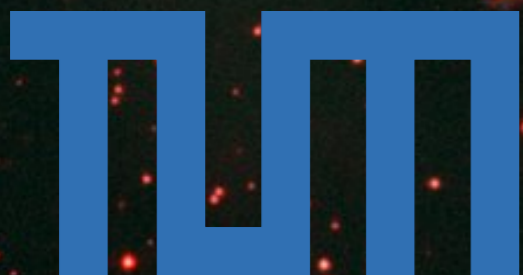


The stellar graveyard as a BSM laboratory

5th NPKI Workshop

Andreas Weiler (TUM)
6/6/23



work in collaboration with:

Reuven Balkin (Technion -> UC Santa Cruz),

Javi Serra (IFT Madrid),

Stefan Stelzl (EPFL),

Konstantin Springmann (TUM -> Weizman/DESY)



Plan for the next 32 Minutes

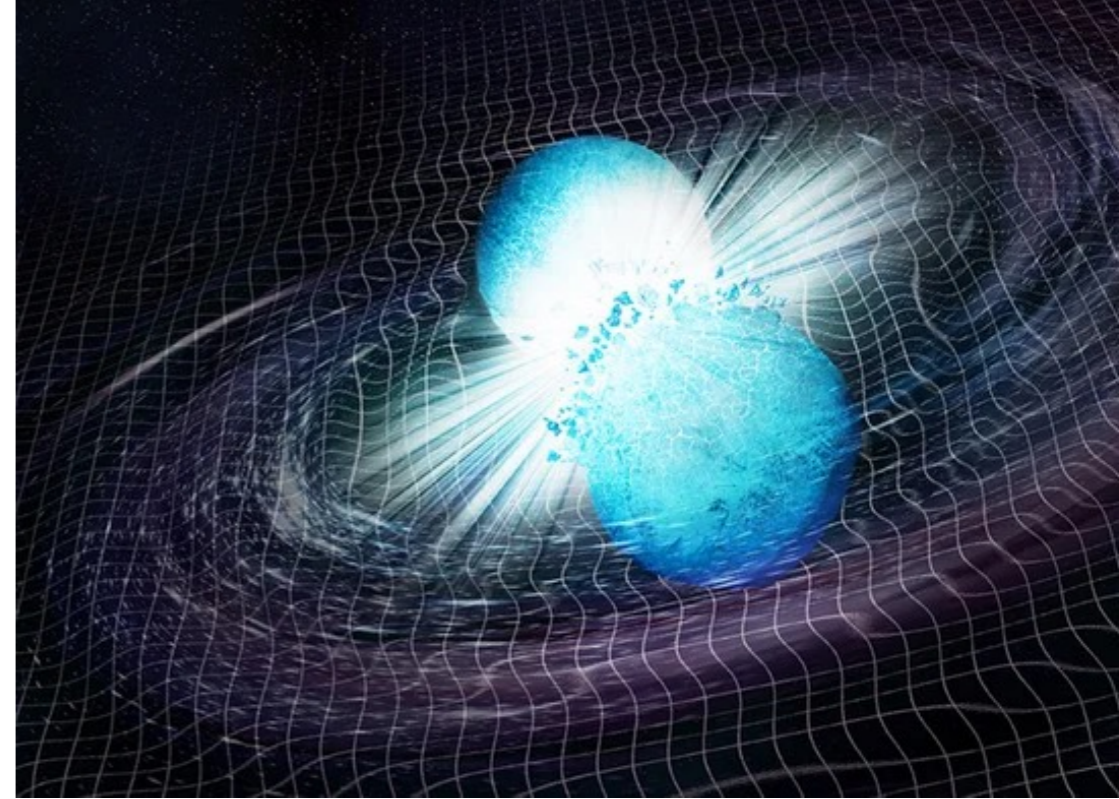
- White dwarfs and neutron stars simplified
- What happens to a (light) QCD axion inside them?
- General theory for light scalars in dense stars

Neutron Stars as a laboratory for fundamental physics

- Densest stars in the universe,

$$\epsilon \sim \frac{M_{\odot}}{\frac{4\pi}{3} (10 \text{ km})^3} \sim 10^{14} \text{ g/cm}^3 \sim (200 \text{ MeV})^4$$

- Strongest magnetic fields in the universe.
- Very precise measurements of their spin (pulsars).
- Observed in merger events via gravitational waves.



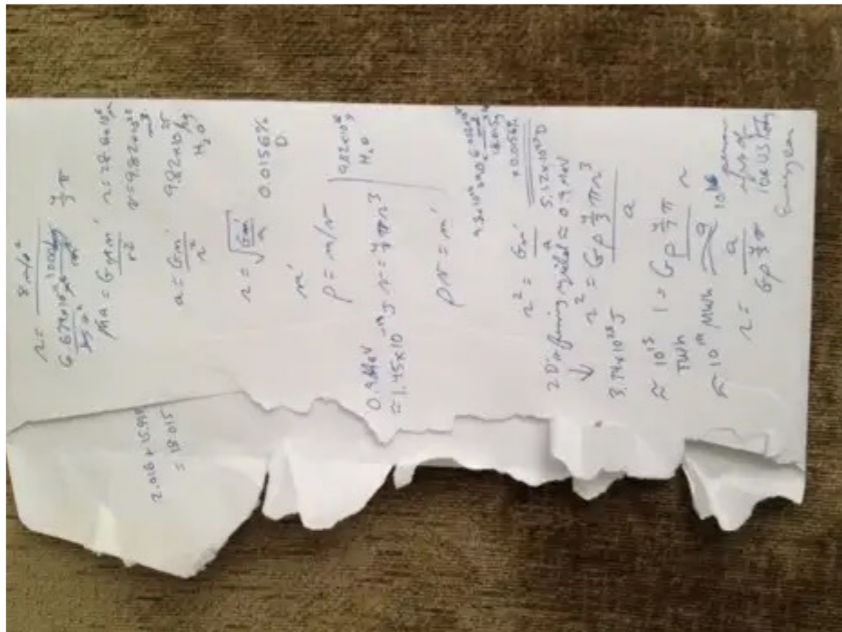
Neutron Stars

A central neutron star is depicted as a glowing, textured sphere with a color gradient from red to yellow. It is surrounded by a complex, blue, filamentary magnetic field structure that radiates outwards, resembling a tangled web of energy lines. The background is a dark, starry space.

how big? how massive? how dense?

Back of the envelope ...

... power of theory (laziness as a virtue)



vs.

$$\begin{aligned}\phi'' \left[1 - \frac{2GM}{r} \right] + \frac{2}{r} \phi' \left[1 - \frac{GM}{r} - 2\pi G r^2 (\varepsilon - p) \right] &= \frac{\partial V}{\partial \phi} + \rho \frac{\partial m_{\psi}^*(\phi)}{\partial \phi} \equiv U(\phi, \rho), \\ p' &= -\frac{GM\varepsilon}{r^2} \left[1 + \frac{p}{\varepsilon} \right] \left[1 - \frac{2GM}{r} \right]^{-1} \left[1 + \frac{4\pi r^3}{M} \left(p + \frac{(\phi')^2}{2} \left\{ 1 - \frac{2GM}{r} \right\} \right) \right] - \phi' U(\phi, \rho), \\ M' &= 4\pi r^2 \left[\varepsilon + \frac{1}{2} \left(1 - \frac{2GM}{r} \right) (\phi')^2 \right].\end{aligned}$$

Neutron stars simplified

$$E_{\text{Fermi}} \approx E_{\text{Gravity}}$$

$$E_{\text{Fermi}}/N = m_N \quad (\text{nucleon mass})$$

$$E_{\text{Gravity}} = -\frac{3}{5} \frac{GM_{\text{NS}}^2}{R_{\text{NS}}} \sim \frac{M_{\text{NS}}^2}{M_{\text{planck}}^2 R_{\text{NS}}} \sim \frac{N^2 m_N^2}{M_{\text{planck}}^2 R_{\text{NS}}}$$

with $N \sim R_{\text{NS}}^3 \cdot n \sim R_{\text{NS}}^3 m_N^3$

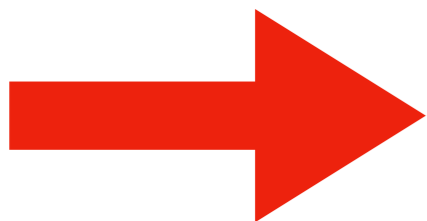
Neutron stars simplified

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with $N \sim R_{\text{NS}}^3 \cdot n \sim R_{\text{NS}}^3 m_N^3$



$$R_{\text{NS}} \sim \frac{M_{\text{Planck}}}{m_N^2}$$

$$M_{\text{NS}} \sim \frac{M_{\text{Planck}}^3}{m_N^2}$$

Neutron star estimate

$$R_{\text{NS}} \sim \frac{M_{\text{Planck}}}{m_N^2} \quad M_{\text{NS}} \sim \frac{M_{\text{Planck}}^3}{m_N^2}$$

$$R_{\text{NS}} \sim (\text{few}) \text{ km} \quad M_{\text{NS}} \sim (\text{few}) 10^{30} \text{ kg} \sim M_{\odot}$$

Mass of the sun within a few km

Planck scale

$$M_{\text{Planck}} \approx 10^{19} \text{ GeV}$$

nucleon mass

$$M_N \approx 1 \text{ GeV}$$

Comparison to data



Jocelyn Bell Burnell

Neutron stars: mass vs. radius

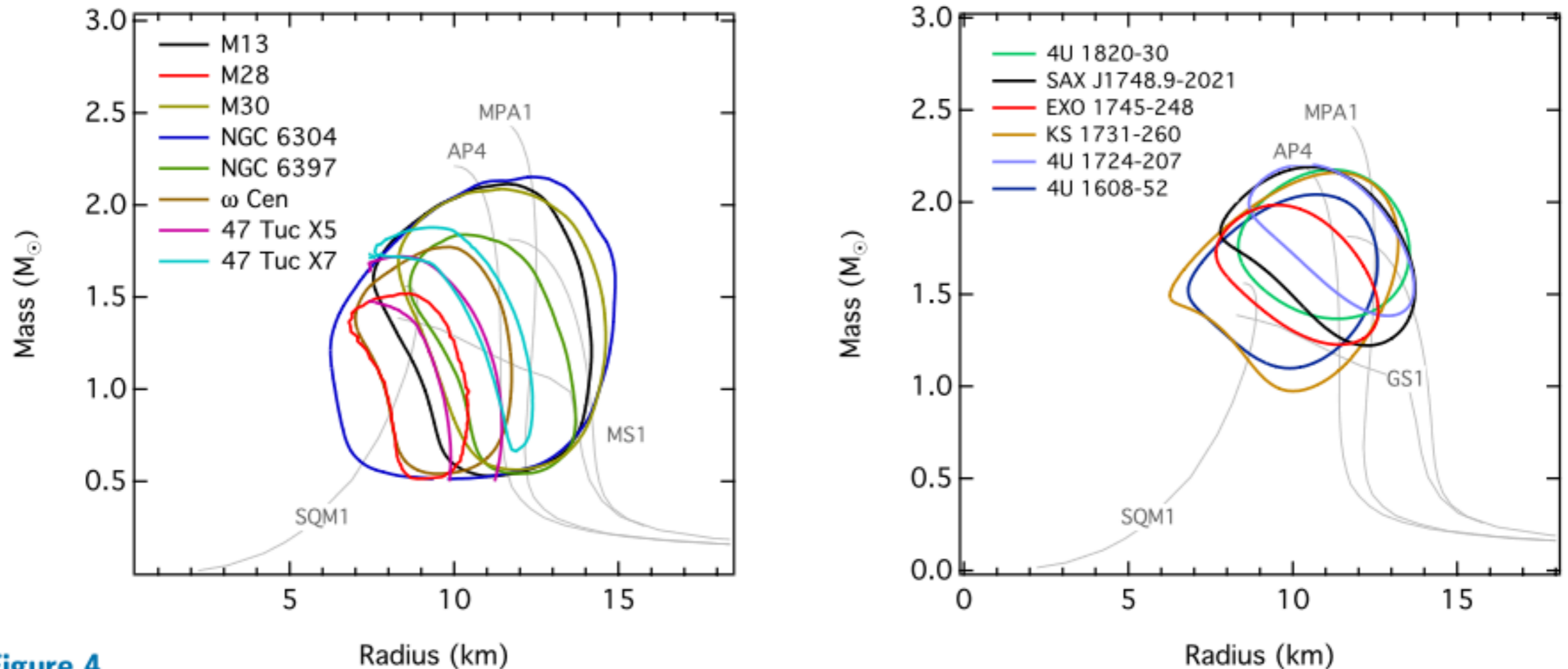
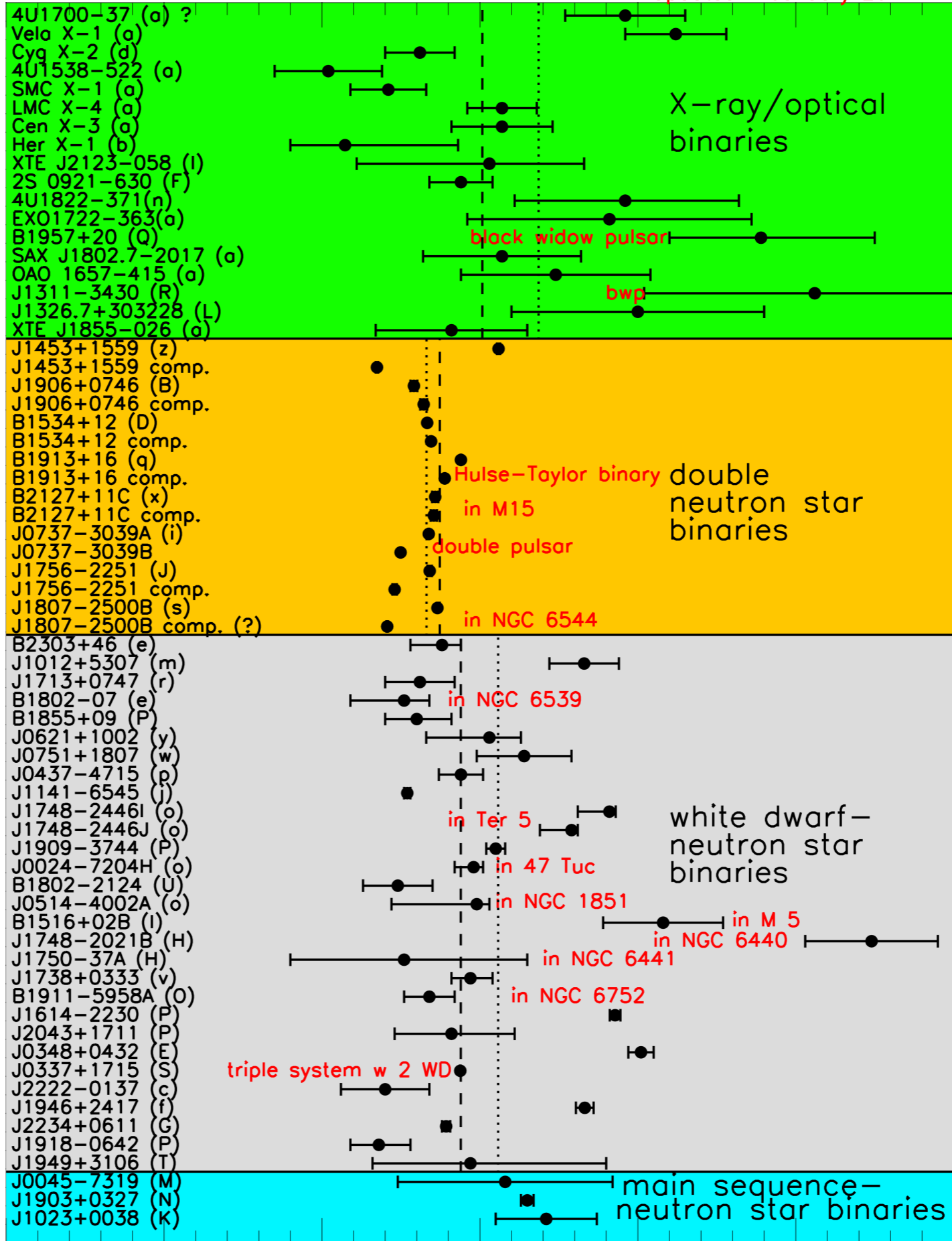


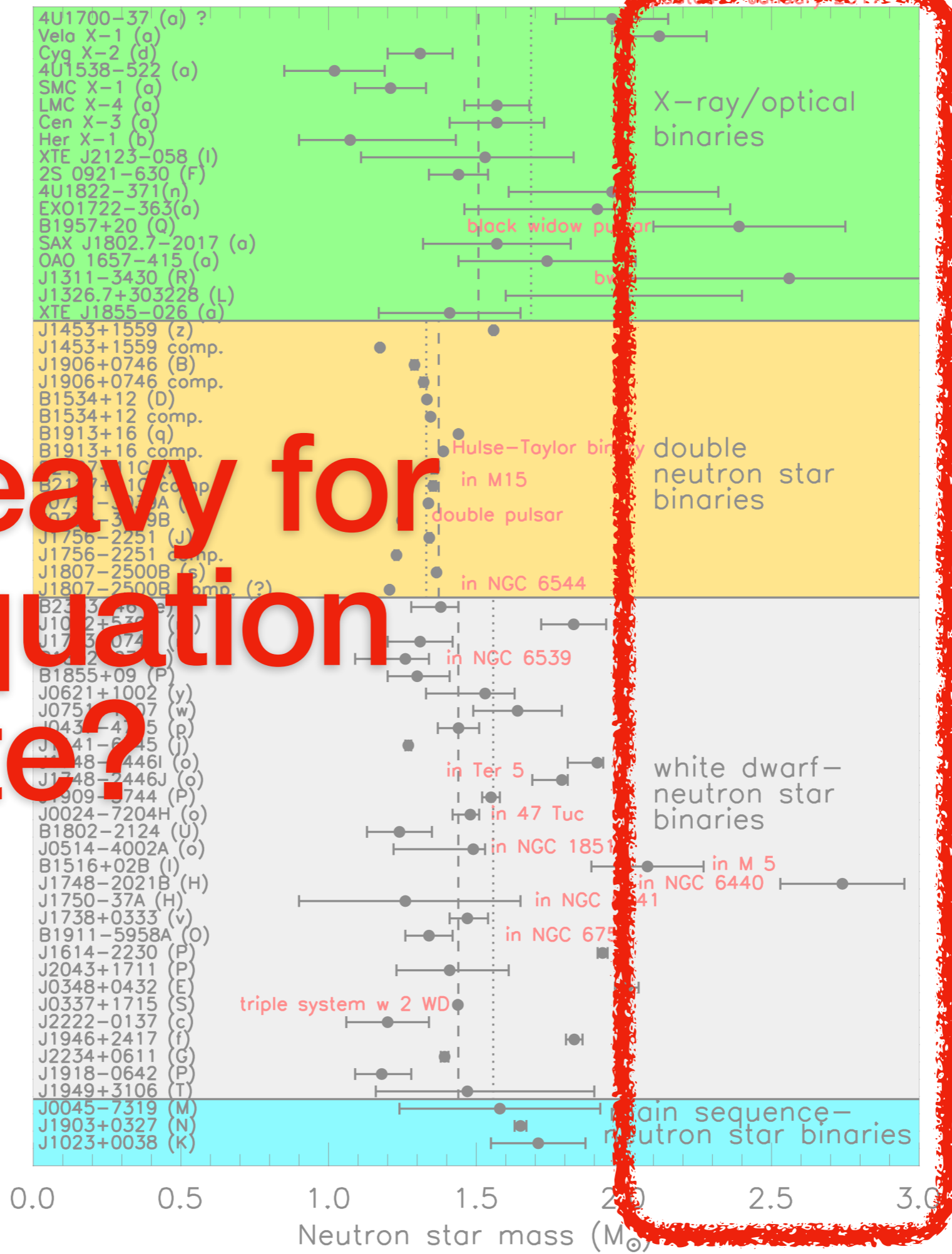
Figure 4

The combined constraints at the 68% confidence level over the neutron star mass and radius obtained from (Left) all neutron stars in low-mass X-ray binaries during quiescence (Right) all neutron stars with thermonuclear bursts. The light grey lines show mass-relations corresponding to a few representative equations of state (see Section 4.1 and Fig. 7 for detailed descriptions.)



0.0 0.5 1.0 1.5 2.0 2.5 3.0
Neutron star mass (M_{\odot})

Too heavy for
SM equation
of state?

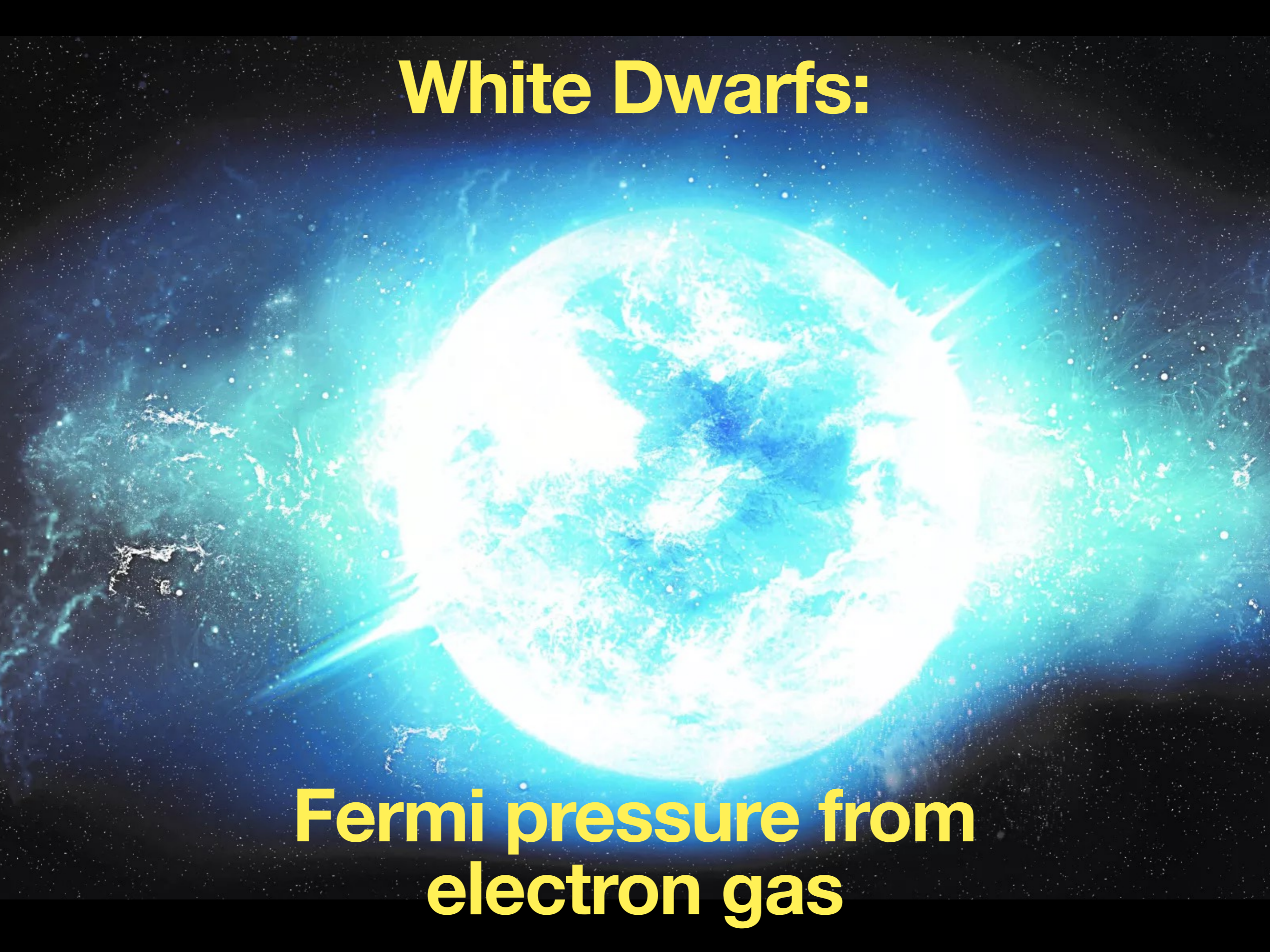




**You need some
BSM?**

White Dwarfs:

Fermi pressure from
electron gas



White dwarfs simplified

$$E_{\text{Fermi}} \approx E_{\text{Gravity}}$$

$$E_{\text{Fermi}}/N = m_e \quad (\text{electron mass})$$

$$E_{\text{Gravity}} = -\frac{3}{5} \frac{GM_{\text{WD}}^2}{R_{\text{WD}}} \sim \frac{M_{\text{WD}}^2}{M_{\text{planck}}^2 R_{\text{WD}}} \sim \frac{N^2 m_N^2}{M_{\text{planck}}^2 R_{\text{WD}}}$$

$$N \sim R_{\text{WD}}^3 n \sim R_{\text{WD}}^3 m_e^3$$

White dwarfs simplified

$$E_{\text{Fermi}} \approx E_{\text{Gravity}}$$

$$E_{\text{Fermi}}/N = m_e \quad (\text{electron mass})$$

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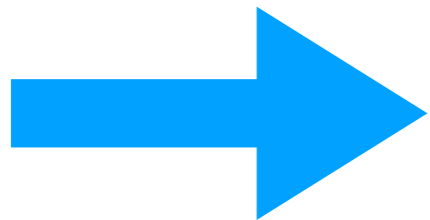
$$N \sim R_{\text{WD}}^3 n \sim R_{\text{WD}}^3 m_e^3$$

$$R_{\text{WD}} \sim \frac{M_{\text{planck}}}{m_e m_N} \sim \frac{m_N}{m_e} R_{\text{NS}}$$

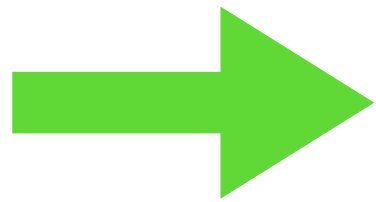
larger radius

$$M_{\text{WD}} \sim \frac{M_{\text{planck}}^3}{m_N^2} \sim M_{\text{NS}}$$

same mass



White dwarf estimate



$$R_{\text{WD}} \sim \frac{M_{\text{planck}}}{m_e m_N} \sim \frac{m_N}{m_e} R_{\text{NS}}$$

$$M_{\text{WD}} \sim \frac{M_{\text{planck}}^3}{m_N^2} \sim M_{\text{NS}}$$

$$M_{\text{WD}} \sim (\text{few}) 10^{30} \text{ kg} \sim M_{\odot}$$

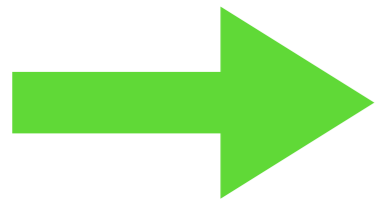
$$R_{\text{WD}} \sim (\text{few}) 10000 \text{ km}$$

electron mass $m_e = 0.51099895000(15) \text{ MeV}$

Planck scale $M_{\text{Planck}} \approx 10^{19} \text{ GeV}$

nucleon mass $M_N \approx 1 \text{ GeV}$

White dwarf estimate



$$R_{\text{WD}} \sim \frac{M_{\text{planck}}}{m_e m_N} \sim \frac{m_N}{m_e} R_{\text{NS}}$$

$$M_{\text{WD}} \sim \frac{M_{\text{planck}}^3}{m_N^2} \sim M_{\text{NS}}$$

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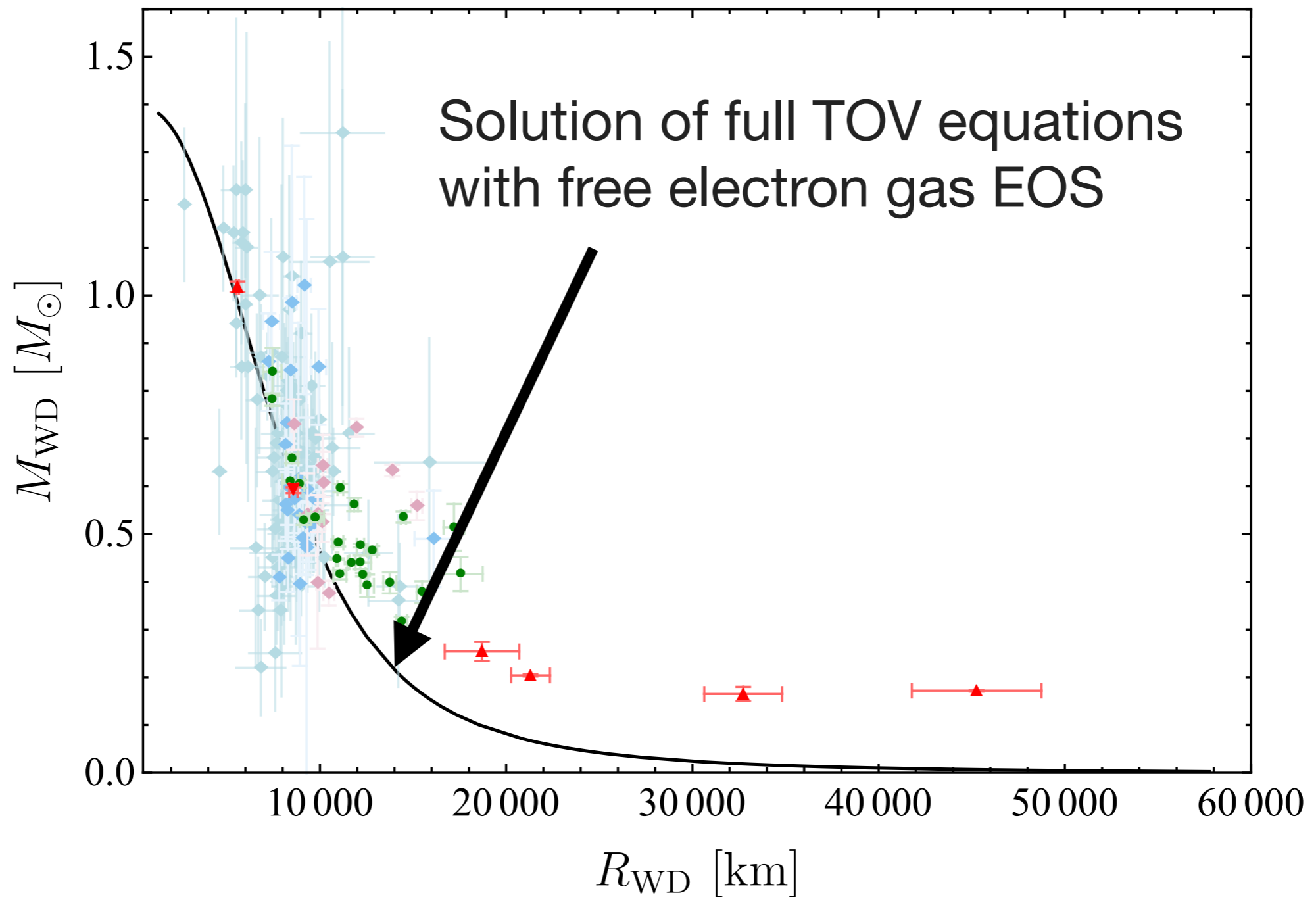
Mass of the sun at the size of the earth.

electron mass $m_e = 0.51099895000(15) \text{ MeV}$

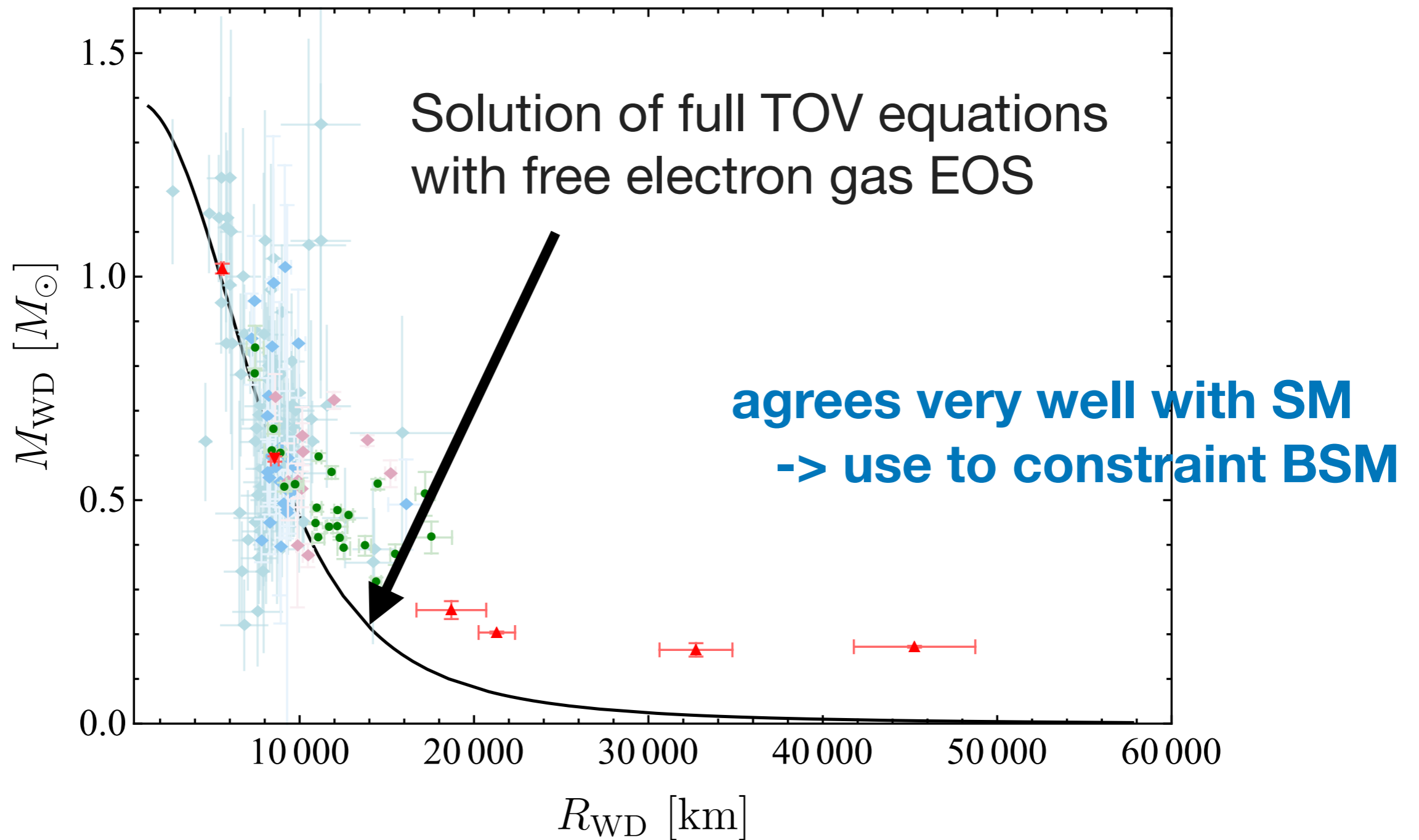
Planck scale $M_{\text{Planck}} \approx 10^{19} \text{ GeV}$

nucleon mass $M_N \approx 1 \text{ GeV}$

White dwarf mass-radius curve



White dwarf mass-radius curve



**We will study these very dense objects
in the presence of light QCD axions
(as well as other light scalars)**

**We will not have to assume that they
are the dark matter.**

QCD axion: motivated by the strong CP problem

CP violation in the strong sector

$$\mathcal{L}_{\text{QCD}} = \sum_q \bar{q} (i\not{D} - m_q e^{i\theta_q}) q - \frac{1}{4} G_a^{\mu\nu} G_{\mu\nu}^a - \theta \frac{\alpha_s}{8\pi} G_a^{\mu\nu} \tilde{G}_{\mu\nu}^a$$

Predicts a neutron EDM:

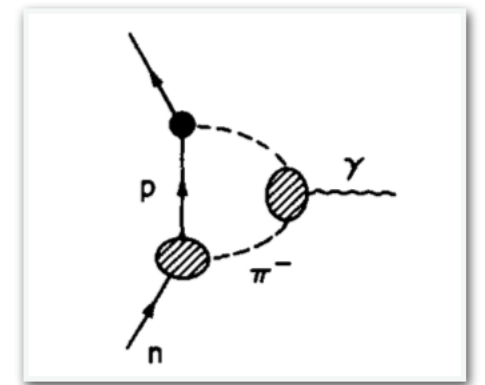
QCD axion: motivated by the strong CP problem

CP violation in the strong sector

$$\mathcal{L}_{\text{QCD}} = \sum_q \bar{q} (i\not{D} - m_q e^{i\theta_q}) q - \frac{1}{4} G_a^{\mu\nu} G_{\mu\nu}^a - \theta \frac{\alpha_s}{8\pi} G_a^{\mu\nu} \tilde{G}_{\mu\nu}^a$$

Predicts a neutron EDM:

$$\mathcal{L}_\chi \supset d_n \bar{n} \sigma^{\mu\nu} \gamma_5 n F_{\mu\nu}$$

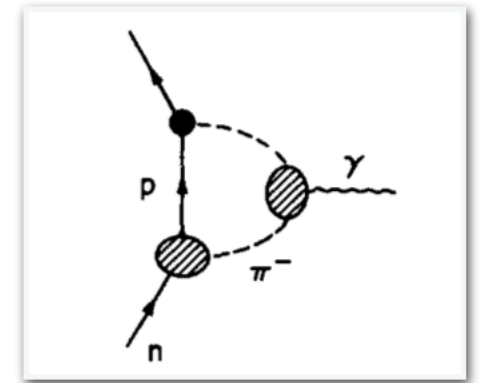


$$d_n \approx \frac{e |\bar{\theta}| m_\pi^2}{m_n^3} \approx 10^{-16} |\bar{\theta}| e \text{ cm}$$

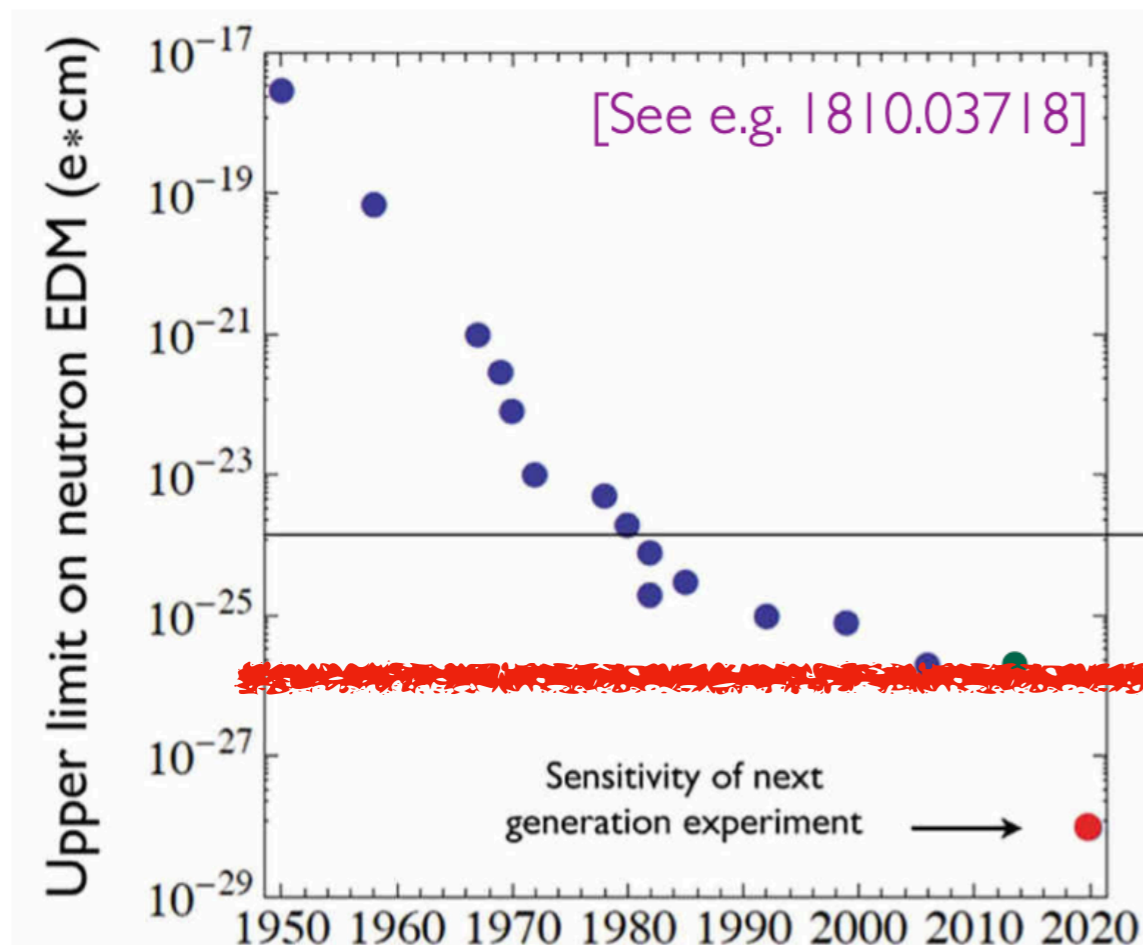
QCD axion: motivated by the strong CP problem

CP violation in the strong sector

$$\mathcal{L}_{\text{QCD}} = \sum_q \bar{q} (i\not{D} - m_q e^{i\theta_q}) q - \frac{1}{4} G_a^{\mu\nu} G_{\mu\nu}^a - \theta \frac{\alpha_s}{8\pi} G_a^{\mu\nu} \tilde{G}_{\mu\nu}^a$$



Predicts a neutron EDM: $\mathcal{L}_\chi \supset d_n \bar{n} \sigma^{\mu\nu} \gamma_5 n F_{\mu\nu}$



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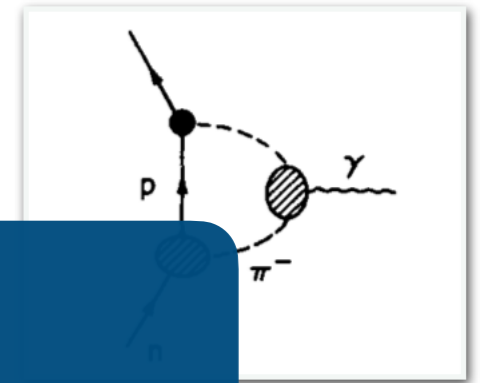
Why so small ?

$$|\bar{\theta}| \lesssim 10^{-10}$$

QCD axion: motivated by the strong CP problem

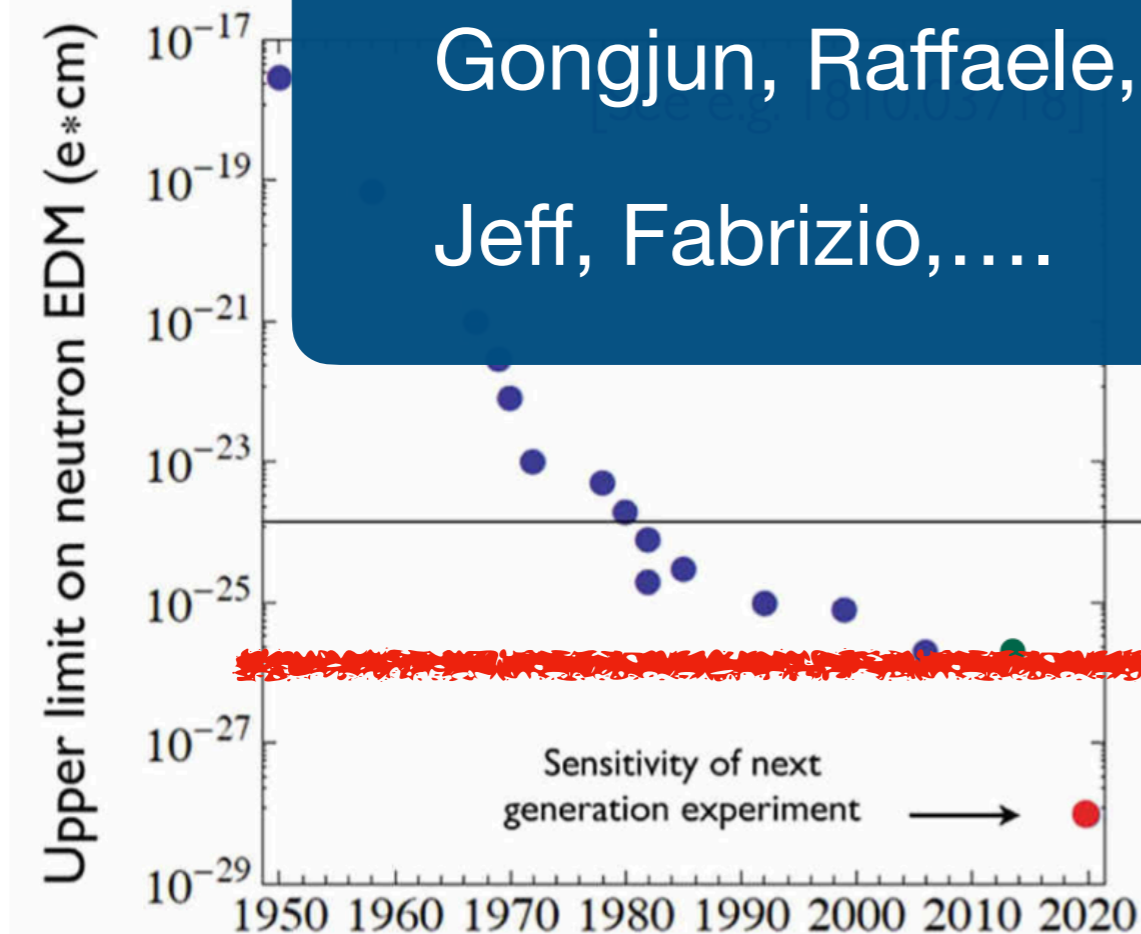
CP violation in the strong sector

$$\mathcal{L}_{\text{QCD}} = \sum_q \bar{q} (i\not{D} - m_q e^{i\theta_q}) q - \frac{1}{4} G_a^{\mu\nu} G_{\mu\nu}^a - \theta \frac{\alpha_s}{8\pi} G_a^{\mu\nu} \tilde{G}_{\mu\nu}^a$$



Predicts

See talks by: Surjeet, Tony, Sungwoo, Gongjun, Raffaele, Michael, Maximilian, Jeff, Fabrizio,....



10⁻²⁶

Why so small ?

$$|\bar{\theta}| \lesssim 10^{-10}$$

$$|\bar{\theta}| e \text{ cm}$$

Axion Solution



Roberto Peccei



Helen Quinn

- PQ symmetry $U(1)_{\text{PQ}}$: spontaneously broken at scale $\sim f_a$
- explicitly broken at the quantum level by a mixed QCD anomaly
- pseudo-Nambu-Goldstone boson (pNG) arises: the QCD axion $\phi(x)$

$$\mathcal{L} = \left(\frac{\phi(x)}{f_a} - \bar{\theta} \right) \frac{\alpha_s}{8\pi} G^{\mu\nu a} \tilde{G}_{\mu\nu}^a$$

- non-perturbative axion potential: axion **relaxes to CP-conserving minimum**

$$\bar{\theta}_{\text{eff}} \equiv \langle \phi \rangle / f_a - \bar{\theta} \rightarrow 0$$

Axion Solution



Roberto Peccei



Helen Quinn

- PQ symmetry $U(1)_{\text{PQ}}$: spontaneously broken at scale $\sim f_a$
- explicitly broken at the quantum level by a mixed QCD anomaly
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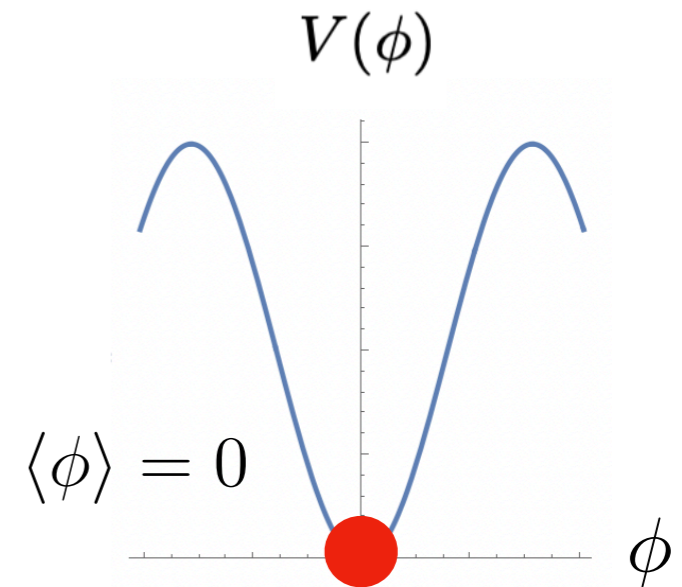
See talks by: Surjeet, Tony, Sungwoo,
Gongjun, Raffaele, Michael, Maximilian,
Jeff, Fabrizio,....

- non-perturbative potential: axion relaxes to CP-conserving minimum

$$\bar{\theta}_{\text{eff}} \equiv \langle \phi \rangle / f_a - \bar{\theta} \rightarrow 0$$

QCD Axions

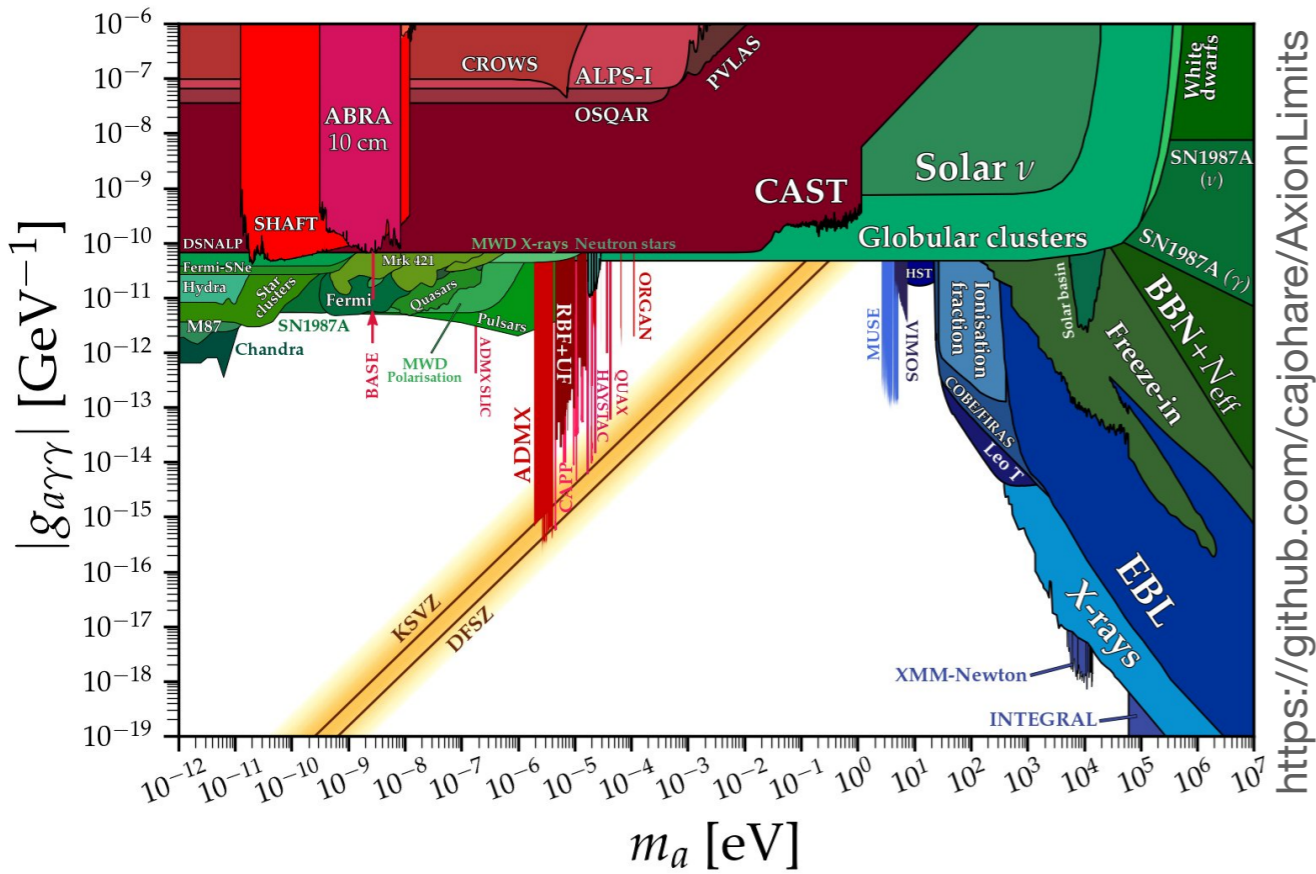
The QCD axion potential at low energy



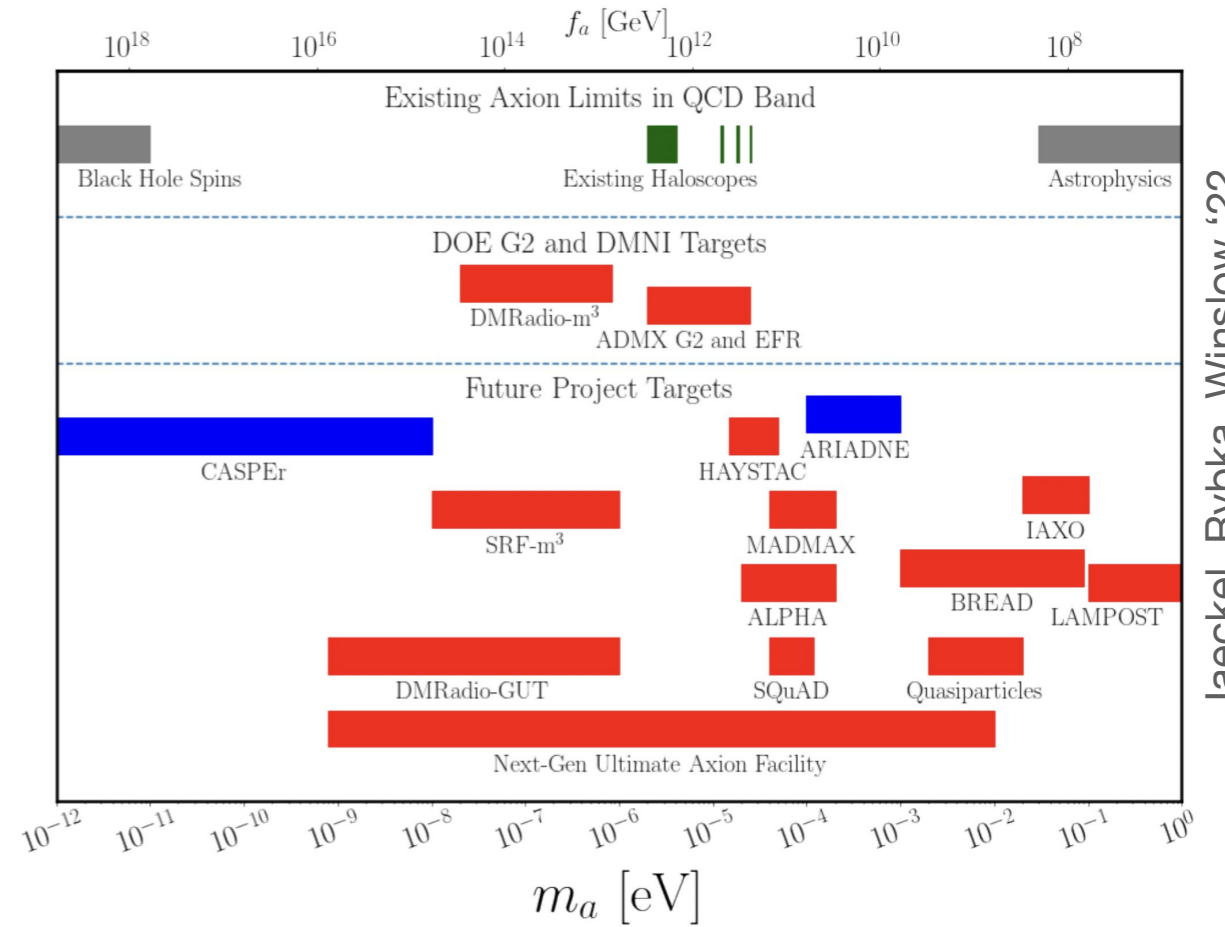
$$\frac{g_s^2}{32\pi^2} \frac{\phi}{f} G\tilde{G} \xrightarrow{\text{UV} \rightarrow \text{IR}} V(\phi) \simeq -\frac{m_\pi^2 f_\pi^2}{4} \left[\cos\left(\frac{\phi}{f}\right) - 1 \right]$$

The QCD axion couplings to photons, nucleons, electrons determined by scale **f**

Axion: plethora of new searches



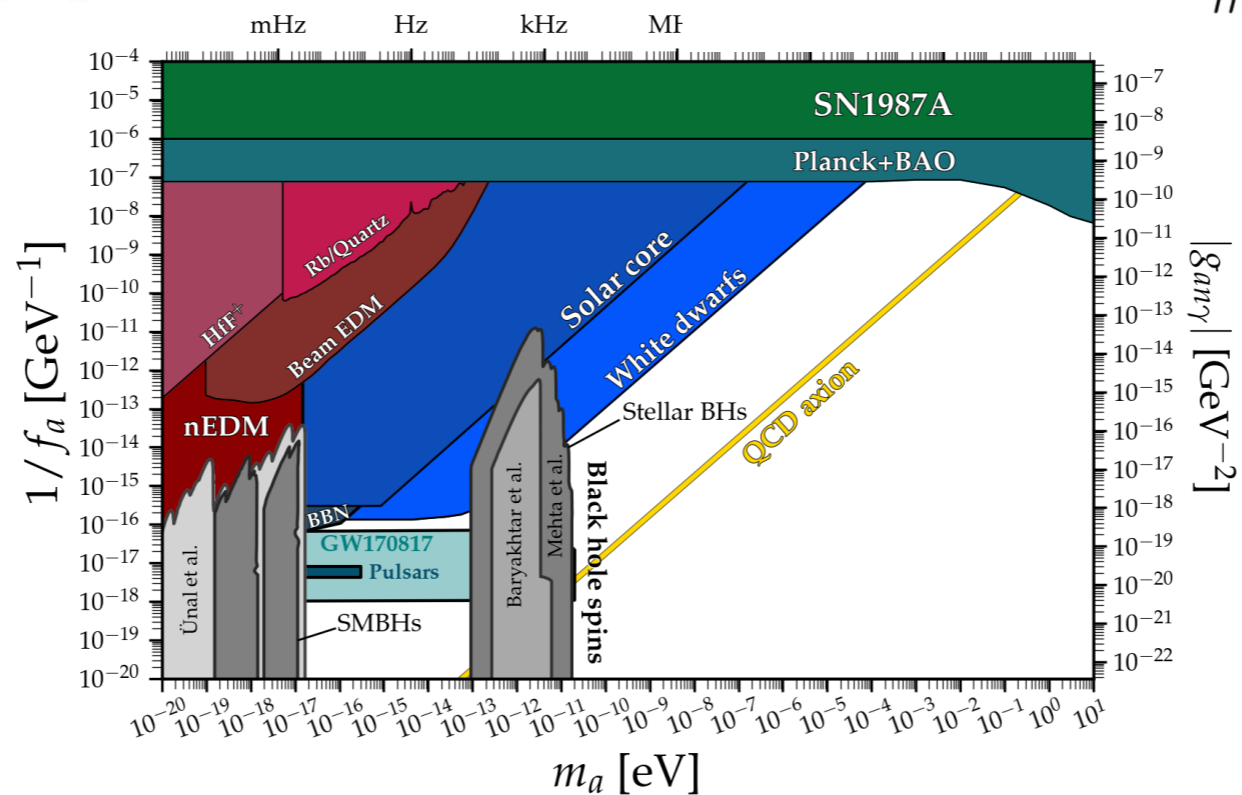
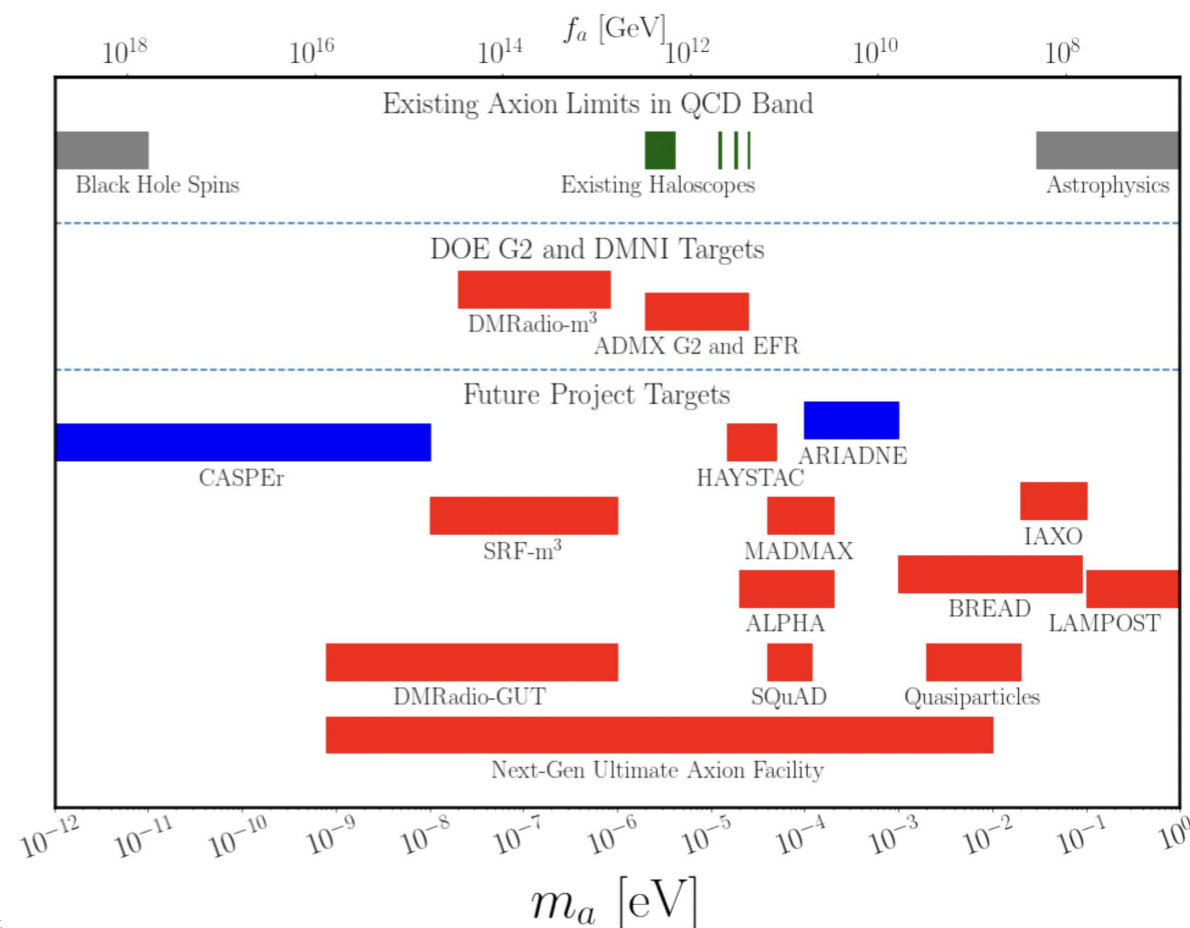
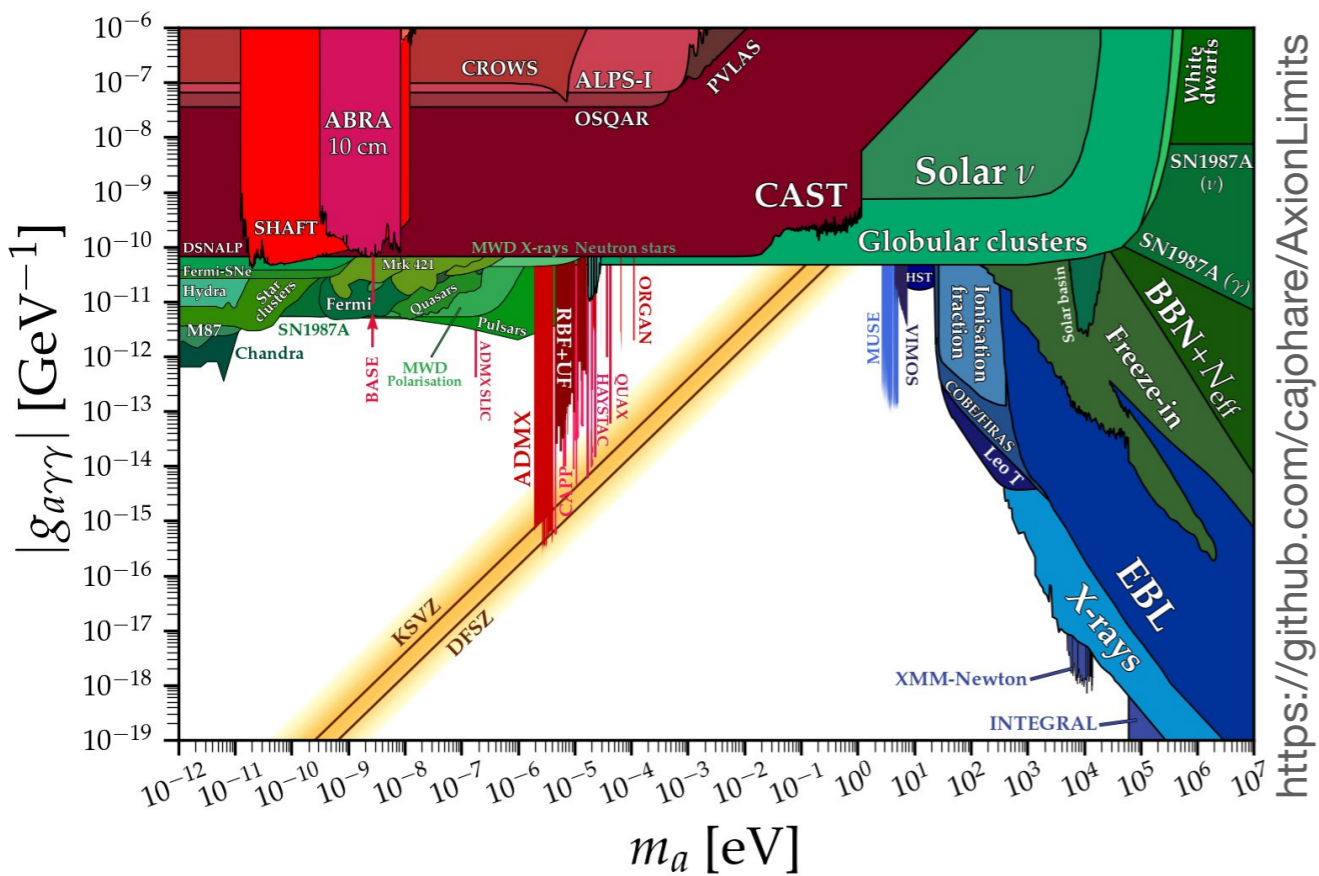
<https://github.com/cajohare/AxionLimits>



Jaeckel, Rybka, Winslow '22

see e.g., Surjeet's talk

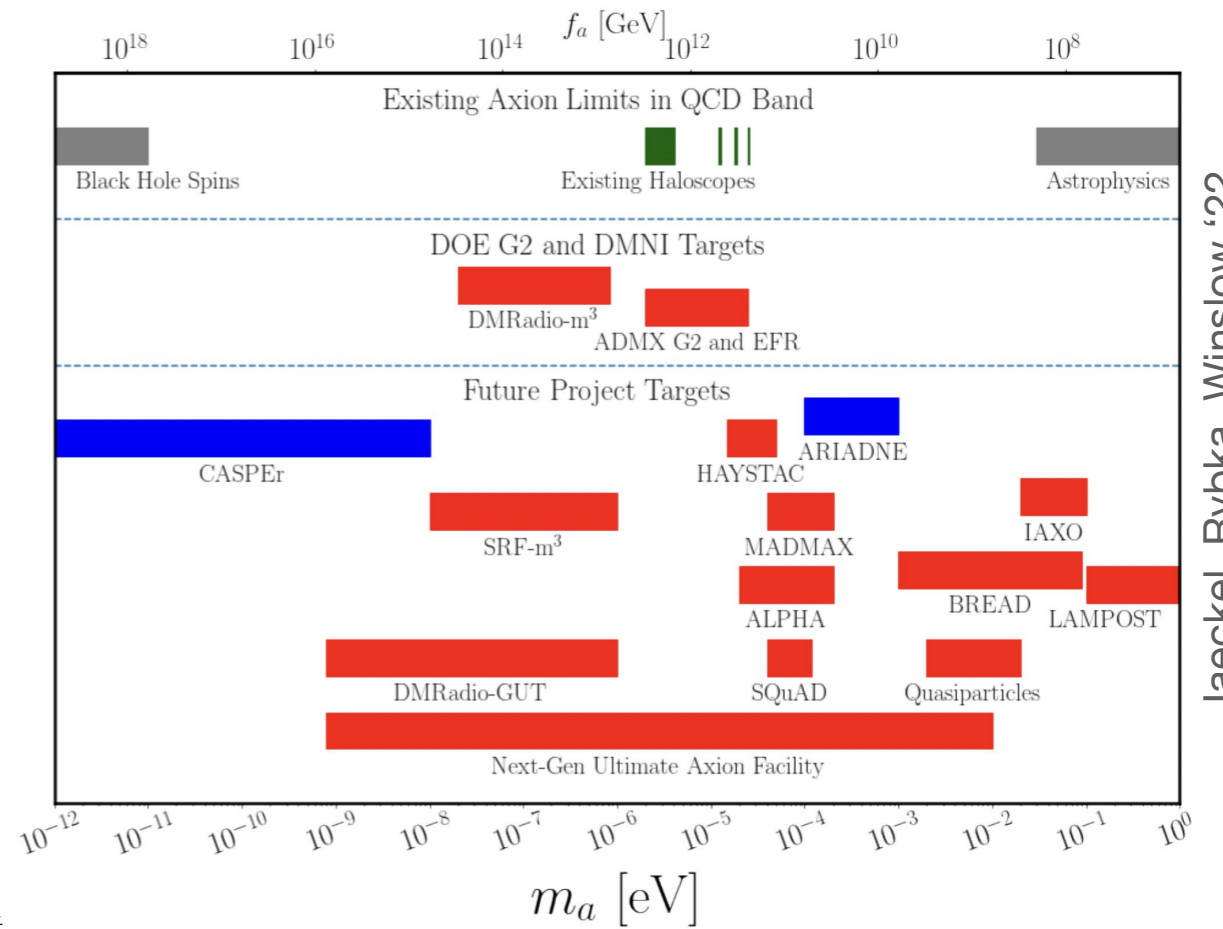
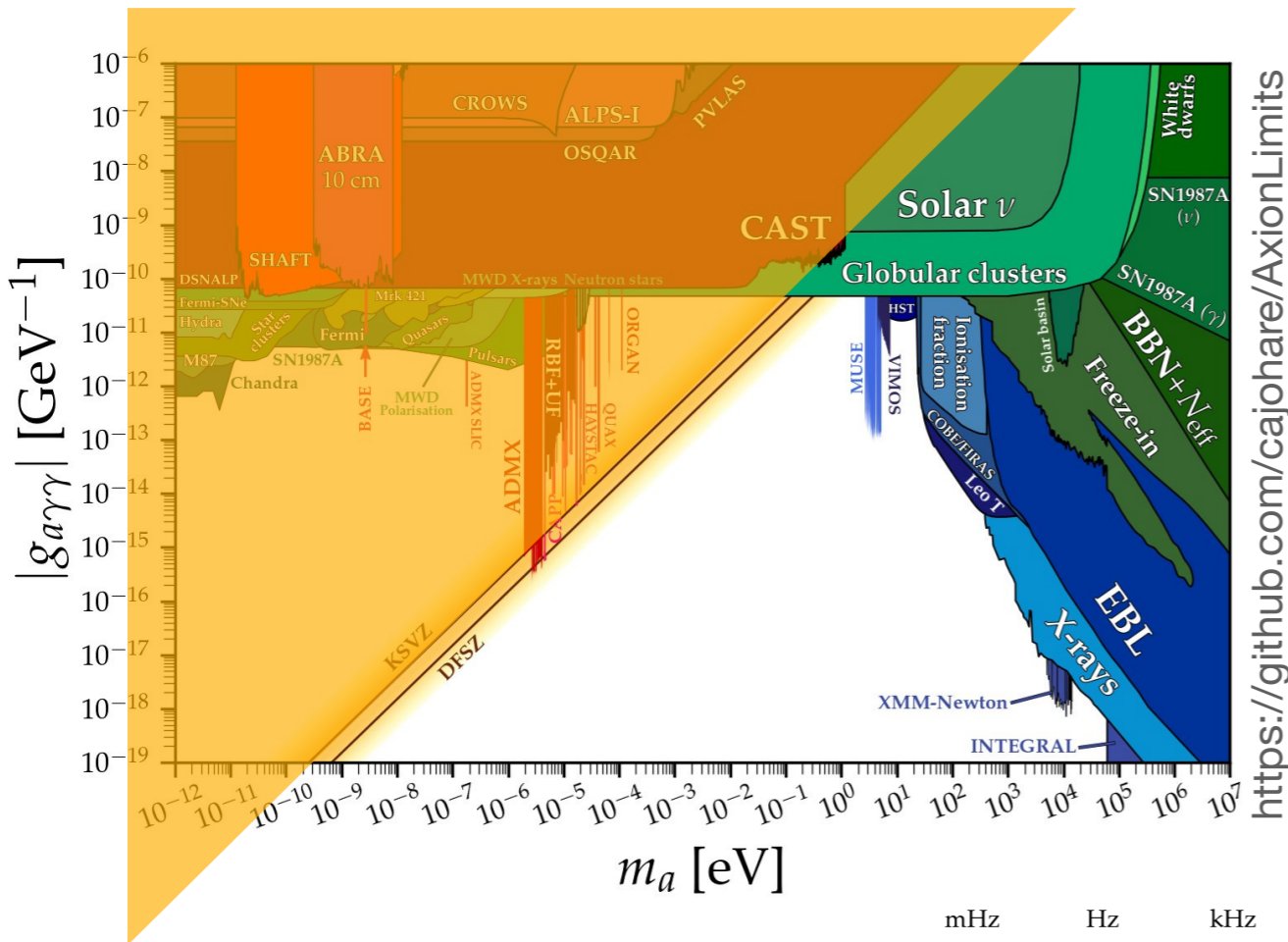
Axion: plethora of new searches



Jaeckel, Rybka, Winslow '22

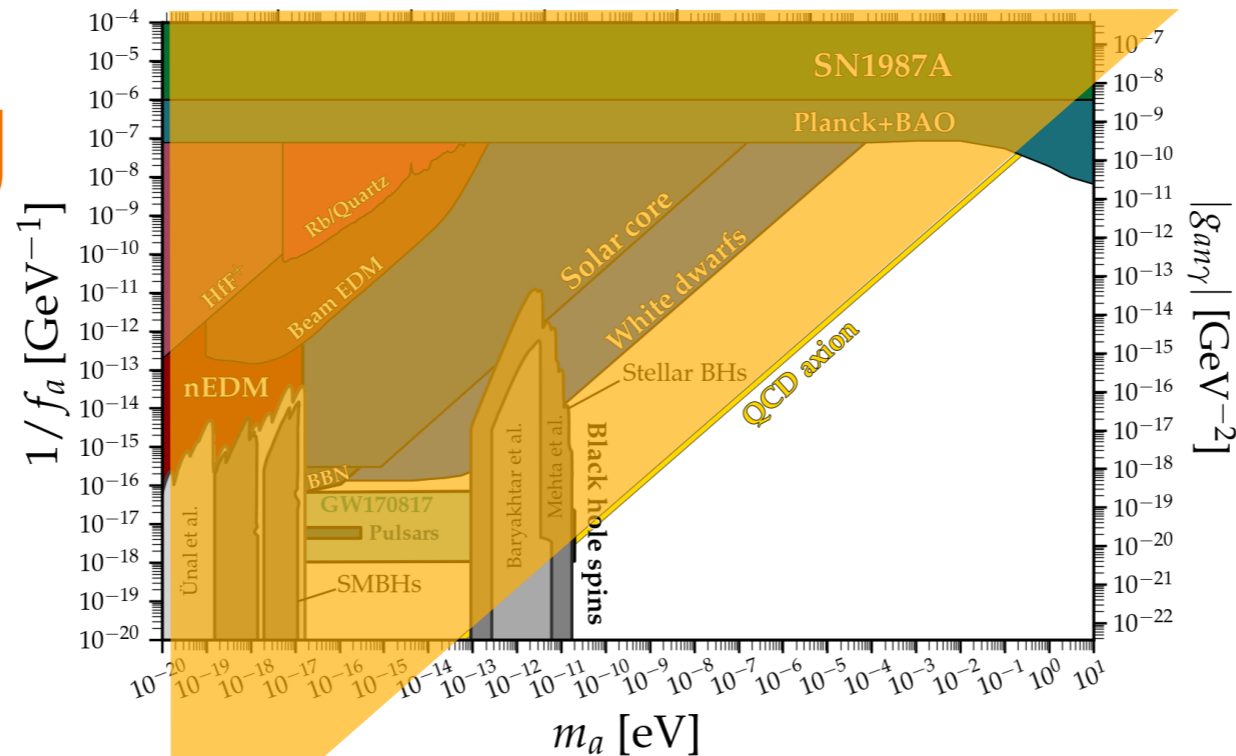
see e.g., Surjeet's talk

Axion: plethora of new searches



Jaeckel, Rybka, Winslow '22

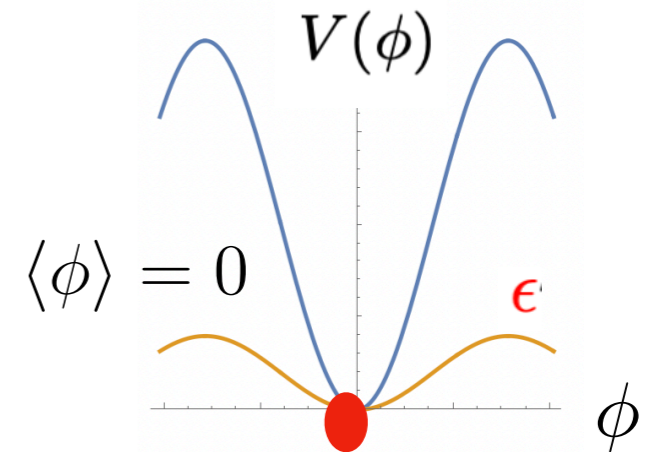
Interesting region



see e.g., Surjeet's talk

Light QCD Axions

The QCD axion potential at low energy



$$\frac{g_s^2}{32\pi^2} \frac{\phi}{f} G\tilde{G}$$

UV



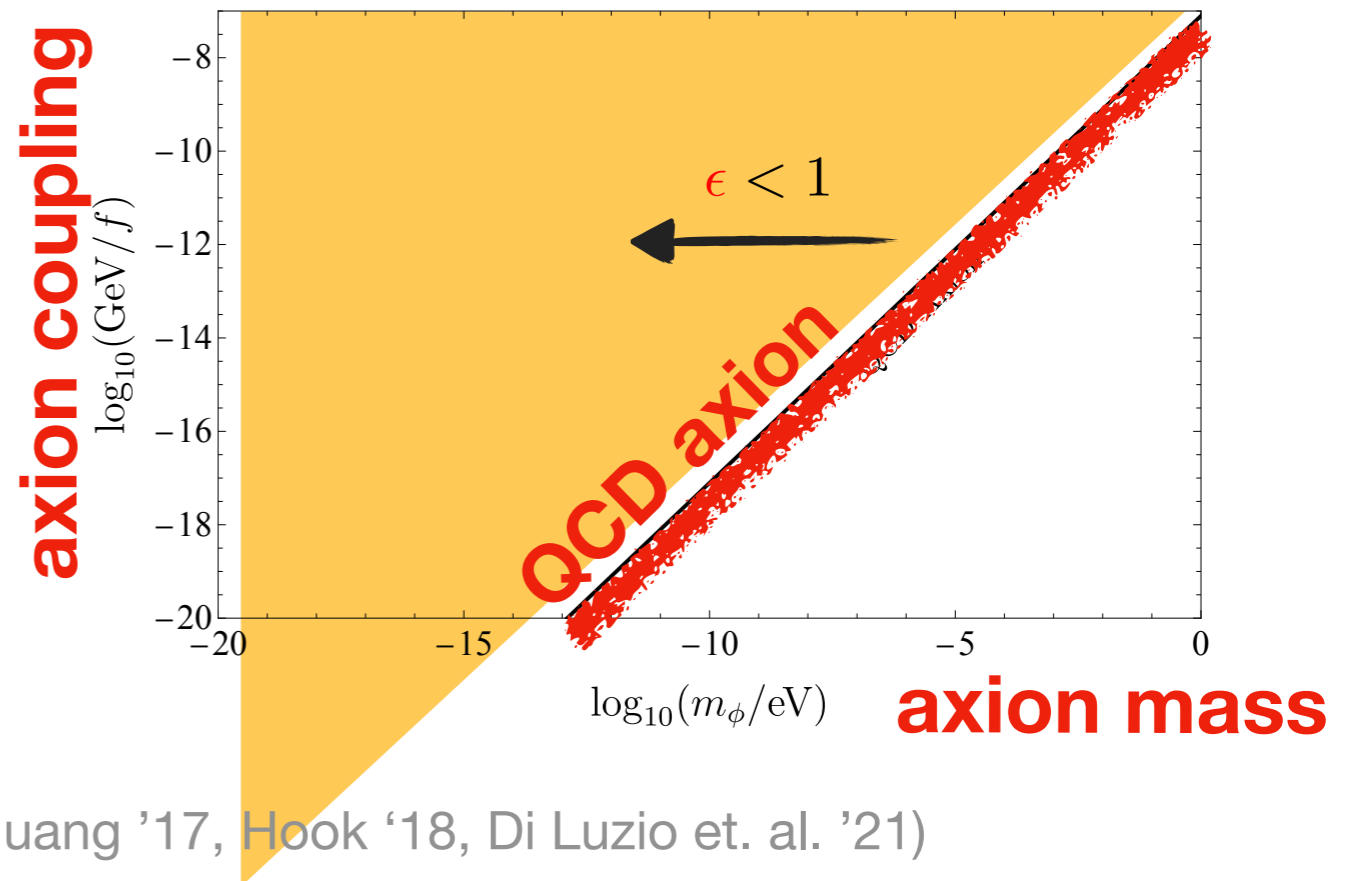
IR

$$V(\phi) \simeq -\frac{m_\pi^2 f_\pi^2}{4} \left[\cos\left(\frac{\phi}{f}\right) - 1 \right]$$

... with a smaller mass:

$$V(\phi) \simeq -\frac{\epsilon m_\pi^2 f_\pi^2}{4} \left[\cos\left(\frac{\phi}{f}\right) - 1 \right]$$

above QCD axion line



ε symmetry realization, see (Hook, Huang '17, Hook '18, Di Luzio et. al. '21)

QCD Axion: Coupling to Nucleons

Nuclear Chiral Perturbation theory

$$\mathcal{L}_{\chi PT} = \text{Tr} [U M_q e^{i\phi/f} + \text{h.c.}] \bar{N}N + \dots$$

leads to **non-derivative coupling of axions to nucleons:**

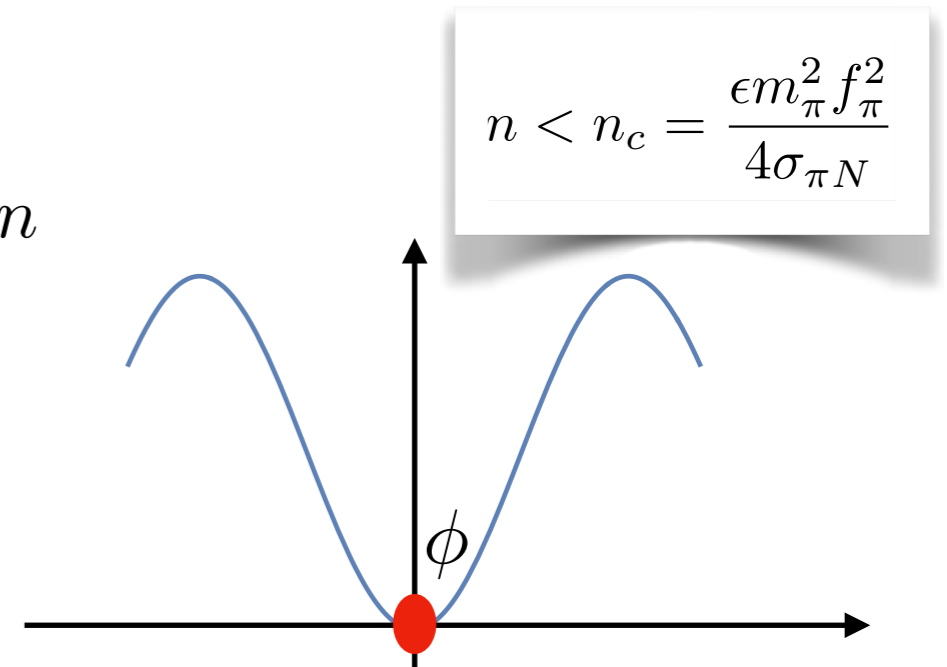
$$\mathcal{L} \supset -m_N(\phi) \bar{N}N \quad \text{with} \quad m_N(\phi) \equiv m_N \left[1 + \frac{\sigma_{\pi N}}{2m_N} \left(\cos \frac{\phi}{f} - 1 \right) \right]$$

such that $m_N(0) = m_N$, $\sigma_{\pi N} \simeq 50 \text{ MeV}$

Light QCD Axion at Finite Density

Turn on baryon density background $\langle \bar{N}N \rangle = n$

$$V(\phi) \simeq - \left[\frac{\epsilon m_\pi^2 f_\pi^2}{4} - \sigma_{\pi N} n \right] \left(\cos \frac{\phi}{f} - 1 \right)$$

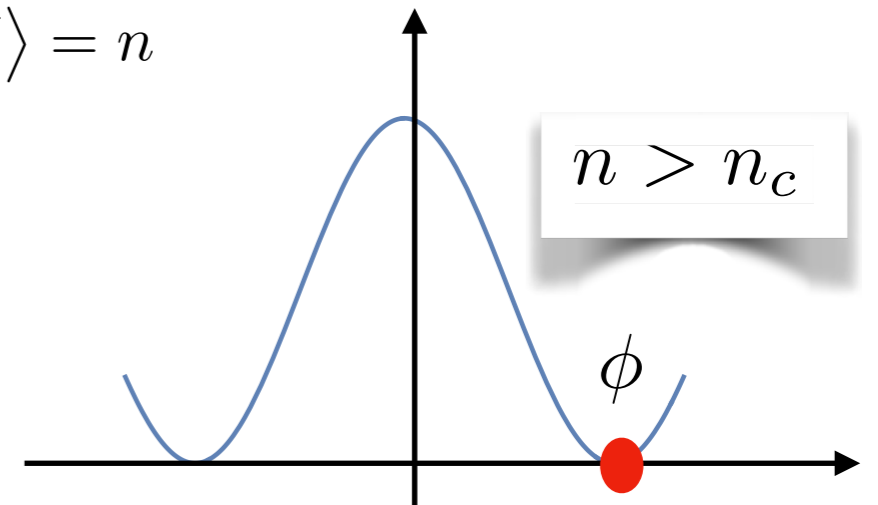


Light QCD Axion at finite density

$$n_c = \frac{\epsilon m_\pi^2 f_\pi^2}{4\sigma_{\pi N}}$$

Turn on baryon density background $\langle \bar{N}N \rangle = n$

$$V(\phi) \simeq - \left[\frac{\epsilon m_\pi^2 f_\pi^2}{4} - \sigma_{\pi N} n \right] \left(\cos \frac{\phi}{f} - 1 \right)$$

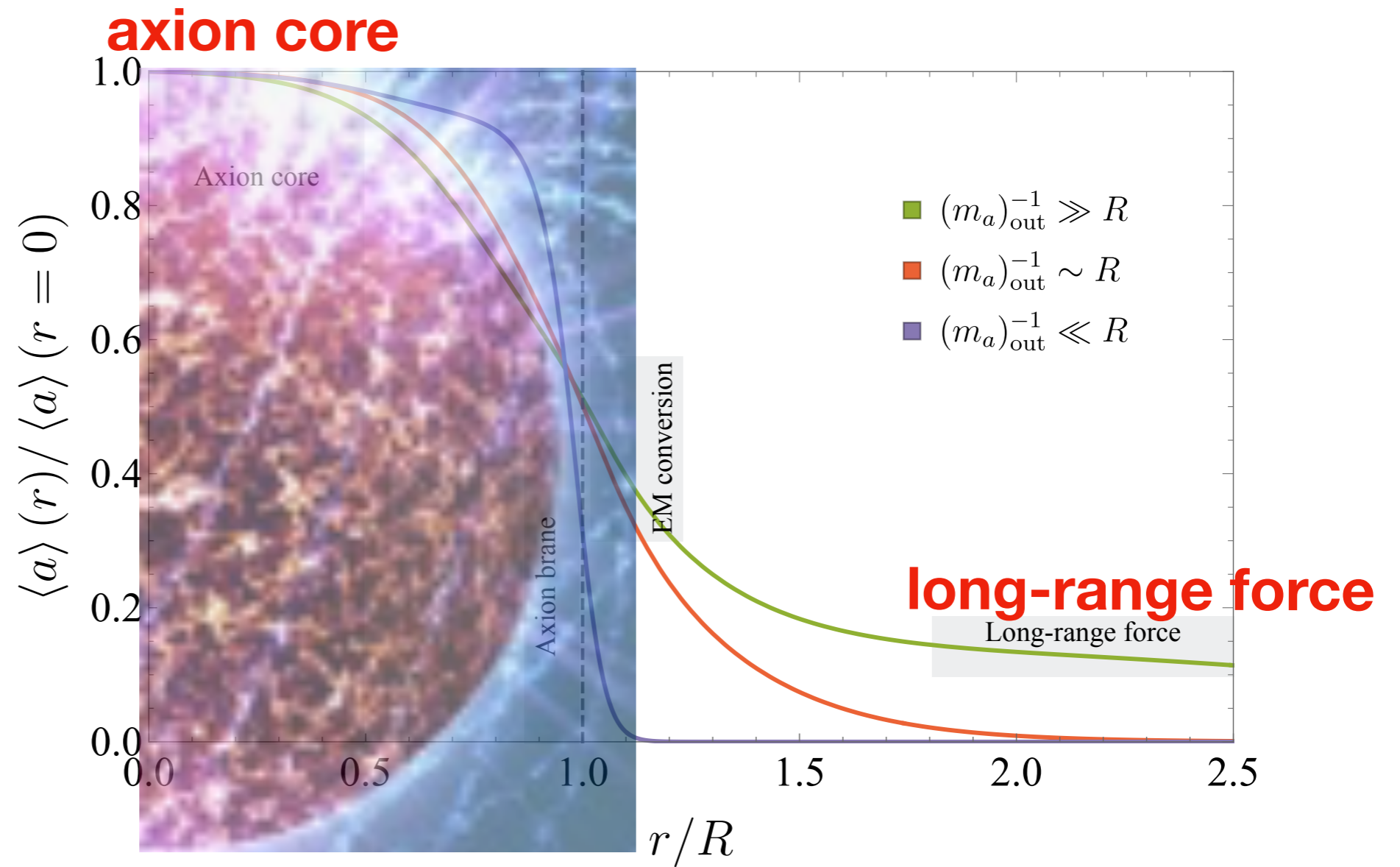
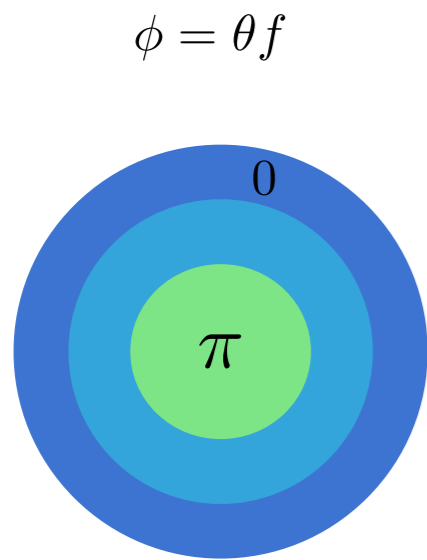


Above **critical density** $n > n_c$:

new minimum at $\langle \phi \rangle = \pi f$

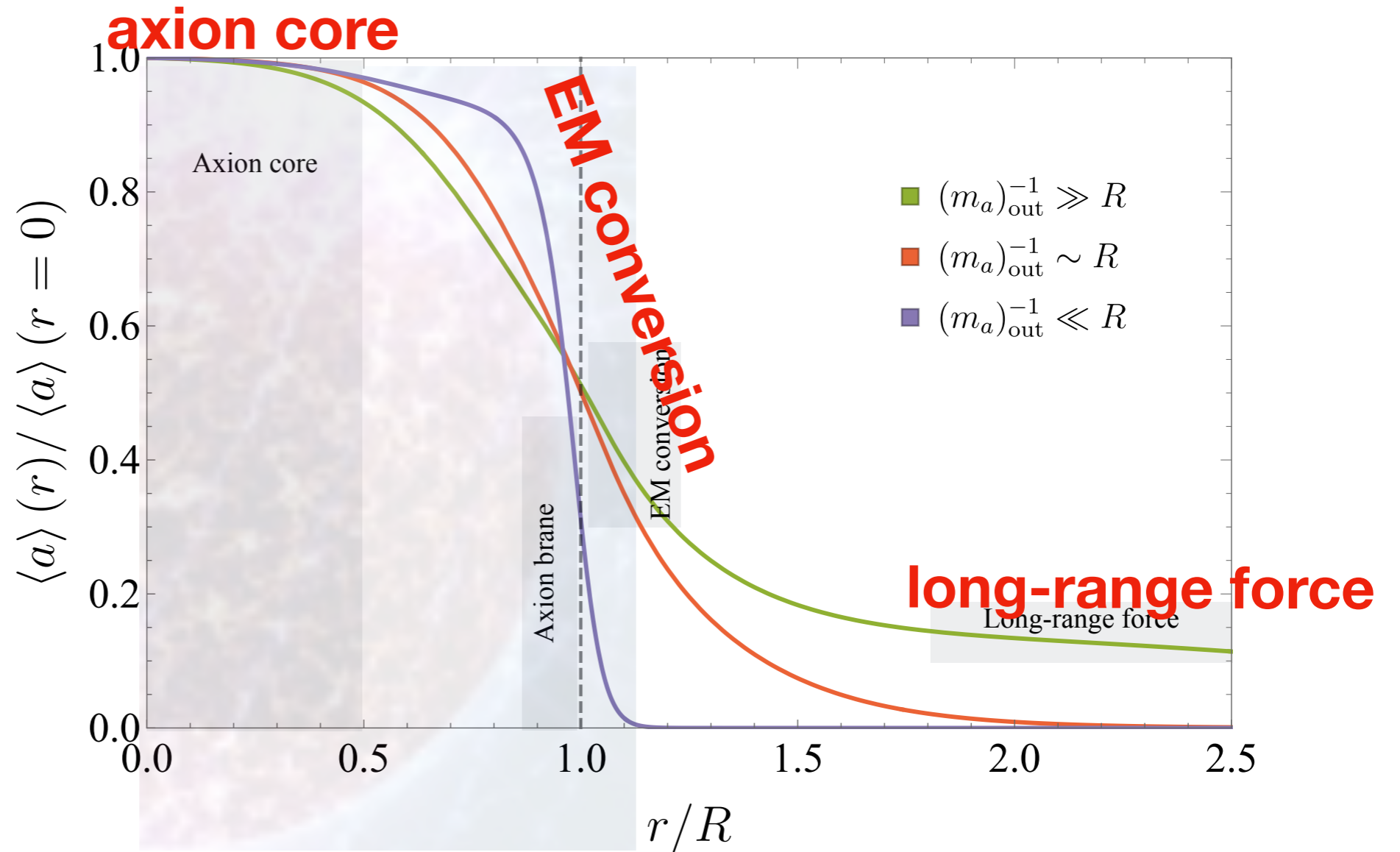
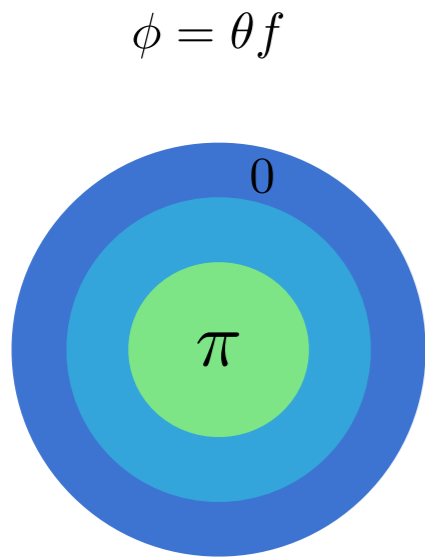
Dramatic effects appear once ϕ develops a **non-trivial profile**

Axion Profile



see Hook, Huang '17 and Balkin, Serra, Springmann, AW '20

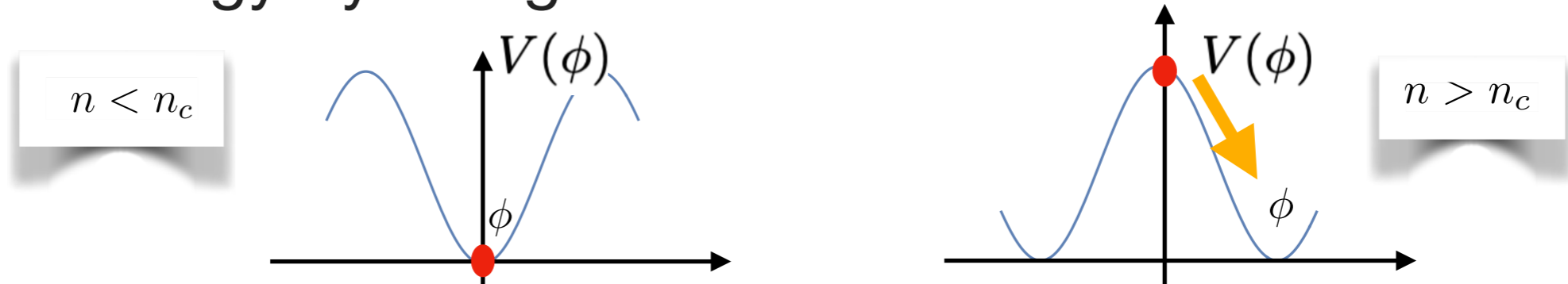
Axion Profile



see Hook, Huang '17 and Balkin, Serra, Springmann, AW '20

Why does this not affect large nucleons?

We gain energy by being in the **true vacuum** inside dense object.



Field theory potential energy includes **gradient** term!

$$E = \frac{1}{2}(\partial_t \phi)^2 + U(\phi)$$

$$U = \frac{1}{2}(\nabla \phi)^2 + V(\phi)$$

Resists change in profile

(“string does not want to be bend”)



Why does this not affect large nucleons?

Condition for non-trivial profile:

Potential gain... $m_\pi^2 f_\pi^2 \left(\epsilon - \frac{\sigma_N n_N}{m_\pi^2 f_\pi^2} \right)$

... outweighs gradient energy price $(\nabla\phi)^2 \sim f^2/r^2$

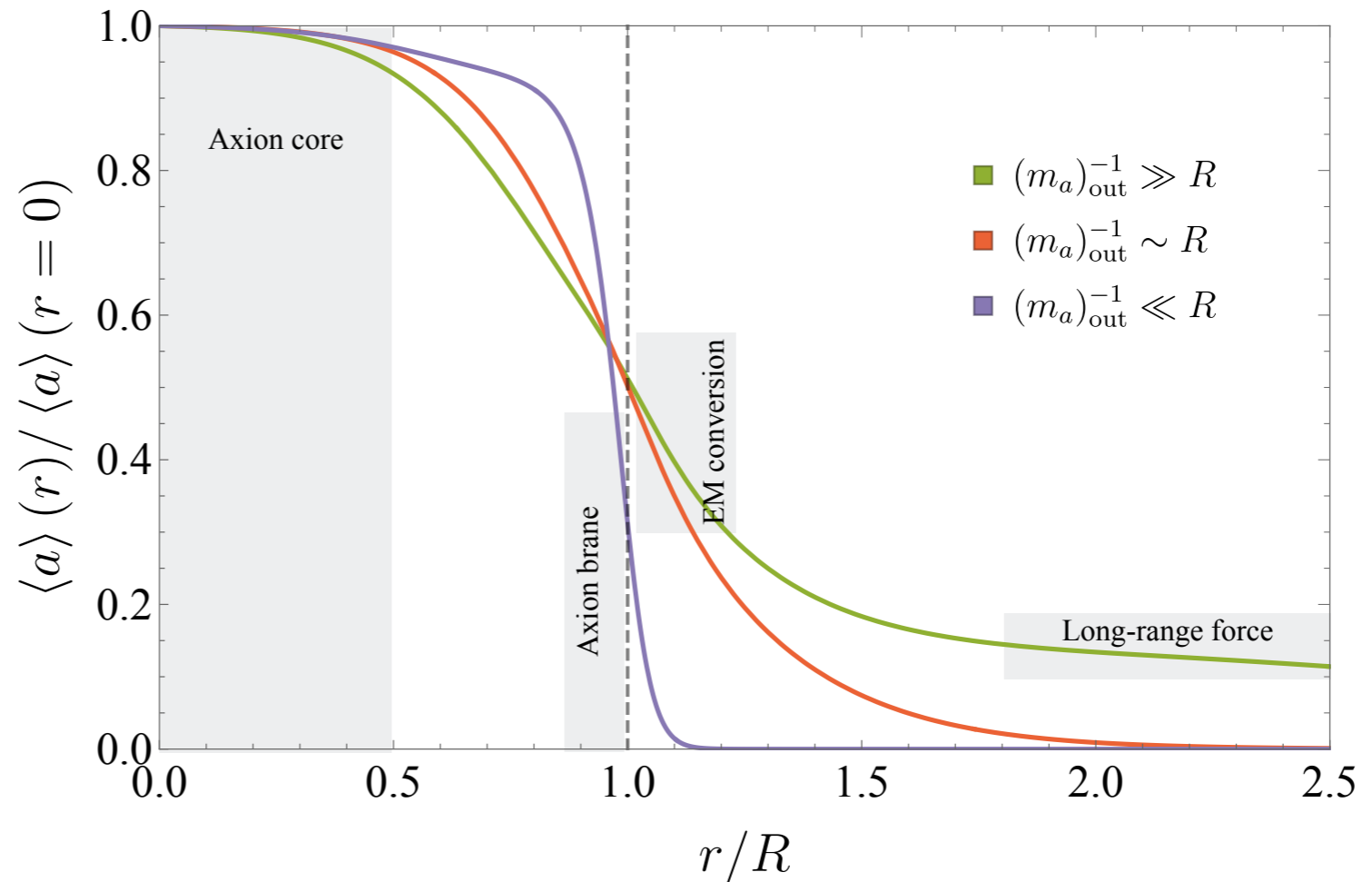
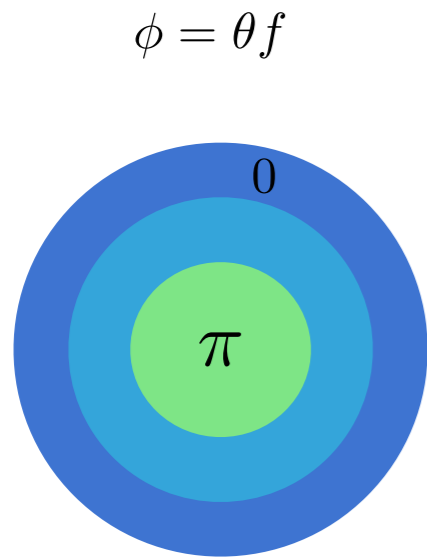
$$r_{\text{critical}} > 1/m_\phi^{\text{inside}}$$

e.g. $f \sim 10^{12} \text{ GeV}$

$$r_{\text{critical}} \sim 0.2 \text{ cm}$$

Objects must be **large** enough. No effects in particle physics experiments.

Axion Profile



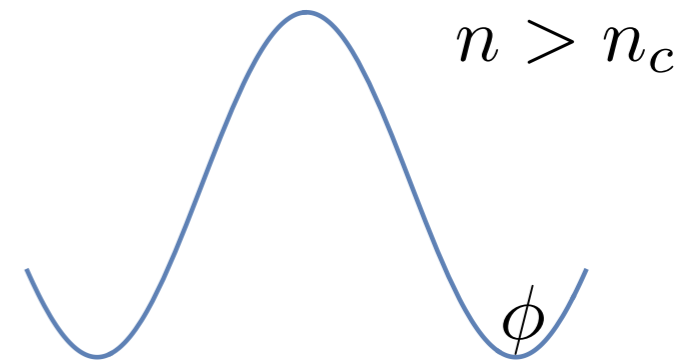
What happens to neutrons stars and white dwarfs with an axion sourced inside?

Stars with a sourced axion profile

1) Nucleon mass is **reduced** once the axion is at $\langle \phi \rangle = \pi f$

$$m_N(\phi) \equiv m_N \left[1 + \frac{\sigma_{\pi N}}{2m_N} \left(\cos \frac{\phi}{f} - 1 \right) \right]$$

$$m_N(\pi f) \simeq m_N - \sigma_{\pi N}$$

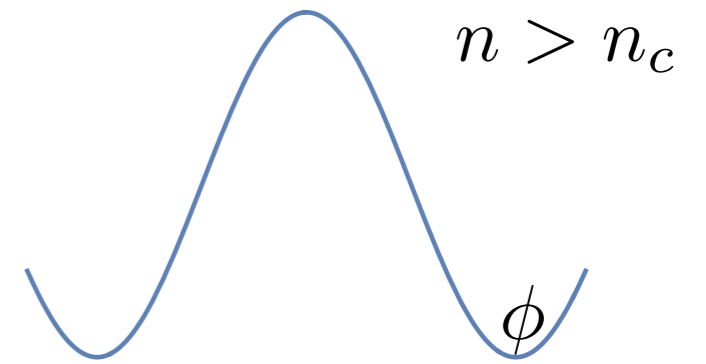


Stars with a sourced axion

1) Nucleon mass is **reduced** once the axion is at $\langle \phi \rangle = \pi f$

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2) Energy density of the potential acts as vacuum energy (like a CC)

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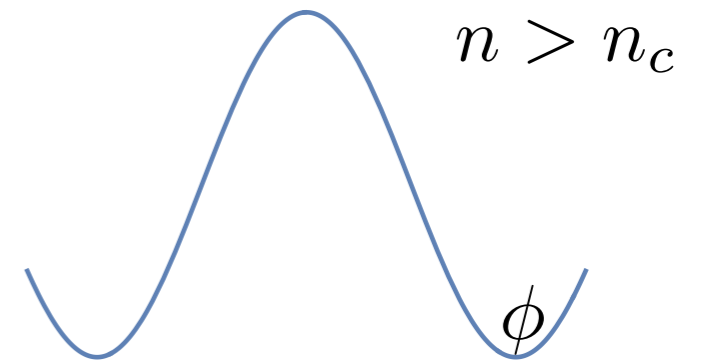
see Bellazzini et. al. '15 and Csaki et. al. '18

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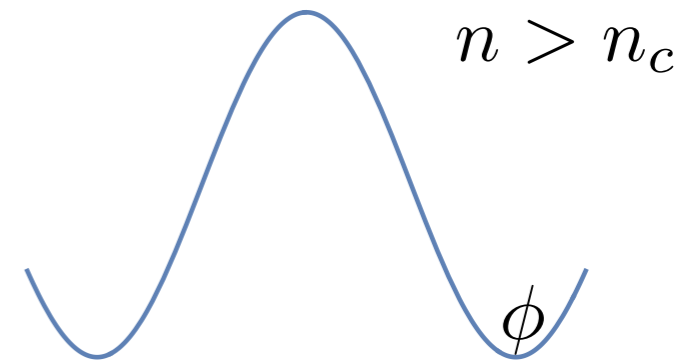
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dominant effect here

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Free Fermi Gas of Neutrons with an Axion

Minimizing the action $\frac{\delta S}{\delta g_{\mu\nu}} = \frac{\delta S}{\delta \phi} = 0$

$$\mathcal{L}_{N\phi} = \sqrt{-g} \left[\bar{N} (i g^{\mu\nu} \gamma_\mu D_\nu - m_N^*(\phi)) N + \frac{1}{2} g^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi) - V(\phi) \right],$$

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$$\begin{aligned} \phi'' \left[1 - \frac{2GM}{r} \right] + \frac{2}{r} \phi' \left[1 - \frac{GM}{r} - 2\pi G r^2 (\varepsilon - p) \right] &= \frac{\partial V}{\partial \phi} + \rho \frac{\partial m_N^*(\phi)}{\partial \phi} \equiv U(\phi, \rho), \\ p' &= -\frac{GM\varepsilon}{r^2} \left[1 + \frac{p}{\varepsilon} \right] \left[1 - \frac{2GM}{r} \right]^{-1} \left[1 + \frac{4\pi r^3}{M} \left(p + \frac{(\phi')^2}{2} \left\{ 1 - \frac{2GM}{r} \right\} \right) \right] - \phi' U(\phi, \rho), \\ M' &= 4\pi r^2 \left[\varepsilon + \frac{1}{2} \left(1 - \frac{2GM}{r} \right) (\phi')^2 \right]. \end{aligned}$$

coupled system

Einstein equations and axion EOM

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can be solved numerically, very **involved**

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coupled system

Einstein equations and axion EOM

can be solved numerically, very **involved**

Luckily, there is often a simplifying limit!

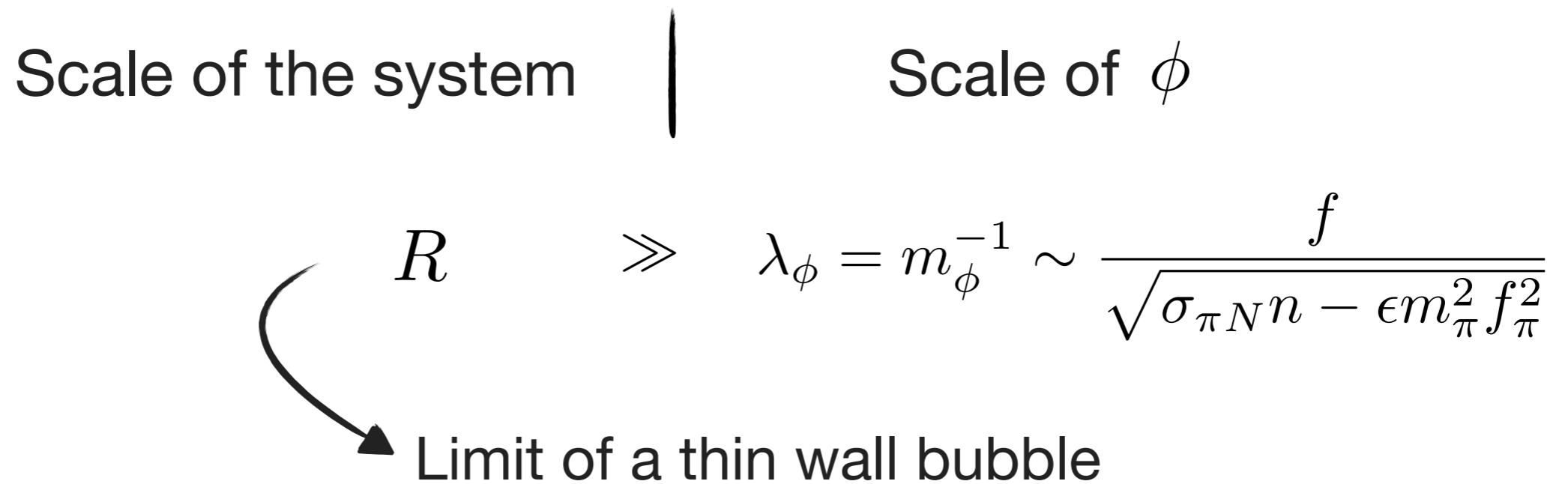
Zero Gradient Limit

Scale hierarchy

$$\begin{array}{ccc} \text{Scale of the system} & | & \text{Scale of } \phi \\ R & \gg & \lambda_\phi = m_\phi^{-1} \sim \frac{f}{\sqrt{\sigma_{\pi N n} - \epsilon m_\pi^2 f_\pi^2}} \end{array}$$

Zero Gradient Limit

Scale hierarchy



Gradient energy becomes negligible: $\phi'(r) = 0$

The system effectively decouples: Solve for EOS



Solve pressure gravity equations

Equation of state

$\frac{\partial \varepsilon}{\partial \phi} = 0$ minimising the potential energy

$$\varepsilon(n, \phi) = \varepsilon_N(n, \phi) + V(\phi)$$

$$p(n, \phi) = p_N(n, \phi) - V(\phi)$$

$$\frac{\partial V}{\partial \phi} + n \frac{\partial m_N(\phi)}{\partial \phi} = 0$$

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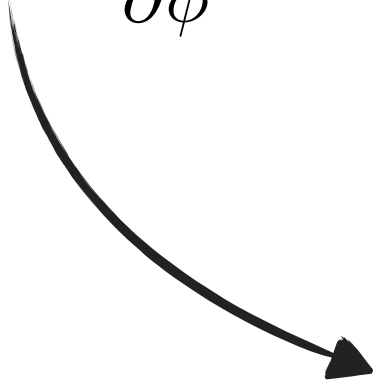
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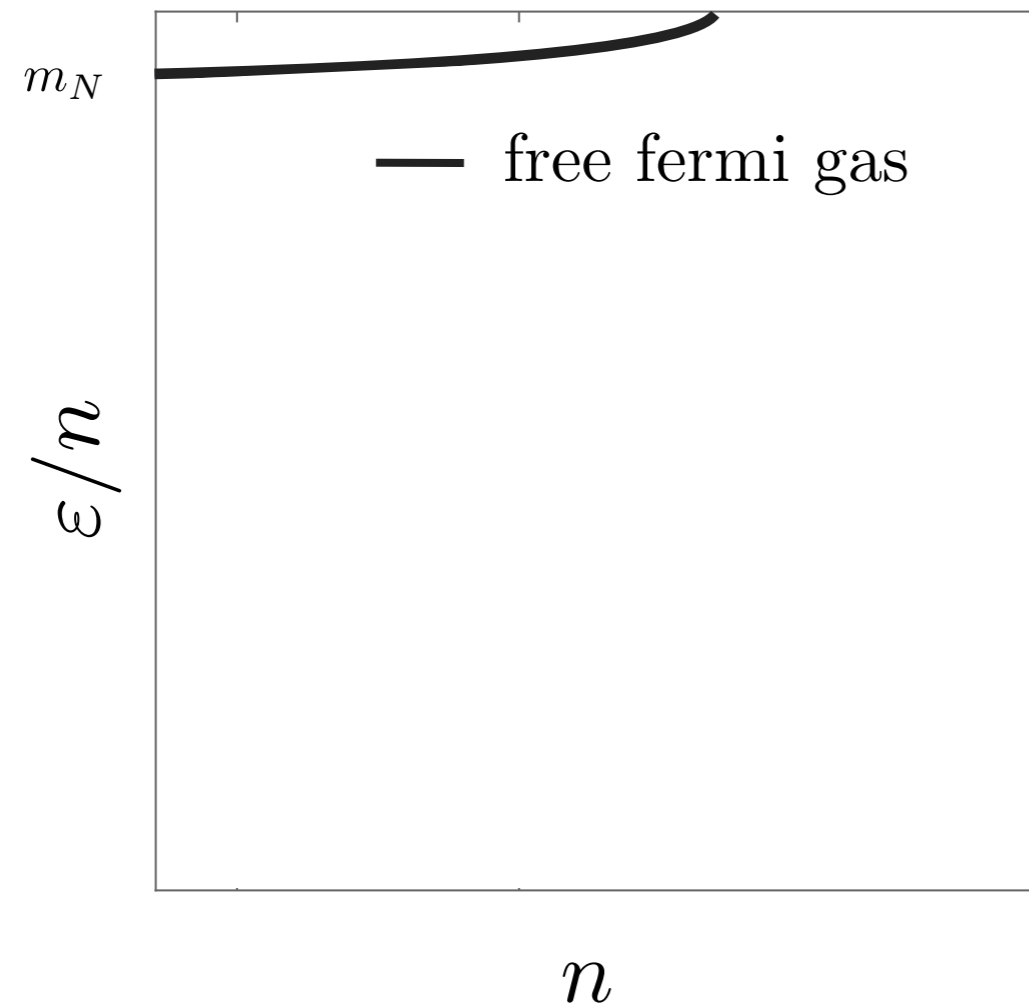
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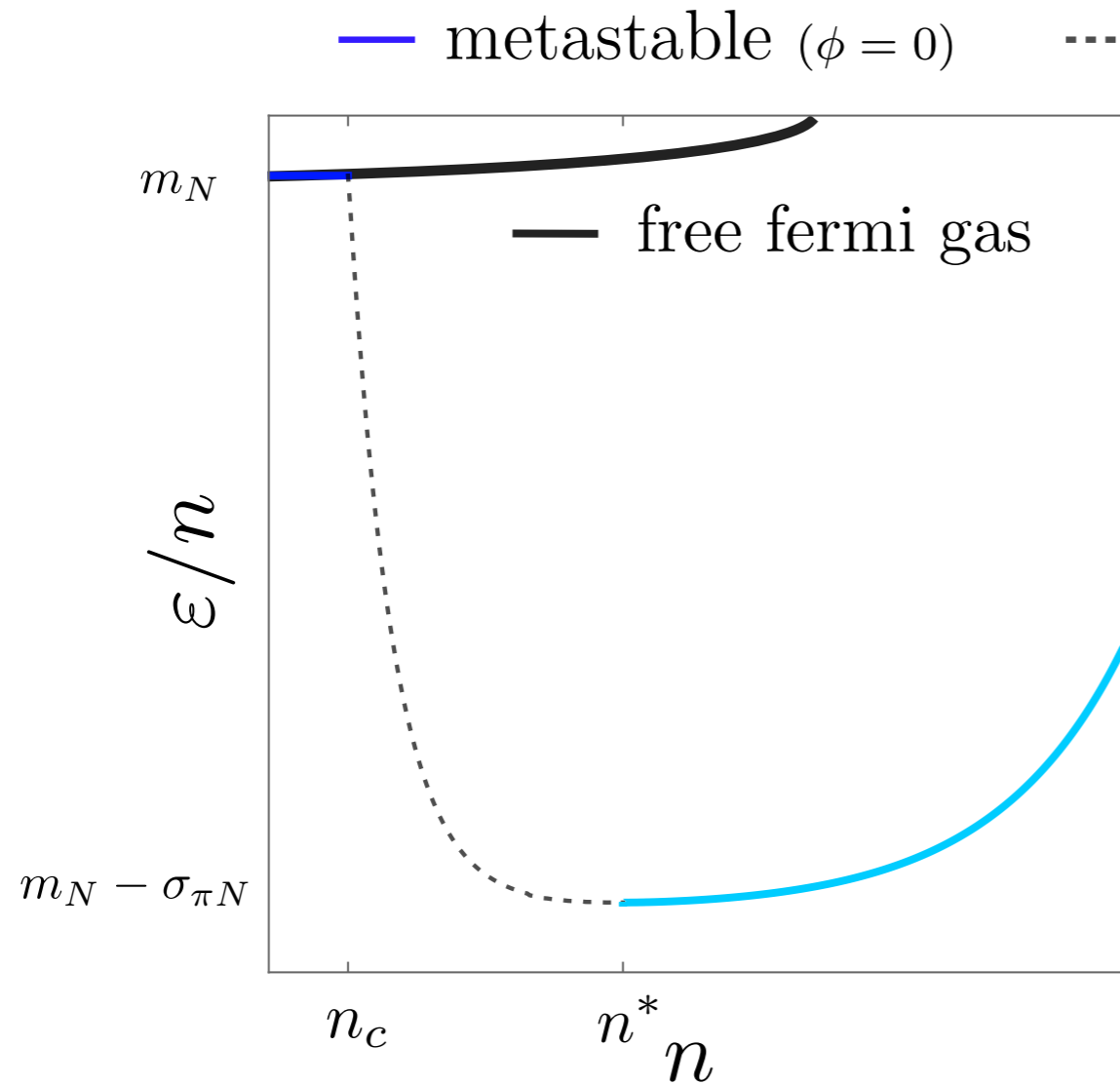

$$\varepsilon(n), p(n)$$

equation of state

Energy per particle



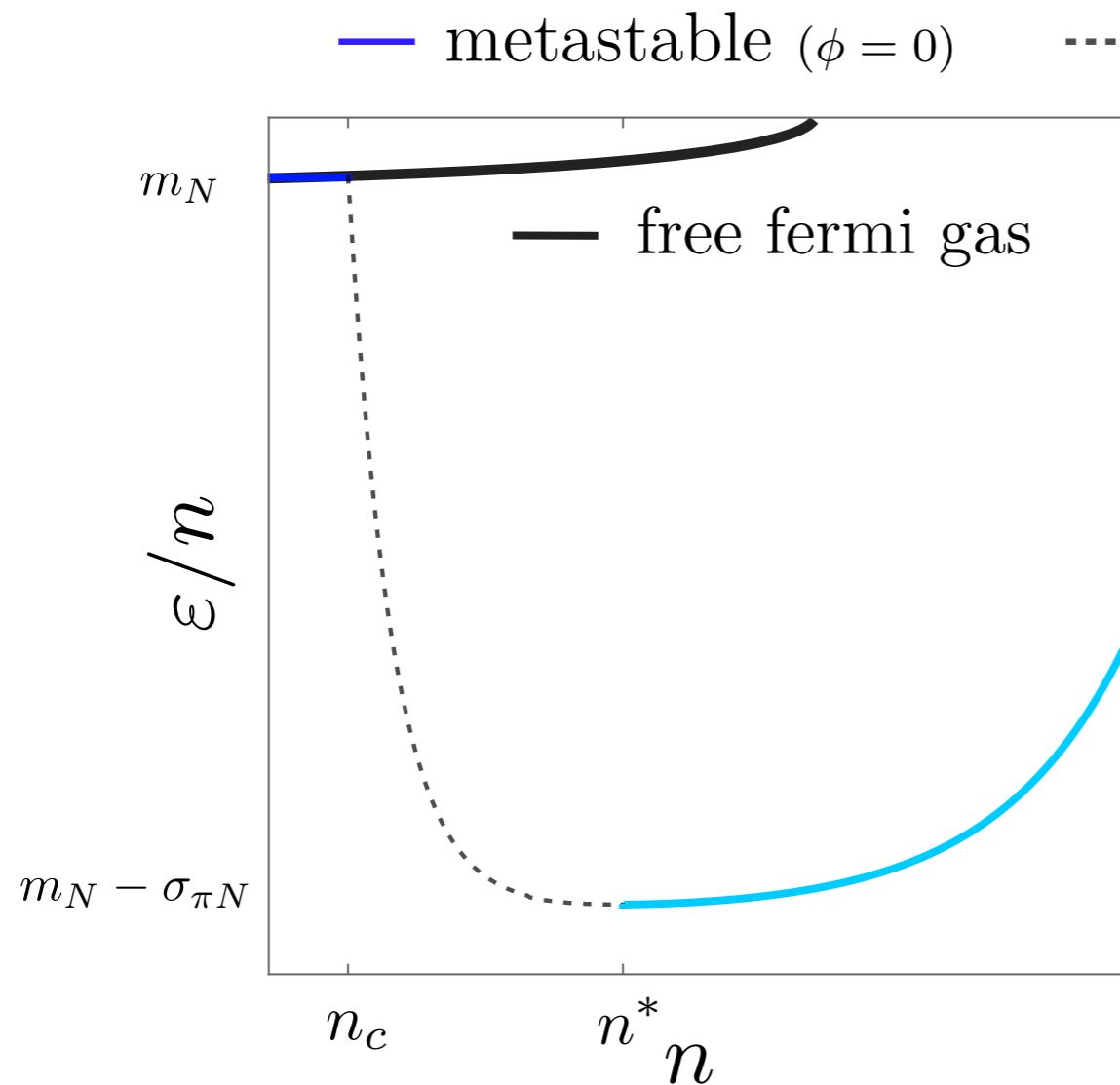
Energy per particle



Energy per particle is related to pressure

$$p = n^2 \frac{\partial(\epsilon/n)}{\partial n} = p_N - V$$

Energy per particle



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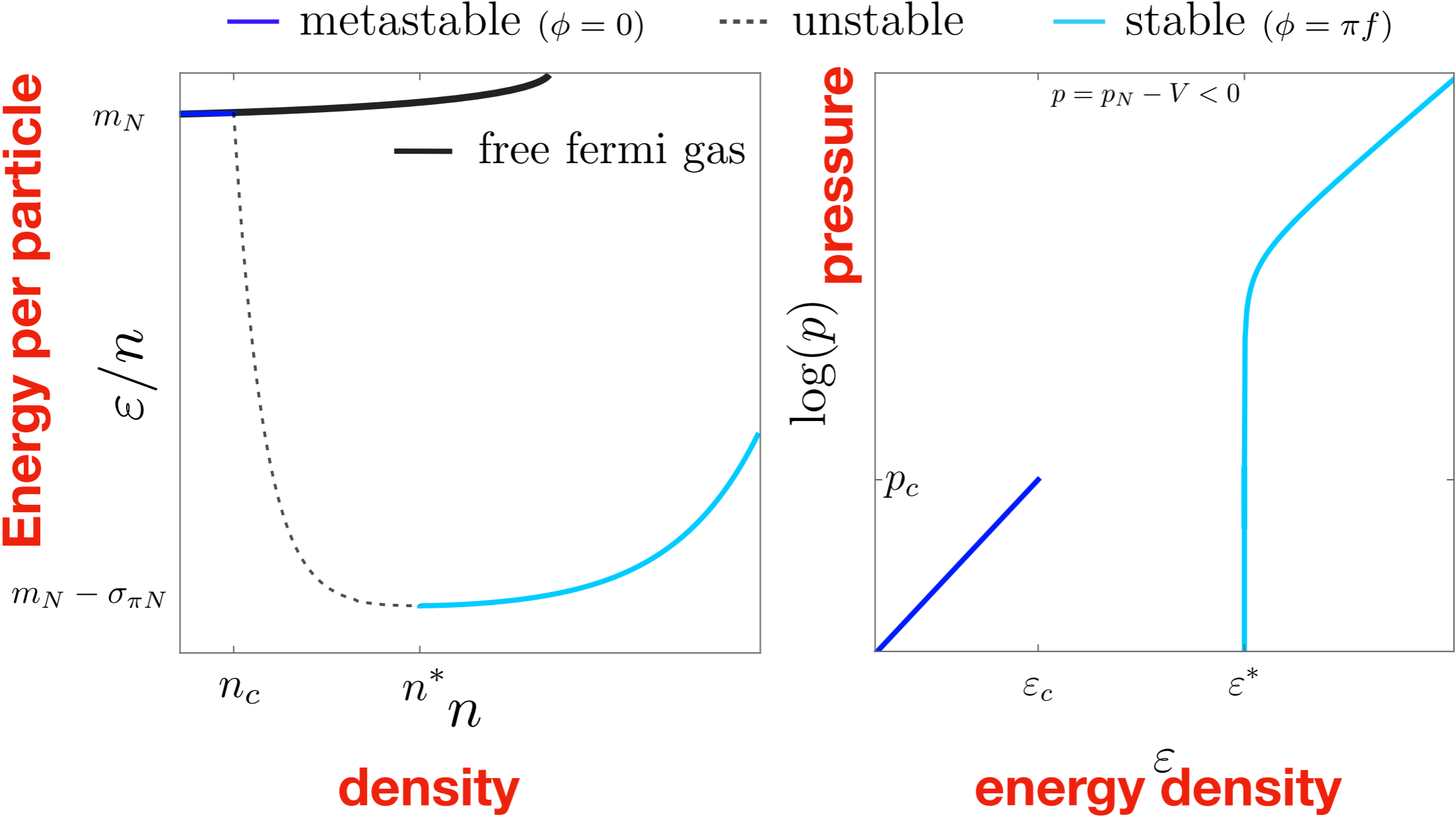
Negative pressure for

$$n_c < n < n^*$$

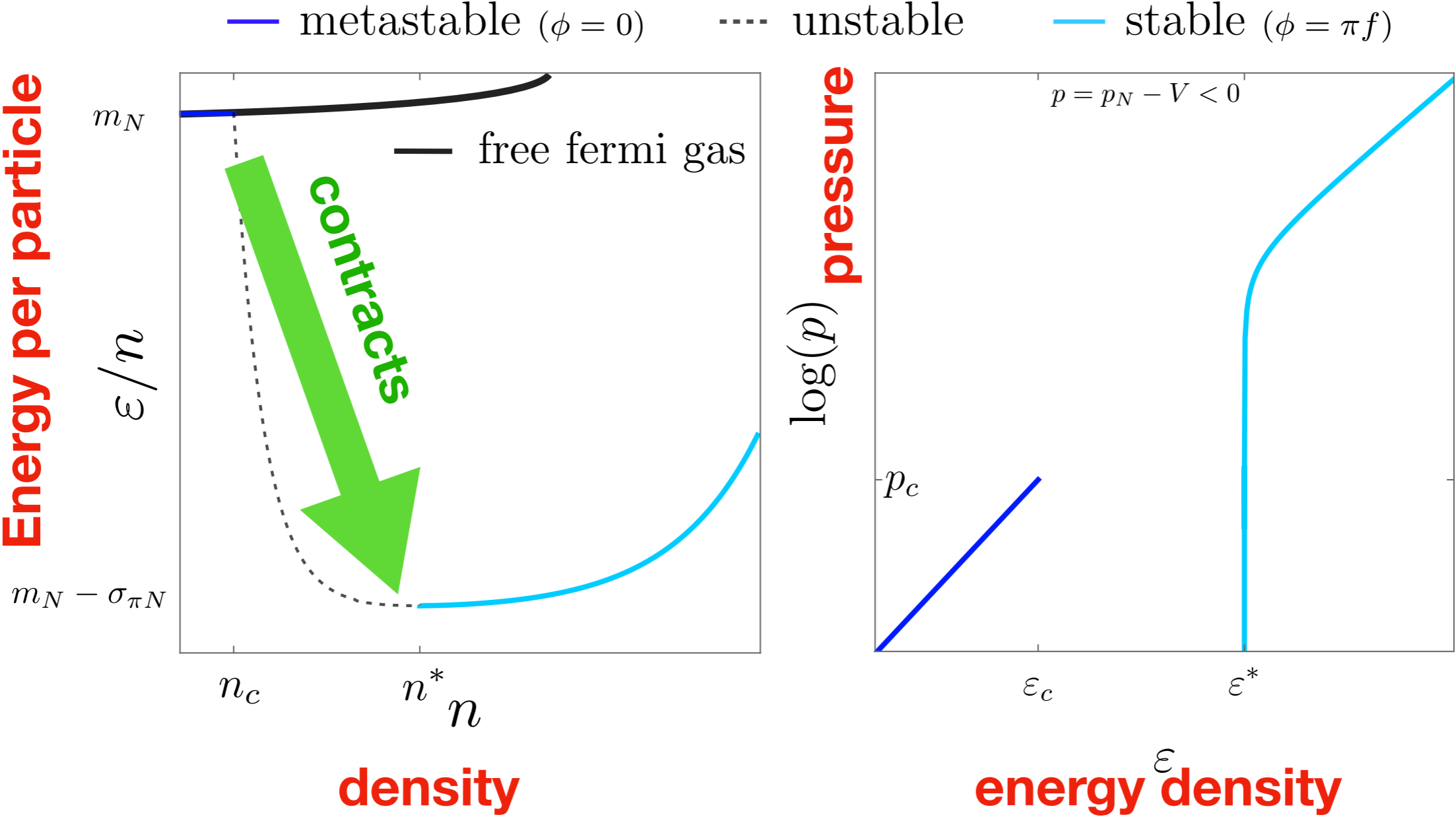
Defines n^* as

$$p(n^*) = p_N(n^*) - V = 0$$

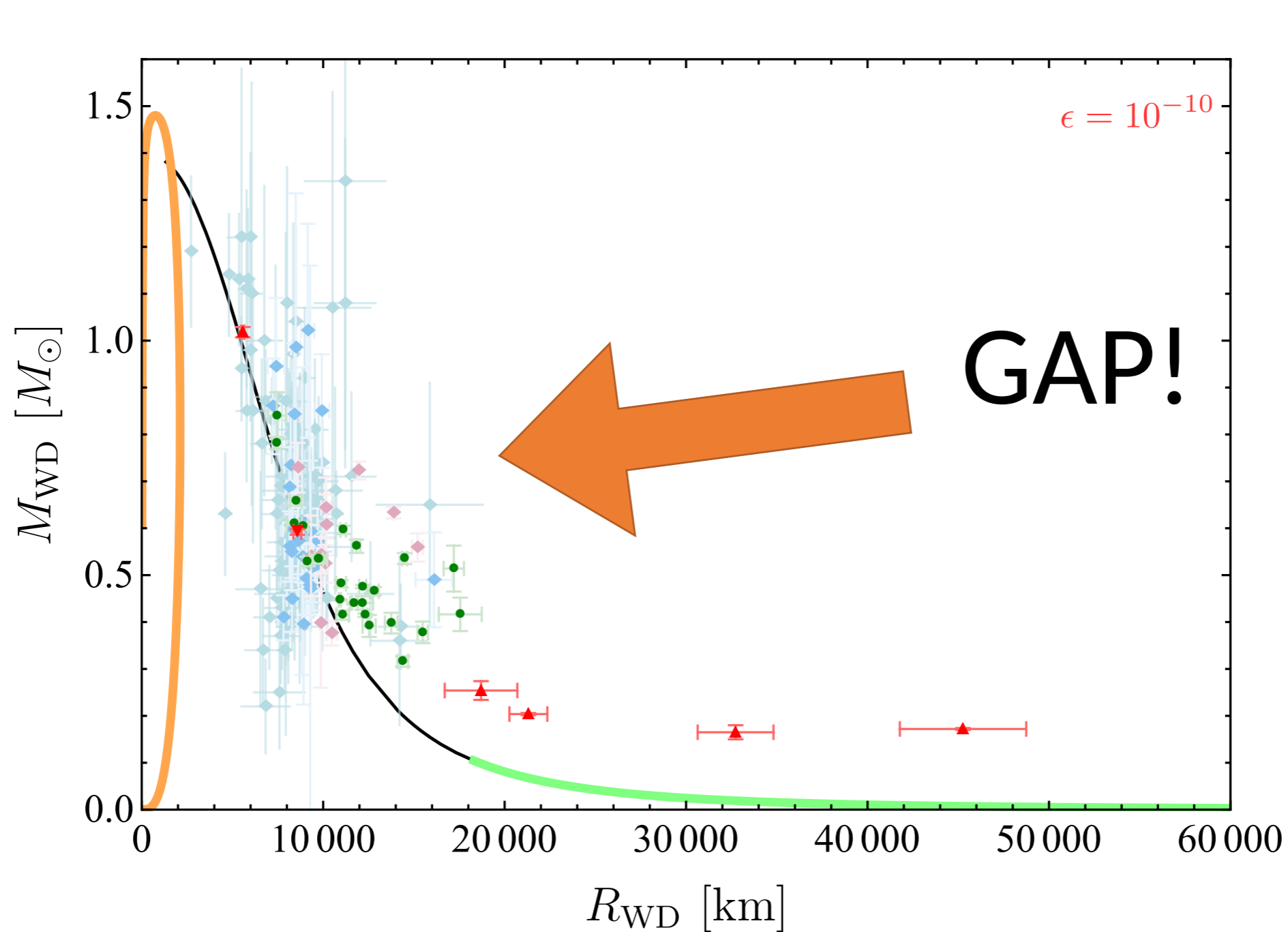
Energy per particle vs pressure inside



Energy per particle vs pressure inside

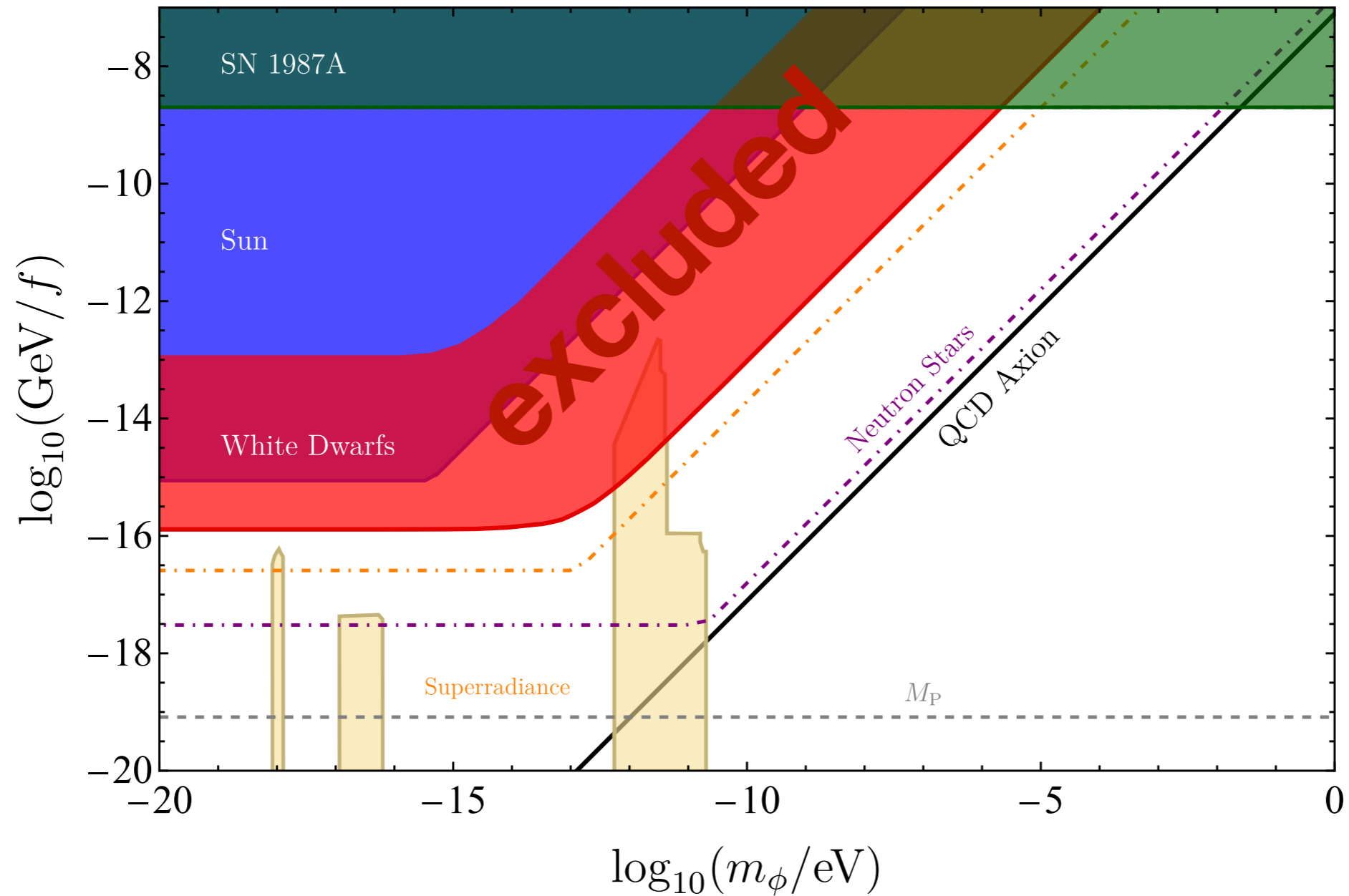


White Dwarfs with a light Axion

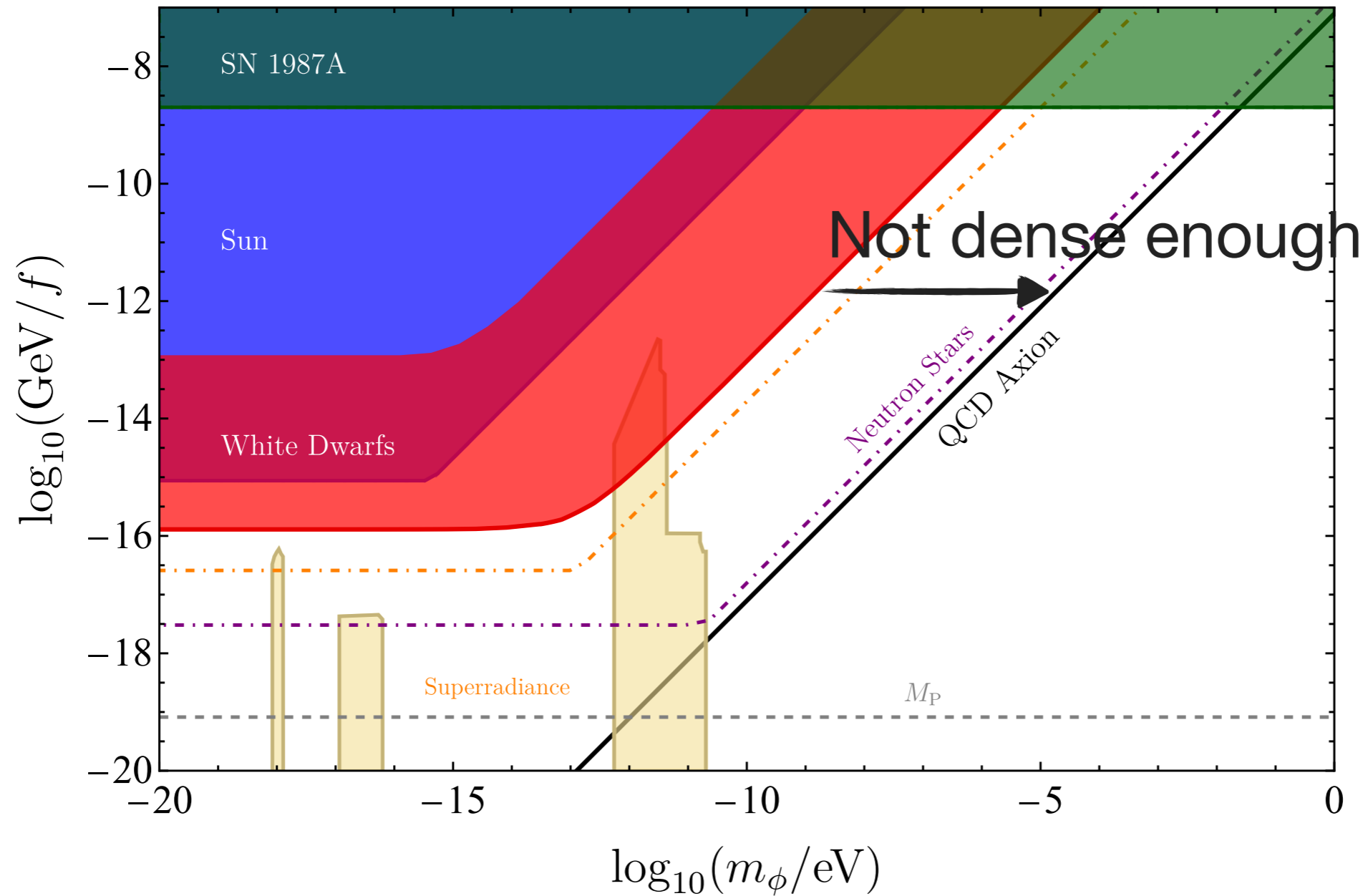


$$n_c = \frac{\epsilon m_{\pi}^2 f_{\pi}^2}{4\sigma_{\pi N}}$$

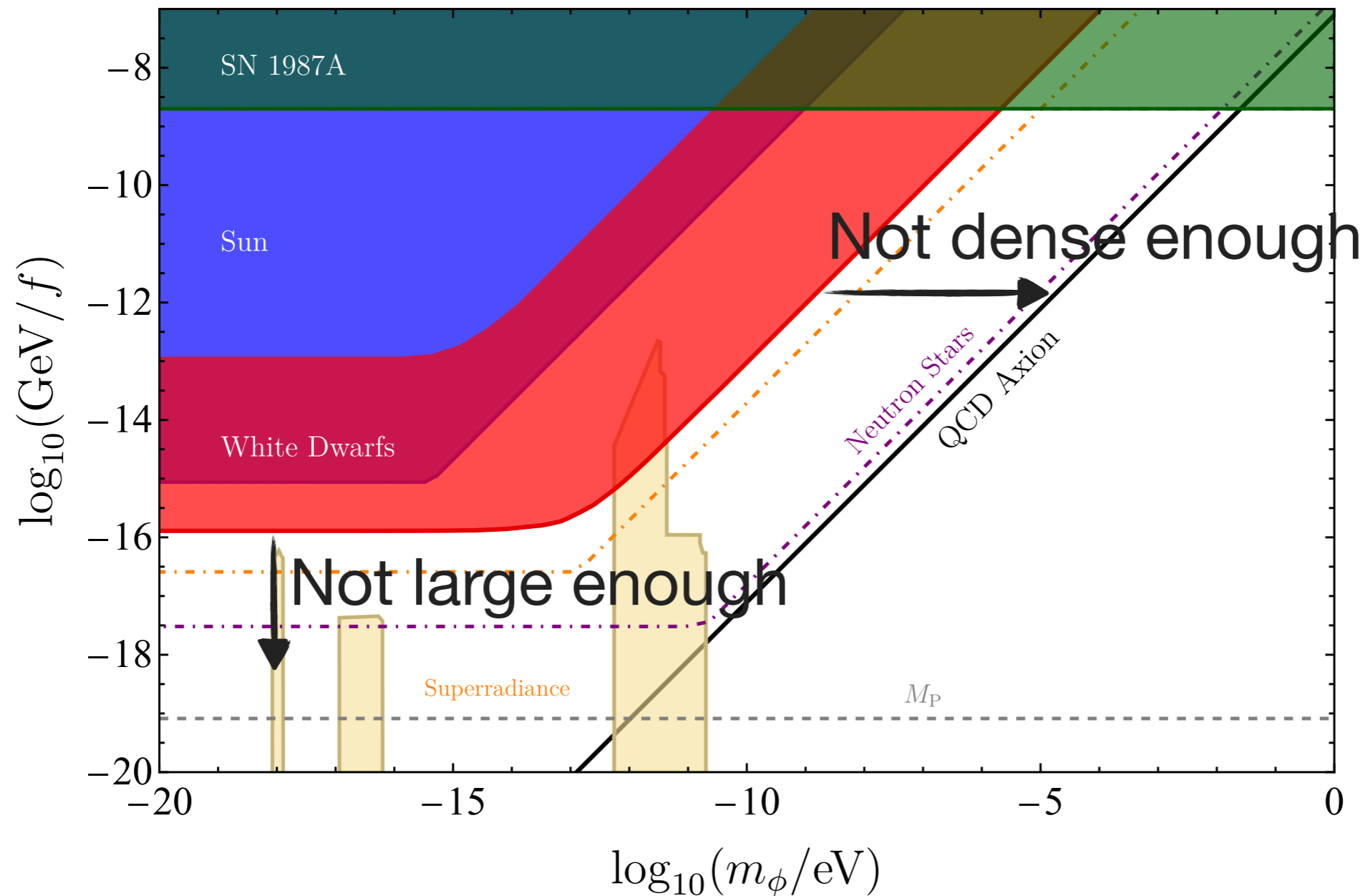
Axion Parameter Space



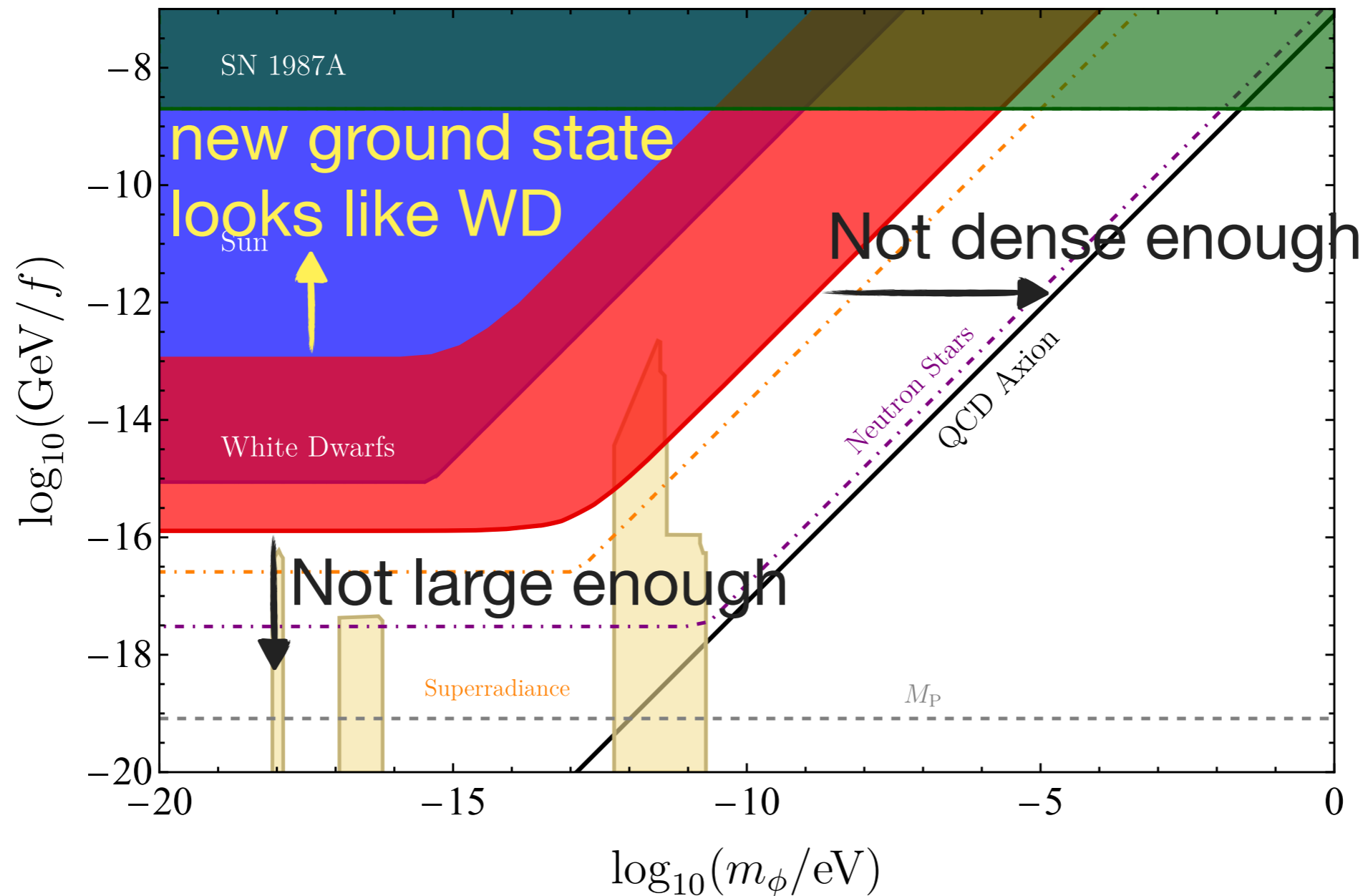
Axion Parameter Space



Axion Parameter Space



Axion Parameter Space



Note: no need to assume that axions constitute dark matter.

Heavy Neutron Stars from light Scalars

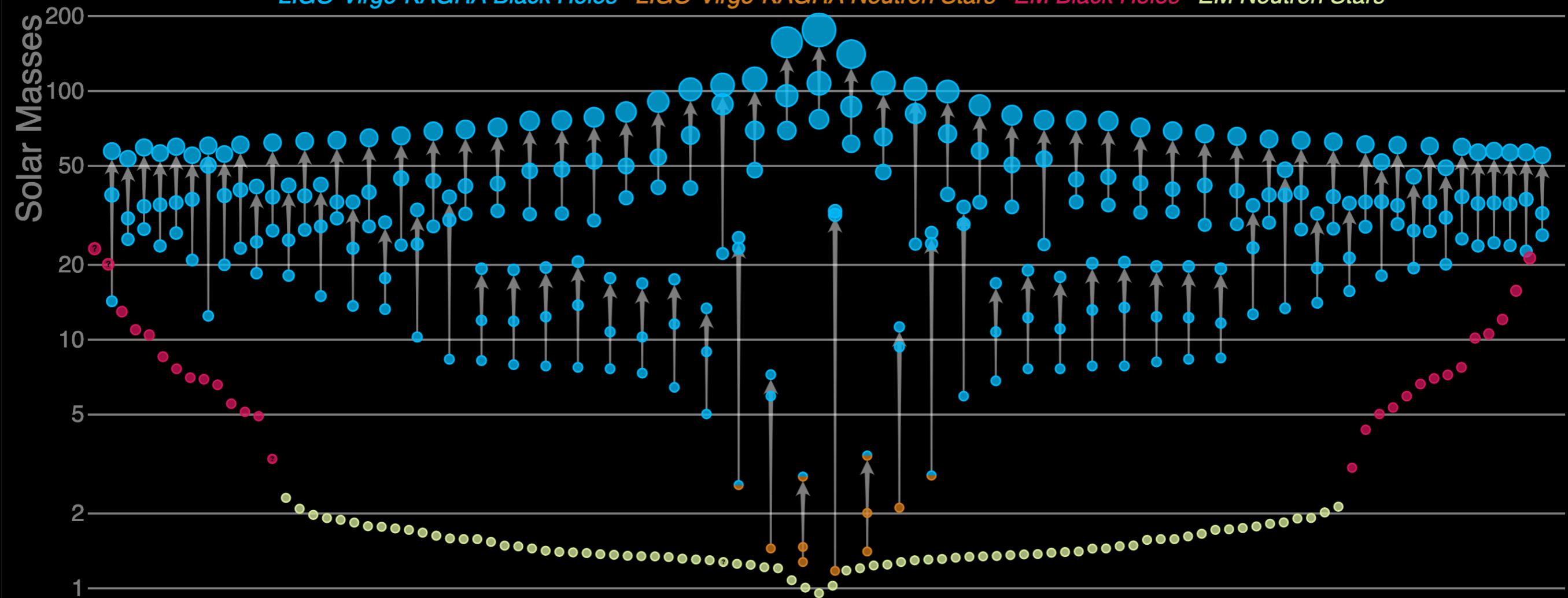
... or Fat Zombies in the Stellar Graveyard



Motivation:

Masses in the Stellar Graveyard

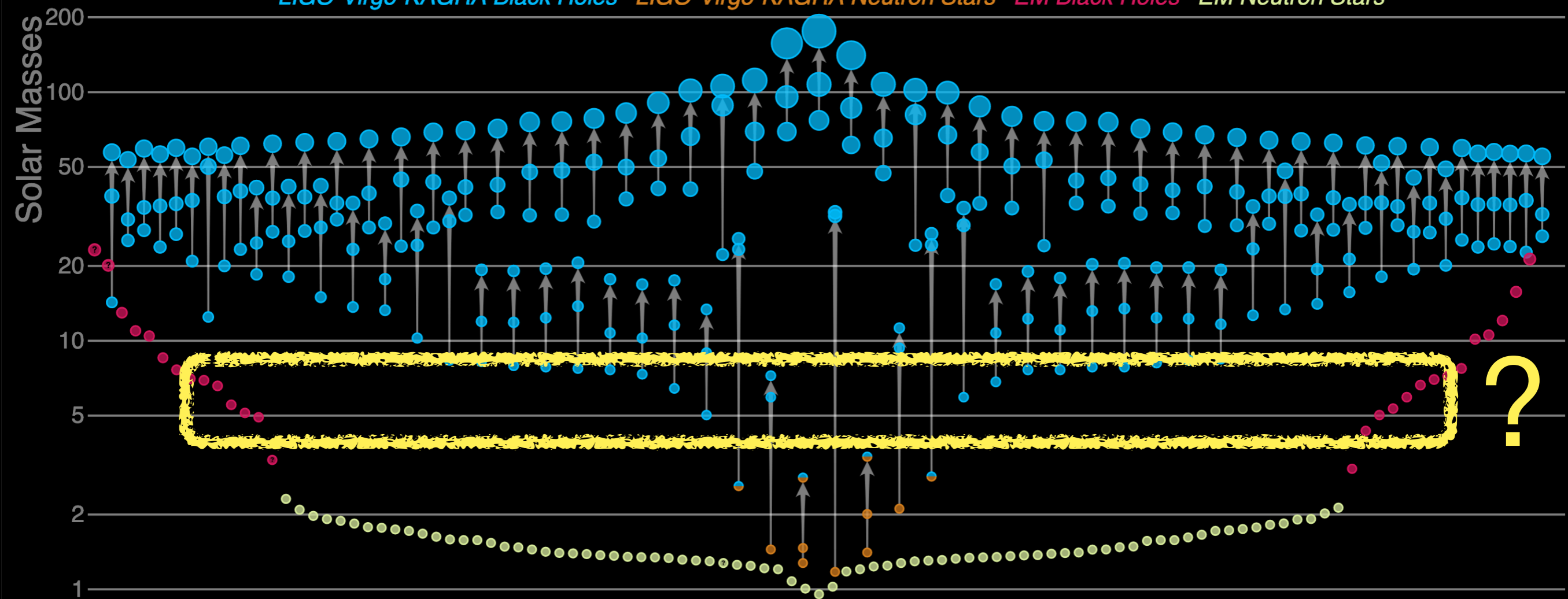
LIGO-Virgo-KAGRA Black Holes *LIGO-Virgo-KAGRA Neutron Stars* *EM Black Holes* *EM Neutron Stars*



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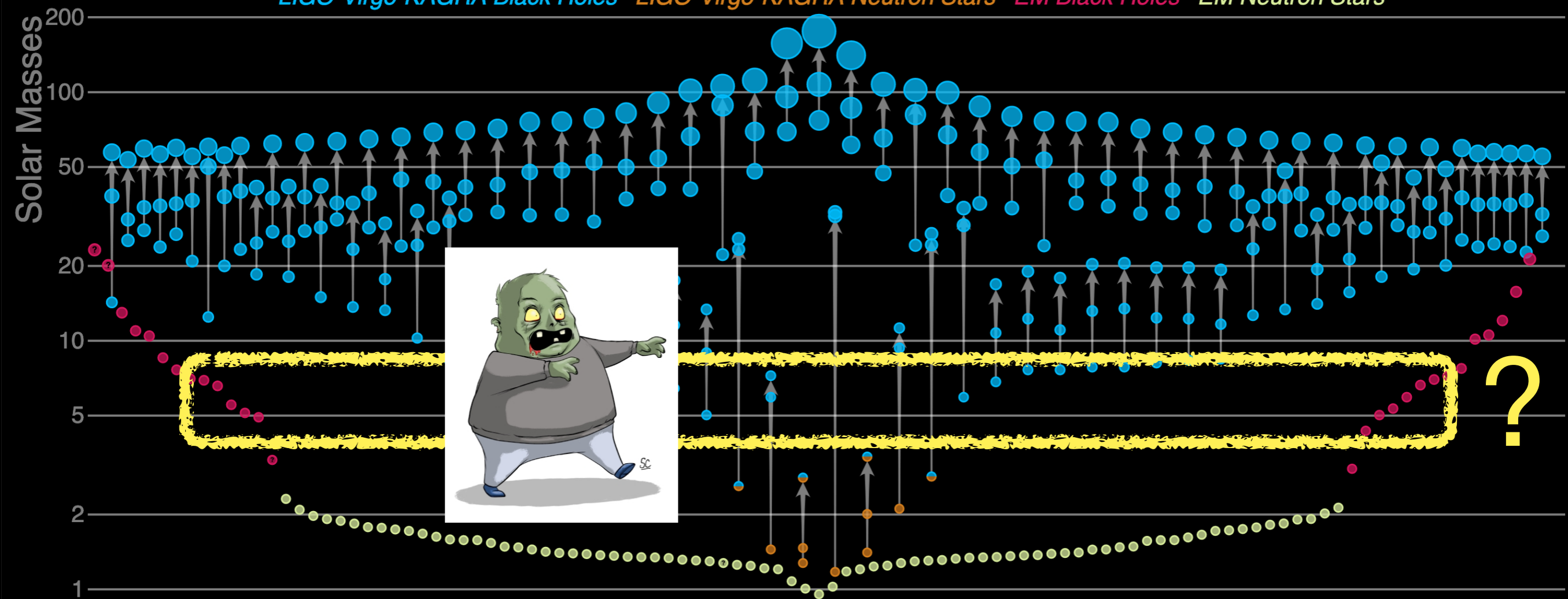
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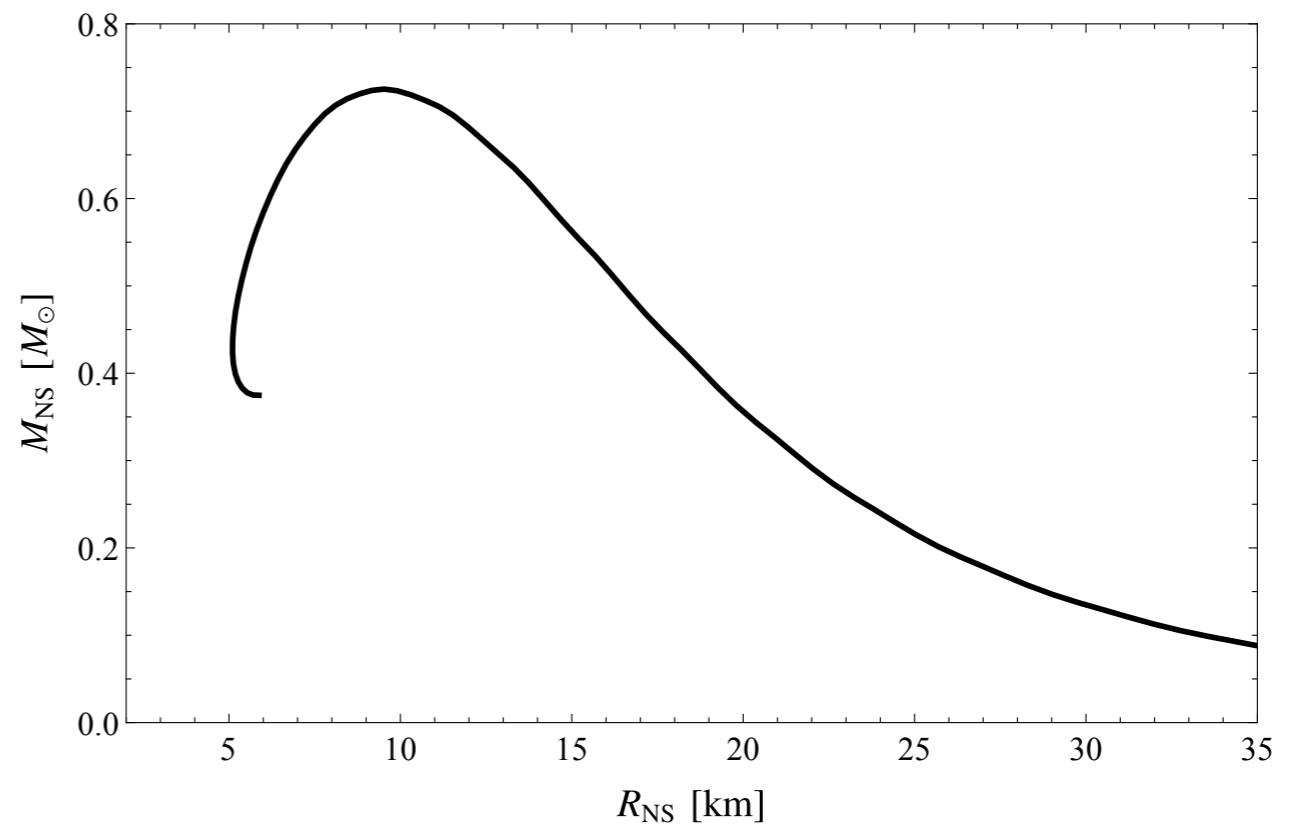


Free fermi-gas toy model

Consider non-interacting Fermi gas of neutrons

$$\Rightarrow M_{\max} \sim 0.7M_{\odot}$$

$$\Rightarrow R_{\max} \simeq 10 \text{ km}$$

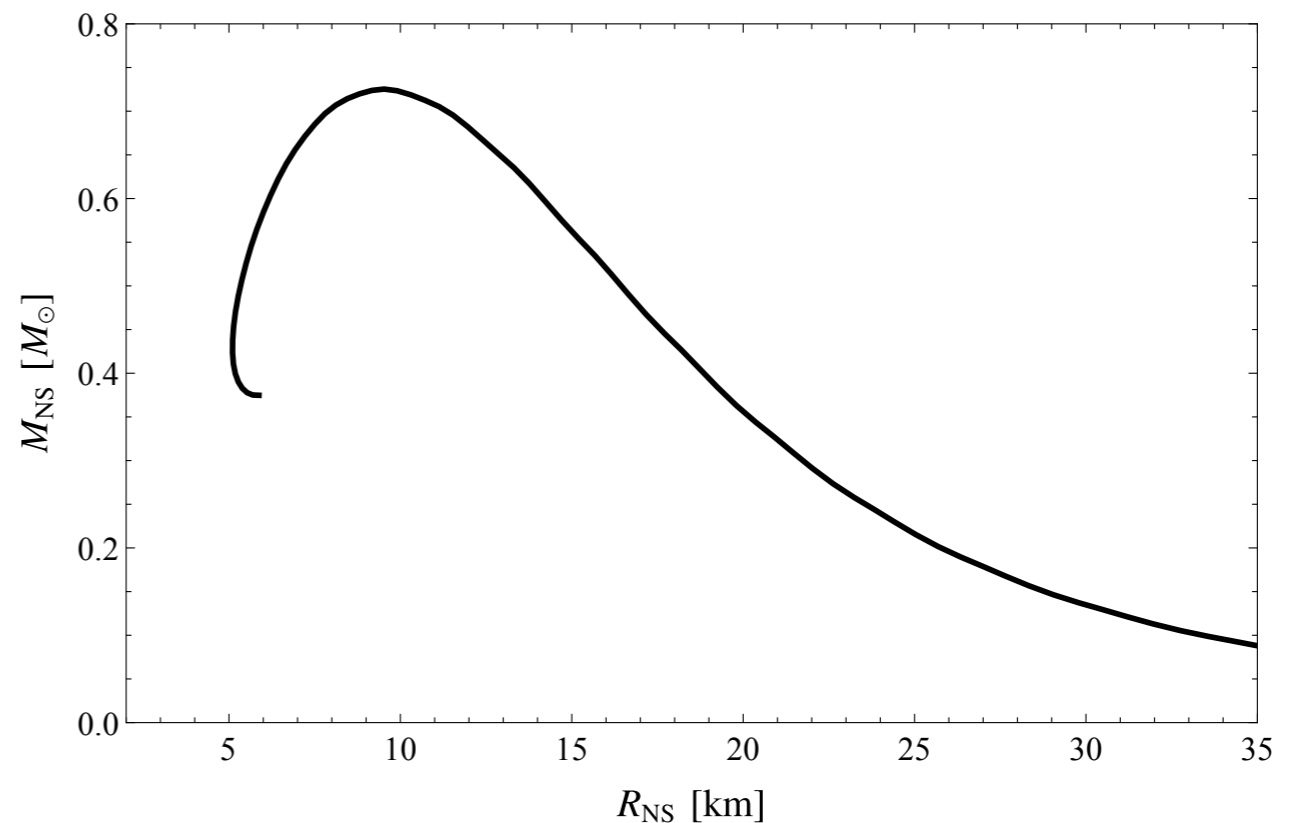


Free fermi-gas toy model

Consider non-interacting Fermi gas of neutrons

$$\Rightarrow M_{\max} \sim 0.7 \left(\frac{m_N}{m} \right)^2 M_{\odot}$$

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Free fermi-gas toy model

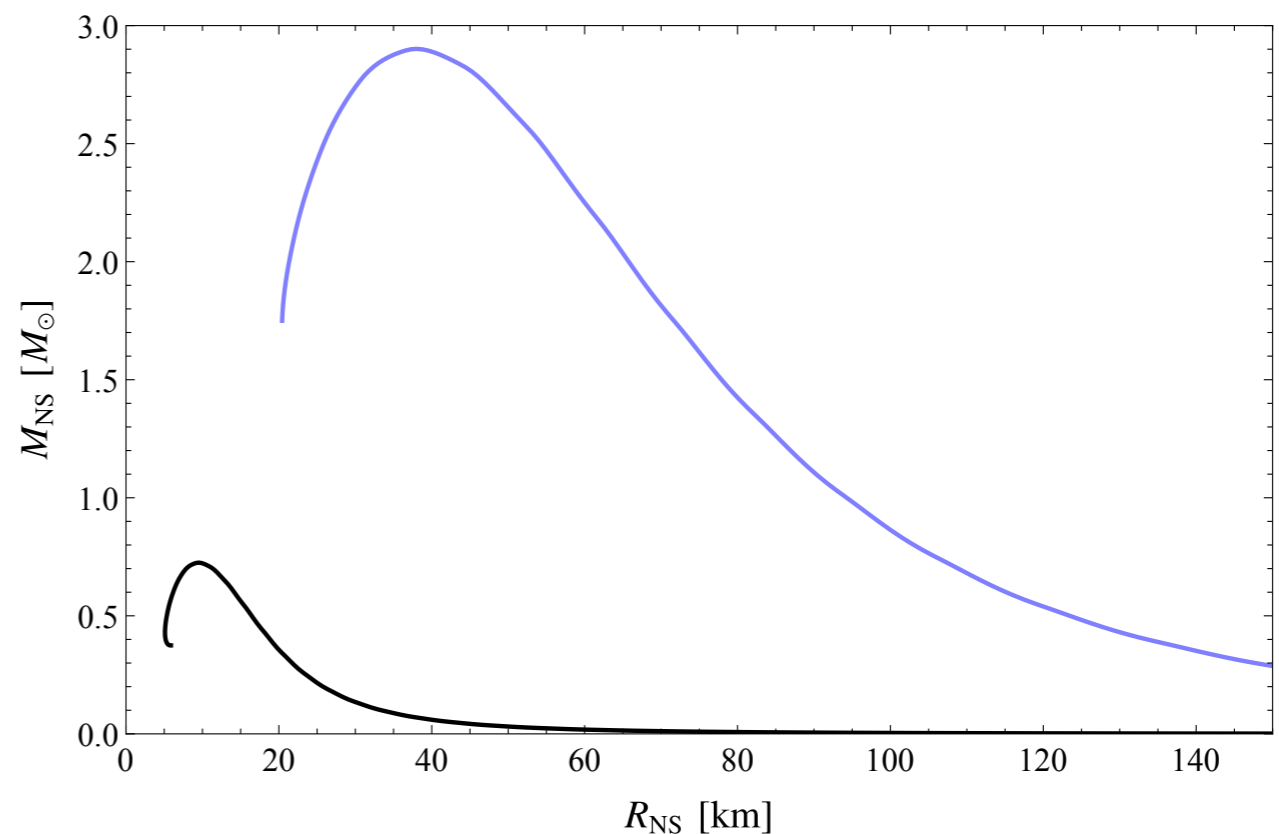
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For lighter neutrons

$$m \sim m_N/3 \rightarrow \mathcal{O}(10)$$



Recalling earlier neutron star estimates, we find same scaling:

$$R_{\text{NS}} \sim \frac{M_{\text{Planck}}}{m_N^2} \quad M_{\text{NS}} \sim \frac{M_{\text{Planck}}^3}{m_N^2}$$

Free fermi-gas toy model

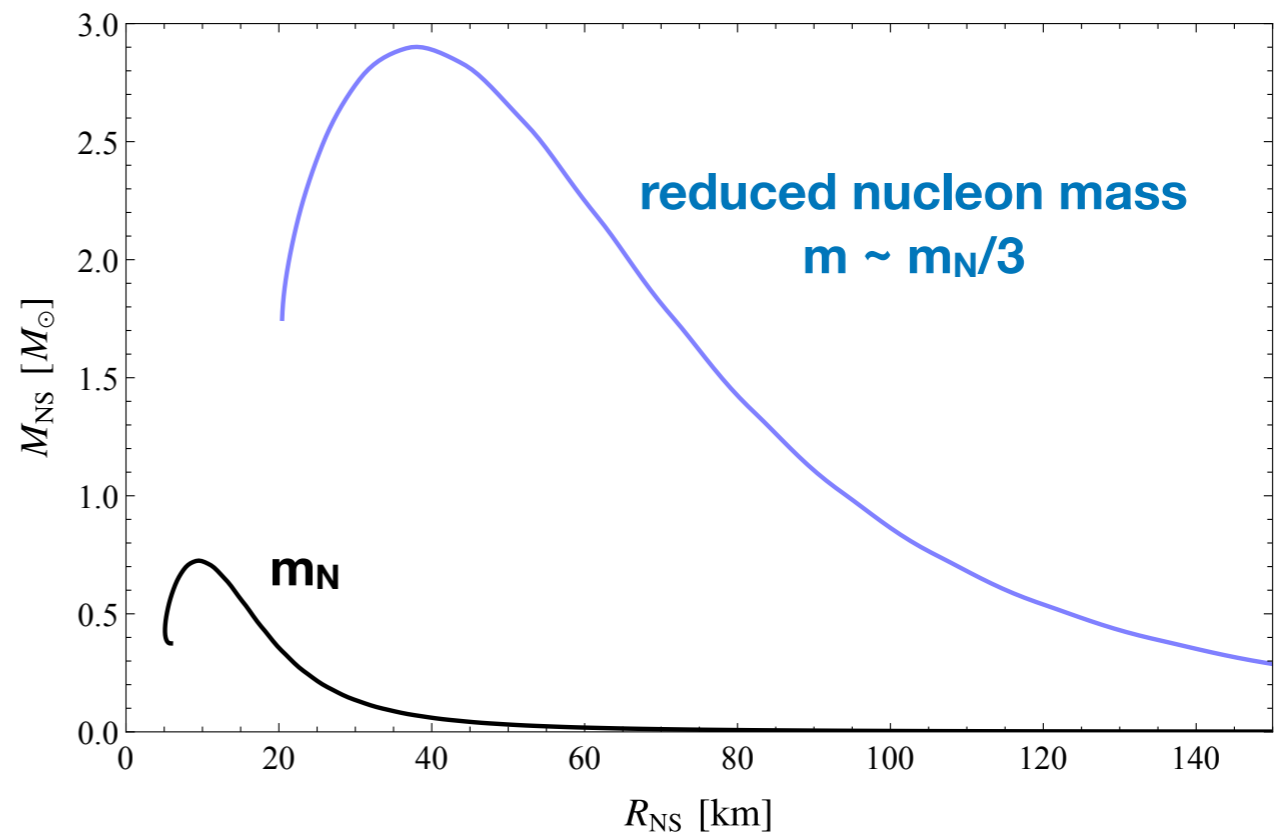
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Also obvious from our neutron star estimates:

$$R_{\text{NS}} \sim \frac{M_{\text{Planck}}}{m_N^2} \quad M_{\text{NS}} \sim \frac{M_{\text{Planck}}^3}{m_N^2}$$

Light scalars coupled to nucleons

Scalar potential $V(\phi) = -\Lambda^4 (\cos(\phi/f) - 1)$

Matter coupling $\mathcal{O}_{\phi N} = \frac{g m_N}{2} \bar{N} N \cos\left(\frac{\phi}{f}\right)$ 

Effective nucl. mass $m_N^* = \begin{cases} m_N & \phi = 0 \\ m_N(1 - g) & \phi = \pi \end{cases} \quad 1 > g > 0$

What kind of EOS?

1) Mass reduction $m_N^* < m_N$ **stiffens** the EOS $\varepsilon = \text{const.} = m_N^* \rho$
 $\Rightarrow M_{\text{max}} \sim 0.7 \left(\frac{m_N}{m}\right)^2 M_\odot$


2) Vacuum energy $V(\pi f) = 2\Lambda^4$ **softens** the EOS

additional energy density gravitates

see Bellazzini et. al. '15 and Csaki et. al. '18

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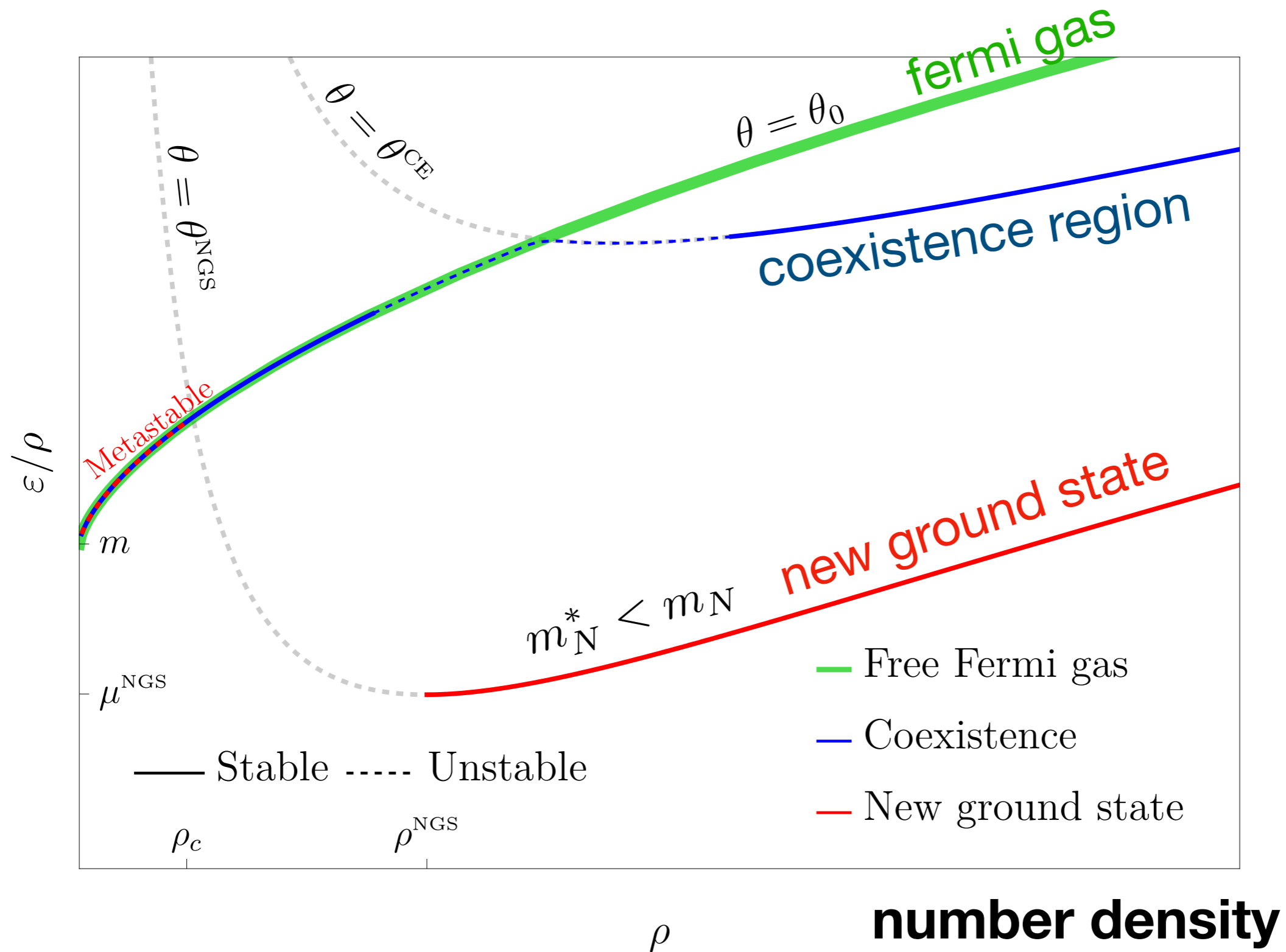
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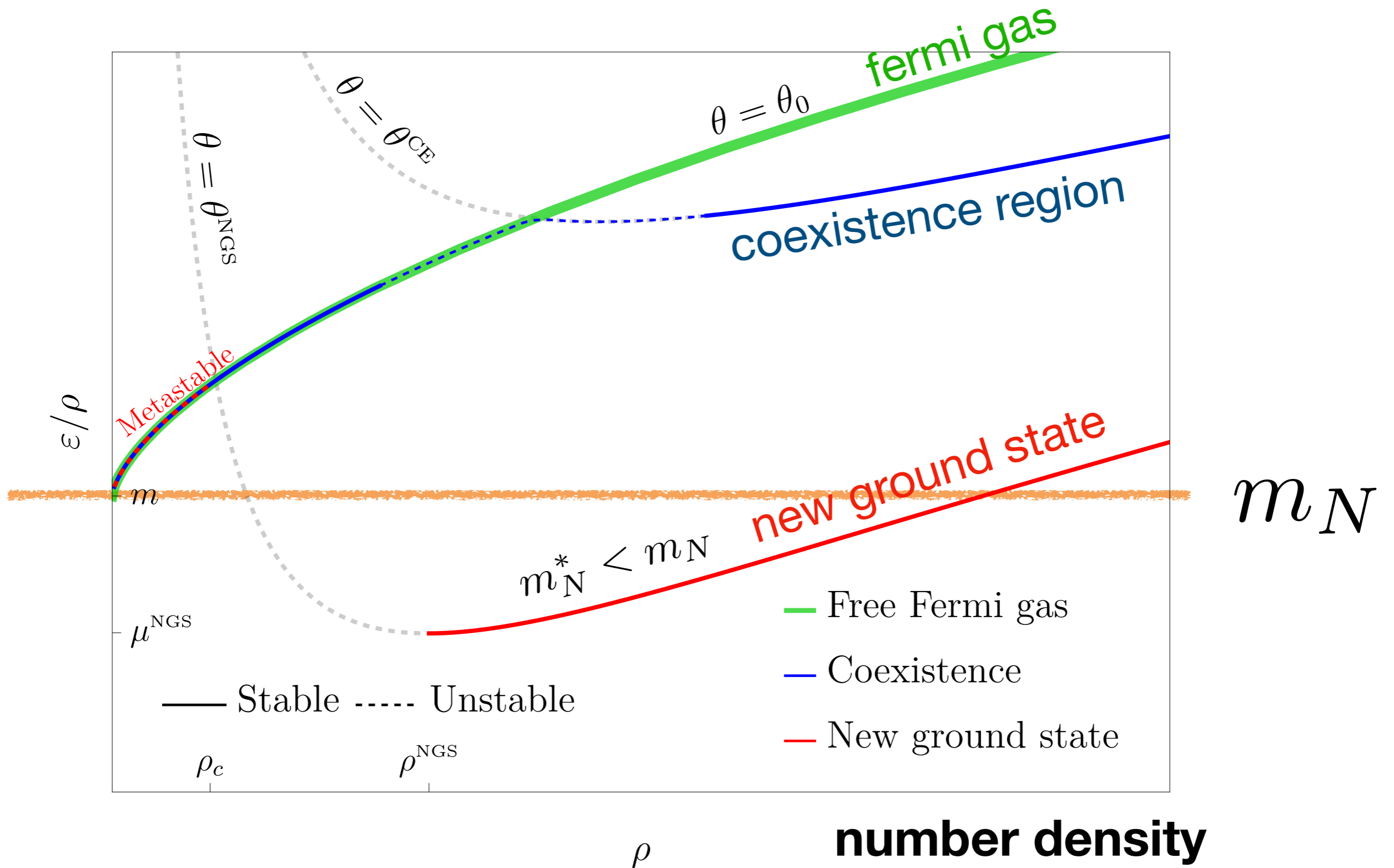
see Bellazzini et. al. '15 and Csaki et. al. '18

could ignore in white dwarfs

Energy per particle



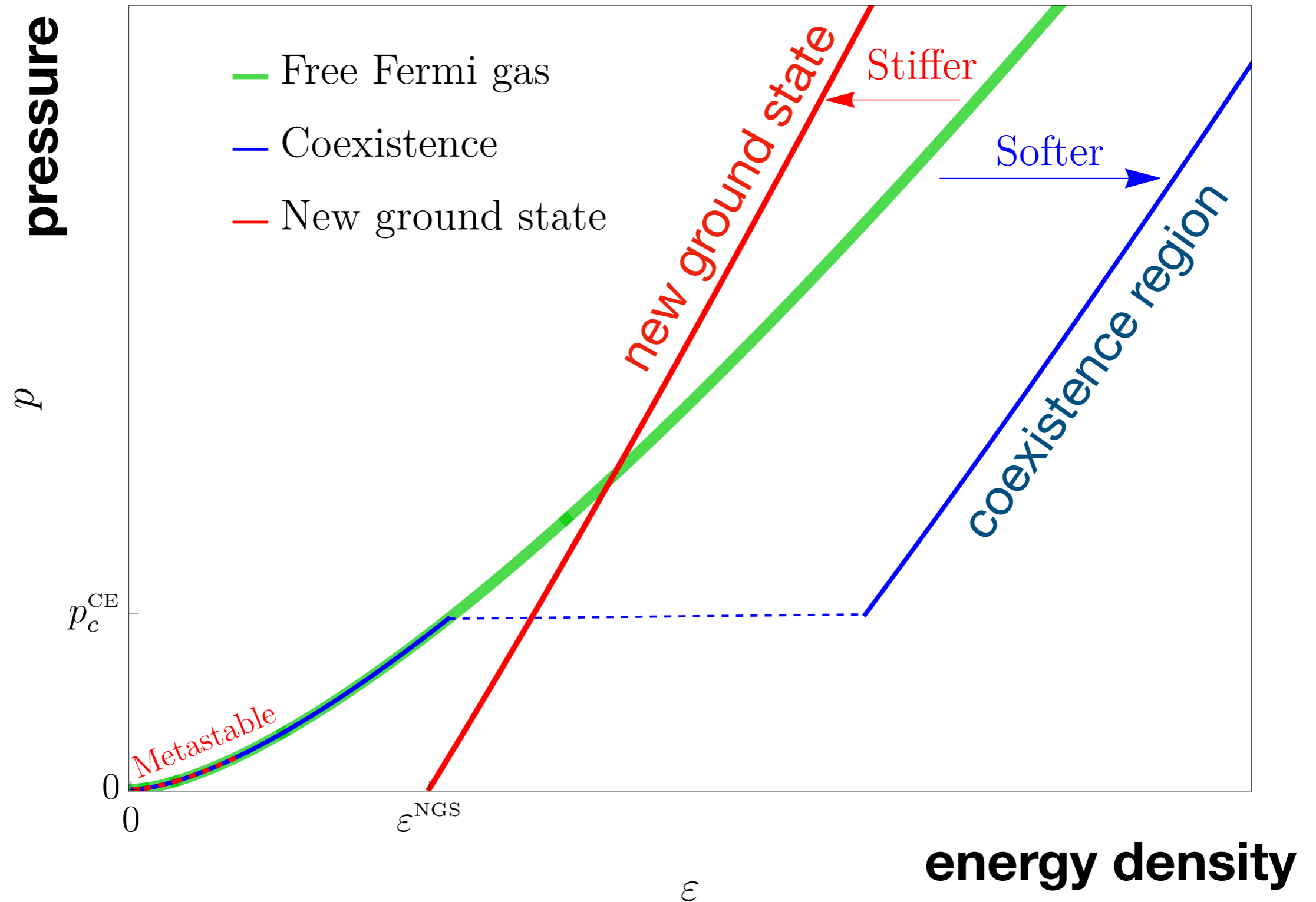
Energy per particle



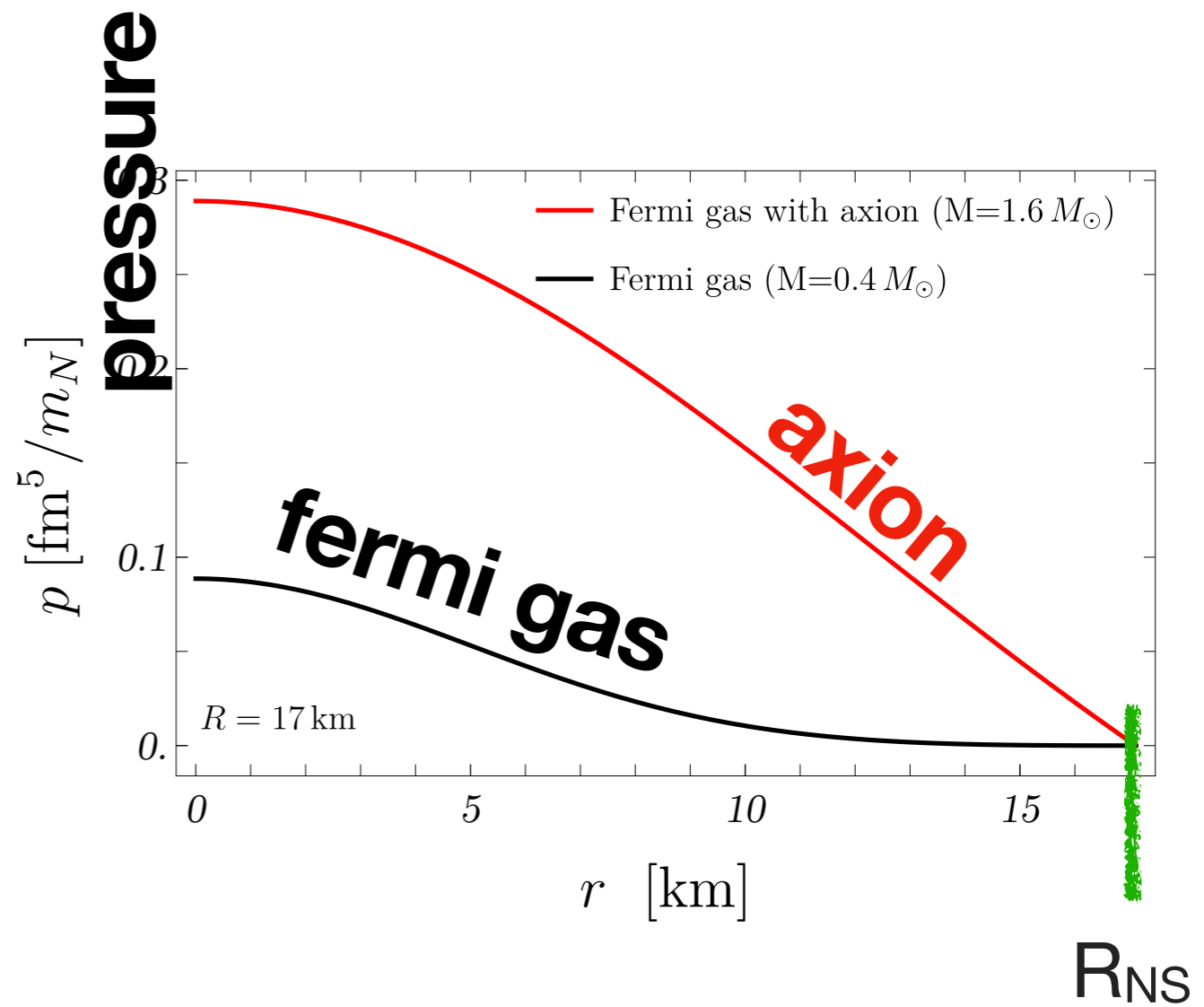
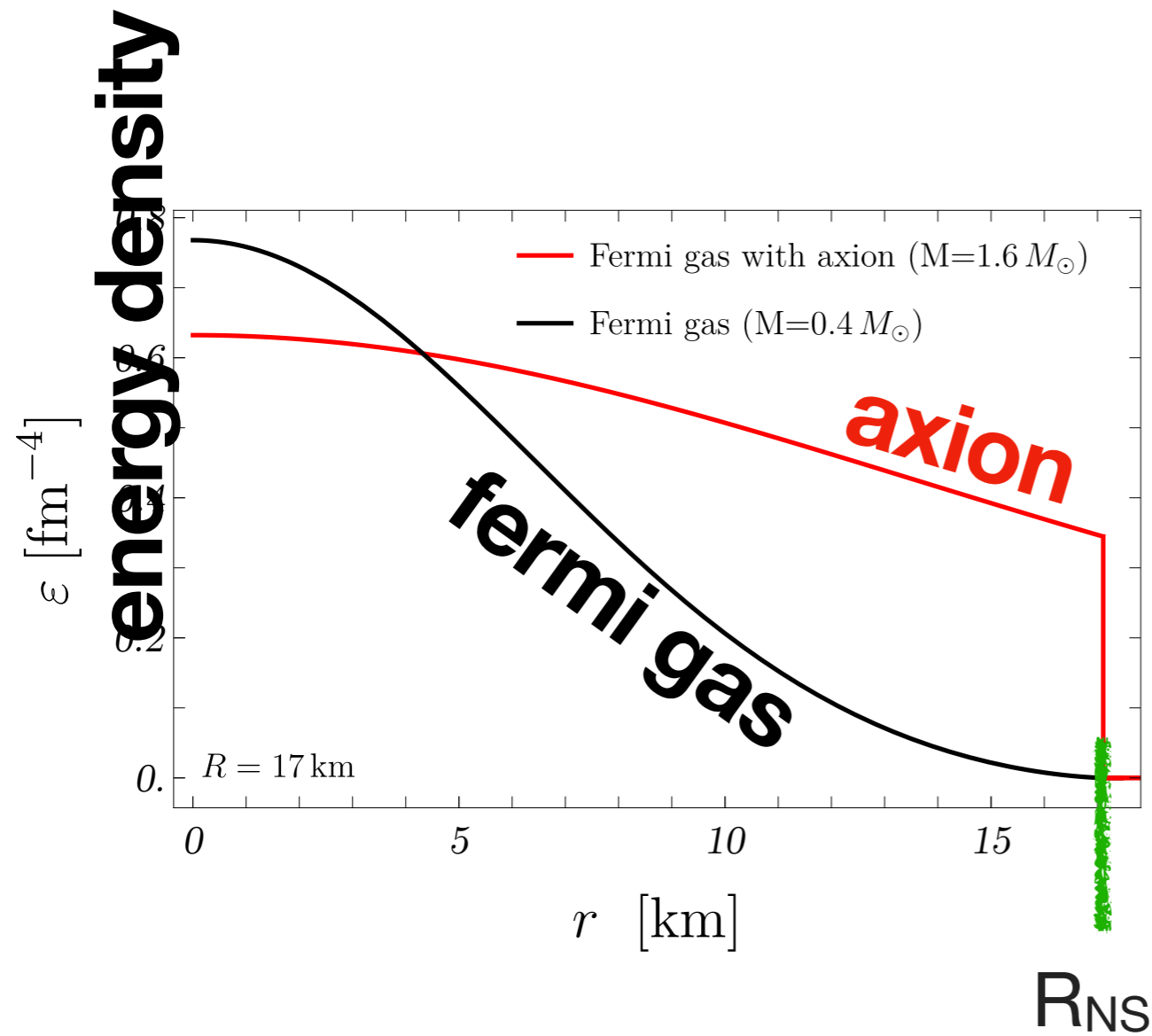
→ Parameter space is split

New ground state	$p = 0, \varepsilon = \varepsilon^{\text{NGS}}$
Coexistence	first order PT

Equation of state



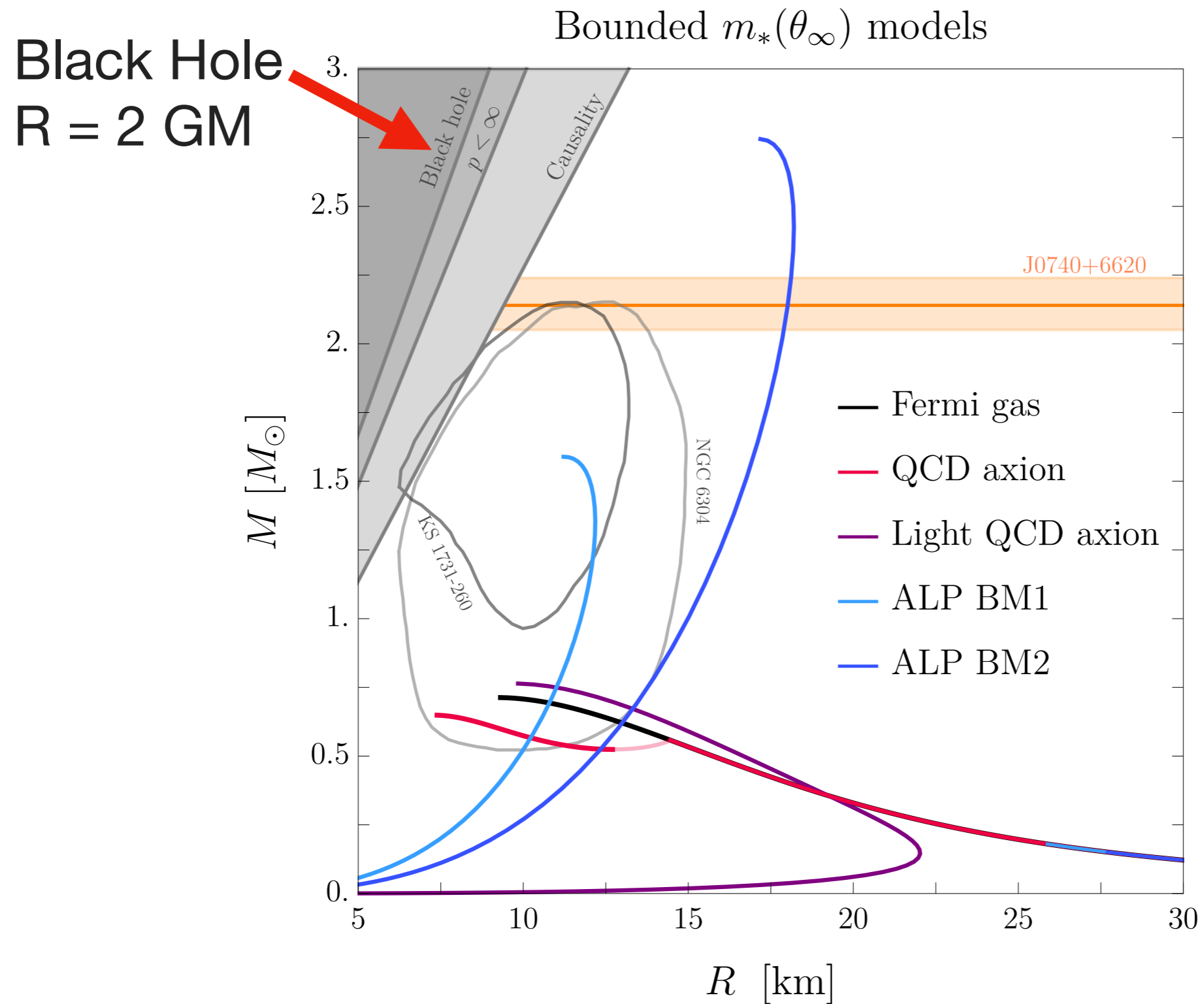
Axion effect on NS profile



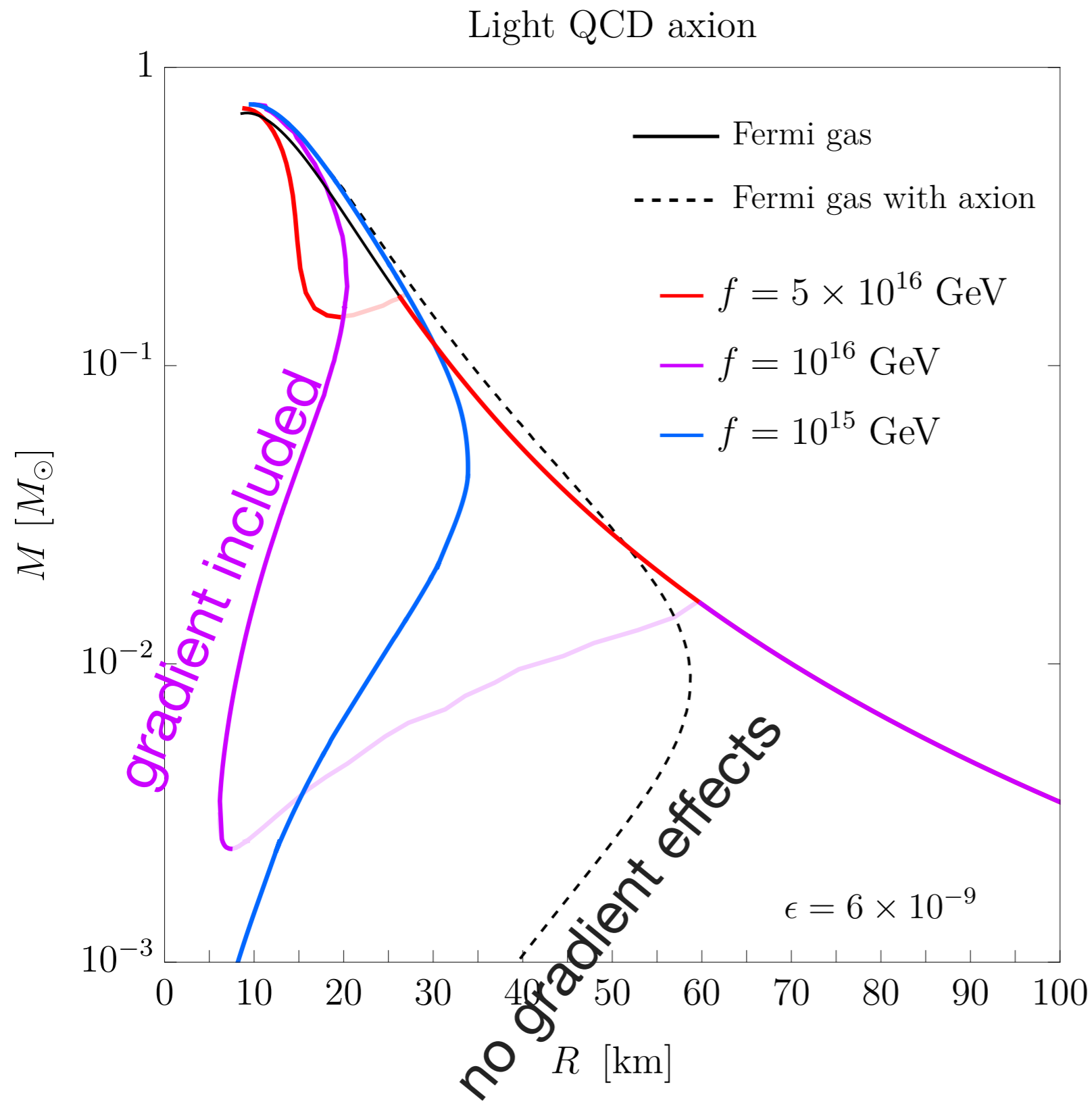
axion NS: $M = 1.6 M_{\odot}$

fermi gas NS $M = 0.4 M_{\odot}$

Mass vs. Radius of NS



Gradient Effects



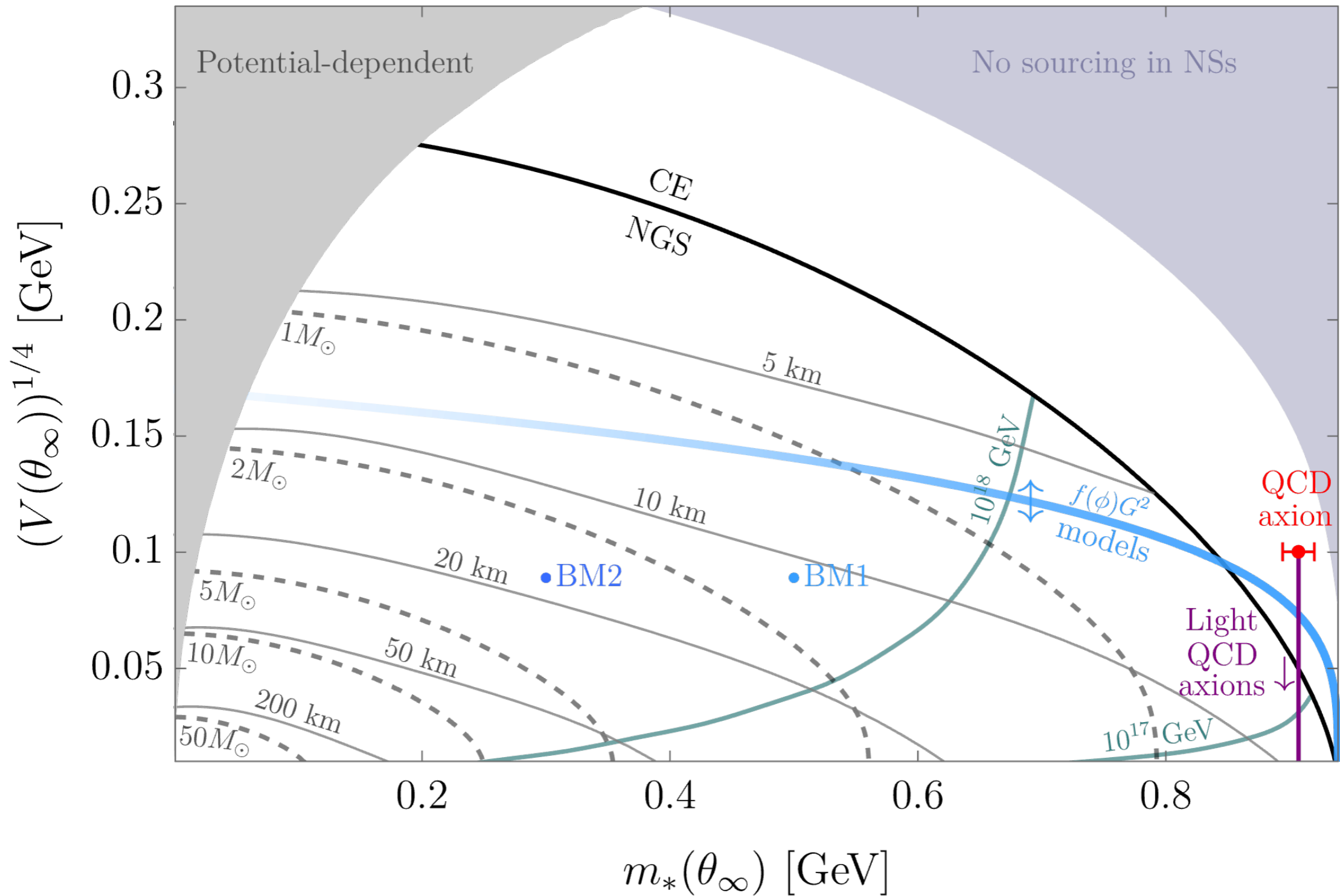
O(1) changes in
axion profile and M-R

Note: important also for
bounds on light axions
derived from GW170817

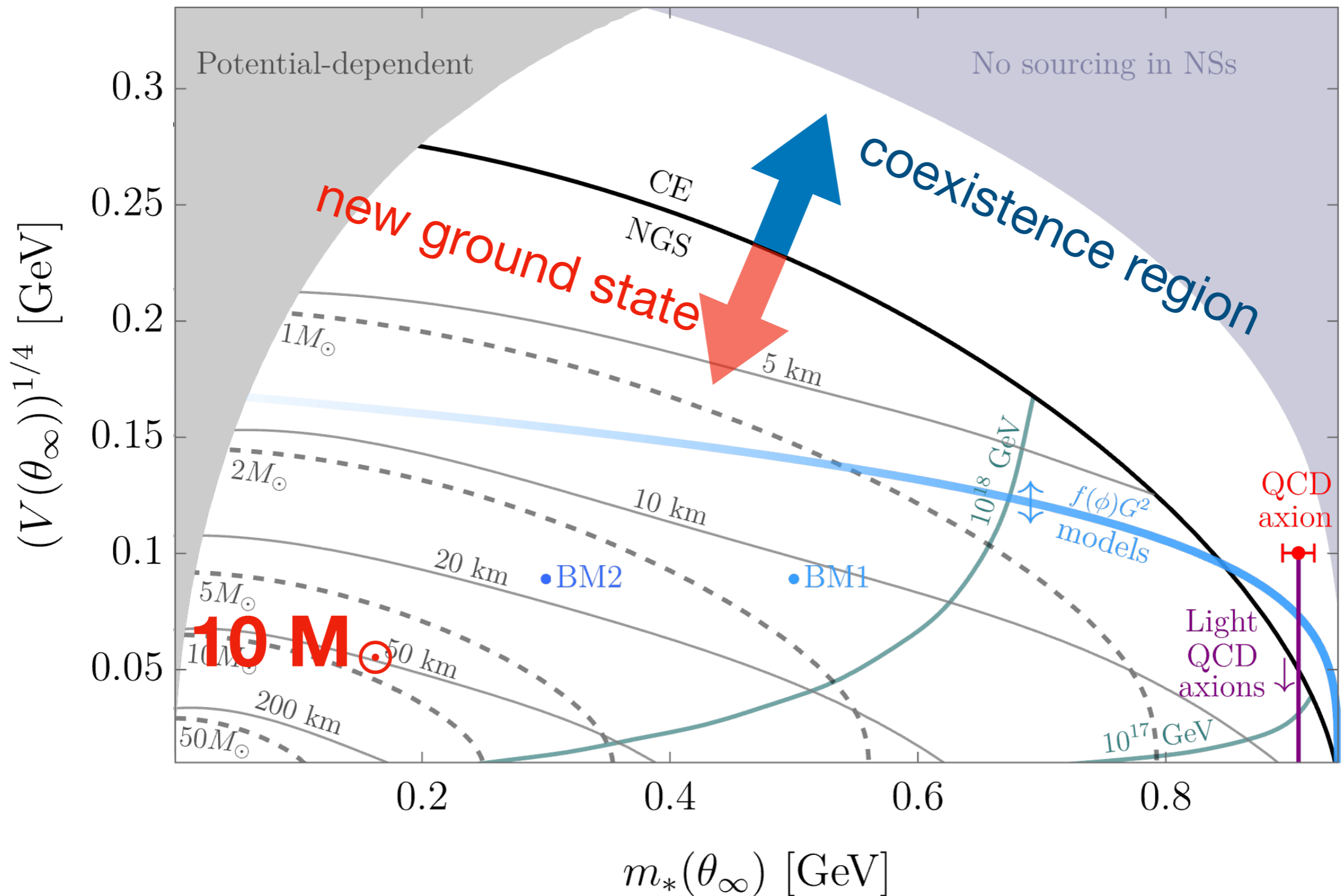
Weakening of the GW bound
at large f by a factor of a few.

Full parameter space

Neutron stars with light scalars



Neutron stars with light scalars

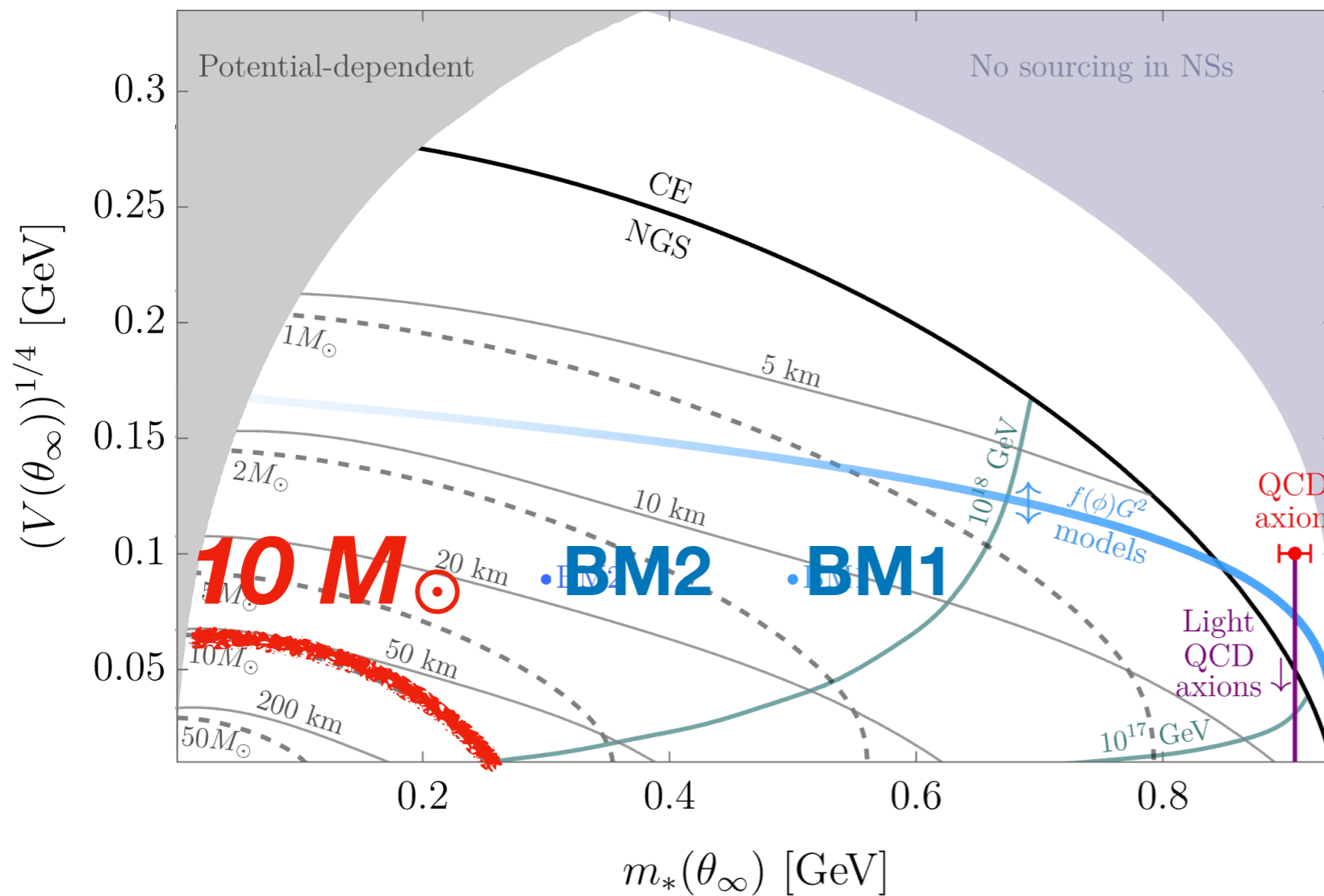


Neutron stars with light scalars

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$$\mathcal{O}_{\phi N} = \frac{g m_N}{2} \bar{N} N \cos\left(\frac{\phi}{f}\right)$$

$$m_N^* = \begin{cases} m_N & \phi = 0 \\ m_N(1-g) & \phi = \pi \end{cases} \quad 1 > g > 0$$



Two benchmarks

BM1 $m_*(\theta_\infty) = m_N/2$

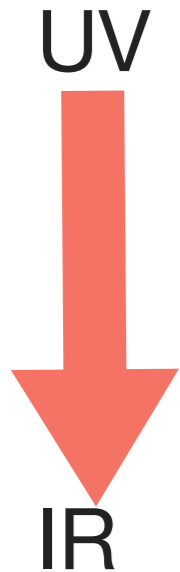
BM2 $m_*(\theta_\infty) = m_N/3$

$$V(\theta_\infty) = 2 \times (0.075 \text{ GeV})^4$$

Can we really populate the full parameter range?

Large m_N -reduction toy model

$$\mathcal{L}_\phi = \frac{1}{2}(\partial_\mu\phi)^2 - \alpha_s g f(\phi) G^{\mu\nu} G_{\mu\nu} - \epsilon g \alpha_s \frac{M_{\text{UV}}^4}{16\pi^2} f(\phi)$$



$$\langle N | \alpha_s G^{\mu\nu} G_{\mu\nu} | N \rangle = \frac{4\alpha_s^2}{\beta(\alpha_s)} m_0 \quad m_0 = 869.5 \text{ MeV}$$

$$\mathcal{L}_{\phi, \text{IR}} \supset -m_N \bar{N} N \left(1 - f(\phi) \frac{g}{g_*} \right) - \left(\epsilon \frac{M_{\text{UV}}^4}{16\pi^2} + c \frac{M_{\text{QCD}}^4}{4\pi} \right) g f(\phi)$$

$$g_* \equiv \frac{|\beta(\alpha_s)|}{4\alpha_s^2} \frac{m_N}{m_0} \sim 0.39$$

Decoding scalar-matter interactions

$$m_*(\phi)/m = 1 - \left(\frac{\phi}{M_\phi}\right)^n \left[1 + O\left(\frac{\phi}{F_\phi}\right)\right] \quad \text{scalar matter coupling}$$

$$V(\phi) = \frac{1}{2}m_\phi^2\phi^2 \left[1 + O\left(\frac{\phi}{f_\phi}\right)\right] \quad \text{scalar potential}$$

If higher order effects are unimportant:

$$F_\phi, f_\phi \gg M_\phi \quad \phi \sim M_\phi \text{ and } m_*(M_\phi)/m \ll 1$$

Screening by higher order effects:

$$\phi \sim f_\phi \ll M_\phi \quad \text{chameleon screening}$$

$$\phi \sim F_\phi \ll M_\phi \quad \text{QCD axion and (light) axion}$$

Side-remark: scalar-tensor gravity theories are in this class

scalar-nucleon coupling:

$$S_{\text{matter}}[\psi, \phi; g_{\mu\nu}] \equiv \int d^4x e \bar{\psi} \left([i/2] e^{\mu a} \gamma_a \overleftrightarrow{D}_\mu - m_*(\phi) \right) \psi$$

$$\square\phi = \frac{dV}{d\phi} + T \Big|_{\text{on-shell}} \frac{d \log m_*(\phi)}{d\phi}$$

chameleon model:

$$S_{\text{matter}}[\psi_*; A^2(\phi)g_{\mu\nu}].$$

$$e_{a\mu} \rightarrow \tilde{e}_{a\mu} = A e_{a\mu} \quad \omega_\mu^{ab} \rightarrow \tilde{\omega}_\mu^{ab} = \omega_\mu^{ab} + (e^{\alpha b} e_\mu^a - e^{\alpha a} e_\mu^b) \partial_\alpha \log A$$

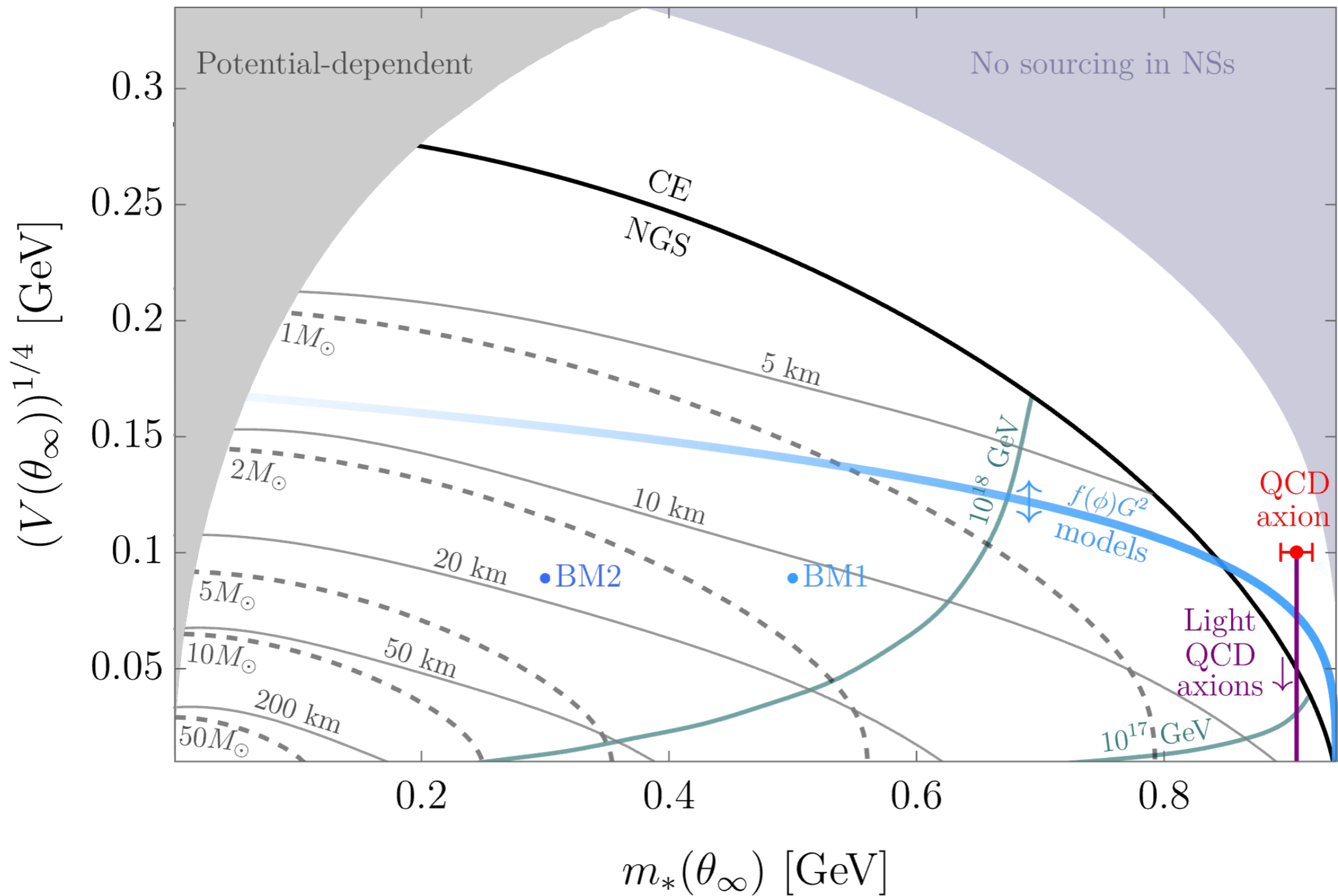
$$e(A^{3/2}\bar{\psi}_*)e^{\mu a}\gamma_a D_\mu(A^{3/2}\psi_*)$$

canonical normalization

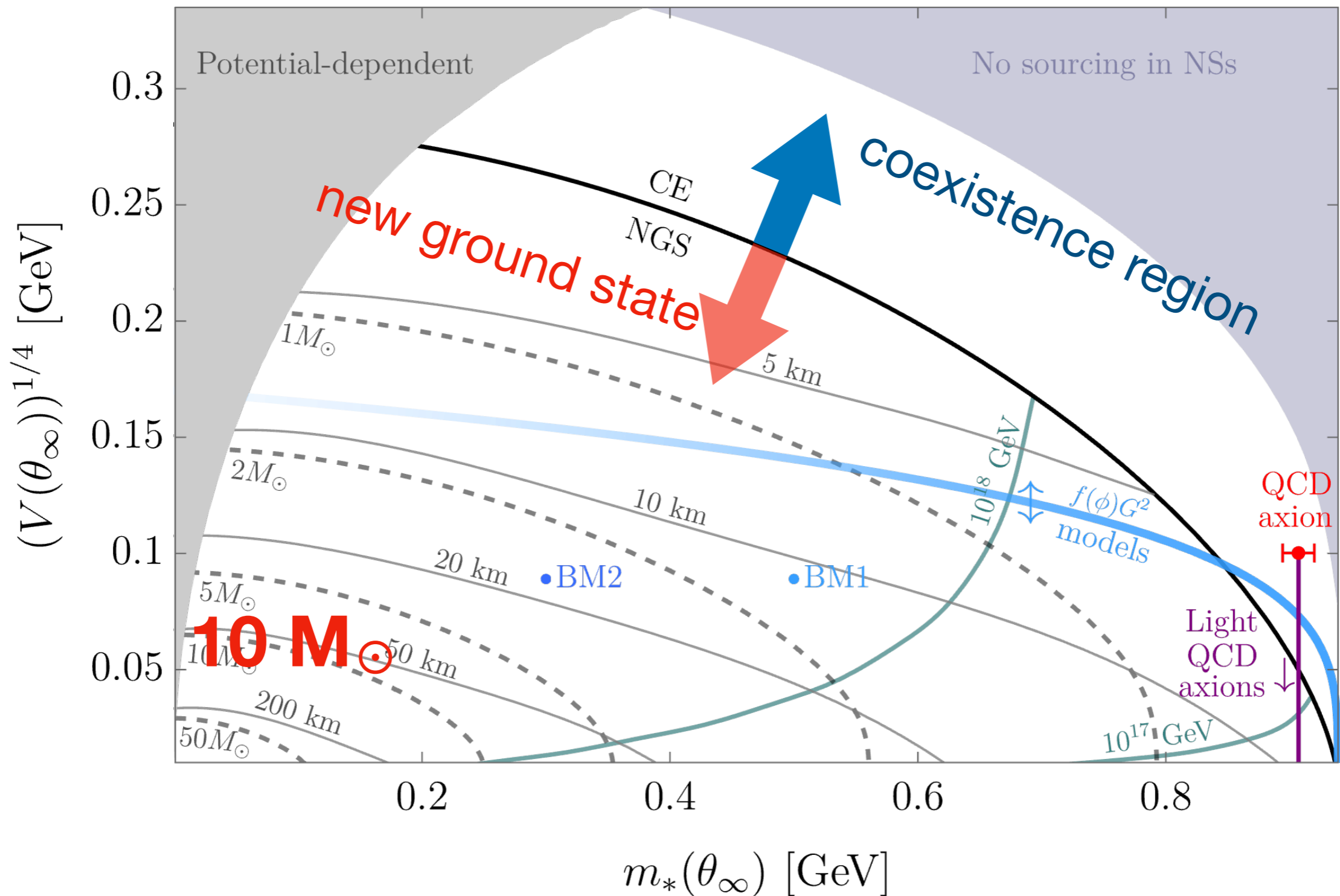
... are equivalent (modulo interactions):

$$\mathcal{L}_{\text{matter}}[\psi_*; A^2(\phi)g_{\mu\nu}] \rightarrow S_{\text{matter}}[\psi, \phi; g_{\mu\nu}] = \bar{\psi} (ie^{a\nu}\gamma_a D_\nu - A(\phi)m) \psi$$

Neutron stars with light scalars



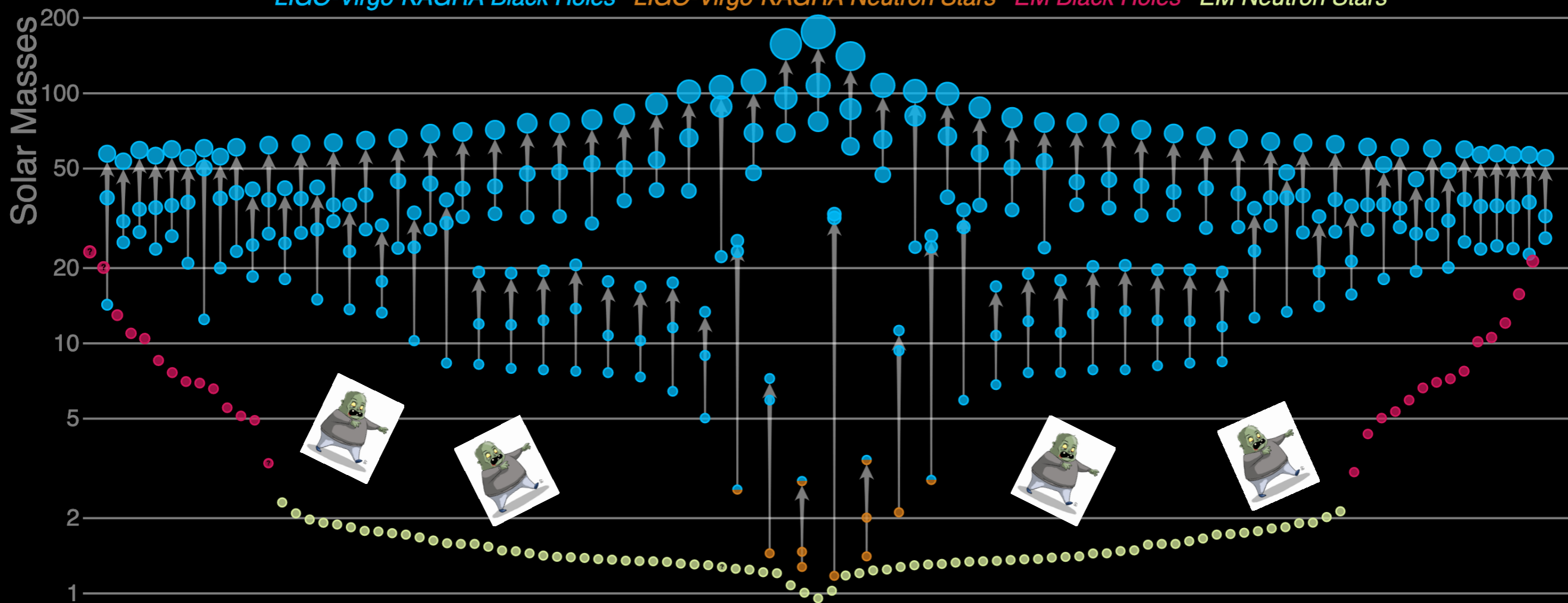
Neutron stars with light scalars



Masses in the Stellar Graveyard

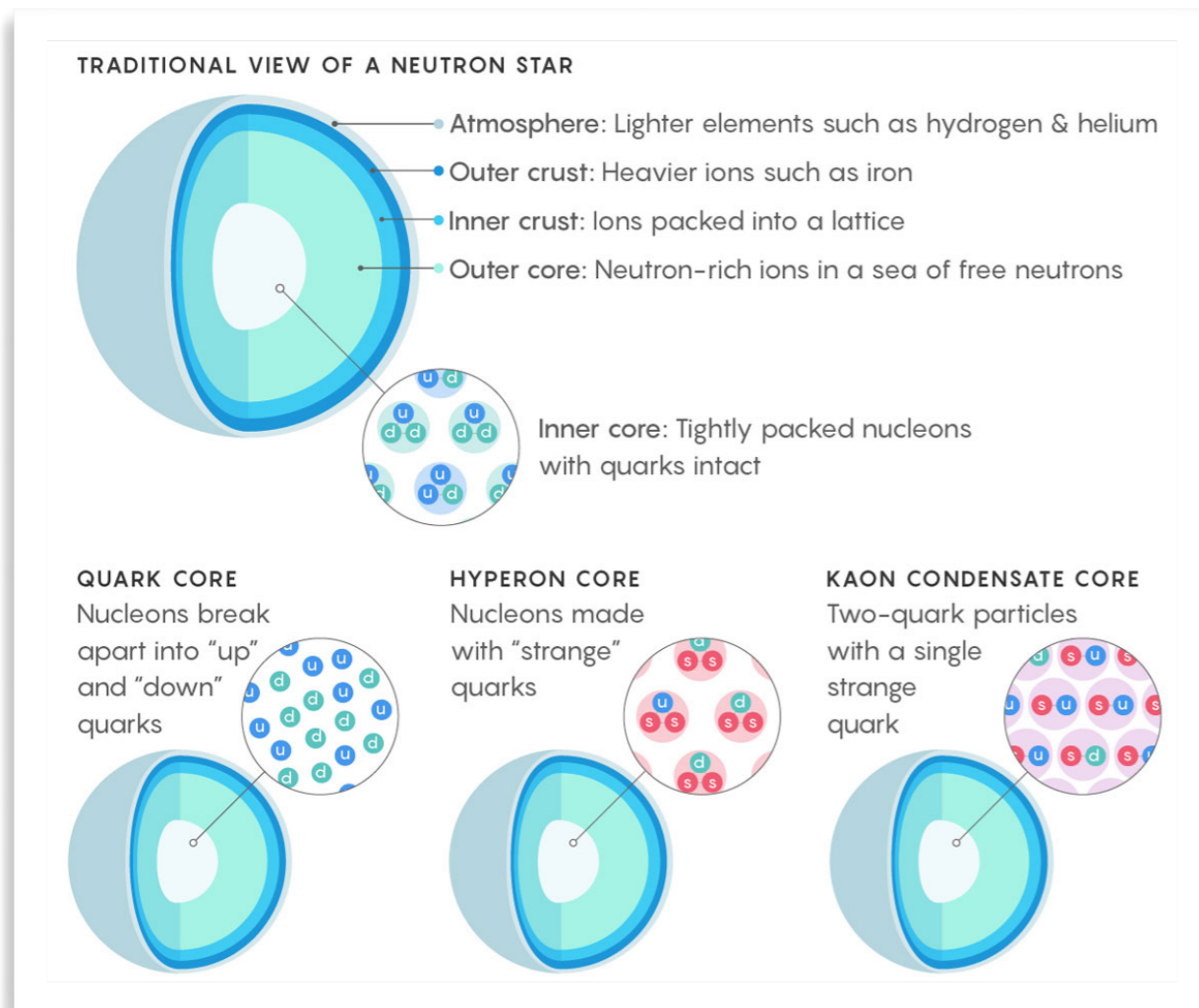


LIGO-Virgo-KAGRA Black Holes *LIGO-Virgo-KAGRA Neutron Stars* *EM Black Holes* *EM Neutron Stars*



Outlook

Actual QCD Axion inside neutron stars

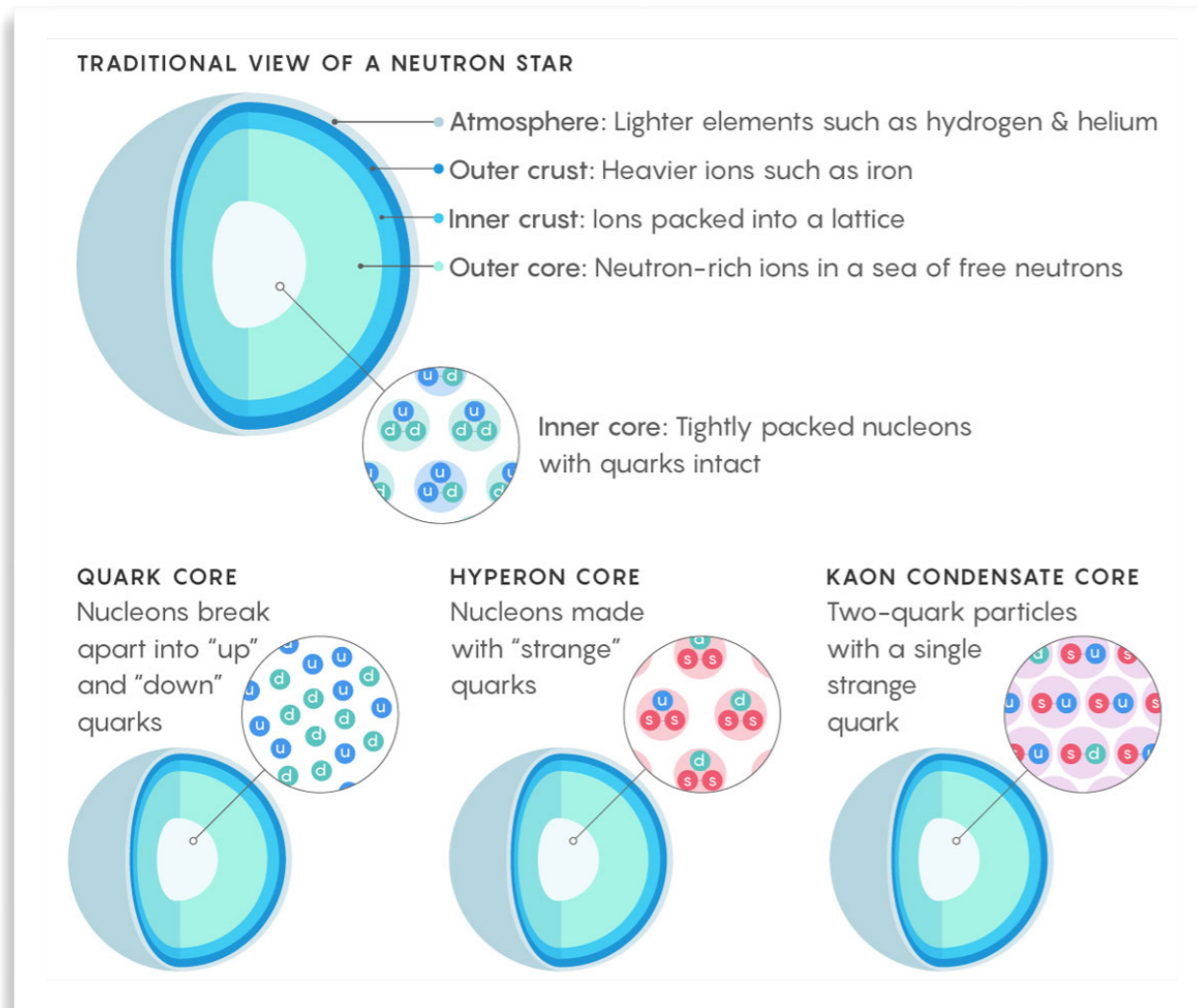


QCD phase transition sources axion:

Kaon condensation

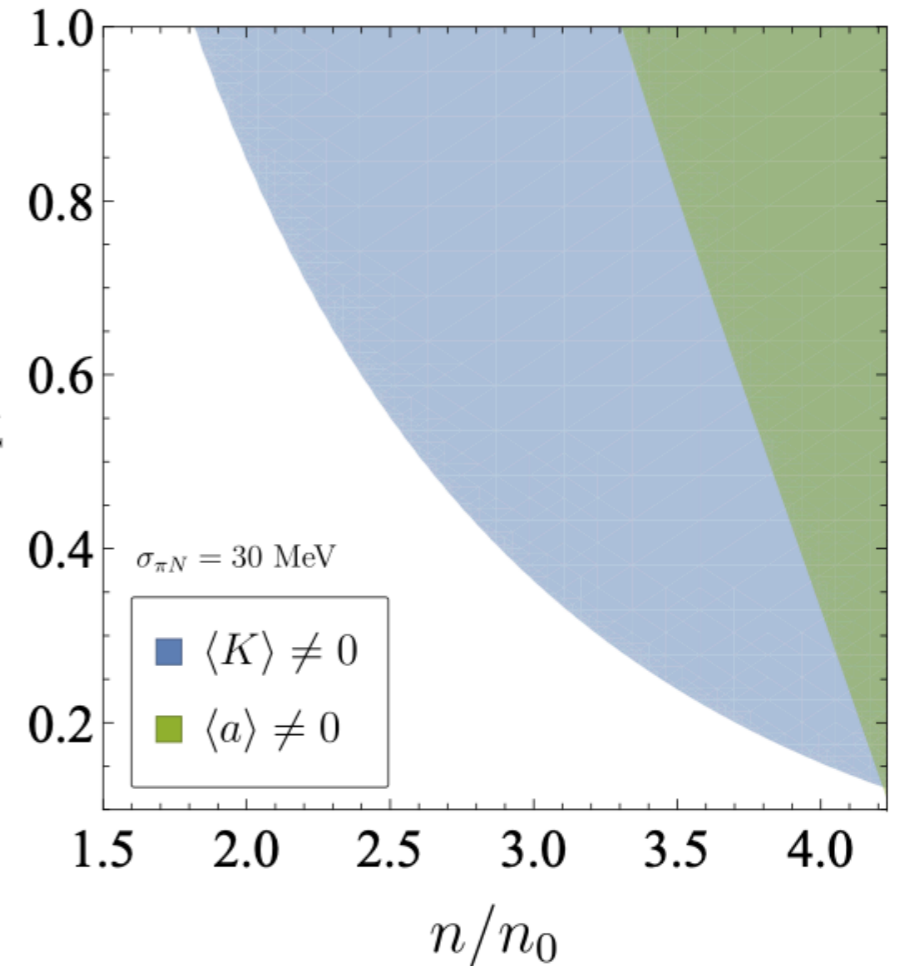
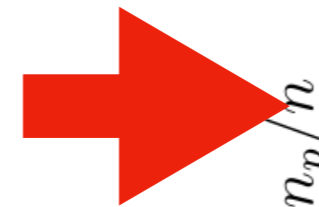
color-flavor locking at asymptotic densities

Actual QCD Axion inside neutron stars



usual QCD axion-mass relation

$$\epsilon \approx 1$$



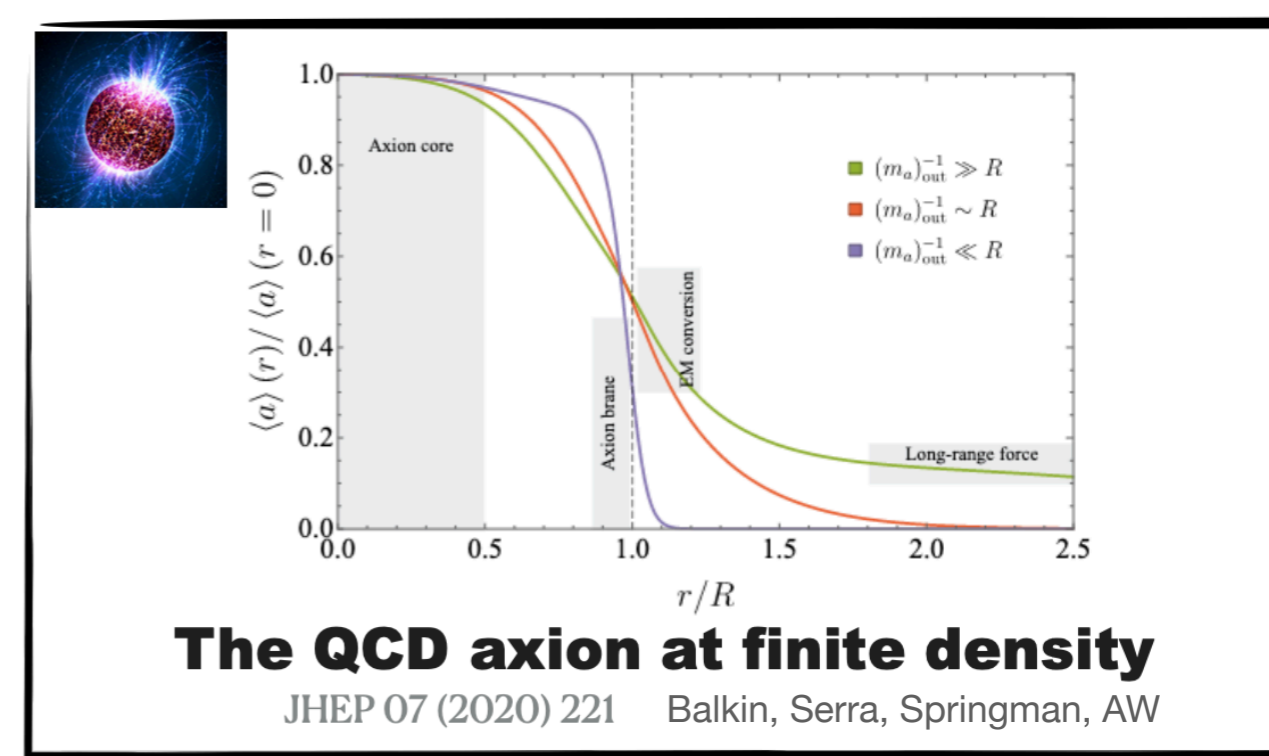
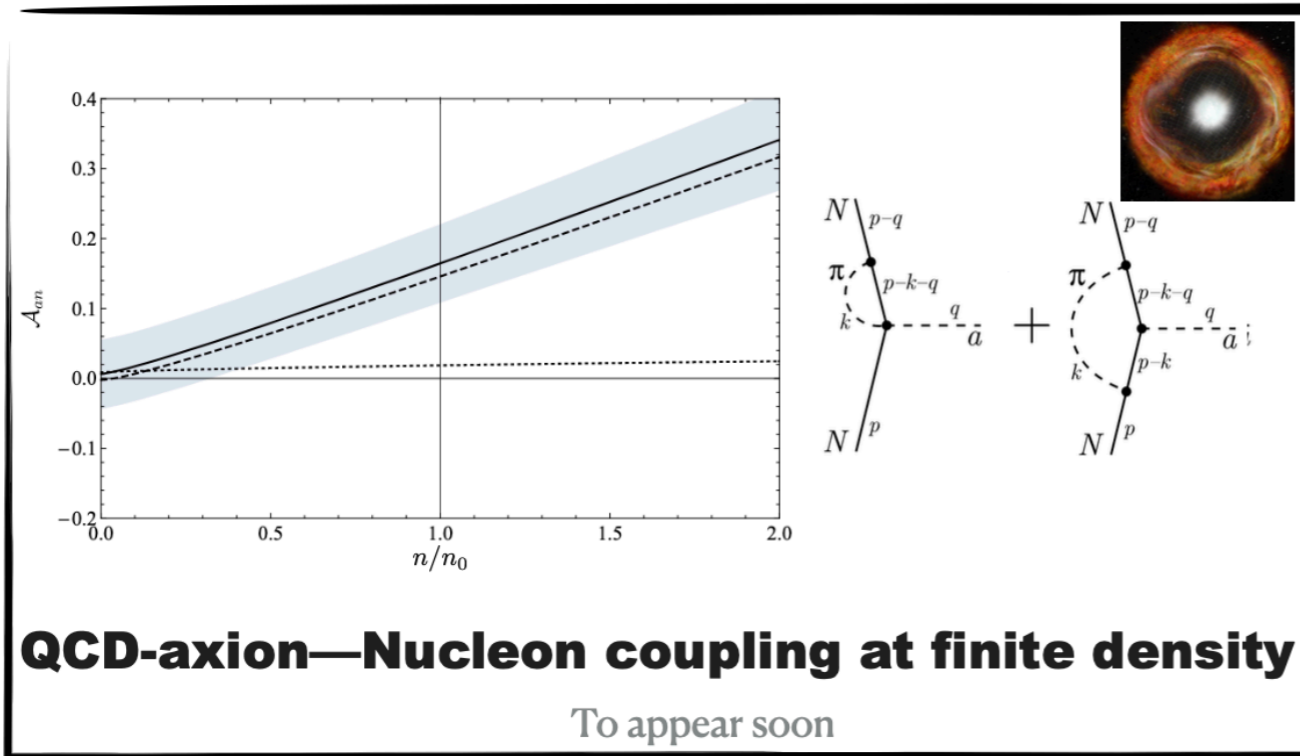
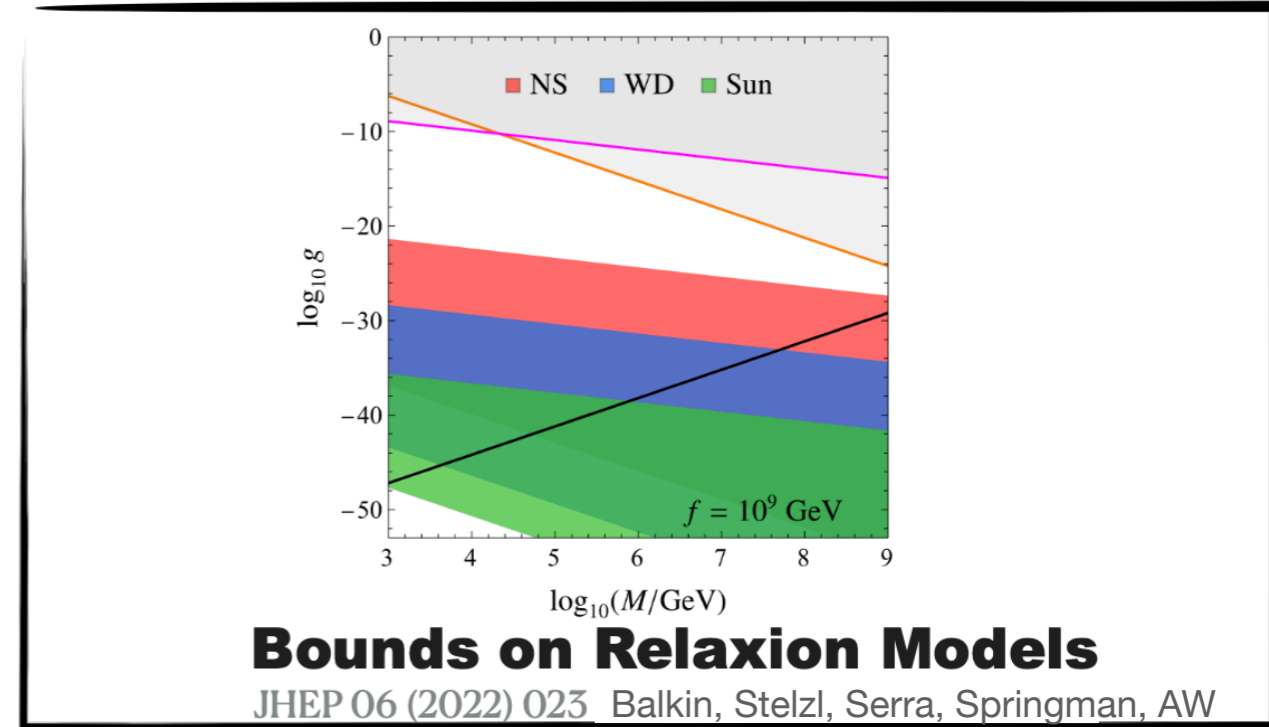
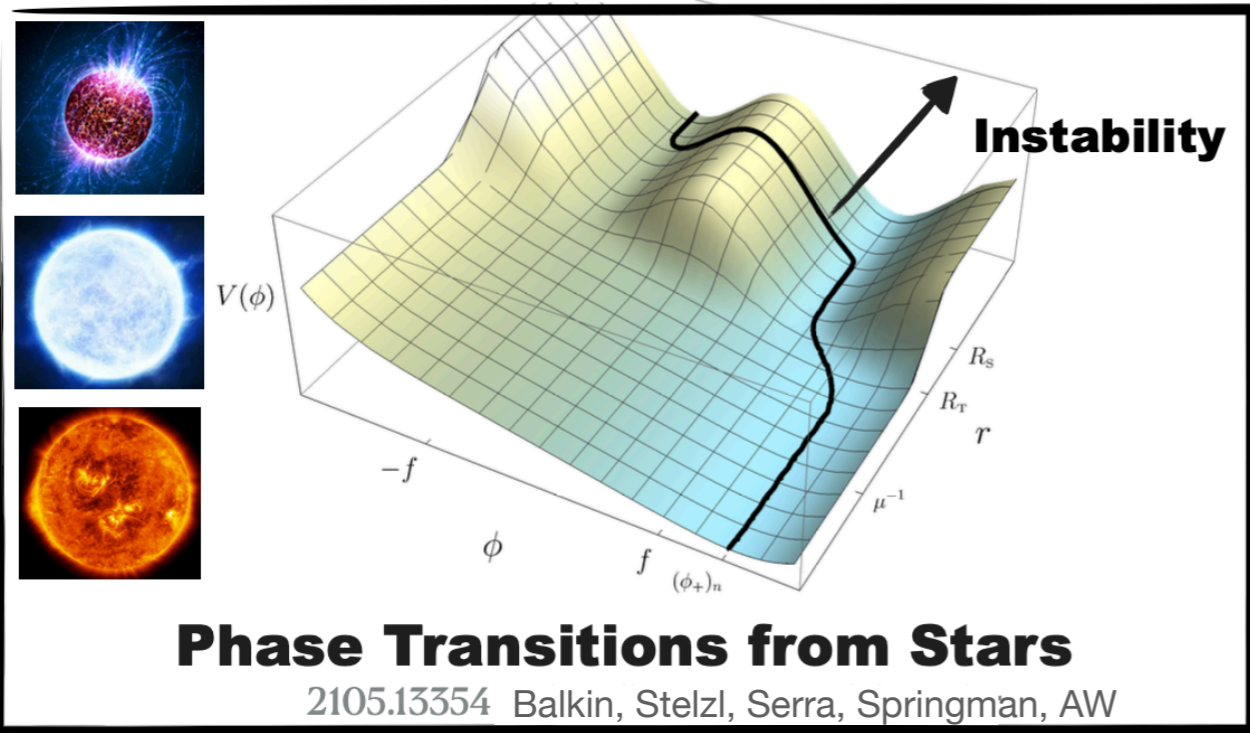
QCD phase transition sources axion:

Kaon condensation

color-flavor locking at asymptotic densities

Balkin, Serra, Springmann, AW '20

Studying density effects is fun and rewarding



Conclusions

White dwarfs as a probe of light QCD axions

- Light QCD axions can get sourced in White Dwarfs leading to a new ground state (NGS)
- There is a gap in densities between the Meta Stable and the NGS branch...
- ... which translates to a gap in the M-R curve
- Allows to exclude large chunk of parameter space
- This does **not** rely on the axion being DM

Heavy Neutron Stars from light Scalars

- Light Scalars with non-derivative coupling to nucleons make them lighter
- Coexistence Phase: Hybrid stars with softer EOS
- New ground state: Large effects on maximal mass

More to do

Self bound objects as DM, Coupling to electrons, GW from 1st order PT, what about Supernovae?...



Thanks