Parton Cascade

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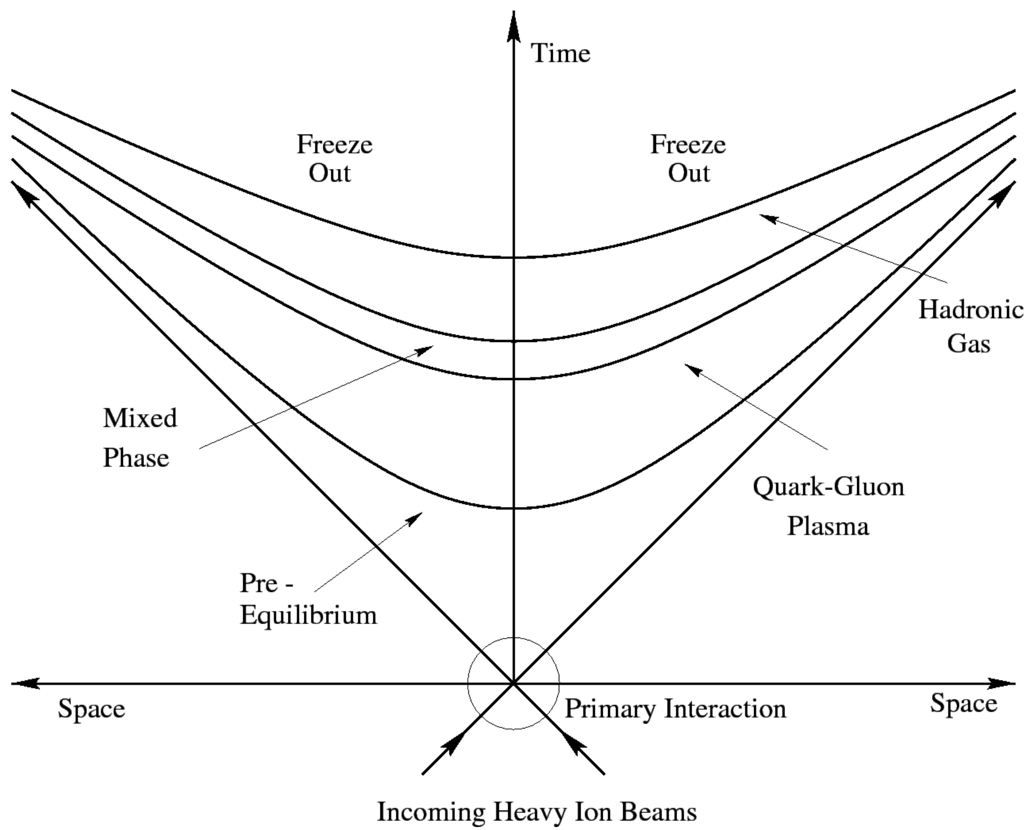
Contents

1. Introduction
2. Initial Phase Space of HIC
3. Parton cascade
4. Some Results
5. Discussion

1. Introduction

- Ambitious proposals: Understanding from the start to the end of HIC:



- **The dream we have is to describe all the way from the first contact of two colliding nuclei to hadronic detections**
  - What is the phase space of incoming nucleus (parton distribution)?
  - How does the degree of freedom of nuclei make collisions and be liberated?
    - How many?
    - What is the virtuality?
    - What is the phase space?
  - What is the mechanism to thermalize them?
  - How the partons evolve(in QGP)?
    - What is the color force?
  - How do the partons become hadrons?
  - How do hadrons make expansion?

2. Initial Phase Space

- We make simple:
  - Projectile and target nucleus are made of nucleons(protons and neutrons) and nucleons are made of partons (gluons, valance quarks, sea quarks and anti-quarks)
  - These quasi-particles (parton) will make collisions while the two nuclei passing through with the assumption of frozen transversal momentum
  - If transversal momentum after collision is greater than p_T(min), we assume the quasi-particle is liberated from the mother nucleus and become on-shell particle.

2.1 Nucleus Phase Space

- The phase space can be factorized into spatial and momentum distribution;

$$F(x, p) = \rho(x)\, f(p)$$

- Spatial distribution:
  – Wood-Saxon model:
  – Hard sphere model:

- Momentum distribution:
  – CGC (KNV)
  – cteq6
  – grv
  – ABM
  – HERA
  – MRST/MSTW
  – NNPDF

  – EKS for shadowing effect

2.2 initial phase space

- Thus, the number of events involving parton k;

$$\frac{dN_k}{dy\,dp_T} = K\,T_{AB}(b) \int dy_2\ \frac{2\,\pi p_T}{\hat{s}} \sum_{ij,kl} \frac{1}{1+\delta_{ij}} \{$$

$$x_1\, f_i^A\,(x_1, Q_0)\, x_2\, f_j^B(x_2, Q_0)\, \sigma_{ij\to kl}(\hat{s}, \hat{t}, \hat{u})$$

$$+\, x_1\, f_j^A(x_1, Q_0)\, x_2\, f_i^B(x_2, Q_0)\, \sigma_{ij\to kl}(\hat{s}, \hat{u}, \hat{t}) \quad \}$$

  - where x_1, x_2 are Bjorken variables of projectile and target respectively
  - y , y_2 are rapidities of produced partons
  - K is K-factor to include higher order diagrams

- $T_{AB}(b) = \int dx_T \, T_A(x_T) T_B(b - x_T)$; Nuclear overlap function
- $T_A(\vec{x}_T) = \int dx \; \rho_A(\vec{x}_T, z)$, nuclear profile function
- $f_i^A \,(x_1, Q_0)$, distribution of parton i in nucleus A
  - $f_i^A = f_i^{n\, or\, p} * EKS$
  - Where we use CTEQ6 for f_i^p

- Kinematics and allowed ranges

$$x_1 = p_T(e^{y_1} + e^{y_2})/\sqrt{s},$$

$$x_2 = p_T(e^{-y_1} + e^{-y_2})/\sqrt{s},$$

$$\hat{s} = x_1 x_2 s,$$

$$\hat{t} = -p_T^2(1 + e^{y_2 - y_1}),$$

$$\hat{u} = -p_T^2(1 + e^{y_1 - y_2}).$$

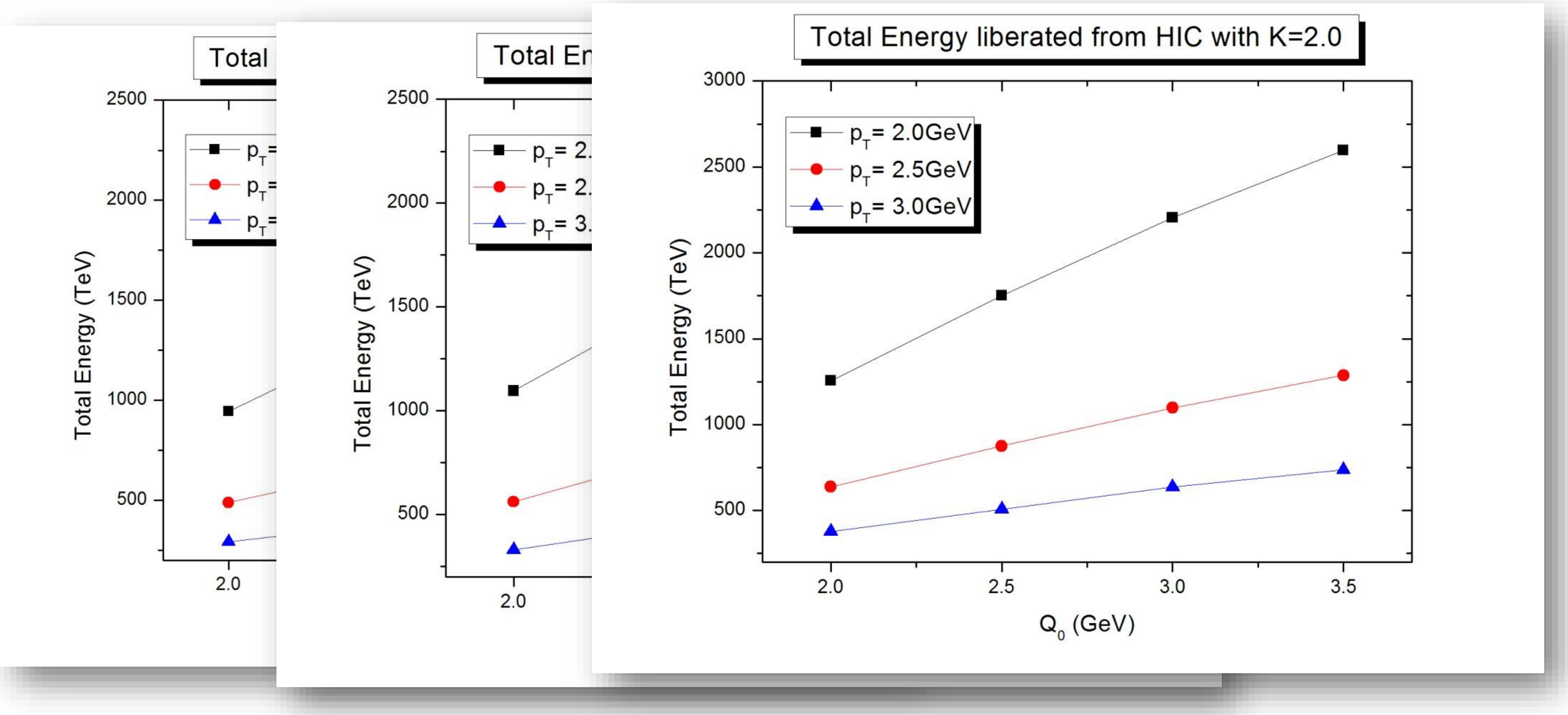
$$Q_0{}^2 \leq p_T{}^2 \leq (\frac{\sqrt{s}}{2\cosh y})^2,$$

$$-\log(\frac{\sqrt{s}}{p_T} - e^{-y}) \leq y_4 \leq \log(\frac{\sqrt{s}}{p_T} - e^{-y}),$$

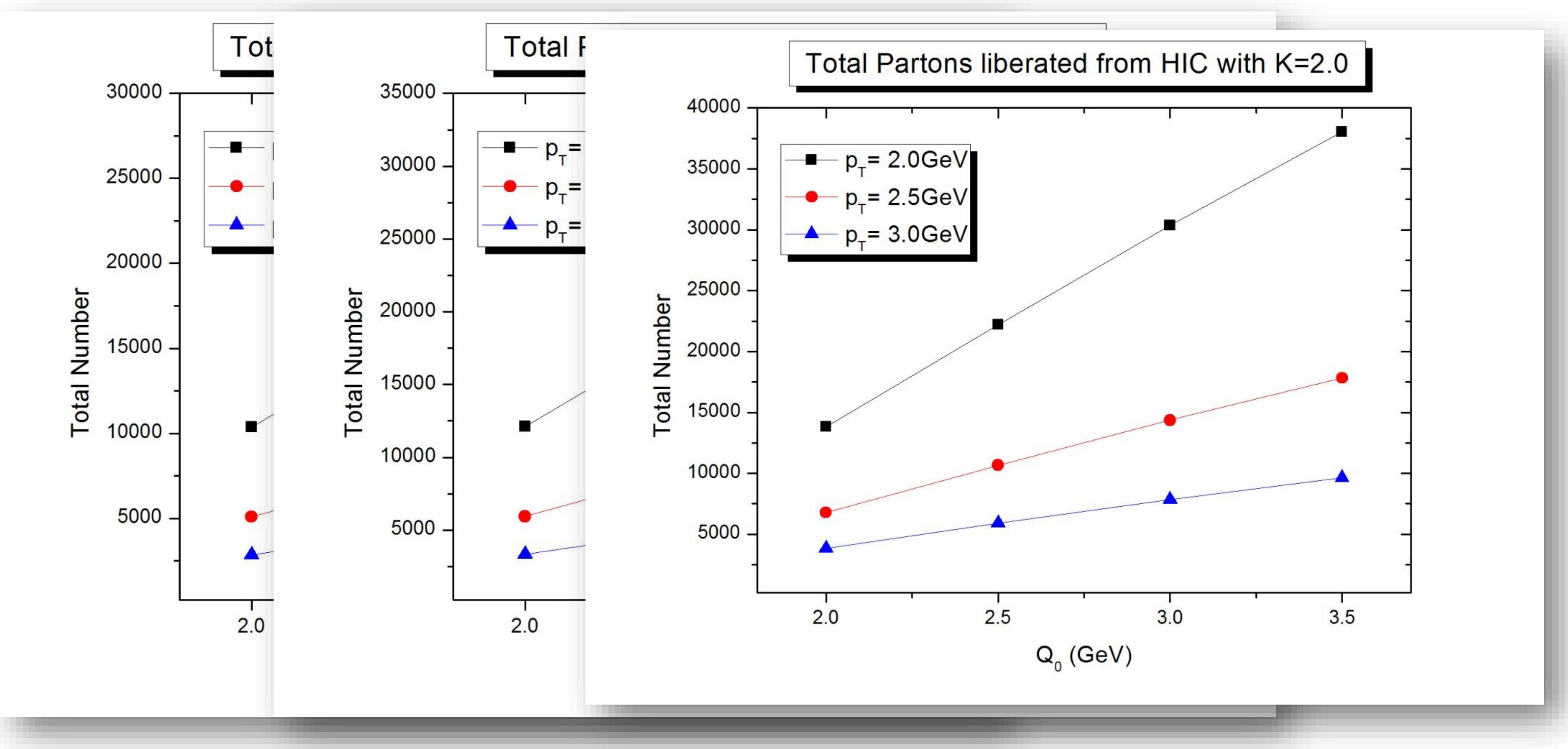
$$|y| \leq \log(\frac{\sqrt{s}}{2Q_0} + \sqrt{\frac{s}{4Q_0{}^2} - 1}).$$

2.3 examples of initial phase space

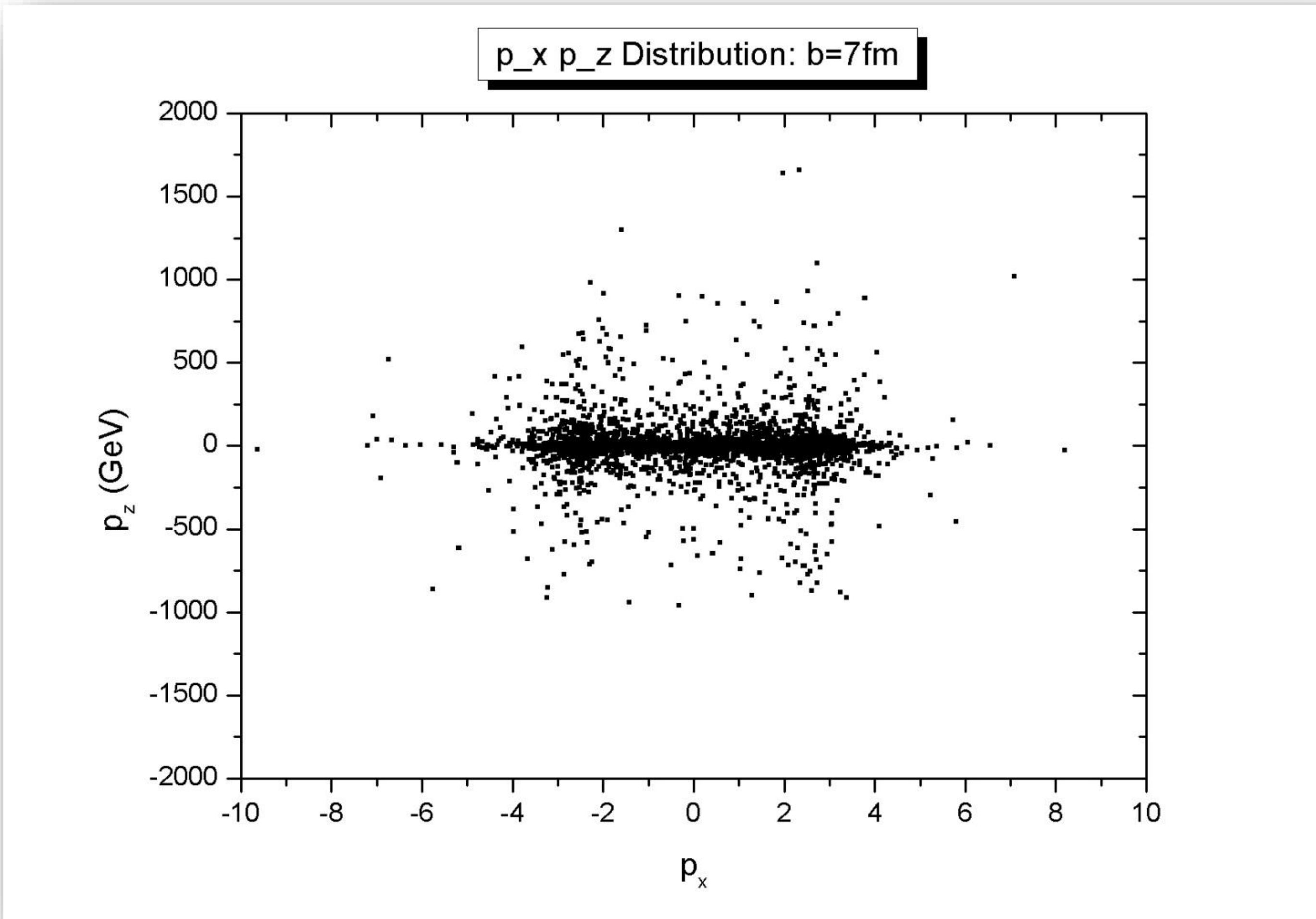
- Total Energy liberated: as a function of Q_0, p_T(cut), K-factor

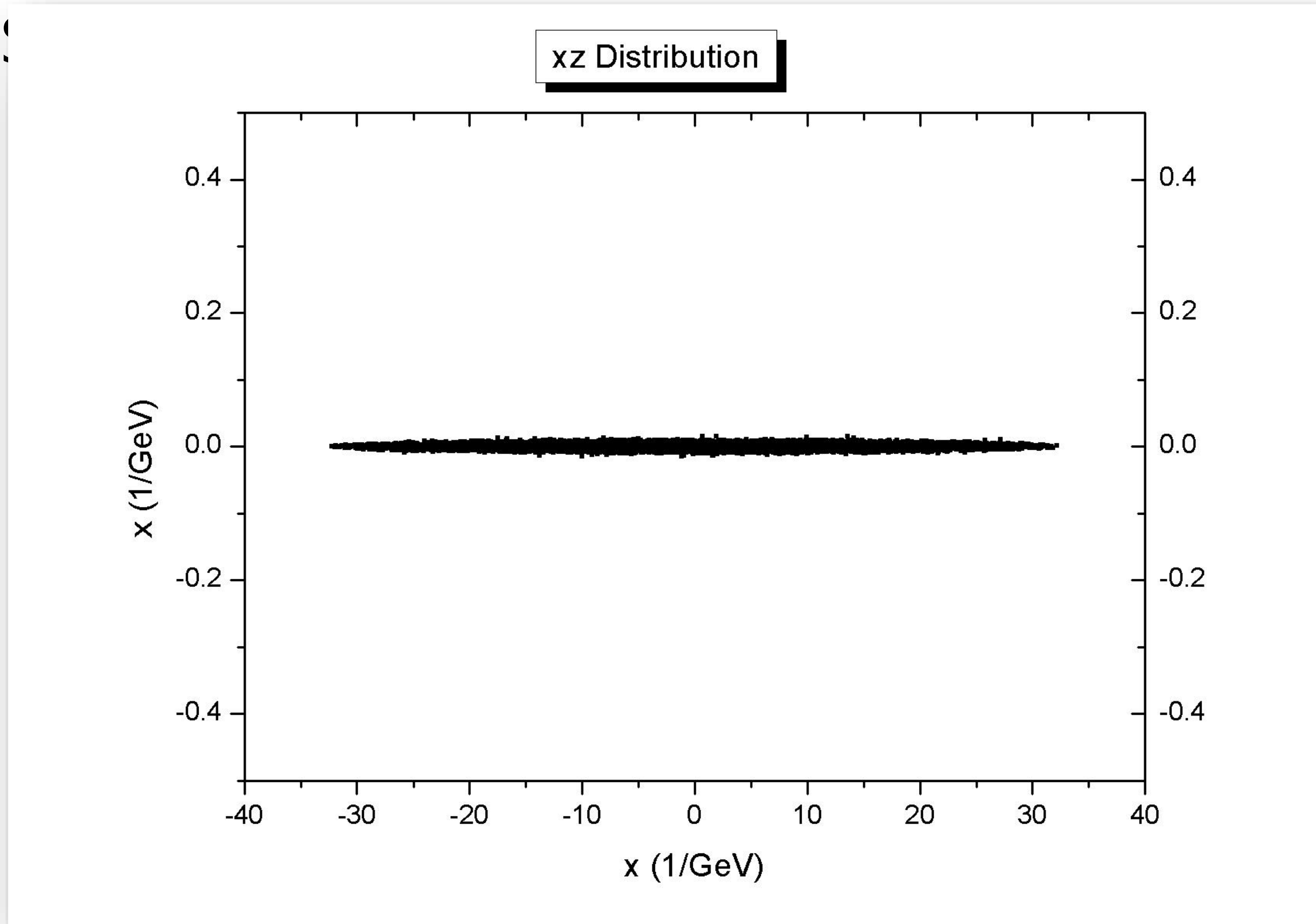


- Number of partons:



- Momentum distribution:




xz Distribution
0.4
0.2
0.0
-0.2
-0.4
x (1/GeV)
-40
-30
-20
-10
0
10
20
30
40
x (1/GeV)

3. Parton Cascade

- We solve the Boltzmann Equation:

$$\frac{df}{dt} + \frac{\vec{p}}{m} \cdot \frac{df}{d\vec{r}} + \frac{d\vec{p}}{dt} \cdot \frac{df}{d\vec{p}} = C(f)$$

Where $C(f)$ is collision channels.

3.1 Collision channels

$$\begin{aligned} gg &\leftrightarrow gg, q\bar{q}, \\ gq &\leftrightarrow gq, \\ g\bar{q} &\leftrightarrow g\bar{q}, \\ q^a q^b &\leftrightarrow q^c q^d, \\ q\bar{q} &\leftrightarrow q\bar{q}, \\ \bar{q}^a \bar{q}^b &\leftrightarrow \bar{q}^c \bar{q}^d, \\ gg &\rightarrow ggg. \end{aligned}$$

3.2 Cross Section

$$\frac{d\sigma^{gg\to gg}}{dt} = \frac{9\pi\alpha_s^2}{2s^2}\left(3 - \frac{tu}{s^2} - \frac{su}{t^2} - \frac{st}{u^2}\right)$$

$$\frac{d\sigma^{gg\to q_a\bar{q}_b}}{dt} = \frac{\pi\alpha_s^2}{6s^2}\delta_{ab}\left(\frac{u}{t} + \frac{t}{u} - \frac{9}{4}\frac{t^2+u^2}{s^2}\right)$$

$$\frac{d\sigma^{gq\to gq}}{dt} = \frac{4\pi\alpha_s^2}{9s^2}\left(-\frac{u}{s} - \frac{s}{u} + \frac{9}{4}\frac{s^2+u^2}{t^2}\right)$$

$$\frac{d\sigma^{q_a q_b\to q_a q_b}}{dt} = \frac{4\pi\alpha_s^2}{9s^2}\left[\frac{s^2+u^2}{t^2} + \delta_{ab}\left(\frac{t^2+s^2}{u^2} - \frac{2}{3}\frac{s^2}{ut}\right)\right]$$

$$\frac{d\sigma^{q_a\bar{q}_b\to q_c\bar{q}_d}}{dt} = \frac{4\pi\alpha_s^2}{9s^2}\left[\delta_{ac}\delta_{bd}\frac{s^2+u^2}{t^2} + \delta_{ab}\delta_{cd}\frac{t^2+u^2}{s^2} - \delta_{abcd}\frac{2}{3}\frac{u^2}{st}\right]$$

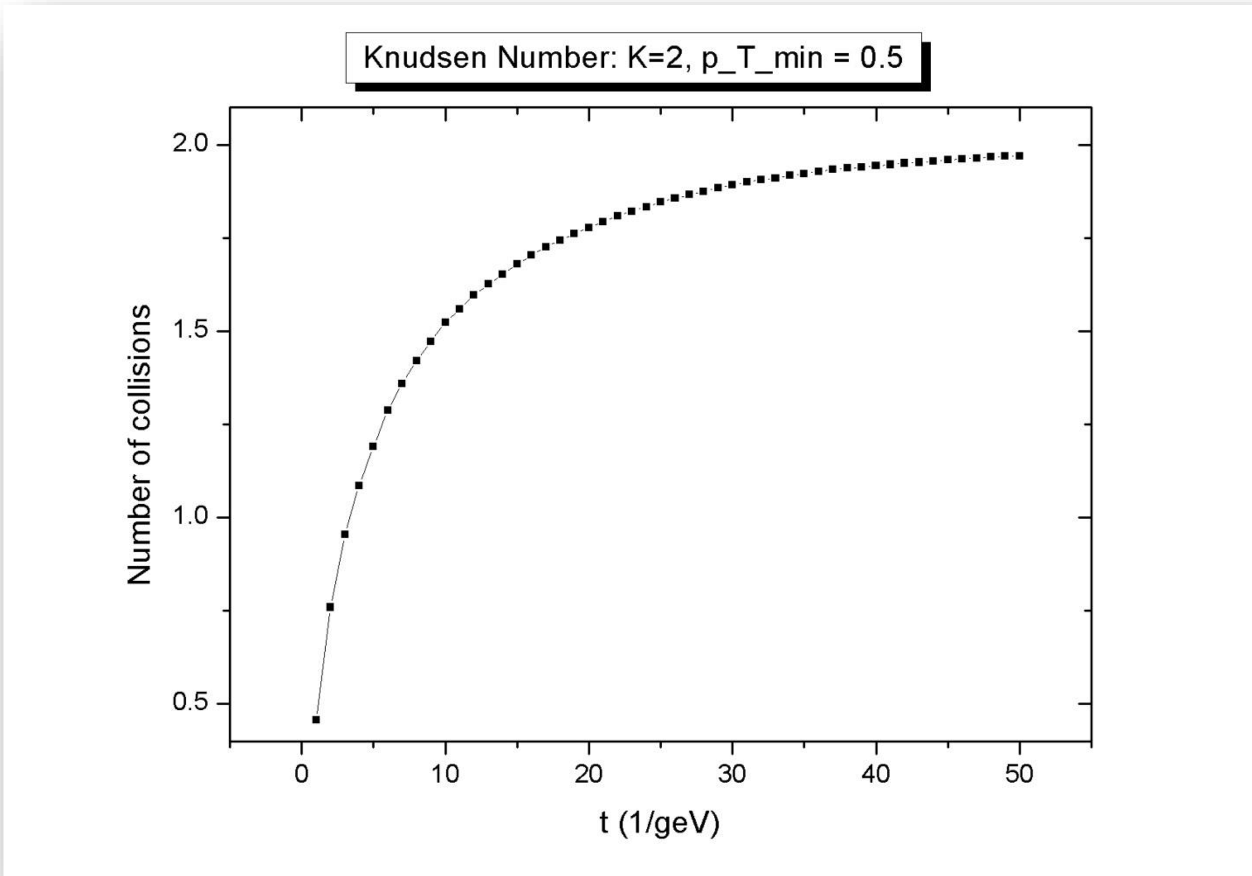
$$\frac{d\sigma^{q_a\bar{q}_b\to gg}}{dt} = \frac{32\pi\alpha_s^2}{27s^2}\delta_{ab}\left[\frac{u}{t} + \frac{t}{u} - \frac{9}{4}\frac{t^2+u^2}{s^2}\right]$$

$$\frac{d\sigma^{gg\to ggg}}{dq_\perp^2\,dy\,dk_\perp^2} = \frac{9C_A\alpha_s^3}{2}\frac{q_\perp^2}{(q_\perp^2+\mu_D^2)^2}\cdot \frac{\Theta(k_\perp\lambda_f - \cosh y)\Theta(\sqrt{s} - k_\perp\cosh y)}{k_\perp^2\sqrt{(k_\perp^2+q_\perp^2+\mu_D^2)^2 - 4k_\perp^2 q_\perp^2}}$$

- K-factor = 1.5 ~ 2
- Minimum momentum transfer:
  - $p_T(minimum) = 0.5\ GeV$
- Coupling constant:
  - $\alpha_s = 0.3$

4. Some Results

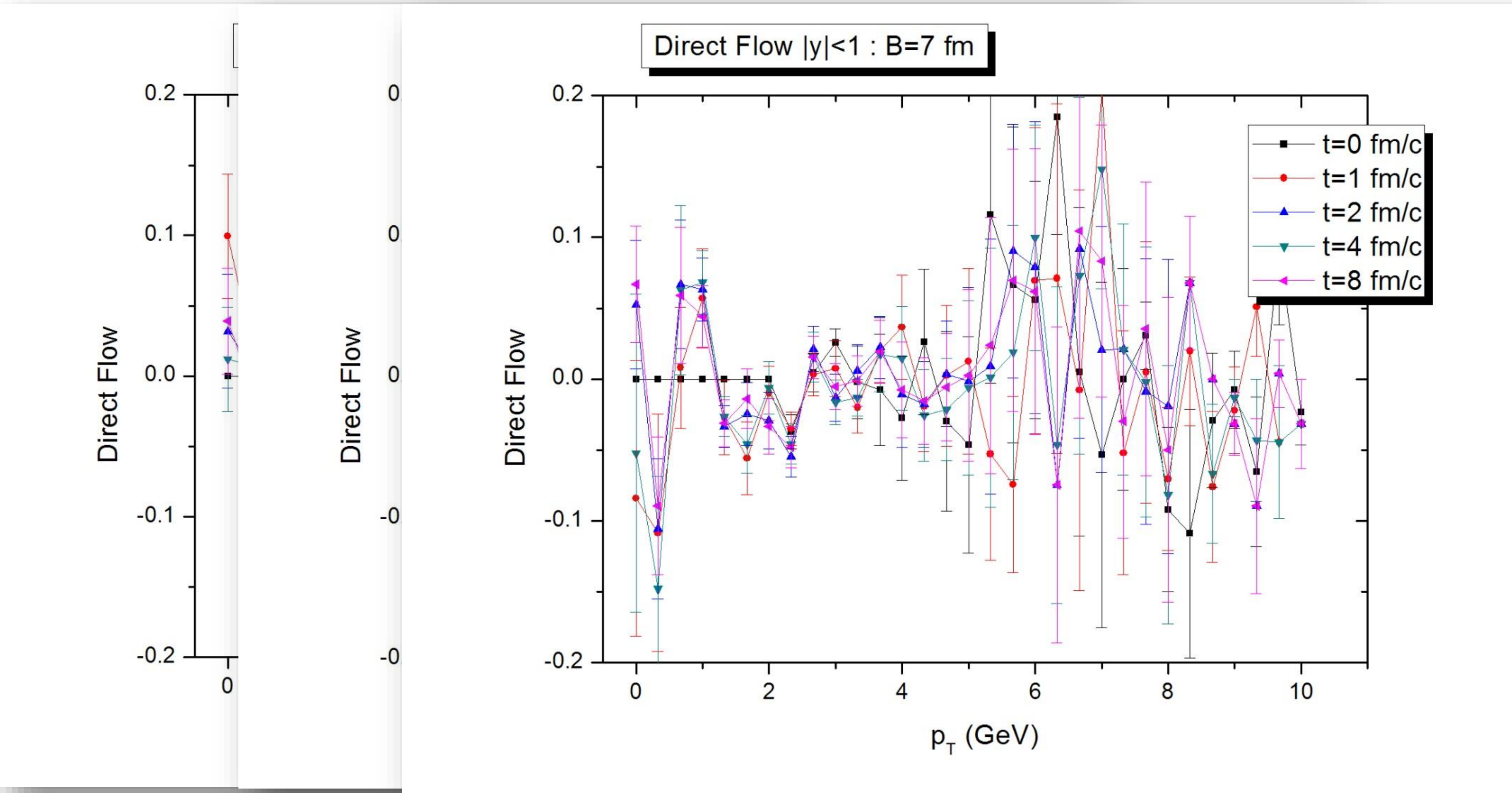
1) Knudsen Number:



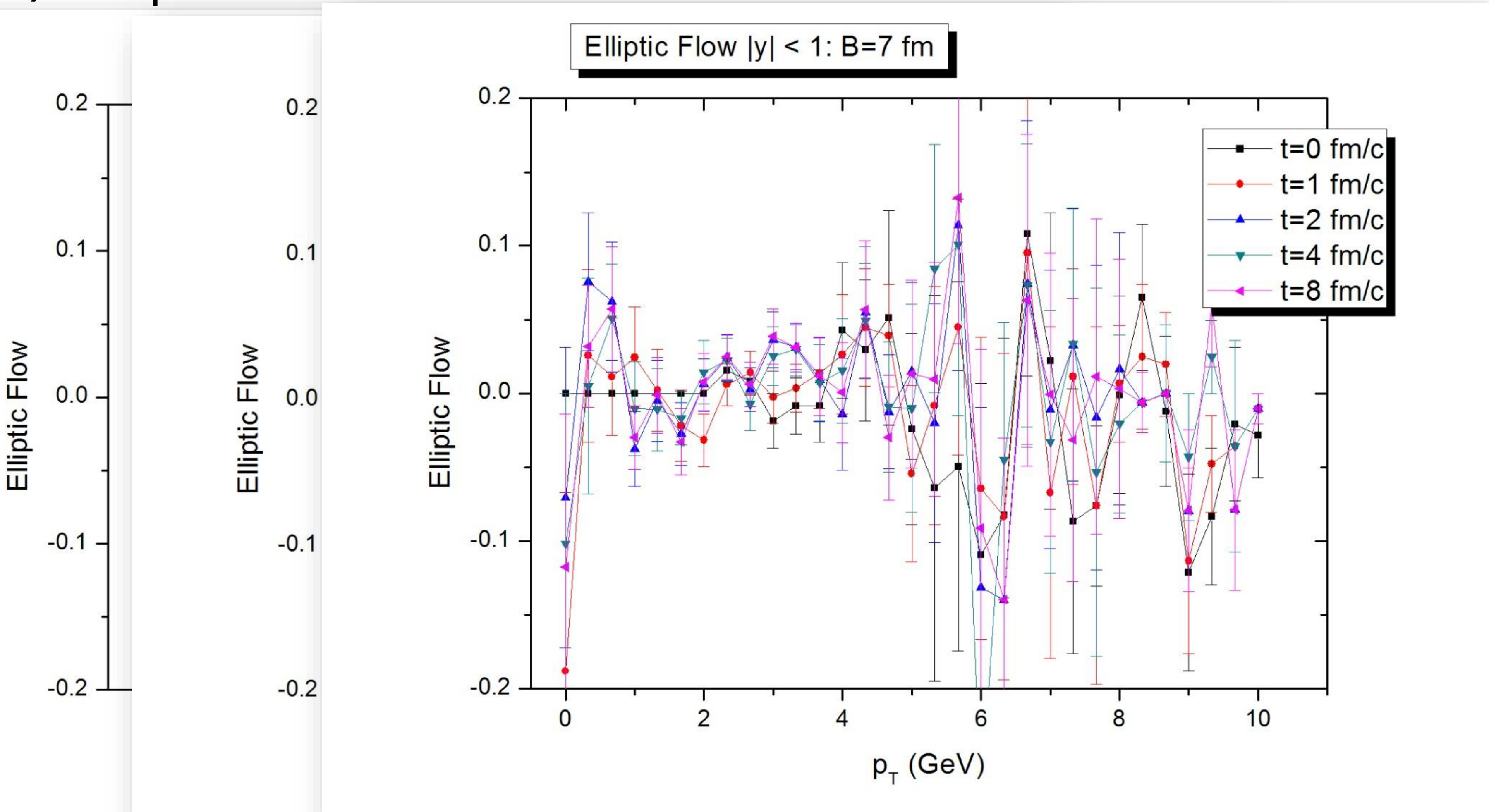
- Note that

  We have only 20 runs (pretty bad statistics ; statistical error $= \sigma_s/\sqrt{N-1}$**) and almost all of partons are gluons so that flow data should be similar to hadron data).**

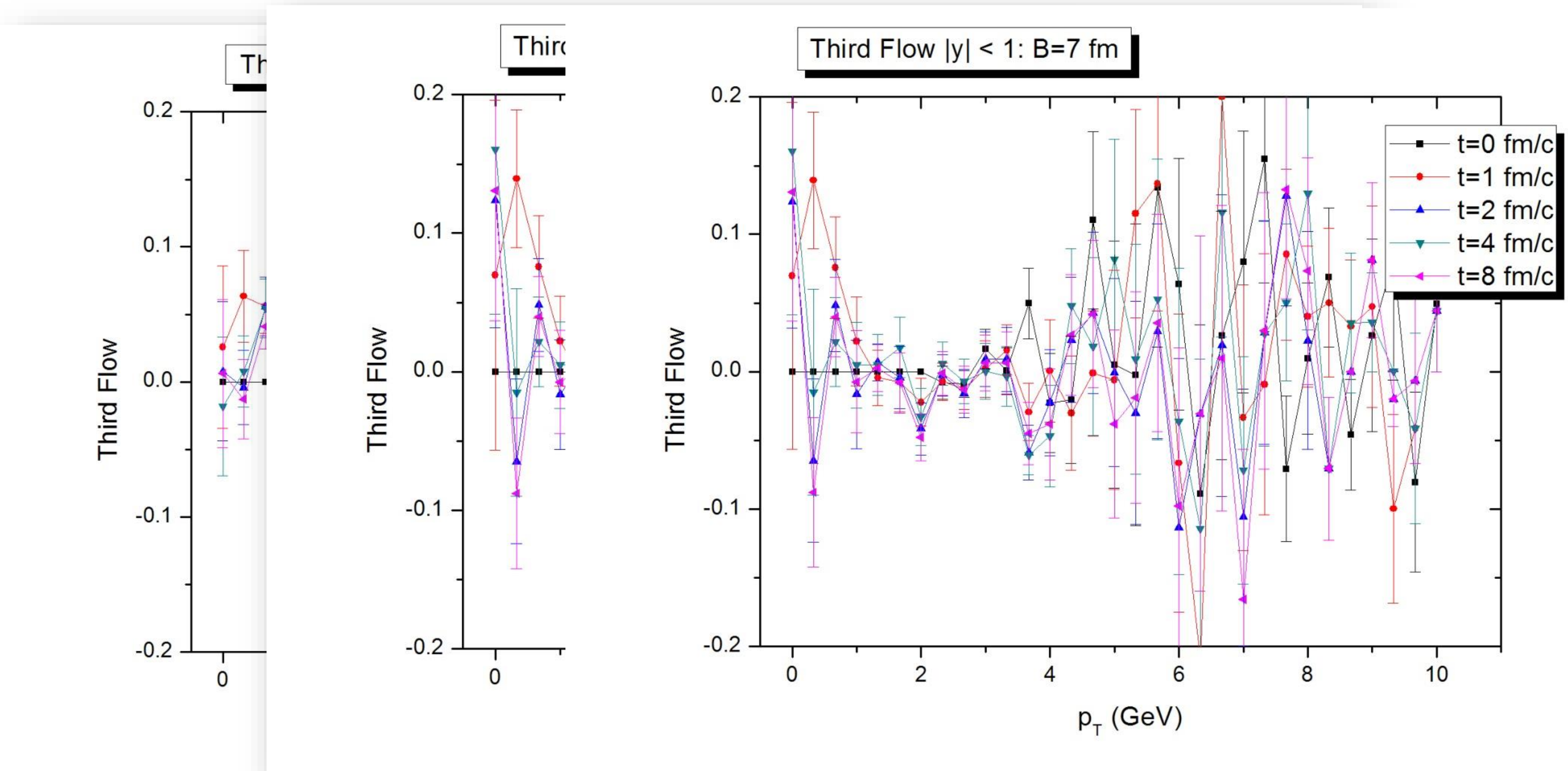
2) Direct Flow:



3) Elliptic Flow:

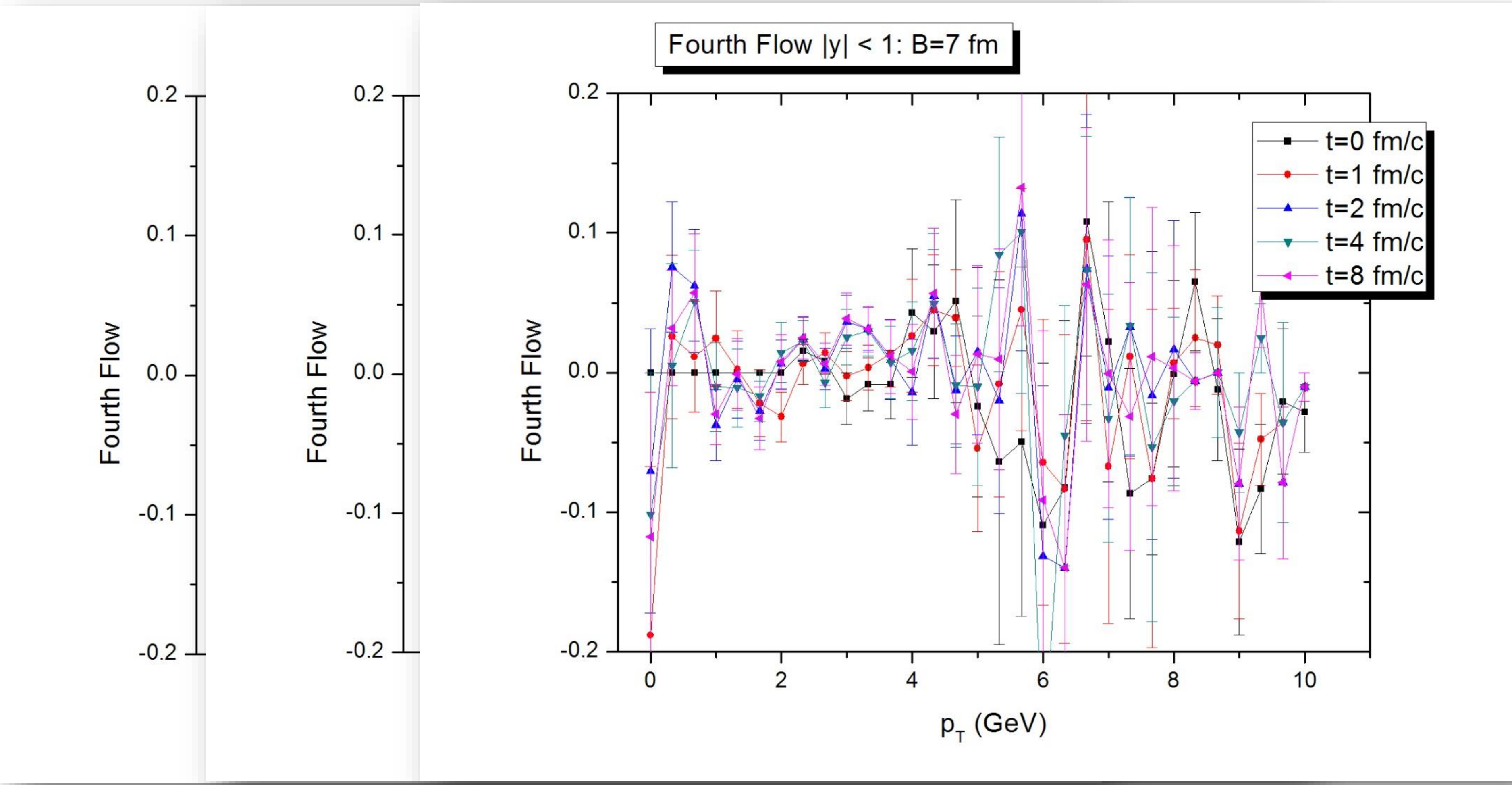


4) Third Flow



Third Flow |y| < 1: B=7 fm
0.2
0.1
0.0
-0.1
-0.2
Third Flow
0
2
4
6
8
10
p_T (GeV)
t=0 fm/c
t=1 fm/c
t=2 fm/c
t=4 fm/c
t=8 fm/c

5) Fourth Flow:



5. Discussions

- I have seen the elliptic flow at RHIC energy but not at LHC. Why? Partial answer: Those partons at LHC have much higher longitudinal momentum.
- How to improve?
  - Virtuality
  - Better parameters:
    - K-factor
    - LPM effect: formation time
    - parton mass
    - coupling constant
    - Minimum momentum transfer
  - Color Force: MAJOR PROBLEM

- Hadronization: to compare to hadron data

Color force

- Color charge:
  - RGB is not color charge but just color index
  - Color charge can be defined by continuity equation: $\partial_\mu J^\mu = 0$ and J^0 is charge density and integration over volume is total charge at least in QED, $Q = \int J^0 \, dx$.

  - Quark continuity equation:
    - $D_\mu[A] \, J^{\mu a} = 0$ where $D_\mu^{ab}[A] = \delta^{ab} \, \partial_\mu + g \, f^{acb} \, A_\mu^c(x)$
    - $J_\mu^a(x) = g \, \bar{\varphi}_i(x) \gamma_\mu \, T_{ij}^a \, \varphi_j(x)$, a=1,2, 8. i, j=1,2,3
    - Color charges are not constant but function of t
  - Gluon continuity equation:

- It is difficult to find E and B surrounding a (anti) quark. In fact there is no E & B field for single parton.
- Even if we have E and B, it is not easy to get color Lorentz force $F = qE + q\ v\ X\ B$ since q is not constant
- Remember that some color factor is positive and some negative which means some attractive and some repulsive

- What is reasonable approximation?

- Related studies:
  - Wong: Drive gluon and quark field equations from QCD Lagrangian
    - Identify Hamiltonian from quark field equation
    - Using the Hamiltonian, derive the equations of motion for (x, p, I)
    - Include the gluon field equation as classical
  - Haller: total color charge is conserved
    - $J^a_{q,\mu} = g\,\bar{\varphi}\,\gamma_\mu\,T^a\,\varphi$
    - $J^a_{g,\mu} = g\,f^{abc}\,A^b_j\,F^a_{j\mu}$ in temporal gauge $A^a_0(x) = 0$
    - $\partial_\mu\,J^{\mu,total} = 0$
  - Nayak:
  - Geiger:
  - Mrowczynski:

- Possible answer to the last question:
  - We may start from the equation by Haller and separate the gluon field into soft and hard part. Treat the hard part as classical particles and derive their equations of motion, for example, Wong equation, for quark and hard gluon. And solve (soft, abelian, classical) gluon field equation.
  - Average over color charge for the given parton

- We do not know the answer BUT the NATURE does.