

# New 5D Perspectives on the QCD axion

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# Strong CP problem

$$\mathcal{L}_{QCD} \supset \theta G\tilde{G}$$

CP-odd term

Basis independent:  $\bar{\theta} = \theta + \arg(\det \mathcal{M}_q)$

*Observable effect:*

Neutron electric dipole moment  $d_N \simeq (5 \times 10^{-16} e \cdot \text{cm}) \bar{\theta}$

$$|d_N| \lesssim 3 \times 10^{-26} e \cdot \text{cm} \quad \rightarrow \quad \boxed{\bar{\theta} \lesssim 10^{-10}}$$

## Why is $\bar{\theta}$ so small?

$\bar{\theta}$  does not appear to be “anthropic”


Our Universe possible for  $0 \lesssim \bar{\theta} \lesssim 0.1$  [Lee, Meissner, Olive, Shifman, Vonk: 2006.12321]

# Dynamical Solution: PQ mechanism and axion

Axion:  $\Phi = \frac{1}{\sqrt{2}}(f_a + \rho)e^{i\frac{a}{f_a}}$   $U(1)_{PQ}$  spontaneously broken  $\langle \Phi \rangle = \frac{f_a}{\sqrt{2}}$

Axion Lagrangian:  $\mathcal{L}_a = \frac{1}{2}(\partial_\mu a)^2 + \underbrace{\frac{a}{f_a} \frac{\alpha_s}{8\pi} G\tilde{G}}_{\text{dim 5 term}} + \frac{1}{4} a \underbrace{g_{a\gamma\gamma} F\tilde{F}}_{\text{diphoton coupling}} + \frac{1}{f_a} J^\mu \underbrace{\partial_\mu a}_{\text{axial current coupling}}$

Axion mass:  $m_a^2 = \frac{\mathcal{T}}{f_a^2}$   $\mathcal{T} \equiv -i \int d^4x \langle 0|T \left[ \frac{1}{32\pi^2} G_{\mu\nu}^a \tilde{G}^{a\mu\nu}(x), \frac{1}{32\pi^2} G_{\rho\sigma}^b \tilde{G}^{b\rho\sigma}(0) \right] |0\rangle$  topological susceptibility

  $m_a^2 f_a^2 \sim \underbrace{\frac{1}{8} \Lambda_{\text{QCD}}^4}_{\text{light-quark contribution}}$  [or precisely,  $m_a = 5.70(7) \mu\text{eV} \left( \frac{10^{12} \text{ GeV}}{f_a} \right)$ ] [Cortona, Hardy, Pardy Vega, Villadoro, 1511.02867]

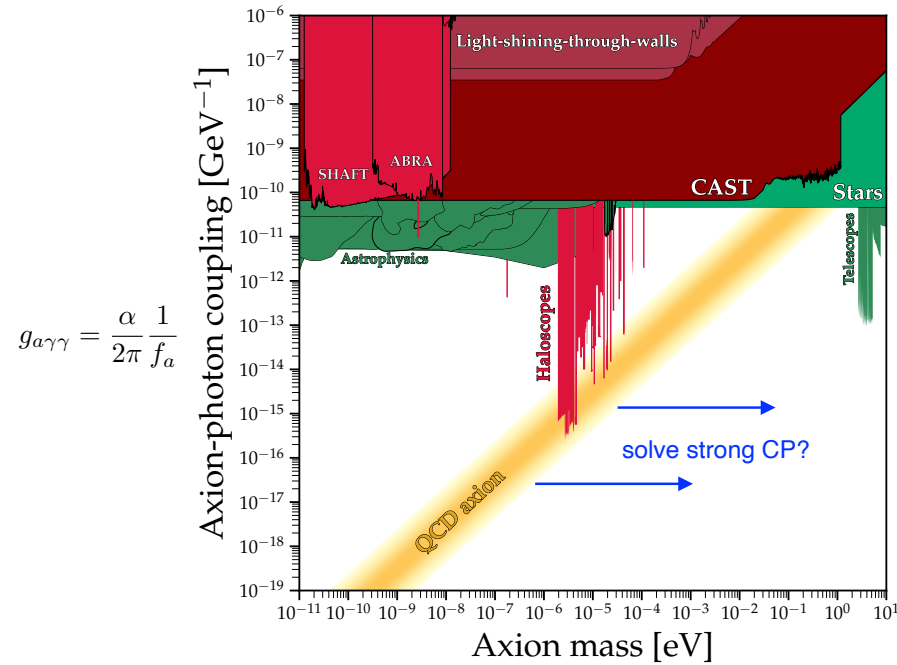
Axion quality: Gravitational violation of  $U(1)_{PQ}$   $\frac{c_n}{M_P^{n-4}} \Phi^n + h.c.$

Terms must be suppressed to very high order! ( $c_n \sim 1, n \geq 10$ )

[however, if only gravitational instantons  $c_n \sim e^{-S} \rightarrow S \geq 200$  ]

# Questions

- How to solve the axion quality problem?
- Can the QCD axion mass be different?



➡ Use 5th dimension to address these questions!

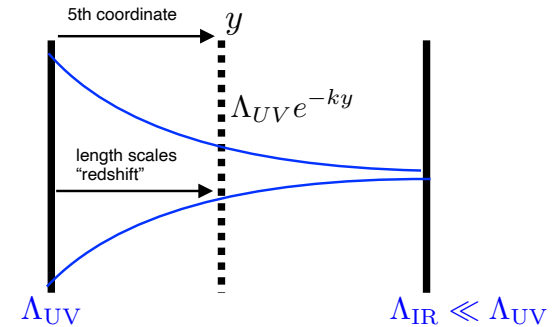


# Why use the 5th dimension?

- ◆ Can generate a hierarchy of scales!



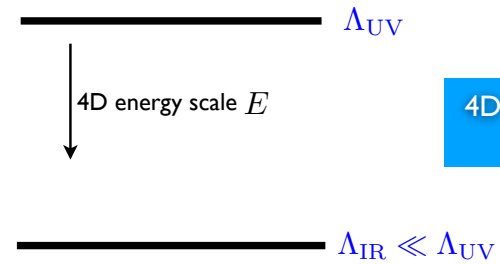
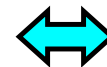
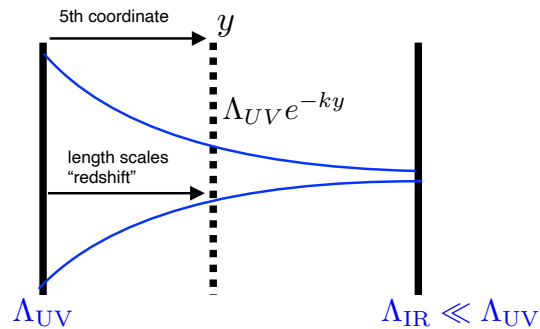
5D AdS



- ◆ “Warped” dimension is dual to 4D strong dynamics!

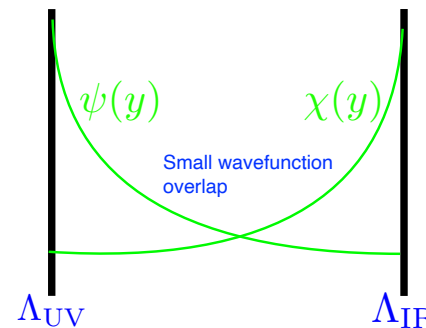


5D AdS



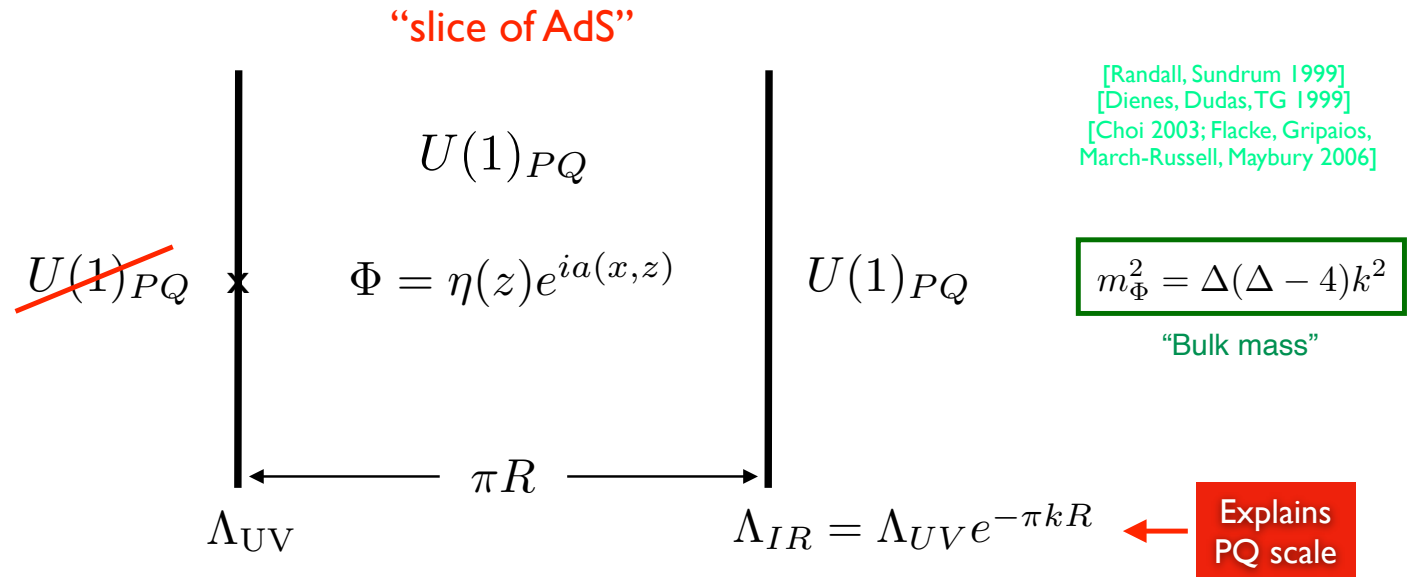
4D strongly-coupled gauge theory

- ◆ Can generate small couplings!



# I. Axion Quality Problem [Cox, TG, Nguyen 1911.09385]

5D metric:  $ds^2 = A^2(z)(dx^2 + dz^2) \equiv g_{MN}dx^M dx^N$   $A(z) = \frac{1}{kz}$   
AdS curvature scale



PQ symmetry breaking



$$\eta(z) = k^{3/2}(\lambda(kz)^{4-\Delta} + \sigma(kz)^\Delta)$$

“Bulk VEV”


$$\lambda = \frac{\ell_{UV}}{\Delta - 4 + b_{UV}}(kz_{UV})^{\Delta-4},$$

“explicit” breaking

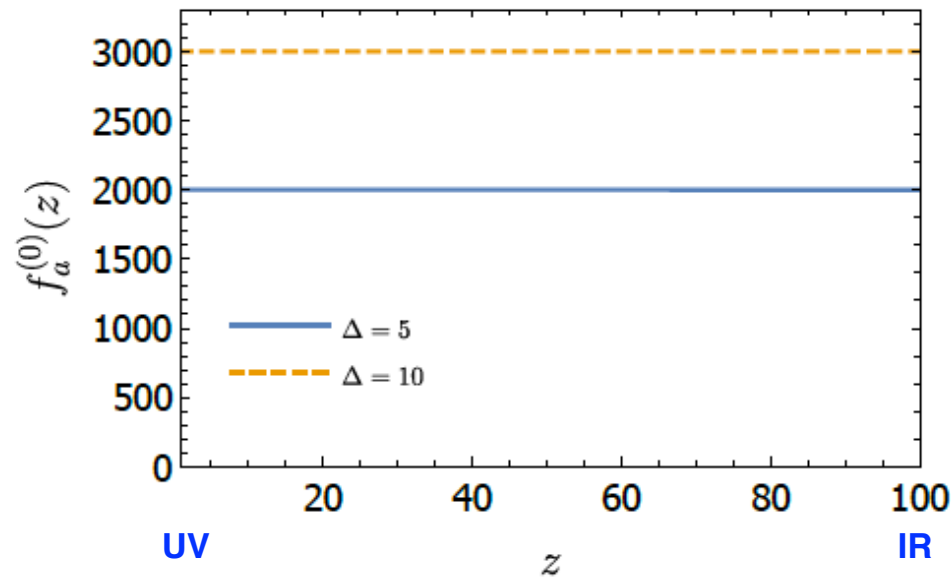
$$\sigma = \sqrt{v_{IR}^2 - \frac{\Delta}{2\lambda_{IR}}}(kz_{IR})^{-\Delta} \equiv \sigma_0(kz_{IR})^{-\Delta}$$

“spontaneous” breaking

# No UV PQ-breaking $(\lambda = 0)$

  
 $(z_{IR} \gg z_{UV})$

$$f_a^{(0)}(z) \simeq \frac{z_{IR}}{\sigma_0} \sqrt{\Delta - 1} \left( 1 + \frac{g_5^2 k \sigma_0^2}{4\Delta(\Delta - 1)} \left( \frac{(\Delta - 1)^2}{2\Delta - 1} + \frac{z^2}{z_{IR}^2} \left( \left( \frac{z}{z_{IR}} \right)^{2(\Delta - 1)} - \Delta \right) \right) + \mathcal{O}(\sigma_0^4) \right)$$




Global  $U(1)_{PQ}$  symmetry:

$$a^{(0)}(x^\mu) \rightarrow a^{(0)}(x^\mu) + \alpha_0$$

Massless axion

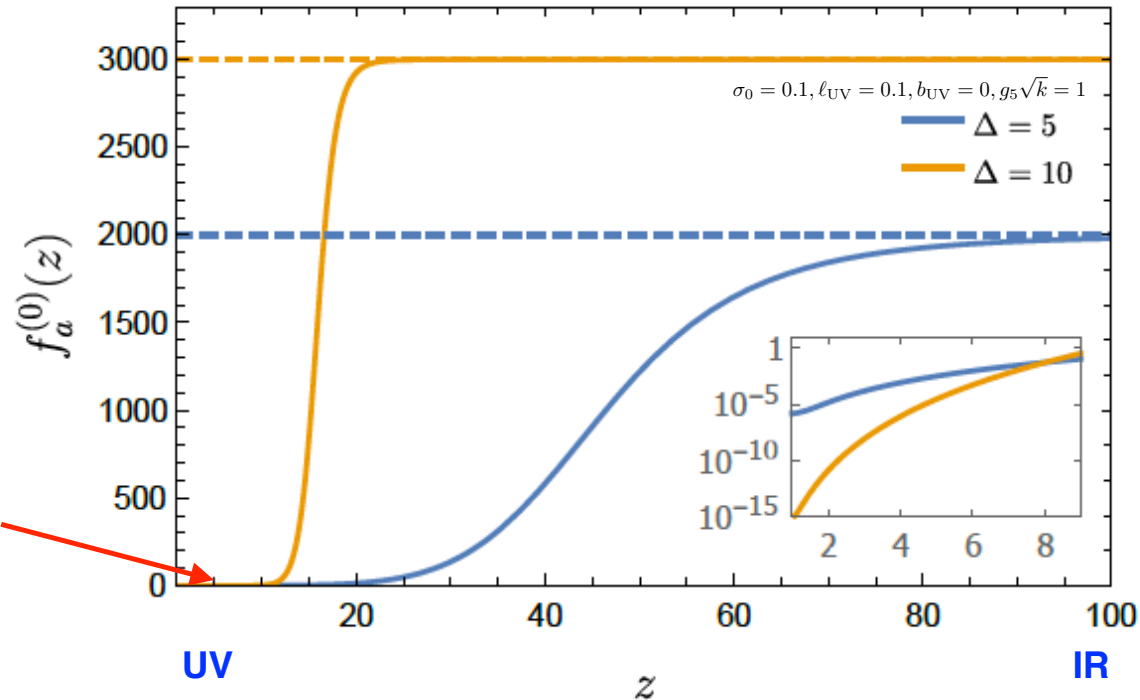
# UV PQ-breaking ( $\lambda \neq 0$ )

$$U_{UV}(\Phi) \supset -2\ell_{UV}k^{5/2}\eta \cos(a) = -2\ell_{UV}k^{5/2}\eta \left(1 - \frac{1}{2}a^2 + \dots\right)$$

  
 $(z_{IR} \gg z_{UV})$

$$f_a^{(0)}(z) \simeq z_{IR} \frac{k^{3/2}}{\eta(z)} \sqrt{\Delta - 1} \left(\frac{z}{z_{IR}}\right)^\Delta \left[ 1 + \frac{2\lambda(\Delta - 2)(kz_{UV})^\Delta (kz)^{2(2-\Delta)}}{\ell_{UV} + 2\sigma_0(\Delta - 2)(z_{UV}/z_{IR})^\Delta} \right]$$

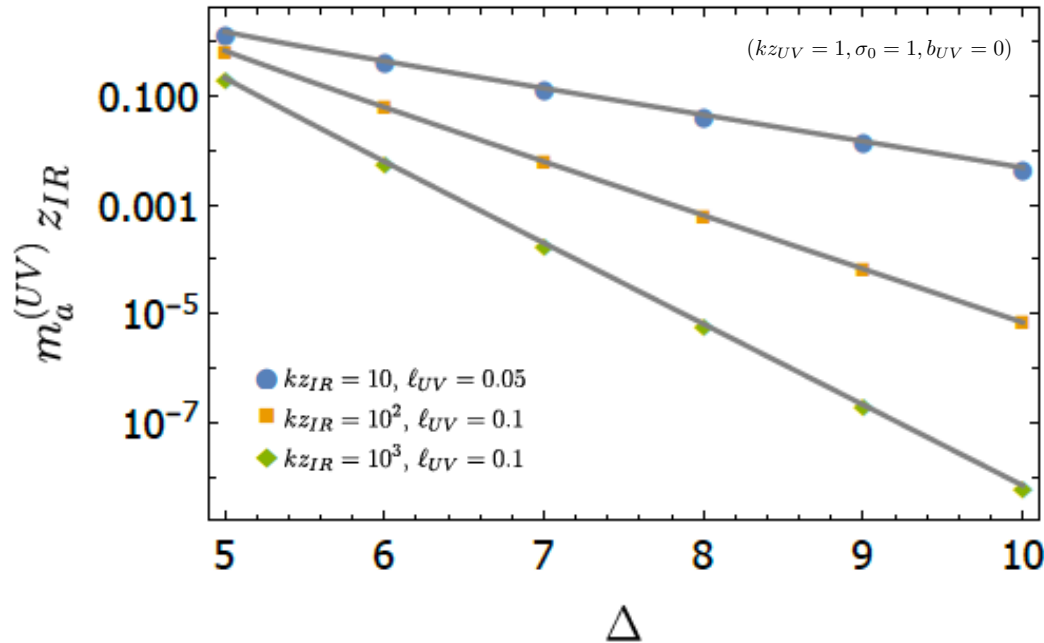
[Cox, TG, Nguyen 1911.09385]



**Axion profile suppressed!**

# UV axion mass ( $\lambda \neq 0$ )

[Cox, TG, Nguyen 1911.09385]



UV axion mass suppressed for large  $\Delta$

Bulk Chern-Simons term:

$$-\frac{\kappa}{32\pi^2} \int_{z_{UV}}^{z_{IR}} d^5x \epsilon^{MNPQR} V_M G_{NP}^a G_{QR}^a$$



generates axion-gluon coupling



$$(m_a^{(UV)})^2 = \frac{4\ell_{UV}\sigma_0(\Delta-2)}{\kappa^2(\Delta-4+b_{UV})} \left(\frac{\kappa\sqrt{\Delta-1}}{\sigma_0}\right)^\Delta \left(\frac{F_a}{\Lambda_{UV}}\right)^{\Delta-4} F_a^2$$

where

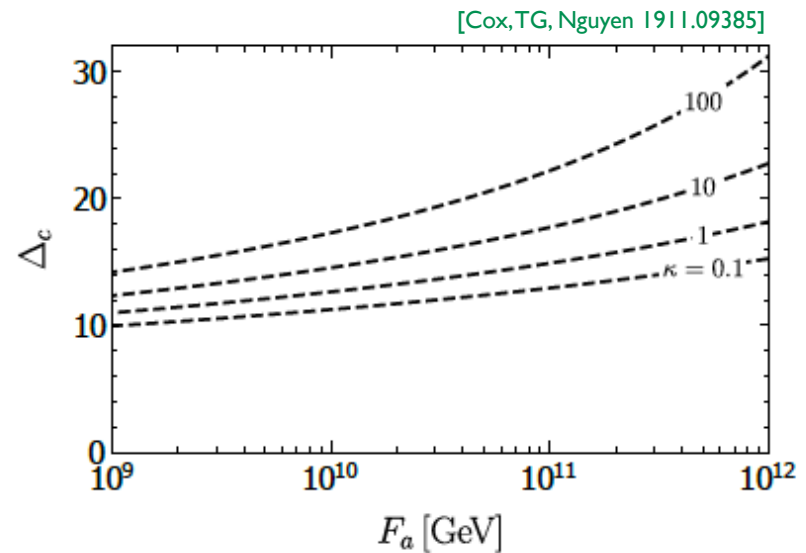
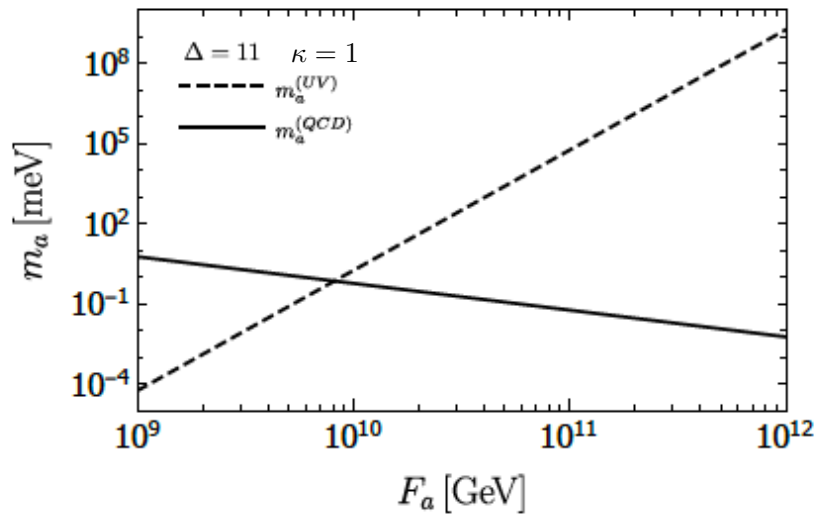
$$F_a \simeq \frac{1}{\kappa} \frac{\sigma_0}{\sqrt{\Delta-1}} z_{IR}^{-1}$$

# Axion potential:

$$V(a^{(0)}) \simeq -(m_a^{(QCD)})^2 F_a^2 \cos\left(\frac{a^{(0)}}{F_a} + \bar{\theta}\right) - \overset{\text{UV explicit PQ violation}}{(m_a^{(UV)})^2 F_a^2 \cos\left(\frac{a^{(0)}}{F_a} + \delta\right)}$$

relative phase

Require:  $(m_a^{(UV)})^2 \lesssim 10^{-10} (m_a^{(QCD)})^2$



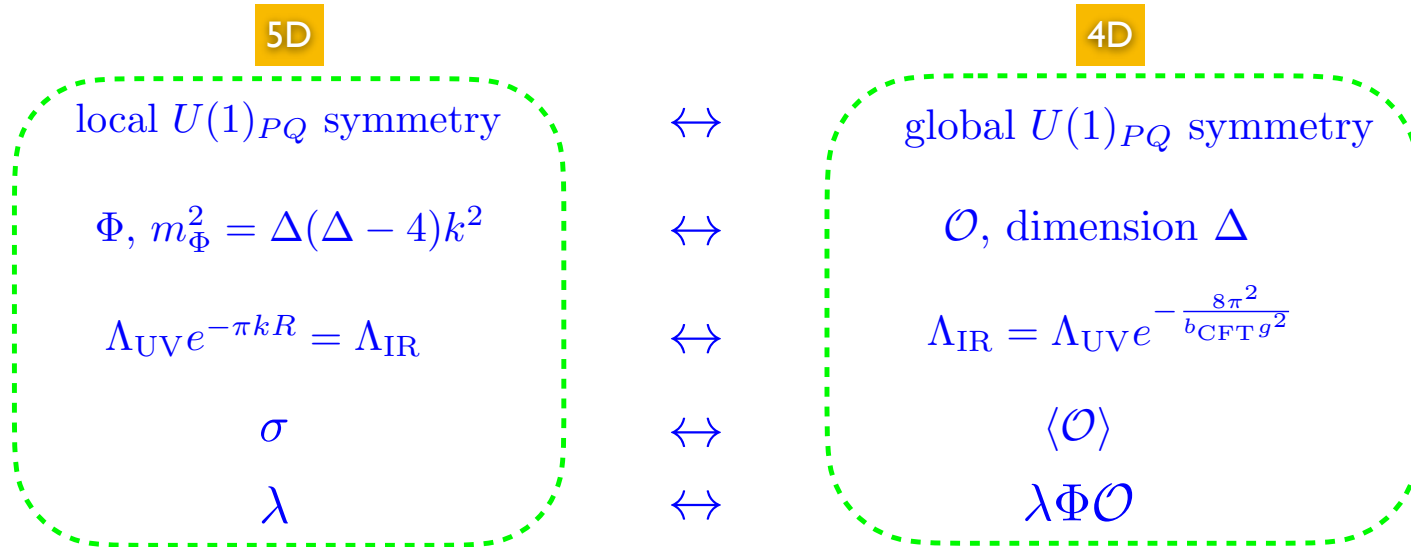
$$10^9 \text{ GeV} \lesssim F_a \lesssim 10^{12} \text{ GeV}$$



$$\Delta_c \gtrsim 10$$

Solves axion-quality problem!

# AdS/CFT dictionary



*Holographic interpretation:*

5D axion,  
local U(1) PQ symmetry



4D composite axion,  
accidental global  
U(1) PQ symmetry

*Axion quality:*  $\frac{c_n}{M_P^{n-4}} \Phi^n \quad (n \gtrsim 10)$



$\frac{c_\Delta}{M_P^{\Delta-4}} \mathcal{O} \sim \frac{c_\Delta}{M_P^{\Delta-4}} (\underbrace{\psi_i \psi_j \dots \psi_k}_{\text{due to gauge + Lorentz symmetry!}}) \quad (\Delta \gtrsim 10)$

**Examples:** [Kim 1985; Choi, Kim 1985];  
[Randall 1992]; [Redi, Sato 2016]; [Lillard, Tait: 1811.03089]; [Gavela, Ibe, Quilez, Yanagida: 1812.08174]





# Numerical results

Flavor-violating couplings:

$$\frac{1}{(F_u^{V,A})_{ij}} \equiv \frac{(c_u^{V,A})_{ij}}{F_a}$$

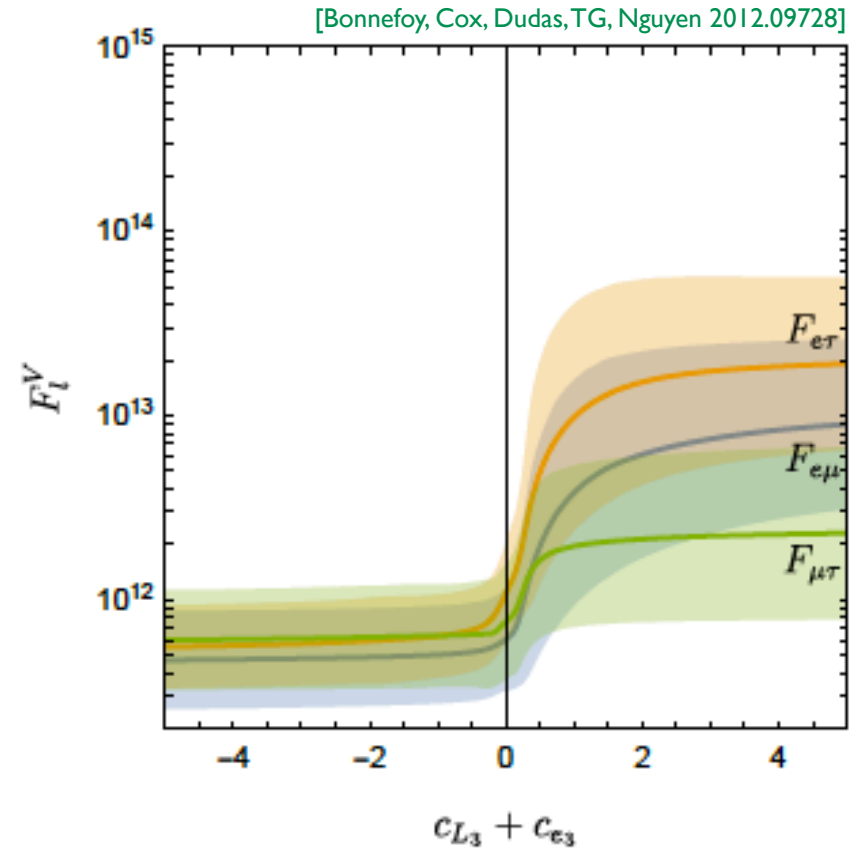
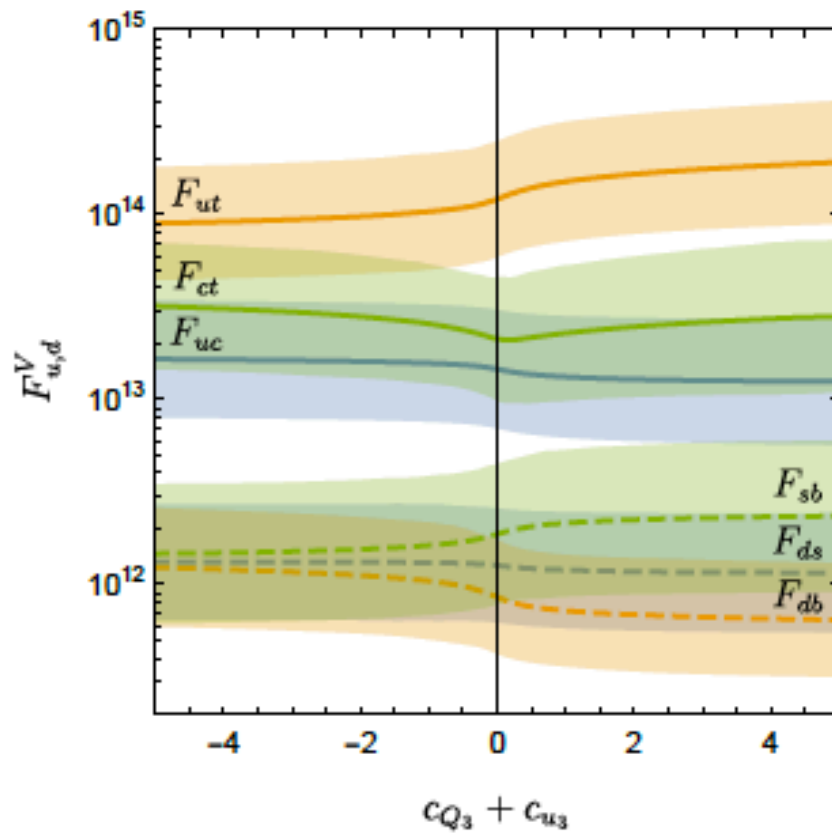
$$F_{u,d,\ell}^A \approx \mathcal{O}(F_{u,d,\ell}^V)$$

$$kz_{\text{IR}} = 10^{10}, g_5^2 k = 1.$$

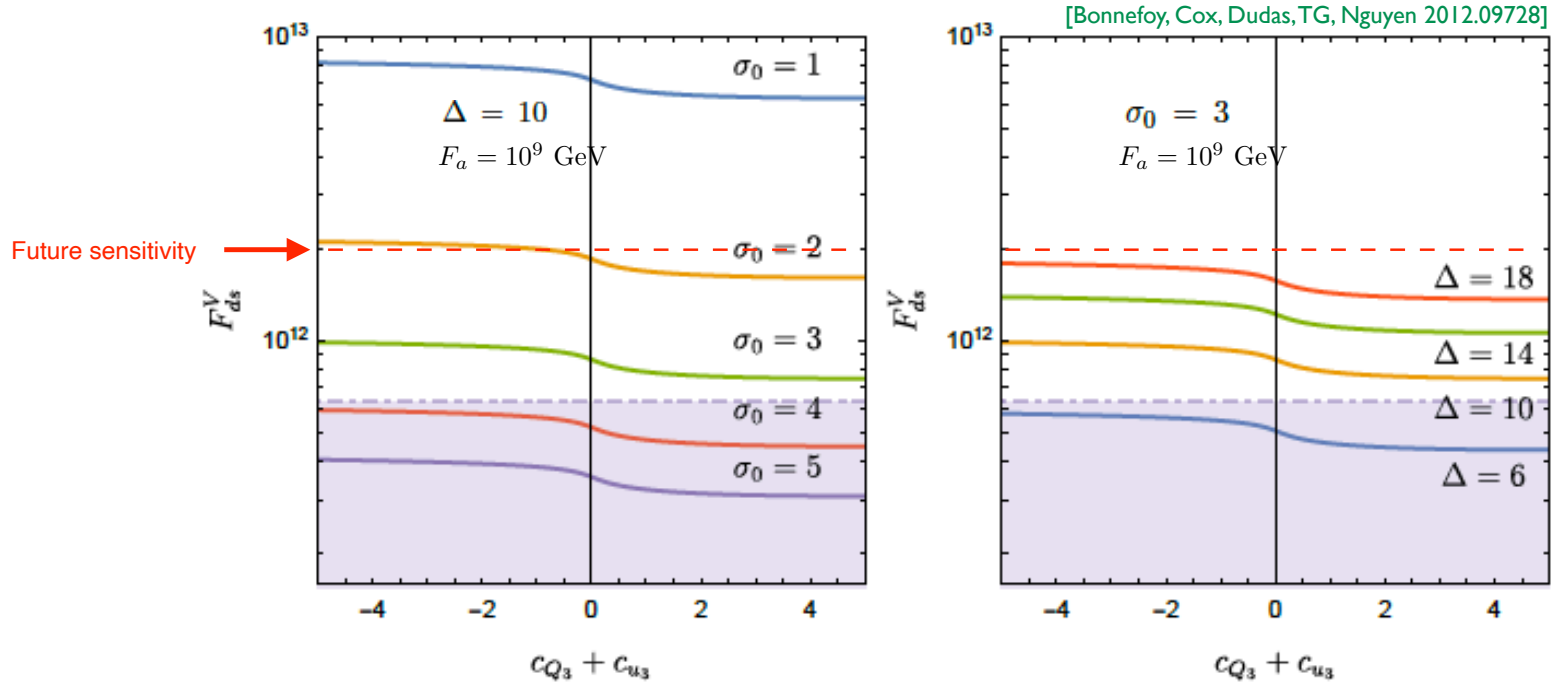
$$\Delta = 10 \quad \sigma_0 = 3.$$

$$F_a \simeq 10^9 \text{ GeV}.$$

Scan over  $y_{u,d,e}^{(5)} \sim 1$



[Bonnefoy, Cox, Dudas, TG, Nguyen 2012.09728]



Experimental limits :  
 [Martin Camalich, Pospelov, Vuong, Ziegler, Zupan 2002.04623]

$$(F_d^V)_{12} \gtrsim 6.8 \times 10^{11} \text{ GeV}$$

( $K^+ \rightarrow \pi^+ a$  decays)  
 [  $(8.3 \times 10^{11} \text{ GeV})$  Bauer, Neubert, Renner, Schnubel, Thamm 2102.13112; 2110.10698 ]

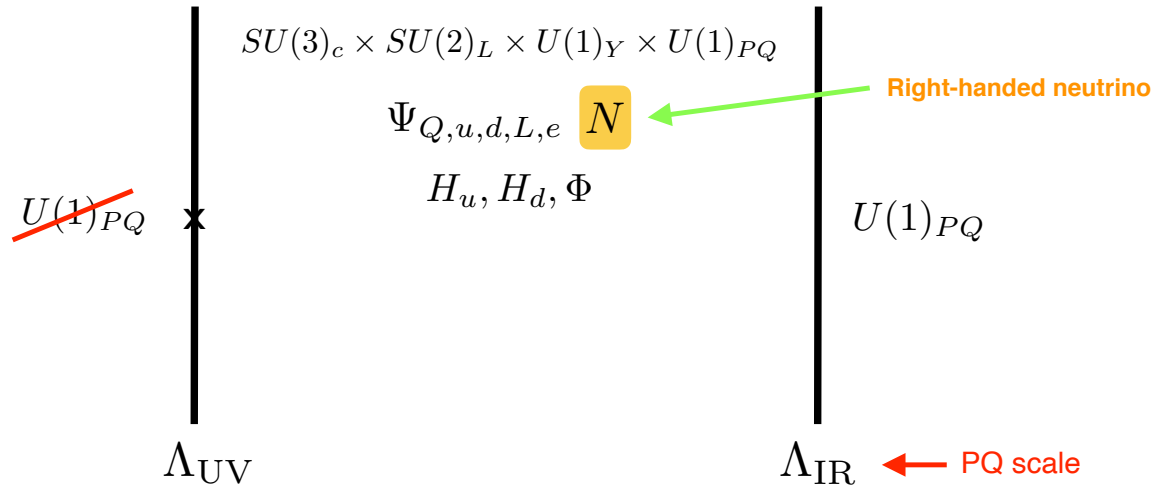
➡

$$\sigma_0 \gtrsim 4, \quad \Delta \gtrsim 6$$

# Neutrino-Axion Model

[Cox, TG, Nguyen: 2107.14018]

[Also Mohapatra, Senjanovic 1983; Holman, Lazarides, Shafi 1983; Langacker, Peccei, Yanagida 1986; Bertolini et al 2015; Clarke, Volkas 2016]



Bulk Yukawa coupling:  $\frac{1}{\sqrt{k}} \left( y_{\nu,ij}^{(5)} \bar{L}_i N_j H_u + y_{e,ij}^{(5)} \bar{L}_i E_j H_d + \text{h.c.} \right)$  ← PQ charge of N forbids bulk Majorana terms

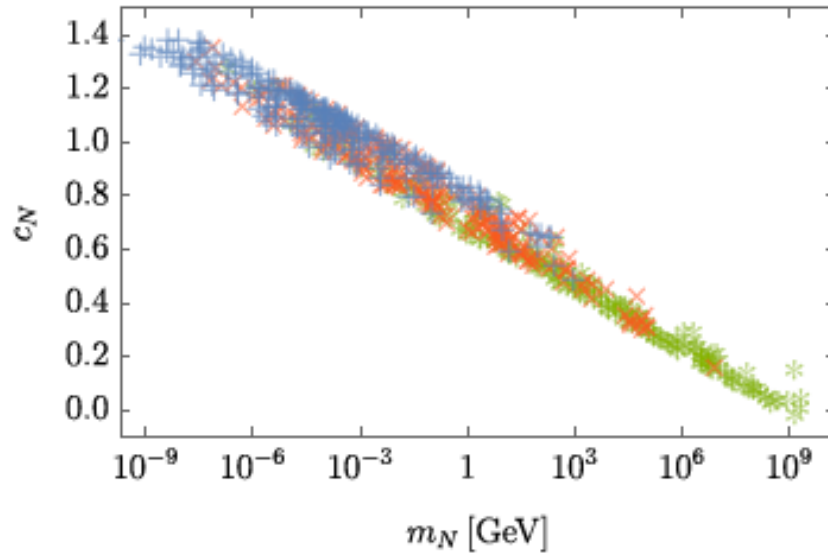
UV boundary:  $\frac{1}{2} \left( b_{N,ij} \bar{N}_i^c N_j + \frac{y_{N,ij}^{(5)}}{k^{3/2}} \Phi \bar{N}_i^c N_j + \text{h.c.} \right)$

# Predictions:

Parameters chosen to explain axion quality, neutrino mass differences and PMNS angles

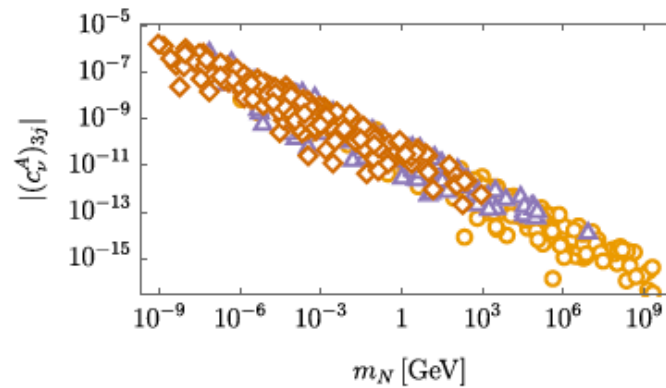
$$\sigma_0 = 0.1, \lambda = 0.1, \Delta = 10, \tan \beta = 3, kz_{\text{IR}} = 10^7 \quad \rightarrow \quad F_a \simeq 8.12 \times 10^9 \text{ GeV}$$

## ◆ Light sterile neutrinos!



$\rightarrow$   $1 \text{ eV} \lesssim m_N \lesssim 10^9 \text{ GeV}$

## ◆ Axion-neutrino couplings



$\rightarrow$  Probe local DM halo with neutrino oscillations NEW  
[TG, Shkerin: 2305.06441]

# Local Dark Matter Halo [TG, Shkerin: 2305.06441]

Ultra-light particles (e.g. axion) = dark matter

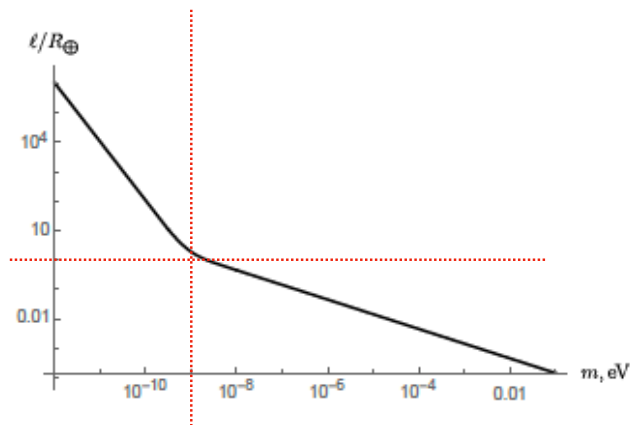
➡ can form dense compact objects  $\rho_{\text{local}} \gg \rho_{\text{average}}$

Consider a local halo surrounding the Earth:

Nonrelativistic ansatz:  $\varphi(r, t) = \sqrt{\frac{2c}{m}} \left( \Psi(r, t) e^{-imc^2 t} + c.c. \right)$        $\Psi(r, t) = \psi(r) e^{-iEt}$

$$-\frac{1}{x^2} \frac{d}{dx} \left( x^2 \frac{d\psi}{dx} \right) + 2(\mathcal{M}\Phi - \mathcal{E})\psi = 0$$

$x = r/R, \quad \mathcal{M} = Gm^2 MR, \quad \mathcal{E} = EmR^2$



HALO SIZE

$$\ell \sim R_{\oplus} \left( \frac{10^{-9} \text{ eV}}{m} \right)^2 \quad m \ll 10^{-9} \text{ eV}$$

$$\ell \sim R_{\oplus} \left( \frac{10^{-9} \text{ eV}}{m} \right)^{1/2} \quad m \gg 10^{-9} \text{ eV}$$

# How to probe halo? Use neutrino oscillations!

Neutrino evolution (Assume 2 flavour, ultrarelativistic, neglect decoherence and dispersion)

$$i \frac{d\nu_a}{dz} = H_{ab} \nu_b \quad H = \frac{1}{2E} U_0 \text{diag}(0, \Delta m_{0,21}^2, \dots) U_0^\dagger + \Delta H$$

## Dark matter-neutrino interaction

$$\mathcal{L}_{5,\text{int}} = -\frac{g_{ab}}{\Lambda_5} \partial_\mu \varphi \bar{\psi}_{La} \gamma^\mu \psi_{Lb} \quad \Rightarrow \quad \Delta H_5 = \frac{m}{\Lambda_5} g \varphi$$

where  $\varphi(r, t) = f(r) \cos(mt + \delta)$   
 $(\dot{\varphi} \sim m\varphi \gg \nabla\varphi)$

## Survival probability

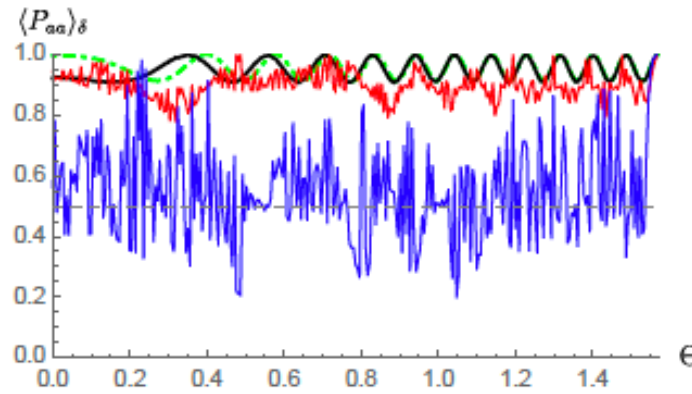
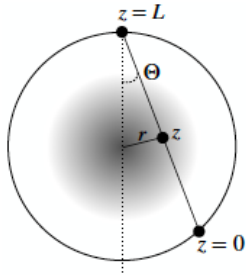
z-dependent eigenvalues affect survival probability!

$$P_{ab}(L) = \left| \sum_i U_{ai}(0) e^{-\frac{i}{2E} \int_0^L dz m_i^2(z)} U_{bi}^*(L) \right|^2$$

$$\epsilon \equiv \frac{\beta f_0}{m} \sim \left( \frac{\beta}{10^{-22}} \right) \left( \frac{m}{10^{-10} \text{ eV}} \right) \left( \frac{M_{\text{halo}}}{10^{15} \text{ kg}} \right)^{1/2} \quad \text{expansion parameter}$$

$$\eta \equiv \frac{mE}{\Delta m_0^2} = \left( \frac{2.5 \times 10^{-3} \text{ eV}^2}{\Delta m_0^2} \right) \left( \frac{m}{10^{-10} \text{ eV}} \right) \left( \frac{E}{25 \text{ MeV}} \right) \quad \text{number of halo oscillations per neutrino oscillation}$$

# Small halo ( $\ell < R_\oplus$ )



$$m = 3 \times 10^{-9} \text{ eV}, E = 1 \text{ GeV}$$

$$\Delta m_0^2 \approx 2.5 \times 10^{-3} \text{ eV}, \sin^2 2\theta_0 \approx 0.087.$$

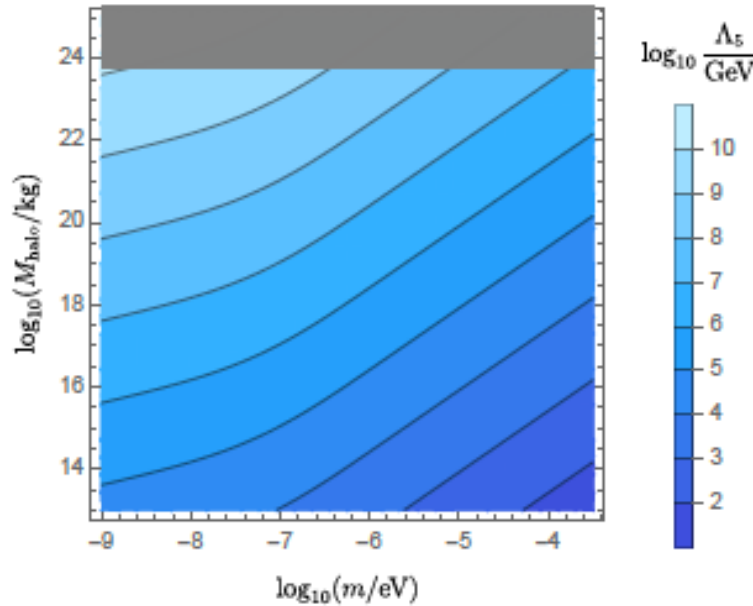
$$\epsilon = 0.1 \text{ (black line)}$$

$$\epsilon = 0.5 \text{ (thin, red line)}$$

$$\epsilon = 2 \text{ (thin, blue line)}$$

$$\epsilon \sim \left( \frac{\beta}{10^{-23}} \right) \left( \frac{10^{-9} \text{ eV}}{m} \right)^{5/4} \left( \frac{M_{\text{halo}}}{10^{15} \text{ kg}} \right)^{1/2}$$

(10% deviation)



$$E = 1 \text{ GeV (atmospheric neutrinos)}$$

$$\Delta m_0^2 = 2.5 \times 10^{-3} \text{ eV}^2$$

$$\Theta_{\text{res}} = 30^\circ$$

$$\eta = 400 \left( \frac{2.5 \times 10^{-3} \text{ eV}^2}{\Delta m_0^2} \right) \left( \frac{m}{10^{-9} \text{ eV}} \right) \left( \frac{E}{1 \text{ GeV}} \right)$$

Can probe scales up to  $\Lambda_5 \sim 10^{10} \text{ GeV}$

# 2. Axion mass from 5D small instantons

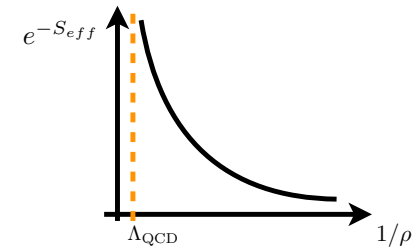
[TG, Khoze, Pomarol, Shirman: 2001.05610]

QCD axion mass:

$$m_a^2 = \frac{\mathcal{T}}{f_a^2} \quad \mathcal{T} \equiv -i \int d^4x \langle 0 | T \left[ \frac{1}{32\pi^2} G_{\mu\nu}^a \tilde{G}^{a\mu\nu}(x), \frac{1}{32\pi^2} G_{\rho\sigma}^b \tilde{G}^{b\rho\sigma}(0) \right] | 0 \rangle \quad \text{“topological susceptibility”}$$

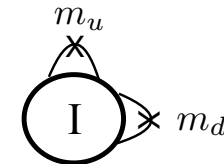
Dilute instanton gas approximation:

$$\mathcal{T} \propto \int \frac{d\rho}{\rho^5} C[3] \left( \frac{2\pi}{\alpha_s(1/\rho)} \right)^6 e^{-\frac{2\pi}{\alpha_s(1/\rho)}}$$



QCD asymptotically free  $\Rightarrow \mathcal{T} \propto \Lambda_{QCD}^4$  “Large instantons”  $\rho \sim 1/\Lambda_{QCD}$

Fermion zero modes:  $(\rho m_f)^{N_f} \longrightarrow$  suppression  $\frac{\prod_f m_f}{\Lambda_{QCD}^{N_f}}$

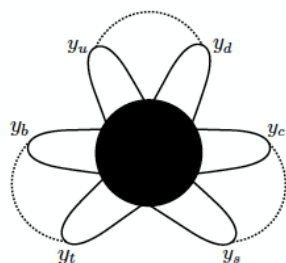


$$\Rightarrow \boxed{m_{a,QCD}^2 = \frac{m_u m_d}{(m_u + m_d)^2} \frac{m_\pi^2 f_\pi^2}{f_a^2}}$$



# How to enhance QCD axion mass?

- Change QCD coupling in UV  $\alpha_s(1/\rho) \sim 1$  “Small instantons”  $\rho \sim 1/\Lambda_{UV}$
- Close fermion loops with Higgs boson



$$\kappa_f = \frac{y_u y_d y_c y_s y_t y_b}{4\pi 4\pi 4\pi 4\pi 4\pi 4\pi} \approx 10^{-23} \quad (\text{otherwise } \frac{m_u m_d m_c m_s m_b m_t}{\Lambda_{UV}^6})$$



$$m_a^2 f_a^2 \sim \frac{1}{8} \Lambda_{\text{QCD}}^4 + \Lambda_I^4$$

new contribution

where  $\Lambda_I \gg \Lambda_{\text{QCD}}$

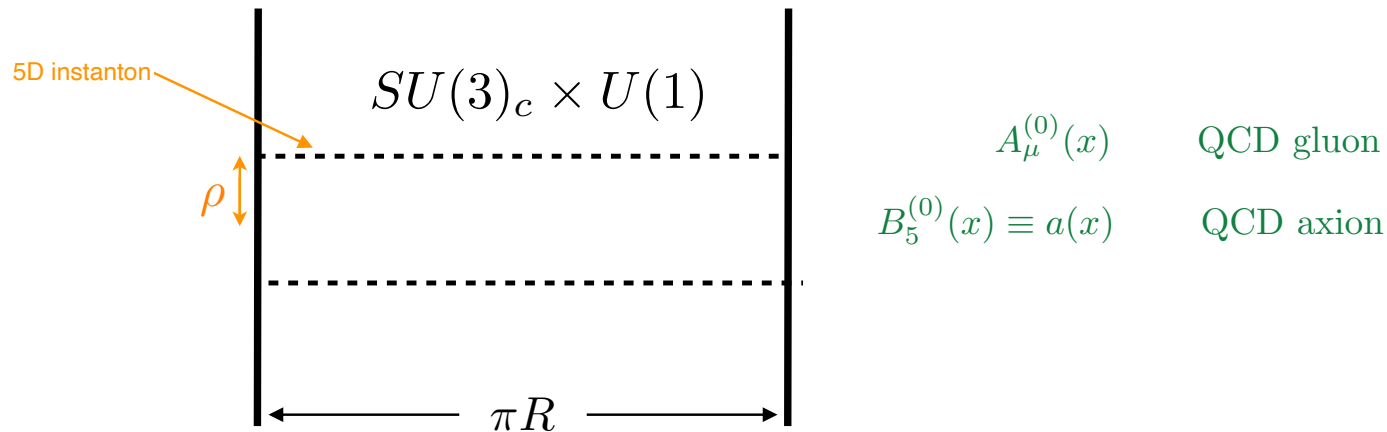
**Use 5th dimension to make QCD axion heavy!**

# QCD in 5D

Flat space 5D metric:

$$ds^2 = dx^2 + dy^2$$

$$S_5 = - \int d^4x \int_0^L dy \left( \frac{1}{4g_5^2} \text{Tr}[G_{MN}^2] + \frac{b_{CS}}{32\pi^2} \epsilon^{MNRST} B_M \text{Tr}[G_{NR}G_{ST}] + \frac{1}{4g_5^2} F_{MN}^2 + \dots \right)$$



5D instanton:

$$A_\mu^a(x, y) = A_\mu^{(I)a}(x) = \frac{2\eta_{a\mu\nu}(x-x_0)_\nu}{(x-x_0)^2 + \rho^2}, \quad A_5^a(x, y) = 0$$

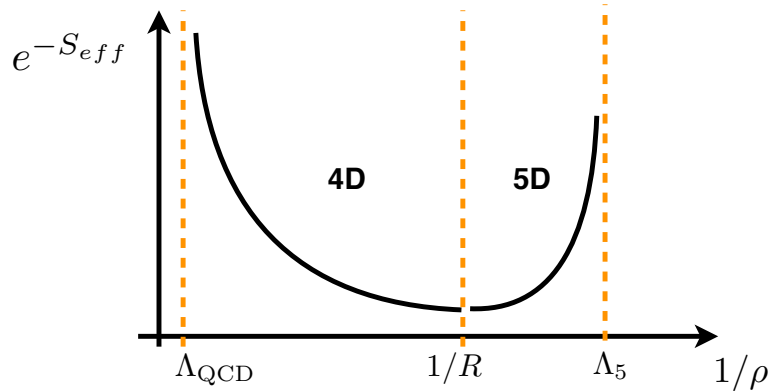


$$S_5^{(I)} = \frac{8\pi^3 R}{g_5^2} = \frac{2\pi}{\alpha_s}$$

Finite action

# 5D small instantons

Fluctuations + Kaluza-Klein contributions



$$\mathcal{T} \sim \int_{1/\Lambda_5}^R \frac{d\rho}{\rho^5} C[3] \left( \frac{2\pi}{\alpha_s(1/R)} \right)^6 e^{-S_{\text{eff}}} \equiv \frac{K}{R^4} \propto m_a^2 f_a^2$$

small instantons!



$$S_{\text{eff}} = \frac{2\pi}{\alpha_s(1/R)} - 3\xi(R/\rho) \frac{R}{\rho} + b_0 \ln \frac{R}{\rho}$$

power law term!

$$\xi(R/\rho) \sim 1/3$$

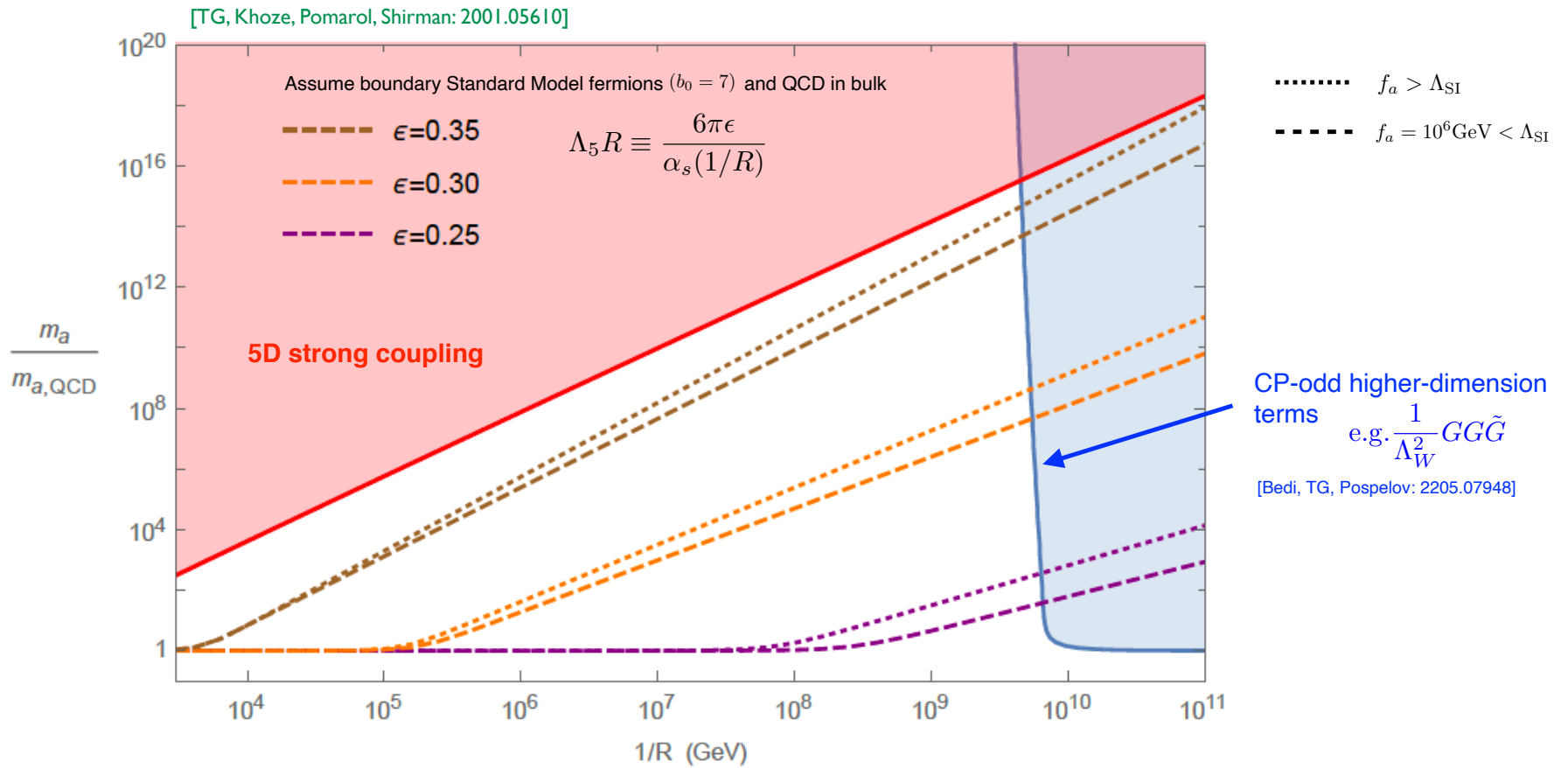
$$R/\rho \gtrsim 20$$



$$K \simeq C[3] \left( \frac{2\pi}{\alpha_s(1/R)} \right)^6 (\Lambda_5 R)^{3-b_0} e^{-\frac{2\pi}{\alpha_s(1/R)} + \Lambda_5 R}$$

power law contribution can overcome suppression

Valid up to  $\frac{g_5^2 \Lambda_5}{24\pi^3} \sim 1$  or  $\Lambda_5 R \lesssim \frac{6\pi}{\alpha_s}$



Small 5D instantons can dominate for  $\frac{1}{R} \gtrsim 100 \text{ TeV}$

# Enhanced EDMs

[Bedi, TG, Pospelov: 2205.07948]

Higher-dimension CP-odd sources are enhanced by small instantons

Weinberg operator:  $\mathcal{L} \supset \frac{1}{\Lambda_W^2} GG\tilde{G}$  [ Also fermion operator:  $\mathcal{L} \supset \sum_{ijkl} \frac{\lambda_{ijkl}}{\Lambda_F^2} \bar{\psi}_i \psi_j \bar{\psi}_k \psi_l$  ]

→  $V(a) = \chi_W(0) \left(\frac{a}{f_a}\right) + \frac{1}{2} \chi(0) \left(\frac{a}{f_a}\right)^2$

where  $\chi_W(0) = -i \lim_{k \rightarrow 0} \int d^4x e^{ikx} \langle 0 | T \left\{ \frac{1}{32\pi^2} GG\tilde{G}(x), \frac{1}{\Lambda_W^2} GG\tilde{G}(0) \right\} | 0 \rangle$

→  $\left\langle \frac{a}{f_a} \right\rangle \equiv \theta_{\text{ind}} = -\frac{\chi_W(0)}{\chi(0)} \propto \frac{\Lambda_{\text{SI}}^2}{\Lambda_W^2}$  ← Enhanced by  $\frac{\Lambda_{\text{SI}}}{\Lambda_{\text{QCD}}}$

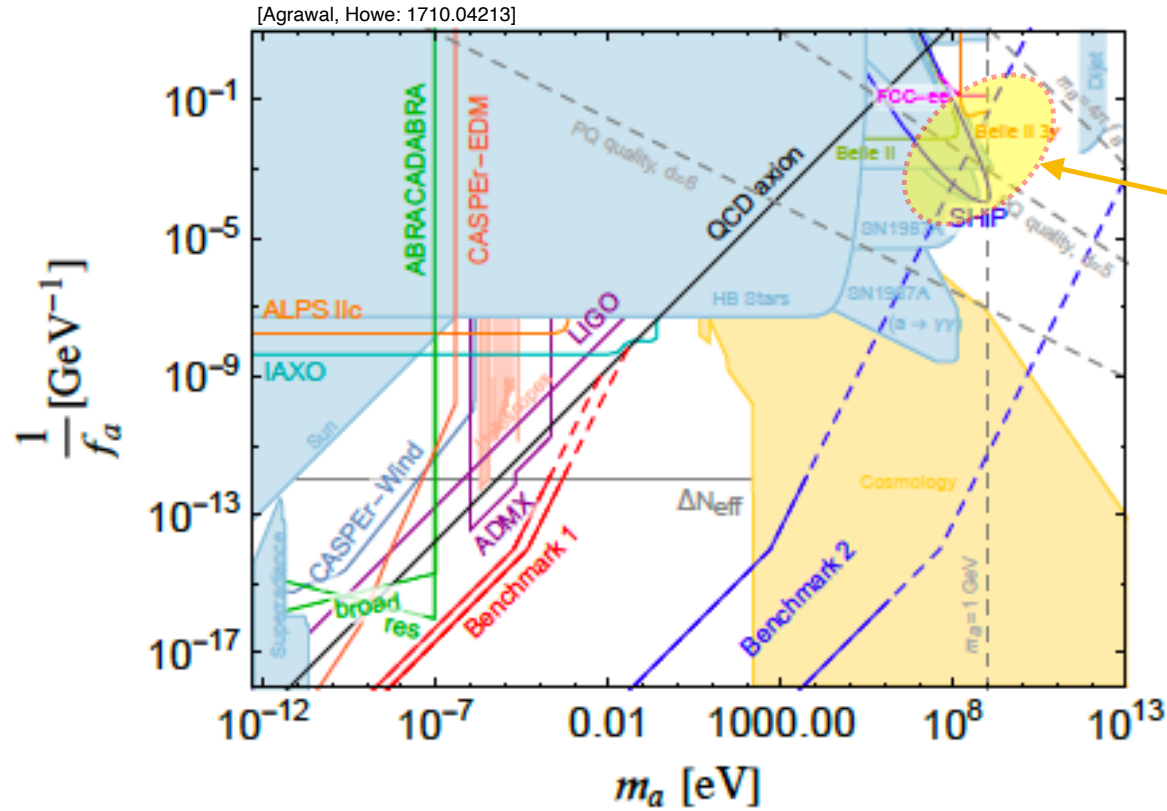
Neutron EDM:

$|\theta_{\text{ind}}| \lesssim 10^{-10}$

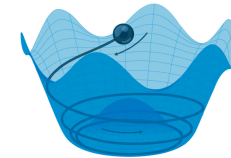


$\frac{\Lambda_{\text{SI}}}{\Lambda_W} \lesssim 10^{-6}$

# Heavy Axion Limits




Favoured by axiogenesis!  
[Co, TG, Harigaya, 2206.00678]



**Heavy QCD axion** (1 MeV – 10 GeV)  
**solves strong CP + baryon asymmetry!**

# Other possibilities:

 **Strong QCD** [Holdom, Peskin 1982] [Flynn, Randall 1987]

 **Enlarge QCD color**

$$SU(3 + N') \rightarrow SU(3)_c \times SU(N')$$


[Dimopoulos, Susskind '79; Dimopoulos '79]

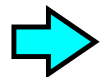
$$SU(3 + N) \times SU(N)' \rightarrow SU(3)_c \times SU(N)_D$$

[TG, Nagata, Shifman: 1604.01127]  
[Gaillard, Gavela, Houtz, Quilez, del Rey: 1805.06465]  
[Valenti, Vecchi, Xu: 2206.04077]

$$SU(3)_1 \times SU(3)_2 \times \cdots \times SU(3)_k \rightarrow SU(3)_c$$

[Agrawal, Howe 1710.04213]  
[Csaki, Ruhdorfer, Shirman 1912.02197]

 **Mirror QCD** [Rubakov '97] [Bereziani, Gianfagna, Gianotti '00]  
[Dimopoulos, Hook, Huang, Marques-Tavares: 1606.03097]  
[Hook, Kumar, Liu, Sundrum: 1911.12364]



**Axion mass is sensitive to UV completion!**

# Summary

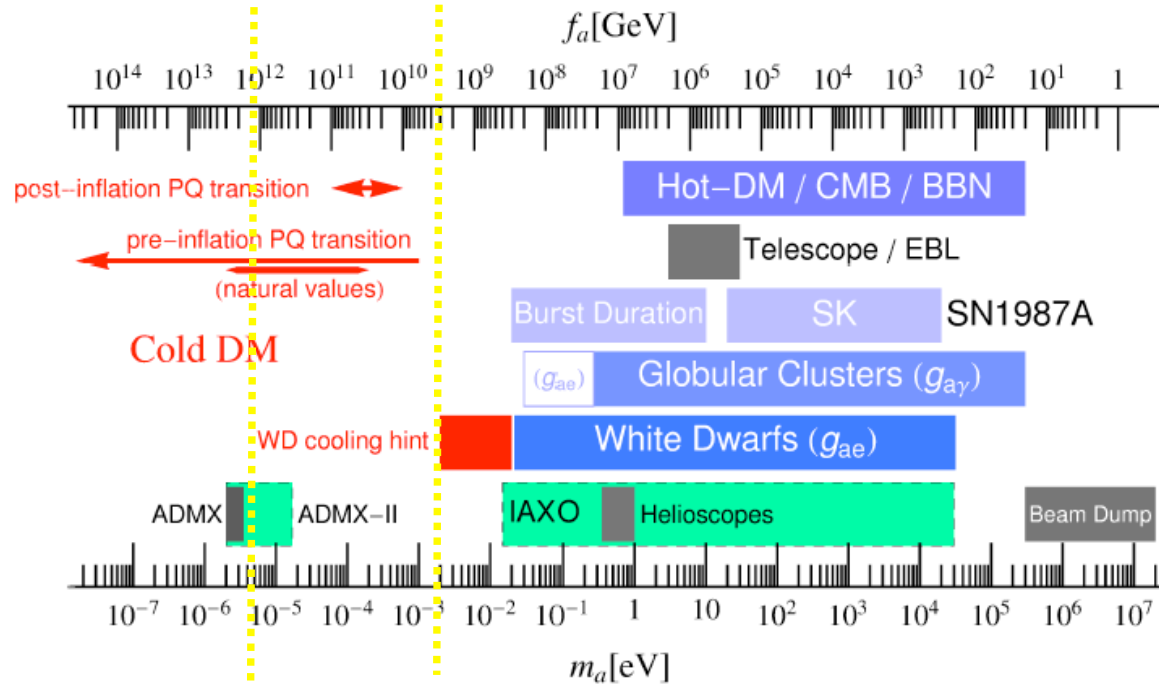
- Axion quality problem can be solved in 5D warped dimension
  - *dual to 4D dynamical axion with accidental PQ symmetry*
- “Flavored” warped axion
  - *solves axion quality and explains fermion mass hierarchy*
  - *predicts flavor-violating axion-fermion couplings*
  - *light sterile neutrinos*
- 5D small instantons
  - *can enhance axion mass and not spoil strong CP solution*
  - *axion mass could be a sensitive probe of UV physics!*





# Extra Slides

# QCD axion limits



$$10^9 \text{ GeV} \lesssim f_a \lesssim 10^{12} \text{ GeV}$$

$$\frac{1}{f_a} J^\mu \partial_\mu a$$



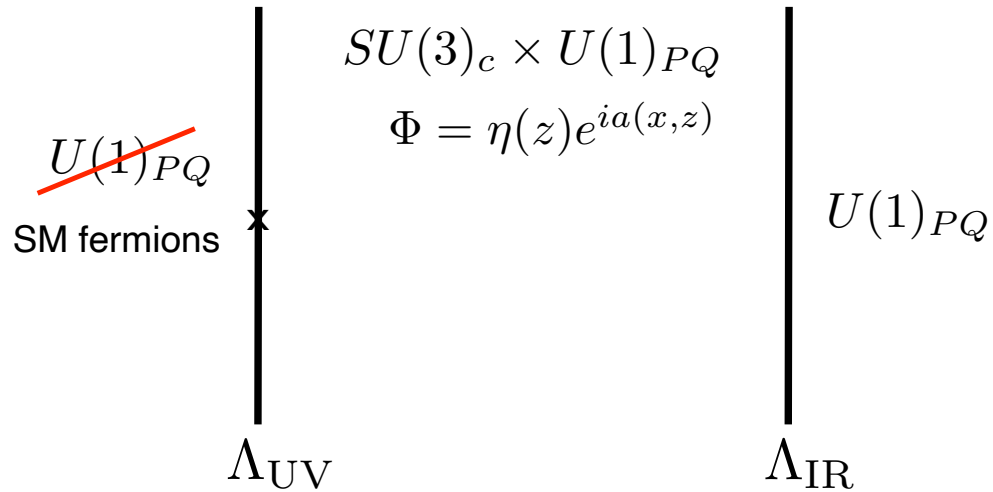
axion weakly-coupled - “invisible”

$$m_a^2 f_a^2 \sim \frac{1}{8} \Lambda_{\text{QCD}}^4$$



$$10^{-5} \text{ eV} \lesssim m_a \lesssim 10^{-3} \text{ eV}$$

# Axion-Gluon Coupling



Bulk Chern-Simons term: 
$$-\frac{\kappa}{32\pi^2} \int_{z_{UV}}^{z_{IR}} d^5x \epsilon^{MNPQR} V_M G_{NP}^a G_{QR}^a$$
 ← generates axion-gluon coupling

Under 5D gauge transformation:  $V_M \rightarrow V_M + \partial_M \alpha$

→ 
$$\delta S = -\frac{\kappa}{32\pi^2} \left[ \int_{z_{UV}}^{z_{IR}} d^4x \alpha(x^\mu, z) \epsilon^{\mu\nu\rho\sigma} G_{\mu\nu}^a G_{\rho\sigma}^a \right]$$

Add IR boundary term:

$$\frac{\kappa}{32\pi^2} \int d^4x a G\tilde{G} \Big|_{z_{IR}}$$

Obtain:

$$\mathcal{S}_{eff} = \int d^4x \left( \frac{1}{2} a^{(0)} (\square - m_a^2) a^{(0)} + \frac{g_s^2}{32\pi^2 F_a} a^{(0)} G\tilde{G} \right)$$

where  $F_a \simeq \frac{1}{\kappa} \frac{\sigma_0}{\sqrt{\Delta - 1}} z_{IR}^{-1}$



$$(m_a^{(UV)})^2 = \frac{4l_{UV}\sigma_0(\Delta - 2)}{\kappa^2(\Delta - 4 + b_{UV})} \left( \frac{\kappa\sqrt{\Delta - 1}}{\sigma_0} \right)^\Delta \underbrace{\left( \frac{F_a}{\Lambda_{UV}} \right)^{\Delta - 4}}_{\text{(suppression for } F_a \ll \Lambda_{UV} \text{ and } \Delta > 4)} F_a^2$$

(suppression for  $F_a \ll \Lambda_{UV}$  and  $\Delta > 4$ )

# Numerical results

$$\frac{1}{(F_u^{V,A})_{ij}} \equiv \frac{(c_u^{V,A})_{ij}}{F_a}$$

$$kz_{\text{IR}} = 10^{10}, g_5^2 k = 1.$$

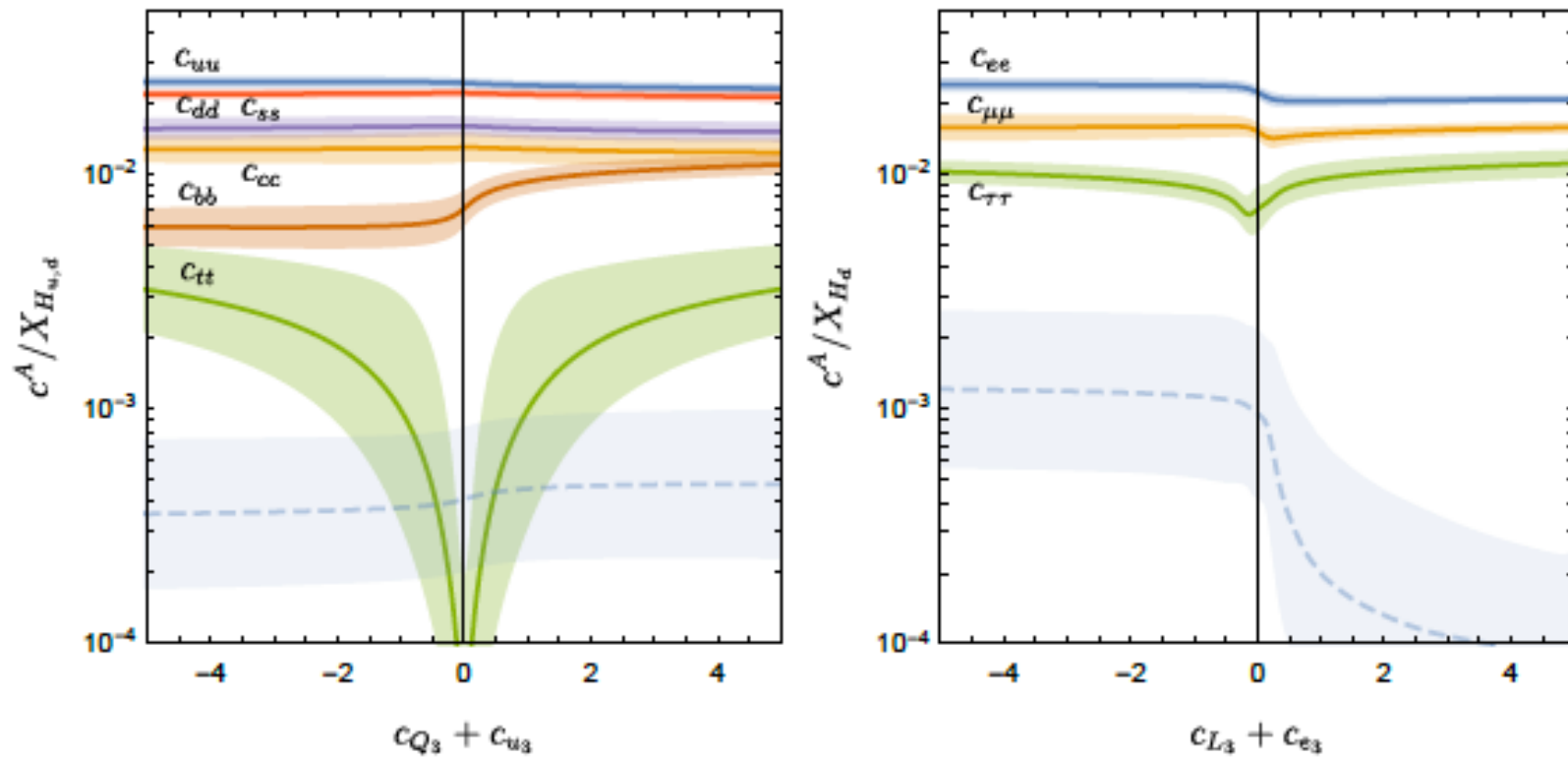
$$\Delta = 10 \quad \sigma_0 = 3.$$

$$F_a \simeq 10^9 \text{ GeV}.$$

Scan over  $y_{u,d,e}^{(5)} \sim 1$

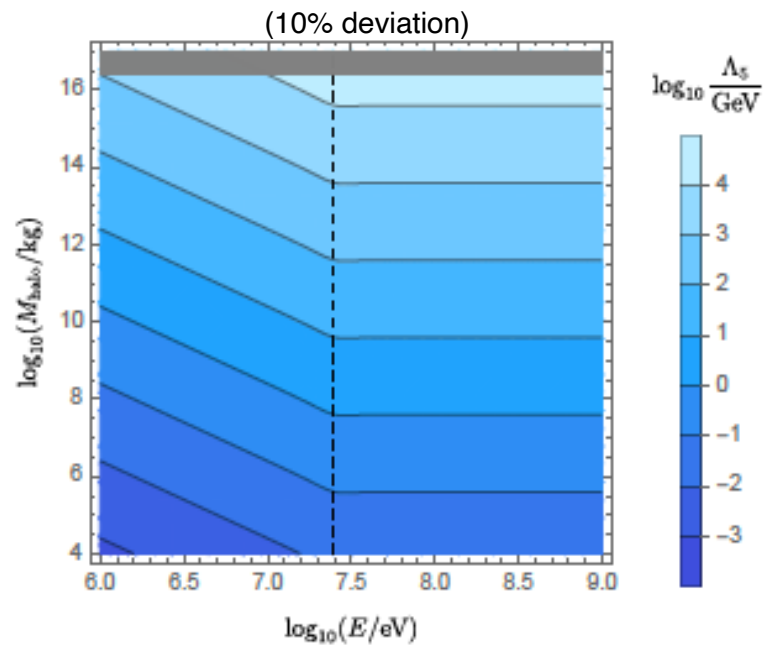
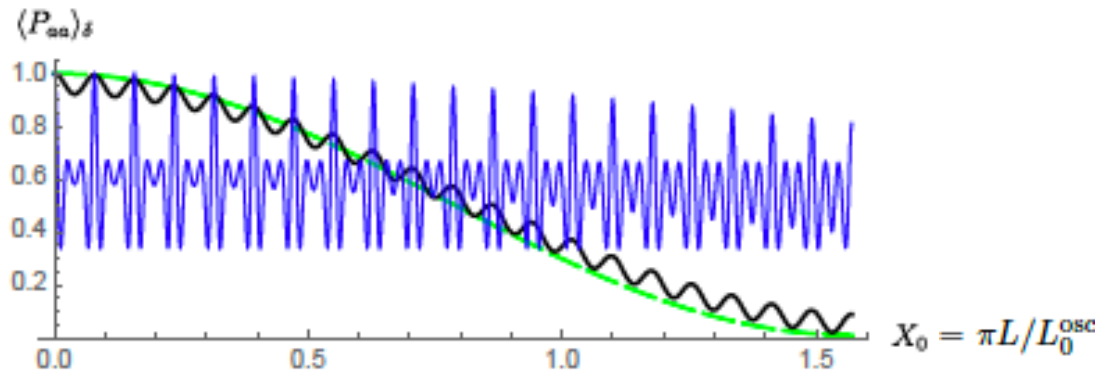
## Flavor-preserving couplings:

[Bonnefoy, Cox, Dudas, TG, Nguyen 2012.09728]



# Results

Big halo ( $l \sim R_{\oplus}$ )



$m = 10^{-10} \text{eV}$   
 $\Delta m_0^2 = 2.5 \times 10^{-3} \text{eV}^2$   
 $\eta = 1$  (dashed line)

## Higher dimension terms:

$$\Delta S_5 = -\frac{1}{4g_5^2} \int d^4x \int_0^L dy \frac{c_6}{\Lambda_5^2} \text{Tr} G_{MN} \square G^{MN}$$



$$S_{\text{eff}} = \frac{2\pi}{\alpha_s} + \frac{3\pi}{\alpha_s} \frac{c_6}{(\Lambda_5 \rho)^2} - 3\xi(R/\rho) \frac{R}{\rho} + \dots$$

Higher dimension contribution

Extremum:  
( $c_6 > 0$ )

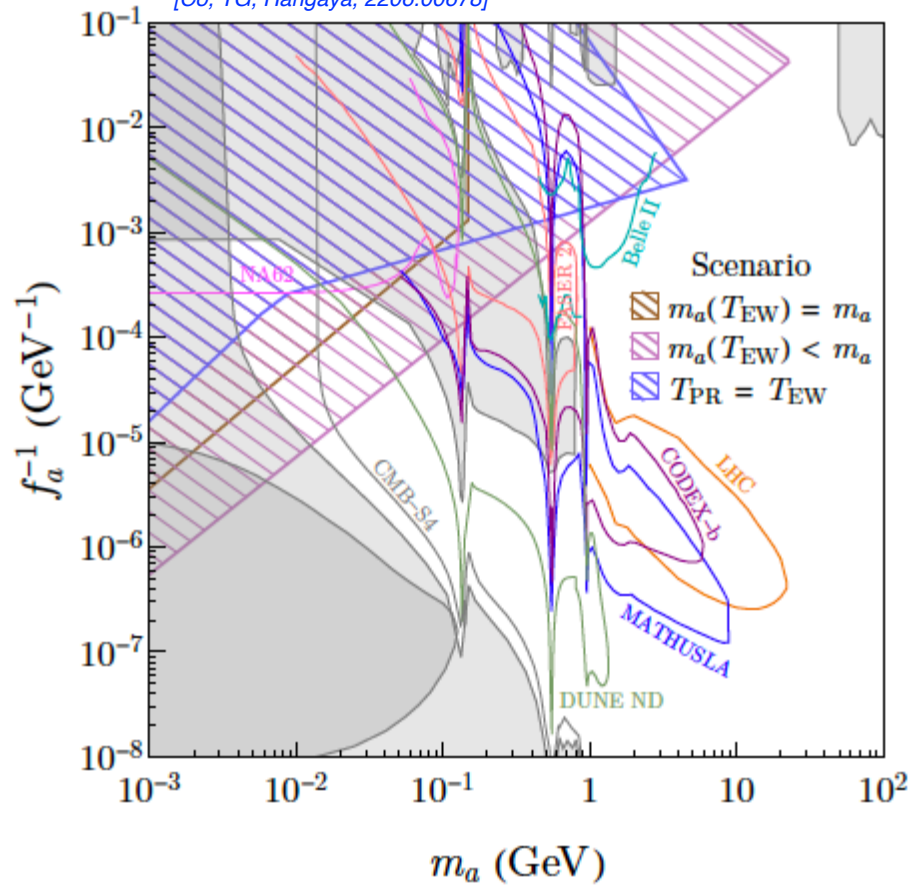
$$\frac{1}{\rho_*} \simeq \frac{3}{c_6} \xi(R/\rho) \left( \frac{g_5^2 \Lambda_5}{24\pi^3} \right) \Lambda_5$$

Provided  $\frac{g_5^2 \Lambda_5}{24\pi^3} \ll 1 \quad \Rightarrow \quad \rho_* \gg \frac{1}{\Lambda_5}$

i.e. instantons of size near UV cutoff ( $\Lambda_5$ ) are suppressed



[Co, TG, Harigaya, 2206.00678]



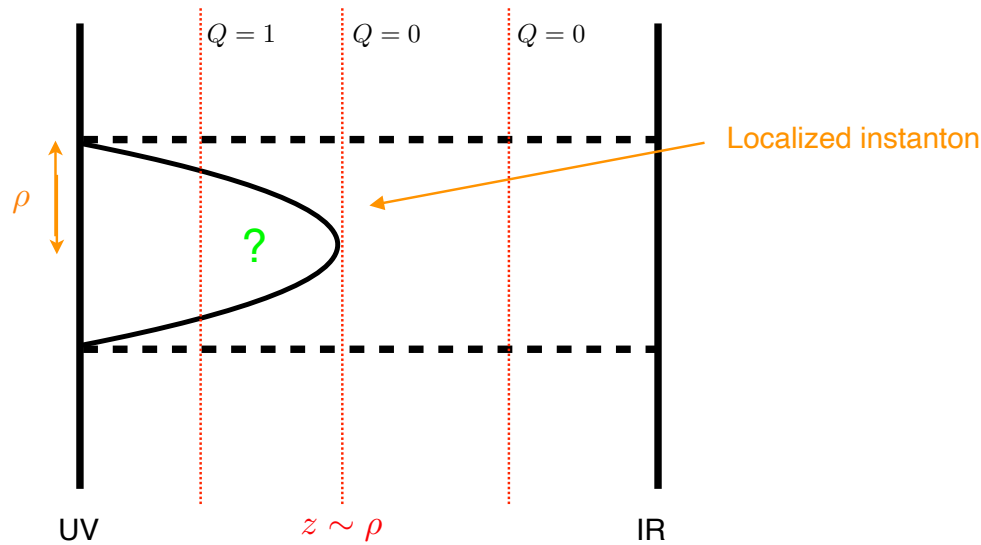
**Heavy QCD axion solves strong CP + baryon asymmetry!**

# Localized Instanton?

[TG, Pomarol: 2110.01762]

Running coupling:  $\frac{1}{g_4^2(\mu)} = \frac{1}{g_4^2} - \frac{L}{g_5^2} \log(\mu z_{UV})$

Topological charge:  $Q = \frac{1}{32\pi^2} \int d^4x G_{\mu\nu}^a \tilde{G}^{a\mu\nu}$



Axion mass:  $m_a^2 f_a^2 \propto e^{-S}$

$S_{\text{local}} = \frac{8\pi^2}{g_4^2(1/\rho)} \ll \frac{8\pi^2}{g_4^2(z_{IR}^{-1})}$

Larger axion mass enhancement compared to flat space?

However, topological charge conservation



NO localized instanton!

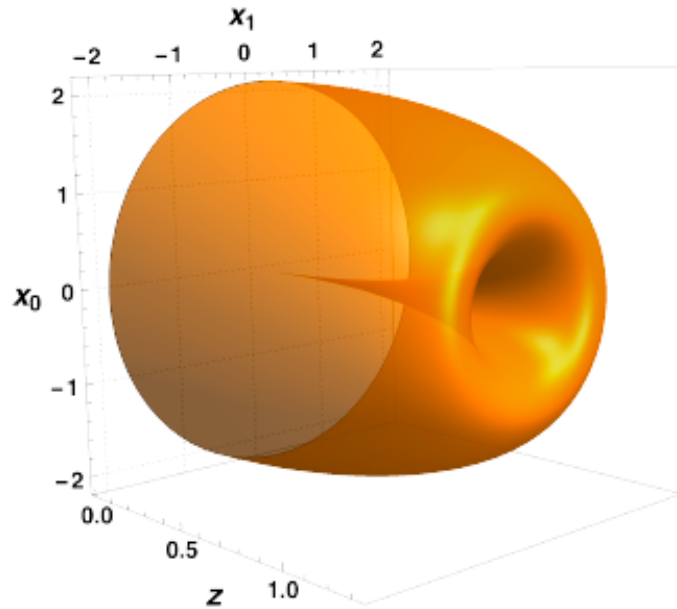


CFT dual *necessarily* contains colored fermions!

Instead, obtain instanton-anti-instanton solution:

$$A_\mu^a(x, z) = 2\eta_{\mu\nu}^a \frac{x_\nu}{x^2} f(x, z), \quad A_5^a(x, z) = 0$$

where  $f(x, z) = \frac{(x^2 + z^2)^2}{x^2 \rho^2 + (x^2 + z^2)^2}$



$$S = \frac{16\pi^2}{g_4^2(1/\rho)} \quad Q = 0 !$$

Also, obtain multi-instanton and new meron solutions [TG, Pomarol: 2110.01762]

e.g. 5D meron:  $Q = \frac{1}{2}$   $f(x, z) = \frac{1}{2} \frac{x^2}{x^2 + \frac{3}{4}z^2}$