Multi-dimensional angular distributions made easy

J. A. Aguilar Saavedra
Instituto de Física Téorica, Madrid

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Disclaimers

Old wine in new bottles?

- The formalism is general and allows for easy escalability to high dimensions

  → multi-dim studies possible at Run 3 and HL-LHC

- The formalism offers more insight on the nature of the observables measured.

  → Every measured parameter has a meaning.

- And the methods employed for the extraction of the observables from data are different → Practical differences

Here: translation of previous work done for tops, Ws, Zs and Hs.

- Examples for b decays are work in progress

- Several other possibilities not studied, and no analogue for Ws

Density operators

Density operators

When one particle decays, the rest-frame distributions of its daughter particles are determined by the spin state [not necessarily pure] of the parent.

The [possibly mixed] spin state is given by a density operator in spin space.

→ a square matrix

- spin 1/2: the matrix is 2×2
- spin 1: the matrix is 3×3
- …

Not any operator can be a density operator. A valid density operator has several characteristics:

- Unit trace
- Hermitian
- Positive semidefinite: eigenvalues ≥ 0

Density operators

A Hermitian unit-trace matrix can be *expanded* in terms of a convenient reference set of matrices.

For spin 1/2, the choice is obvious: the identity plus the Pauli matrices

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\rho = \frac{1}{2}\left(1_{2\times 2} + a_i \sigma_i\right)$$

3 arbitrary coefficients

Because the Pauli matrices are precisely the spin operators (up to 1/2) the coefficients a_i have a simple \`spin interpretation´: they are the so-called polarisations P_i

Density operators

For spin 1, the choice is less obvious. It is very convenient to use operators with well-defined transformation under rotations:

😱 irreducible tensor operators 😱

$$T^1_1 = \sqrt{\frac{3}{2}} \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{pmatrix} \quad T^1_0 = \sqrt{\frac{3}{2}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$T^2_2 = \sqrt{3} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad T^1_2 = \sqrt{\frac{3}{2}} \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$T^2_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

T^1 are the spin operators in the spherical basis with a convenient normalisation

$$T^1_{-1} = -(T^1_1)^\dagger$$
$$T^2_{-2} = -(T^2_2)^\dagger$$
$$T^2_{-1} = -(T^2_1)^\dagger$$

8 arbitrary coefficients

$$\rho = \frac{1}{3}\left(1_{3\times 3} + A_{LM} T^L_M\right)$$

Density operators

For systems with more than one particle, the spin space is the direct product of the individual spaces.

Example: W^+W^- pair [replace Ws by your favourite spin-1 hadrons]

$$\rho = \frac{1}{9}\left(1_{9\times 9} + A^1_{LM} T^L_M \otimes 1_{3\times 3} + A^2_{LM} 1_{3\times 3} \otimes T^L_M + C_{L_1 M_1 L_2 M_2} T^{L_1}_{M_1} \otimes T^{L_2}_{M_2}\right)$$

8 coefficients corresponding to W^+ polarisation

8 coefficients corresponding to W^- polarisation

64 spin correlation coefficients

The parameterisation of the W^+W^- spin state, involving 80 independent parameters, is remarkably simple

Decay distributions

Decay distributions

The spin state of a decaying particle leaves its imprint in the rest-frame distributions of its daughter particles.

In full generality, the decay distribution can be obtained by multiplying, entry by entry, the spin density matrix by a `decay density matrix´ Γ

$$\left.\begin{matrix}\rho \\ \Gamma\end{matrix}\right\} \Rightarrow \quad \frac{d\sigma}{d\Omega} \propto \sum_{i,j} \rho_{ij} \Gamma_{ij} = \mathrm{Tr}(\rho\, \Gamma^T)$$

The [Hermitian] matrices Γ have a definite form with a few free parameters that depend on the spins of the daughter particles, their masses, etc.

For a system of two particles, the product is done independently for each particle, if there is no interference between decay products.

For decays such as $ZZ \to e^+e^- \, e^+e^-$ things are more complicated. We will not cover this case.

Spin 1/2 particles, 2-body decays

Boudjema, Singh 0903.4705

$$\Gamma_{11} = \frac{1}{2}\left[1 + \alpha \cos\theta\right]$$

$$\Gamma_{12} = \frac{1}{2}\alpha \sin\theta e^{i\phi}$$

$$\Gamma_{22} = \frac{1}{2}\left[1 - \alpha \cos\theta\right]$$

Spin 1 particles, 2-body decays

$$\Gamma_{11} = \frac{1}{4}\left[1 + \delta + (1-3\delta)\cos^2\theta + 2\alpha\cos\theta\right]$$

$$\Gamma_{12} = \frac{1}{2\sqrt{2}}\left[\alpha + (1-3\delta)\cos\theta\right]\sin\theta e^{-i\phi}$$

$$\Gamma_{13} = \frac{1}{4}(1-3\delta)\sin^2\theta e^{i2\phi}$$

$$\Gamma_{22} = \delta + \frac{1}{2}(1-3\delta)\sin^2\theta$$

$$\Gamma_{23} = \frac{1}{2\sqrt{2}}\left[\alpha - (1-3\delta)\cos\theta\right]\sin\theta e^{i\phi}$$

$$\Gamma_{33} = \frac{1}{4}\left[1 + \delta + (1-3\delta)\cos^2\theta - 2\alpha\cos\theta\right]$$

Ω=(θ,ϕ) is the polar angle of one decay product in the rest frame of the mother particle [as usual]

α, δ: constants

Due to the symmetry, the resulting equations are simple and beautiful

spin 1/2 $$\frac{1}{\sigma}\frac{d\sigma}{d\Omega} = \frac{1}{4\pi}\left[1 + \alpha\vec{P}\cdot\hat{n}\right]\,, \quad \hat{n} = (s_\theta c_\phi, s_\theta s_\phi, c_\theta)$$

spin 1 $$\frac{1}{\sigma}\frac{d\sigma}{d\Omega} = \frac{1}{4\pi}\left[1 + B_L A_{LM} Y_L^M(\Omega)\right]$$

$$B_1 = \sqrt{2\pi}\alpha$$

$$B_2 = \sqrt{\frac{2\pi}{5}}(1-3\delta)$$

The constants in red depend on decay mode. The parameters in blue describe the spin state.

The advantage of this parameterisation becomes obvious when one considers more than one particle. For a system of two spin-1 particles

JAAS et al. 2209.13441

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega_1 d\Omega_2} = \frac{1}{(4\pi)^2}\Big[1 + B^1_{L_1} A^1_{L_1M_1} Y_{L_1}^{M_1}(\Omega_1) + B^2_{L_2} A^2_{L_2M_2} Y_{L_2}^{M_2}(\Omega_2) + B^1_{L_1} B^2_{L_2} C_{L_1M_1L_2M_2} Y_{L_1}^{M_1}(\Omega_1) Y_{L_2}^{M_2}(\Omega_2)\Big]$$

If you are scared by the previous equation, you may want to consider the multi-page alternative in the literature

$$
\frac{1}{\sigma}\frac{d^2\sigma}{d\Omega_a d\Omega_b} = \frac{3}{4\pi}\frac{3}{4\pi}\sum_{\lambda_A,\lambda'_A,\lambda_B,\lambda'_B} P_{AB(2,2)}\left(\lambda_A,\lambda'_A,\lambda_B,\lambda'_B\right)\times\Gamma_{A(2)}\left(\lambda_A,\lambda'_A\right)\times\Gamma_{B(2)}\left(\lambda_B,\lambda'_B\right)
$$

$$
\begin{aligned}
&= \frac{1}{16\pi^2}\Bigg[1+\frac{3}{2}\alpha_{A/B}\,p_i^{A/B}c_i^{a/b}+\sqrt{\frac{3}{2}}(1-3\delta_{A/B})T_{ij}^{A/B}c_i^{a/b}c_j^{a/b}\ (i\neq j)\\
&+\frac{1}{2}\sqrt{\frac{3}{2}}(1-3\delta_{A/B})\underbrace{\left(T_{11}^{A/B}-T_{22}^{A/B}\right)}_{T^{A/B}_{11-22}}\underbrace{\left((c_1^{a/b})^2-(c_2^{a/b})^2\right)}_{\sin^2\theta_{a/b}\cos(2\phi_{a/b})}\\
&+\frac{1}{2}\sqrt{\frac{3}{2}}(1-3\delta_{A/B})T_{33}^{A/B}\left(3(c_3^{a/b})^2-1\right)\\
&+\alpha_A\alpha_B\,pp_{ij}^{AB}c_i^a c_j^b\\
&+\alpha_A(1-3\delta_B)pT_{ijk}^{AB}c_i^a c_j^b c_k^b+\alpha_B(1-3\delta_A)pT_{ijk}^{BA}c_i^b c_j^a c_k^a\ (j\neq k)\\
&+\frac{1}{2}\alpha_A(1-3\delta_B)\underbrace{\left(pT_{i11}^{AB}-pT_{i22}^{AB}\right)}_{pT^{AB}_{i(11-22)}}c_i^a\left((c_1^b)^2-(c_2^b)^2\right)\\
&+\frac{1}{2}\alpha_B(1-3\delta_A)\underbrace{\left(pT_{i11}^{BA}-pT_{i22}^{BA}\right)}_{pT^{BA}_{i(11-22)}}c_i^b\left((c_1^a)^2-(c_2^a)^2\right)\\
&+\frac{1}{2}\alpha_A(1-3\delta_B)pT_{i33}^{AB}c_i^a\left(3(c_3^b)^2-1\right)+\frac{1}{2}\alpha_B(1-3\delta_A)pT_{i33}^{BA}c_i^b\left(3(c_3^a)^2-1\right)\\
&+(1-3\delta_A)(1-3\delta_B)TT_{ijkl}^{AB}c_i^a c_j^a c_k^b c_l^b\ (i\neq j,\ k\neq l)\\
&+\frac{1}{2}(1-3\delta_A)(1-3\delta_B)\underbrace{\left(TT_{ij11}^{AB}-TT_{ij22}^{AB}\right)}_{TT^{AB}_{ij(11-22)}}c_i^a c_j^a\left((c_1^b)^2-(c_2^b)^2\right)\ (i\neq j)\\
&+\frac{1}{2}(1-3\delta_A)(1-3\delta_B)\underbrace{\left(TT_{11ij}^{AB}-TT_{22ij}^{AB}\right)}_{TT^{AB}_{(11-22)ij}}c_i^b c_j^b\left((c_1^a)^2-(c_2^a)^2\right)\ (i\neq j)\\
&+\frac{1}{2}(1-3\delta_A)(1-3\delta_B)TT_{ij33}^{AB}c_i^a c_j^a\left(3(c_3^b)^2-1\right)\ (i\neq j)\\
&+\frac{1}{2}(1-3\delta_A)(1-3\delta_B)TT_{33ij}^{AB}c_i^b c_j^b\left(3(c_3^a)^2-1\right)\ (i\neq j)\\
&+\frac{1}{4}(1-3\delta_A)(1-3\delta_B)\underbrace{\left(TT_{1111}^{AB}-TT_{1122}^{AB}-TT_{2211}^{AB}+TT_{2222}^{AB}\right)}_{TT^{AB}_{(11-22)(11-22)}}\\
&\times\left((c_1^a)^2-(c_2^a)^2\right)\left((c_1^b)^2-(c_2^b)^2\right)\\
&+\frac{1}{4}(1-3\delta_A)(1-3\delta_B)\underbrace{\left(TT_{1133}^{AB}-TT_{2233}^{AB}\right)}_{TT^{AB}_{(11-22)33}}\left((c_1^a)^2-(c_2^a)^2\right)\left(3(c_3^b)^2-1\right)\\
&+\frac{1}{4}(1-3\delta_A)(1-3\delta_B)\underbrace{\left(TT_{3311}^{AB}-TT_{3322}^{AB}\right)}_{TT^{AB}_{33(11-22)}}\left((c_1^b)^2-(c_2^b)^2\right)\left(3(c_3^a)^2-1\right)\\
&+\frac{1}{4}(1-3\delta_A)(1-3\delta_B)TT_{3333}^{AB}\left(3(c_3^a)^2-1\right)\left(3(c_3^b)^2-1\right)\Bigg]. \qquad (29)
\end{aligned}
$$

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Decay distributions 5/6

Simplicity alone would suffice to favour this parameterisation…

multi-page
equations

spherical
harmonics

Decay distributions

… but there is more! It is quite easy and straightforward to measure the *A* and *C coefficients* in data.

Because spherical harmonics are orthogonal functions, to pick a selected term in the distribution one just has to take the average

[after background subtraction and correction for detector effects]

of the appropriate Y*(Ω)

$$\int \frac{1}{\sigma}\frac{d\sigma}{d\Omega_1 d\Omega_2} Y_{L_1}^{M_1}(\Omega_1)^* Y_{L_2}^{M_2}(\Omega_2)^* = \frac{1}{(4\pi)^2} B_{L_1} B_{L_2} C_{L_1 M_1 L_2 M_2}$$

constants you can calculate

data follow this distribution

calculate the average of this quantity on your data

the quantity you want

Example #1

$B^0 \rightarrow K^{0*}\gamma$

Example #1: $B^0 \rightarrow K^{0*}\gamma$

This is a decay $0 \rightarrow 1 + 1$. Angular momentum conservation implies that many A and C coefficients are zero. The non-zero ones are(*)

you can measure γ polarisation by looking at K^{0*}

$$A^1_{10} = -A^2_{10}\,, \quad A^1_{20} = A^2_{20}$$

$$C_{1010}\,, \quad C_{2020}\,, \quad C_{1020}\,, \quad C_{2010}$$

$$C_{111-1} = C^*_{1-111}\,, \quad C_{222-2} = C^*_{2-222}\,, \quad C_{212-1} = C^*_{2-121}\,,$$

$$C_{112-1} = C^*_{1-121}\,, \quad C_{211-1} = C^*_{2-111}$$

The decay $K^{0*} \rightarrow K\pi$ is $1 \rightarrow 0 + 0$

the Γ matrix for this decay has $\alpha=0$, $\delta=1$

$$B^1_1 = 0\,, \quad B^1_2 = -2\sqrt{\frac{2\pi}{5}}$$

some of the above spin coefficients terms do not appear in the distribution (!!!)

This K^{0*} decay is insensitive to γ polarisation

(*) This is found for example by calculating the decay using helicity formalism and matching to the general expression

Example #1: $B^0 \to K^{0*}\gamma$

Let us now take $B^0 \to K^{0*}\gamma^* \to K^{0*}e^+e^-$ at low $m(e^+e^-)$ so that the decay is dominated by photon contributions.

For $\gamma^* \to e^+e^-$

α=0 due to the vector γe^+e^- coupling

δ=0 for massless leptons.

$$B_1^2 = 0\,, \quad B_2^2 = \sqrt{\frac{2\pi}{5}}$$

small because photon is nearly on shell

Among the 18 nonzero coefficients, only 7

$$A_{20}^1 = A_{20}^2\,, \quad C_{2020}\,, \quad C_{222-2} = C^*_{2-222}\,, \quad C_{212-1} = C^*_{2-121}$$

contribute to the angular distribution, which was [reminder]

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega_1 d\Omega_2} = \frac{1}{(4\pi)^2}\Big[1 + B^1_{L_1} A^1_{L_1 M_1} Y_{L_1}^{M_1}(\Omega_1) + B^2_{L_2} A^2_{L_2 M_2} Y_{L_2}^{M_2}(\Omega_2)$$
$$+ B^1_{L_1} B^2_{L_2} C_{L_1 M_1 L_2 M_2} Y_{L_1}^{M_1}(\Omega_1) Y_{L_2}^{M_2}(\Omega_2)\Big]$$

Example #1: $B^0 \to K^{0*}\gamma$

80 terms → 7 terms. Much ado about nothing?

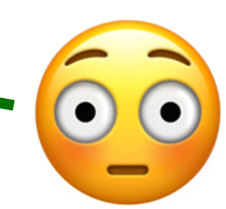
The resulting distribution is

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega_1 d\Omega_2} = \frac{1}{(4\pi)^2}\Big[1 + B_2^1 A_{20}^1 Y_2^0(\Omega_1) + B_2^2 A_{20}^2 Y_2^0(\Omega_2) + B_2^1 B_2^2 C_{2020} Y_2^0(\Omega_1) Y_2^0(\Omega_2)$$
$$+ (B_2^1 B_2^2 C_{222-2} Y_2^2(\Omega_1) Y_2^{-2}(\Omega_2) + B_2^1 B_2^2 C_{212-1} Y_2^1(\Omega_1) Y_2^{-1}(\Omega_2) + \mathrm{CC}\Big]$$

to be matched to the one used by LHCb

LHCb 2010.06011

$$\frac{1}{\mathrm{d}(\Gamma+\bar{\Gamma})/\mathrm{d}q^2}\frac{\mathrm{d}^4(\Gamma+\bar{\Gamma})}{\mathrm{d}q^2\,\mathrm{d}\cos\theta_\ell\,\mathrm{d}\cos\theta_K\,\mathrm{d}\tilde{\phi}} = \frac{9}{16\pi}\Big[\tfrac{3}{4}(1-F_\mathrm{L})\sin^2\theta_K + F_\mathrm{L}\cos^2\theta_K$$
$$+\tfrac{1}{4}(1-F_\mathrm{L})\sin^2\theta_K\cos 2\theta_\ell - F_\mathrm{L}\cos^2\theta_K\cos 2\theta_\ell$$
$$+(1-F_\mathrm{L})A_\mathrm{T}^\mathrm{Re}\sin^2\theta_K\cos\theta_\ell$$
$$+\tfrac{1}{2}(1-F_\mathrm{L})A_\mathrm{T}^{(2)}\sin^2\theta_K\sin^2\theta_\ell\cos 2\tilde{\phi}$$
$$+\tfrac{1}{2}(1-F_\mathrm{L})A_\mathrm{T}^\mathrm{Im}\sin^2\theta_K\sin^2\theta_\ell\sin 2\tilde{\phi}\Big].$$



There is not a great simplification due to the low number of parameters, but there is more insight on their origin, and extraction from data may be easier: no need of a global fit.

Example #2

$$\Lambda_b^0 \rightarrow \gamma \Lambda^0$$

Example #2: $\Lambda_b^0 \rightarrow \gamma\Lambda^0$

We revisit the cascade decay $\Lambda_b^0 \rightarrow \gamma\Lambda^0$ followed by $\Lambda^0 \rightarrow p\pi$ using the helicity formalism.

1/2 1 1/2

$\Lambda_b^0 \rightarrow \gamma + \Lambda^0$

$\Lambda^0 \rightarrow p + \pi$

1/2 1/2 0

Λ_b^0 density matrix

$\Omega=(\theta,\phi)$ photon direction in Λ_b^0 rest frame

$\Omega^*=(\theta^*,\phi^*)$ proton direction in Λ^0 rest frame

$$\frac{d\sigma}{d\Omega d\Omega^*} \propto \sum_{MM'\lambda_2\lambda_3} \rho_{MM'} |a_{\lambda_1\lambda_2}|^2 |b_{\lambda_3}|^2 D^{1/2*}_{ML}(\phi,\theta,0) D^{1/2}_{M'L}(\phi,\theta,0) D^{1/2*}_{\lambda_2\lambda_3}(\phi^*,\theta^*,0) D^{1/2}_{\lambda_2\lambda_3}(\phi^*,\theta^*,0)$$

amplitudes for $\Lambda_b^0 \rightarrow \gamma\Lambda^0$

amplitudes for $\Lambda^0 \rightarrow p\pi$

these *D*´s are Wigner functions we do not need to care about now

Example #2: $\Lambda_b^0 \to \gamma \Lambda^0$

Using the expressions for *D* functions and polishing, we arrive at

$$\frac{1}{\sigma}\frac{d\sigma}{d\cos\theta^* d\cos\theta d\phi} = \frac{1}{8\pi}\left[(1+\eta_a\eta_b\cos\theta^*) + \vec{P}_{\Lambda_b}\cdot\hat{p}_\gamma(\eta_a+\eta_b\cos\theta^*)\right]$$

$-a_\gamma$

$-a_\Lambda$

Λ_b^0 polarisation

photon momentum in Λ_b^0 rest frame

to be compared to the one used by LHCb for the measurement of the photon polarisation

LHCb 2111.10194

$$\frac{d\Gamma}{d(\cos\theta_p)} \propto 1 - \alpha_\gamma\alpha_\Lambda\cos\theta_p, \qquad (2)$$

under the reasonable assumption of vanishing Λ_b^0 polarisation, and using a_Λ from other experiments.

In principle, high-statistics data could allow to drop these assumptions with a fully-differential measurement.

Example #2: $\Lambda_b^0 \to \gamma \Lambda^0$

A relatively new method to do this is to expand the differential distribution in terms of a set of orthogonal functions. JAAS, Escobar et al. 1702.03297

The method has been succesfully used by ATLAS in the triple-differential measurement of the top decay. ATLAS 1707.05393

And is being used for the upcoming 4-d differential measurement.

In short:

Instead of rewriting the *D* functions in terms of sines and cosines [previous slides] one arranges the angular distribution in terms of one *D* per decay

$$\frac{d\sigma}{d\Omega d\Omega^*} \propto \sum_{MM'\lambda_2\lambda_3} \rho_{MM'} |a_{\lambda_1\lambda_2}|^2 |b_{\lambda_3}|^2 D^{1/2*}_{ML}(\phi,\theta,0) D^{1/2}_{M'L}(\phi,\theta,0) D^{1/2*}_{\lambda_2\lambda_3}(\phi^*,\theta^*,0) D^{1/2}_{\lambda_2\lambda_3}(\phi^*,\theta^*,0)$$

make one *D*

make another *D*

Example #2: $\Lambda_b^0 \to \gamma\Lambda^0$

The product of two *D* functions gives single *D*s

$$D^{j_1}_{m'_1 m_1}(\alpha,\beta,\gamma) D^{j_2}_{m'_2 m_2}(\alpha,\beta,\gamma) = \sum_{j=|j_1-j_2|}^{j_1+j_2} \langle j_1 m'_1 j_2 m'_2 | j m' \rangle \langle j_1 m_1 j_2 m_2 | j m \rangle D^{j}_{m' m}(\alpha,\beta,\gamma)$$

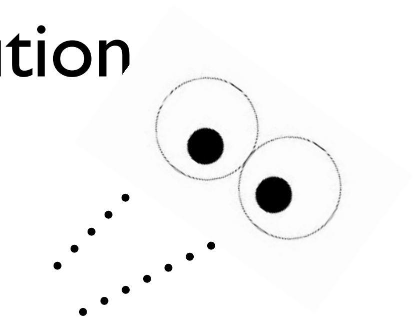
Also, *D*s* can be cast into *D*s

$$D^{j}_{m' m}(\alpha,\beta,\gamma)^* = (-1)^{m-m'} D^{j}_{-m' -m}(\alpha,\beta,\gamma)$$

Therefore, each for each decay \`i´ the squared matrix element $|M|^2$ contains a linear combination of *D*s evaluated at Ω_i.

Example #2: $\Lambda_b^0 \to \gamma \Lambda^0$ 5/7

Side note: if one looks at the decay distribution



$$\frac{1}{\sigma}\frac{d\sigma}{d\cos\theta^* d\cos\theta d\phi} = \frac{1}{8\pi}\left[(1+\eta_a\eta_b\cos\theta^*) + \vec{P}_{\Lambda_b}\cdot\hat{p}_\gamma(\eta_a+\eta_b\cos\theta^*)\right]$$

The *D* functions for spin-1/2 particle decays have $\cos\frac{\theta^*}{2}, \sin\frac{\theta^*}{2}$ etc.

Where did they go? They nicely combine into cos θ, sin θ, cos θ*

Example #2: $\Lambda_b^0 \to \gamma \Lambda^0$

The differential distribution can then be expanded as

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega d\Omega^*} = \sum_{j_1 j_2 m_1 m_2} c^{j_1 j_2}_{m_1 m_2} M^{j_1 j_2}_{m_1 m_2}$$

coefficients to measure

angular dependence

$$M^{j_1 j_2}_{m_1 m_2}(\phi, \theta, \phi^*, \theta^*) = \frac{1}{4\pi}(2j_1+1)^{1/2}(2j_2+1)^{1/2} \times D^{j_1}_{m_1 m_2}(\phi, \theta, 0) D^{j_2}_{m_2 0}(\phi^*, \theta^*, 0)$$

For $\Lambda_b^0 \to \gamma \Lambda^0$ there are 8 non-zero coefficients

$$c^{00}_{00}, \quad c^{10}_{10}, \quad c^{10}_{00}, \quad c^{10}_{-10}$$
$$c^{01}_{00}, \quad c^{11}_{10}, \quad c^{11}_{00}, \quad c^{11}_{-10}$$

that can be easily extracted from data because *M* functions are orthogonal

Example #2: $\Lambda_b^0 \to \gamma \Lambda^0$

- If you want to measure the photon polarisation [η_a] taking the value of η_b [$= -a_\Lambda$] from BES:

  measure $$c_{00}^{01} = \frac{1}{4\sqrt{3}\pi}\eta_a\eta_b$$ ➡ get η_a

  Does not require assumptions on Λ_b^0 polarisation

- If you want to measure the Λ_b^0 polarisation taking the value of η_b from BES:

  measure $$c_{00}^{11} = \frac{1}{12\pi}\eta_b P_3$$ ➡ get P_3

- If you want to measure the photon and Λ_b^0 polarisations, as well as η_b:

  measure $$c_{00}^{01}\,, \quad c_{00}^{11}\,, \quad c_{00}^{10} = \frac{1}{4\sqrt{3}\pi}\eta_a P_3$$

Discussion

[no more eqs]

Discussion

To take away:

- For 1-d distributions there is not much room for creativeness and innovation.
- On the other hand, for higher dimensions there are alternative ways to parameterise distributions.
- Parameterisations of multi-dim distributions in terms of orthogonal functions are desirable for:
  - ☑ Easy [better?] extraction of observables from data
  - ☑ Possibility to extract a single observable without global fit
- Calculations for $H \to ZZ$ showed that integration with spherical harmonics gives a better statistical precision in Cs than ad-hoc FB asymmetries.

To take away:

- The method has been successfully used by ATLAS for multi-dim measurements.
- As mentioned at the beginning, parameterisations in terms of well-behaved functions give more physics insight.
- There is the possibility to study even higher particle multiplicities

$$A \to B(\to b_1 b_2)C(\to c_1 c_2)D \to d_1 d_2)$$

- Not aware of any relevant such process for *B* physics
- Spin-2 particles, 3-body decays, … can also be studied.

To take away:

- These measurements require high statistics, because distributions are multi-dimensional

→ You can notice the small numerical coefficients of the c´s : $1/(4\sqrt{3}\pi)$, etc. Yet, trivial integrations over ϕ, ϕ^* or both give 2π factors for free

End