned}$$

- Averaged single nucleon energy

$$\overline{E} = \frac{1}{\int d^3k_n d^3k_p} \int d^3k_n d^3k_p E(\rho_n, \rho_p) \rightarrow \left(m_0 + \frac{1}{4}\rho(a+b) + \frac{1}{4}\rho I(b-a) \right) + \cdots$$

Mean field approximation

- Quasi-particle on the Fermi sea



$$\overline{E} = \frac{1}{\int d^3k_n d^3k_p} \int d^3k_n d^3k_p \boxed{E(\rho_n, \rho_p)}$$

$$\Rightarrow E_{sym} = \frac{1}{2I} \cdot (\bar{E}_n - \bar{E}_p)$$

(Up to linear density order)

- Nucleon propagator in nuclear medium

$$G(q) = -i \int d^4x e^{iqx} \langle \Psi_0 | \mathrm{T}[\psi(x)\bar{\psi}(0)] | \Psi_0 \rangle$$

$$= \frac{1}{\not{q} - M_n - \Sigma(q)}$$

$$\to \lambda^2 \frac{\not{q} + M^* - \not{u}\Sigma_v}{(q_0 - E_q)(q_0 - \bar{E}_0)}$$

- How we can get nucleon self energies in the fundamental principle?

- **QCD Sum Rule** is a well established method for investigating quasi-particle state in medium

QCD Sum Rule

- Correlation function

$$\Pi(q) \equiv i \int d^4x e^{iqx} \langle \Psi_0 | \mathrm{T}[\eta(x)\bar{\eta}(0)] | \Psi_0 \rangle$$

Contains all possible resonance states

$$\eta(x) = \epsilon_{abc}[u_a^T(x) C \gamma_\mu u_b(x)] \gamma_5 \gamma^\mu d_c(x)$$

Ioffe's interpolating field for proton

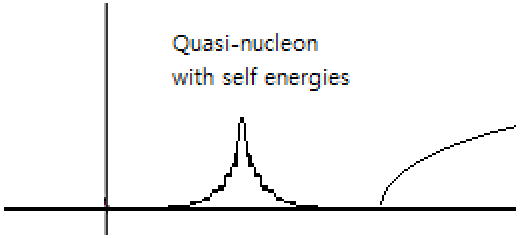
- Vacuum sum rule for nucleon

$$\Pi_{ij}(q) \equiv i \int d^4x \, e^{iq\cdot x} \langle 0 | T[\eta_i(x) \bar{\eta}_j(0)] | 0 \rangle$$
$$\equiv \Pi_s(q^2)\delta_{ij} + \Pi_q(q^2) \not{q}_{ij} \ .$$

- In-medium sum rule for quasi-nucleon

$$\Pi(q) \equiv \Pi_s(q^2, q\cdot u) + \Pi_q(q^2, q\cdot u)\not{q} + \boxed{\Pi_u(q^2, q\cdot u)}\not{u}.$$

In-medium vector part



QCD Sum Rule

- Phenomenological ansatz

$$\Pi(q_0,|\vec{q}|) \sim \frac{1}{(q^\mu - \tilde{\Sigma}_v^\mu)\gamma_\mu - M_N^*}$$

$$\boxed{\begin{aligned} \Pi_s(q_0,|\vec{q}|) &= -\lambda_N^{*2}\frac{M_N^*}{(q_0-E_q)(q_0-\bar{E}_q)} + \cdots \\ \Pi_q(q_0,|\vec{q}|) &= -\lambda_N^{*2}\frac{1}{(q_0-E_q)(q_0-\bar{E}_q)} + \cdots \\ \Pi_u(q_0,|\vec{q}|) &= +\lambda_N^{*2}\frac{\Sigma_v}{(q_0-E_q)(q_0-\bar{E}_q)} + \cdots \end{aligned}}$$

One can find **self energies near pole**

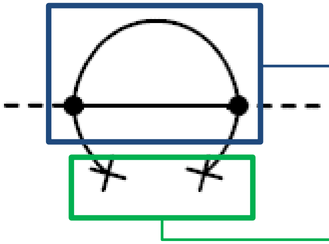
Kinetic part is excluded

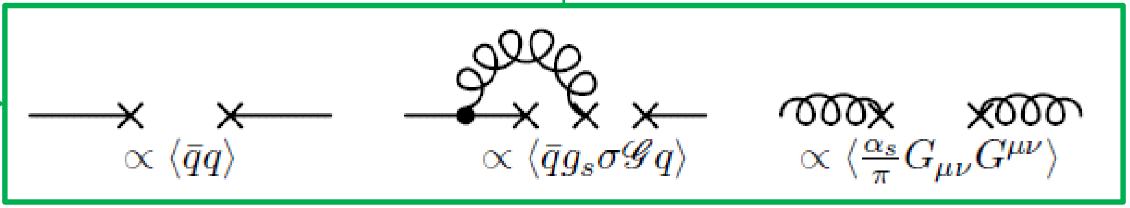
- At short distance, Wilson coefficient can be obtained by perturbative calculation

$$\Pi_i(q^2,q_0) = \sum_n \boxed{C_n^i(q^2,q_0)}\boxed{\langle\hat{O}_n\rangle_{\rho,I}}$$

Not be dependent on external momentum

For example,





(These figures are quoted from Ph.D. thesis of Thomas Hilger)

Iso-spin relation for 2-quark condensates

- In-medium condensate in asymmetric nuclear matter

$$\langle \hat{O}\rangle_{\rho,I} = \langle \hat{O}\rangle_{vac} + \langle n|\hat{O}|n\rangle\rho_n + \langle p|\hat{O}|p\rangle\rho_p$$

can be estimated with nucleon expectation value and each nucleon density

- Iso-scalar/vector operators in light quark flavor

$$\boxed{\hat{O}_0 \equiv \frac{1}{2}(\hat{O}^u + \hat{O}^d)} \longleftrightarrow \boxed{\hat{O}_1 \equiv \frac{1}{2}(\hat{O}^u - \hat{O}^d)}$$

Can be related with ratio factor

Well known from many previous studies

- 2-quark condensates

$$\langle q^a_\alpha(x)\bar{q}^b_\beta(0)\rangle_{\rho_N} = -\frac{\delta^{ab}}{12}\sum_{n=0}\frac{1}{n!}[x^n\boxed{\langle\bar{q}D^n q\rangle_{\rho_N}}\delta_{\alpha\beta} + x^n\boxed{\langle\bar{q}\gamma_\mu D^n q\rangle_{\rho_N}}\gamma^\mu_{\alpha\beta}]$$

Each 2 type of condensate need different ratio factor

Iso-spin relation for condensates

- Ratio factor for $\langle\bar q\gamma_\mu D^n q\rangle_p$ type

$$\frac{\langle u^\dagger D^n q\rangle_p}{\langle d^\dagger D^n d\rangle_p} \rightarrow \frac{\langle p|u^\dagger u|p\rangle}{\langle p|d^\dagger d|p\rangle} = 2 \qquad \langle[q^\dagger D^n q]_1\rangle_p = \frac{1}{3}\langle[q^\dagger D^n q]_0\rangle_p$$

In medium rest frame

- Ratio factor for $\langle\bar q D^n q\rangle_p$ type

$$\boxed{\langle[\bar q D^n q]_1\rangle_p = \frac{\mathcal{R}_-(m_q)}{\mathcal{R}_+(m_q)}\cdot\langle[\bar q D^n q]_0\rangle_p}$$

This relation can be estimated from lower lying baryon octet mass relation

- Lower lying baryon octet mass relation comes from baryon expectation value of energy-momentum tensor

$$m_N\bar\psi\psi = \langle N|\theta^\mu_\mu|N\rangle$$

Phenomenological description for nucleon mass term

Mass terms

- Trace part of QCD energy momentum tensor

$$\theta^\mu_\mu = m_u \bar{u}u + m_d \bar{d}d + m_s \bar{s}s + \sum m_h \bar{h}h$$

$$= \boxed{-\frac{1}{4\alpha_s}(9 - \frac{2}{3}n_h)\left(\frac{\alpha_s^2}{2\pi}\right)\mathcal{G}^2} + m_u \bar{u}u + m_d \bar{d}d + m_s \bar{s}s + \boxed{\sum m_h \bar{h}h}$$

$$\boxed{-\frac{2}{3}\frac{\alpha_s}{8\pi}n_h\mathcal{G}^2 + O(\mu^2/4m_h^2) + \cdots}$$

With renormalizing scheme (Trace anomaly)

Heavy quark expansion

$$= -\frac{9}{4}\frac{\alpha_s}{2\pi}\mathcal{G}^2 + m_u \bar{u}u + m_d \bar{d}d + m_s \bar{s}s$$

- For proton

$$m_p \bar{\psi}\psi = -\frac{9}{4}\left\langle \frac{\alpha_s}{2\pi}\mathcal{G}^2 \right\rangle + m_u \langle p|\bar{u}u|p\rangle + m_d \langle p|\bar{d}d|p\rangle + m_s \langle p|\bar{s}s|p\rangle$$

(M.A.Shifman, Physics Letters B 78B, 443 (1978))

Ratio factor from baryon octet

- Lower lying Baryon octet mass relation

$$m_p \bar{\psi}\psi = -\frac{9}{4}\left\langle \frac{\alpha_s}{2\pi}\mathcal{G}^2 \right\rangle + m_u \langle p|\bar{u}u|p\rangle + m_d \langle p|\bar{d}d|p\rangle + m_s \langle p|\bar{s}s|p\rangle$$

$$m_n \bar{\psi}\psi = -\frac{9}{4}\left\langle \frac{\alpha_s}{2\pi}\mathcal{G}^2 \right\rangle + m_d \langle p|\bar{u}u|p\rangle + m_u \langle p|\bar{d}d|p\rangle + m_s \langle p|\bar{s}s|p\rangle$$

$$m_{\sigma^+} \bar{\psi}\psi = -\frac{9}{4}\left\langle \frac{\alpha_s}{2\pi}\mathcal{G}^2 \right\rangle + m_u \langle p|\bar{u}u|p\rangle + m_s \langle p|\bar{d}d|p\rangle + m_d \langle p|\bar{s}s|p\rangle$$

$$m_{\sigma^-} \bar{\psi}\psi = -\frac{9}{4}\left\langle \frac{\alpha_s}{2\pi}\mathcal{G}^2 \right\rangle + m_d \langle p|\bar{u}u|p\rangle + m_s \langle p|\bar{d}d|p\rangle + m_u \langle p|\bar{s}s|p\rangle$$

$$m_{\Xi^0} \bar{\psi}\psi = -\frac{9}{4}\left\langle \frac{\alpha_s}{2\pi}\mathcal{G}^2 \right\rangle + m_s \langle p|\bar{u}u|p\rangle + m_u \langle p|\bar{d}d|p\rangle + m_d \langle p|\bar{s}s|p\rangle$$

$$m_{\Xi^-} \bar{\psi}\psi = -\frac{9}{4}\left\langle \frac{\alpha_s}{2\pi}\mathcal{G}^2 \right\rangle + m_s \langle p|\bar{u}u|p\rangle + m_d \langle p|\bar{d}d|p\rangle + m_u \langle p|\bar{s}s|p\rangle$$

(S.H.Choi, Master thesis, Yonsei University. (1991))

With this relation…

Ratio factor from baryon octet

- Iso-vector part, $\langle [\bar{q}q]_1 \rangle_p$ can be written as,

$$\langle p|\bar{u}u|p\rangle - \langle p|\bar{d}d|p\rangle = \frac{(m_{\Xi^0} + m_{\Xi^-}) - (m_{\Sigma^+} - m_{\Sigma^-})}{2m_s - 2m_q}$$

$$m_{\Xi^0} = 1315\ \text{MeV}, \quad m_{\Xi^-} = 1321\ \text{MeV}$$

$$m_{\Sigma^+} = 1190\ \text{MeV}, \quad m_{\Sigma^-} = 1197\ \text{MeV}$$

$$m_q \equiv \frac{1}{2}(m_u + m_d).$$

Strange quark mass = 150 MeV

- Iso-scalar part, $\langle [\bar{q}q]_0 \rangle_p$ comes from pion-nucleon sigma term

$$\langle p|\bar{u}u|p\rangle + \langle p|\bar{d}d|p\rangle = 2\langle [\bar{q}q]_0 \rangle_p = \frac{\sigma_N}{m_q}$$

- Ratio factor for $\langle \bar{q}D^n q \rangle_p$ type

$$\langle p|\bar{u}u|p\rangle \pm \langle p|\bar{d}d|p\rangle$$

$$= \boxed{\left(1 \pm \frac{\frac{45\ \text{MeV}}{m_q} + \frac{249\ \text{MeV}}{300\ \text{MeV} - 2m_q}}{\frac{45\ \text{MeV}}{m_q} - \frac{249\ \text{MeV}}{300\ \text{MeV} - 2m_q}}\right)} \cdot \langle p|\bar{u}u|p\rangle \equiv \boxed{\mathcal{R}_\pm(m_q)} \cdot \langle p|\bar{u}u|p\rangle$$

4-quark Operator Product Expansion

- So far, there was no systematic description for four-quark condensates in the nucleon sum rule

$$\langle u^a_\alpha \bar{u}^b_\beta u^c_\gamma \bar{u}^d_\delta \rangle_{\rho,I} \simeq \langle u^a_\alpha \bar{u}^b_\beta \rangle_{\rho,I} \langle u^c_\gamma \bar{u}^d_\delta \rangle_{\rho,I} - \langle u^a_\alpha \bar{u}^d_\delta \rangle_{\rho,I} \langle u^c_\gamma \bar{u}^b_\beta \rangle_{\rho,I},$$

$$\langle u^a_\alpha \bar{u}^b_\beta d^c_\gamma \bar{d}^d_\delta \rangle_{\rho,I} \simeq \langle u^a_\alpha \bar{u}^b_\beta \rangle_{\rho,I} \langle d^c_\gamma \bar{d}^d_\delta \rangle_{\rho,I}.$$

Simple factorization scheme was used as in the vacuum saturation hypothesis

- To express four-quark condensates in terms of linear combination of independent operators, one can use **Fierz rearrangement**

$$\langle \bar{q}^a_i q^b_j \bar{q}^c_k q^d_l \rangle_p = \frac{1}{16} \Gamma^o_{ij} \Gamma^m_{kl} \langle (\bar{q}^a \Gamma_o q^b)(\bar{q}^c \Gamma_m q^d) \rangle_p$$

- **Constraint** from **`Zero Identity'** for pure quark flavor case which comes from forbidden di-quark structure of quark model for hadron

$$\epsilon_{abc}\epsilon_{a'b'c}(u_a^T C \gamma_\mu \gamma_5 u_b)(\bar{u}_{b'}^T \gamma_\nu \gamma_5 C \bar{u}_{a'}) = 0$$

And with assumed P, T symmetry of medium ground state…

4-quark Operator Product Expansion

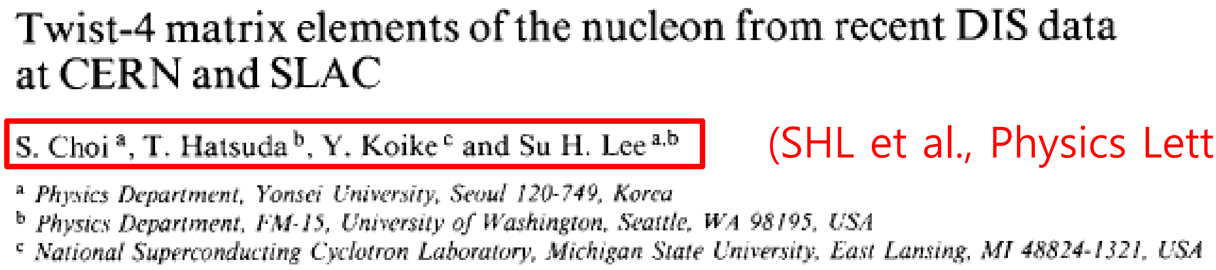
- 4-qaurk OPE of the nucleon sum rule can be expressed with only few condensates as below

$\langle\bar{q}_1 q_1 \bar{q}_2 q_2\rangle_p$	$q_1 = q_2$	$q_1 \neq q_2$
Twist 4 (Dimension 6 spin2)	$\langle(\bar{q}_1^a\gamma^\alpha q_1^{a'})(\bar{q}_2^b\gamma^\beta q_2^{b'})\rangle_p\vert_{s,t}$ $\langle(\bar{q}_1^a\gamma^\alpha\gamma_5 q_1^{a'})(\bar{q}_2^b\gamma^\beta\gamma_5 q_2^{b'})\rangle_p\vert_{s,t}$ (4 independent Ops.)	$\langle(\bar{q}_1^a\gamma^\alpha q_1^{a'})(\bar{q}_2^b\gamma^\beta q_2^{b'})\rangle_p\vert_{s,t}$ $\langle(\bar{q}_1^a\gamma^\alpha\gamma_5 q_1^{a'})(\bar{q}_2^b\gamma^\beta\gamma_5 q_2^{b'})\rangle_p\vert_{s,t}$ (4 independent Ops.)
Dimension 6 scalar	$\langle(\bar{q}_1^a\gamma^\alpha q_1^{a'})(\bar{q}_2^b\gamma_\alpha q_2^{b'})\rangle_p$	$\langle(\bar{q}_1^a\gamma^\alpha q_1^{a'})(\bar{q}_2^b\gamma_\alpha q_2^{b'})\rangle_p$
Dimension 6 vector		$\langle(\bar{q}_1^a q_1^{a'})(\bar{q}_2^b\gamma^\alpha q_2^{b'})\rangle_p$

Can be combined into scalar and vector condensates

- Among these, except for $\langle(\bar{u}\gamma^\alpha u)(\bar{d}\gamma^\beta d)\rangle_p$, $\langle(\bar{u}\gamma^\alpha\gamma_5 u)(\bar{d}\gamma^\beta\gamma_5 d)\rangle_p$ all twist 4 operators can be estimated from experiment (DIS)

Twist 4 operator from DIS data



Twist-4 matrix elements of the nucleon from recent DIS data at CERN and SLAC

S. Choi [a], T. Hatsuda [b], Y. Koike [c] and Su H. Lee [a,b]

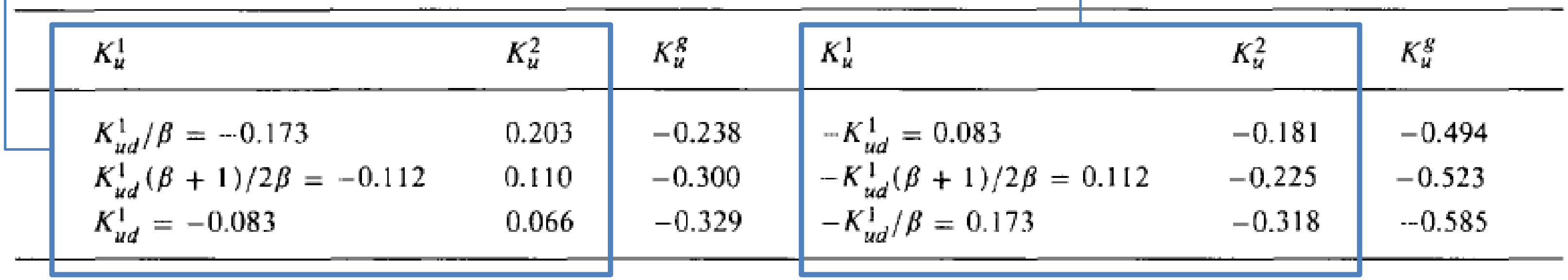
[a] Physics Department, Yonsei University, Seoul 120-749, Korea
[b] Physics Department, FM-15, University of Washington, Seattle, WA 98195, USA
[c] National Superconducting Cyclotron Laboratory, Michigan State University, East Lansing, MI 48824-1321, USA

(SHL et al., Physics Letter B 312 (1993) 351-357)

$$\frac{1}{2}M_N(u_\alpha u_\beta - \frac{1}{4}g_{\alpha\beta})K_u^1 = \langle(\bar{u}\gamma_\alpha\gamma_5\tau^a u)(\bar{u}\gamma_\alpha\gamma_5\tau^a u)\rangle_p + \langle(\bar{u}\gamma_\alpha\gamma_5\tau^a u)(\bar{d}\gamma_\alpha\gamma_5\tau^a d)\rangle_p$$

$$\frac{1}{2}M_N(u_\alpha u_\beta - \frac{1}{4}g_{\alpha\beta})K_u^2 = \langle(\bar{u}\gamma_\alpha\tau^a u)(\bar{u}\gamma_\alpha\tau^a u)\rangle_p + \langle(\bar{u}\gamma_\alpha\tau^a u)(\bar{d}\gamma_\alpha\tau^a d)\rangle_p$$

Where, K_u, K_d are

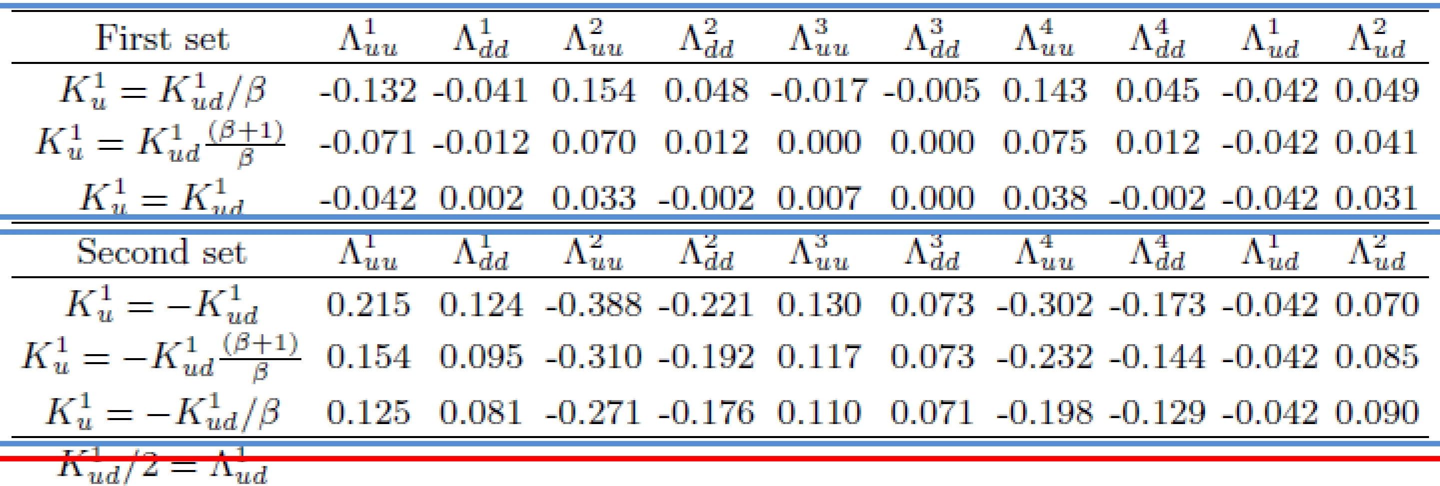


K_u^1	K_u^2	K_u^g	K_u^1	K_u^2	K_u^g
$K_{ud}^1/\beta = -0.173$	0.203	−0.238	$-K_{ud}^1 = 0.083$	−0.181	−0.494
$K_{ud}^1(\beta+1)/2\beta = -0.112$	0.110	−0.300	$-K_{ud}^1(\beta+1)/2\beta = 0.112$	−0.225	−0.523
$K_{ud}^1 = -0.083$	0.066	−0.329	$-K_{ud}^1/\beta = 0.173$	−0.318	−0.585

From these result…

Twist 4 operator from DIS data

- Table for twist 4 matrix elements for the nucleon SR



First set	Λ^1_{uu}	Λ^1_{dd}	Λ^2_{uu}	Λ^2_{dd}	Λ^3_{uu}	Λ^3_{dd}	Λ^4_{uu}	Λ^4_{dd}	Λ^1_{ud}	Λ^2_{ud}
$K^1_u = K^1_{ud}/\beta$	-0.132	-0.041	0.154	0.048	-0.017	-0.005	0.143	0.045	-0.042	0.049
$K^1_u = K^1_{ud}\frac{(\beta+1)}{\beta}$	-0.071	-0.012	0.070	0.012	0.000	0.000	0.075	0.012	-0.042	0.041
$K^1_u = K^1_{ud}$	-0.042	0.002	0.033	-0.002	0.007	0.000	0.038	-0.002	-0.042	0.031

Second set	Λ^1_{uu}	Λ^1_{dd}	Λ^2_{uu}	Λ^2_{dd}	Λ^3_{uu}	Λ^3_{dd}	Λ^4_{uu}	Λ^4_{dd}	Λ^1_{ud}	Λ^2_{ud}
$K^1_u = -K^1_{ud}$	0.215	0.124	-0.388	-0.221	0.130	0.073	-0.302	-0.173	-0.042	0.070
$K^1_u = -K^1_{ud}\frac{(\beta+1)}{\beta}$	0.154	0.095	-0.310	-0.192	0.117	0.073	-0.232	-0.144	-0.042	0.085
$K^1_u = -K^1_{ud}/\beta$	0.125	0.081	-0.271	-0.176	0.110	0.071	-0.198	-0.129	-0.042	0.090

$K^1_{ud}/2 = \Lambda^1_{ud}$

$$\langle(\bar{q}_1\gamma^{\bar{\alpha}}\gamma_5 t^A q_1)(\bar{q}_2\gamma^{\alpha}\gamma_5 t^A q_2)\rangle_p|_{s,t} = \frac{1}{4\pi\alpha_s}\frac{M_N}{2}\left(u^{\bar{\alpha}}u^{\alpha} - \frac{1}{4}g^{\bar{\alpha}\alpha}\right)\Lambda^1_{q_1q_2}$$

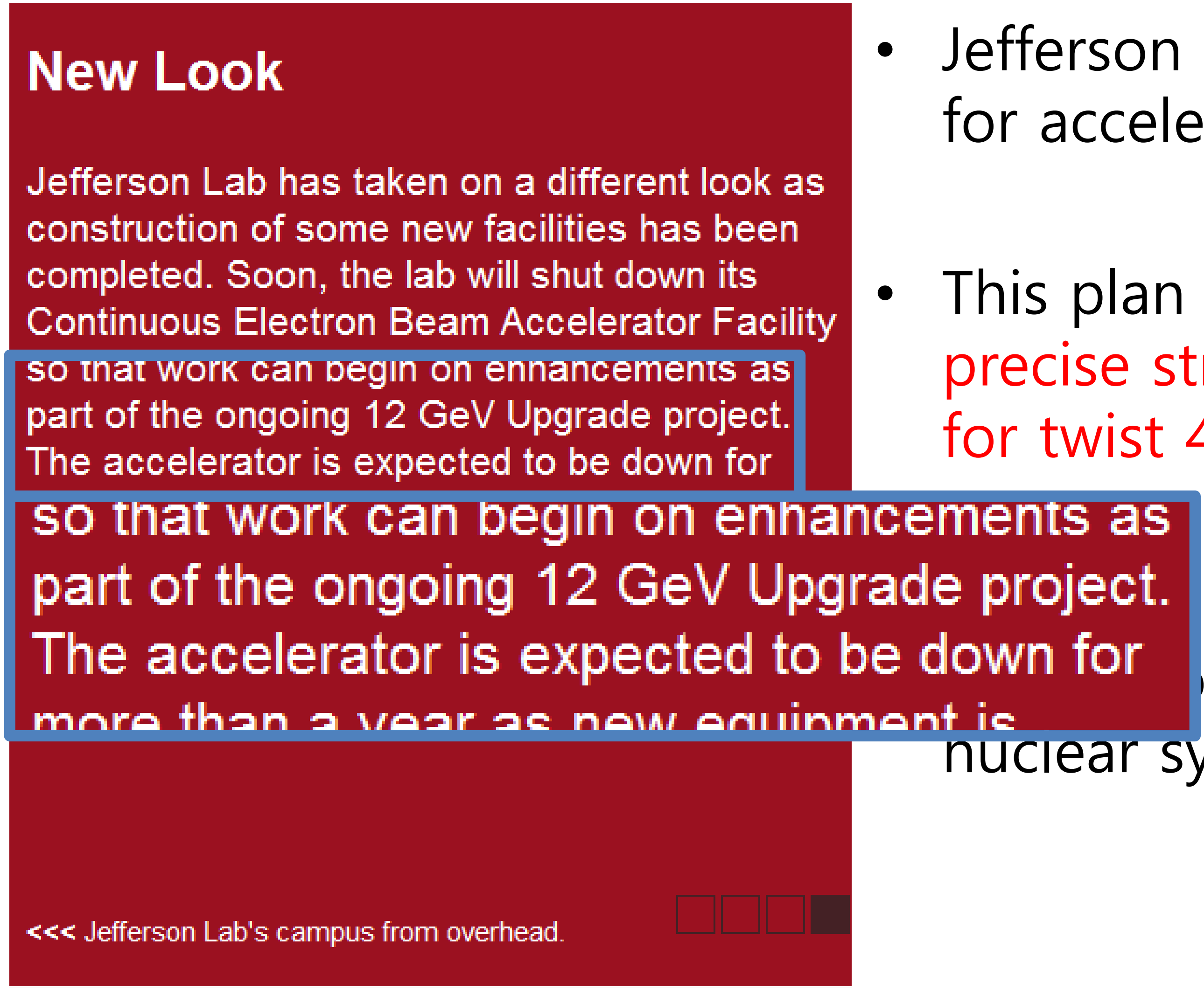
$$\langle(\bar{q}_1\gamma^{\bar{\alpha}} t^A q_1)(\bar{q}_2\gamma^{\alpha} t^A q_2)\rangle_p|_{s,t} = \frac{1}{4\pi\alpha_s}\frac{M_N}{2}\left(u^{\bar{\alpha}}u^{\alpha} - \frac{1}{4}g^{\bar{\alpha}\alpha}\right)\Lambda^2_{q_1q_2}$$

$$\langle(\bar{q}\gamma^{\bar{\alpha}}\gamma_5 q)(\bar{q}\gamma^{\alpha}\gamma_5 q)\rangle_p|_{s,t} = \langle(\bar{q}\gamma^{\bar{\alpha}} q)(\bar{q}\gamma^{\alpha} q)\rangle_p|_{s,t}$$
$$= \frac{1}{4\pi\alpha_s}\frac{M_N}{2}\left(q^{\bar{\alpha}}q^{\alpha} - \frac{1}{4}g^{\bar{\alpha}\alpha}\right)\Lambda^3_{qq},$$

First set and second set give different contribution to Nuclear Symmetry Energy

DIS data will be sharpened

- Table for twist 4 matrix elements has some ambiguity



New Look

Jefferson Lab has taken on a different look as construction of some new facilities has been completed. Soon, the lab will shut down its Continuous Electron Beam Accelerator Facility so that work can begin on enhancements as part of the ongoing 12 GeV Upgrade project. The accelerator is expected to be down for

so that work can begin on enhancements as part of the ongoing 12 GeV Upgrade project. The accelerator is expected to be down for more than a year as new equipment is

<<< Jefferson Lab's campus from overhead.

- Jefferson Lab has a plan for accelerator upgrade
- This plan will lead to more precise structure functions for twist 4 matrix elements
- et more precise ntributions to the nuclear symmetry energy

Borel transformation

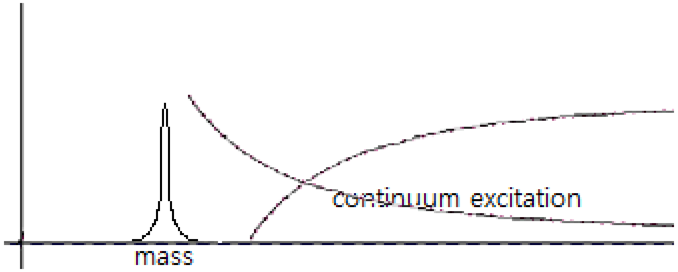
- Borel transformation to exclude quasi-hole and continuum excitation

- Differential operator for OPE side

$$\boxed{\mathcal{B}[f(q_0^2,|\vec{q}|)] \equiv \lim_{\substack{-q_0^2,n\to\infty \\ -q_0^2/n=M^2}} \frac{(-q_0^2)^{n+1}}{n!}\left(\frac{\partial}{\partial q_0^2}\right)^n f(q_0^2,|\vec{q}|)}$$

$$\Pi_i(q_0^2,|\vec{q}|) \Rightarrow \bar{\mathcal{B}}[\Pi_i(q_0^2,|\vec{q}|)]$$

- Weight function for phenomenological side



With weight function

$$\boxed{W(\omega) = (\omega - \bar{E}_q)e^{-\omega^2/M^2}}$$

Self energies with OPEs

- Self energies can be obtained by taking ratio

$$\frac{\mathcal{B}[\Pi_s(q_0^2,|\vec{q}|)]}{\mathcal{B}[\Pi_q(q_0^2,|\vec{q}|)]} = \frac{\lambda_N^{*2} M_p^* e^{-(E_q^2-\vec{q}^2)/M^2}}{\lambda_N^{*2} e^{-(E_q^2-\vec{q}^2)/M^2}} = M_p^*$$

$$\frac{\mathcal{B}[\Pi_u(q_0^2,|\vec{q}|)]}{\mathcal{B}[\Pi_q(q_0^2,|\vec{q}|)]} = \frac{\lambda_N^{*2} \Sigma_v^p e^{-(E_q^2-\vec{q}^2)/M^2}}{\lambda_N^{*2} e^{-(E_q^2-\vec{q}^2)/M^2}} = \Sigma_v^p$$

To treat self energies in terms of density order and asymmetric factor, self energies need re-arrangement

- New symbols for self energies

$$\begin{aligned}\mathcal{N}^{n,p}(\rho) \equiv& \bar{\mathcal{B}}[\Pi_s^{n,p}(q_0^2,|\vec{q}|)] + \bar{\mathcal{B}}[\Pi_u^{n,p}(q_0^2,|\vec{q}|)]\\ =&\mathcal{N}_{(\rho^0,I^0)}^{n,p} + \mathcal{N}_{(\rho,I^0)}^{n,p}\rho + [\mathcal{N}_{(\rho,I)}^{n,p}\rho]I,\end{aligned}$$

Numerator of total self energy

$$\begin{aligned}\mathcal{D}^{n,p}(\rho) \equiv& \bar{\mathcal{B}}[\Pi_q^{n,p}(q_0^2,|\vec{q}|)]\\ =&\mathcal{D}_{(\rho^0,I^0)}^{n,p} + \mathcal{D}_{(\rho,I^0)}^{n,p}\rho + [\mathcal{D}_{(\rho,I)}^{n,p}\rho]I,\end{aligned}$$

Denominator of total self energy

Power of ρ in the first index represents the density power order
Power of I in the second index represents the iso-spin order

QCD sum rule Formula

- General expression

$$E^{V}_{sym}(\rho) = \boxed{\frac{1}{2}\Big[\frac{1}{2}\rho\cdot\left(E^{n}_{(\rho,I)} - E^{p}_{(\rho,I)}\right)} + \frac{1}{3}\rho^{2}\cdot\left(E^{n}_{(\rho^{2},I)} - E^{p}_{(\rho^{2},I)}\right) + \frac{1}{4}\rho^{3}\cdot\left(E^{n}_{(\rho^{3},I)} - E^{p}_{(\rho^{3},I)}\right) + \cdots\Big]$$
$$+\frac{1}{2}\left[\frac{1}{3}\rho^{2}\cdot\left(E^{n}_{(\rho^{2},I^{2})} + E^{p}_{(\rho^{2},I^{2})}\right) + \frac{1}{4}\rho^{3}\cdot\left(E^{n}_{(\rho^{3},I^{2})} + E^{p}_{(\rho^{3},I^{2})}\right) + \cdots\right].$$

- Expression up to linear density order

$$\boxed{E^{V}_{sym}(\rho) = \frac{1}{4}\rho\cdot\left[\frac{1}{\mathcal{D}^{p}_{(\rho^{0},I^{0})}}\cdot\left[-2\mathcal{N}^{p}_{(\rho,I)}\right] - \frac{\mathcal{N}^{p}_{(\rho^{0},I^{0})}}{\left(\mathcal{D}^{p}_{(\rho^{0},I^{0})}\right)^{2}}\cdot\left[-2\mathcal{D}^{p}_{(\rho,I)}\right]\right]}$$

- We need both sum rule

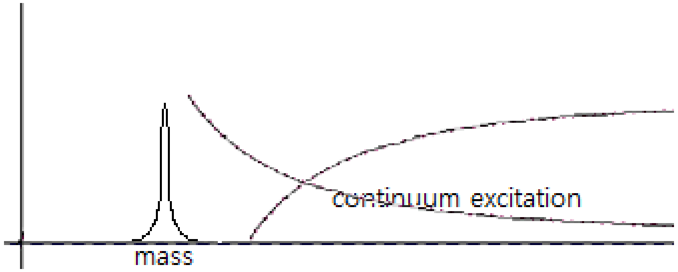
$$\langle\hat{O}\rangle_{\rho,I} = \langle\hat{O}\rangle_{vac} + \langle n|\hat{O}|n\rangle\rho_{n} + \langle p|\hat{O}|p\rangle\rho_{p}$$

In medium condensates are estimated up to linear order

General expression which contains up to 2nd order density is needed to check higher density behavior of Nuclear Symmetry Energy

Sum Rule analysis up to dimension 5

- Borel window



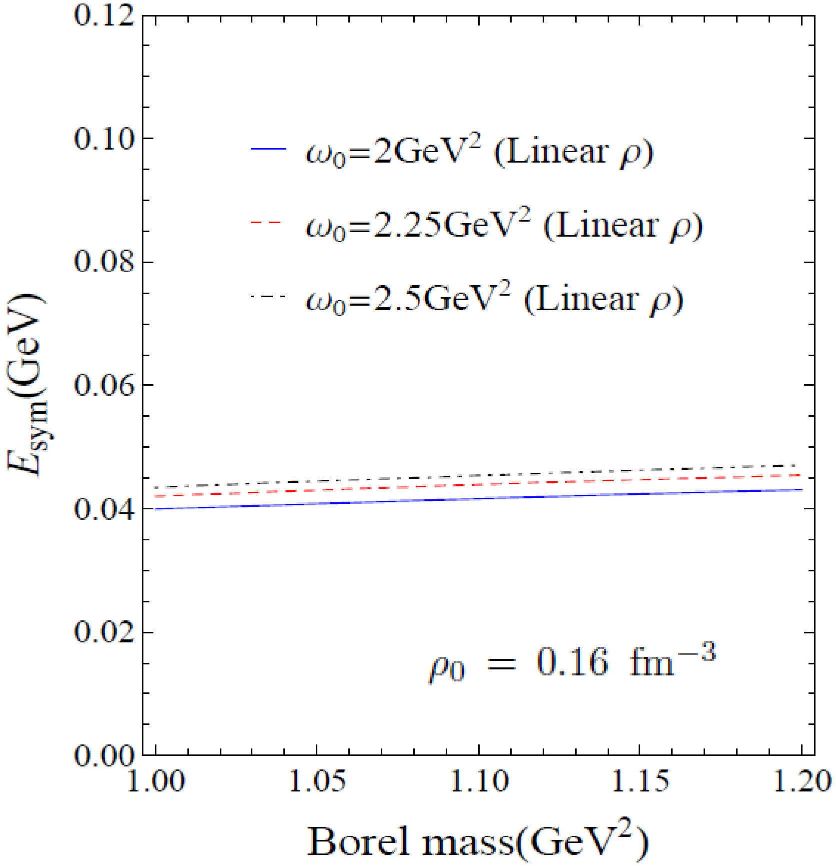
To make pole contribution be more than 50% and
Highest mass dim condensate contribution be less than 50%,

We will see our sum rule in

$$1.0 \text{ GeV}^2 \leq M^2 \leq 1.2 \text{ GeV}^2$$

(B.L.Ioffe and A.V.Smilga, NPB232 (1984),109)

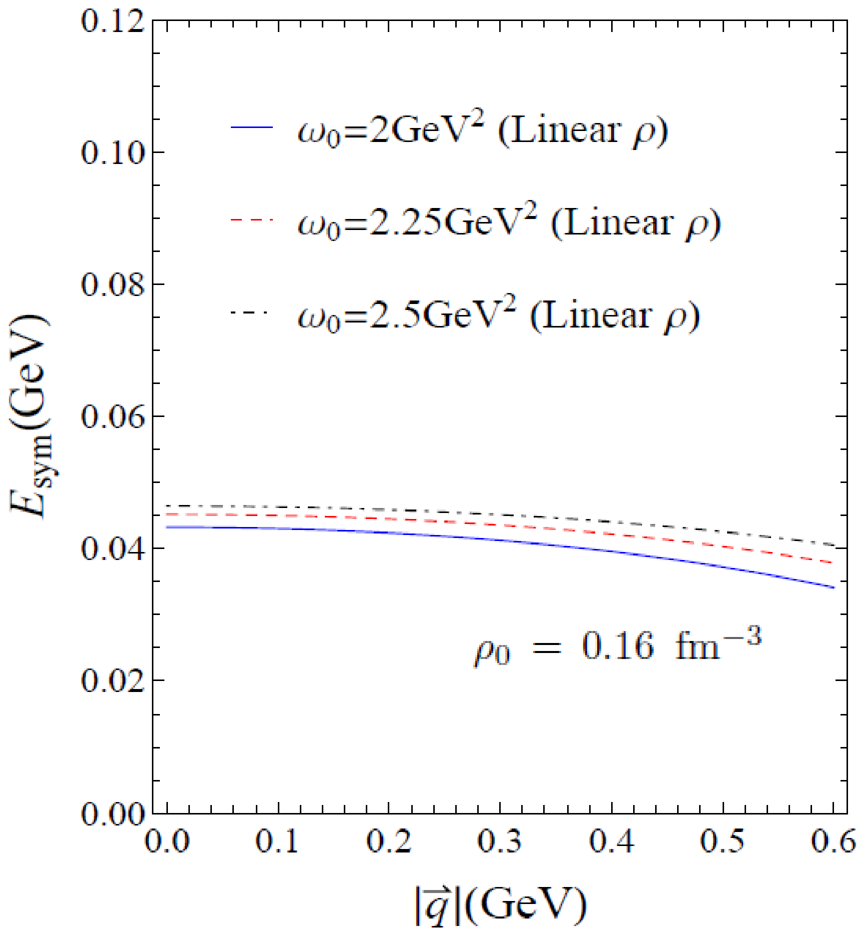
- Nuclear Symmetry Energy



Nuclear Symmetry Energy is **~40 MeV**
This result shows consistency with previous Nuclear Symmetry Energy study in order of magnitude

Sum Rule analysis up to dimension 5

- Main ingredient?



- Nuclear Symmetry Energy do not **strongly depend on quasi nucleon three-momentum** in 0 ≤ **q** ≤ 0.5 GeV
- Our sum rule result mainly consists of ``Potential like" part
- Kinetic part of Nuclear Symmetry Energy is known as

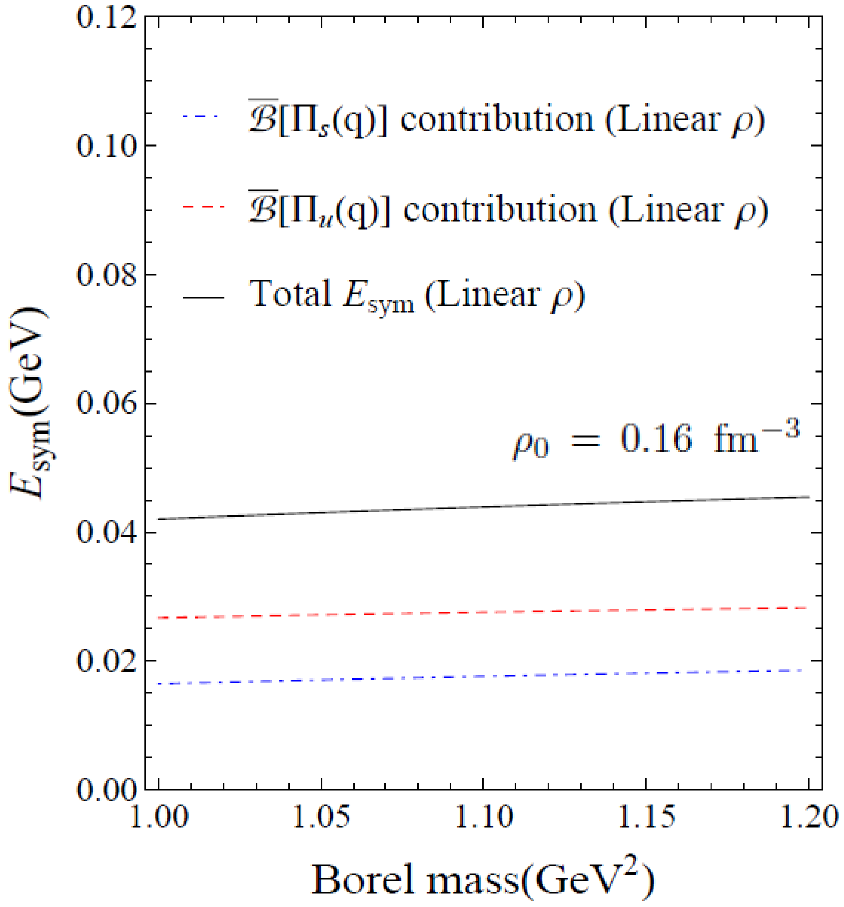
$$E_{sym}^{kin} = \frac{1}{6}\frac{k_F^2}{\sqrt{k_F^2 + M_N^{*2}}}$$

in mean field type calculation

(S.Kubis et al., Physics Letters B 399 (1997) 191-195)

Comparison to RMFT

- Scalar/Vector self energy contribution



- RMFT result

$$E_{\rm sym} = \frac{1}{6}\frac{k_F^2}{E_F^*} + \frac{1}{2}\left[f_\rho - f_\delta\left(\frac{m^*}{E_F^*}\right)^2\right]\rho_B$$
$$\simeq \frac{1}{6}\frac{k_F^2}{E_F^*} + \Sigma_\rho^0 + \frac{m^*}{E_F^*}\Sigma_\delta$$

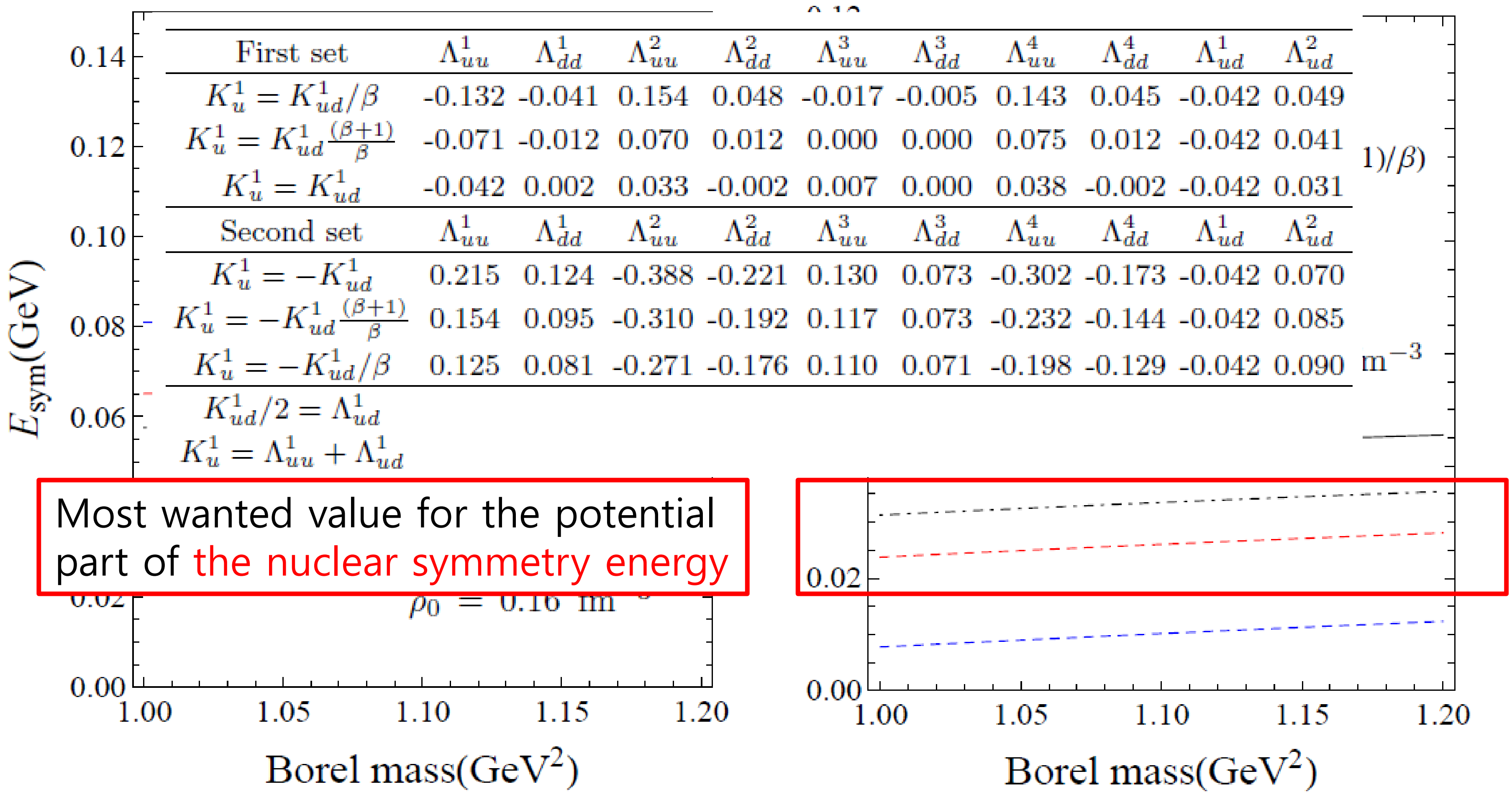
Vector meson exchange -> Repulsive
Scalar meson exchange -> Attractive

(V.Baran, et al. Physics Reports 410 (2005) 335–466)

- In our result, both self energies give **positive contribution**
- **Which part gives reducing contribution in our sum rule?**

Nuclear Symmetry Energy with twist 4 op.

- Contribution from first set
- Contribution from second set



First set	Λ^1_{uu}	Λ^1_{dd}	Λ^2_{uu}	Λ^2_{dd}	Λ^3_{uu}	Λ^3_{dd}	Λ^4_{uu}	Λ^4_{dd}	Λ^1_{ud}	Λ^2_{ud}
$K^1_u = K^1_{ud}/\beta$	-0.132	-0.041	0.154	0.048	-0.017	-0.005	0.143	0.045	-0.042	0.049
$K^1_u = K^1_{ud}\frac{(\beta+1)}{\beta}$	-0.071	-0.012	0.070	0.012	0.000	0.000	0.075	0.012	-0.042	0.041
$K^1_u = K^1_{ud}$	-0.042	0.002	0.033	-0.002	0.007	0.000	0.038	-0.002	-0.042	0.031
Second set	Λ^1_{uu}	Λ^1_{dd}	Λ^2_{uu}	Λ^2_{dd}	Λ^3_{uu}	Λ^3_{dd}	Λ^4_{uu}	Λ^4_{dd}	Λ^1_{ud}	Λ^2_{ud}
$K^1_u = -K^1_{ud}$	0.215	0.124	-0.388	-0.221	0.130	0.073	-0.302	-0.173	-0.042	0.070
$K^1_u = -K^1_{ud}\frac{(\beta+1)}{\beta}$	0.154	0.095	-0.310	-0.192	0.117	0.073	-0.232	-0.144	-0.042	0.085
$K^1_u = -K^1_{ud}/\beta$	0.125	0.081	-0.271	-0.176	0.110	0.071	-0.198	-0.129	-0.042	0.090

$K^1_{ud}/2 = \Lambda^1_{ud}$

$K^1_u = \Lambda^1_{uu} + \Lambda^1_{ud}$

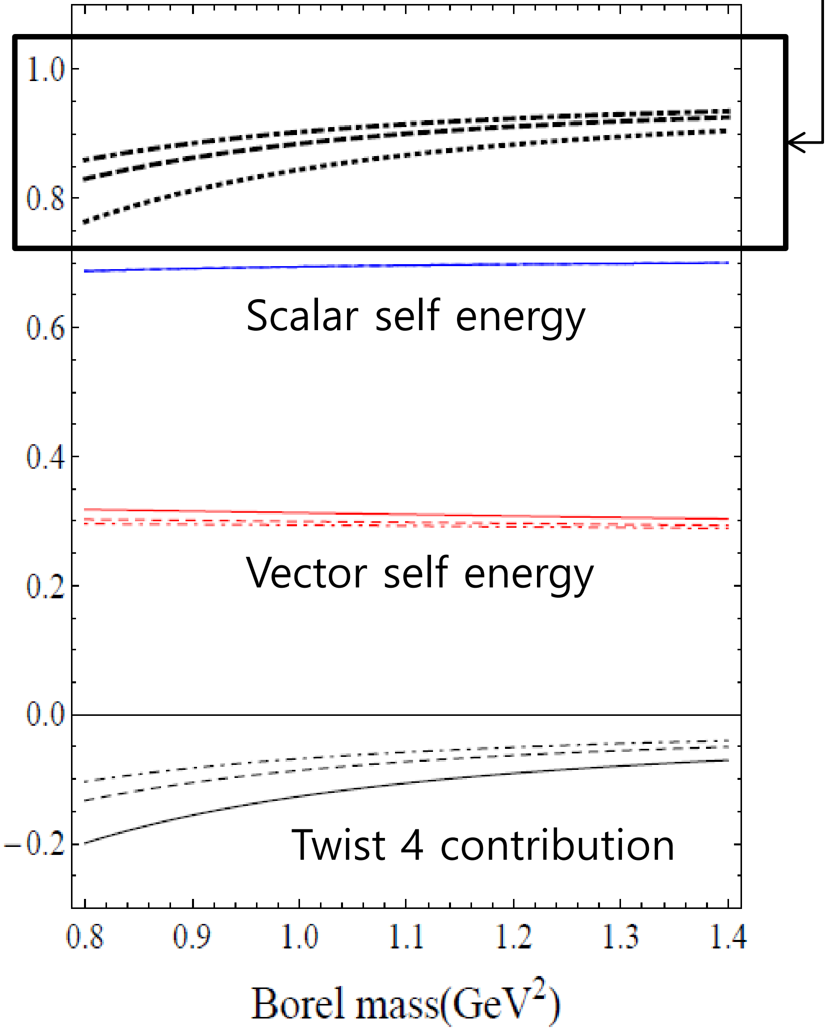
Most wanted value for the potential part of the nuclear symmetry energy

20 MeV~40 MeV enhanced

20 MeV~30 MeV reduced

Nuclear Symmetry Energy with twist 4 op.

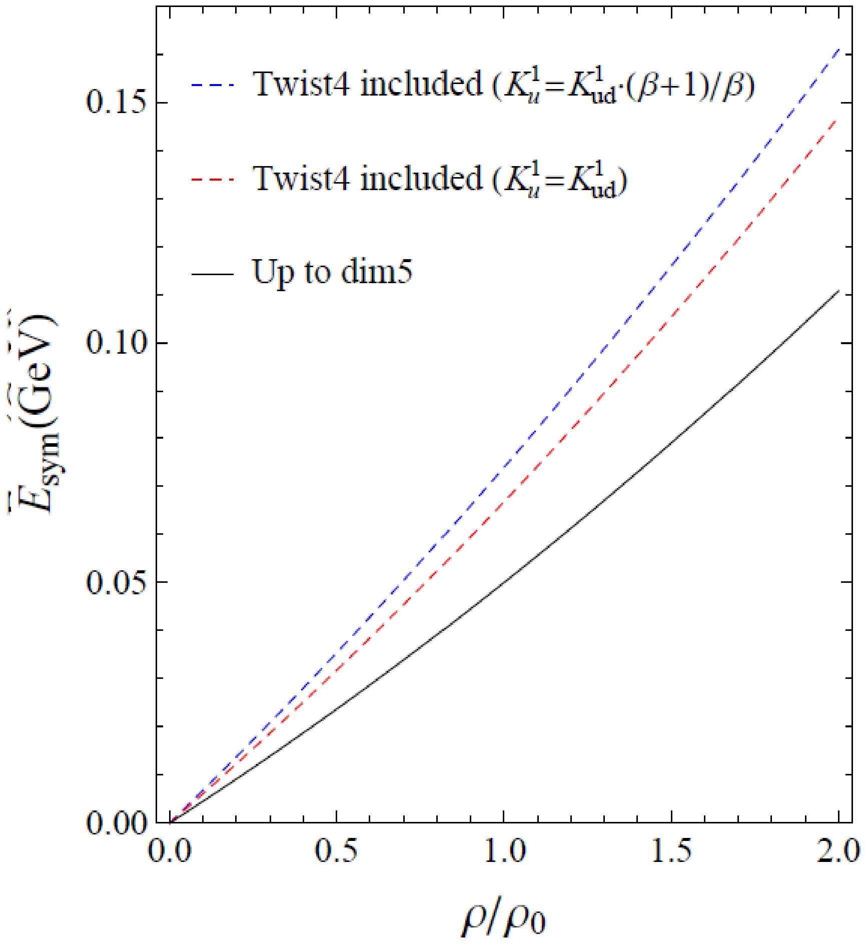
- In-medium nucleon sum rule with twist 4 condensates



- Thick black lines are total quasi-nucleon self energy
- The first set gives quasi-nucleon energy over than the nucleon mass in vacuum
- But **the second set gives plausible quasi-nucleon energy**, slightly smaller than the nucleon mass in vacuum
- So hereafter **we conclude that** the second set may be more plausible choice

Nuclear Symmetry Energy with twist 4 op.

- Density dependence with twist-4 contribution



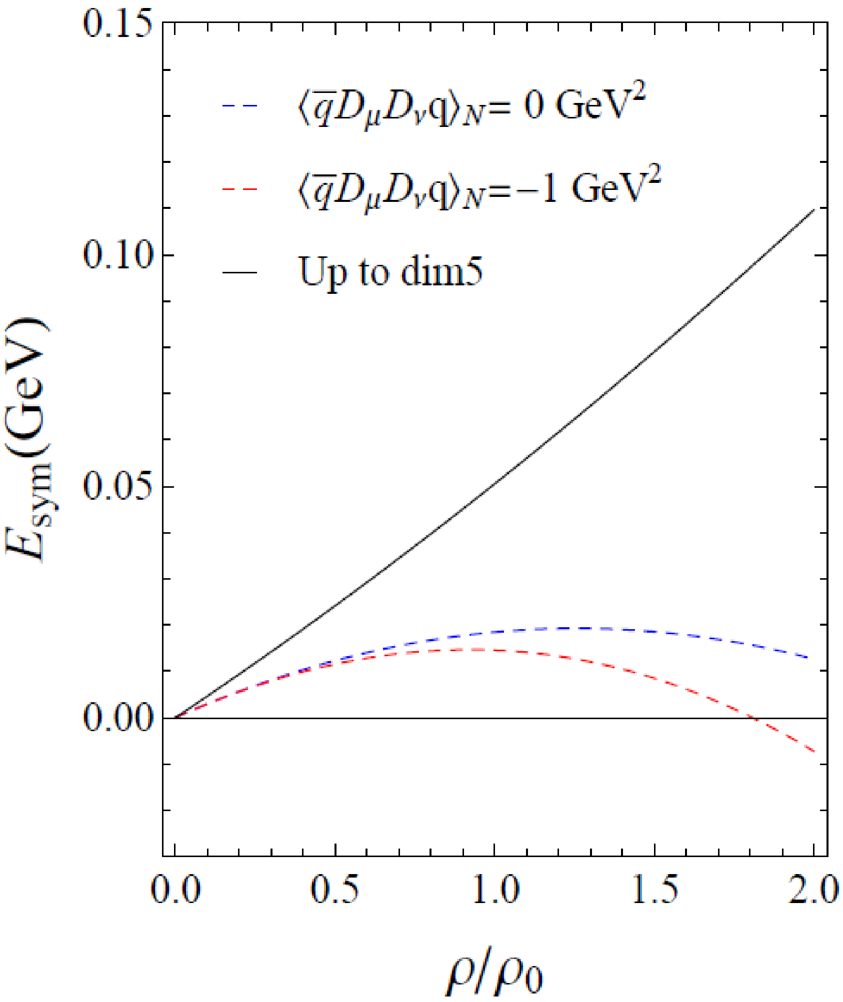
- The result with the **first set gives strong enhancement and** **second set gives strong reduction** as nuclear matter becomes higher density

- In dense matter, **twist-4 condensates determine** higher density behavior of the Nuclear Symmetry Energy

- **Multi-quark operators** may determine the asymmetric nuclear bulk properties in dense condition

Nuclear Symmetry Energy <-> DIS

- Contributions of 2-quark dimension 5 operators



- Our sum rule shows that **precise measurement of Nuclear Symmetry Energy** can give us a meaningful constraint to unknown higher twist operators

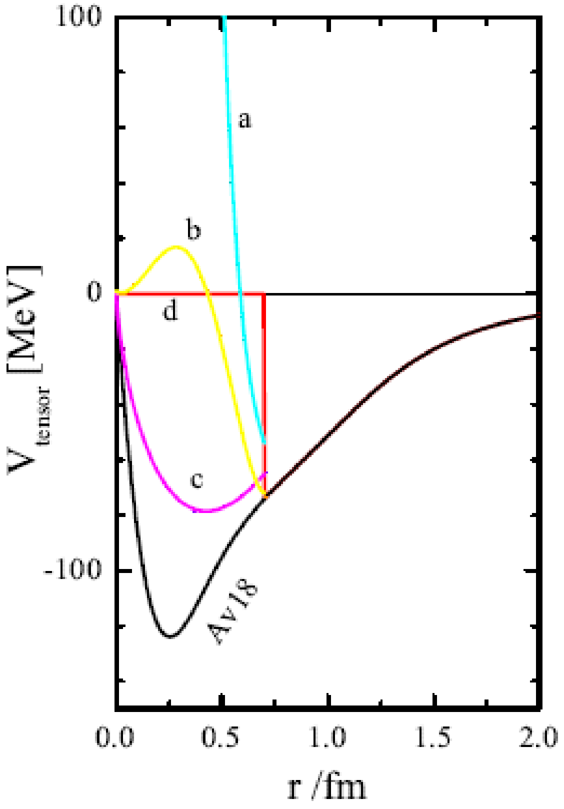
- From these facts, we can re-write effective expression for Nuclear Symmetry Energy in our sum rule context

$$E_{sym}(\rho) \simeq \frac{1}{6}\frac{k_F^2}{E_F^*} + \left[c_{s,\langle \bar{q}\Gamma q\rangle}(\rho) + c_{v,\langle q^\dagger \Gamma q\rangle}(\rho)\right]$$
$$\pm \left[\tilde{c}_{t,Twist4}(\rho) + \tilde{c}_{t,2q\ tensor}(\rho)\right]$$

Scalar-Vector Cond.=> Enhancing
Pure Tensor Cond. => Reducing

At Extremely high density?

- Model prediction with effective nucleon-nucleon-tensor potential



(Bao An Li: arXiv:1107.0496v1)

$$V_{t\pi} = -\frac{f_\pi^2}{4\pi} m_\pi (\tau_1 \cdot \tau_2) S_{12}$$
$$[\frac{1}{(m_\pi r)^3} + \frac{1}{(m_\pi r)^2} + \frac{1}{3 m_\pi r}] \exp(-m_\pi r)$$

$$S_{12} = 3 \frac{(\sigma_1 \cdot r)(\sigma_1 \cdot r)}{r^2} - (\sigma_2 \cdot \sigma_2)$$

- At high density, phase of nuclear matter may become to quark phase, and mean distance between quarks will be shortened

- We may make naïve guess **that this kind of prediction for dense condition might be justified by QCD via studying twist 4 of multi quark condensates at high density**

Conclusion

- We have successfully reproduced numerical value of the Nuclear Symmetry Energy of previous studies and found exact contribution of twist 4 matrix elements to the nucleon sum rule
- Twist 4 matrix elements give non-negligible contribution to Nuclear Symmetry Energy. Reducing contribution may be more plausible
- Nuclear Symmetry Energy can be understood via QCD, and also constraints for twist-n and multi-quark condensates can be studied from Nuclear Symmetry Energy
- Extremely high density behavior remains unclear, but this also might be understood via QCD

Back up slides

4 quark Operator Product Expansion

- So far, there was no systematic description for four-quark condensates in the nucleon sum rule

$$\langle u^a_\alpha \bar{u}^b_\beta u^c_\gamma \bar{u}^d_\delta \rangle_{\rho,I} \simeq \langle u^a_\alpha \bar{u}^b_\beta \rangle_{\rho,I} \langle u^c_\gamma \bar{u}^d_\delta \rangle_{\rho,I} - \langle u^a_\alpha \bar{u}^d_\delta \rangle_{\rho,I} \langle u^c_\gamma \bar{u}^b_\beta \rangle_{\rho,I} ,$$

$$\langle u^a_\alpha \bar{u}^b_\beta d^c_\gamma \bar{d}^d_\delta \rangle_{\rho,I} \simeq \langle u^a_\alpha \bar{u}^b_\beta \rangle_{\rho,I} \langle d^c_\gamma \bar{d}^d_\delta \rangle_{\rho,I} .$$

Simple factorization scheme was used as in the vacuum saturation hypothesis

- To express four-quark condensates in terms of linear combination of independent operators, one can use **Fierz rearrangement**

$$\epsilon_{abc}\epsilon_{a'b'c}(u_a^T C \gamma_\mu u_b)(\bar{u}_{b'}^T \gamma_\nu C \bar{u}_{a'})$$

(For pure quark flavor case)

$$= \epsilon_{abc}\epsilon_{a'b'c} \frac{1}{16} (\bar{u}_{a'} \Gamma^o u_a)(\bar{u}_{b'} \Gamma^k u_b) \cdot \mathrm{Tr}\left[\gamma_\mu \Gamma_k \gamma_\nu C \Gamma_o^T C\right]$$

$$\epsilon_{abc}\epsilon_{a'bc'} \ \gamma^5 \gamma^\mu d_c \bar{d}_{c'}^T \gamma^\nu \gamma^5 \ (u_a^T C \gamma_\mu \not{q} \gamma_\nu C \bar{u}_{a'})$$

(For mixed quark flavor case)

$$= \epsilon_{abc}\epsilon_{a'bc'} \ \frac{1}{16} (\gamma^5 \gamma^\mu \Gamma_k \gamma^\nu \gamma^5)(\bar{u}_{a'} \Gamma^o u_a)(\bar{d}_{c'} \Gamma^k d_c) \cdot \mathrm{Tr}\left[\gamma_\mu \not{q} \gamma_\nu C \Gamma_o C\right]$$

4 quark Operator Product Expansion

- And one may use `zero identity' for pure quark flavor case which comes from forbidden di-quark structure of quark model for hadron

$$\epsilon_{abc}\epsilon_{a'b'c}(u_a^T C\gamma_\mu\gamma_5 u_b)(\bar{u}_{b'}^T\gamma_\nu\gamma_5 C\bar{u}_{a'}) = 0$$

- Finally, with this constraint, one can find that it needs only for 2type of twist-4 condensates and their trace part to express the 4q OPE of the nucleon sum rule (for pure flavor case)

$$\begin{aligned}
&\epsilon_{abc}\epsilon_{a'b'c}(u_a^T C\gamma_\mu u_b)(\bar{u}_{b'}^T\gamma_\nu C\bar{u}_{a'}) \\
&\quad = \epsilon_{abc}\epsilon_{a'b'c}\,\frac{1}{16}\,\Big\{8S_{\mu\alpha\nu\bar{\alpha}}\cdot\Big((\bar{u}^{a'}\gamma^{\bar{\alpha}}u^a)(\bar{u}^{b'}\gamma^{\alpha}u^b) - (\bar{u}^{a'}\gamma^{\bar{\alpha}}\gamma_5 u^a)(\bar{u}^{b'}\gamma^{\alpha}\gamma_5 u^b)\Big) \\
&\qquad\qquad \boxed{+\,16i\epsilon_{\mu\alpha\nu\bar{\alpha}}\,(\bar{u}^{a'}\gamma^{\bar{\alpha}}u^a)(\bar{u}^{b'}\gamma^{\alpha}\gamma_5 u^b)}\Big\},
\end{aligned}$$

Will be dropped with assumed P, T symmetry of nuclear medium

4 quark Operator Product Expansion

- For mixed quark flavor case, with assumed P, T symmetry of nuclear medium ground state, the OPE can be simplified as

$$
\begin{aligned}
&\epsilon_{abc}\epsilon_{a'bc'}\ \gamma^5\gamma^\mu d_c \bar{d}^T_{c'}\gamma^\nu\gamma^5\ (u^T_a C\gamma_\mu \not{q}\gamma_\nu C\bar{u}_{a'}) \\
&\qquad = \epsilon_{abc}\epsilon_{a'bc'}\ \frac{1}{16}(\gamma^5\gamma^\mu\Gamma_k\gamma^\nu\gamma^5)(\bar{u}_{a'}\Gamma^o u_a)(\bar{d}_{c'}\Gamma^k d_c)\cdot \mathrm{Tr}\left[\gamma_\mu \not{q}\gamma_\nu C\Gamma_o C\right] \\
&\qquad \Rightarrow \epsilon_{abc}\epsilon_{a'bc'}\ \frac{1}{16}\Big\{-8q_\alpha(\bar{u}_{a'}\gamma^\alpha u_a)(\bar{d}_{c'}d_c) - 8(q_{\bar{\alpha}}\gamma_\alpha + g_{\alpha\bar{\alpha}}\not{q})(\bar{u}_{a'}\gamma^\alpha u_a)(\bar{d}_{c'}\gamma^{\bar{\alpha}}d_c) \\
&\qquad\qquad\qquad + 8(q_{\bar{\alpha}}\gamma_\alpha - g_{\alpha\bar{\alpha}}\not{q})(\bar{u}_{a'}\gamma^\alpha\gamma_5 u_a)(\bar{d}_{c'}\gamma^{\bar{\alpha}}\gamma_5 d_c)\Big\},
\end{aligned}
$$

- So now we have got exact four-quark OPE for the nucleon sum rule with Ioffe current

- These four-quark OPE gives important contribution to nucleon in the **symmetric**/**asymmetric** nuclear matter