Non-Invertible Naturalness

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(2211.07639 + Work in Progress)
With Clay Córdova, Seth Koren, and Kantaro Ohmori

The 5th NPKI Workshop

Generalized Global Symmetries!

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Introduction to Generalized Global Symmetries in QFT and Particle Physics

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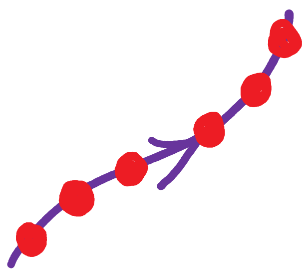
ABSTRACT: Generalized symmetries (also known as categorical symmetries) is a newly developing technique for studying quantum field theories. It has given us new insights into the structure of QFT and many new powerful tools that can be applied to the study of particle phenomenology. In these notes we give an exposition to the topic of generalized/categorical symmetries for high energy phenomenologists although the topics covered may be useful to the broader physics community. Here we describe generalized symmetries without the use of category theory and pay particular attention to the introduction of discrete symmetries and their gauging.

Generalized Global Symmetries!

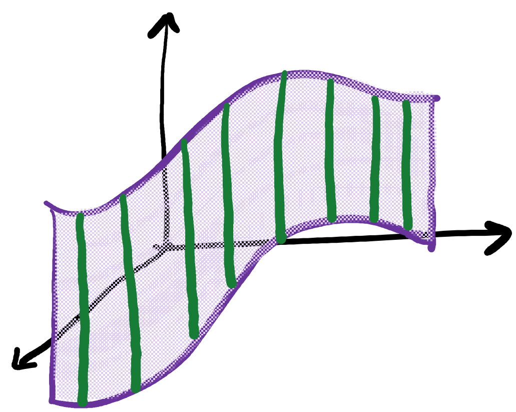
Higher-form symmetries

Various **extended objects** appear in broad class of theories.

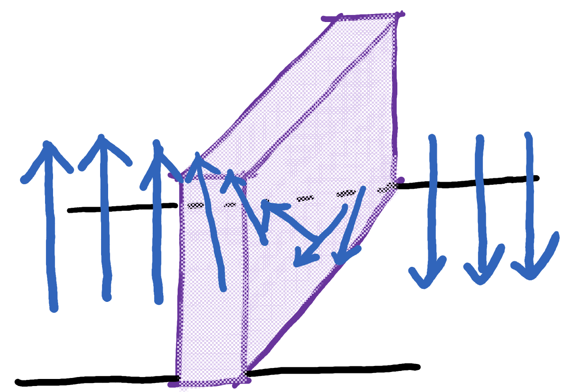
Local operator
e.g. particle
0-form symmetry



Line operator
e.g. Wilson loop
't Hooft loop
1-form symmetry



Surface operator
e.g. Cosmic string
2-form symmetry



Volume operator
e.g. Domain Wall
3-form symmetry

Non-Invertible Symmetry

Consider a massless QED: ψ_-, ψ_+ charged under gauged $U(1)$

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∃ global $U(1)_A$ with Adler-Bell-Jackiw (ABJ) anomaly

$$\psi_- \to e^{i\alpha}\psi_-, \quad \psi_+ \to e^{i\alpha}\psi_+ \quad \Rightarrow \quad \partial^\mu J_\mu = \frac{N_f}{32\pi^2}\epsilon^{\alpha\beta\gamma\delta}F_{\alpha\beta}F_{\gamma\delta}$$

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Now, we say that $U(1)_A \to Z_{N_f}$ (invertible) + {non-invertible}

2205.05086 (Yichul Choi, Ho Tat Lam, Shu-Heng Shao),
2205.06243 (Clay Córdova, Kantaro Ohmori)

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Under $\alpha = \frac{2\pi}{k}, \qquad S \to S + \frac{2\pi i N_f}{k} \int_{M_4} \frac{F \wedge F}{8\pi^2}$

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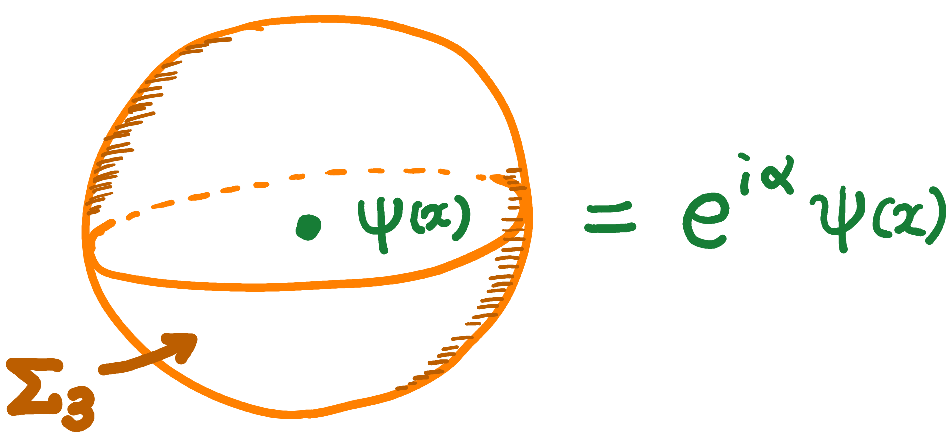

$$Q(\Sigma_3) = \int_{\Sigma_3} d^3x\, J^0 = \int_{\Sigma_3} * J_1$$

$$U(\alpha, \Sigma_3) = e^{i\alpha Q(\Sigma_3)}$$

$$\langle U(\alpha, \Sigma_3)\psi(x)\rangle \sim e^{i\alpha}\psi(x)$$

"Symmetry Defect Operator"

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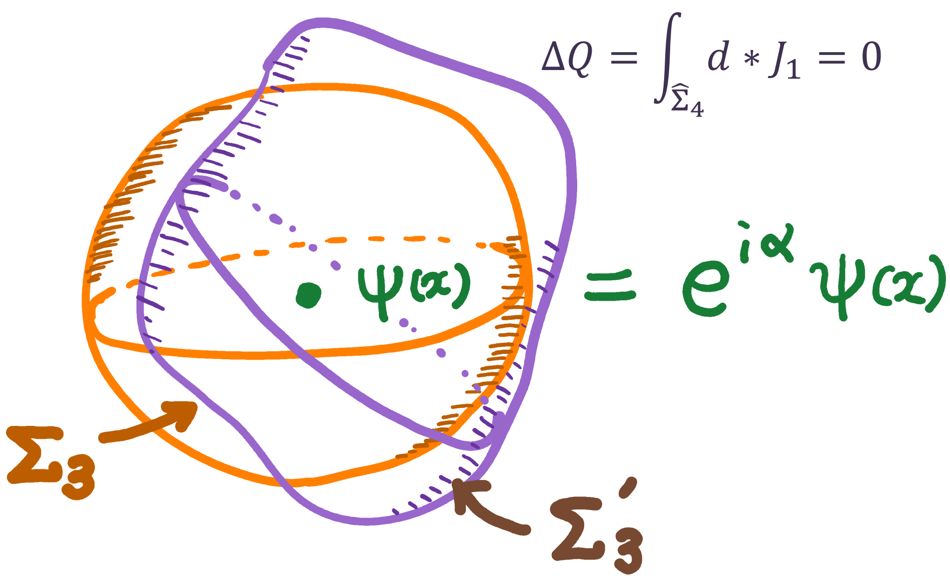

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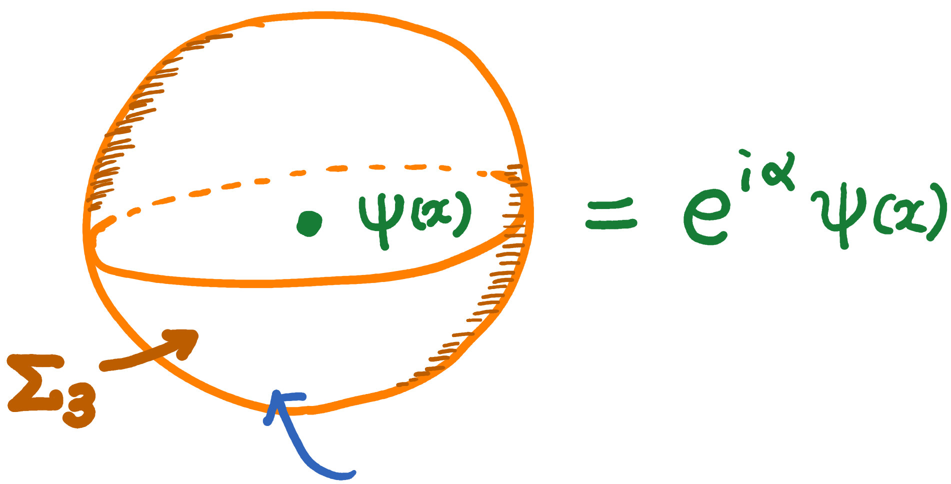

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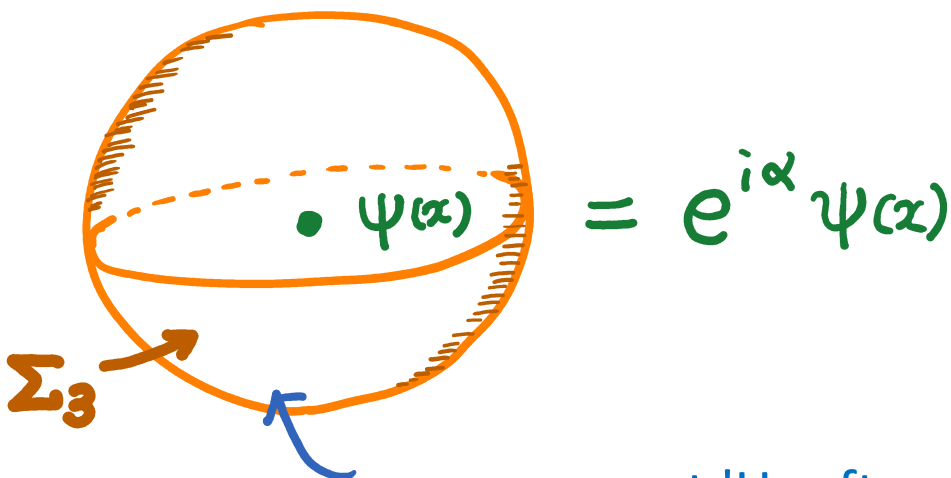

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t 'Hooft anomaly of $Z_N^{(1)}$ 1-form global symmetry

$$S_{defect} = \frac{iN}{4\pi} \int_{\Sigma_3} C \wedge dC$$

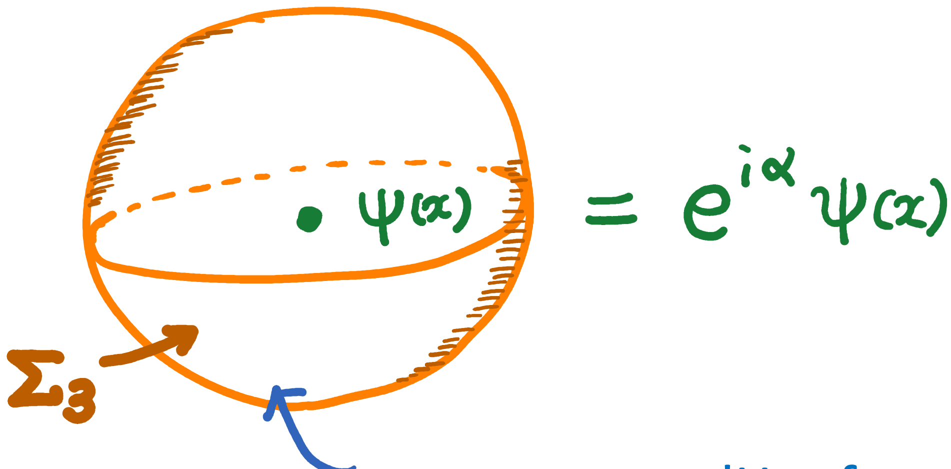
$$+ \frac{iN}{2\pi} \int_{\Sigma_3} C \wedge B_2$$

$$S_{inflow} = -\frac{2\pi i p}{N} \int_{M_4} \frac{\mathcal{P}(B_2)}{2}$$

$$C \to C + \frac{1}{N}\epsilon_1, \int \frac{\epsilon_1}{2\pi} \in Z$$

Non-Invertible Symmetry

Under $\alpha = \frac{2\pi}{k}$, $\quad S \to S + \frac{2\pi i N_f}{k} \int_{M_4} \frac{F \wedge F}{8\pi^2} - \frac{2\pi i p}{N} \int_{M_4} \frac{F \wedge F}{8\pi^2}$



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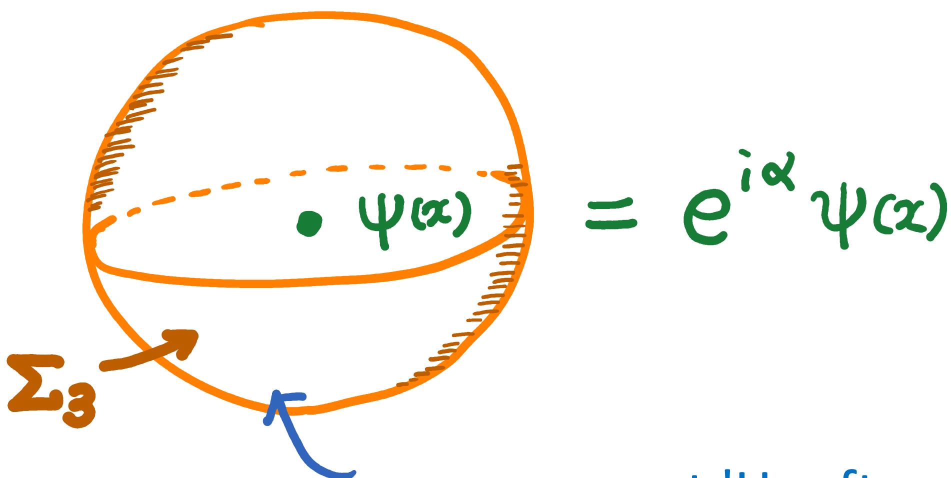

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Gauging 1-form $Z_N^{(1)}$ of 3d TQFT through the dynamical gauge field $F = dA$

= identifying 1-form $Z_N^{(1)}$ symmetry with "bulk" magnetic 1-form symmetry

Non-Invertible Symmetry

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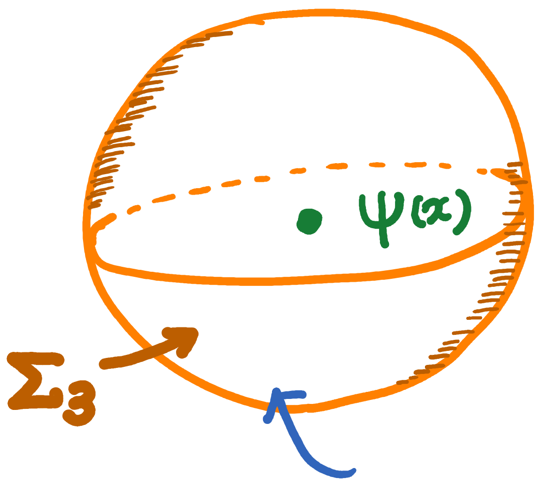

$= e^{i\alpha}\psi(x)$

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Non-Invertible Symmetry

Similar construction possible for non-abelian gauge theory with 1-form magnetic center symmetry

e.g. $G = SU(N)$

electric 1-form: Z_N
magnetic 1-form: none

Non-Invertible Symmetry

Similar construction possible for non-abelian gauge theory with 1-form magnetic center symmetry

e.g. $G = SU(N)/Z_L$

electric 1-form: $Z_{N/L}$

magnetic 1-form: Z_L

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$\in Z$ $\qquad \in Z_L$

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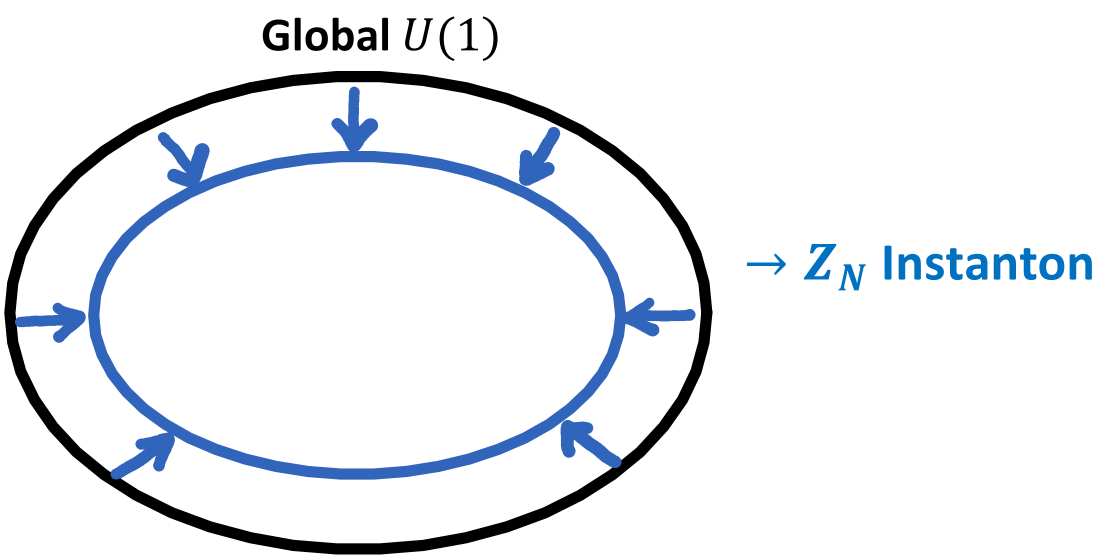

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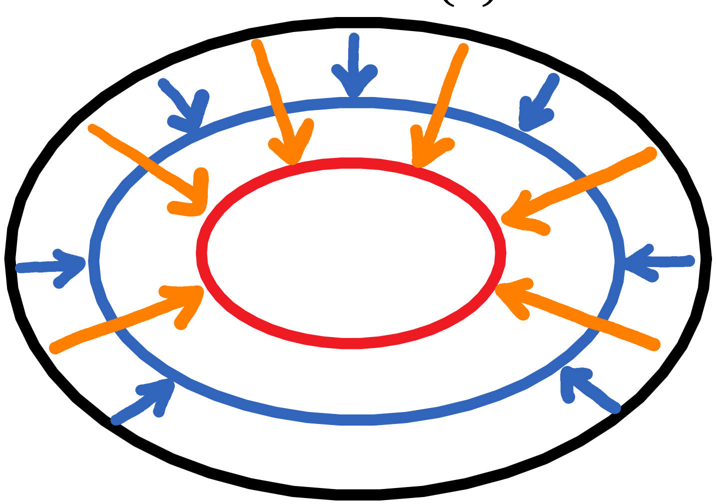
Global $U(1)$



→ Z_N Instanton

→ Z_L (fractional) Instanton

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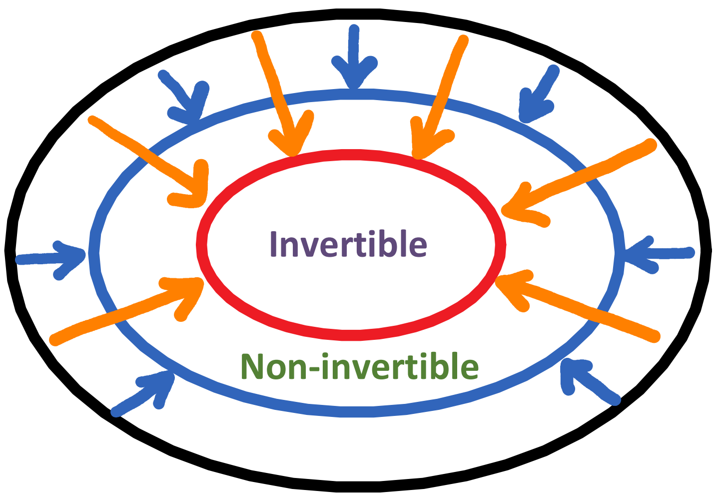
$\in Z$ $\in Z_L$



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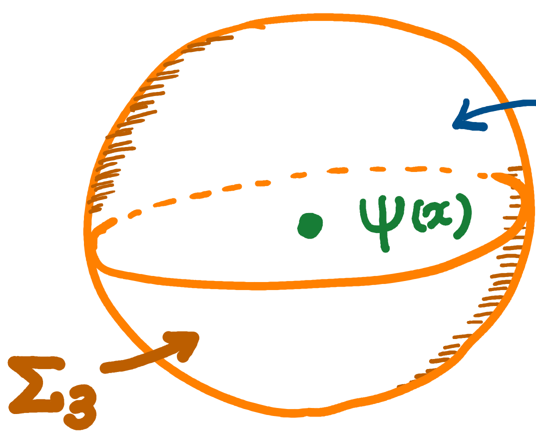

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Non-Invertible Symmetry

1. $D_k = U\left(\frac{2\pi}{k}, \Sigma_3\right) \times \mathcal{A}^{N,p}\left(\frac{F}{2\pi}\right)$ does not have an inverse operation

$$D_k \times \overline{D}_k \sim \sum_S \xi(S) \exp\left(\frac{i}{2\pi N}\int_S F\right)$$

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2. Breaking of non-invertible symmetry

   Existence of non-invertible chiral symmetry relies on
   1-form magnetic symmetry $dF = 0$ (vs $d * F = j$)

t 'Hooft line

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Monopole in UV $\rightarrow$ 1-form magnetic symm broken

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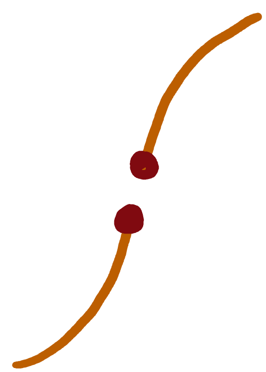

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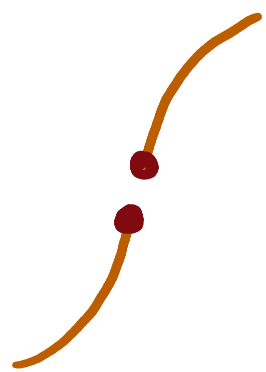

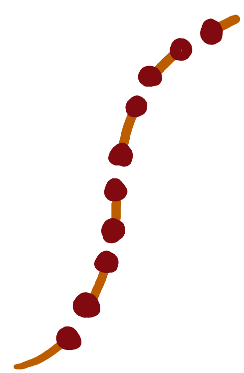

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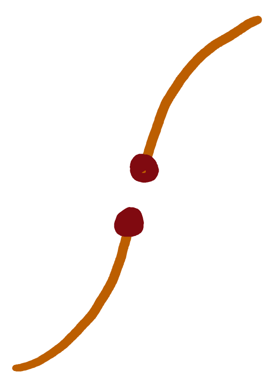
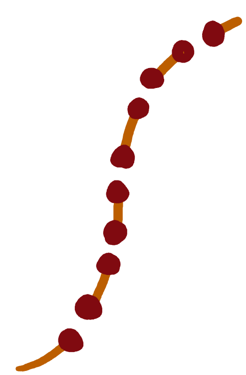
Monopole in UV $\rightarrow$ 1-form magnetic symm broken
$\rightarrow$ non-invertible symm in IR broken





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$\rightarrow$ non-invertible symm in IR broken

$$M_{mon} \sim \frac{v}{g}\,, \qquad \Lambda \sim gv$$

$$\delta\mathcal{L} \sim \exp(-S_{mon}) \sim \exp(-M_{mon}\delta t)$$

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Existence of non-invertible chiral symmetry relies on
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Monopole in UV $\rightarrow$ 1-form magnetic symm broken
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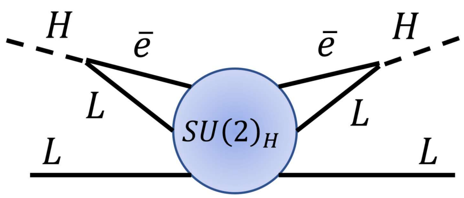
(with Clay Córdova, Seth Koren, and Kantaro Ohmori)



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Solving Strong CP Problem Non-invertibly

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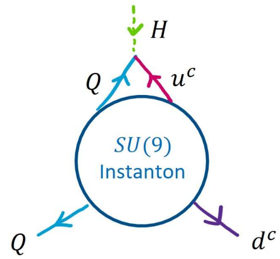

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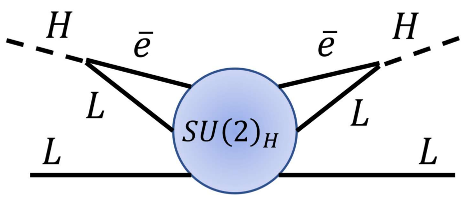

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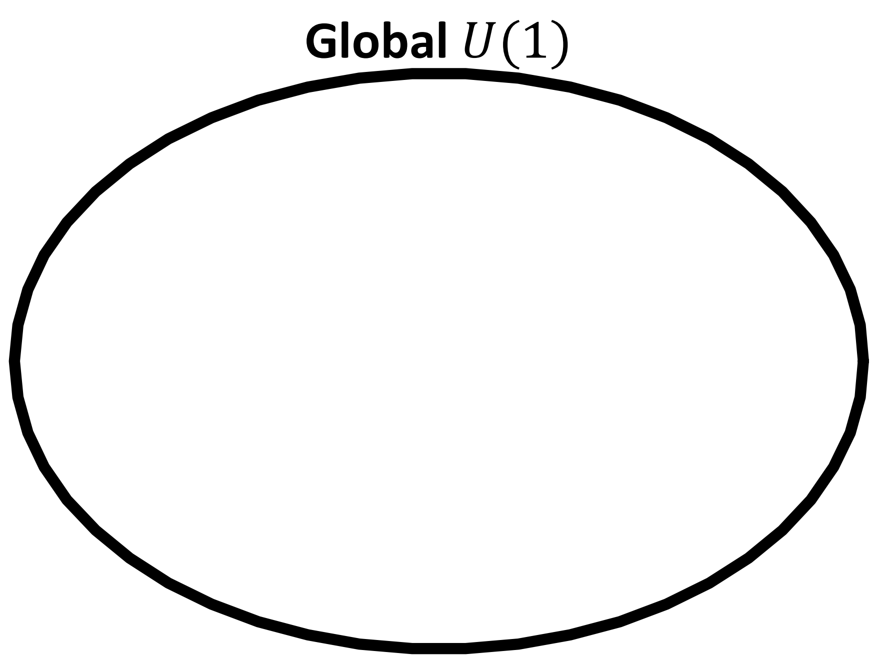

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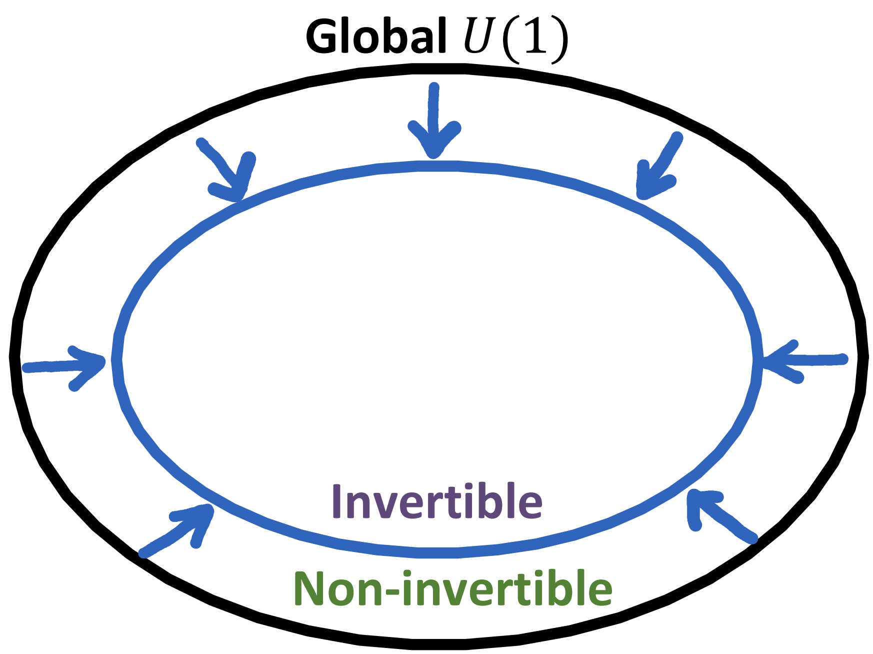

$\rightarrow U(1)_Y$ Instanton

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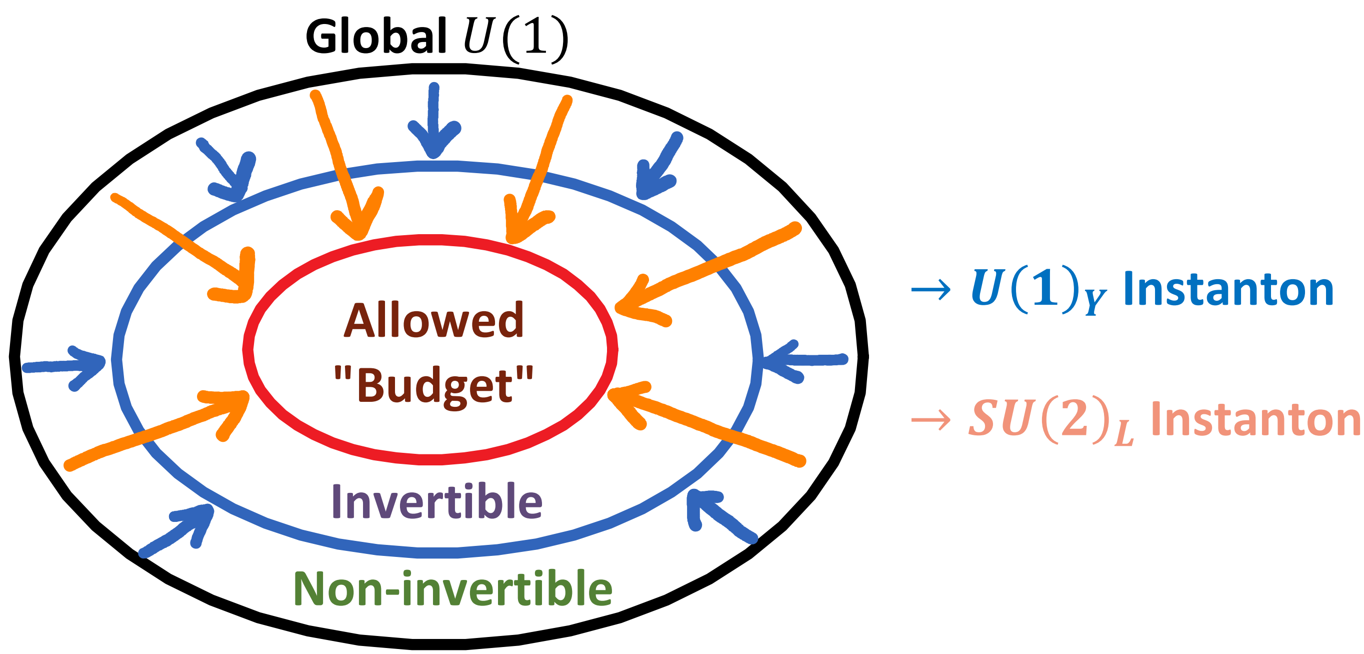

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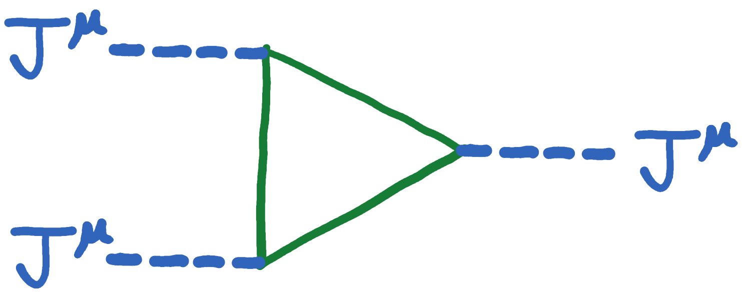

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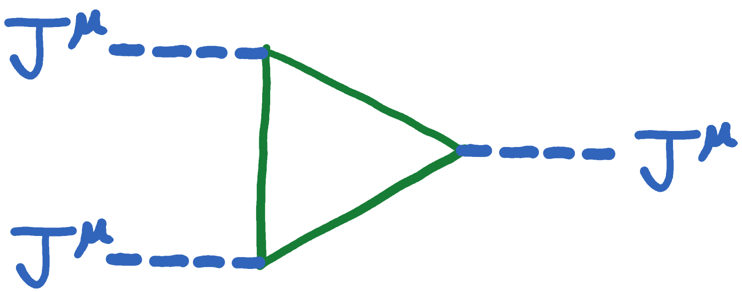

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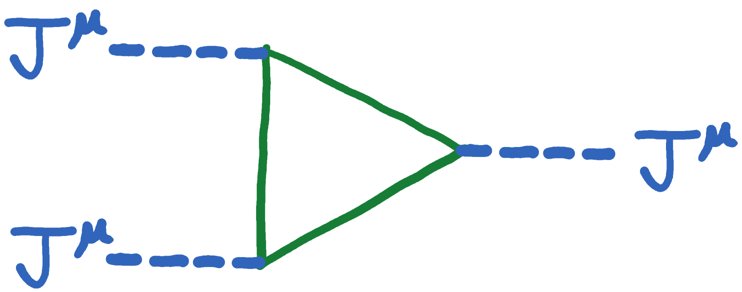

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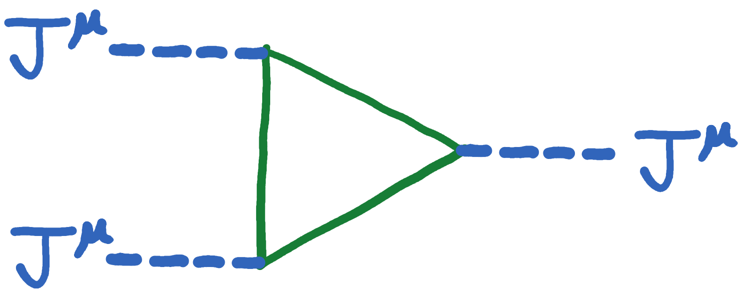

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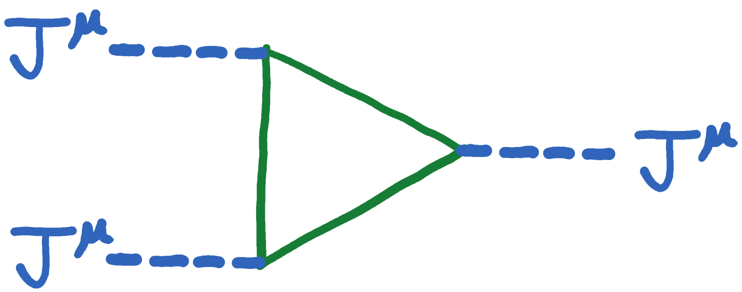

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	$SU(2)_H$	$U(1)_Z$	$L_\mu - L_\tau$	$U(1)_L$
Φ	2	-1	$\begin{bmatrix}\Phi_e \\ \Phi_\tau\end{bmatrix} = \begin{bmatrix}0 \\ -1\end{bmatrix}$	0
$L_{\mu e}$	2	$+1$	$\begin{bmatrix}L_\mu \\ L_{e_1}\end{bmatrix} = \begin{bmatrix}+1 \\ 0\end{bmatrix}$	$+1$
$L_{E\tau}$	2	-1	$\begin{bmatrix}L_{e_2} \\ L_\tau\end{bmatrix} = \begin{bmatrix}0 \\ -1\end{bmatrix}$	$+1$
ψ_L	$-$	0	0	-1
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- ABJ-anomalies:
  - $U(1)_L \to Z^L_{N_g-1} \; (SU(2)_H \; inst)$
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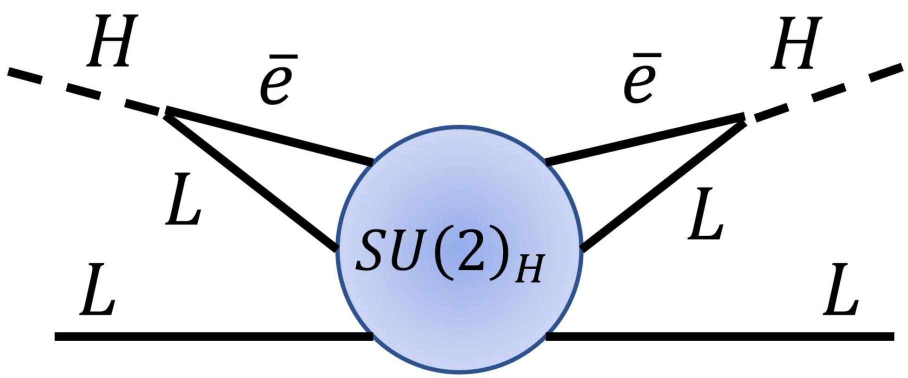

$\boldsymbol{U(1)_L}\, SU(2)_H^2 = 2$

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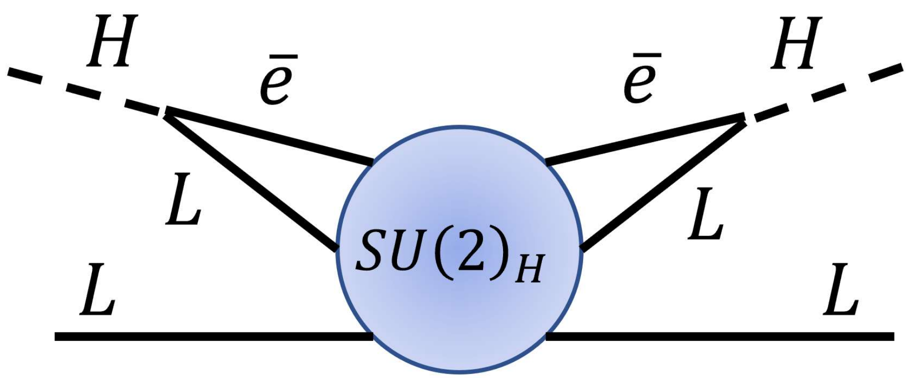

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$$\to\; y_\tau y_\mu \frac{v^2}{v_\Phi}\, e^{-\frac{2\pi}{\alpha_H}} \left[\nu_\mu \nu_\tau - \frac{1}{2} \sin 2\theta_L \nu_e \nu_e\right]$$

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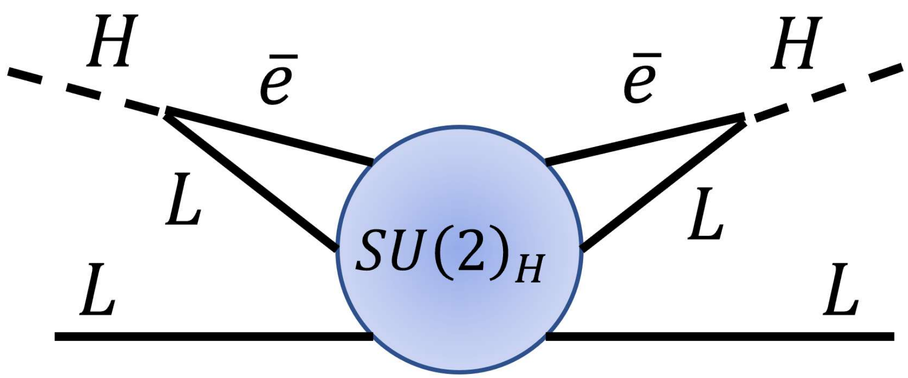

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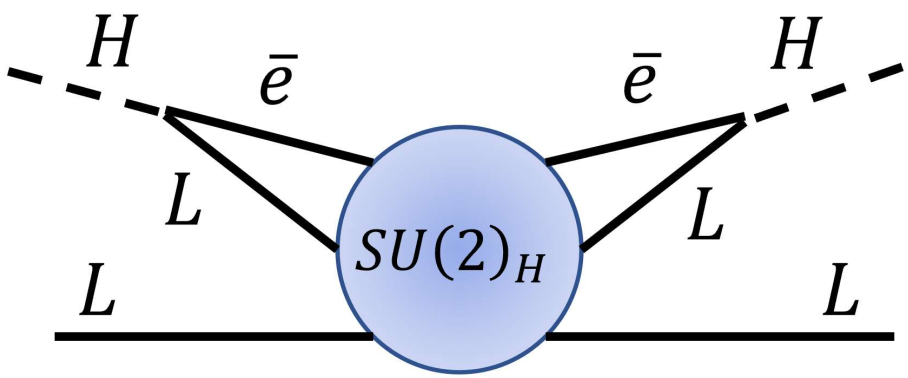

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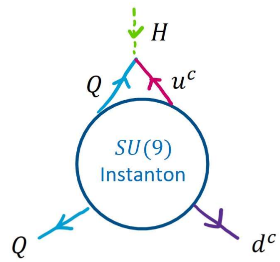

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2. In nature, up quark seems to be massive

   e.g. Chiral-PT + observed hadron mass : $m_u/m_d \sim 0.6$

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non-perturbative contribution to mass renormalization is real.
(H.Georgi, I.N.McArthur (1981), K.Choi, C.W.Kim, W.K.Sze (1988))

$$\mu \frac{d}{d\mu} \det(m) \supset c_0 \left(\frac{8\pi^2}{g^2}\right)^6 e^{-\frac{8\pi^2}{g^2}} \mu^{n_f-2} \det(m^\dagger m)\, Tr(m^\dagger m)^{-1}$$

Basic Idea: Massless up quark solution

3. Consistent story:

- $m_u = 0$ at some UV scale (symmetry!)
- $m_u \neq 0$ generated by non-pertubative (instanton) effects

non-perturbative contribution to mass renormalization is real.
(H.Georgi, I.N.McArthur (1981), K.Choi, C.W.Kim, W.K.Sze (1988))

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- QCD instanton calculation not under analytic control
- Lattice QCD : QCD instanton not sufficient

Solving Strong CP with Non-invertible Symmetry

1. **IR** : Start with SM with only y_u (massless down quark solution)

2. No non-invertible symmetry in quark sector of SM
   $(\widetilde{\mathrm{B}}_i = Q_i - u_i^c)$

	$U(1)_{\tilde{B}_1}$	$U(1)_{\tilde{B}_2}$	$U(1)_{\tilde{B}_3}$	$U(1)_{d_1}$	$U(1)_{d_2}$	$U(1)_{d_3}$
$[SU(3)_c]^2$	1	1	1	1	1	1
$[SU(2)_L]^2$	N_c	N_c	N_c	0	0	0
$[U(1)_Y]^2$	$-14N_c$	$-14N_c$	$-14N_c$	$4N_c$	$4N_c$	$4N_c$

3. This is true regardless of "global structure" of G_{SM}

$$SU(3)_C \times SU(2)_L \times U(1)_Y/\Gamma \;, \qquad \Gamma = 1, Z_2, Z_3, Z_6$$

Solving Strong CP with Non-invertible Symmetry

4. **Intermediate scale** : NIS from Embedding

Goal: forbid $y_d H Q d^c$ by non-invertible symmetry

(1) $\frac{SU(3)_C \times U(1)_H}{Z_{N_C}} \times SU(2)_L \times U(1)_Y \quad (H = B_1 + B_2 - 2B_3)$

- Z_3 (CFU) Fractional Instantons $\Rightarrow$ NIS from $U(1)_{d_3-d_1}$ and $U(1)_{d_3-d_2}$

  $(U(1)_{d_3-d_1}[CFU]^2 = 1,\ U(1)_{d_3-d_2}[CFU]^2 = 1)$

- $\mathcal{L}_{y_d} = y_d^{ij} H Q_i d_j^c$
  - $U(1)_H$ gauge-inv: $Q_1 d_1, Q_2 d_2, Q_3 d_3,\ Q_1 d_2, Q_2 d_1$
  - All these components are forbidden by non-invertible symmetries

Solving Strong CP with Non-invertible Symmetry

4. **Intermediate scale** : NIS from Embedding

Goal: forbid $y_d H Q d^c$ by non-invertible symmetry

(2) $\frac{SU(3)_C \times SU(3)_H}{Z_{N_C}} \times SU(2)_L \times U(1)_Y$ $\quad$ ($SU(3)_H =$ (horizontal) flavor)

	$SU(3)_C$	$SU(3)_H$	$U(1)_{\tilde{B}}$	$U(1)_d$
Q	3	3	1	0
u^c	$\overline{3}$	$\overline{3}$	-1	0
d^c	$\overline{3}$	$\overline{3}$	0	1

$\tilde{B} = Q - u^c$

- w/o Z_3modding $\Rightarrow$ $U(1)_{\tilde{B}} \to Z_3$, $U(1)_d \to Z_3$

  $\mathcal{L} = y_d H Q d^c$ forbidden by these invertible symmetries

Solving Strong CP with Non-invertible Symmetry

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Q	3	3	1	0
u^c	$\bar{3}$	$\bar{3}$	-1	0
d^c	$\bar{3}$	$\bar{3}$	0	1

$\tilde{B} = Q - u^c$

- with Z_3 modding $\Rightarrow$ Z_3 magnetic 1-form
  $\Rightarrow$ fractional instanton breaks $U(1)_{\tilde{B}}, U(1)_d$ completely

$\mathcal{L} = y_d H Q d^c$ forbidden by Z_3 non-invertible symmetries

Solving Strong CP with Non-invertible Symmetry

5. **UV scale** : Breaking of NIS by non-perturbative effects

Goal: generate $y_d H Q d^c$ by breaking non-invertible symmetry

UV embedding: $\frac{SU(3)_c \times SU(3)_H}{Z_{N_c}} \subset SU(9)$

	$SU(9)$	$U(1)_{\tilde{B}}$	$U(1)_d$
Q	9	1	0
u^c	$\bar{9}$	-1	0
d^c	$\bar{9}$	0	1

$\tilde{B} = Q - u^c$

$$\mathcal{L}_{UV} \supset y_u \tilde{H} Q u^c + \cdots$$

$y_d H Q d^c$ forbidden by $\left(d \text{ or } \tilde{B} + d\right)$

Solving Strong CP with Non-invertible Symmetry

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	$SU(9)$	$U(1)_{\tilde{B}}$	$U(1)_d$
Q	9	1	0
u^c	$\bar{9}$	-1	0
d^c	$\bar{9}$	0	1

$U(1)_{B=\tilde{B}-d}[SU(9)]^2 = 0$

$U(1)_{\tilde{B}+d}[SU(9)]^2 = 2$

(or $U(1)_d[SU(9)]^2 = 1$*)*

$$\mathcal{L}_{UV} \supset y_u \widetilde{H} Q u^c + \cdots$$

Solving Strong CP with Non-invertible Symmetry

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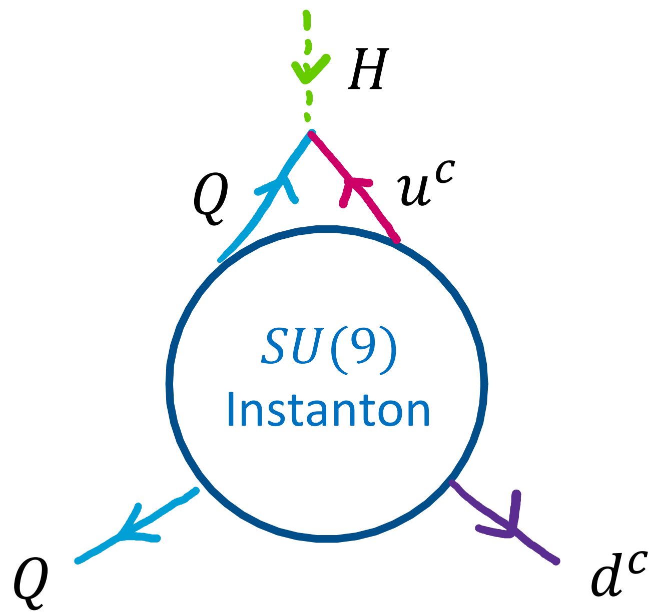
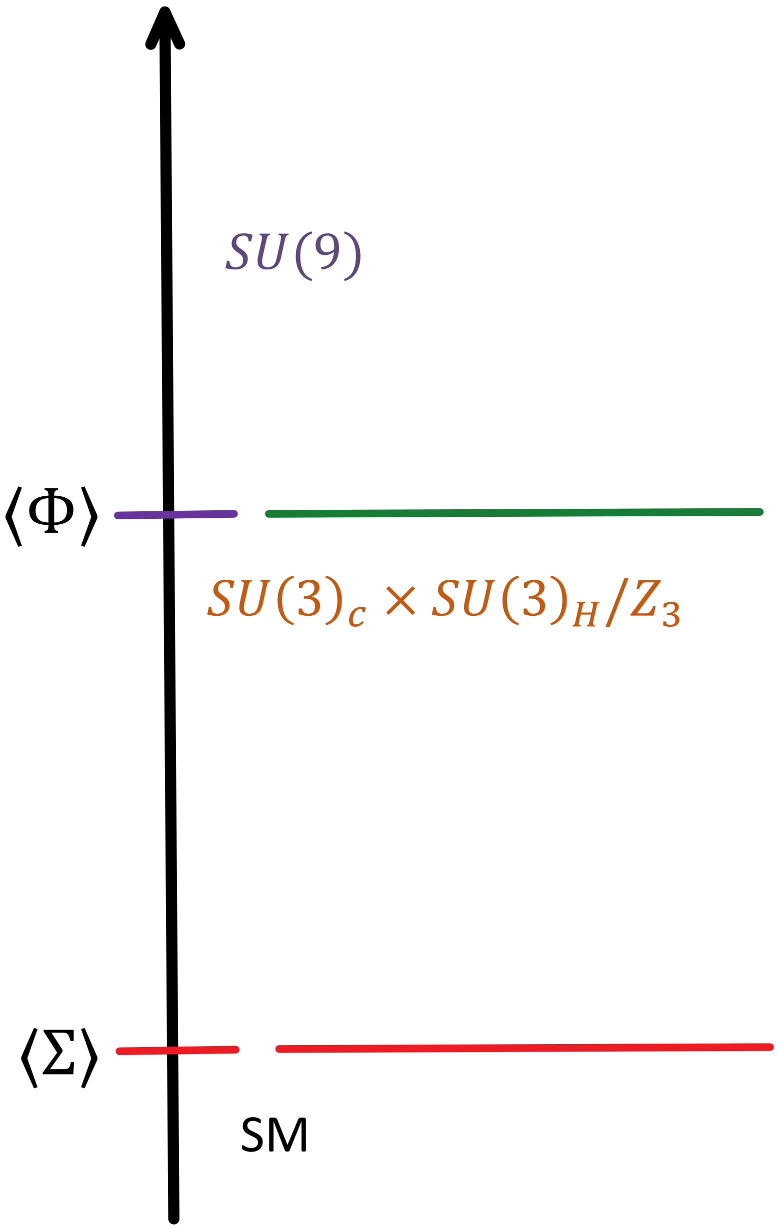
Goal: generate $y_d H Q d^c$ by breaking non-invertible symmetry



$$\mathcal{L}_{UV} \supset y_u \widetilde{H} Q u^c + y_u e^{-\frac{2\pi}{\alpha_9}} H Q d^c + \cdots$$

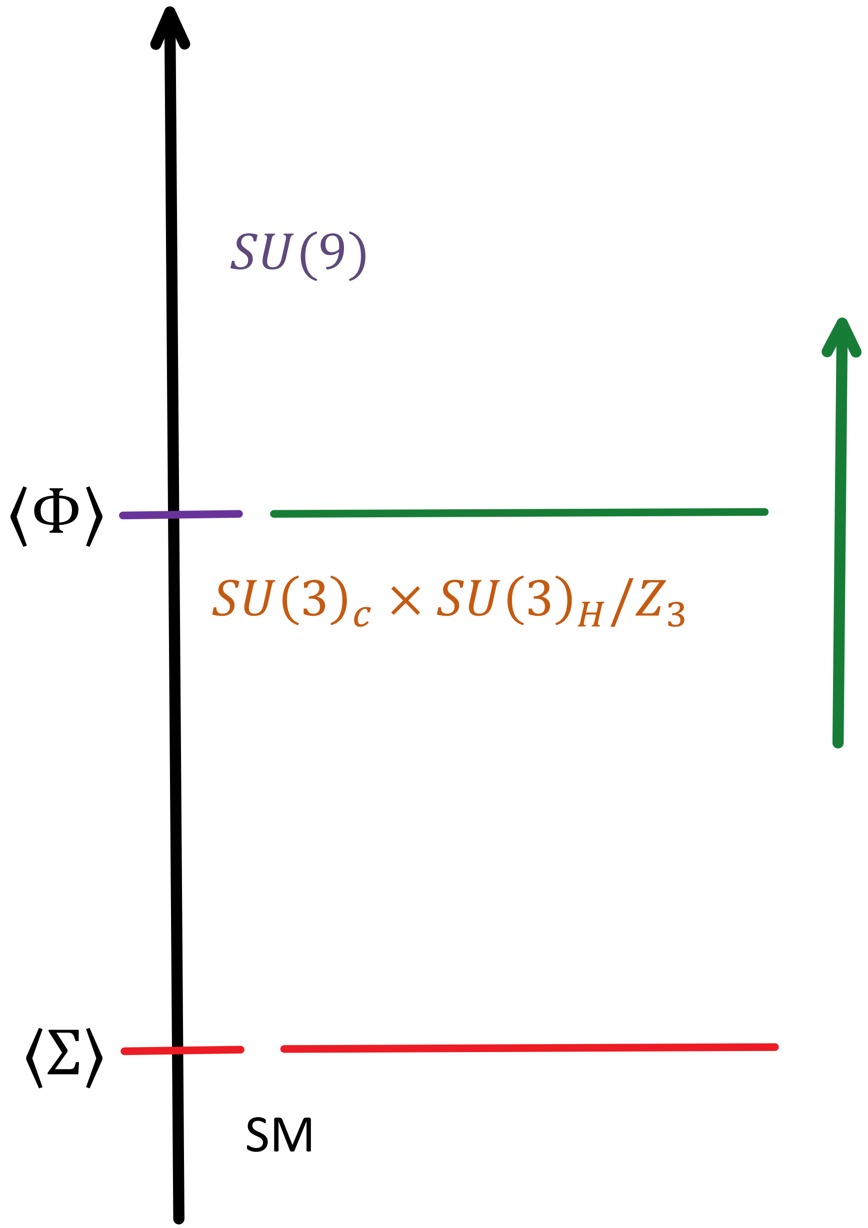
In IR, $\frac{y_b}{y_t} \sim \frac{1}{40} \Rightarrow \alpha_9 \sim 1.5$

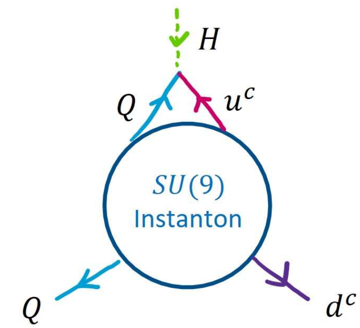
Solving Strong CP with Non-invertible Symmetry



- Start with only $y_u \widetilde{H} Q u^c$
- Z_3^d **NIS** from fractional CFU instantons
- $y_d H Q d^c$ protected by **NIS**
- Any $y_d H Q d^c$ must be from UV instanton (i.e. breaking of NIS)

Solving Strong CP with Non-invertible Symmetry





$$\mathcal{L}_{UV} \supset y_u \widetilde{H} Q u^c + y_u e^{-\frac{2\pi}{\alpha_9}} H Q d^c + \cdots$$

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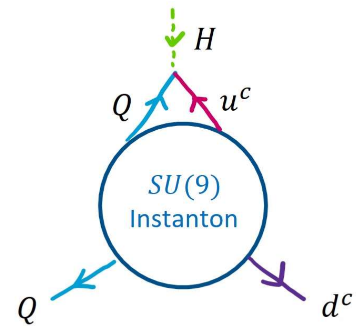
Solving Strong CP with Non-invertible Symmetry



$$\mathcal{L}_{UV} \supset y_u \widetilde{H} Q u^c + y_u e^{-\frac{2\pi}{\alpha_9}} H Q d^c + \cdots$$

$SU(9)$

$\langle \Phi \rangle$

$SU(3)_c \times SU(3)_H / Z_3$

- Start with only $y_u \widetilde{H} Q u^c$
- Z_3^d **NIS** from fractional CFU instantons
- $y_d H Q d^c$ protected by **NIS**
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$\langle \Sigma \rangle$

SM

Yukawa texture and CKM

THANK YOU

FOR

YOUR ATTENTION!

Small M_ν from Generalized Symmetry Breaking

5. M_ν^{ij} from non-invertible symmetry $\Rightarrow$ new physics scale v_Φ

Small M_ν from Generalized Symmetry Breaking

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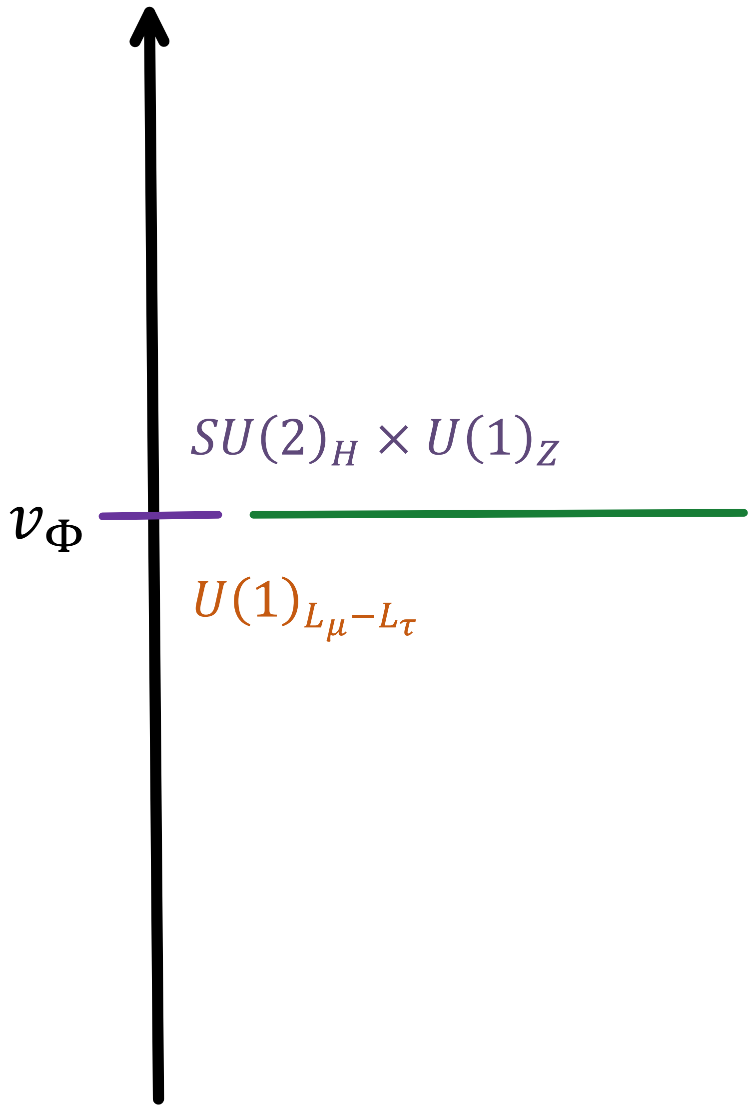
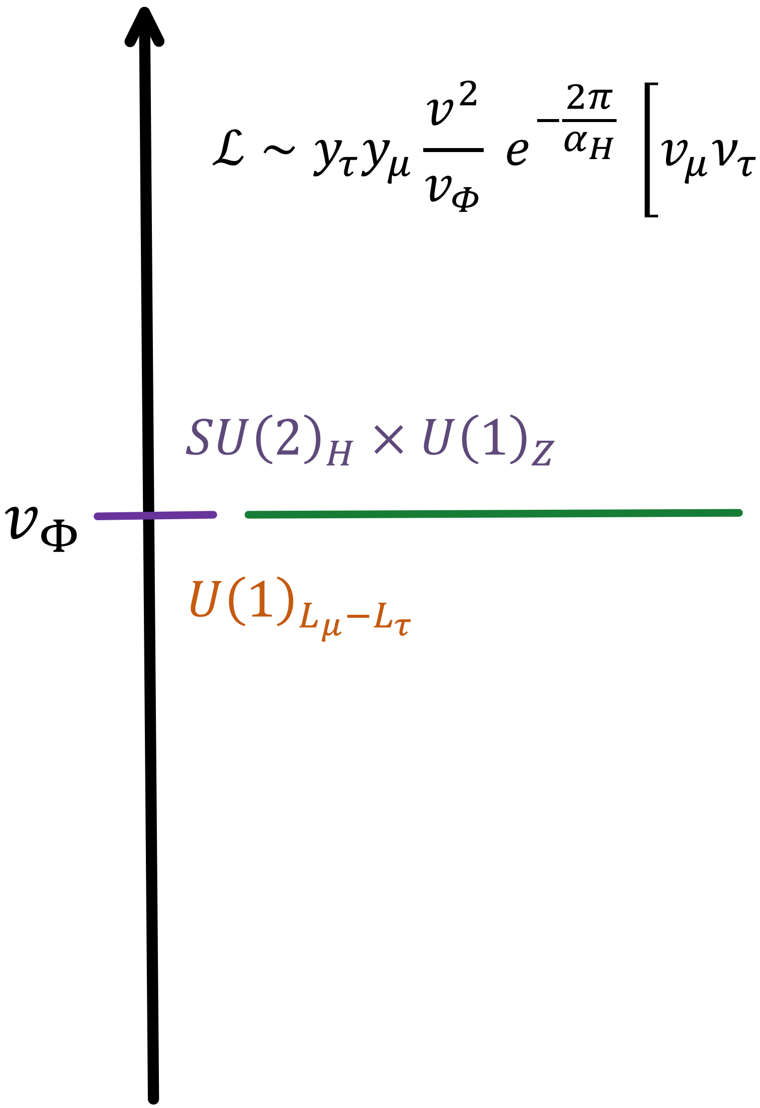
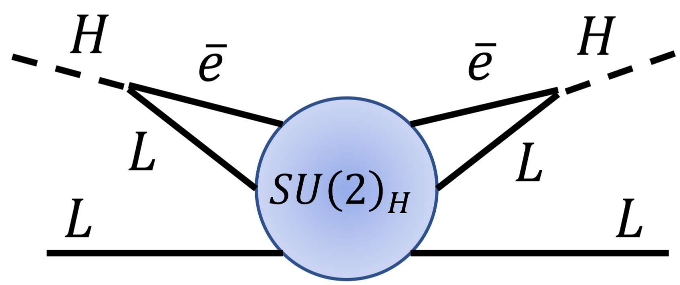


Small M_ν from Generalized Symmetry Breaking

5. M_ν^{ij} from non-invertible symmetry $\Rightarrow$ new physics scale v_Φ

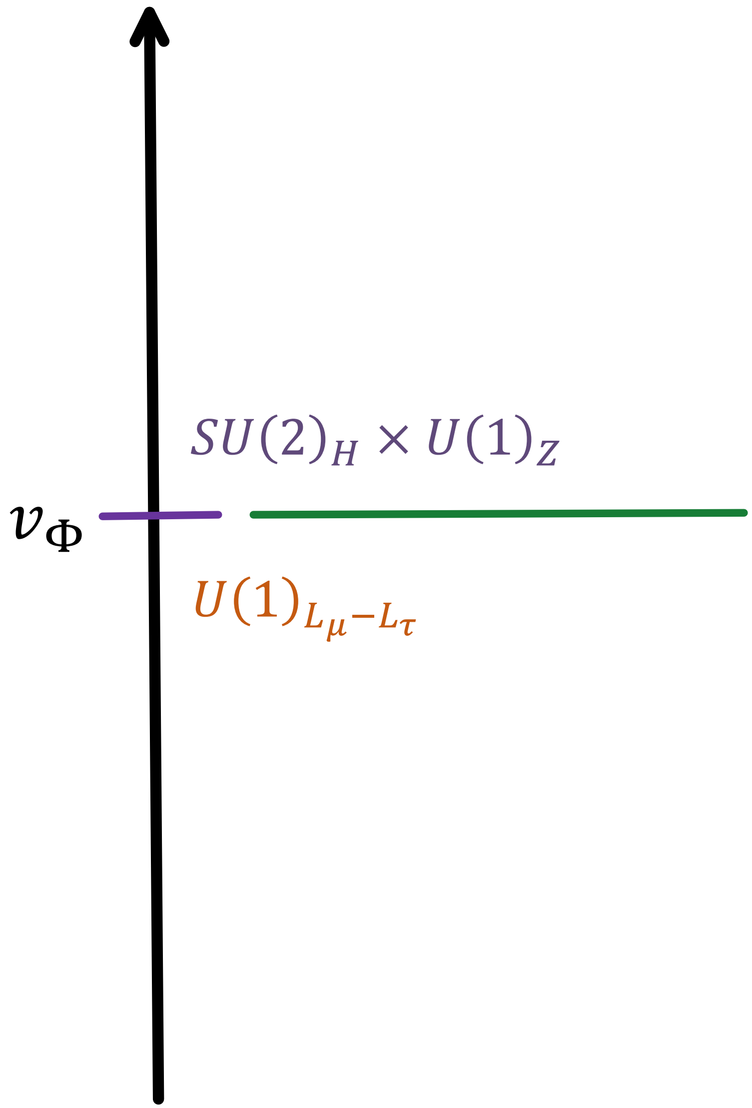
$$\mathcal{L} \sim y_\tau y_\mu \frac{v^2}{v_\Phi}\, e^{-\frac{2\pi}{\alpha_H}} \left[\nu_\mu \nu_\tau - \frac{1}{2}\sin 2\theta_L \nu_e \nu_e \right] \rightarrow m_\nu \sim \frac{m_\tau m_\mu}{v_\Phi}\, e^{-\frac{2\pi}{\alpha_H(v_\Phi)}}$$





Small M_ν from Generalized Symmetry Breaking

5. M_ν^{ij} from non-invertible symmetry $\Rightarrow$ new physics scale v_Φ



$$m_\nu \sim \frac{m_\tau m_\mu}{v_\Phi} e^{-\frac{2\pi}{\alpha_H(v_\Phi)}}$$

$$\alpha_H = \frac{g_H^2}{4\pi}, \qquad g_{\mu\tau}(v_\Phi) = g_H(v_\Phi) \sin\theta_H$$

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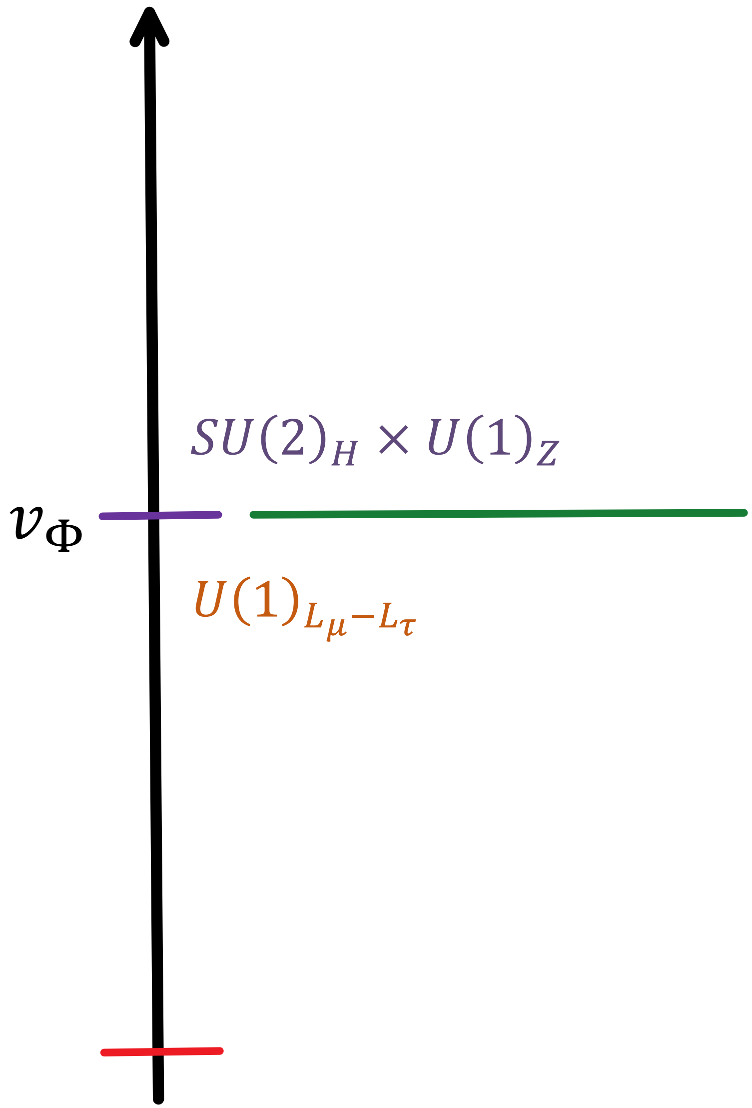

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$$M_{Z'}, \alpha_{\mu\tau}(M_{Z'}^2)$$

Small M_ν from Generalized Symmetry Breaking

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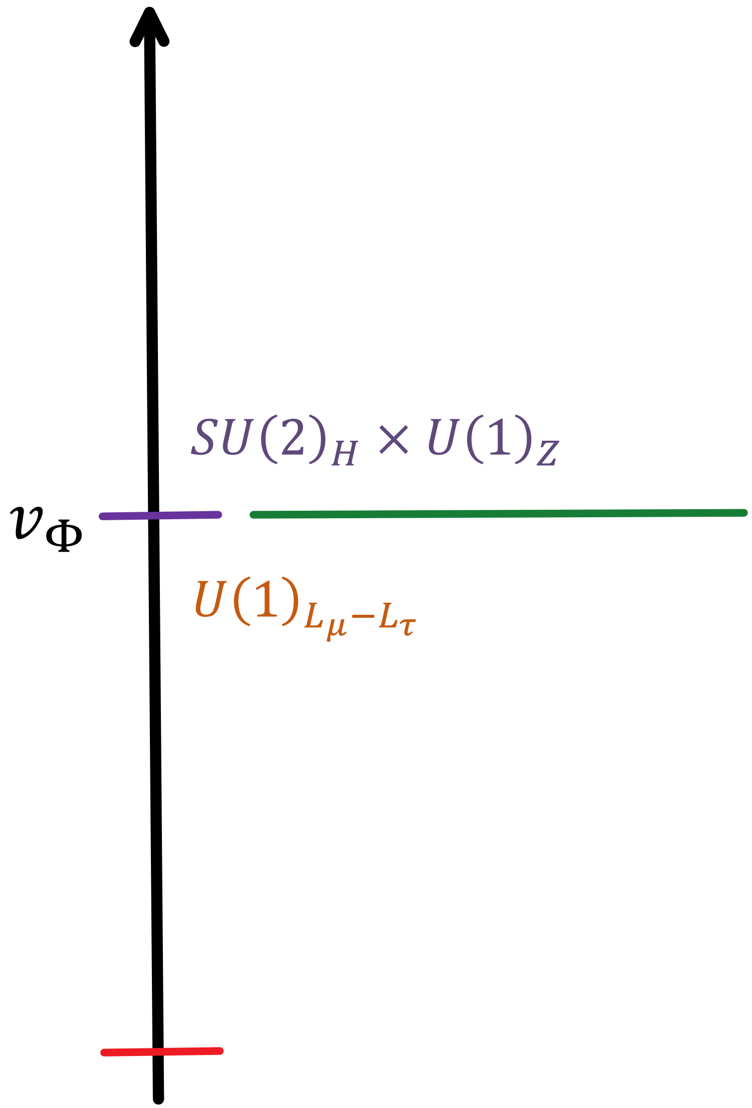

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$$\alpha_H = \frac{g_H^2}{4\pi}, \qquad g_{\mu\tau}(v_\Phi) = g_H(v_\Phi)\sin\theta_H$$

$$\alpha_{\mu\tau}(\mu^2)^{-1} = \alpha_{\mu\tau}(M_{Z'}^2)^{-1} - \frac{1}{\pi}\log\frac{\mu^2}{M_{Z'}^2}$$

$M_{Z'}, \alpha_{\mu\tau}(M_{Z'}^2)$

Small M_ν from Generalized Symmetry Breaking

5. M_ν^{ij} from non-invertible symmetry $\Rightarrow$ new physics scale v_Φ

$$v_\Phi^{4s_H^2-1} \sim M_{Z'}^{4s_H^2-1} \left(\frac{M_{Z'} m_\nu}{m_\mu m_\tau}\right) \exp \frac{2\pi s_H^2}{\alpha_{\mu\tau}\left(M_{Z'}^2\right)}$$

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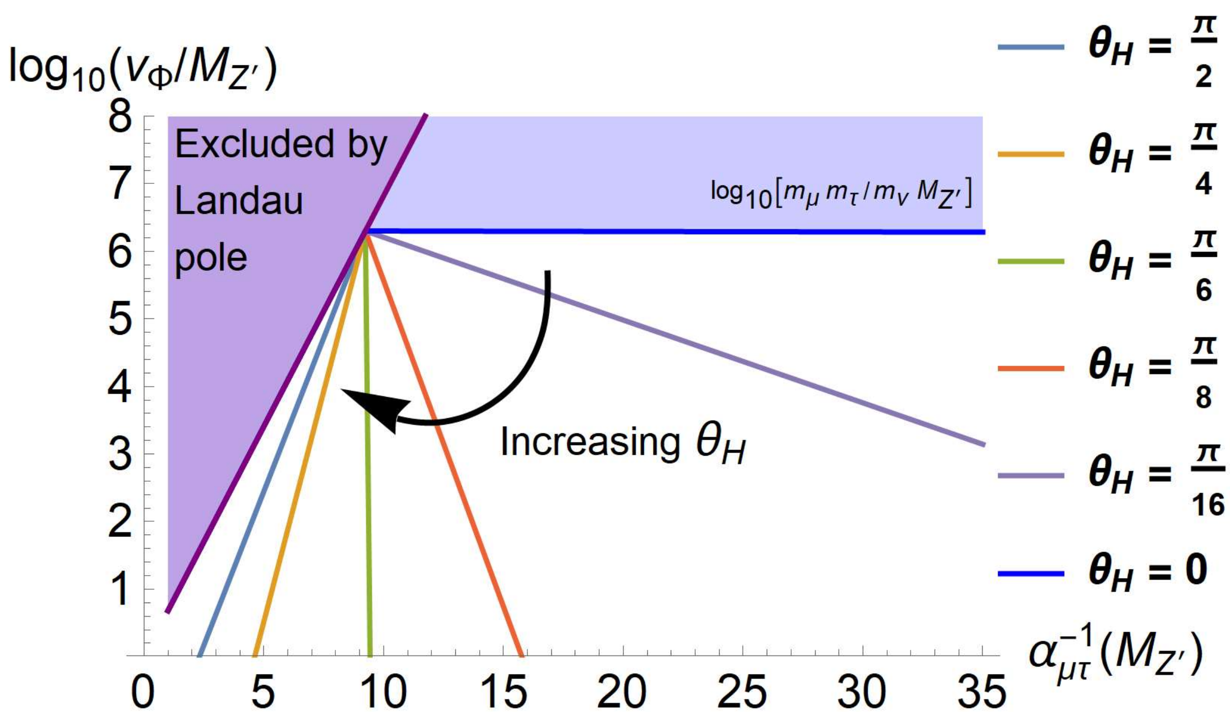

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Cf) For Dirac mass case

$$v_\Phi^2 \sim M_{Z'}^2\left(\frac{m_\nu}{m_\tau}\right)^{\frac{3}{2}}\exp\frac{3\pi}{4\alpha_{\mu\tau}(M_{Z'}^2)}$$

Small M_ν from Generalized Symmetry Breaking

5. M_ν^{ij} texture through RG-flow

$$\mathcal{L} \sim y_\tau y_\mu \frac{v^2}{v_\Phi} e^{-\frac{2\pi}{\alpha_H}} \left[\nu_\mu \nu_\tau - \frac{1}{2} \sin 2\theta_L \nu_e \nu_e\right]$$

Small M_ν from Generalized Symmetry Breaking

5. M_ν^{ij} texture through RG-flow

$$\mathcal{L} \sim y_\tau y_\mu \frac{v^2}{v_\Phi}\, e^{-\frac{2\pi}{\alpha_H}} \left[\nu_\mu \nu_\tau - \frac{1}{2} \sin 2\theta_L \nu_e \nu_e\right]$$

- Below $E < v_\Phi$ we have $G_{SM} \times U(1)_{L_\mu - L_\tau}$ theory
- Let φ be the (charge 1) scalar that Higgses $U(1)_{L_\mu - L_\tau}$
- RG-flow below $E < v_\Phi$ generates textures

$$\mathcal{L} \sim y_\tau y_\mu \frac{v^2}{v_\Phi}\, e^{-\frac{2\pi}{\alpha_H}} \left[\nu_e \nu_\tau \frac{\varphi}{v_\Phi} + \nu_e \nu_\mu \frac{\varphi^\dagger}{v_\Phi} + \nu_\tau \frac{\varphi}{v_\Phi} \nu_\tau \frac{\varphi}{v_\Phi} + \nu_\mu \frac{\varphi^\dagger}{v_\Phi} \nu_\mu \frac{\varphi^\dagger}{v_\Phi}\right]$$