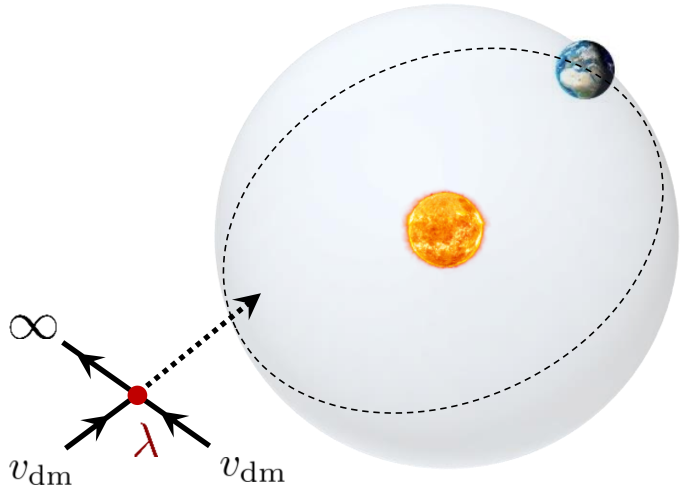
Formation of Ultralight Dark Matter Solar Halos



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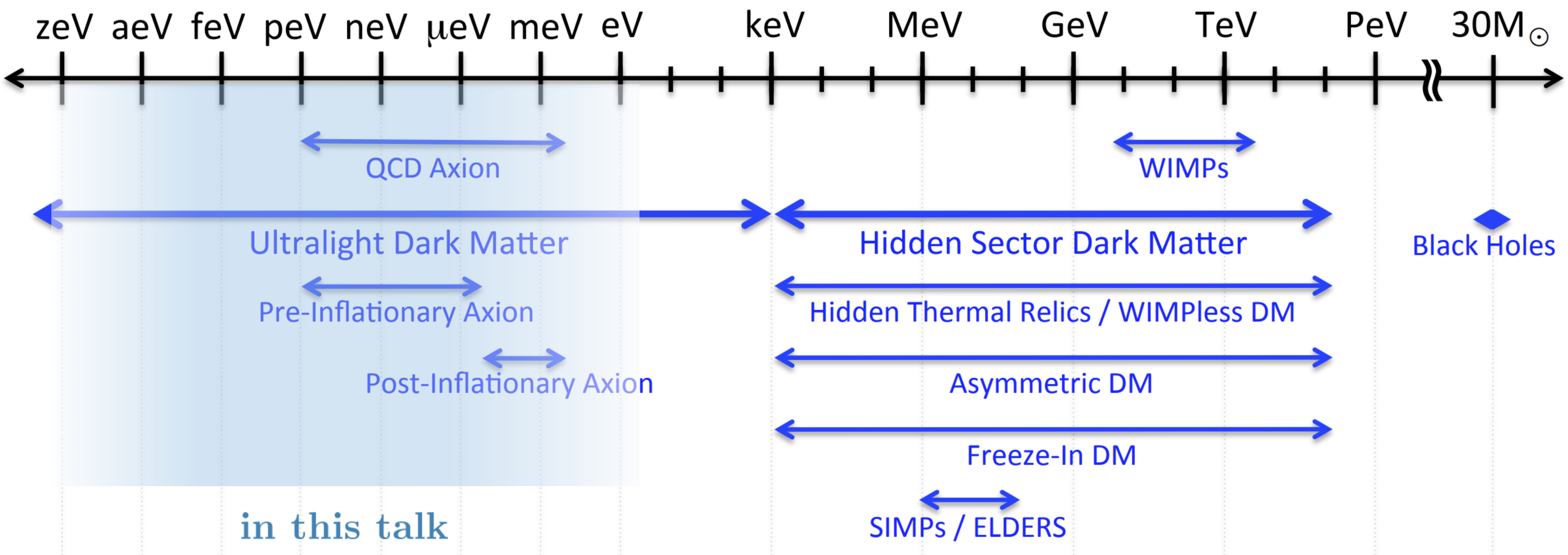
with **Budker, Eby, Jiang, Perez**

[to appear]

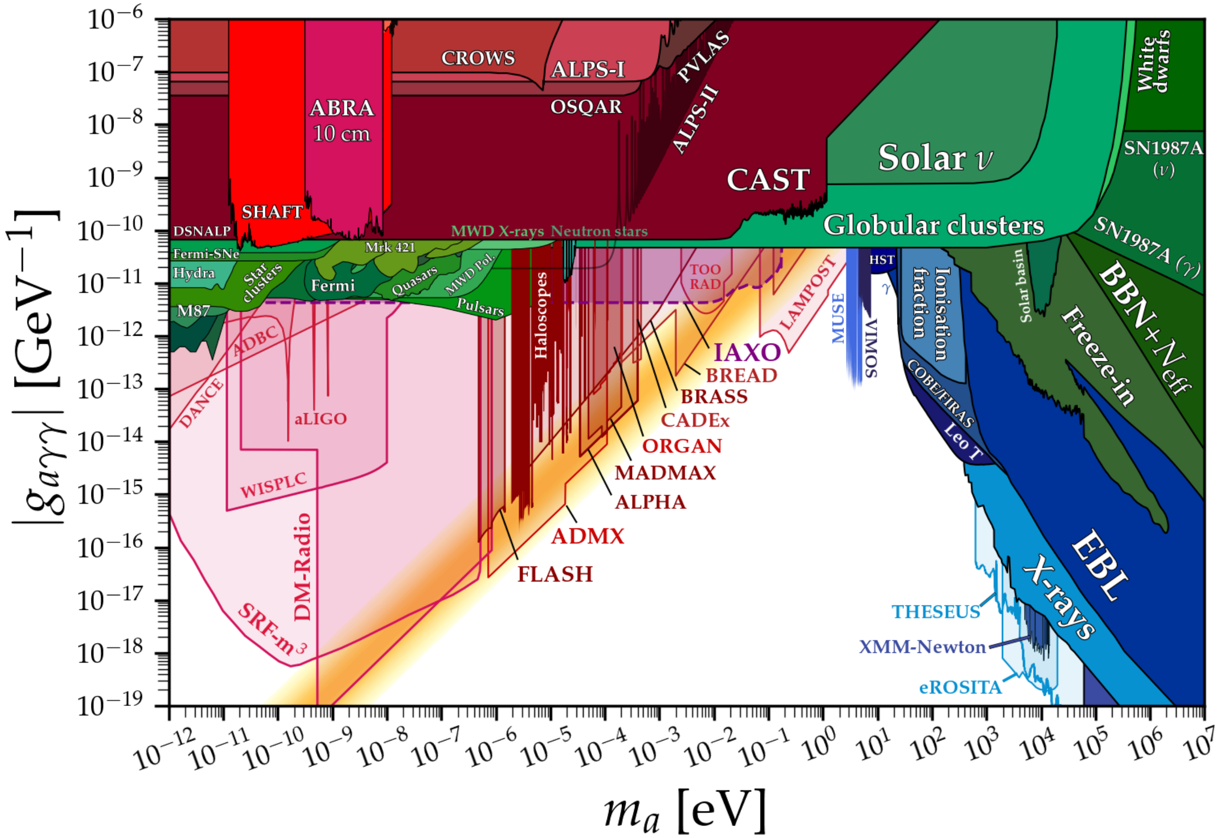
Dark Matter Candidates



- Occupation number $\gg 1 \implies$ boson, ϕ
- Classical equations of motion
- Automatically produced as dark matter relics

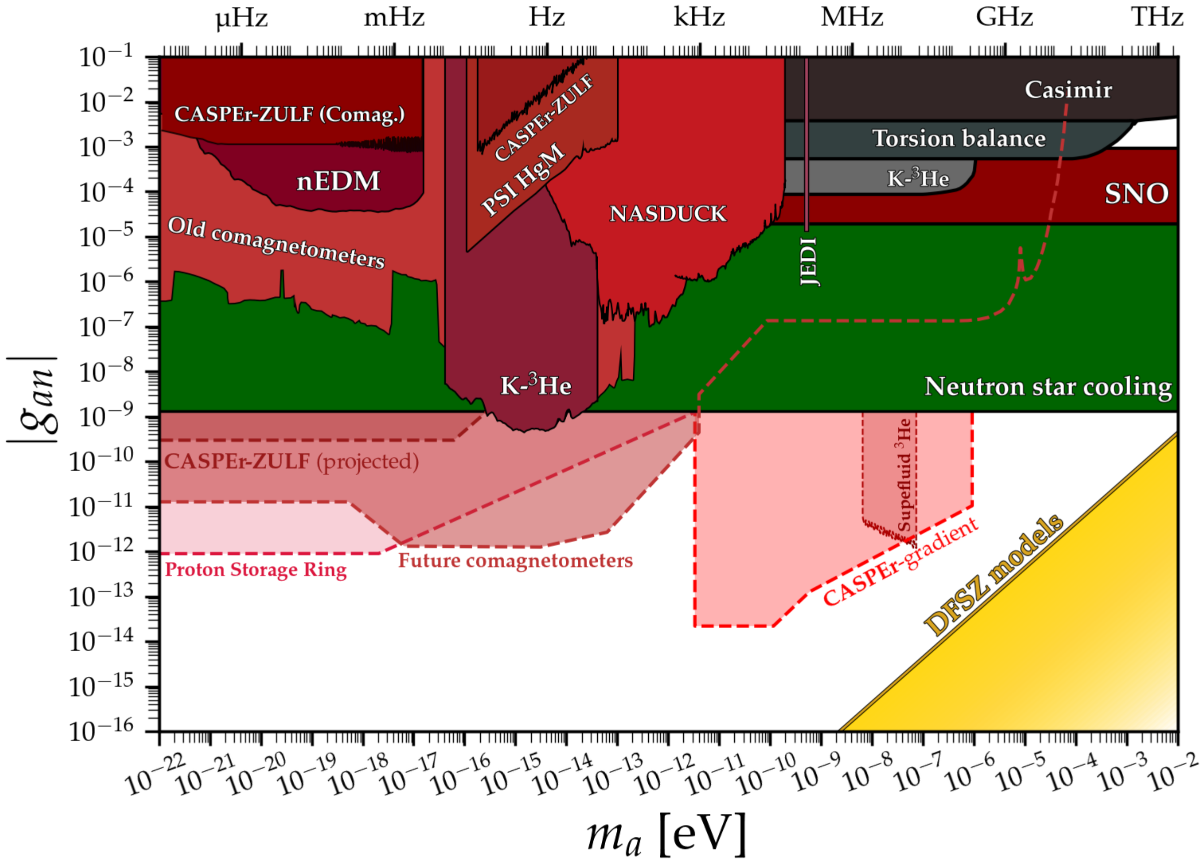
coupling to photons

$$\mathcal{L} \supset \frac{1}{4} g_{a\gamma\gamma} \phi F_{\mu\nu} \tilde{F}^{\mu\nu}$$



coupling to neutrons

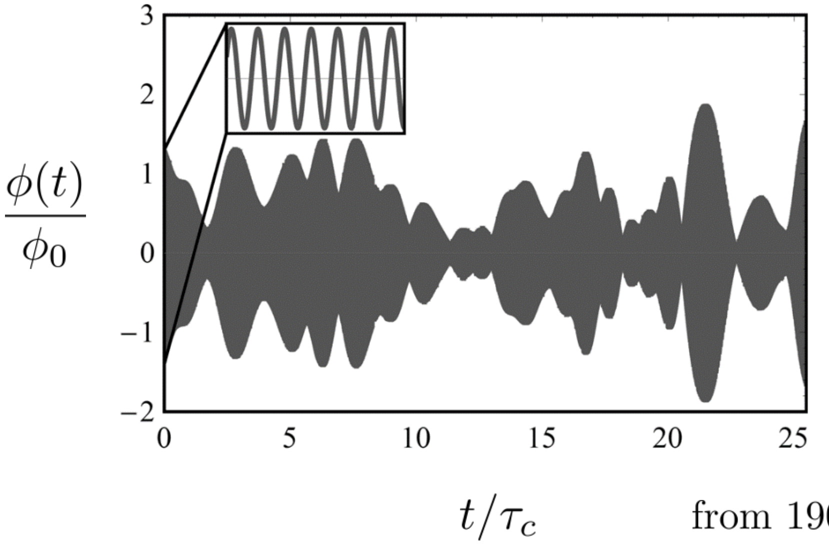
$$\mathcal{L} \supset g_{an} \frac{\partial_\mu \phi}{2 f_a} \bar{n} \gamma^\mu \gamma^5 n$$



https://cajohare.github.io/AxionLimits

Dark matter detection prospects depend on:

- Local density ρ in the neighborhood of the Sun
- Coherence time, τ_c



from 1905.13650

In this talk:

- ϕ is the dark matter
- $V(\phi) = \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4 + \ldots$

$\neq 0$ $\quad |\lambda| \lll 1$

axion $\rightarrow$ $\quad \lambda = -\frac{m^2}{f_a^2}$

$\rightarrow$ misalignment fixes $f_a = f_a(m; \theta_0)$

relation between m and f_a unfixed

Outline

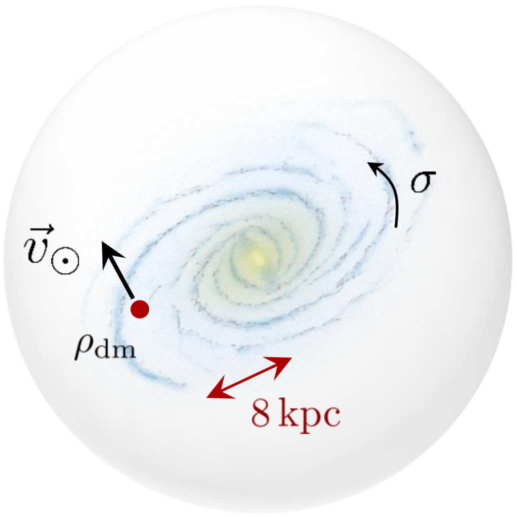
- **ULDM distribution and bound states**
- **Formation of the gravitational atom**
- **Some implication for direct detection**

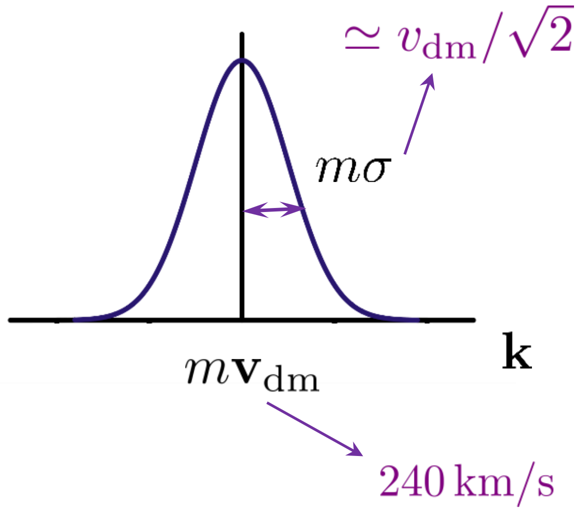
Local dark matter distribution

'Standard halo' model

- Local density $\rho_{\rm dm} \simeq 0.3 \div 0.4\,{\rm GeV/cm^3}$

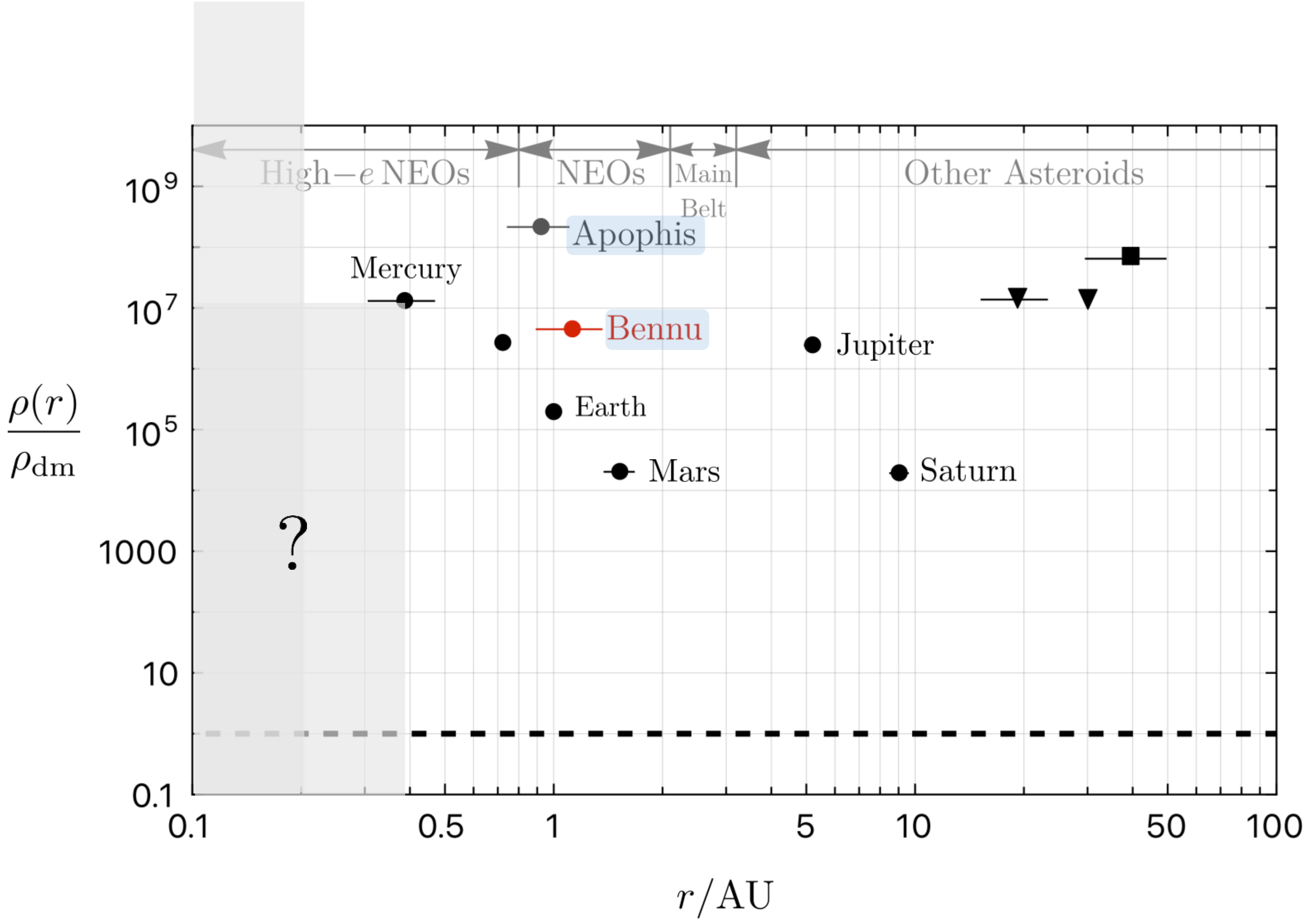
- Dark matter velocity in the frame of the Sun:
  - average $\mathbf{v}_{\rm dm} = -\mathbf{v}_\odot \simeq 240\,{\rm km/s}$
  - dispersion $\sigma \simeq 160\,{\rm km/s}$ $\simeq v_{\rm dm}/\sqrt{2}$

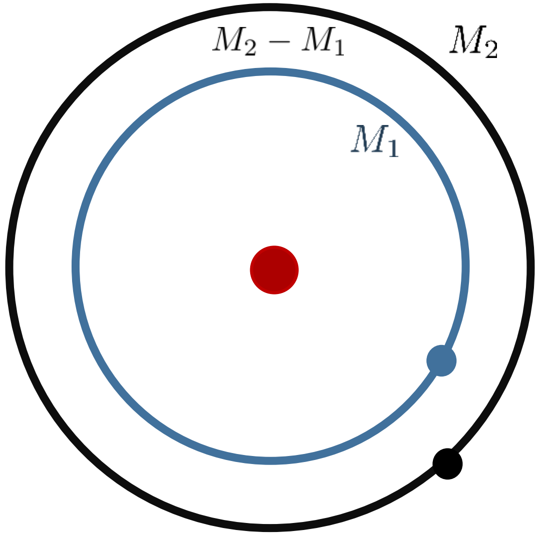




inferred from measurements on galactic scales $\longrightarrow$ insensitive to the 'very local' density

Gravitational bounds from planetary and asteroid motion





orbit comparison bounds $M_2 - M_1$

- processes inducing the capture of ULDM?

[Pitjev and Pitjeva '13; Tsai et al '22; Gron and Soleng '96; Anderson et al. '95]

ULDM bound states

EoM: $$(g^{\mu\nu} D_\mu \partial_\nu + m^2)\phi = -\frac{1}{3}\lambda\phi^3 + \cdots$$

$$g_{00} = 1 + 2\Phi$$

$$\simeq \Phi_{\rm ex} = -\frac{GM}{r}$$

external mass, e.g. Sun

$$\phi \equiv \frac{1}{\sqrt{2m}}(\psi e^{-imt} + \text{c.c.})$$

non-relativistic field

Schroedinger: $$\left(i\partial_t + \frac{\nabla^2}{2m} + \frac{\alpha}{r}\right)\psi = g|\psi|^2\psi + \cdots$$

$$-m\Phi$$

- gravitational coupling

$$\boxed{\alpha \equiv GMm}$$

- dimensionful self-coupling

$$\boxed{g \equiv \frac{\lambda}{8m^2}} = -\frac{1}{8f_a^2}$$

$$\left(i\partial_t + \frac{\nabla^2}{2m} + \overset{-m\Phi}{\overbrace{\frac{\alpha}{r}}}\right)\psi = \cancel{g|\psi|^2\psi} \; + \cdots \quad = 0$$

$$\frac{1}{R_\star^2 m} \simeq \frac{\alpha}{R_\star}$$

- hydrogen atom on the Sun

$\vert\psi\vert^2$	$\longleftrightarrow$	number density (could be large)	$\longleftrightarrow$	QM probability density
$\alpha = GMm$		gravitational coupling		fine structure constant

density $\quad \rho \equiv m|\psi|^2 \quad \overset{\text{if locally}}{\ll} \quad \rho_{\rm crit} \equiv \frac{m^2\Phi}{|g|} = \frac{8m^4\Phi}{|\lambda|}$

self-interactions negligible
$\rightarrow$ free EoM

solutions:
bound
unbound

- Ground state $\quad \psi = \psi_{100}(\vec{x})e^{-i\omega_1 t}$

$$\psi_{100}(\vec{x}) \propto e^{-\frac{r}{R_\star}} \qquad \omega_1 \to -\frac{m\alpha^2}{2}$$

binding energy

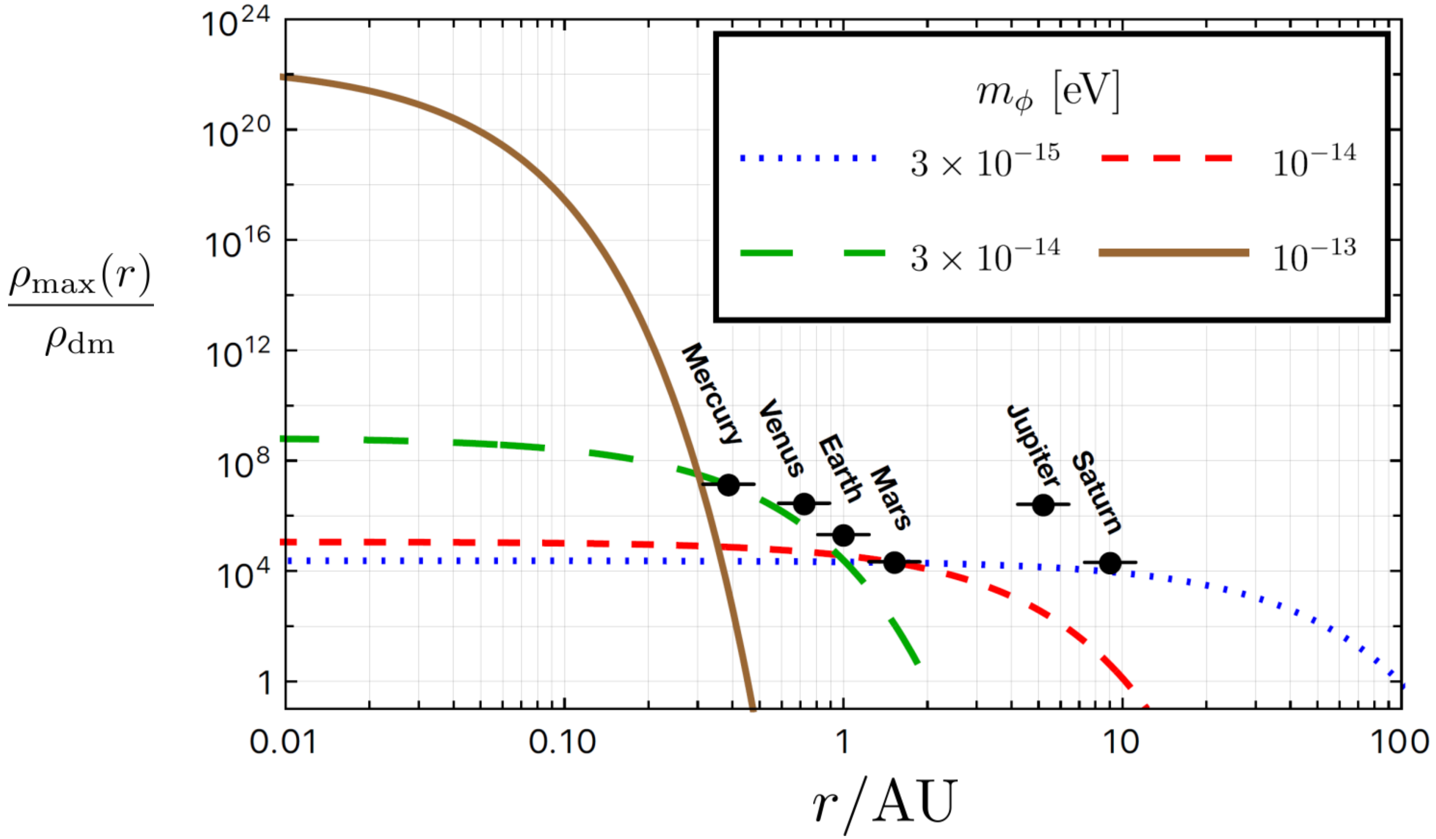
'gravitational' Bohr radius

$$R_\star = \frac{1}{m\alpha} = 1\,\mathrm{AU}\left[\frac{1.3\cdot 10^{-14}\,\mathrm{eV}}{m}\right]^2\left[\frac{M_\odot}{M}\right]$$

$$\psi = \sqrt{\frac{\boxed{M_\star}}{\pi R_\star^3 m}}\, e^{-\frac{r}{R_\star}}\, e^{-i\omega_1 t}$$

bound mass $\lll M_\odot$

$$R_\star = 1\,\mathrm{AU}\left[\frac{1.3\cdot 10^{-14}\,\mathrm{eV}}{m}\right]^2\left[\frac{M_\odot}{M}\right]$$



m/eV : 2×10^{-13} 1.3×10^{-14} 3×10^{-15}

$R_\star$: $R_\odot$ AU Saturn orbit

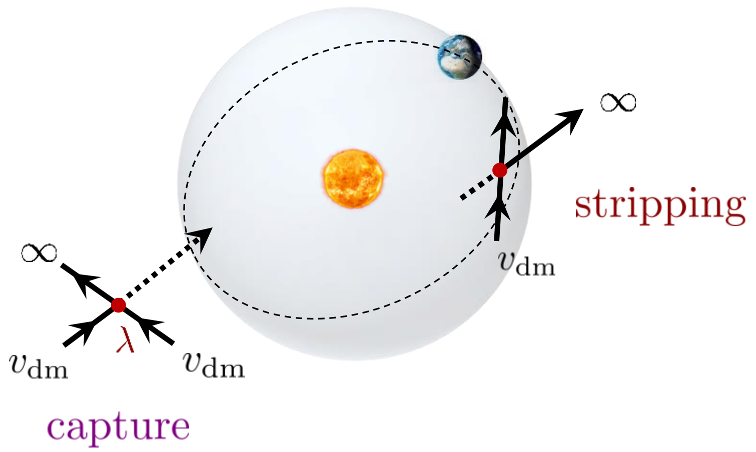
Formation of the gravitational atom

1) initially, DM in the continuum: $M_\star = 0$

2) $$\frac{dM_\star}{dt} = \text{(capture)} - \text{(stripping)}$$

$$= C + \Gamma_1 M_\star - \Gamma_2 M_\star$$

$$= C + \underbrace{(\Gamma_1 - \Gamma_2)}_{\Gamma \gtrless 0} M_\star$$

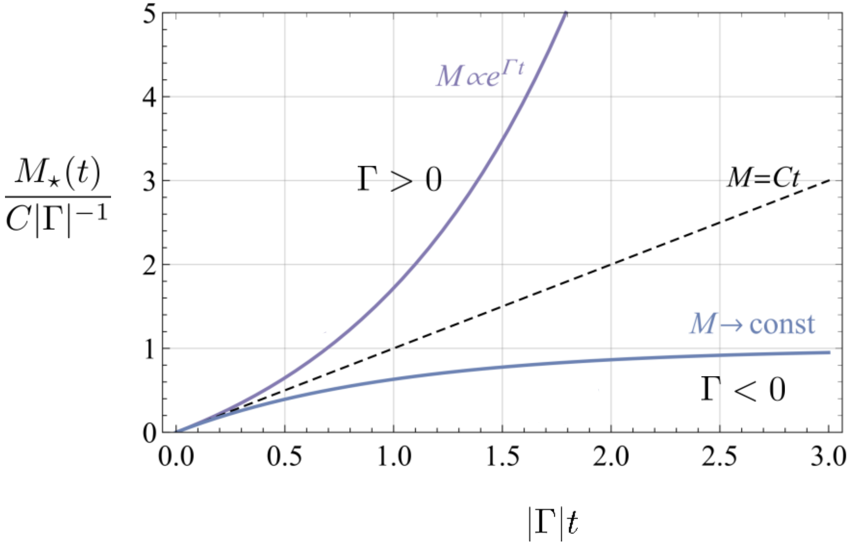


C = direct capture

Γ_1 = stimulated capture $\longleftrightarrow$ Bose enhancement

Γ_2 = stripping, via inverse process

all > 0 and $\propto g^2 \propto \lambda^2$



For $g = 0$, DM in the galaxy halo is

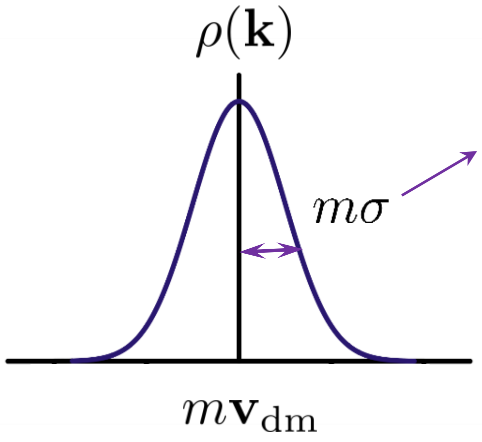
$$\psi_w(t, \mathbf{x}) \equiv \int \frac{d^3k}{(2\pi)^3} \underbrace{f(\mathbf{k})}_{\text{momentum distribution}} e^{-i\overbrace{\omega_k}^{\frac{k^2}{2m}} t} \psi_{\mathbf{k}}(\mathbf{x})$$

unbound solutions of the atom:
'scattering states' or 'waves', $\mathbf{k}$

$$\left(i\partial_t + \frac{\nabla^2}{2m} + \frac{\alpha}{r} \right) \psi_{\mathbf{k}} = 0$$

- $f(\mathbf{k})$, standard halo model

$$\langle f^*(\mathbf{k}) f(\mathbf{k}') \rangle = (2\pi)^3 \rho(\mathbf{k}) \delta(\mathbf{k} - \mathbf{k}')$$



$\simeq v_{\rm dm}/\sqrt{2}$

$\mathbf{k} = (k_1, k_2, k_3)$

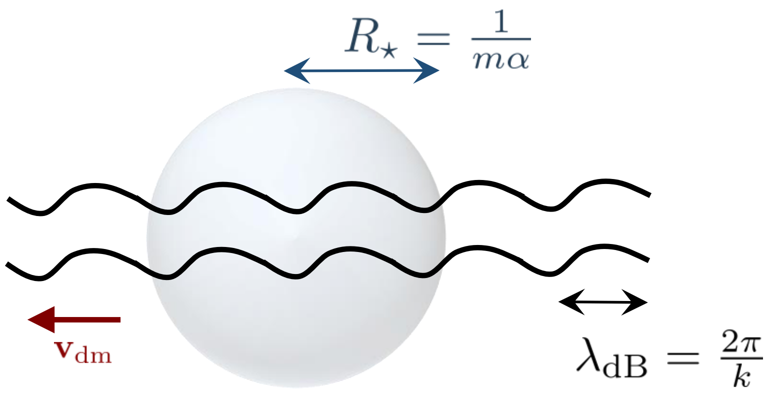
240 km/s

$$\rho(\mathbf{k}) = \frac{\overset{0.4\,\mathrm{GeV/cm^3}}{\overbrace{\rho_{\rm dm}}}}{\sigma^3 m^4} e^{-\frac{(\mathbf{k} - m\mathbf{v}_{\rm dm})^2}{2m^2\sigma^2}}$$

- Scattering states $\psi_{\mathbf{k}}(\mathbf{x})$

$$\boxed{\xi_{\rm foc}(k) \equiv \frac{\lambda_{\rm dB}}{R_\star}} = \frac{2\pi}{\underset{\swarrow\; mv}{k}}\,\frac{1}{\underset{\searrow\; (m\alpha)^{-1}}{R_\star}} = \frac{2\pi\alpha}{v}$$

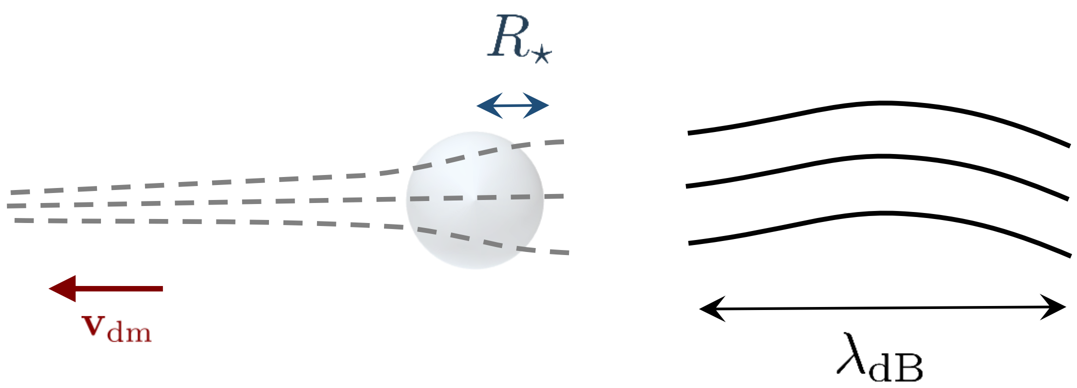
$$\boxed{\xi_{\rm foc}(k) \ll 1 \quad \leftrightarrow v \gg 2\pi\alpha}$$



$$\psi_{\mathbf{k}} \to e^{i\mathbf{k}\cdot\mathbf{x}}$$

large velocity

$$\boxed{\xi_{\rm foc}(k) \gg 1 \quad \leftrightarrow v \ll 2\pi\alpha}$$



small velocity

- Scattering states $\psi_{\mathbf{k}}(\mathbf{x})$

$$\boxed{\xi_{\rm foc}(k) \equiv \frac{\lambda_{\rm dB}}{R_\star}} = \frac{2\pi}{k}\,\frac{1}{R_\star} = \frac{2\pi\alpha}{v}$$

$mv \qquad (m\alpha)^{-1}$

$\boxed{\xi_{\rm foc}(k) \ll 1 \quad \leftrightarrow v \gg 2\pi\alpha}$

$$\psi_{\mathbf{k}} \to e^{i\mathbf{k}\cdot\mathbf{x}}$$

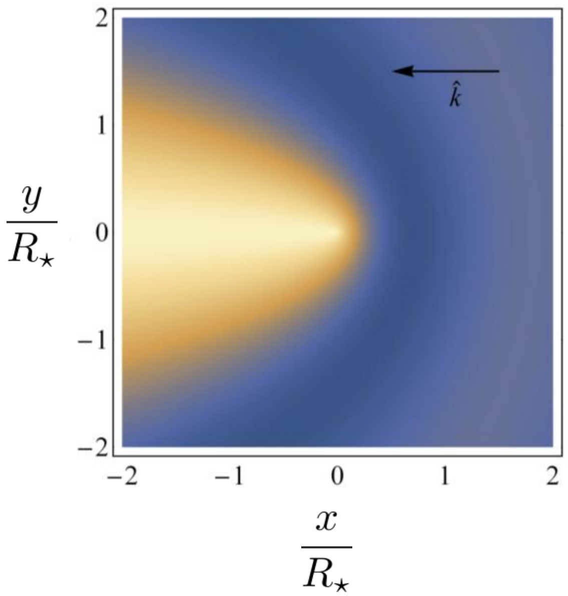
energy $\gg$ binding energy

$\frac{mv^2}{2} \qquad -\frac{m\alpha^2}{2}$

$\boxed{\xi_{\rm foc}(k) \gg 1 \quad \leftrightarrow v \ll 2\pi\alpha}$

gravitational focusing

$$|\psi_{\mathbf{k}}|^2 \to$$



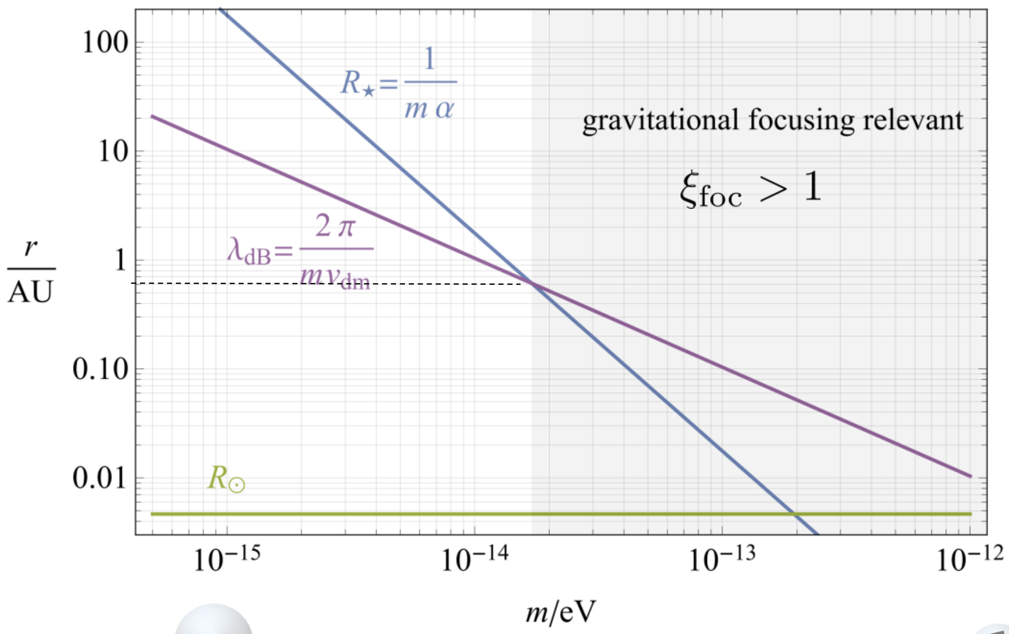
$$\dot{M}_\star = C + (\Gamma_1 - \Gamma_2) M_\star$$

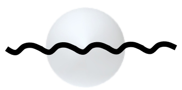
$\Gamma_1 < \Gamma_2$

stripping dominates

$\Gamma_1 > \Gamma_2$

stimulated capture dominates





in the Solar System $$\xi_{\mathrm{foc}} = \frac{2\pi\alpha}{v_{\mathrm{dm}}} \simeq \left[\frac{m}{1.7\times 10^{-14}\,\mathrm{eV}}\right]\left[\frac{M}{M_\odot}\right]\left[\frac{240\,\mathrm{km/s}}{v_{\mathrm{dm}}}\right] \gtrsim 1$$

i.e. if $m \gtrsim 1.7\cdot 10^{-14}\,\mathrm{eV}$

$R_\star \lesssim \mathrm{AU}!$

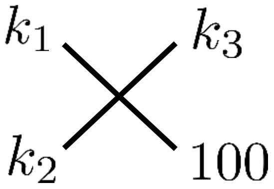
Coincidence: $$v_c \equiv \sqrt{\frac{GM}{R_\star}} = \sqrt{GMm\alpha} = \sqrt{\alpha^2} = \alpha$$

at $R_\star = 1\,\mathrm{AU}$ $\quad v_c \simeq 30\,\mathrm{km/s} \quad \rightarrow \quad 2\pi v_c \simeq 200\,\mathrm{km/s} \simeq v_{\mathrm{dm}}$

Bound state formation: derivation

$$S_{\rm non-rel} = -\int dt\, d^3x \left[\frac{i}{2}(\dot{\psi}^*\psi - \psi^*\dot{\psi}) + \frac{1}{2m}|\nabla\psi|^2 + m\Phi_{\rm ex}|\psi|^2 + \underbrace{\frac{1}{2}g|\psi|^4}_{\mathcal{H}_{\rm int}}\right]$$

$$\boxed{k_1 + k_2 \to k_3 + 100}$$



$$\hat{\psi}(x)\hat{\psi}(x)\,|k_1 k_2\rangle = 2\psi_{\mathbf{k_1}}(\mathbf{x})\psi_{\mathbf{k_2}}(\mathbf{x})e^{-i(\omega_{k_1}+\omega_{k_2})t}\,|0\rangle$$

$$\mathcal{A} = \langle k_3\, nlm|\, T[\hat{H}_{int}]\,|k_1 k_2\rangle = 2g(2\pi)\delta(\Delta\omega)\mathcal{M}$$

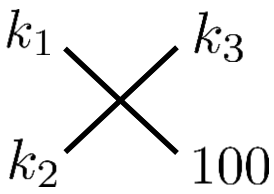
$$\Delta\omega \equiv \omega_{k_1} + \omega_{k_2} - \omega_{k_3} - \omega_1$$

$$= \frac{k_1^2}{2m} + \frac{k_2^2}{2m} - \frac{k_3^2}{2m} + \frac{m\alpha^2}{2}$$

$$\mathcal{M} = \int d^3x\, \psi_{100}^*\psi_{\mathbf{k_3}}^*\psi_{\mathbf{k_1}}\psi_{\mathbf{k_2}}$$

$$P_{k_1+k_2\to k_3+100} = (2\pi)\delta(\Delta\omega)4g^2|\mathcal{M}|^2$$

$$P_{k_1+k_2\to k_3+100} = (2\pi)\delta(\Delta\omega)4g^2|\mathcal{M}|^2$$



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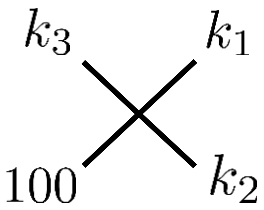


subtract inverse process

Bose enhancement:

$$P_{\text{indist}} = (N+1)P_{\text{dist}}$$

$$\frac{dN_0}{dt} = \frac{4g^2}{2}\int [dk_1][dk_2][dk_3]\left[(\rho(\mathbf{k_3})+1)(N_0+1)\rho(\mathbf{k_1})\rho(\mathbf{k_2}) - \rho(\mathbf{k_3})N_0(\rho(\mathbf{k_1})+1)(\rho(\mathbf{k_2})+1)\right](2\pi)\delta(\Delta\omega)|\mathcal{M}|^2$$

initial state density

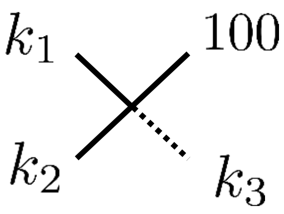
$$= 2g^2\int [dk_1][dk_2][dk_3]\left\{\rho(\mathbf{k_1})\rho(\mathbf{k_2})\rho(\mathbf{k_3}) + N_0\left[\rho(\mathbf{k_1})\rho(\mathbf{k_2}) - 2\rho(\mathbf{k_2})\rho(\mathbf{k_3})\right]\right\}(2\pi)\delta(\Delta\omega)|\mathcal{M}|^2$$

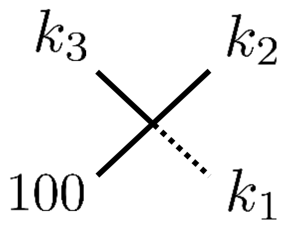
$M_\star = mN_0 \quad \rightarrow$

$$\dot{M}_\star = C + (\Gamma_1 - \Gamma_2)M_\star$$

stimulated capture stripping

$$\Gamma \equiv \Gamma_1 - \Gamma_2 = g^2 \int [dk_{1,2,3}]\delta(\Delta\omega)|\mathcal{M}|^2 \{\ \rho(\mathbf{k_1})\rho(\mathbf{k_2}) - 2\rho(\mathbf{k_2})\rho(\mathbf{k_3})\ \}$$

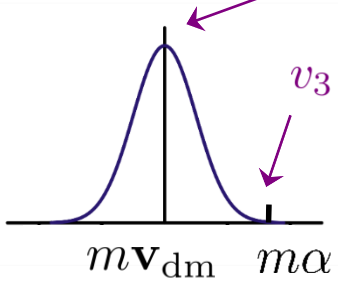




$$\xi_{\rm foc} = \frac{2\pi\alpha}{v_{\rm dm}} \gg 1$$

$$m \gtrsim 10^{-14}\,\mathrm{eV}$$

v_1 or v_2



$$v_3 = \sqrt{v_1^2 + v_2^2 + \alpha^2} \gtrsim \alpha$$

$$2\pi\alpha \gg v_{\rm dm}$$

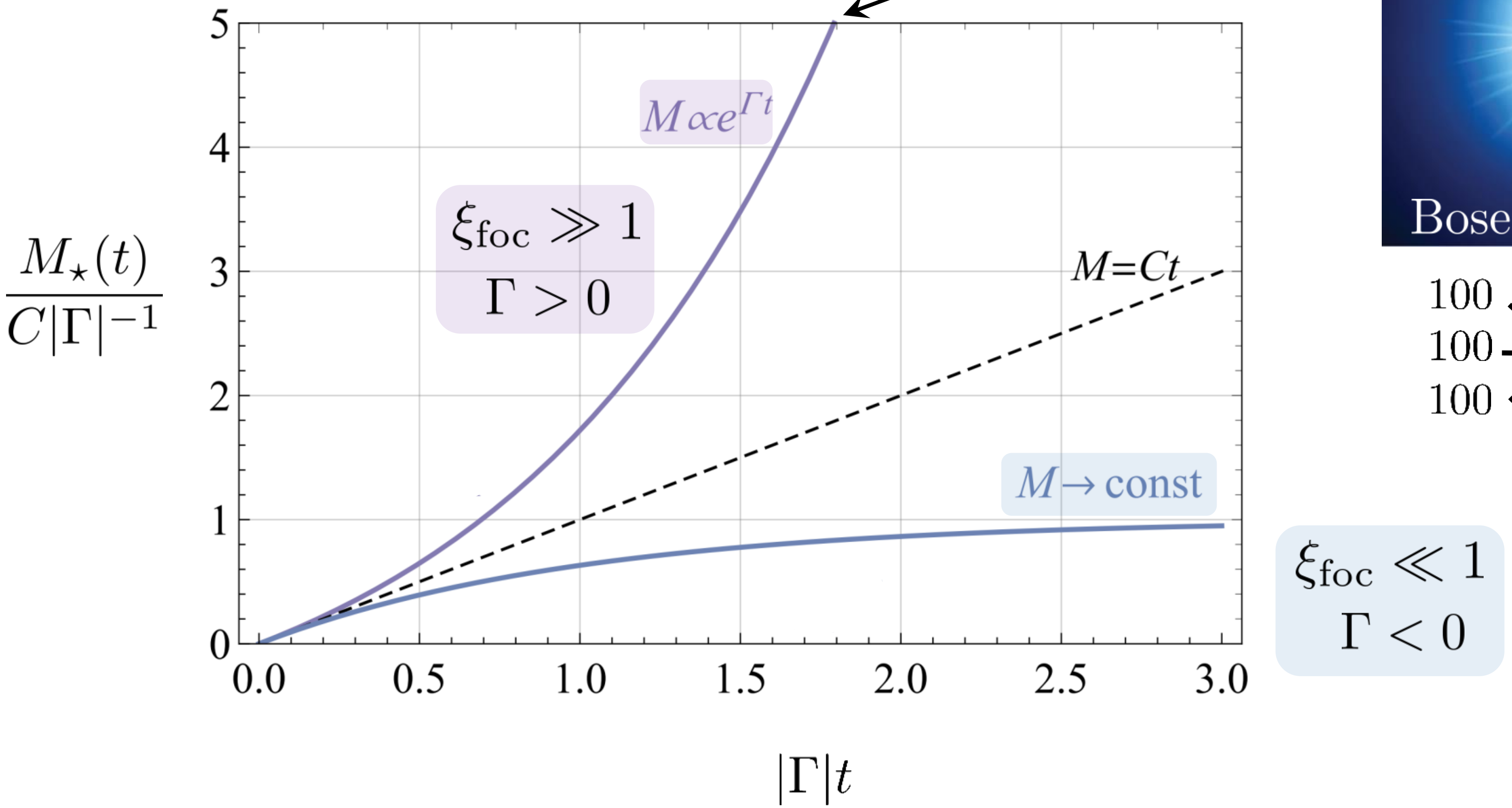
$$\Gamma_1 \simeq \mathrm{const}$$

$$\Gamma_2 \simeq e^{-\xi_{\rm foc}}$$

$$\rightarrow \Gamma > 0$$

Phases of formation

$$\dot{M}_\star = C + \underbrace{(\Gamma_1 - \Gamma_2)}_{\Gamma} M_\star$$

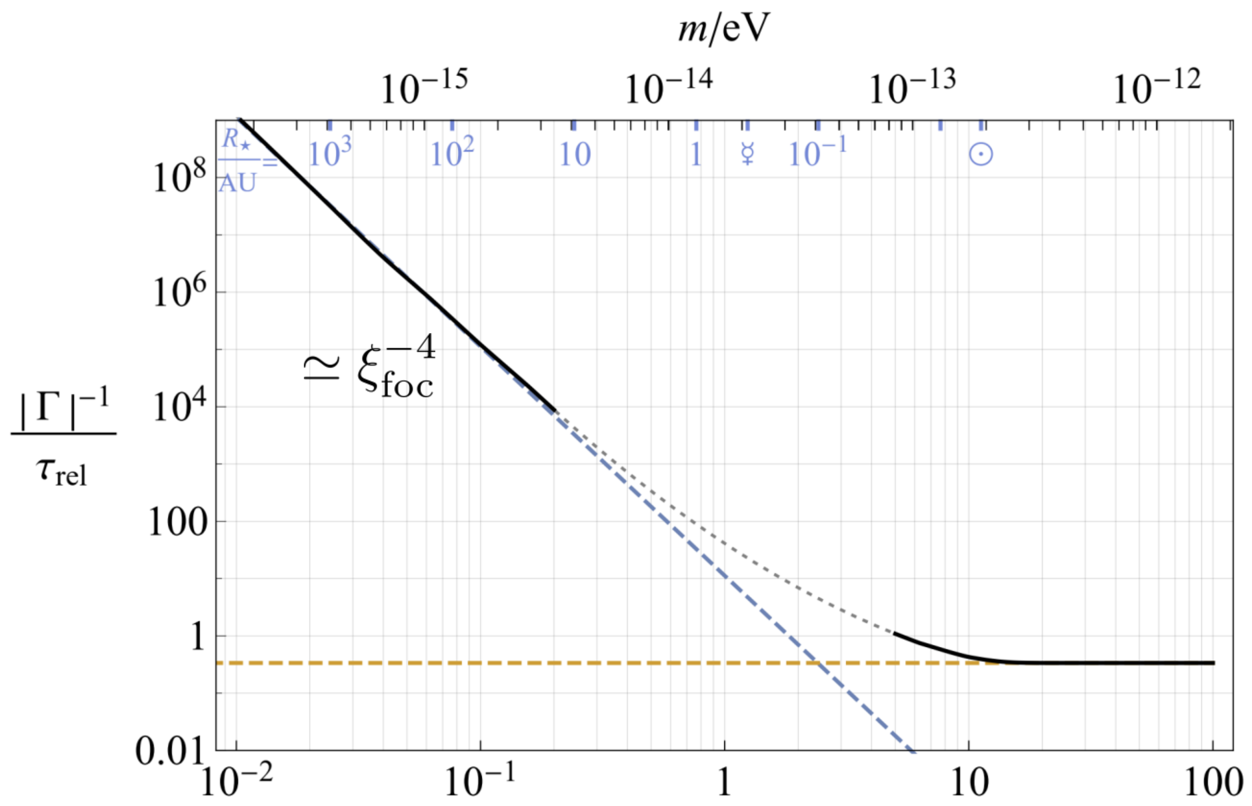


100
100 ...... relativistic
100

$$\rho_{\rm crit} \equiv \frac{2\Phi_{\rm ex} m^2}{|g|} \simeq 2\frac{\alpha^2 m^2}{|g|} \simeq 6\cdot 10^4 \rho_{\rm dm} \left[\frac{f_a}{5\cdot 10^7\,{\rm GeV}}\right]^2 \left[\frac{m}{1.7\cdot 10^{-14}\,{\rm eV}}\right]^4 \left[\frac{M}{M_\odot}\right]^2 \left[\frac{0.4\,{\rm GeV/cm^3}}{\rho_{\rm dm}}\right]$$

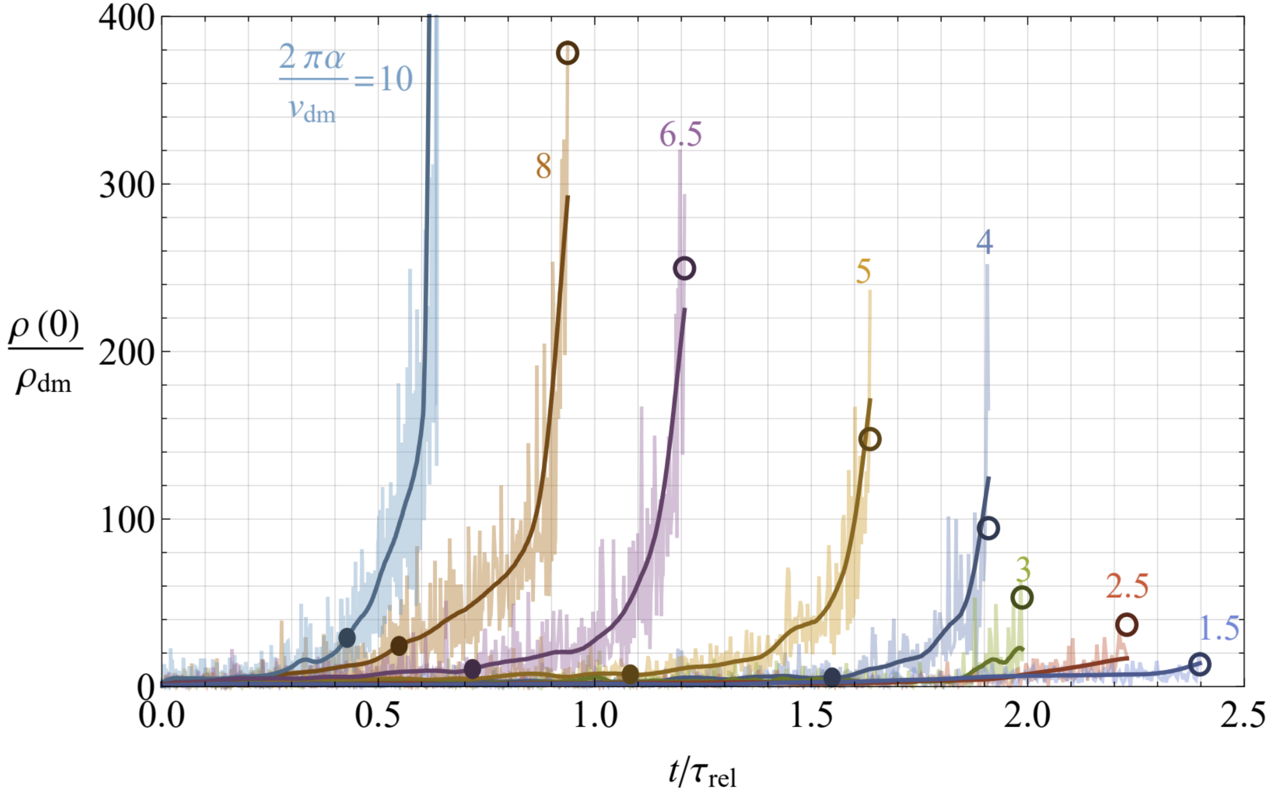
$\Gamma^{-1} \leftrightarrow$

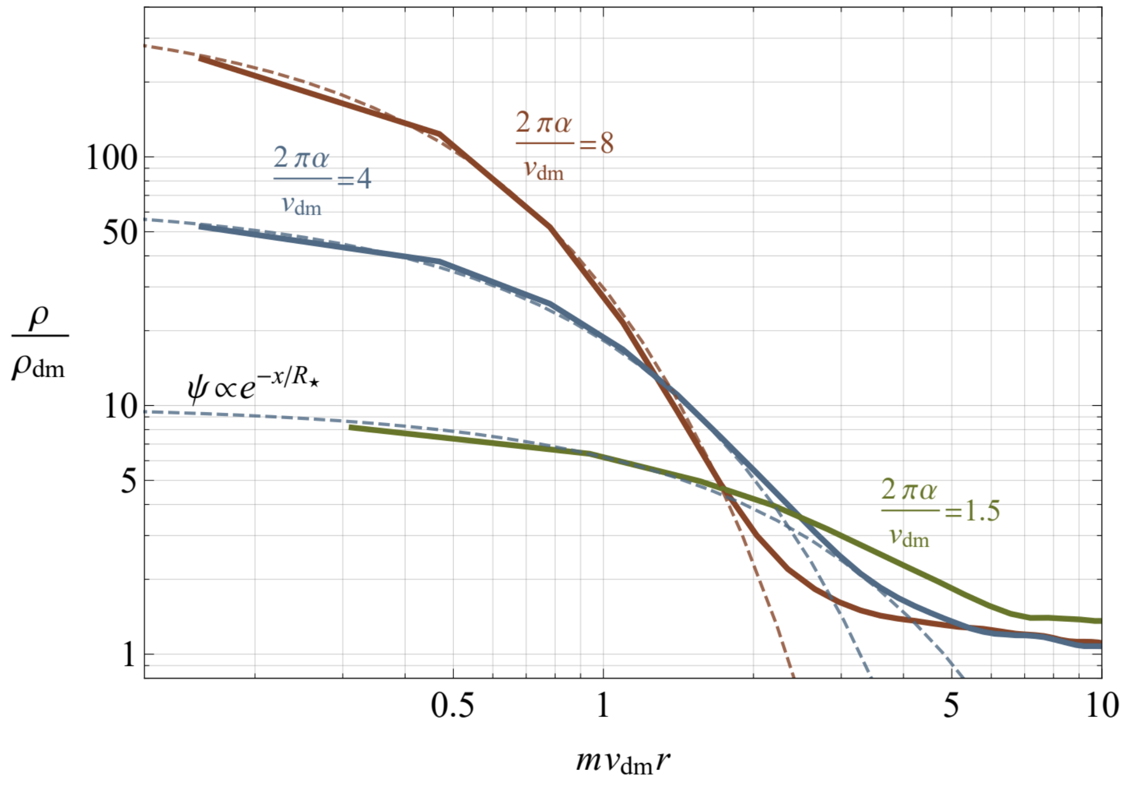
relaxation time $$\tau_{\rm rel} \equiv \frac{m^3 v_{\rm dm}^2}{g^2 \rho_{\rm dm}^2} \simeq 9\,{\rm Gyr} \left[\frac{f_a}{10^8\,{\rm GeV}}\right]^4 \left[\frac{m}{10^{-14}\,{\rm eV}}\right]^3 \left[\frac{0.4\,{\rm GeV/cm^3}}{\rho_{\rm dm}}\right]^2 \left[\frac{v_{\rm dm}}{240\,{\rm km/s}}\right]^2$$



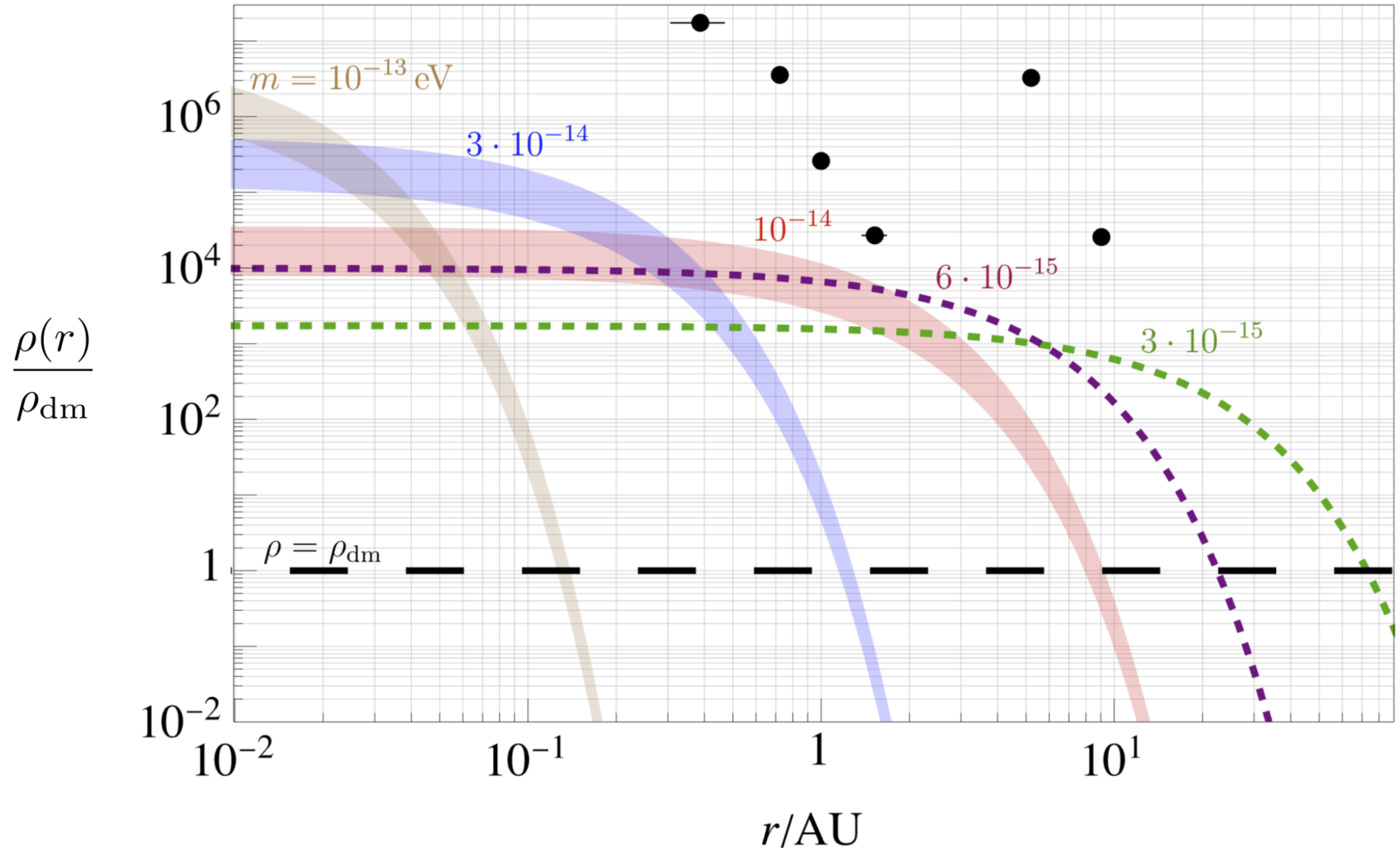
$$\xi_{\rm foc} = \frac{2\pi\alpha}{v_{\rm dm}}$$

Comparison with simulations

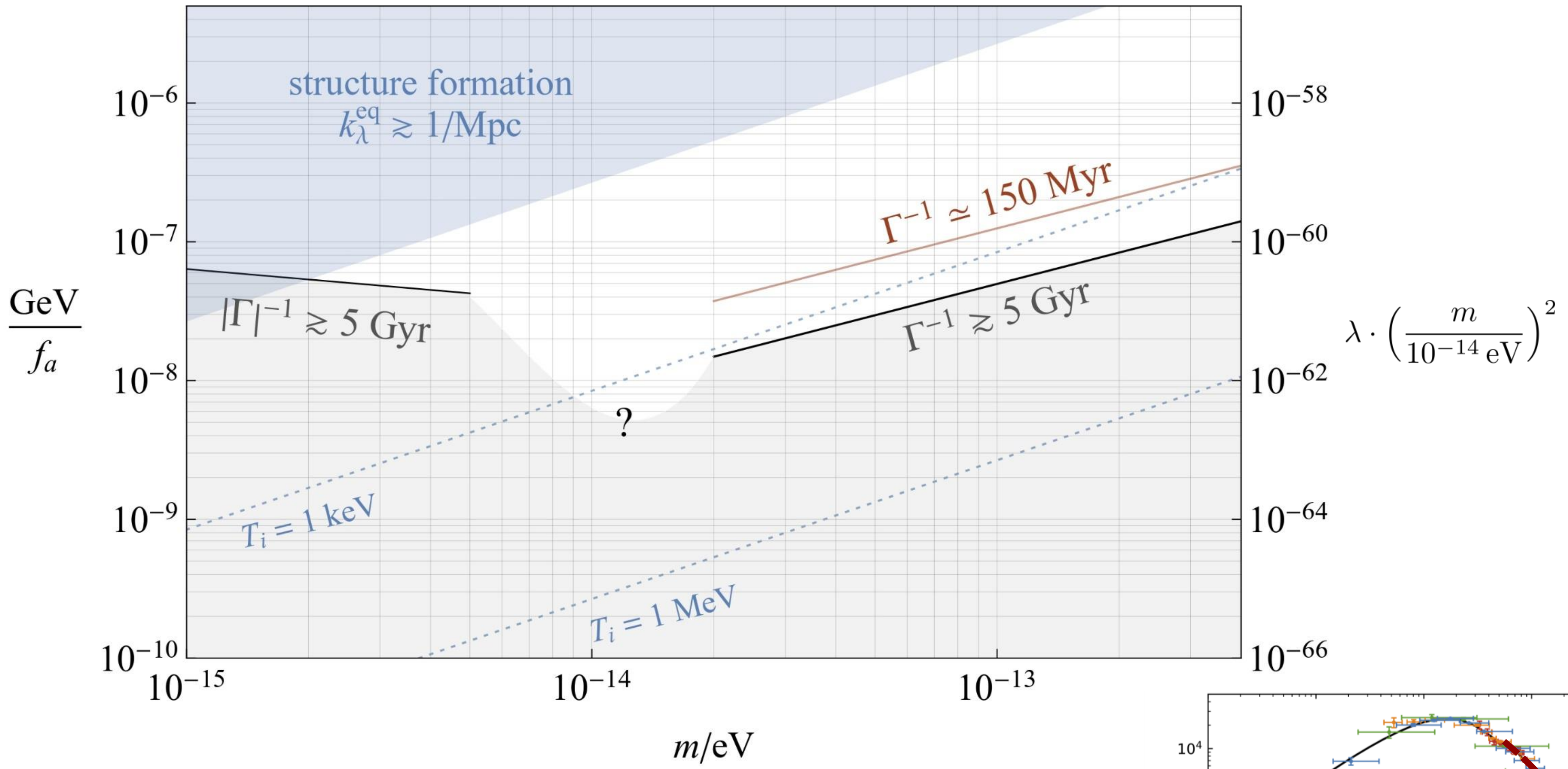


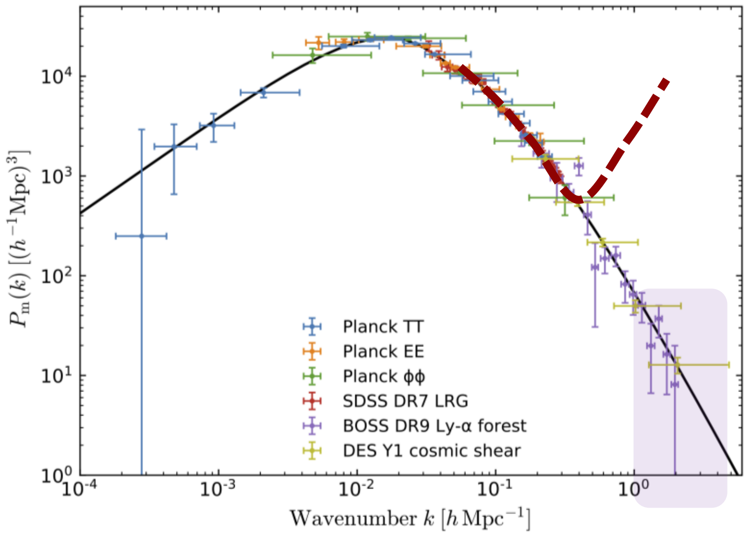


density profile after 5 Gyr



- bands have $v_{\mathrm{dm}} = 50 \div 240\,\mathrm{km/s}$
- f_a (or λ) fixed in $10^7 \div 10^8\,\mathrm{GeV}$



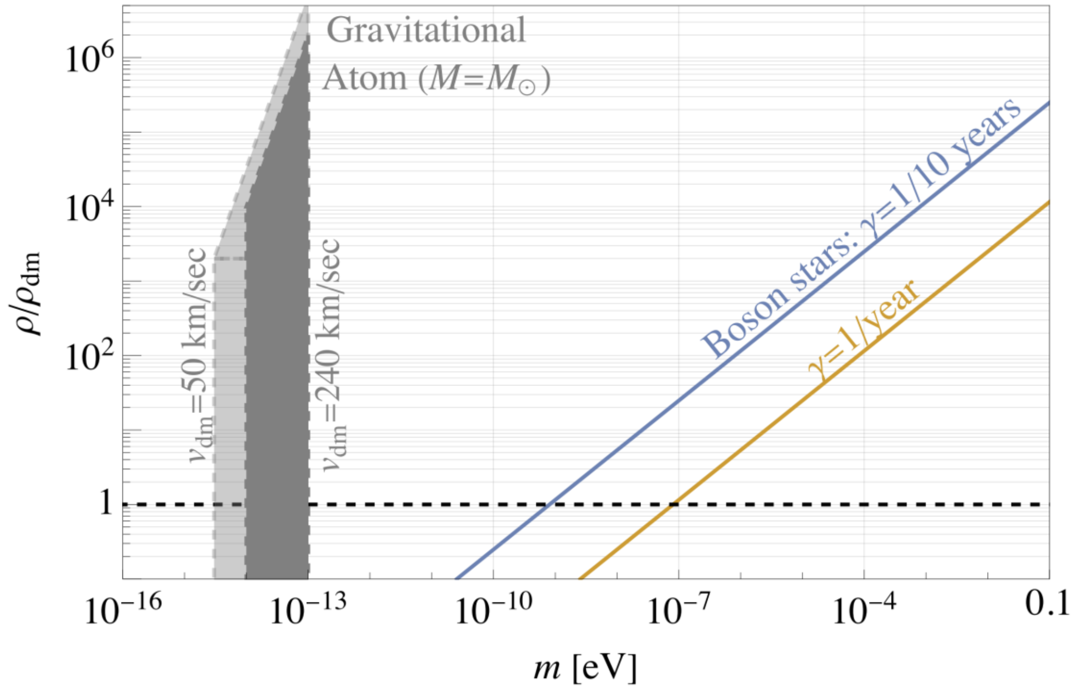


$$\delta \equiv \rho/\bar{\rho} - 1$$

$$\ddot{\delta}_{\mathbf{k}} + 2H\dot{\delta}_{\mathbf{k}} - \underbrace{\left[4\pi G\rho + \frac{k^2}{a^2}\frac{\rho}{8f_a^2 m_a^2}\right]}_{4\pi G\rho\left[1+\left(\frac{k_{\rm today}}{k_\lambda}\frac{a_{\rm eq}}{a}\right)^2\right]}\delta_{\mathbf{k}} = 0$$

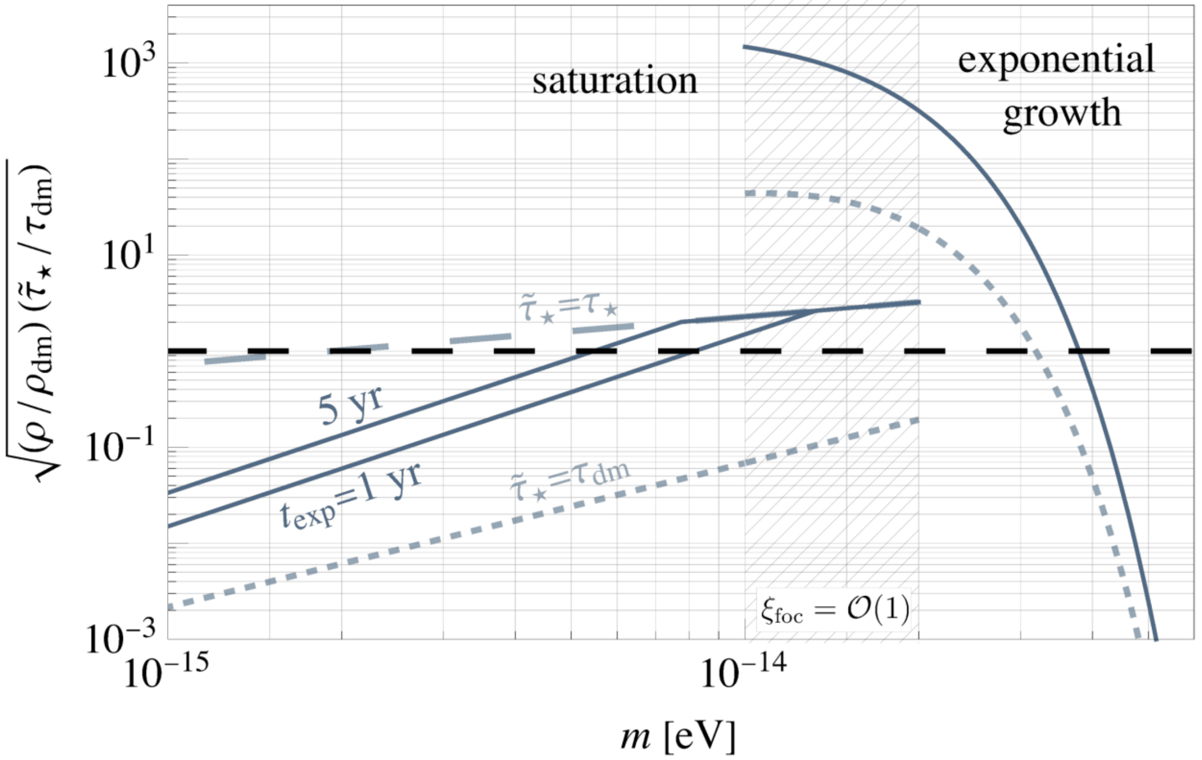
$$k_\lambda \simeq \frac{3.8}{\rm Mpc}\,\frac{f_a}{10^7\,{\rm GeV}}\,\frac{m}{10^{-14}{\rm eV}} \;\lesssim 1\,{\rm Mpc}^{-1}$$

overdensity at the center



effective coherence time

$$\tau_\star \gtrsim \frac{2\pi}{m\alpha^2} = \left[\frac{2\pi}{\xi_{\rm foc}}\right]^2 \tau_{\rm dm} \simeq 1\,\text{year}\left[\frac{1.3\cdot 10^{-14}\,\text{eV}}{m}\right]^3$$



Summary

- **Small ULDM self-interactions induce the capture of the galaxy halo DM**

  $\rightarrow$ happens efficiently when galactic DM is gravitationally focused, i.e. if $\frac{\lambda_{\rm dB}}{R_\star} = \frac{2\pi\alpha}{v_{\rm dm}} \gtrsim 1$

  $\rightarrow$ leads to exponential growth of gravitational atoms bound to the Sun for $m \gtrsim 10^{-14}\,\rm eV$

  $\rightarrow$ speed of the process depends on f_a (or λ) and ends with Bosenova explosions

Outlook

- direct detection on Earth, larger DM density and coherence time
- detection of Bosenova explosions
- apply to other systems, including more massive object and SMBH (smaller m)