



Understanding the quantum structure of the dimension-8 SMEFT

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based on [2106.05291](#), [2110.01264](#), [2301.09995](#) and ongoing work

The 5th NPKI workshop, Busan; June 7, 2023

The SMEFT is the SM extended with effective operators

(Probably) the most reasonable model of new physics

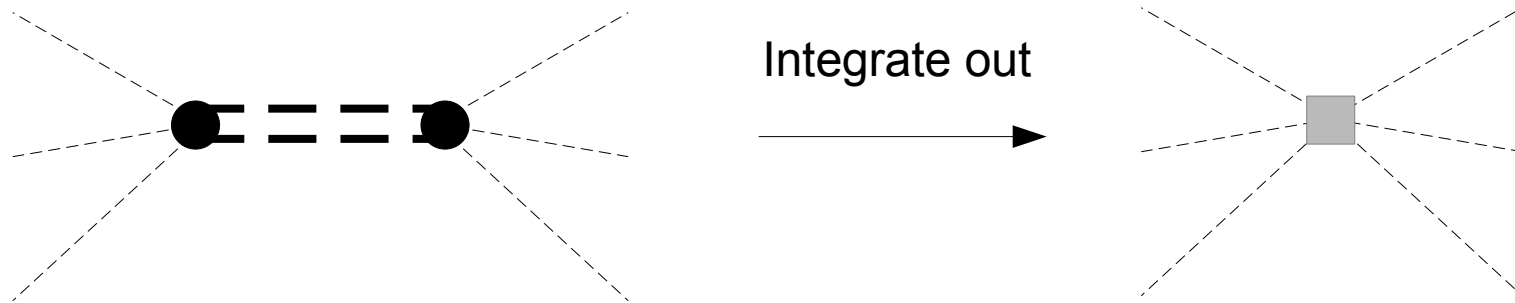
$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \sum_i \frac{c_i^{(6)}}{\Lambda^2} \mathcal{O}_i^{(6)} + \sum_j \frac{c_j^{(8)}}{\Lambda^4} \mathcal{O}_j^{(8)} + \dots$$

If probed by experiments at very different scales, **RGEs of the theory are needed** [Jenkins, Manohar, Trott, Alonso '13].

Interesting theoretical aspects at dimension 8 (positivity, tree-loop mixing, test tools, ...)

Besides pure **theoretical considerations**, anomalous dimensions of dimension-8 operators [Murphy '20; Li, Ren, Shu, Xiao, Yu, Zheng '20] not always phenomenologically irrelevant

Simplest example:



Custodial symmetry violation absent at tree-level dimension-6, one-loop dimension-6 and tree-level dimension-8 [MC, Krause, Nardini '18; Durieux, McCullough, Salvioni '22]

Some partial results:

MC, Guedes, Ramos, Santiago; [2106.05291](#)

Accettulli Huber, De Angelis; [2108.03669](#)

Bakshi, MC, Diaz-Carmona, Guedes; [2205.03301](#)

Helset, Jenkins, Manohar; [2212.03253](#)

Asteriadis, Dawson, Fontes; [2212.03258](#)

Bakshi, Diaz-Carmona; [2301.07151](#)

More generally, [certain aspects of the full anomalous dimension matrix](#) well understood

Craig, Jiang, Li, Sutherland; [2001.00017](#)

| | | | | | | |
|-----------|---|---------|---|---|---|---|
| \bar{w} | 8 | X_L^4 | $X_L^3 H^2,$ $X_L^2 \psi^2 H,$ $X_L \psi^4$ | $X_L^2 H^4,$ $X_L \psi^2 H^3,$ $\psi^4 H^2$ | $\psi^2 H^5$ | H^8 |
| | 6 | | $X_L^2 H^2 D^2,$ $X_L^2 \psi \bar{\psi} D,$ $X_L \psi^2 H D^2,$ $\psi^4 D^2$ | $X_L H^4 D^2,$ $X_L^2 \bar{\psi}^2 H,$ $X_L \psi \bar{\psi} H^2 D,$ $\psi^2 H^3 D^2,$ $X_L \psi^2 \bar{\psi}^2,$ $\psi^3 \bar{\psi} H D$ | $H^6 D^2,$ $\psi \bar{\psi} H^4 D,$ $\psi^2 \bar{\psi}^2 H^2$ | $\bar{\psi}^2 H^5$ |
| | 4 | | | $X_L^2 X_R^2,$ $X_L X_R H^2 D^2,$ $H^4 D^4,$ $X_L X_R \psi \bar{\psi} D,$ $X_R \psi^2 H D^2,$ $X_L \bar{\psi}^2 H D^2,$ $\psi \bar{\psi} H^2 D^3,$ $\psi^2 \bar{\psi}^2 D^2$ | $X_R H^4 D^2,$ $X_R^2 \psi^2 H,$ $X_R \psi \bar{\psi} H^2 D,$ $\bar{\psi}^2 H^3 D^2,$ $X_R \psi^2 \bar{\psi}^2,$ $\psi \bar{\psi}^3 H D$ | $X_R^2 H^4,$ $X_R \bar{\psi}^2 H^3,$ $\bar{\psi}^4 H^2$ |
| | 2 | | | | $X_R^2 H^2 D^2,$ $X_R^2 \psi \bar{\psi} D,$ $X_R \bar{\psi}^2 H D^2,$ $\bar{\psi}^4 D^2$ | $X_R^3 H^2,$ $X_R^2 \bar{\psi}^2 H,$ $X_R \bar{\psi}^4$ |
| | 0 | | | | | X_R^4 |
| | | 0 | 2 | 4 | 6 | 8 |

w

Murphy '20;
based on Craig et al '20

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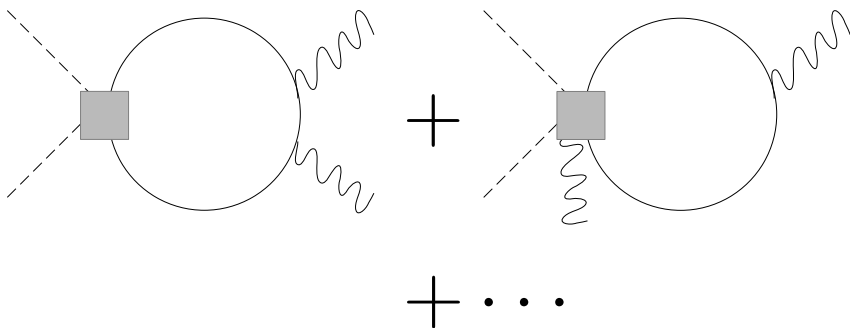
It is obvious that there are zeros in mixing of specific operators of different classes

It is **not so clear how to anticipate them**, not even with amplitude methods

$$\mathcal{O}_{e^2\phi^2 D^3}^{(1)} = i(\bar{e}\gamma^\mu D^\nu e)(D_{(\mu}D_{\nu)}\phi^\dagger\phi) + \text{h.c.}$$

$$\mathcal{O}_{B^2\phi^2 D^2}^{(1)} = (D^\mu\phi^\dagger D^\nu\phi)B_{\mu\rho}B_\nu^\rho$$

$$\mathcal{O}_{e^2\phi^2 D^3}^{(2)} = i(\bar{e}\gamma^\mu D^\nu e)(\phi^\dagger D_{(\mu}D_{\nu)}\phi) + \text{h.c.}$$



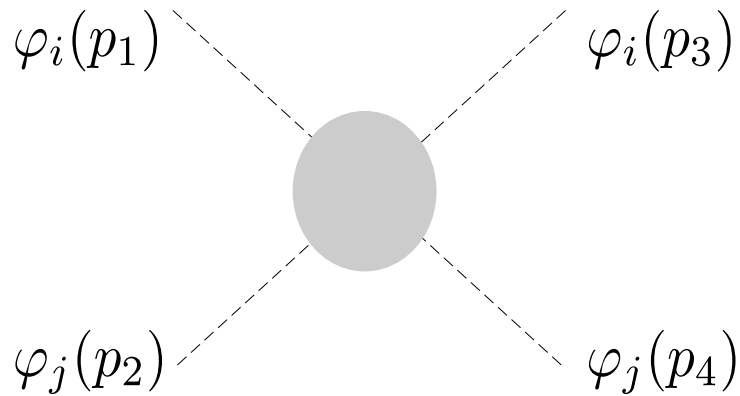
$$\underbrace{\mathcal{O}_{e^2\phi^2 D^3}^{(1)} - \mathcal{O}_{e^2\phi^2 D^3}^{(2)}}_{\tilde{\mathcal{O}}_{e^2\phi^2 D^3}^{(1)}} \not\rightarrow \mathcal{O}_{B^2\phi^2 D^2}^{(1)}$$

$$\begin{array}{ccc}
 \begin{array}{c} 1_0 \\ \diagdown \\ \square \\ \diagup \\ 2_0 \end{array} \begin{array}{c} \text{wavy} \\ \text{lines} \end{array} \begin{array}{c} 3_{+1} \\ \diagup \\ 4_{-1} \end{array} & = \langle 41 \rangle^2 [31]^2 & \begin{array}{c} 1_0 \\ \diagdown \\ \square \\ \diagup \\ 2_0 \end{array} \begin{array}{c} \text{solid} \\ \text{lines} \end{array} \begin{array}{c} 3_{+1/2} \\ \diagup \\ 4_{-1/2} \end{array} & = \langle 43 \rangle \langle 41 \rangle [43] [31]
 \end{array}$$

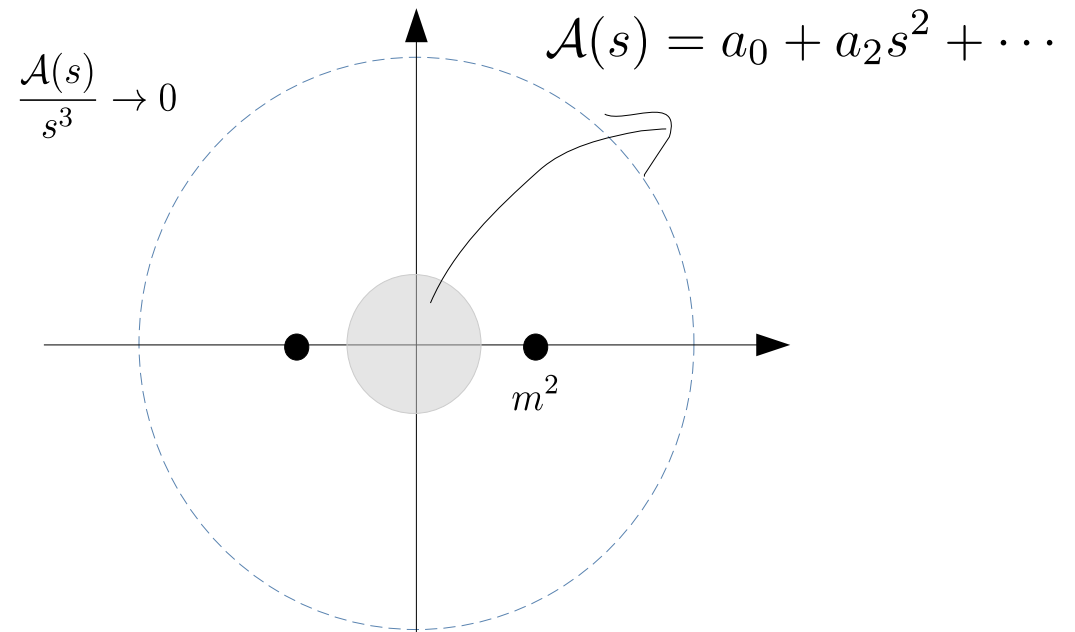
$$\gamma_{\tilde{c}}^{(1)} \Big|_{e^2 \phi^2 D^3} \rightarrow c_{B^2 \phi^2 D^2}^{(1)} \propto \begin{array}{c} 1_0 \\ \diagdown \\ \square \\ \diagup \\ 2_0 \end{array} \begin{array}{c} \text{solid} \\ \text{lines} \end{array} \begin{array}{c} 3'_{+1/2} \\ \diagup \\ 4'_{-1/2} \end{array} \quad \text{---} \quad \begin{array}{c} 3'_{-1/2} \\ \diagdown \\ \text{SM} \\ \diagup \\ 4'_{+1/2} \end{array} \begin{array}{c} \text{wavy} \\ \text{lines} \end{array} \begin{array}{c} 3_{+1} \\ \diagup \\ 4_{-1} \end{array}$$

$$\begin{aligned}
 &= \int d\text{LIPS} \langle 4'3' \rangle \langle 4'1 \rangle [4'3'] [3'1] \frac{\langle 3'4 \rangle^2}{\langle 3'3 \rangle \langle 34' \rangle} \\
 &= \int_0^{2\pi} d\phi \int_0^{\pi/2} d\theta s_\theta c_\theta \left[\#_1 e^{i\phi} + \#_2 e^{2i\phi} + \dots \right]
 \end{aligned}$$

A different perspective: **certain operators are constrained by positivity**, from unitarity+locality
 [Adams, Arkani-Hamed, Nicolis, Rattazzi '06]



$$\mathcal{A}(s) \equiv \mathcal{A}(s, t = 0)$$



$$0 = \sum \text{res} \frac{\mathcal{A}(s)}{s^3} = a_2 - \frac{1}{\pi} \int s \frac{\sigma(s)}{(m^2)^3} \Rightarrow a_2 > 0$$

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$$c_{B^2\phi^2 D^2}^{(1)} \leq 0$$

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But some others are not:

$$\tilde{\mathcal{O}}_{e^2\phi^2 D^3}^{(1)} = \begin{array}{ccc} 1_0 & \text{---} & 3_{+1/2} \\ & \diagdown \quad \diagup & \\ & \square & \\ & \diagup \quad \diagdown & \\ 2_0 & \text{---} & 4_{-1/2} \end{array} = \langle 43 \rangle \langle 41 \rangle [43] [31]$$

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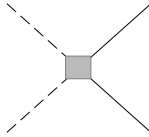
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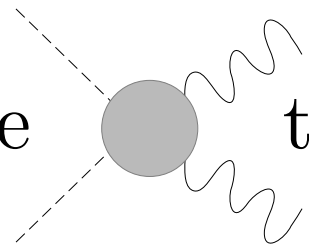
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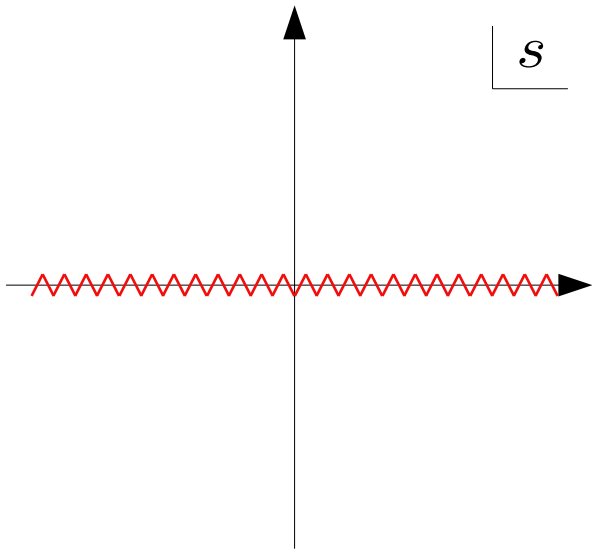
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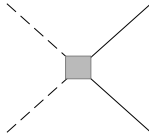
“Therefore”,

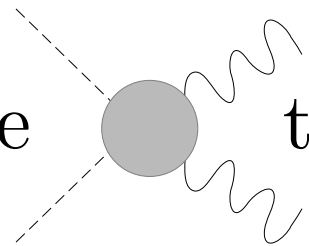
$$\dot{c}_{B^2\phi^2 D^2}^{(1)} = \#_1 \tilde{c}_{e^2\phi^2 D^3}^{(1)} + \dots \Rightarrow \#_1 = 0$$

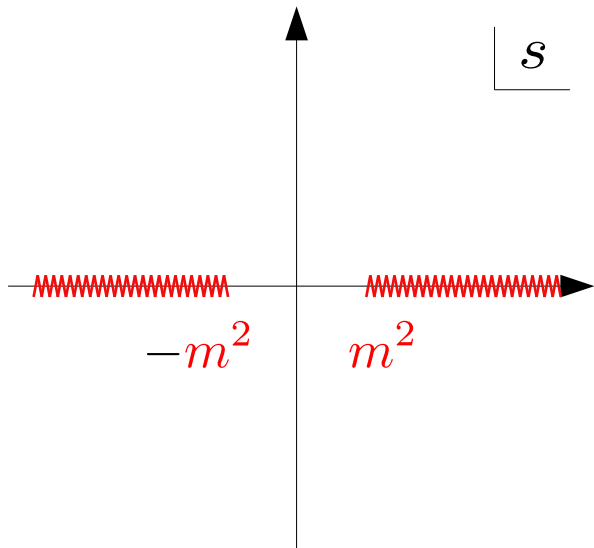
1. For any $(c_{e^2\phi^2 D^3}^{(1)}, c_{e^2\phi^2 D^3}^{(2)})$ compatible with the positivity bounds $(c_{e^2\phi^2 D^3}^{(1)} + c_{e^2\phi^2 D^3}^{(2)} \leq 0)$, there exists UV such that only $(c_{e^2\phi^2 D^3}^{(1)}, c_{e^2\phi^2 D^3}^{(2)})$ (and lower-dimensional  ones) at tree level.

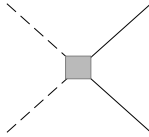
2. Within any such UV, compute  to order $O(g^2)$

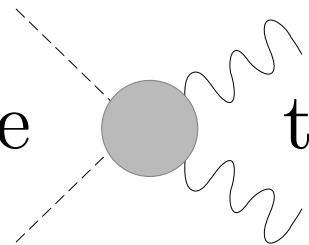


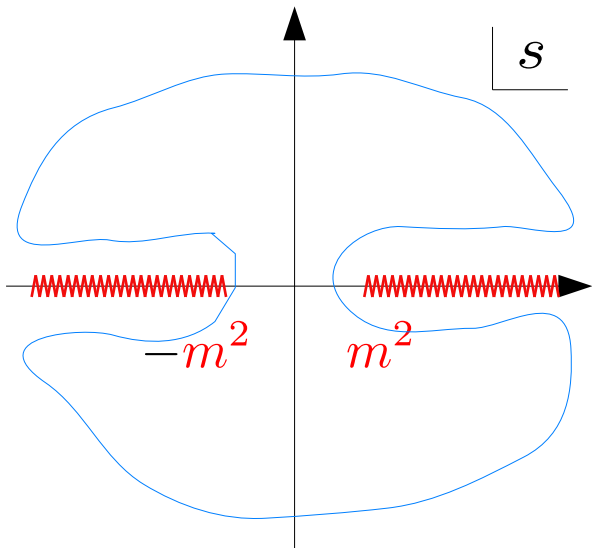
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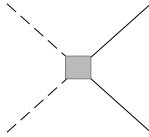


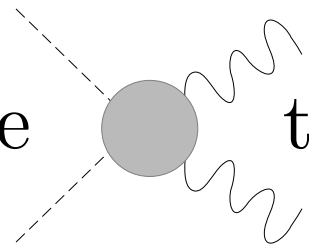
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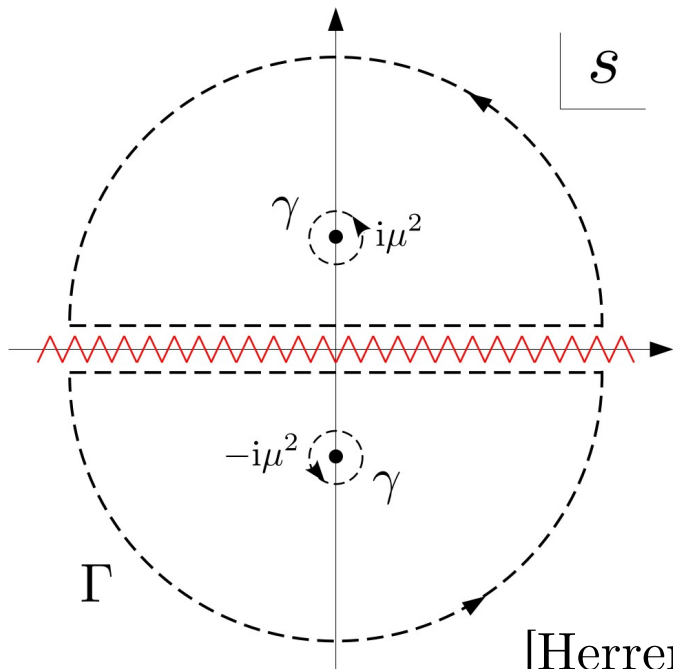
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See Minyuan's talk

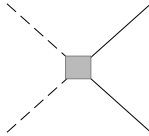
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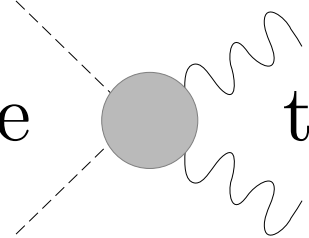
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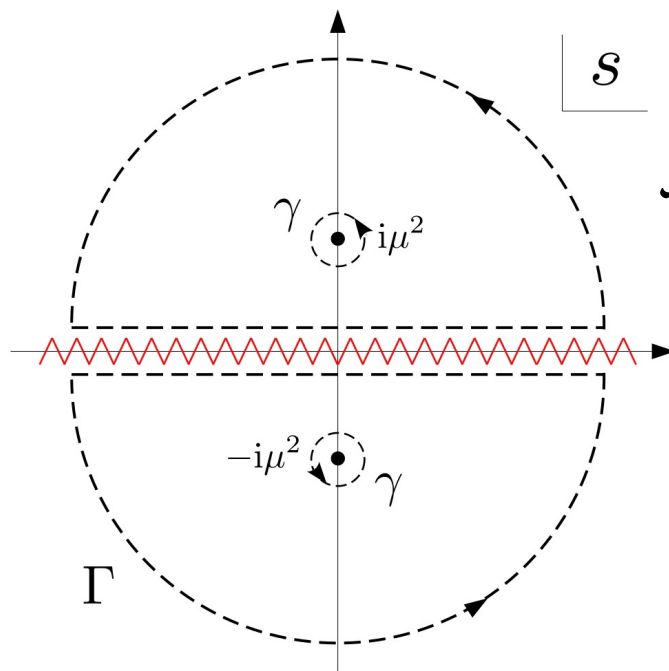


$$\Sigma(\mu) \equiv \frac{1}{2\pi i} \int_{\gamma} \frac{\mathcal{A}(s) s^3}{(s^2 + \mu^4)^3} \geq 0$$

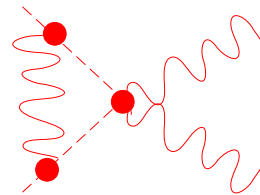
[Herrero-Valea et al '20]

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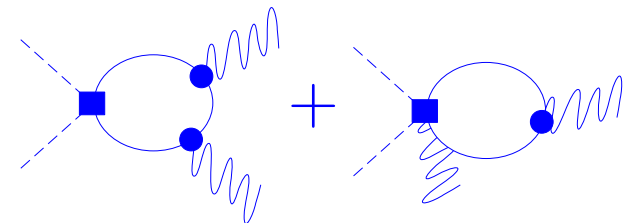
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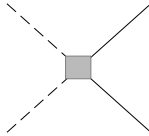
$$\mathcal{A}(s) \sim -(\beta_4 + \beta_8 s^2 + \beta_{12} s^4 + \dots) \log \frac{s}{\Lambda^2}$$

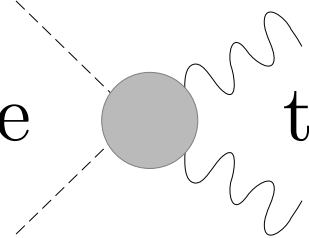


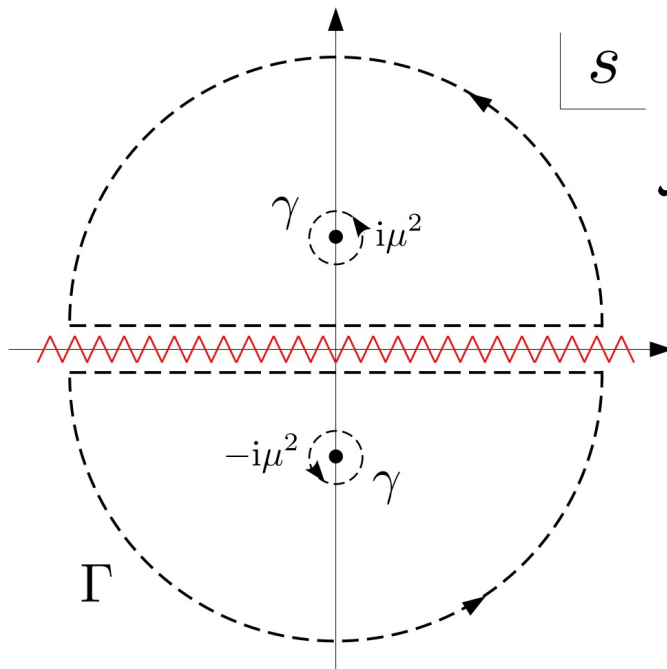
+ ...



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$$\mathcal{A}(s) \sim -(\beta_4 + \beta_8 s^2 + \beta_{12} s^4 + \dots) \log \frac{s}{\Lambda^2}$$

$$\Sigma(\mu) = -\beta_8 + \beta_{12} \mu^4 + \dots$$

$$\Rightarrow \lim_{\mu \rightarrow 0} \Sigma(\mu) = -\beta_8 \geq 0$$

So $\beta_8 \leq 0$ in any of the aforementioned UV, and therefore for all values of $(c_{e^2\phi^2 D^3}^{(1)}, c_{e^2\phi^2 D^3}^{(2)})$ compatible with $c_{e^2\phi^2 D^3}^{(1)} + c_{e^2\phi^2 D^3}^{(2)} \leq 0$

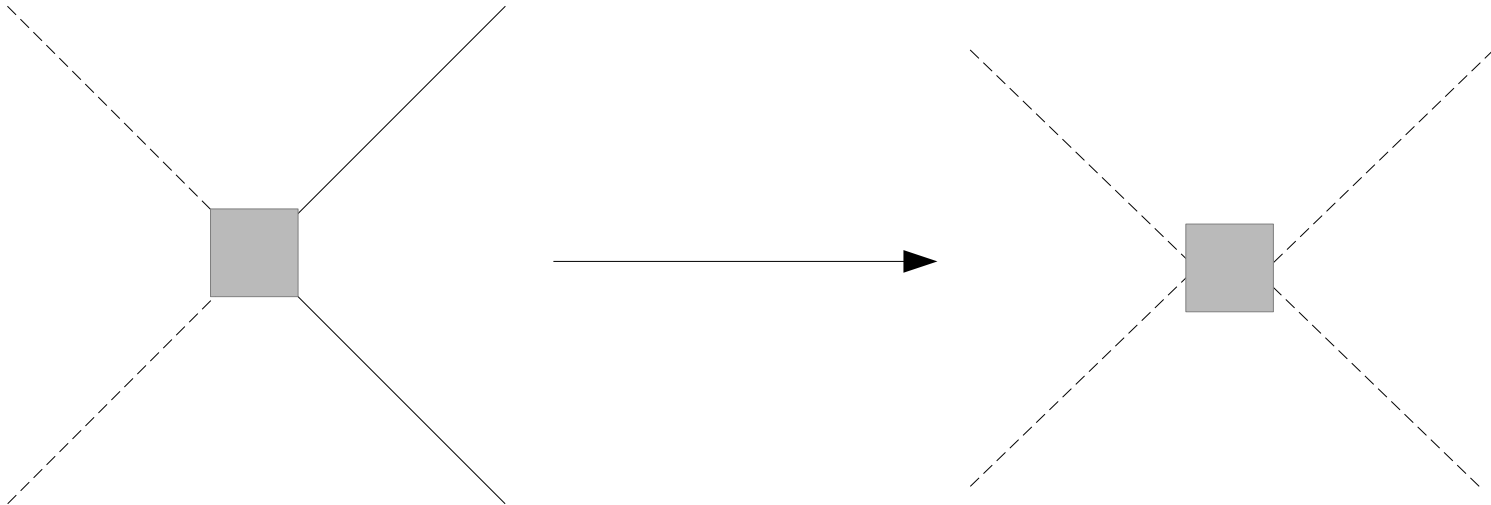
3. The beta function is linear in the Wilson coefficients:

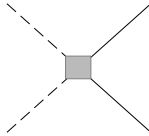
$$\beta_8 = \alpha(c_{e^2\phi^2 D^3}^{(1)} + c_{e^2\phi^2 D^3}^{(2)}), \quad \alpha \geq 0$$

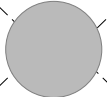
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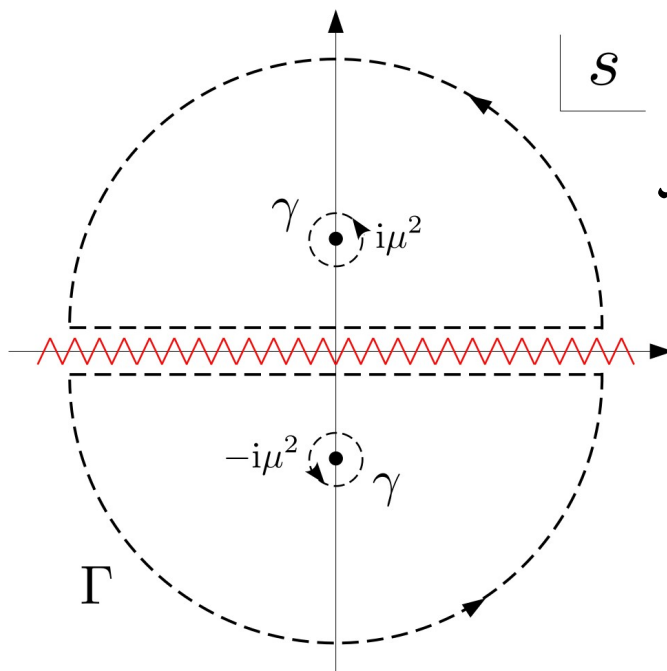
$$\underbrace{\mathcal{O}_{e^2\phi^2 D^3}^{(1)} - \mathcal{O}_{e^2\phi^2 D^3}^{(2)}}_{\tilde{\mathcal{O}}_{e^2\phi^2 D^3}^{(1)}} \xrightarrow{\text{red}} \mathcal{O}_{B^2\phi^2 D^2}^{(1)}$$

How do things change if we consider instead...?

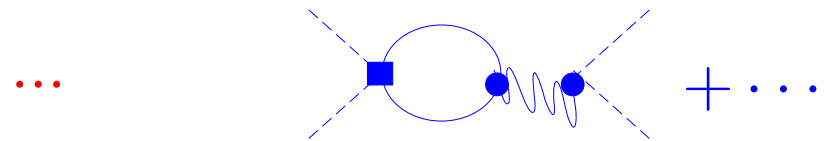


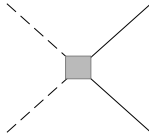
1. For any $(c_{e^2\phi^2 D^3}^{(1)}, c_{e^2\phi^2 D^3}^{(2)})$ compatible with the positivity bounds $(c_{e^2\phi^2 D^3}^{(1)} + c_{e^2\phi^2 D^3}^{(2)} \leq 0)$, there exists UV such that only $(c_{e^2\phi^2 D^3}^{(1)}, c_{e^2\phi^2 D^3}^{(2)})$ (and lower-dimensional  ones) at tree level.

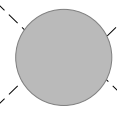
2. Within any such UV, compute  to order $O(g^2)$

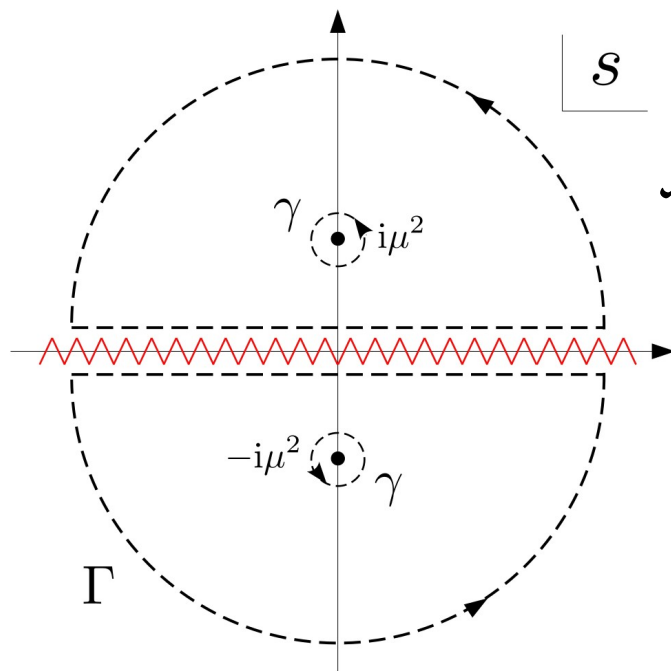


$$\mathcal{A}(s) \sim -(\beta_4 + \beta_8 s^2 + \beta_{12} s^4 + \dots) \log \frac{s}{\Lambda^2}$$

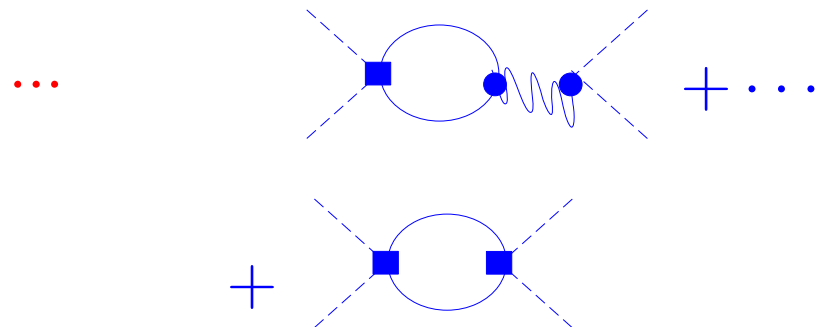


1. For any $(c_{e^2\phi^2 D^3}^{(1)}, c_{e^2\phi^2 D^3}^{(2)})$ compatible with the positivity bounds $(c_{e^2\phi^2 D^3}^{(1)} + c_{e^2\phi^2 D^3}^{(2)} \leq 0)$, there exists UV such that only $(c_{e^2\phi^2 D^3}^{(1)}, c_{e^2\phi^2 D^3}^{(2)})$ (and lower-dimensional  ones) at tree level.

2. Within any such UV, compute  to order $O(g^2)$



$$\mathcal{A}(s) \sim -(\beta_4 + \beta_8 s^2 + \beta_{12} s^4 + \dots) \log \frac{s}{\Lambda^2}$$



The dim-6 squared contributions fulfill positivity:

$$c_{\phi^4}^{(2)} \geq 0, \quad c_{\phi^4}^{(1)} + c_{\phi^4}^{(2)} \geq 0, \quad c_{\phi^4}^{(1)} + c_{\phi^4}^{(2)} + c_{\phi^4}^{(3)} \geq 0$$

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$$c_{\phi^4}^{(2)} \geq 0, \quad c_{\phi^4}^{(1)} + c_{\phi^4}^{(2)} \geq 0, \quad c_{\phi^4}^{(1)} + c_{\phi^4}^{(2)} + c_{\phi^4}^{(3)} \geq 0$$

$$16\pi^2 \beta_{H^4 D^4}^{(1)} = \frac{8}{3} \left[-2(c_{H^4 D^2}^{(1)})^2 - \frac{11}{8}(c_{H^4 D^2}^{(2)})^2 + 4c_{H^4 D^2}^{(1)}c_{H^4 D^2}^{(2)} \right. \\ \left. + \underline{\underline{\underline{3c_{Hd}^2}}} + \underline{\underline{\underline{c_{He}^2}}} + \underline{\underline{\underline{2(c_{Hl}^{(1)})^2}}} - 2(c_{Hl}^{(3)})^2 + \underline{\underline{\underline{6(c_{Hq}^{(1)})^2}}} - 6(c_{Hq}^{(3)})^2 + \underline{\underline{\underline{3c_{Hu}^2}}} - 3c_{Hud}^2 \right],$$

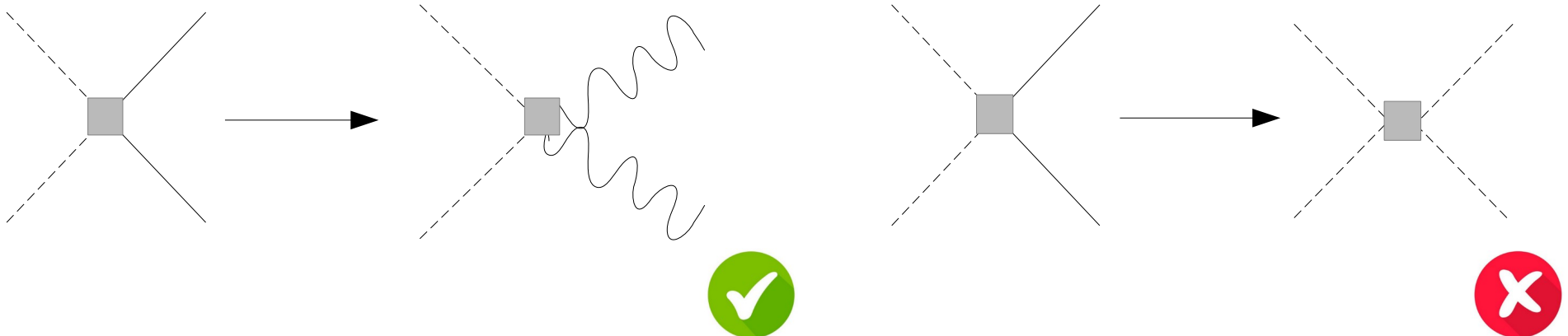
$$16\pi^2 \beta_{H^4 D^4}^{(2)} = \frac{8}{3} \left[-2(c_{H^4 D^2}^{(1)})^2 - \frac{5}{8}(c_{H^4 D^2}^{(2)})^2 - 2c_{H^4 D^2}^{(1)}c_{H^4 D^2}^{(2)} \right. \\ \left. - \underline{\underline{\underline{3c_{Hd}^2}}} - \underline{\underline{\underline{c_{He}^2}}} - \underline{\underline{\underline{2(c_{Hl}^{(1)})^2}}} - 2(c_{Hl}^{(3)})^2 - \underline{\underline{\underline{6(c_{Hq}^{(1)})^2}}} - 6(c_{Hq}^{(3)})^2 - \underline{\underline{\underline{3c_{Hu}^2}}} \right],$$

$$16\pi^2 \beta_{H^4 D^4}^{(3)} = \frac{8}{3} \left[-5(c_{H^4 D^2}^{(1)})^2 + \frac{7}{8}(c_{H^4 D^2}^{(2)})^2 - 2c_{H^4 D^2}^{(1)}c_{H^4 D^2}^{(2)} + 4(c_{Hl}^{(3)})^2 + 12(c_{Hq}^{(3)})^2 + 3c_{Hud}^2 \right]$$

Resorting to the UV to understand the IR is only a trick. In general:

(1) Some tree-level O_i obey $c_i \geq 0$

(2) If O_i involves fields not present in O_j and c_j not constrained by positivity, then $\gamma_{ij} = 0$



Other aspects of anomalous dimensions: signs and inequalities

Let us consider the mixing 

Positivity bounds:

$$c_{\phi^4}^{(2)} \geq 0, \quad c_{\phi^4}^{(1)} + c_{\phi^4}^{(2)} \geq 0, \quad c_{\phi^4}^{(1)} + c_{\phi^4}^{(2)} + c_{\phi^4}^{(3)} \geq 0$$

$$\dot{c}_{B^2\phi^2 D^2}^{(1)} \geq 0$$

From where we obtain:

$$\begin{aligned} \dot{c}_{B^2\phi^2 D^2}^{(1)} &= \alpha(c_{\phi^4}^{(1)} + c_{\phi^4}^{(2)} + c_{\phi^4}^{(3)}) + \beta(c_{\phi^4}^{(1)} + c_{\phi^4}^{(2)}) + \gamma c_{\phi^4}^{(2)} + \dots \\ &= (\alpha + \beta)c_{\phi^4}^{(1)} + (\alpha + \beta + \gamma)c_{\phi^4}^{(2)} + \alpha c_{\phi^4}^{(3)} + \dots, \end{aligned}$$

Other aspects of anomalous dimensions: signs and inequalities

$$\begin{aligned} \dot{c}_{B^2\phi^2 D^2}^{(1)} &= \alpha(c_{\phi^4}^{(1)} + c_{\phi^4}^{(2)} + c_{\phi^4}^{(3)}) + \beta(c_{\phi^4}^{(1)} + c_{\phi^4}^{(2)}) + \gamma c_{\phi^4}^{(2)} + \dots \\ &= (\alpha + \beta)c_{\phi^4}^{(1)} + (\alpha + \beta + \gamma)c_{\phi^4}^{(2)} + \alpha c_{\phi^4}^{(3)} + \dots, \end{aligned}$$

1. The anomalous dimensions are positive

$$\begin{array}{c} \hline c_{\phi^4 D^4}^{(1)} \quad c_{\phi^4 D^4}^{(2)} \quad c_{\phi^4 D^4}^{(3)} \\ c_{B^2\phi^2 D^2}^{(1)} \quad + \quad + \quad + \end{array}$$

2. They fulfill

$$\gamma_{c_{B^2\phi^2 D^2}^{(1)}, c_{\phi^4 D^4}^{(2)}} \geq \gamma_{c_{B^2\phi^2 D^2}^{(1)}, c_{\phi^4 D^4}^{(1)}} \geq \gamma_{c_{B^2\phi^2 D^2}^{(1)}, c_{\phi^4 D^4}^{(3)}}$$

Full electroweak SMEFT (with no flavour)

| | $c_{\phi^4 D^4}^{(1)}$ | $c_{\phi^4 D^4}^{(2)}$ | $c_{\phi^4 D^4}^{(3)}$ | $\tilde{c}_{e^2 \phi^2 D^3}^{(1)}$ | $\tilde{c}_{e^2 \phi^2 D^3}^{(2)}$ | $\tilde{c}_{l^2 \phi^2 D^3}^{(1)}$ | $\tilde{c}_{l^2 \phi^2 D^3}^{(2)}$ | $\tilde{c}_{l^2 \phi^2 D^3}^{(3)}$ | $\tilde{c}_{l^2 \phi^2 D^3}^{(4)}$ | $c_{e^4 D^2}$ | $c_{l^4 D^2}^{(1)}$ | $c_{l^4 D^2}^{(2)}$ | $c_{l^2 e^2 D^2}^{(1)}$ | $c_{l^2 e^2 D^2}^{(2)}$ |
|------------------------------------|------------------------|------------------------|------------------------|------------------------------------|------------------------------------|------------------------------------|------------------------------------|------------------------------------|------------------------------------|---------------|---------------------|---------------------|-------------------------|-------------------------|
| $c_{B^2 \phi^2 D^2}^{(1)}$ | + | + | + | 0 | - | 0 | - | 0 | - | 0 | 0 | 0 | 0 | 0 |
| $c_{W^2 \phi^2 D^2}^{(1)}$ | + | + | + | 0 | 0 | 0 | - | 0 | - | 0 | 0 | 0 | 0 | 0 |
| $\tilde{c}_{e^2 \phi^2 D^3}^{(2)}$ | + | + | + | × | × | 0 | - | 0 | - | - | 0 | 0 | 0 | - |
| $\tilde{c}_{l^2 \phi^2 D^3}^{(2)}$ | + | + | + | 0 | - | × | × | × | × | 0 | - | - | 0 | - |
| $\tilde{c}_{l^2 \phi^2 D^3}^{(4)}$ | + | + | + | 0 | - | × | × | × | × | 0 | - | - | 0 | - |
| $c_{e^2 B^2 D}$ | 0 | 0 | 0 | 0 | - | 0 | 0 | 0 | 0 | - | 0 | 0 | 0 | - |
| $c_{l^2 B^2 D}$ | 0 | 0 | 0 | 0 | 0 | 0 | - | 0 | - | 0 | - | - | 0 | - |
| $c_{e^2 W^2 D}$ | 0 | 0 | 0 | 0 | - | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | - |
| $c_{l^2 W^2 D}^{(1)}$ | 0 | 0 | 0 | 0 | 0 | 0 | - | 0 | - | 0 | - | - | 0 | 0 |
| $c_{l^2 e^2 D^2}^{(2)}$ | 0 | 0 | 0 | 0 | - | 0 | - | 0 | - | - | - | - | × | × |

Full electroweak SMEFT (with no flavour)

| | $c_{\phi^4 D^4}^{(1)}$ | $c_{\phi^4 D^4}^{(2)}$ | $c_{\phi^4 D^4}^{(3)}$ | $\tilde{c}_{e^2 \phi^2 D^3}^{(1)}$ | $\tilde{c}_{e^2 \phi^2 D^3}^{(2)}$ | $\tilde{c}_{l^2 \phi^2 D^3}^{(1)}$ | $\tilde{c}_{l^2 \phi^2 D^3}^{(2)}$ | $\tilde{c}_{l^2 \phi^2 D^3}^{(3)}$ | $\tilde{c}_{l^2 \phi^2 D^3}^{(4)}$ | $c_{e^4 D^2}$ | $c_{l^4 D^2}^{(1)}$ | $c_{l^4 D^2}^{(2)}$ | $c_{l^2 e^2 D^2}^{(1)}$ | $c_{l^2 e^2 D^2}^{(2)}$ |
|------------------------------------|------------------------|------------------------|------------------------|------------------------------------|------------------------------------|------------------------------------|------------------------------------|------------------------------------|------------------------------------|---------------|---------------------|---------------------|-------------------------|-------------------------|
| $c_{B^2 \phi^2 D^2}^{(1)}$ | $\frac{g^2}{3}$ | $\frac{g^2}{2}$ | $\frac{g^2}{6}$ | 0 | - | 0 | - | 0 | - | 0 | 0 | 0 | 0 | 0 |
| $c_{W^2 \phi^2 D^2}^{(1)}$ | + | + | + | 0 | 0 | 0 | - | 0 | - | 0 | 0 | 0 | 0 | 0 |
| $\tilde{c}_{e^2 \phi^2 D^3}^{(2)}$ | + | + | + | × | × | 0 | - | 0 | - | - | 0 | 0 | 0 | - |
| $\tilde{c}_{l^2 \phi^2 D^3}^{(2)}$ | + | + | + | 0 | - | × | × | × | × | 0 | - | - | 0 | - |
| $\tilde{c}_{l^2 \phi^2 D^3}^{(4)}$ | + | + | + | 0 | - | × | × | × | × | 0 | - | - | 0 | - |
| $c_{e^2 B^2 D}$ | 0 | 0 | 0 | 0 | - | 0 | 0 | 0 | 0 | - | 0 | 0 | 0 | - |
| $c_{l^2 B^2 D}$ | 0 | 0 | 0 | 0 | 0 | 0 | - | 0 | - | 0 | - | - | 0 | - |
| $c_{e^2 W^2 D}$ | 0 | 0 | 0 | 0 | - | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | - |
| $c_{l^2 W^2 D}^{(1)}$ | 0 | 0 | 0 | 0 | 0 | 0 | - | 0 | - | 0 | - | - | 0 | 0 |
| $c_{l^2 e^2 D^2}^{(2)}$ | 0 | 0 | 0 | 0 | - | 0 | - | 0 | - | - | - | - | × | × |

Full electroweak SMEFT (with no flavour)

| | $c_{\phi^4 D^4}^{(1)}$ | $c_{\phi^4 D^4}^{(2)}$ | $c_{\phi^4 D^4}^{(3)}$ | $\tilde{c}_{e^2 \phi^2 D^3}^{(1)}$ | $\tilde{c}_{e^2 \phi^2 D^3}^{(2)}$ | $\tilde{c}_{l^2 \phi^2 D^3}^{(1)}$ | $\tilde{c}_{l^2 \phi^2 D^3}^{(2)}$ | $\tilde{c}_{l^2 \phi^2 D^3}^{(3)}$ | $\tilde{c}_{l^2 \phi^2 D^3}^{(4)}$ | $c_{e^4 D^2}$ | $c_{l^4 D^2}^{(1)}$ | $c_{l^4 D^2}^{(2)}$ | $c_{l^2 e^2 D^2}^{(1)}$ | $c_{l^2 e^2 D^2}^{(2)}$ |
|------------------------------------|------------------------|------------------------|------------------------|------------------------------------|------------------------------------|------------------------------------|------------------------------------|------------------------------------|------------------------------------|---------------|---------------------|---------------------|-------------------------|-------------------------|
| $c_{B^2 \phi^2 D^2}^{(1)}$ | $\frac{g^2}{3}$ | $\frac{g^2}{2}$ | $\frac{g^2}{6}$ | 0 | - | 0 | - | 0 | - | 0 | 0 | 0 | 0 | 0 |
| $c_{W^2 \phi^2 D^2}^{(1)}$ | + | + | + | 0 | 0 | 0 | - | 0 | - | 0 | 0 | 0 | 0 | 0 |
| $\tilde{c}_{e^2 \phi^2 D^3}^{(2)}$ | + | + | + | × | × | 0 | $-\frac{4 Y ^2}{3}$ | 0 | - | - | 0 | 0 | 0 | - |
| $\tilde{c}_{l^2 \phi^2 D^3}^{(2)}$ | + | + | + | 0 | - | × | × | × | × | 0 | - | - | 0 | - |
| $\tilde{c}_{l^2 \phi^2 D^3}^{(4)}$ | + | + | + | 0 | - | × | × | × | × | 0 | - | - | 0 | - |
| $c_{e^2 B^2 D}$ | 0 | 0 | 0 | 0 | - | 0 | 0 | 0 | 0 | - | 0 | 0 | 0 | - |
| $c_{l^2 B^2 D}$ | 0 | 0 | 0 | 0 | 0 | 0 | - | 0 | - | 0 | - | - | 0 | - |
| $c_{e^2 W^2 D}$ | 0 | 0 | 0 | 0 | - | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | - |
| $c_{l^2 W^2 D}^{(1)}$ | 0 | 0 | 0 | 0 | 0 | 0 | - | 0 | - | 0 | - | - | 0 | 0 |
| $c_{l^2 e^2 D^2}^{(2)}$ | 0 | 0 | 0 | 0 | - | 0 | - | 0 | - | - | - | - | × | × |

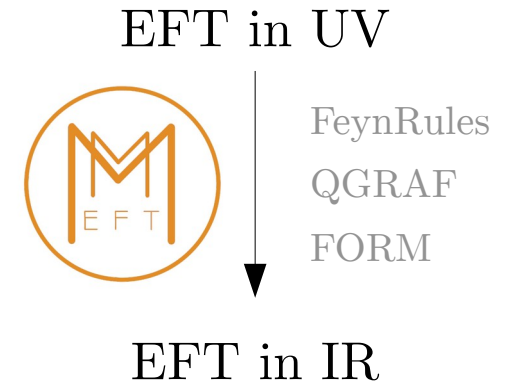
Full electroweak SMEFT (with no flavour)

| | $c_{\phi^4 D^4}^{(1)}$ | $c_{\phi^4 D^4}^{(2)}$ | $c_{\phi^4 D^4}^{(3)}$ | $\tilde{c}_{e^2 \phi^2 D^3}^{(1)}$ | $\tilde{c}_{e^2 \phi^2 D^3}^{(2)}$ | $\tilde{c}_{l^2 \phi^2 D^3}^{(1)}$ | $\tilde{c}_{l^2 \phi^2 D^3}^{(2)}$ | $\tilde{c}_{l^2 \phi^2 D^3}^{(3)}$ | $\tilde{c}_{l^2 \phi^2 D^3}^{(4)}$ | $c_{e^4 D^2}$ | $c_{l^4 D^2}^{(1)}$ | $c_{l^4 D^2}^{(2)}$ | $c_{l^2 e^2 D^2}^{(1)}$ | $c_{l^2 e^2 D^2}^{(2)}$ |
|------------------------------------|------------------------|------------------------|------------------------|------------------------------------|------------------------------------|------------------------------------|------------------------------------|------------------------------------|------------------------------------|---------------|---------------------|---------------------|-------------------------|-------------------------|
| $c_{B^2 \phi^2 D^2}^{(1)}$ | $\frac{g^2}{3}$ | $\frac{g^2}{2}$ | $\frac{g^2}{6}$ | 0 | - | 0 | - | 0 | - | 0 | 0 | 0 | 0 | 0 |
| $c_{W^2 \phi^2 D^2}^{(1)}$ | + | + | + | 0 | 0 | 0 | - | 0 | - | 0 | 0 | 0 | 0 | 0 |
| $\tilde{c}_{e^2 \phi^2 D^3}^{(2)}$ | + | + | + | $g^2 - Y ^2$ | × | 0 | $-\frac{4 Y ^2}{3}$ | 0 | - | - | 0 | 0 | 0 | - |
| $\tilde{c}_{l^2 \phi^2 D^3}^{(2)}$ | + | + | + | 0 | - | × | × | × | × | 0 | - | - | 0 | - |
| $\tilde{c}_{l^2 \phi^2 D^3}^{(4)}$ | + | + | + | 0 | - | × | × | × | × | 0 | - | - | 0 | - |
| $c_{e^2 B^2 D}$ | 0 | 0 | 0 | 0 | - | 0 | 0 | 0 | 0 | - | 0 | 0 | 0 | - |
| $c_{l^2 B^2 D}$ | 0 | 0 | 0 | 0 | 0 | 0 | - | 0 | - | 0 | - | - | 0 | - |
| $c_{e^2 W^2 D}$ | 0 | 0 | 0 | 0 | - | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | - |
| $c_{l^2 W^2 D}^{(1)}$ | 0 | 0 | 0 | 0 | 0 | 0 | - | 0 | - | 0 | - | - | 0 | 0 |
| $c_{l^2 e^2 D^2}^{(2)}$ | 0 | 0 | 0 | 0 | - | 0 | - | 0 | - | - | - | - | × | × |

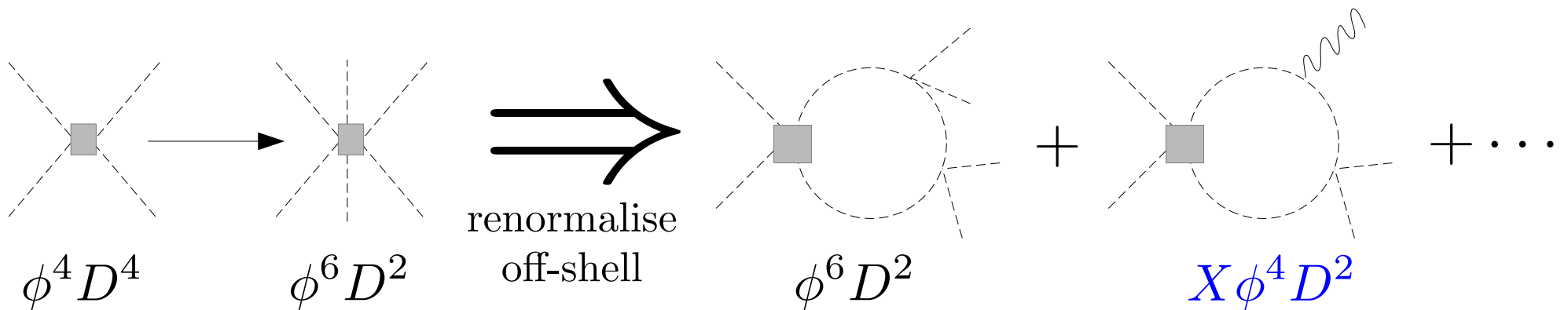
Can't we just compute all anomalous dimensions in
some automated way?

Tools like `matchmakereft` or `matchete`
not yet fully automatic

[Carmona et al '21; Fuentes-Martin et al '22]



Main obstacles: Green's and physical bases [MC, Diaz-Carmona, Guedes '21; Ren, Yu '22; Fonseca]; **field redefinitions** [MC, Santiago]

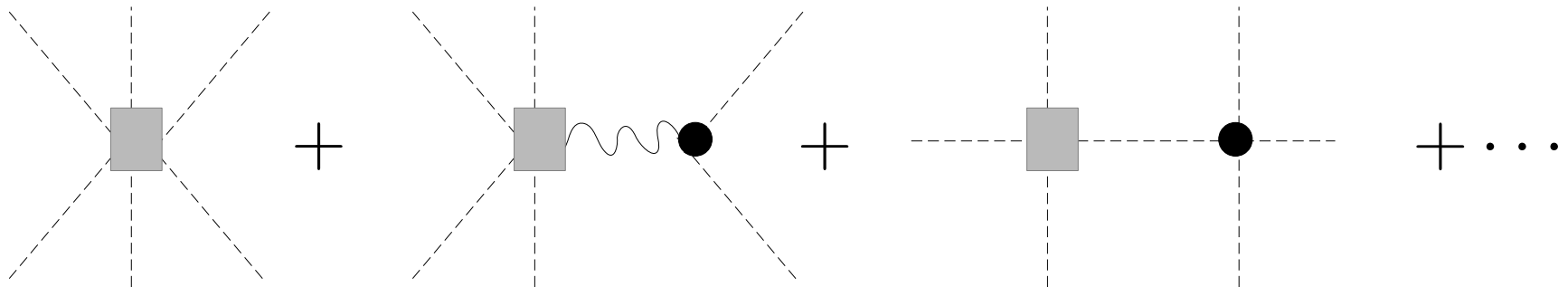


$$L_{\text{div}}^{(\text{local})} \sim \frac{1}{\epsilon} P(p_i \cdot p_j, p_i \cdot \epsilon_j, \dots)$$

Require Lagrangian with redundant operators to provide **same S-matrix** as that without them

Too many constraints on-shell. Solution: **go numerics**

Compute the amplitudes in different Montecarlo physical phase-space points. Problem reduced to linear algebra



Application to the purely Higgs sector [to appear in SMEFT-Tools 2022 proceedings]:

$$c_{\phi\Box} \rightarrow c_{\phi\Box} + \frac{1}{2}r'_{\phi D}, \quad (22)$$

$$c_{\phi^6} \rightarrow c_{\phi^6} + 2\lambda r'_{\phi D}, \quad (23)$$

$$\begin{aligned} c_{\phi^6 D^2}^{(1)} &\rightarrow c_{\phi^6 D^2}^{(1)} + 2\lambda(2r_{\phi^4 D^4}^{(12)} - 2r_{\phi^4 D^4}^{(4)} - r_{\phi^4 D^4}^{(6)}) \\ &\quad - 4c_{\phi\Box}r'_{\phi D} - \frac{1}{2}c_{\phi D}r'_{\phi D} - \frac{7}{4}r_{\phi D}^{\prime 2} + r_{\phi D}^{\prime\prime 2}, \end{aligned} \quad (24)$$

$$c_{\phi^6}^{(2)} \rightarrow c_{\phi^6}^{(2)} + 2\lambda(r_{\phi^4 D^4}^{(12)} - r_{\phi^4 D^4}^{(6)}) - c_{\phi D}r'_{\phi D}. \quad (25)$$

Outlook

Renormalising the SMEFT to dimension 8 is a heavy task, but which can be **automatised**.

Positivity bounds on dimension-8 interactions (which are **important/interesting by themselves**) restrict different aspects of (certain) anomalous dimensions (**zeros, signs, inequalities**).

Further applications include **full SMEFT** [MC, Li 'ongoing work], LEFT and other EFTs.

Phenomenological relevance of dimension-8 quantum corrections still to be fully understood.

Thank you!