Complex dilatations and the S-matrix

Matching EFTs on-shell

SDA, G. Durieux [to appear soon]

Stefano De Angelis - New Physics @ Korean Institute - 07/06/2023

Why a new method for performing matching?

Why a new method for performing matching?

- Why not?

Traditional methods are

- Feynman diagrams expanded in hard-mass region [Appelquist,Carazzone],[Witten],[Collins,Wilczek,Zee] Software: matchmakereft [Carmona,Lazopoulos,Olgoso,Santiago]
- Functional methods (EoM @ LO, functional determinants @ NLO) [Weinberg],[Ovrut,Schnitzer] Software: matchete [Fuentes-Martın,Konig,Pages,Thomsen,Wilsch]

Why a new method for performing matching?

- Why not?

  Traditional methods are

  - Feynman diagrams expanded in hard-mass region [Appelquist,Carazzone],[Witten],[Collins,Wilczek,Zee] Software: matchmakereft [Carmona,Lazopoulos,Olgoso,Santiago]
  - Functional methods (EoM @ LO, functional determinants @ NLO) [Weinberg],[Ovrut,Schnitzer] Software: matchete [Fuentes-Martın,Konig,Pages,Thomsen,Wilsch]

- Possibility of making selection rules/magic zeros manifest [Arkani-Hamed,Harigaya],[Panico,Pomarol,Riembau]

  - In the computation of anomalous dimensions, scattering amplitudes explained the origin of many ZEROs through selection rules. [Elias-Mirò,Espinosa,Pomarol],[Cheung,Shen],[Bern,Parra-Martinez,Sawyer]x2,[Jiang,Shu,Xiao,Zheng],[Chala]
  - It seems that there exist mysterious zeros in the matching and there might be an on-shell explanation from symmetry. [Delle Rose,von Harling,Pomarol]

Why a new method for performing matching?

- Why not?

  Traditional methods are

  - Feynman diagrams expanded in hard-mass region [Appelquist,Carazzone],[Witten],[Collins,Wilczek,Zee] Software: matchmakereft [Carmona,Lazopoulos,Olgoso,Santiago]
  - Functional methods (EoM @ LO, functional determinants @ NLO) [Weinberg],[Ovrut,Schnitzer] Software: matchete [Fuentes-Martın,Konig,Pages,Thomsen,Wilsch]

- Possibility of making selection rules/magic zeros manifest [Arkani-Hamed,Harigaya],[Panico,Pomarol,Riembau]

  - In the computation of anomalous dimensions, scattering amplitudes explained the origin of many ZEROs through selection rules. [Elias-Mirò,Espinosa,Pomarol],[Cheung,Shen],[Bern,Parra-Martinez,Sawyer]x2,[Jiang,Shu,Xiao,Zheng],[Chala]
  - It seems that there exist mysterious zeros in the matching and there might be an on-shell explanation from symmetry. [Delle Rose,von Harling,Pomarol]

- Computational improvements

  - @ integrand level: on-shell techniques (perturbative unitarity and locality) provide a compact reorganisation of the integrands of the Scattering Amplitudes (e.g. gauge theory and gravity amplitudes)
  - @ projecting on operator basis: we deal with S-matrix elements (e.g. no field redefinition)

Matching from Analyticity and Unitarity

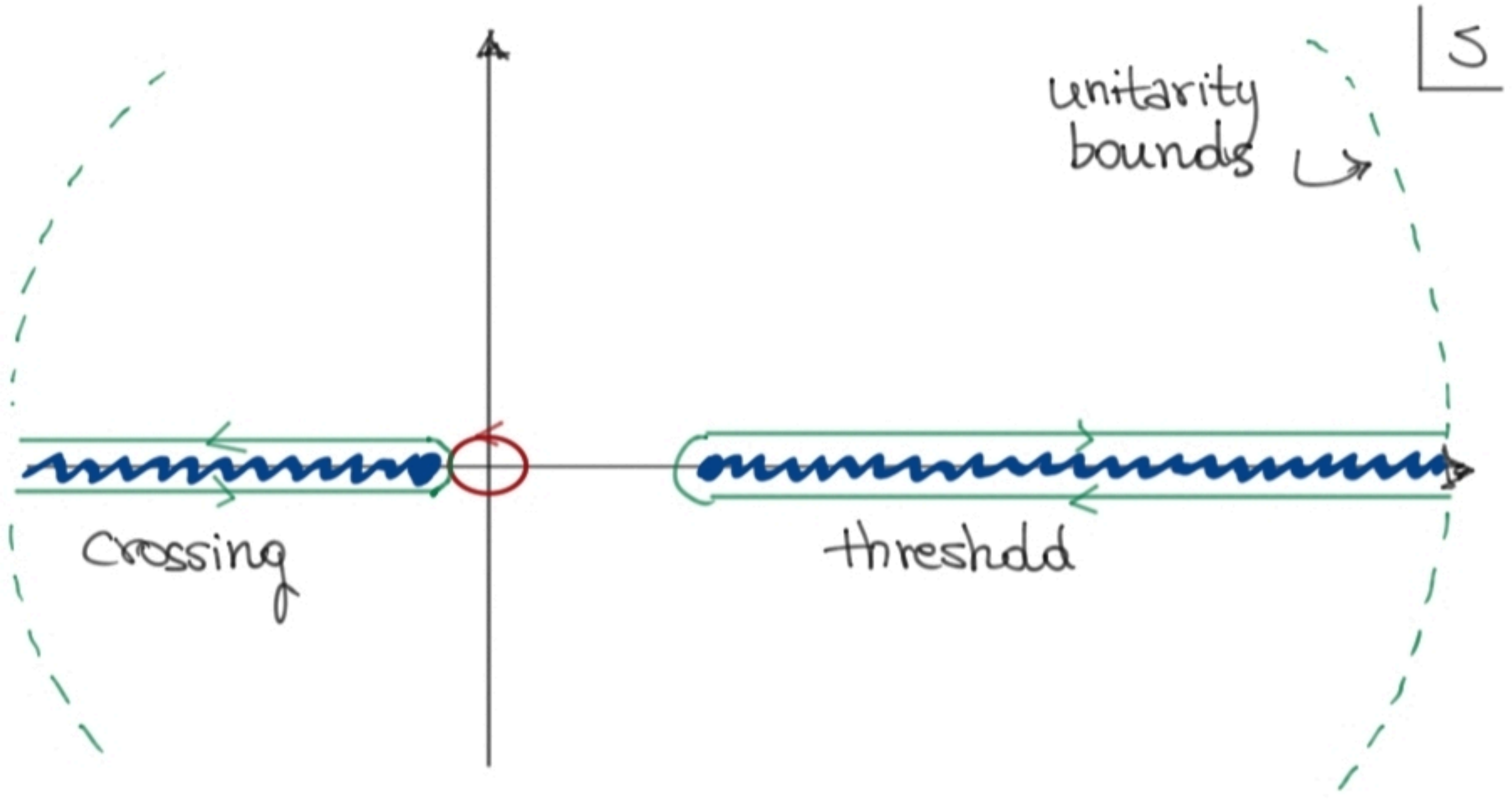
Matching from Analyticity and Unitarity

- We want to use dispersion relations

[Mikael's talk]
[Gell Mann],[Mandelstam]
[Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi]

We were inspired by the approach from analyticity and unitarity used in the context of positivity bounds.

In the forward limit of $2 \to 2$ scattering: $c_n = \oint \frac{ds}{s^{n+1}} \mathscr{A}_n$ and we can deform the contour integration to write the Wilson coefficients in terms of the discontinuities of the amplitude.



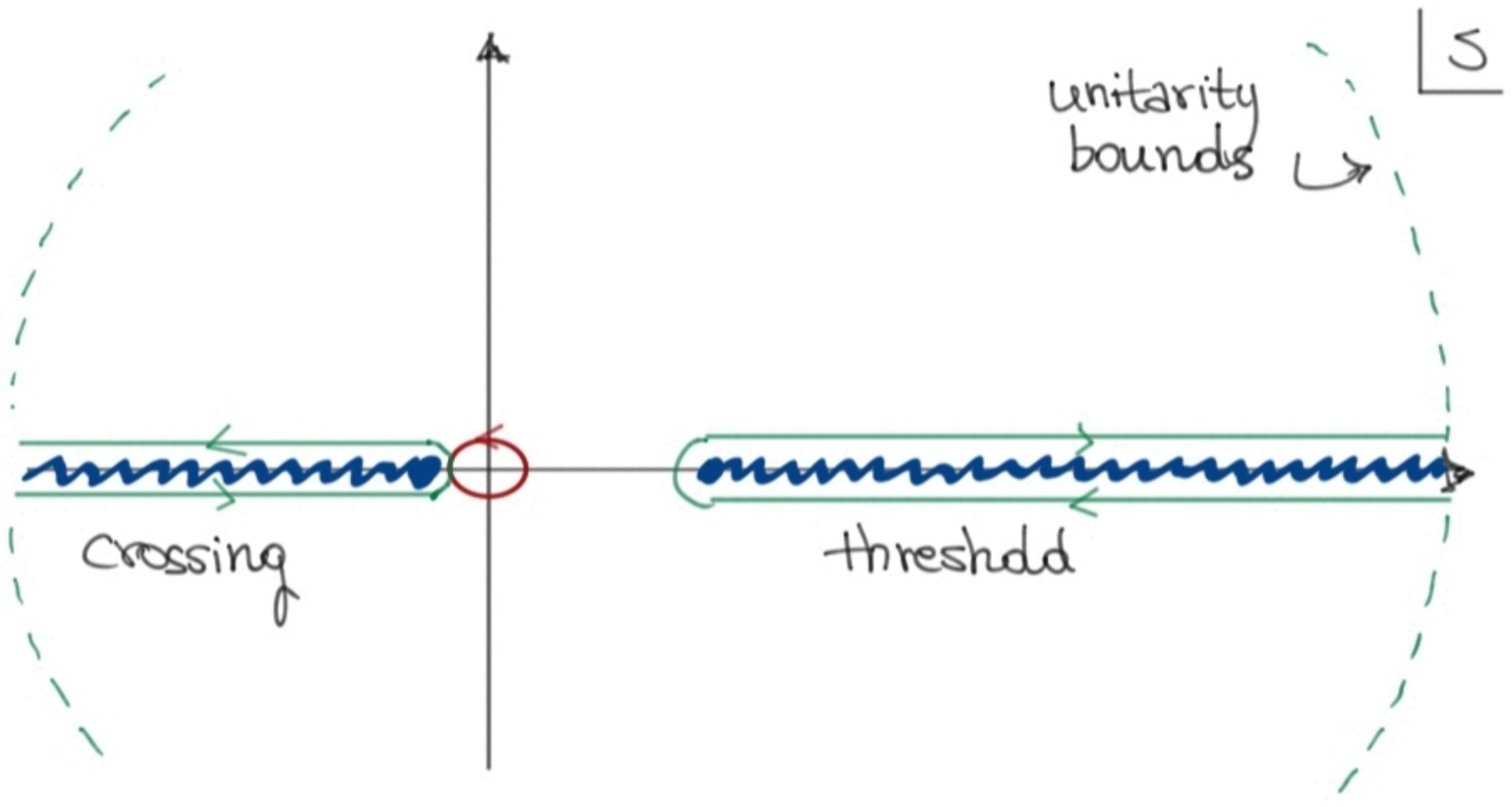
Matching from Analyticity and Unitarity

- We want to use dispersion relations

[Mikael's talk]
[Gell Mann],[Mandelstam]
[Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi]

We were inspired by the approach from analyticity and unitarity used in the context of positivity bounds.

In the forward limit of $2 \to 2$ scattering: $c_n = \oint \frac{ds}{s^{n+1}} \mathcal{A}_n$ and we can deform the contour integration to write the Wilson coefficients in terms of the discontinuities of the amplitude.



- ... beyond four-point scattering:
  - The analytic structure of the S-matrix elements beyond the four-point case is not know. [Bros,Epstein,Glaser]
  - But, we can consider FORM FACTORS!

First studied in the context of $\mathcal{N}$=4 sYM:
[van Neerven],[Brandhuber,Spence,Travaglini,Yang],[Bork,Kazakov,Vartanov]

$$F_{\mathcal{O}}(\vec{m}) = \int d^4x\, e^{ix\cdot q}\, {}_{\text{out}}\langle \psi_{\vec{m}} | \mathcal{O}(x) | 0 \rangle$$

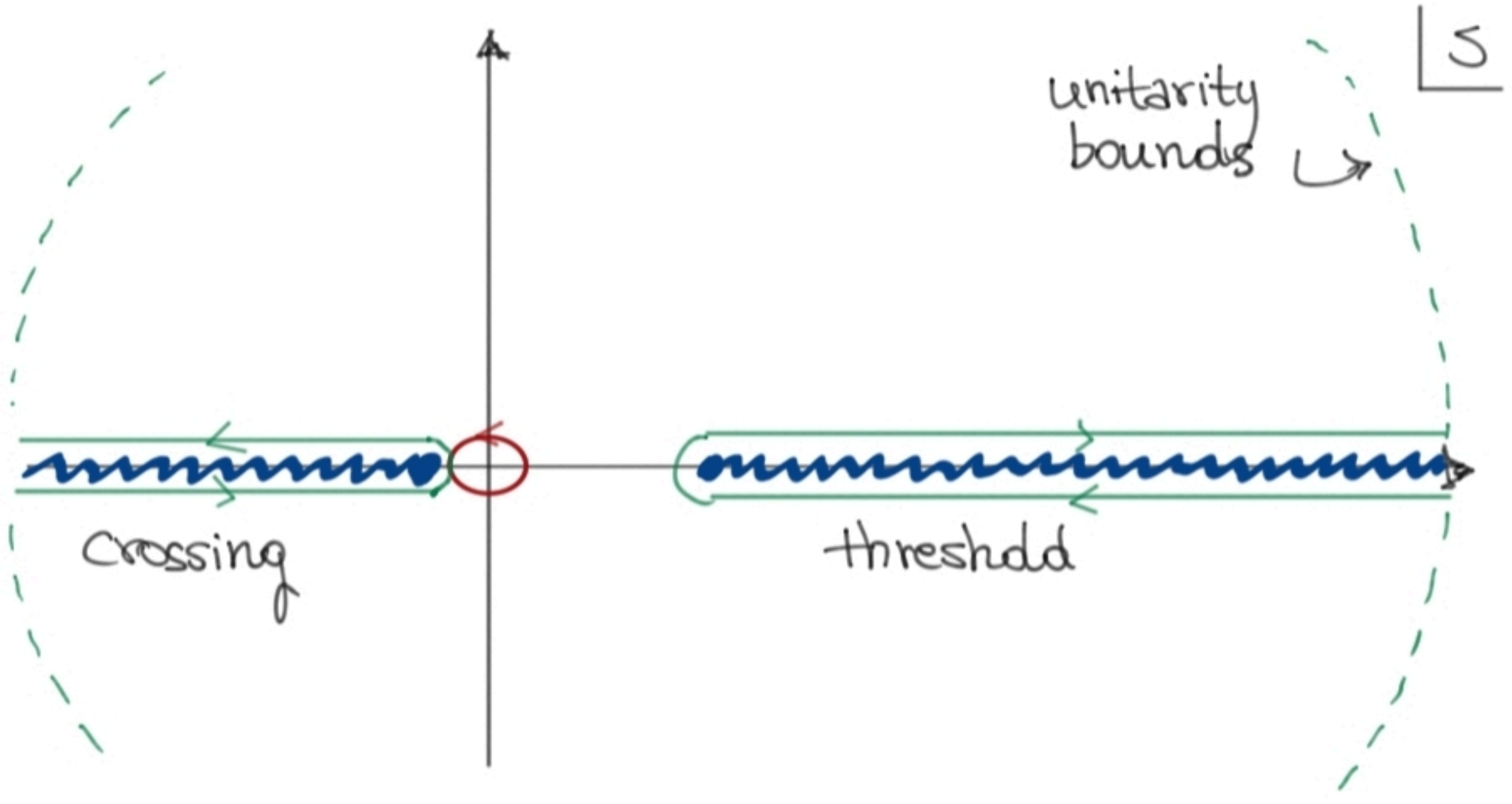
Matching from Analyticity and Unitarity

- We want to use dispersion relations

[Mikael's talk]
[Gell Mann],[Mandelstam]
[Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi]

We were inspired by the approach from analyticity and unitarity used in the context of positivity bounds.

In the forward limit of 2 → 2 scattering: $c_n = \oint \frac{ds}{s^{n+1}} \mathcal{A}_n$ and we can deform the contour integration to write the Wilson coefficients in terms of the discontinuities of the amplitude.



- ... beyond four-point scattering:
  - The analytic structure of the S-matrix elements beyond the four-point case is not know. [Bros,Epstein,Glaser]
  - But, we can consider FORM FACTORS!

First studied in the context of $\mathcal{N}$=4 sYM:
[van Neerven],[Brandhuber,Spence,Travaglini,Yang],[Bork,Kazakov,Vartanov]

$$F_{\mathcal{O}}(\vec{m}) = \int d^4x \; e^{ix\cdot q} {}_{\text{out}}\langle \psi_{\vec{m}} | \mathcal{O}(x) | 0 \rangle$$

- ... and beyond forward limit:
  - This quantities depend on all the Mandelstam invariants (e.g. s, t, u @ three-points)
  - We consider a DILATATION transformation: $p_i \to p_i' = z\, p_i$ and analytically continue in z.

Analytic properties of Form Factors

Analytic properties of Form Factors

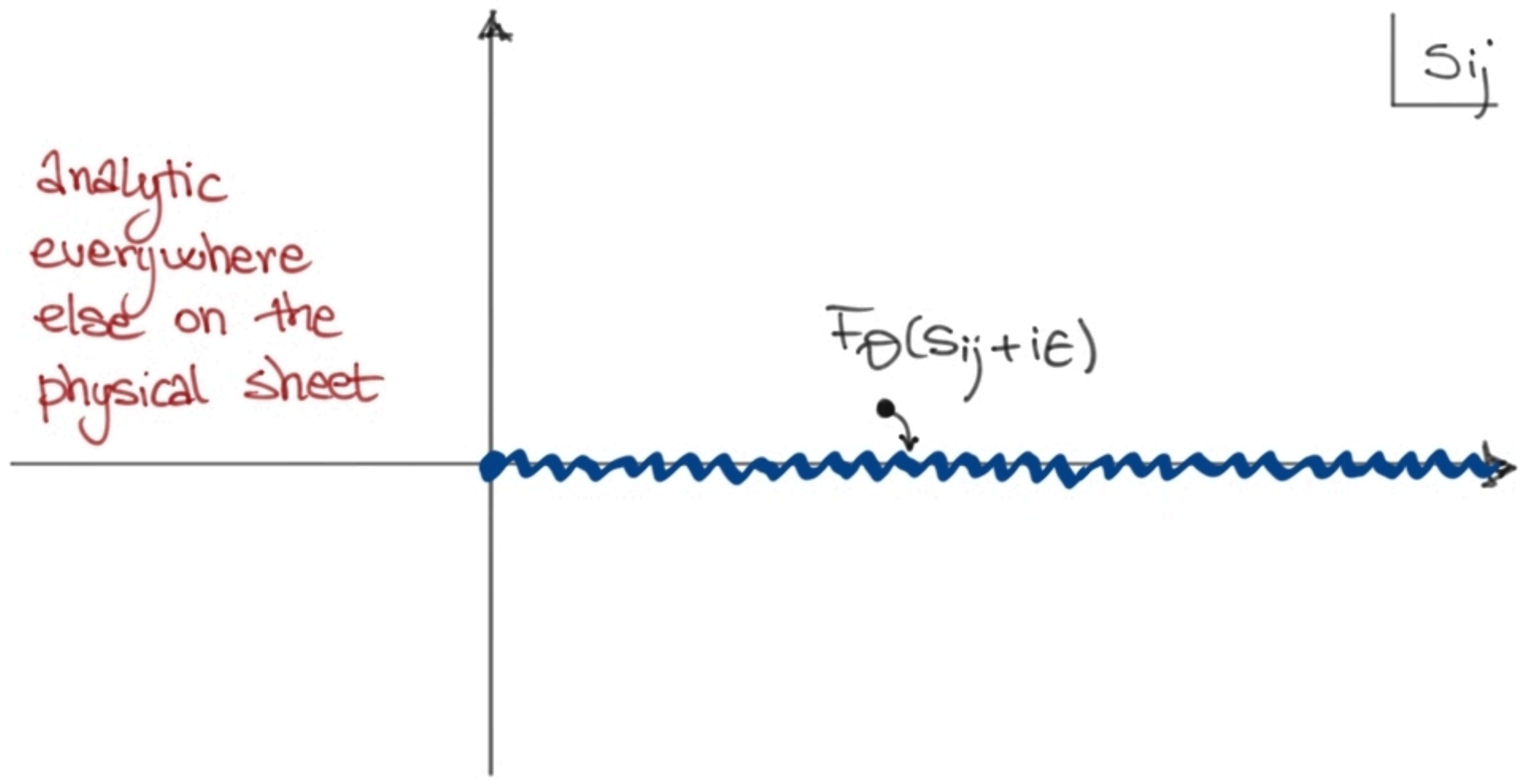
- All the states are outgoing:

$$s_{ij} = 2p_i \cdot p_j = 2E_i E_j (1 - \cos\theta) \geq 0 \qquad \left(\sum_{i=1}^{n} p_i^{\mu} \right)^2 = q^2 > 0$$

Analytic properties of Form Factors

- All the states are outgoing:

$$s_{ij} = 2p_i \cdot p_j = 2E_iE_j(1 - \cos\theta) \geq 0 \qquad \left(\sum_{i=1}^{n} p_i^{\mu}\right)^2 = q^2 > 0$$



- The analytic structure of the form factors is simple

In perturbation theory, it is easy to realise that the form factors in massless theories have singularities only @ $s_{ij} = 0$.

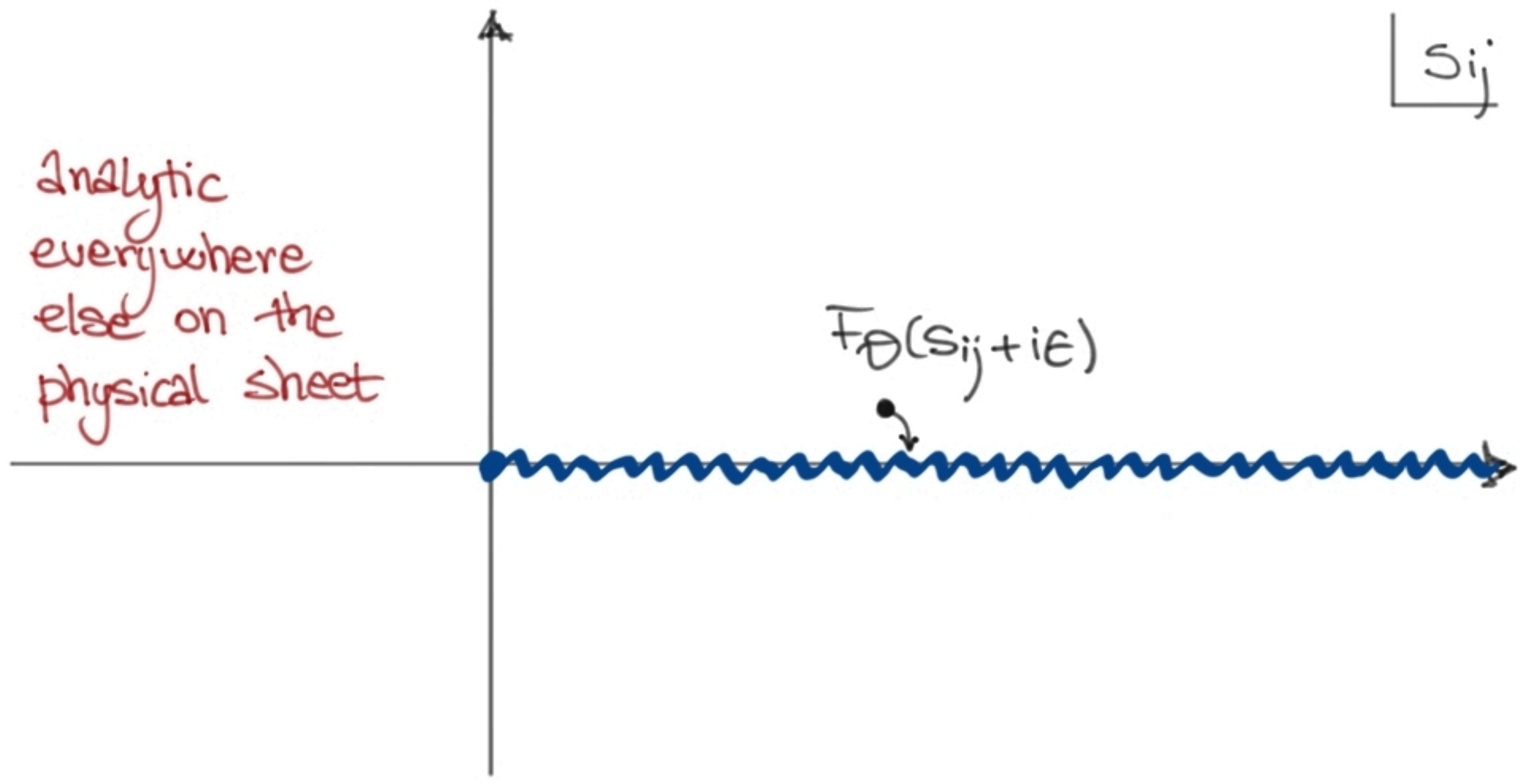
Analytic properties of Form Factors

- All the states are outgoing:

$$s_{ij} = 2p_i \cdot p_j = 2E_iE_j(1 - \cos\theta) \geq 0 \qquad \left(\sum_{i=1}^{n} p_i^\mu\right)^2 = q^2 > 0$$

- The analytic structure of the form factors is simple

In perturbation theory, it is easy to realise that the form factors in massless theories have singularities only @ $s_{ij} = 0$.



$\mathcal{N}$=4: [Zwiebel],[Nandan,Sieg,Wilhelm,Yang],[+Loebbert]
EFTs: [Caron-Huot,Wilhelm],see also [Mirò,Ingoldbya,Riembau],
Partial Waves: [Baratella,Fernandez,Pomarol]
2-loop: [Bern,Sawyer,Parra-Martinez]

Analytic properties of Form Factors

- All the states are outgoing:

$$s_{ij} = 2p_i \cdot p_j = 2E_iE_j(1 - \cos\theta) \geq 0 \qquad \left(\sum_{i=1}^{n} p_i^\mu\right)^2 = q^2 > 0$$

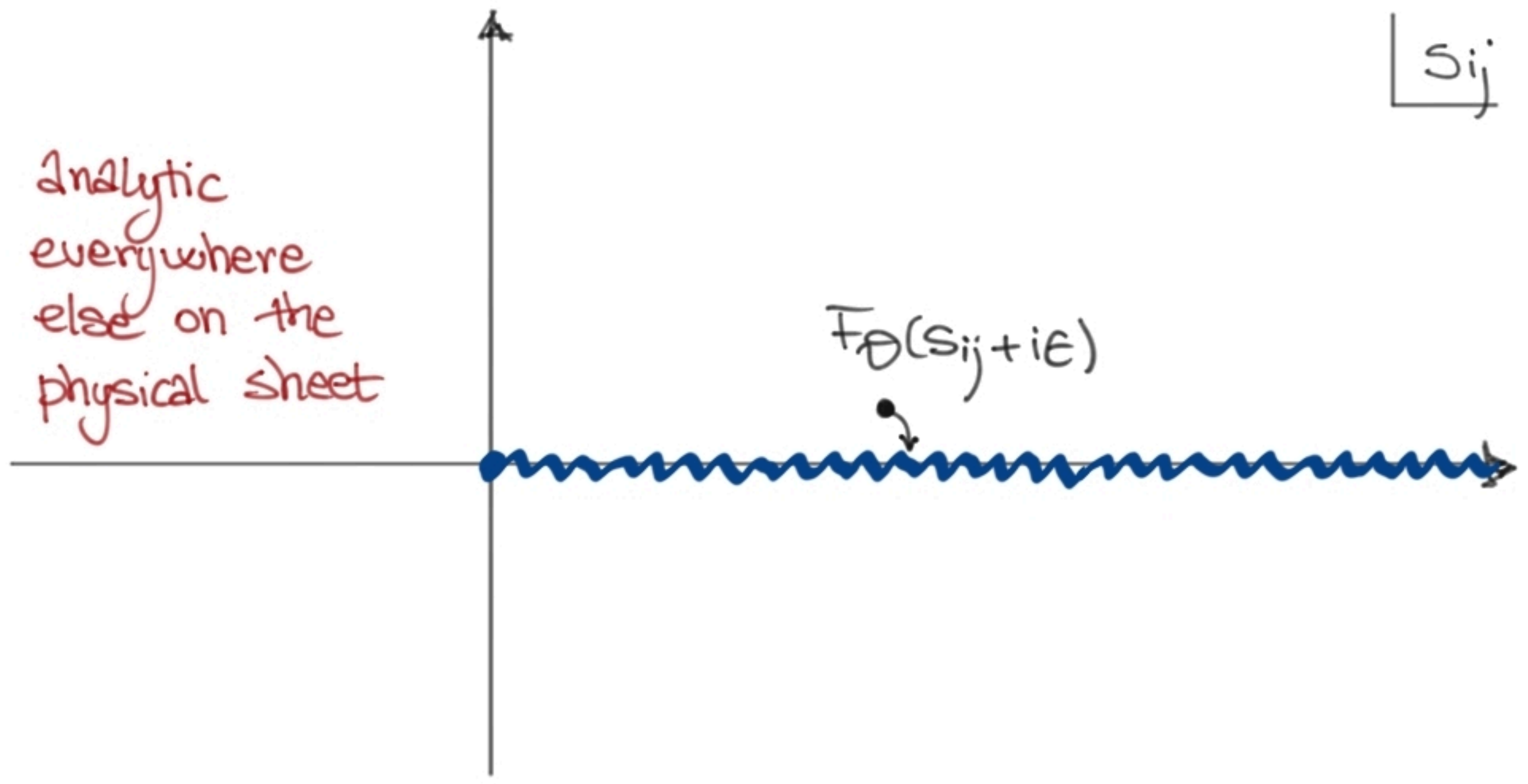
- The analytic structure of the form factors is simple

In perturbation theory, it is easy to realise that the form factors in massless theories have singularities only @ $s_{ij} = 0$.

- ... with discontinuities determined by unitarity!

Similarly to the S-matrix, also form factors satisfy unitarity conditions:

$$S^\dagger S = \mathbf{1} \qquad \longrightarrow \qquad F = S \otimes F^\dagger$$



$\mathcal{N}$=4: [Zwiebel],[Nandan,Sieg,Wilhelm,Yang],[+Loebbert]
EFTs: [Caron-Huot,Wilhelm],see also [Mirò,Ingoldbya,Riembau],
Partial Waves: [Baratella,Fernandez,Pomarol]
2-loop: [Bern,Sawyer,Parra-Martinez]

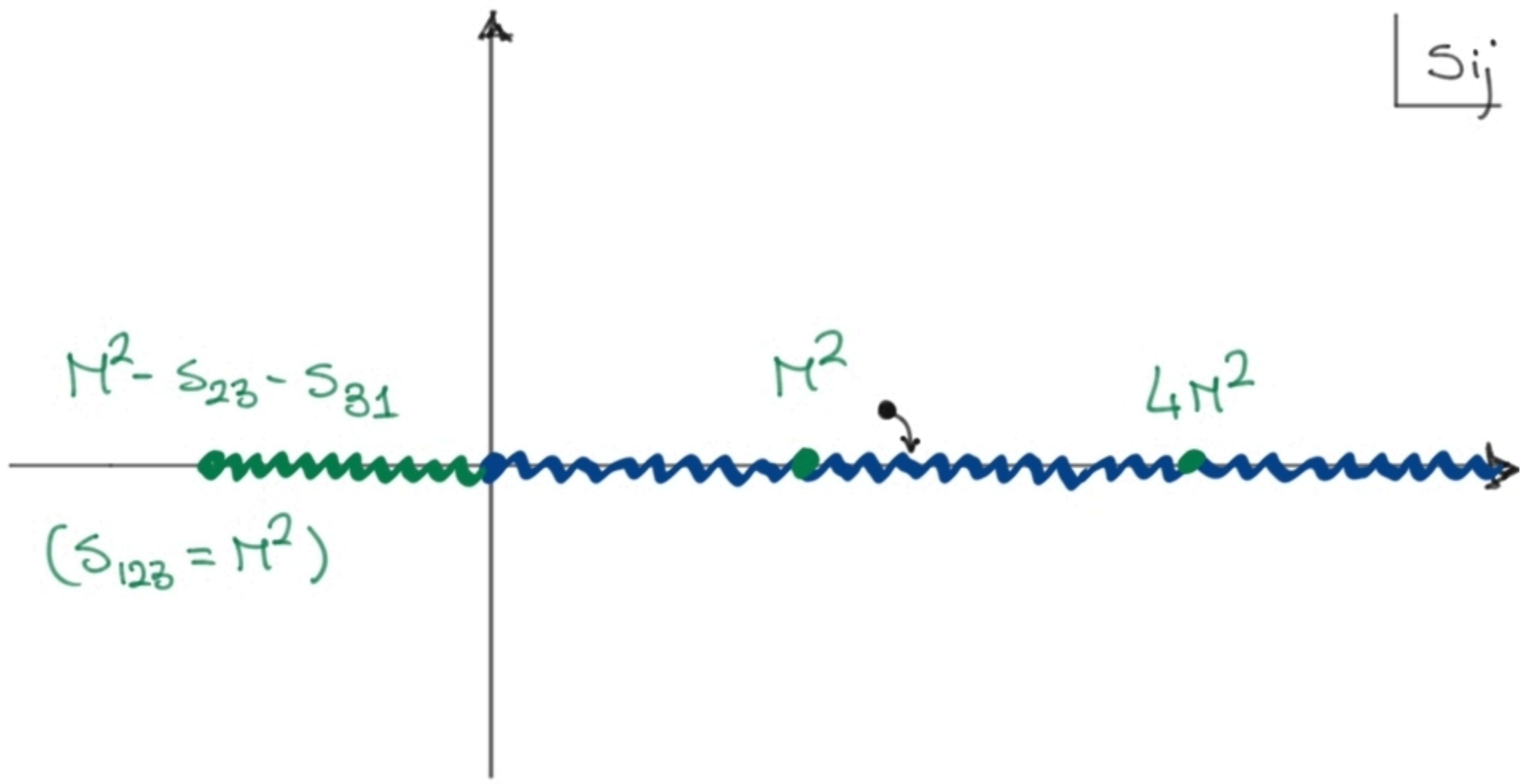
Analytic properties of Form Factors

Analytic properties of Form Factors

- We consider internal massive states.

The analytic structures is modified by virtual or threshold contributions.

We may have contributions in the s_{ij} channels, but also from multi-particle thresholds (e.g. s_{ijk})



Analytic properties of Form Factors

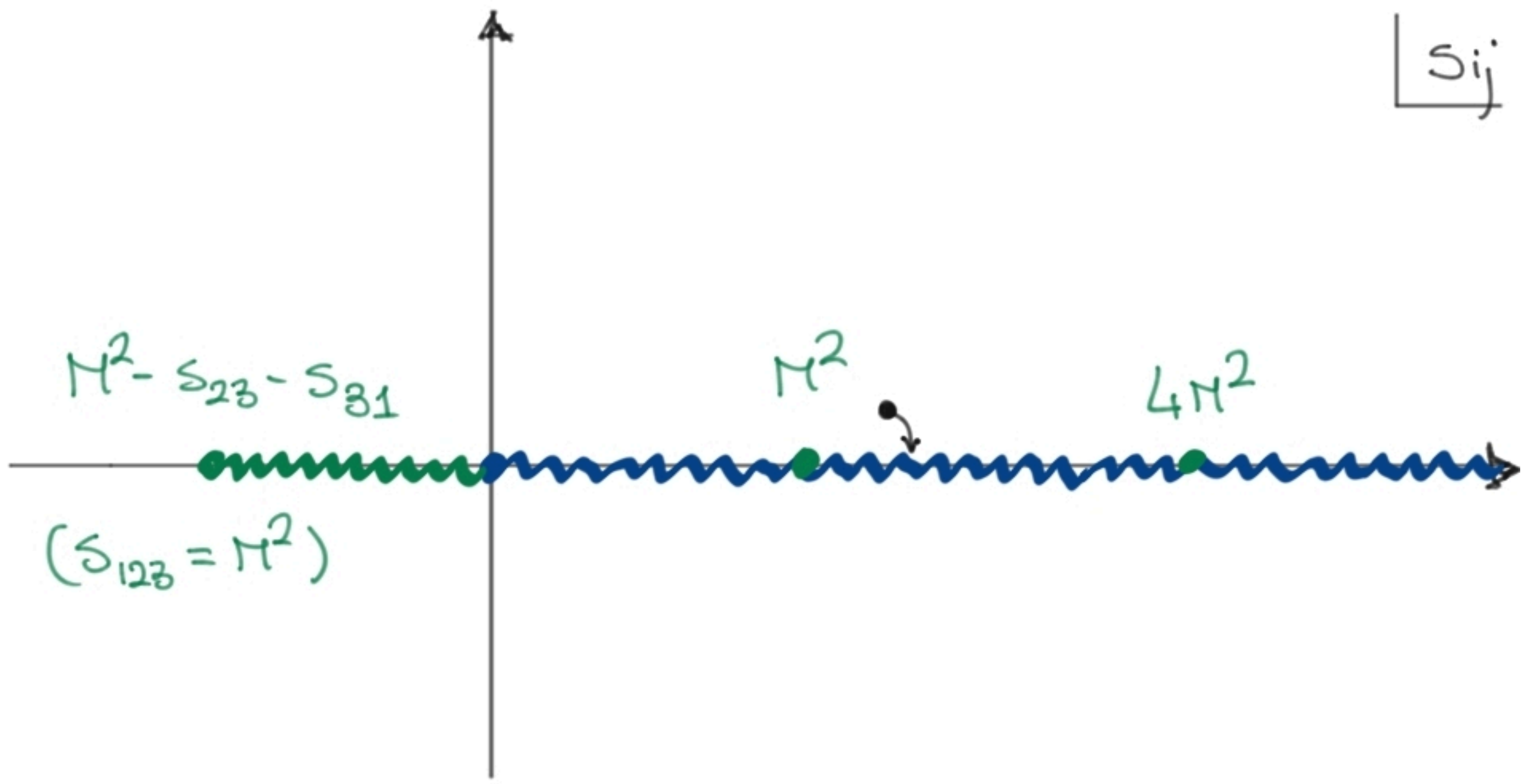
- We consider internal massive states.

  The analytic structures is modified by virtual or threshold contributions.

  We may have contributions in the s_{ij} channels, but also from multi-particle thresholds (e.g. s_{ijk})

- We can work in the space of complex dilatations...

  We can fix the external momenta such that there is no hierarchy between the dynamical variables (e.g. in the four-point case, $s \sim t \sim u \sim q^2$), ...

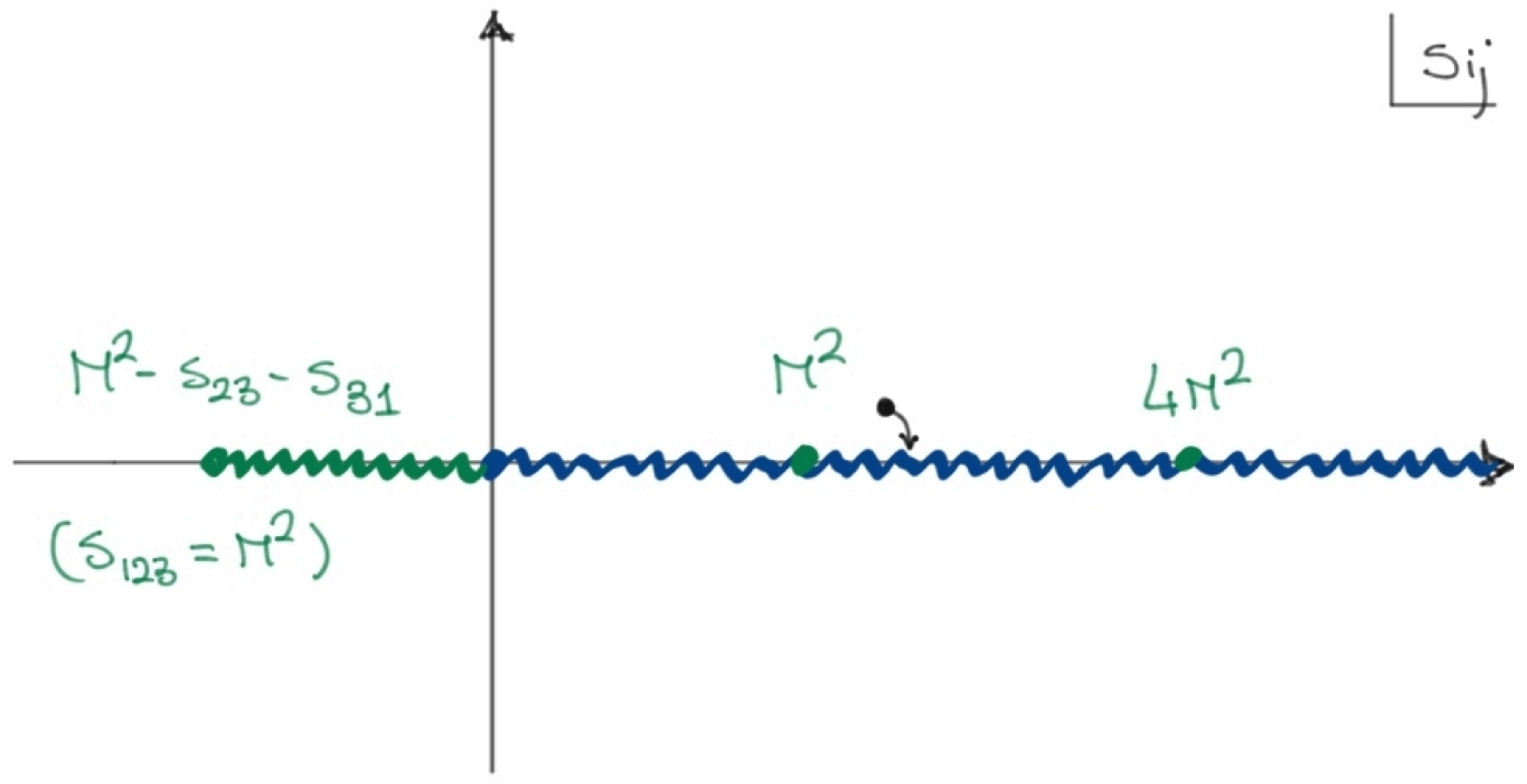


Analytic properties of Form Factors

- We consider internal massive states.

  The analytic structures is modified by virtual or threshold contributions.

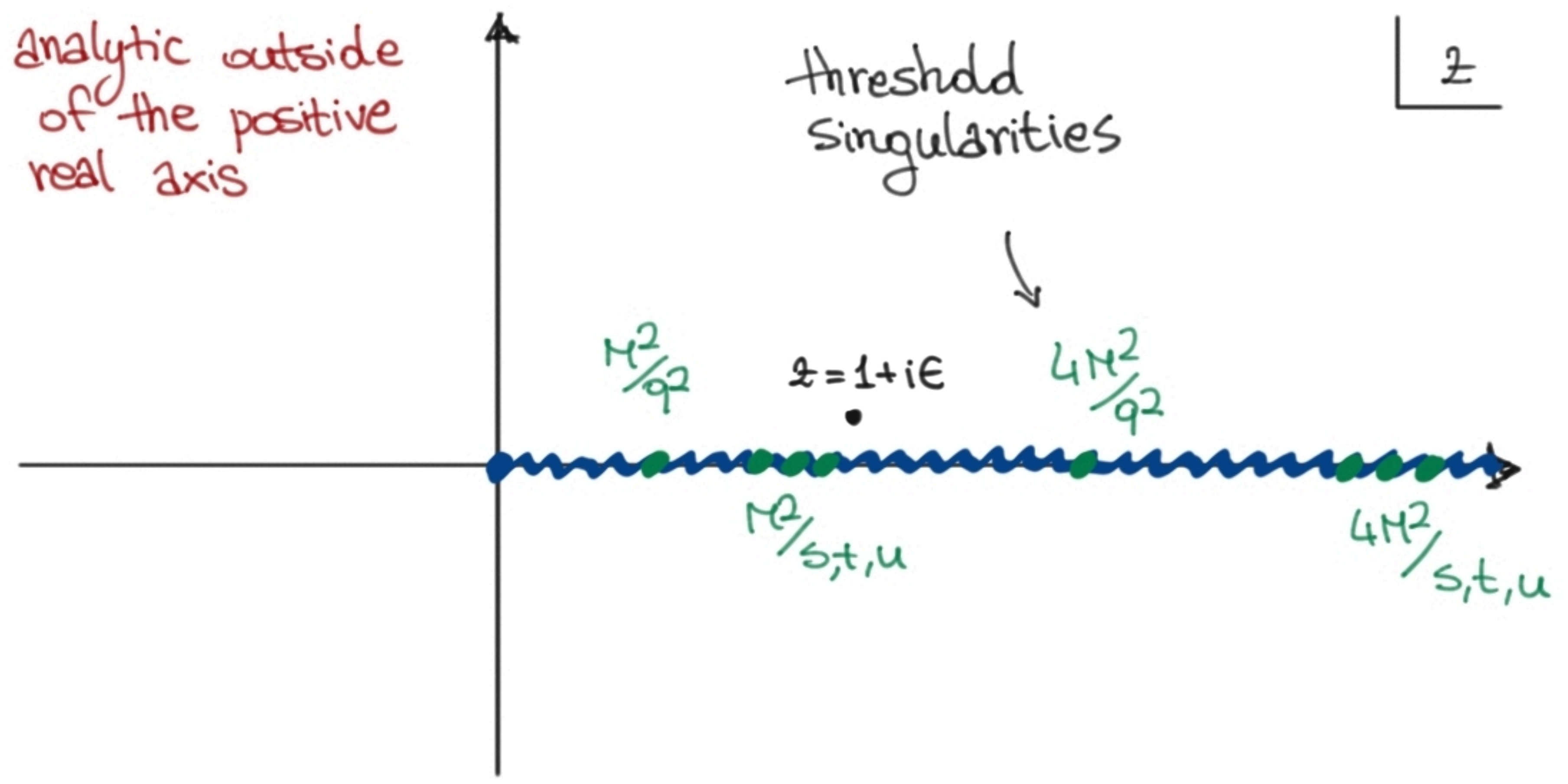
  We may have contributions in the s_{ij} channels, but also from multi-particle thresholds (e.g. s_{ijk})



- We can work in the space of complex dilatations...

  We can fix the external momenta such that there is no hierarchy between the dynamical variables (e.g. in the four-point case, $s \sim t \sim u \sim q^2$), ...

  ... perform a dilatation $p_i \to z\, p_i$ and analytically continue in z.

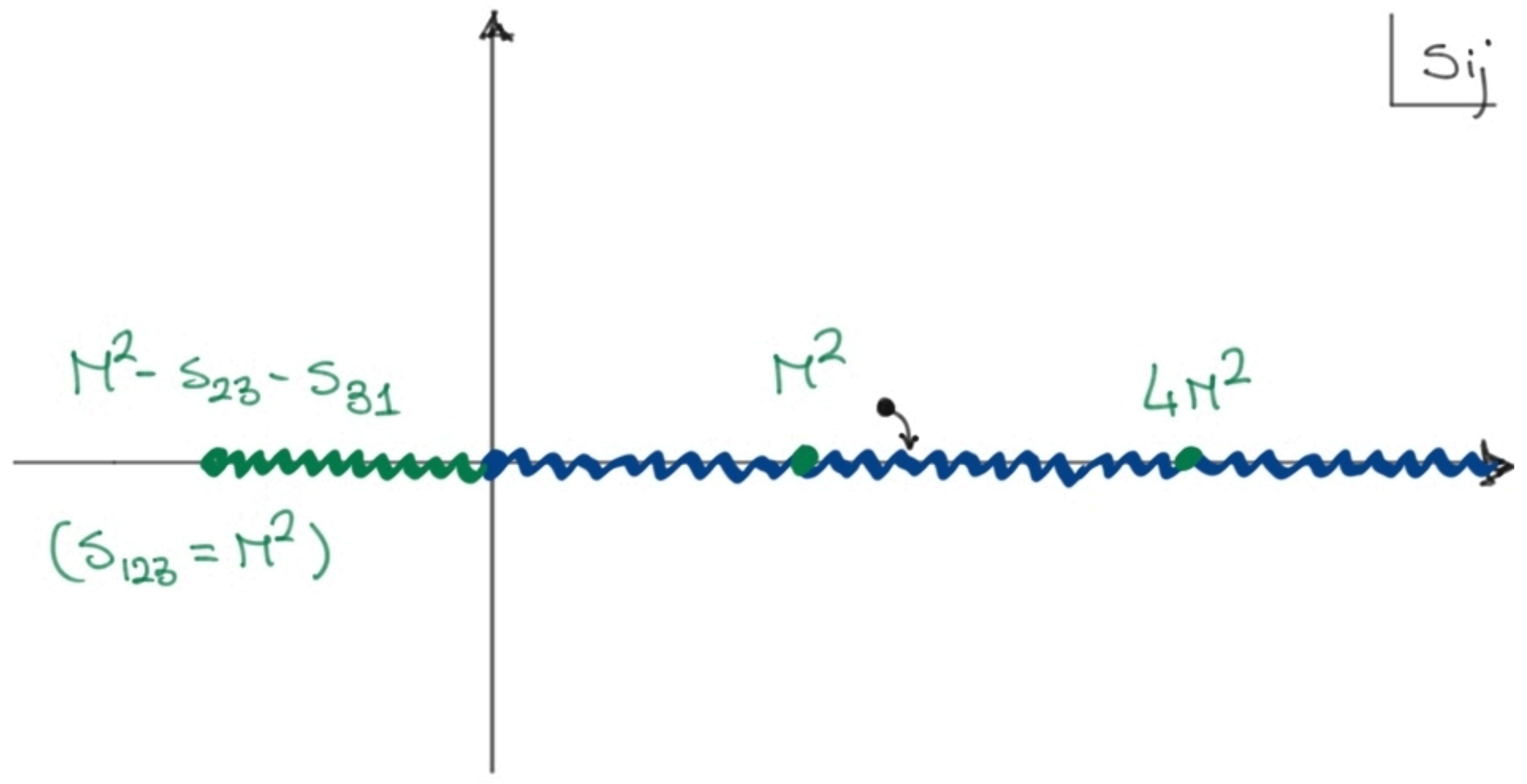


Analytic properties of Form Factors

- We consider internal massive states.

The analytic structures is modified by virtual or threshold contributions.

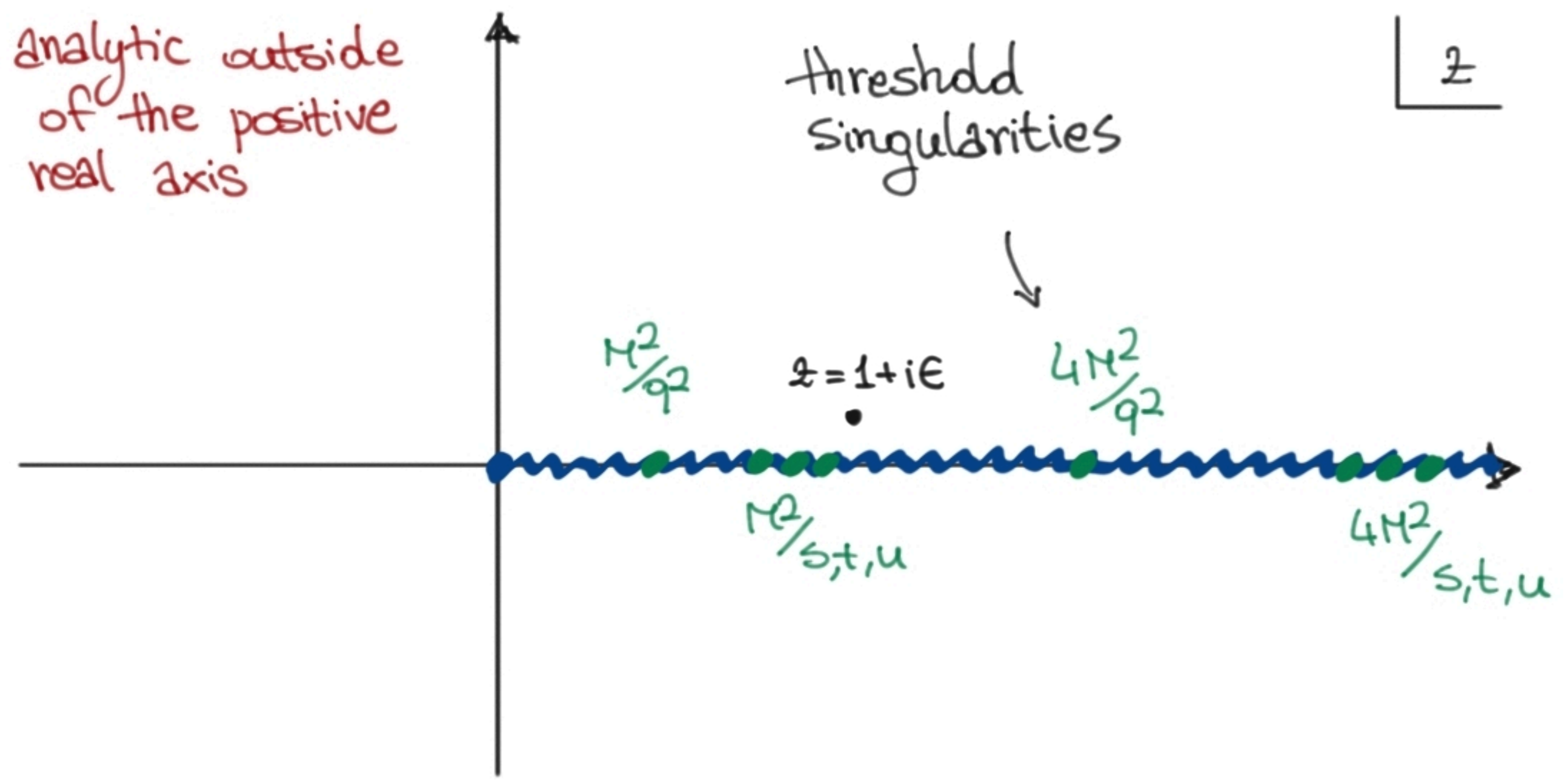
We may have contributions in the s_{ij} channels, but also from multi-particle thresholds (e.g. s_{ijk})



- We can work in the space of complex dilatations...

We can fix the external momenta such that there is no hierarchy between the dynamical variables (e.g. in the four-point case, $s \sim t \sim u \sim q^2$), ...

... perform a dilatation $p_i \to z\, p_i$ and analytically continue in z.



- ... with discontinuities determined by unitarity (again!)

The discontinuities with respect to z correspond to the sum of all the discontinuities:

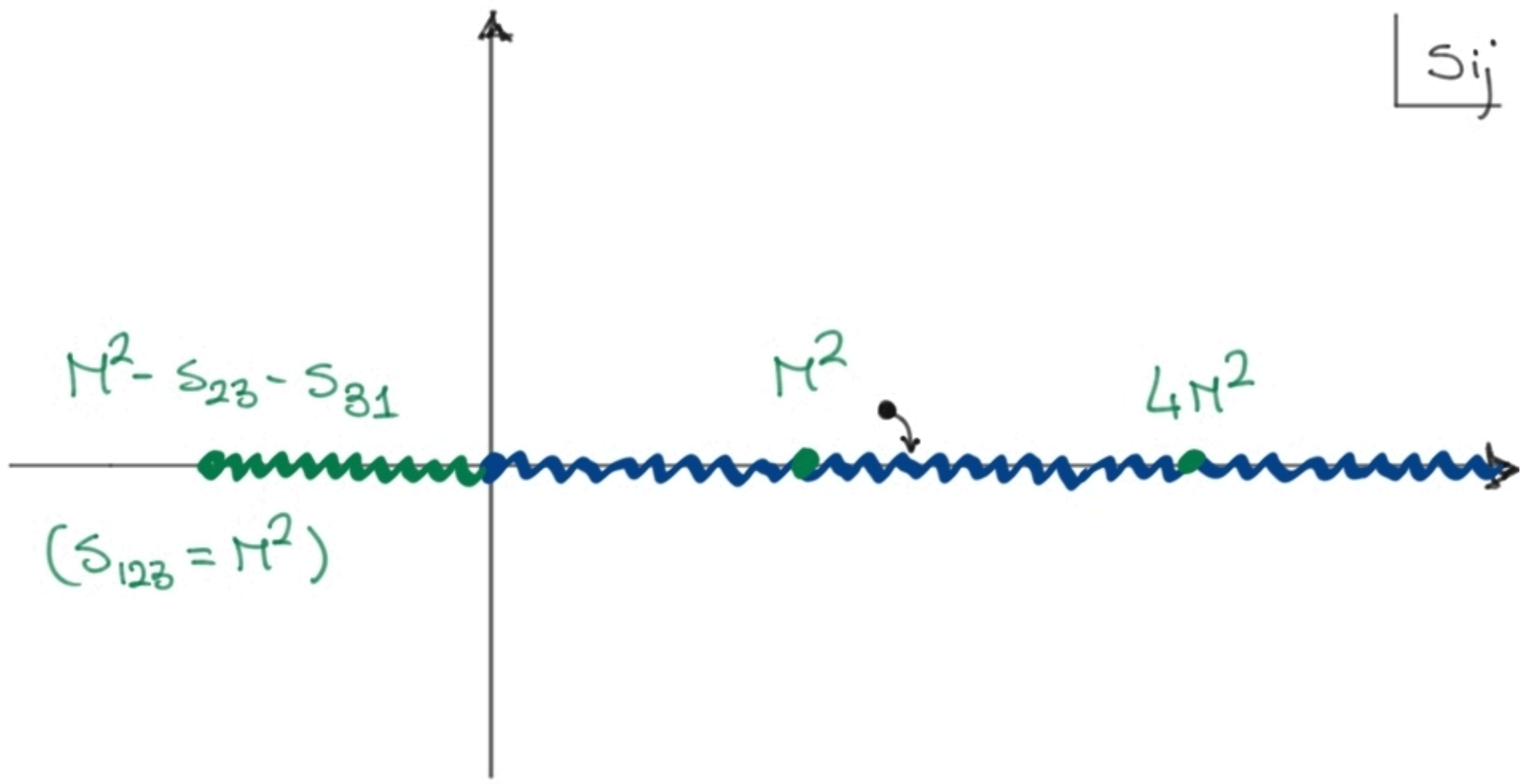
$$\underset{z}{\text{Disc}}\, F_{\mathcal{O}}(\vec{m}) = \sum_{\mathcal{S}_n^{\text{ord}}(\vec{m})} \underset{s_{i_1 \dots i_n}}{\text{Disc}}\, F_{\mathcal{O}}(\vec{m})$$

Analytic properties of Form Factors

- We consider internal massive states.

The analytic structures is modified by virtual or threshold contributions.

We may have contributions in the s_{ij} channels, but also from multi-particle thresholds (e.g. s_{ijk})



- We can work in the space of complex dilatations...

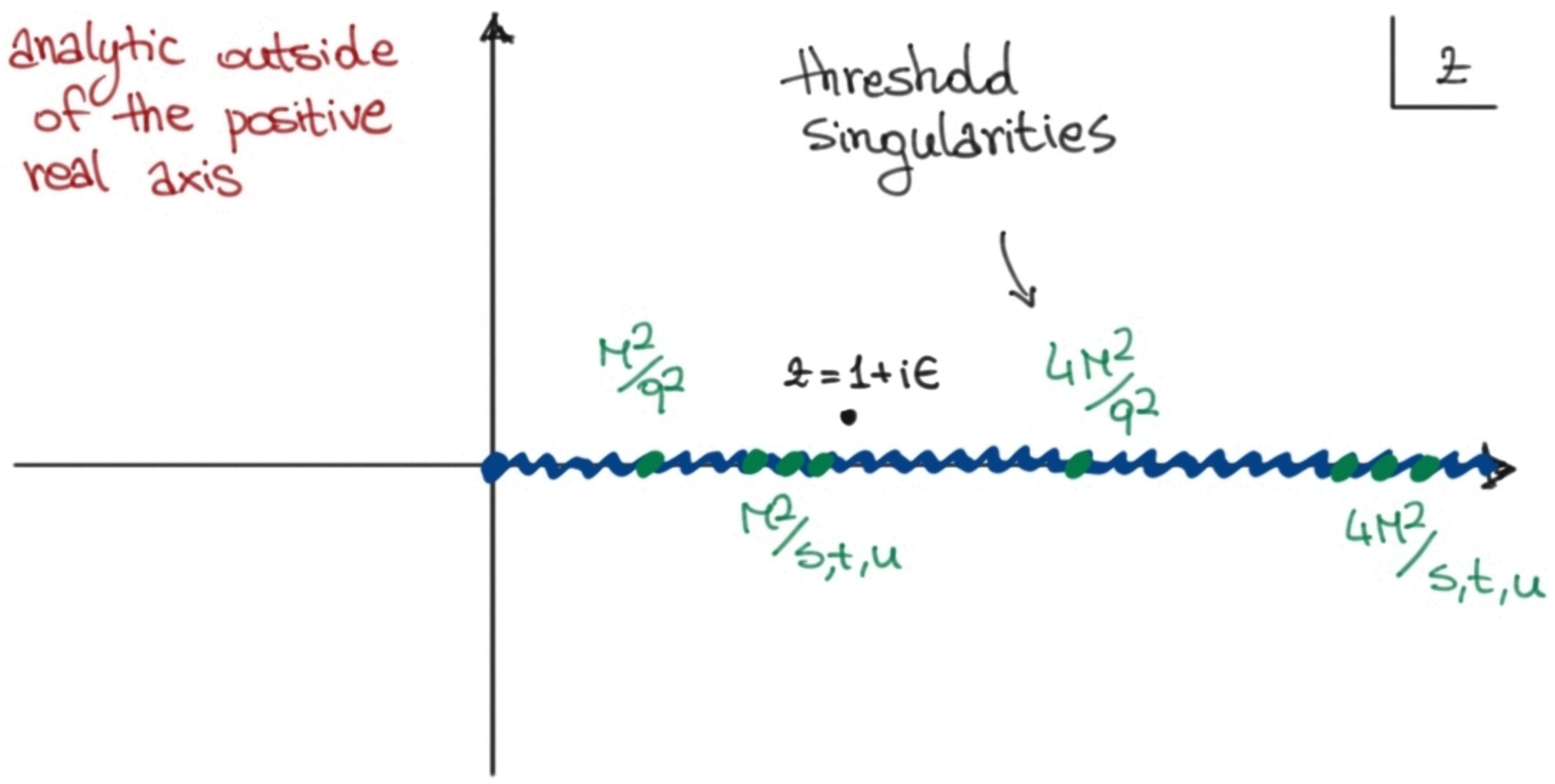
We can fix the external momenta such that there is no hierarchy between the dynamical variables (e.g. in the four-point case, $s \sim t \sim u \sim q^2$), ...

... perform a dilatation $p_i \to z\, p_i$ and analytically continue in z.

- ... with discontinuities determined by unitarity (again!)

The discontinuities with respect to z correspond to the sum of all the discontinuities:

$$\underset{z}{\text{Disc}}\, F_{\mathcal{O}}(\vec{m}) = \sum_{\mathcal{S}_n^{\text{ord}}(\vec{m})} \underset{s_{i_1 \ldots i_n}}{\text{Disc}}\, F_{\mathcal{O}}(\vec{m})$$



If $q^2 < M^2$, the massive state decouples!

Matching in Dilatation Space

Matching in Dilatation Space

- Which form factor? The closest to the S-matrix!

Matching in Dilatation Space

- Which form factor? The closest to the S-matrix!

To perform the matching of an EFT to a possible UV completion we consider the Lagrangian.

$$\mathscr{L}_{\mathrm{IR}} = \mathscr{L}_0 + \sum_{d=5}^{\infty} \sum_{i=1}^{n_d} \frac{c_{d,i}}{\Lambda^{d-4}} \mathscr{O}_{d,i} \,, \qquad \mathscr{L}_{\mathrm{UV}} = \mathscr{L}_0 + \mathscr{L}_{\Phi,\mathrm{kin}} + \mathscr{L}_{\Phi,\mathrm{int}} \,.$$

If we cross some of the out states and take the limit $q \to 0$, we obtain the scattering amplitude.

Matching in Dilatation Space

- Which form factor? The closest to the S-matrix!

To perform the matching of an EFT to a possible UV completion we consider the Lagrangian.

$$\mathscr{L}_{\text{IR}} = \mathscr{L}_0 + \sum_{d=5}^{\infty} \sum_{i=1}^{n_d} \frac{c_{d,i}}{\Lambda^{d-4}} \mathcal{O}_{d,i} \,, \qquad \mathscr{L}_{\text{UV}} = \mathscr{L}_0 + \mathscr{L}_{\Phi,\text{kin}} + \mathscr{L}_{\Phi,\text{int}} \,.$$

If we cross some of the out states and take the limit $q \to 0$, we obtain the scattering amplitude.

- The matching condition: ${}_{\text{out}}\langle \psi_{\vec{n}} | \mathscr{L}_{\text{IR}}(0) | 0 \rangle = {}_{\text{out}}\langle \psi_{\vec{n}} | \mathscr{L}_{\text{UV}}(0) | 0 \rangle$

In the kinematics for which the massive state decouples $q^2 = s_{1,\ldots,n} \leq M^2$ and $|z| \leq 1$, we demand that the form factors computed in the IR is the same as the one in the UV.

Matching in Dilatation Space

- **Which form factor? The closest to the S-matrix!**

To perform the matching of an EFT to a possible UV completion we consider the Lagrangian.

$$\mathscr{L}_{\rm IR} = \mathscr{L}_0 + \sum_{d=5}^{\infty} \sum_{i=1}^{n_d} \frac{c_{d,i}}{\Lambda^{d-4}} \mathscr{O}_{d,i}\,, \qquad \mathscr{L}_{\rm UV} = \mathscr{L}_0 + \mathscr{L}_{\Phi,\rm kin} + \mathscr{L}_{\Phi,\rm int}\,.$$

If we cross some of the out states and take the limit $q \to 0$, we obtain the scattering amplitude.

- **The matching condition:** ${}_{\rm out}\langle \psi_{\vec{n}} | \mathscr{L}_{\rm IR}(0) | 0 \rangle = {}_{\rm out}\langle \psi_{\vec{n}} | \mathscr{L}_{\rm UV}(0) | 0 \rangle$

In the kinematics for which the massive state decouples $q^2 = s_{1,\dots,n} \leq M^2$ and $|z| \leq 1$, we demand that the form factors computed in the IR is the same as the one in the UV.

Choosing the state will select operators and the matching is easier if we choose it carefully.

Matching in Dilatation Space

- Which form factor? The closest to the S-matrix!

To perform the matching of an EFT to a possible UV completion we consider the Lagrangian.

$$\mathscr{L}_{\rm IR} = \mathscr{L}_0 + \sum_{d=5}^{\infty}\sum_{i=1}^{n_d}\frac{c_{d,i}}{\Lambda^{d-4}}\mathscr{O}_{d,i}\,, \qquad \mathscr{L}_{\rm UV} = \mathscr{L}_0 + \mathscr{L}_{\Phi,\rm kin} + \mathscr{L}_{\Phi,\rm int}\,.$$

If we cross some of the out states and take the limit $q \to 0$, we obtain the scattering amplitude.

- The matching condition: ${}_{\rm out}\langle\psi_{\vec{n}}|\mathscr{L}_{\rm IR}(0)|0\rangle = {}_{\rm out}\langle\psi_{\vec{n}}|\mathscr{L}_{\rm UV}(0)|0\rangle$

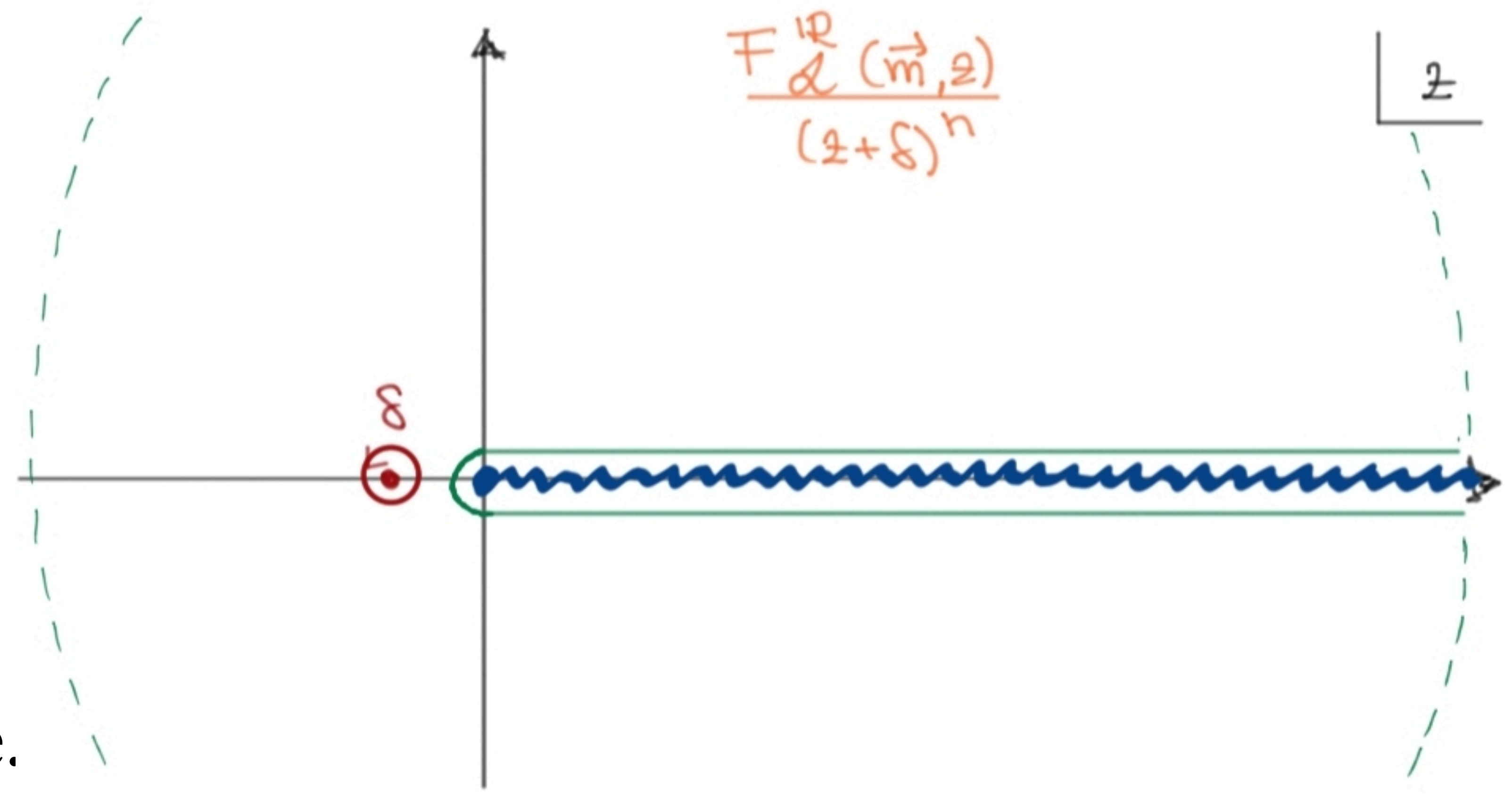
In the kinematics for which the massive state decouples $q^2 = s_{1,\ldots,n} \leq M^2$ and $|z| \leq 1$, we demand that the form factors computed in the IR is the same as the one in the UV.

Choosing the state will select operators and the matching is easier if we choose it carefully.

In the most generic case, we can set $q \to 0$ from the very beginning.

Matching in Dilatation Space

- Which form factor? The closest to the S-matrix!



To perform the matching of an EFT to a possible UV completion we consider the Lagrangian.

$$\mathscr{L}_{\rm IR} = \mathscr{L}_0 + \sum_{d=5}^{\infty}\sum_{i=1}^{n_d}\frac{c_{d,i}}{\Lambda^{d-4}}\mathcal{O}_{d,i}\,, \qquad \mathscr{L}_{\rm UV} = \mathscr{L}_0 + \mathscr{L}_{\Phi,\rm kin} + \mathscr{L}_{\Phi,\rm int}\,.$$

If we cross some of the out states and take the limit $q \to 0$, we obtain the scattering amplitude.

- The matching condition: ${}_{\rm out}\langle \psi_{\vec{n}} | \mathscr{L}_{\rm IR}(0) | 0 \rangle = {}_{\rm out}\langle \psi_{\vec{n}} | \mathscr{L}_{\rm UV}(0) | 0 \rangle$

In the kinematics for which the massive state decouples $q^2 = s_{1,\ldots,n} \leq M^2$ and $|z| \leq 1$, we demand that the form factors computed in the IR is the same as the one in the UV.

Choosing the state will select operators and the matching is easier if we choose it carefully.

In the most generic case, we can set $q \to 0$ from the very beginning.

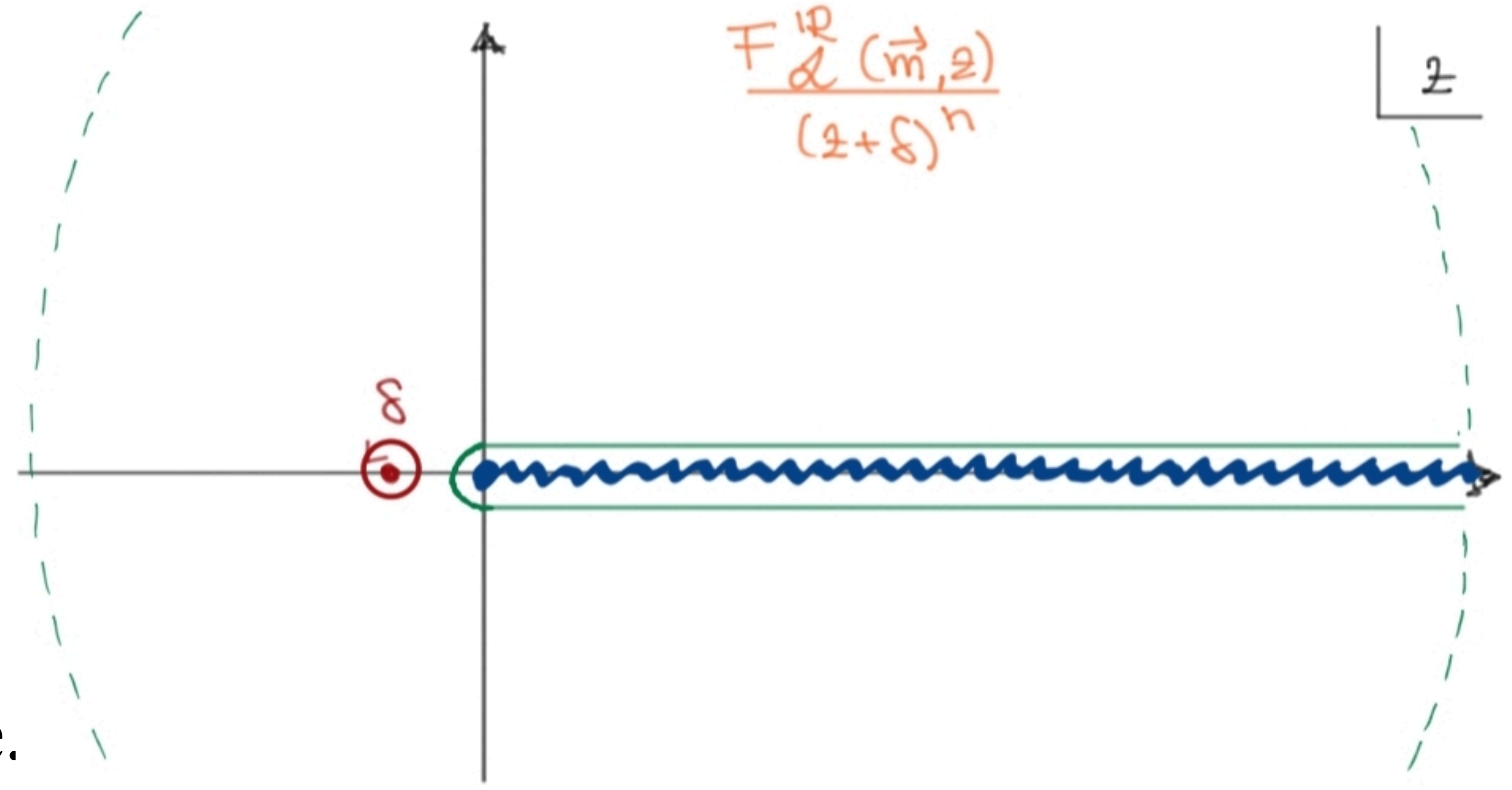
Matching in Dilatation Space

- Which form factor? The closest to the S-matrix!

To perform the matching of an EFT to a possible UV completion we consider the Lagrangian.

$$\mathscr{L}_{\rm IR} = \mathscr{L}_0 + \sum_{d=5}^{\infty} \sum_{i=1}^{n_d} \frac{c_{d,i}}{\Lambda^{d-4}} \mathcal{O}_{d,i} \,, \qquad \mathscr{L}_{\rm UV} = \mathscr{L}_0 + \mathscr{L}_{\Phi,\rm kin} + \mathscr{L}_{\Phi,\rm int} \,.$$

If we cross some of the out states and take the limit $q \to 0$, we obtain the scattering amplitude.



- The matching condition: ${}_{\rm out}\langle \psi_{\vec{n}} | \mathscr{L}_{\rm IR}(0) | 0 \rangle = {}_{\rm out}\langle \psi_{\vec{n}} | \mathscr{L}_{\rm UV}(0) | 0 \rangle$

In the kinematics for which the massive state decouples $q^2 = s_{1,\dots,n} \leq M^2$ and $|z| \leq 1$, we demand that the form factors computed in the IR is the same as the one in the UV.

Choosing the state will select operators and the matching is easier if we choose it carefully.

In the most generic case, we can set $q \to 0$ from the very beginning.

$$c_n \mathscr{P}_{\mathcal{O},n}(\vec{m}) = \frac{1}{2\pi i} \oint \frac{dz}{z^{n+1}} F_{\mathscr{L}_{\rm IR}}(\vec{m}; z) = c_n^{\rm IR} \mathscr{P}_{\mathcal{O},n}(\vec{m})$$

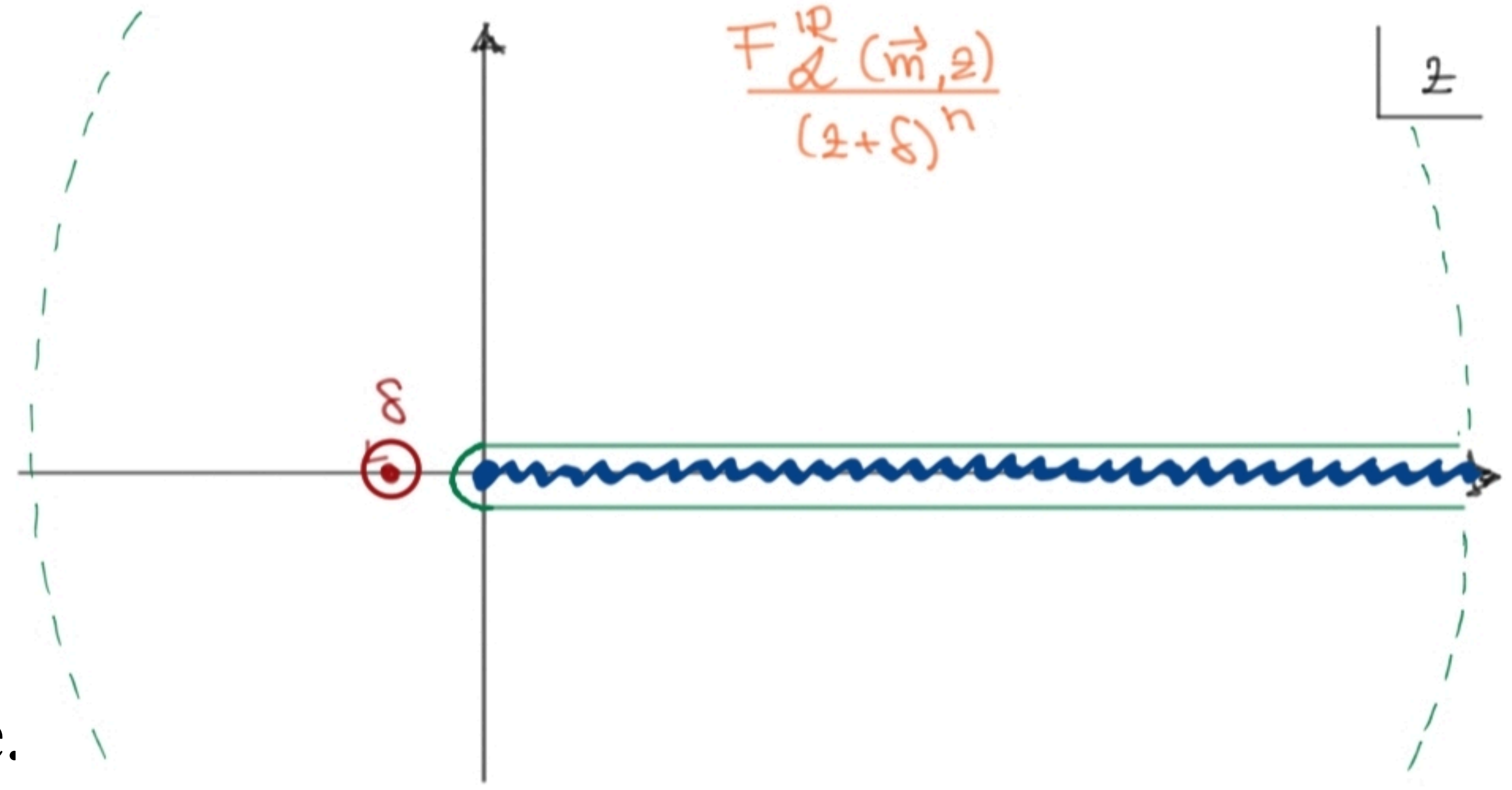
Matching in Dilatation Space

- **Which form factor? The closest to the S-matrix!**

To perform the matching of an EFT to a possible UV completion we consider the Lagrangian.

$$\mathscr{L}_{\mathrm{IR}} = \mathscr{L}_0 + \sum_{d=5}^{\infty} \sum_{i=1}^{n_d} \frac{c_{d,i}}{\Lambda^{d-4}} \mathcal{O}_{d,i} \,, \qquad \mathscr{L}_{\mathrm{UV}} = \mathscr{L}_0 + \mathscr{L}_{\Phi,\mathrm{kin}} + \mathscr{L}_{\Phi,\mathrm{int}} \,.$$

If we cross some of the out states and take the limit $q \to 0$, we obtain the scattering amplitude.



- **The matching condition:** ${}_{\mathrm{out}}\langle \psi_{\vec{n}} | \mathscr{L}_{\mathrm{IR}}(0) | 0 \rangle = {}_{\mathrm{out}}\langle \psi_{\vec{n}} | \mathscr{L}_{\mathrm{UV}}(0) | 0 \rangle$

In the kinematics for which the massive state decouples $q^2 = s_{1,\ldots,n} \leq M^2$ and $|z| \leq 1$, we demand that the form factors computed in the IR is the same as the one in the UV.

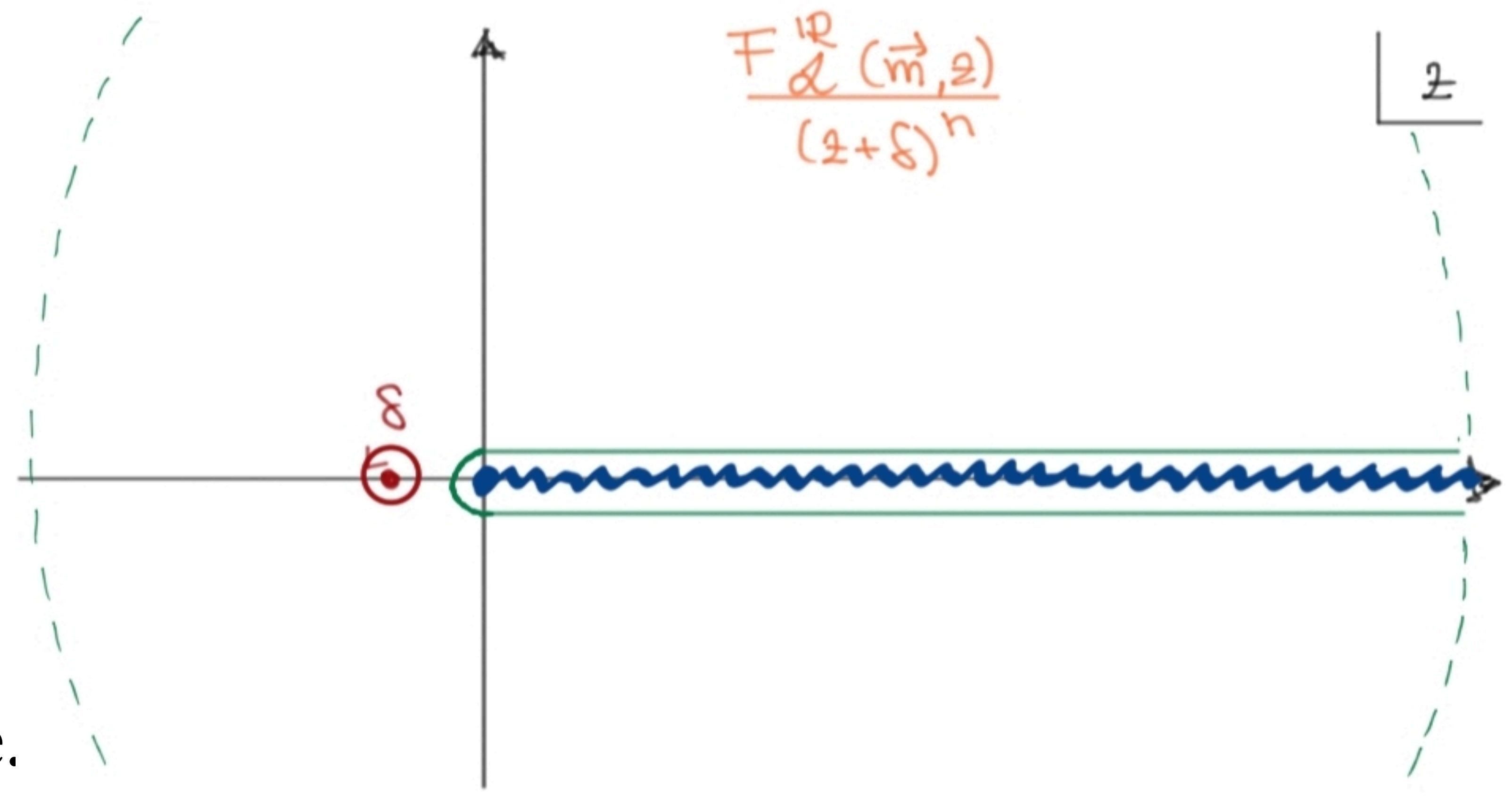
Choosing the state will select operators and the matching is easier if we choose it carefully.

In the most generic case, we can set $q \to 0$ from the very beginning.

$$c_n \mathscr{P}_{\mathcal{O},n}(\vec{m}) = \frac{1}{2\pi i} \oint \frac{dz}{z^{n+1}} F_{\mathscr{L}_{\mathrm{IR}}}(\vec{m}; z) = c_n^{\mathrm{IR}} \mathscr{P}_{\mathcal{O},n}(\vec{m}) - \sum_s \left(\frac{s}{M^2} \right)^{n+1} \operatorname*{Res}_{z=\frac{M^2}{s}} F_{\mathscr{L}_{\mathrm{UV}}}(\vec{m}; z)$$

Matching in Dilatation Space

- **Which form factor? The closest to the S-matrix!**



To perform the matching of an EFT to a possible UV completion we consider the Lagrangian.

$$\mathscr{L}_{\rm IR} = \mathscr{L}_0 + \sum_{d=5}^{\infty}\sum_{i=1}^{n_d} \frac{c_{d,i}}{\Lambda^{d-4}}\mathcal{O}_{d,i}\,, \qquad \mathscr{L}_{\rm UV} = \mathscr{L}_0 + \mathscr{L}_{\Phi,{\rm kin}} + \mathscr{L}_{\Phi,{\rm int}}\,.$$

If we cross some of the out states and take the limit $q \to 0$, we obtain the scattering amplitude.

- **The matching condition:** ${}_{\rm out}\langle \psi_{\vec{n}} | \mathscr{L}_{\rm IR}(0) | 0 \rangle = {}_{\rm out}\langle \psi_{\vec{n}} | \mathscr{L}_{\rm UV}(0) | 0 \rangle$

In the kinematics for which the massive state decouples $q^2 = s_{1,\ldots,n} \leq M^2$ and $|z| \leq 1$, we demand that the form factors computed in the IR is the same as the one in the UV.

Choosing the state will select operators and the matching is easier if we choose it carefully.

In the most generic case, we can set $q \to 0$ from the very beginning.

$$c_n \mathscr{P}_{\mathcal{O},n}(\vec{m}) = \frac{1}{2\pi i}\oint \frac{dz}{z^{n+1}} F_{\mathscr{L}_{\rm IR}}(\vec{m};z) = c_n^{\rm IR}\mathscr{P}_{\mathcal{O},n}(\vec{m}) - \sum_s \left(\frac{s}{M^2}\right)^{n+1} \underset{z=\frac{M^2}{s}}{\rm Res}\, F_{\mathscr{L}_{\rm UV}}(\vec{m};z) + \frac{1}{2\pi i}\int_0^\infty \frac{dz}{z^{n+1}} \underset{z}{\rm Disc}\, F_{\mathscr{L}_{\rm UV}}(\vec{m};z)$$

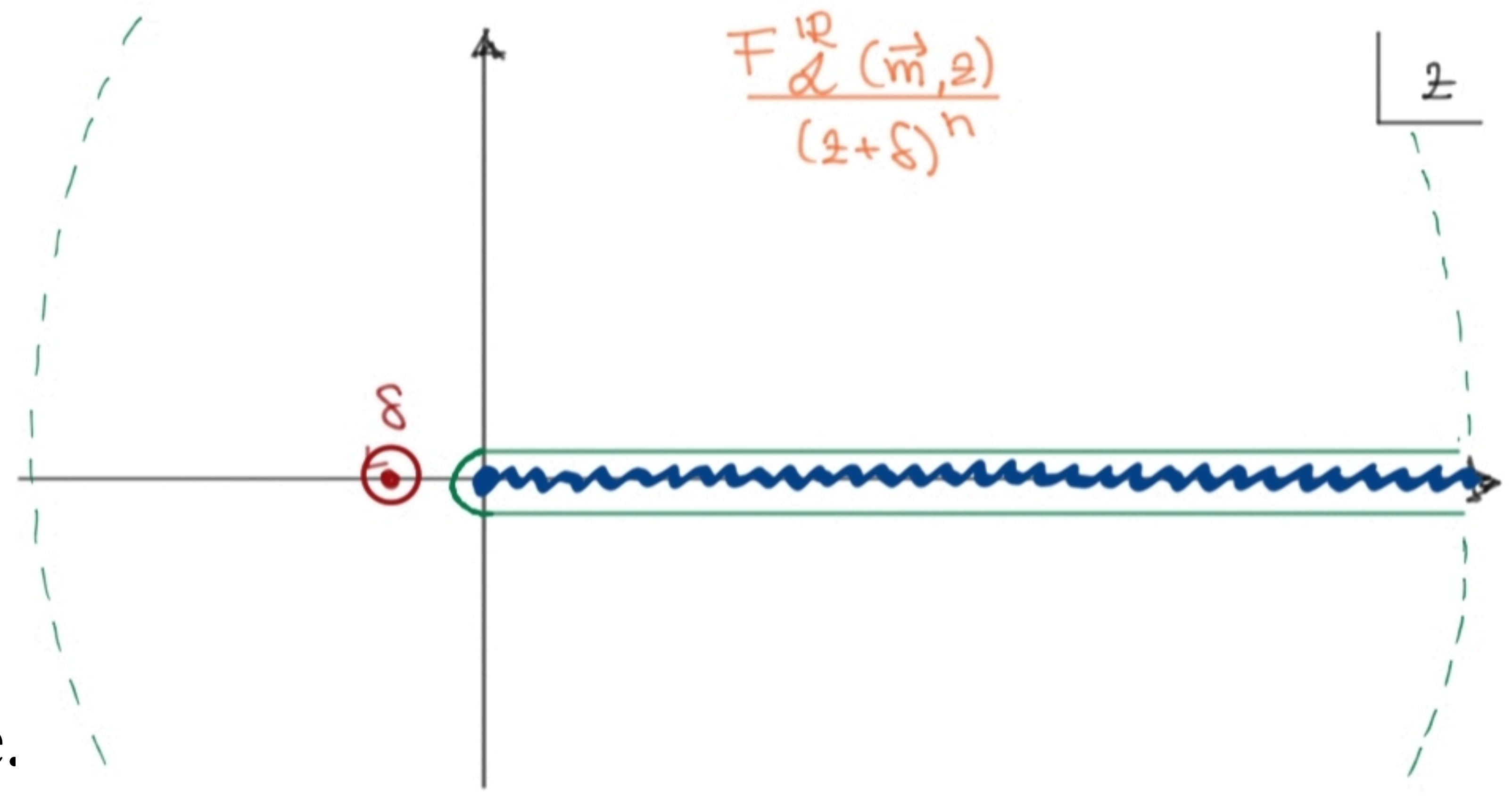
Matching in Dilatation Space

- Which form factor? The closest to the S-matrix!

To perform the matching of an EFT to a possible UV completion we consider the Lagrangian.

$$\mathscr{L}_{\rm IR} = \mathscr{L}_0 + \sum_{d=5}^{\infty}\sum_{i=1}^{n_d} \frac{c_{d,i}}{\Lambda^{d-4}}\mathcal{O}_{d,i}\,, \qquad \mathscr{L}_{\rm UV} = \mathscr{L}_0 + \mathscr{L}_{\Phi,\rm kin} + \mathscr{L}_{\Phi,\rm int}\,.$$

If we cross some of the out states and take the limit $q \to 0$, we obtain the scattering amplitude.



- The matching condition: ${}_{\rm out}\langle \psi_{\vec{n}} | \mathscr{L}_{\rm IR}(0) | 0\rangle = {}_{\rm out}\langle \psi_{\vec{n}} | \mathscr{L}_{\rm UV}(0) | 0\rangle$

In the kinematics for which the massive state decouples $q^2 = s_{1,\dots,n} \le M^2$ and $|z| \le 1$, we demand that the form factors computed in the IR is the same as the one in the UV.

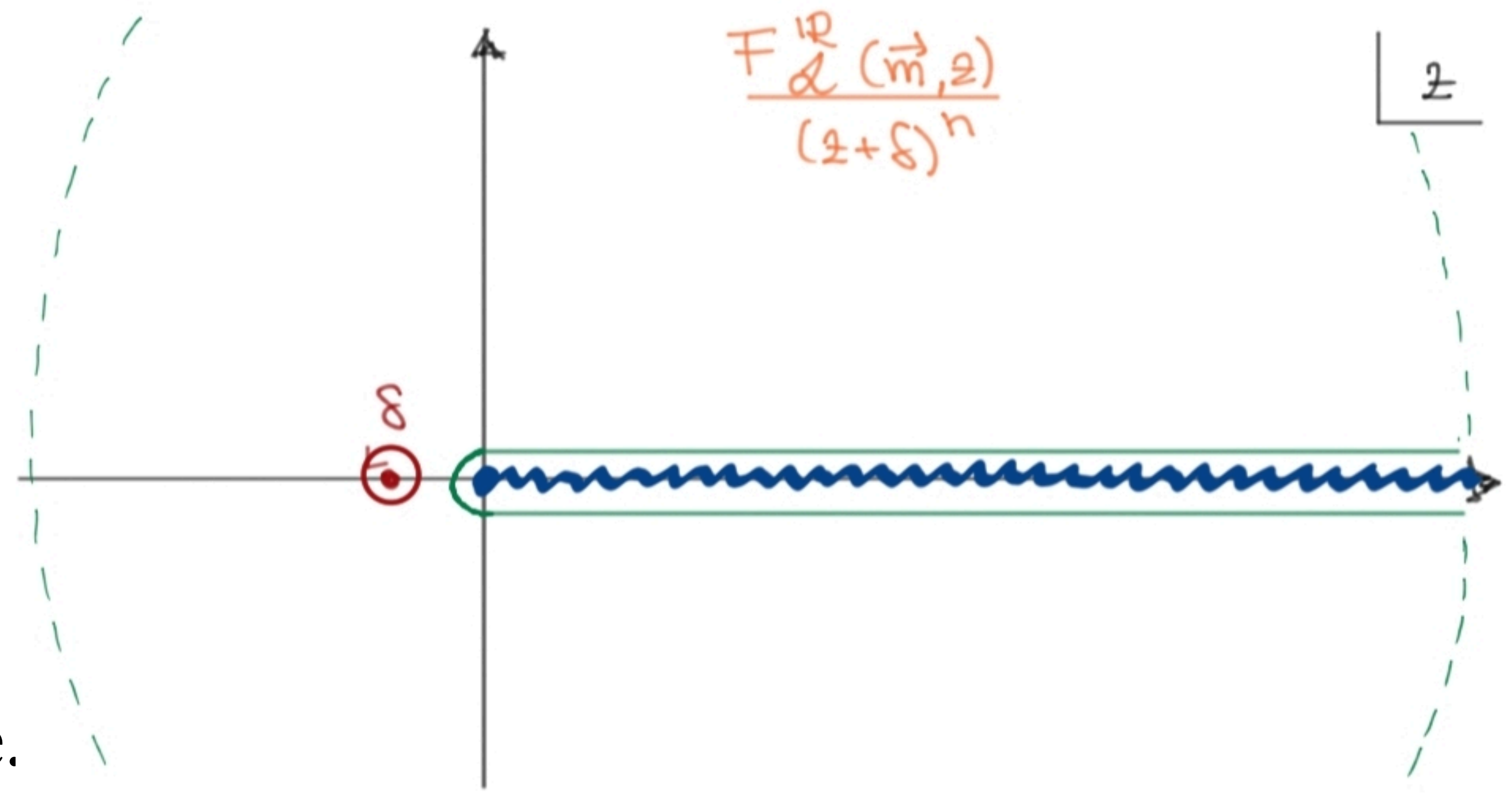
Choosing the state will select operators and the matching is easier if we choose it carefully.

In the most generic case, we can set $q \to 0$ from the very beginning.

$$c_n \mathscr{P}_{\mathcal{O},n}(\vec{m}) = \frac{1}{2\pi i}\oint \frac{dz}{z^{n+1}} F_{\mathscr{L}_{\rm IR}}(\vec{m};z) = c_n^{\rm IR}\mathscr{P}_{\mathcal{O},n}(\vec{m}) - \sum_s \left(\frac{s}{M^2}\right)^{n+1} \underset{z=\frac{M^2}{s}}{\rm Res}\, F_{\mathscr{L}_{\rm UV}}(\vec{m};z) + \frac{1}{2\pi i}\int_0^\infty \frac{dz}{z^{n+1}}\, \underset{z}{\rm Disc}\, F_{\mathscr{L}_{\rm UV}}(\vec{m};z)$$

Matching in Dilatation Space

- Which form factor? The closest to the S-matrix!



To perform the matching of an EFT to a possible UV completion we consider the Lagrangian.

$$\mathscr{L}_{\rm IR} = \mathscr{L}_0 + \sum_{d=5}^{\infty}\sum_{i=1}^{n_d} \frac{c_{d,i}}{\Lambda^{d-4}} \mathscr{O}_{d,i}\,, \qquad \mathscr{L}_{\rm UV} = \mathscr{L}_0 + \mathscr{L}_{\Phi,\rm kin} + \mathscr{L}_{\Phi,\rm int}\,.$$

If we cross some of the out states and take the limit $q \to 0$, we obtain the scattering amplitude.

- The matching condition: ${}_{\rm out}\langle \psi_{\vec{n}} | \mathscr{L}_{\rm IR}(0) | 0 \rangle = {}_{\rm out}\langle \psi_{\vec{n}} | \mathscr{L}_{\rm UV}(0) | 0 \rangle$

In the kinematics for which the massive state decouples $q^2 = s_{1,\ldots,n} \leq M^2$ and $|z| \leq 1$, we demand that the form factors computed in the IR is the same as the one in the UV.

Choosing the state will select operators and the matching is easier if we choose it carefully.

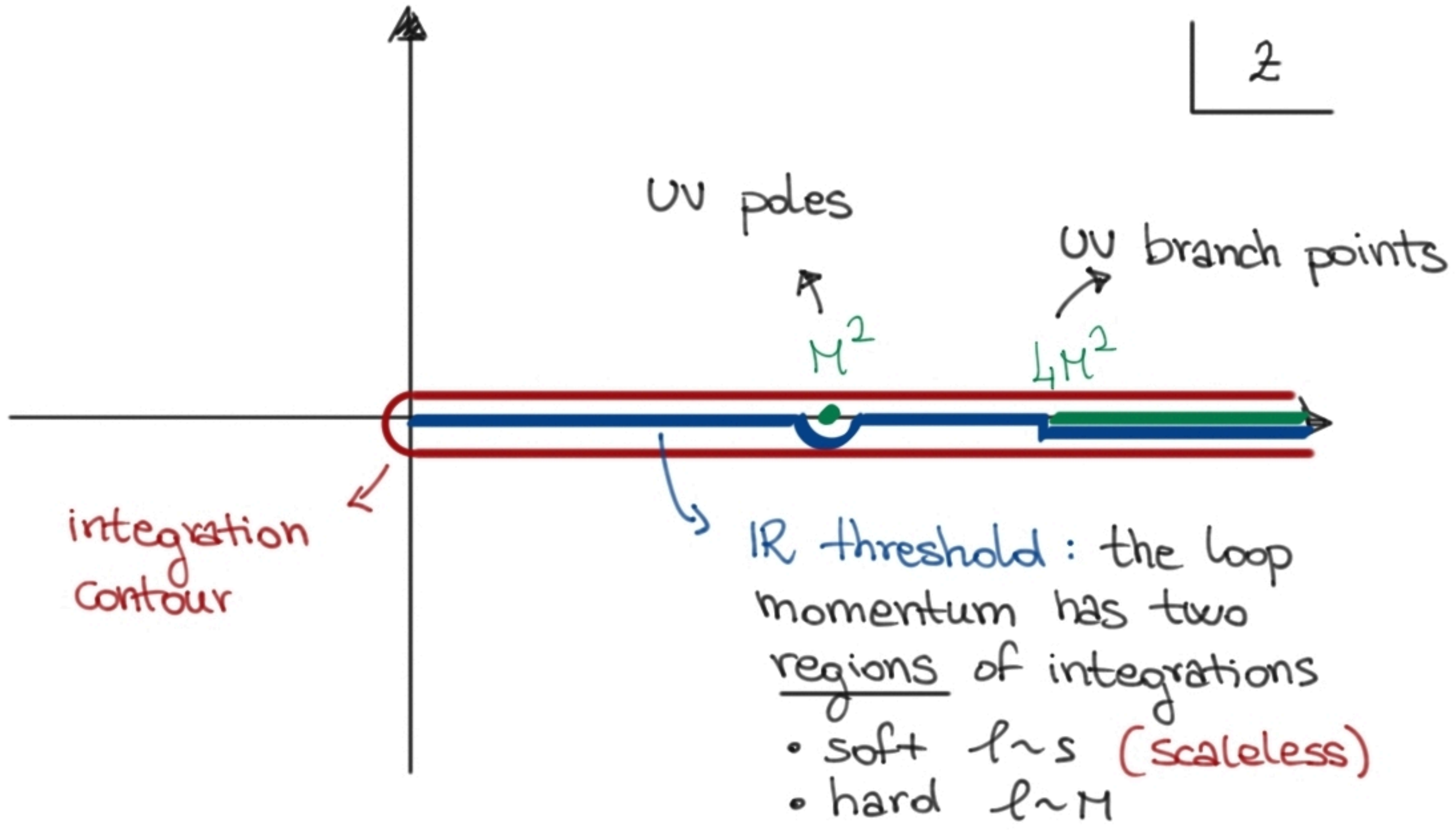
In the most generic case, we can set $q \to 0$ from the very beginning.

BONUS MATERIAL (for discussion or questions): We have just generalised the central equation of [Caron-Huot,Wilhelm] for computing anomalous dimensions to the case of light massive states.

$$c_n \mathscr{P}_{\mathscr{O},n}(\vec{m}) = \frac{1}{2\pi i}\oint \frac{dz}{z^{n+1}} F_{\mathscr{L}_{\rm IR}}(\vec{m};z) = c_n^{\rm IR}\mathscr{P}_{\mathscr{O},n}(\vec{m}) - \sum_s \left(\frac{s}{M^2}\right)^{n+1} \underset{z=\frac{M^2}{s}}{\rm Res}\, F_{\mathscr{L}_{\rm UV}}(\vec{m};z) + \frac{1}{2\pi i}\int_0^\infty \frac{dz}{z^{n+1}} \underset{z}{\rm Disc}\, F_{\mathscr{L}_{\rm UV}}(\vec{m};z)$$

Matching in Dilatation Space

$$c_n \mathscr{P}_{\mathcal{O},n}(\vec{m}) = \frac{1}{2\pi i}\oint \frac{dz}{z^{n+1}} F_{\mathscr{L}_{\rm IR}}(\vec{m};z) = c_n^{\rm IR}\mathscr{P}_{\mathcal{O},n}(\vec{m}) - \sum_s \left(\frac{s}{M^2}\right)^{n+1} \operatorname*{Res}_{z=\frac{M^2}{s}} F_{\mathscr{L}_{\rm UV}}(\vec{m};z) + \frac{1}{2\pi i}\int_0^\infty \frac{dz}{z^{n+1}} \operatorname*{Disc}_z F_{\mathscr{L}_{\rm UV}}(\vec{m};z)$$

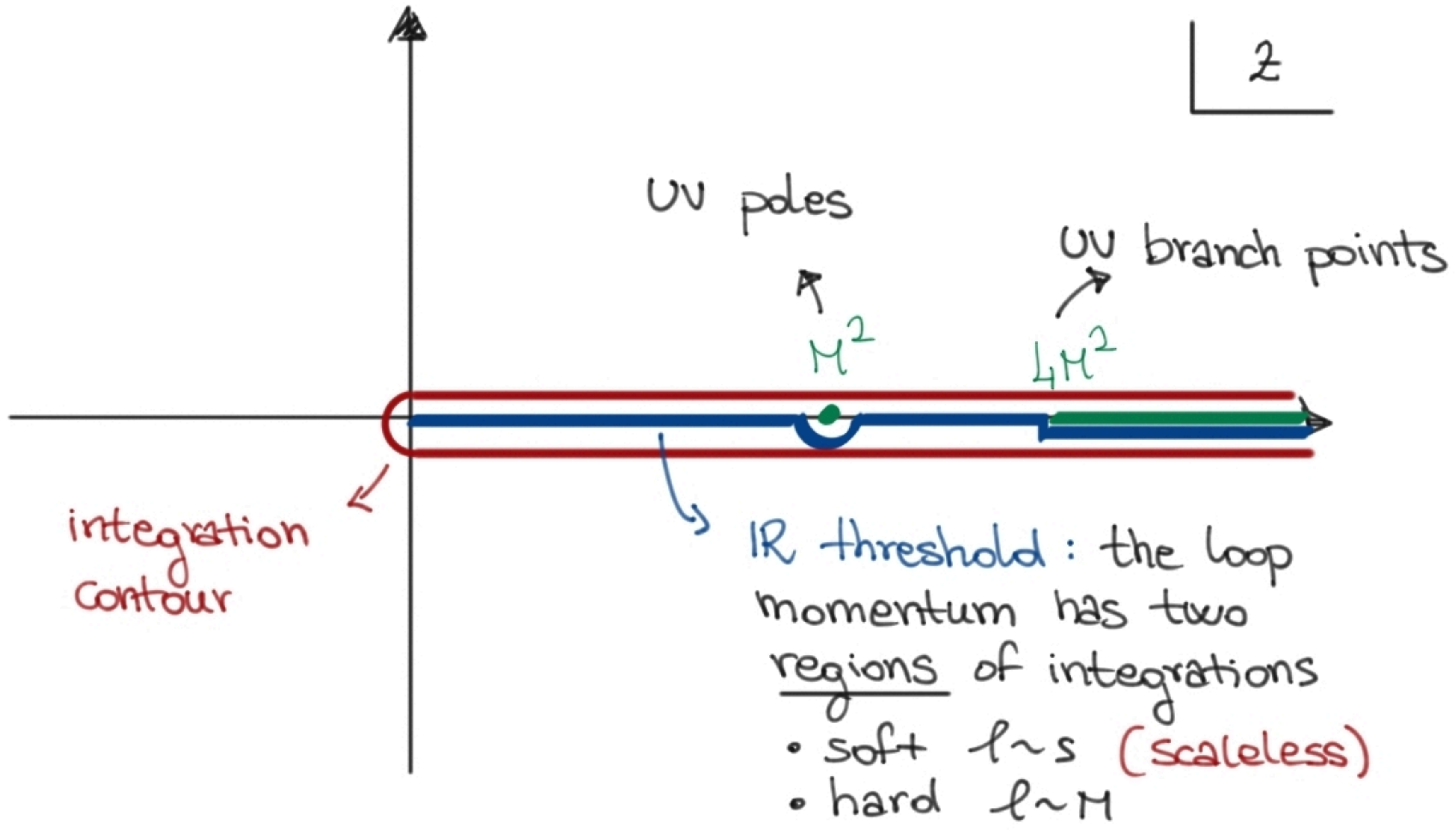


Matching in Dilatation Space

$$c_n \mathscr{P}_{\mathcal{O},n}(\vec{m}) = \frac{1}{2\pi i}\oint \frac{dz}{z^{n+1}} F_{\mathscr{L}_{\rm IR}}(\vec{m};z) = c_n^{\rm IR}\mathscr{P}_{\mathcal{O},n}(\vec{m}) - \sum_s \left(\frac{s}{M^2}\right)^{n+1} \underset{z=\frac{M^2}{s}}{\rm Res}\, F_{\mathscr{L}_{\rm UV}}(\vec{m};z) + \frac{1}{2\pi i}\int_0^\infty \frac{dz}{z^{n+1}}\, \underset{z}{\rm Disc}\, F_{\mathscr{L}_{\rm UV}}(\vec{m};z)$$

- The rational terms do not contribute to the matching.

In [Delle Rose, von Harling, Pomarol] it was argued (based on certain scaling of the Feynman integrals) that rational terms do not contribute to the matching (for 4-point interactions).



Matching in Dilatation Space

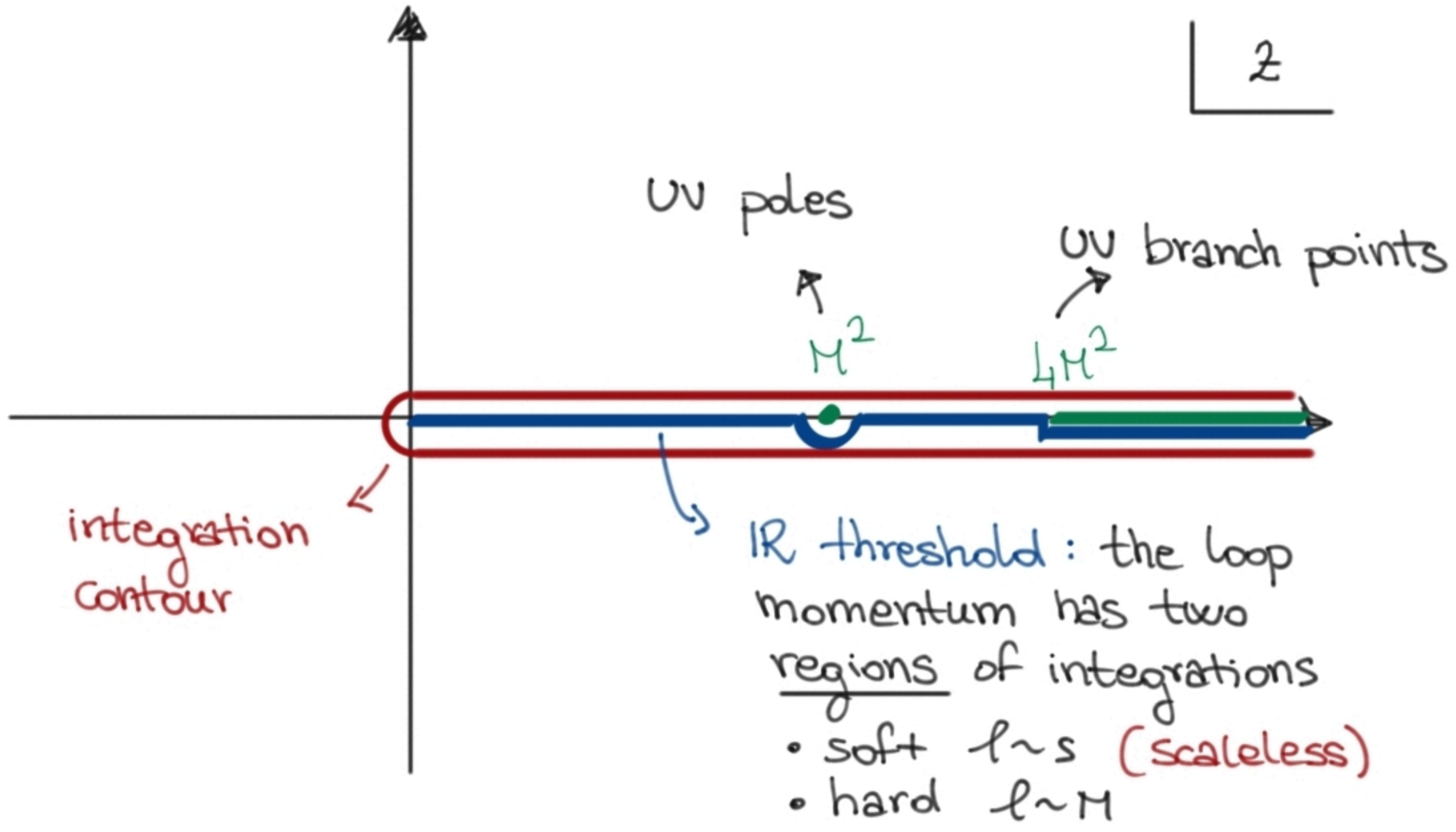
$$c_n \mathcal{P}_{\mathcal{O},n}(\vec{m}) = \frac{1}{2\pi i}\oint \frac{dz}{z^{n+1}} F_{\mathcal{L}_{\rm IR}}(\vec{m};z) = c_n^{\rm IR}\mathcal{P}_{\mathcal{O},n}(\vec{m}) - \sum_s \left(\frac{s}{M^2}\right)^{n+1} \underset{z=\frac{M^2}{s}}{\rm Res}\, F_{\mathcal{L}_{\rm UV}}(\vec{m};z) + \frac{1}{2\pi i}\int_0^\infty \frac{dz}{z^{n+1}}\, \underset{z}{\rm Disc}\, F_{\mathcal{L}_{\rm UV}}(\vec{m};z)$$

- The rational terms do not contribute to the matching.

  In [Delle Rose, von Harling, Pomarol] it was argued (based on certain scaling of the Feynman integrals) that rational terms do not contribute to the matching (for 4-point interactions).

- The projection onto an operator basis is trivial.

  $\mathcal{P}_{\mathcal{O},n}(\vec{m})$ are polynomials (or rational functions) and the projection can be performed numerically ($\sim$ solving a linear system)



Working Example - Scalar Theory

$$\mathscr{L} = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{\lambda}{4!}\phi^4 + \frac{1}{2}\partial_\mu \Phi \partial^\mu \Phi - \frac{1}{2}M^2\Phi^2 - \frac{g_3}{2!}\Phi\phi^2 - \frac{g_4}{3!}\Phi\phi^3$$

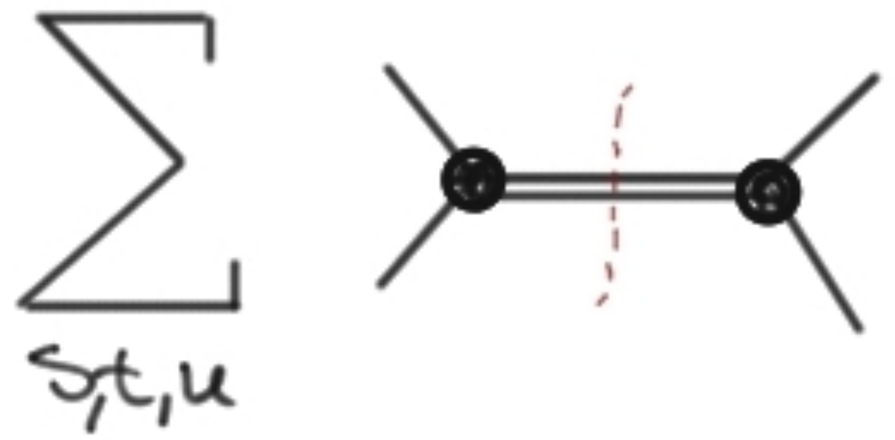
Working Example - Scalar Theory

$$\mathscr{L} = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{\lambda}{4!}\phi^4 + \frac{1}{2}\partial_\mu \Phi \partial^\mu \Phi - \frac{1}{2}M^2\Phi^2 - \frac{g_3}{2!}\Phi\phi^2 - \frac{g_4}{3!}\Phi\phi^3$$

Matching the $\partial^{2n}\phi^4$ interactions:

$$\mathscr{A}^{(0)}_{\mathrm{UV},4} = \lambda - \sum_{s,t,u} \frac{g_3^2}{s_{ij} - M^2}$$

$$\mathscr{A}^{(0)}_{\mathrm{IR},4} = \lambda + \sum_{n=0}^{\infty} g_3^2 \frac{1}{M^{2n+2}}(s^n + t^n + u^n)$$



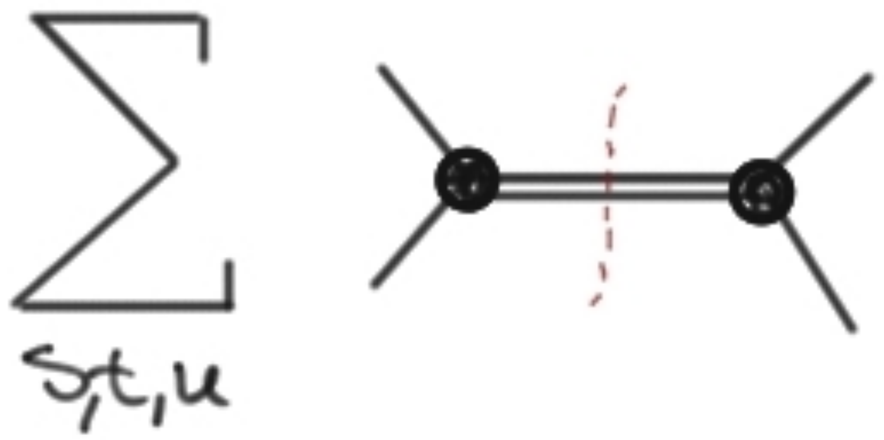
Working Example - Scalar Theory

$$\mathscr{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{\lambda}{4!}\phi^4 + \frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi - \frac{1}{2}M^2\Phi^2 - \frac{g_3}{2!}\Phi\phi^2 - \frac{g_4}{3!}\Phi\phi^3$$

Matching the $\partial^{2n}\phi^4$ interactions:

$$\mathscr{A}^{(0)}_{\mathrm{UV},4} = \lambda - \sum_{s,t,u}\frac{g_3^2}{s_{ij}-M^2}$$

$$\mathscr{A}^{(0)}_{\mathrm{IR},4} = \lambda + \sum_{n=0}^{\infty} g_3^2\frac{1}{M^{2n+2}}(s^n+t^n+u^n)$$



$$\mathscr{A}^{\mathrm{IR}} = \lambda - \sum_{s,t,u}\left(\frac{s_{ij}}{M^2}\right)^{n+1}\underset{z=\frac{M^2}{s_{ij}}}{\mathrm{Res}}\,\mathscr{A}^{\mathrm{UV}}(z) = \lambda + \sum_{s,t,u} g_3^2\frac{s_{ij}^n}{M^{2n+2}}$$

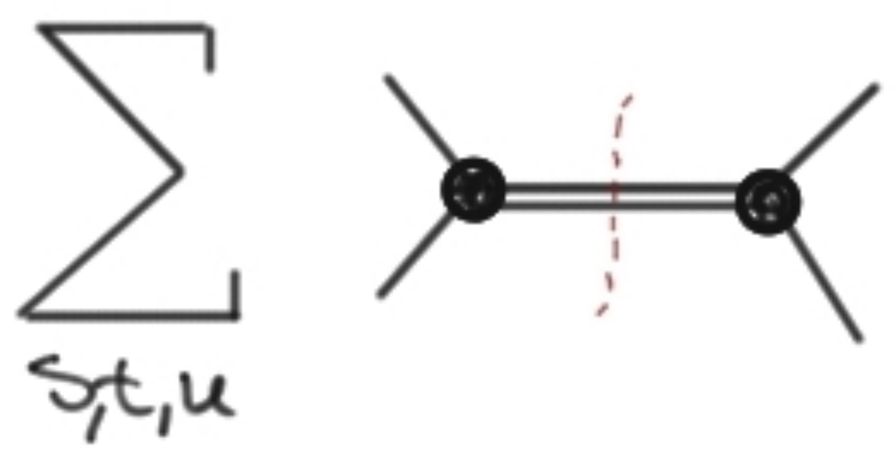
$$\underset{z=\frac{M^2}{s_{ij}}}{\mathrm{Res}}\,A(z) = -\left.\frac{A_L\times A_R}{s_{ij}}\right|_{z=\frac{M^2}{s_{ij}}}$$

Working Example - Scalar Theory

$$\mathscr{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{\lambda}{4!}\phi^4 + \frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi - \frac{1}{2}M^2\Phi^2 - \frac{g_3}{2!}\Phi\phi^2 - \frac{g_4}{3!}\Phi\phi^3$$

Matching the $\partial^{2n}\phi^4$ interactions:

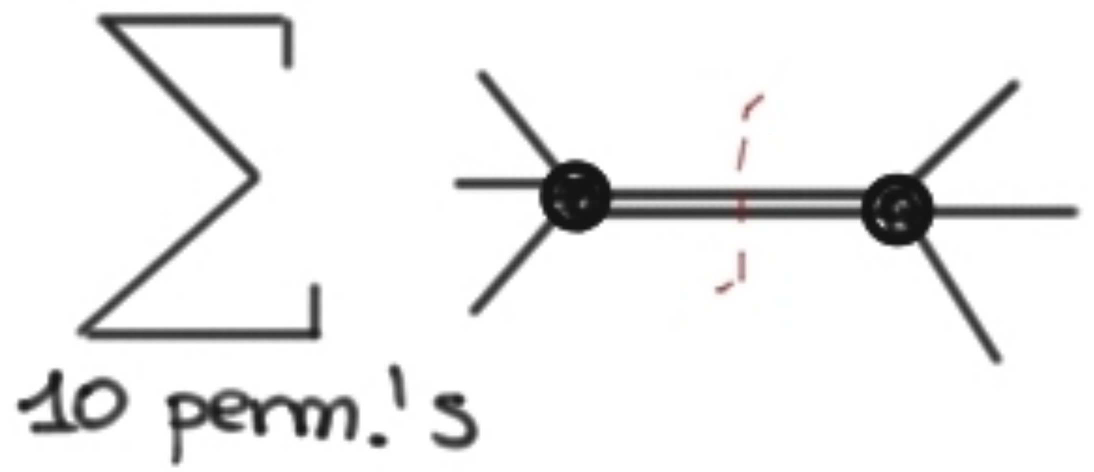
$$\mathscr{A}^{(0)}_{\text{UV},4} = \lambda - \sum_{s,t,u}\frac{g_3^2}{s_{ij} - M^2} \qquad \mathscr{A}^{(0)}_{\text{IR},4} = \lambda + \sum_{n=0}^{\infty} g_3^2 \frac{1}{M^{2n+2}}(s^n + t^n + u^n)$$



$$\mathscr{A}^{\text{IR}} = \lambda - \sum_{s,t,u}\left(\frac{s_{ij}}{M^2}\right)^{n+1} \quad \operatorname*{Res}_{z=\frac{M^2}{s_{ij}}} \mathscr{A}^{\text{UV}}(z) = \lambda + \sum_{s,t,u} g_3^2 \frac{s_{ij}^n}{M^{2n+2}}$$

$$\operatorname*{Res}_{z=\frac{M^2}{s_{ij}}} A(z) = -\left.\frac{A_L \times A_R}{s_{ij}}\right|_{z=\frac{M^2}{s_{ij}}}$$

Matching the $\partial^{2n}\phi^6$ interactions: the result is identical after substituting $g_3 \to g_4$ and $s_{ij} \to s_{ijk}$.



Working Example - Scalar Theory

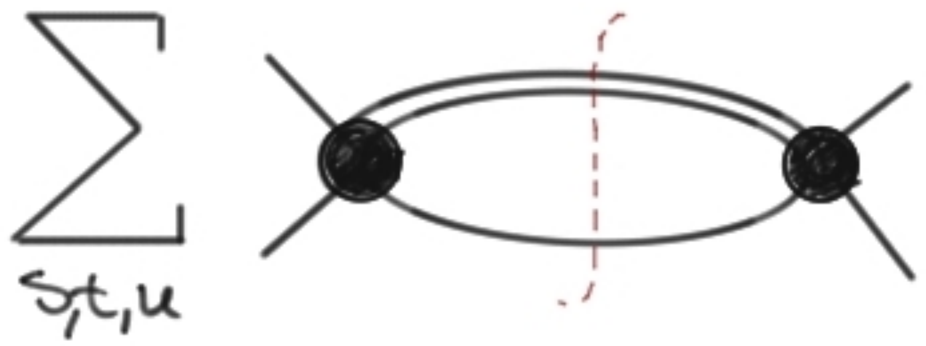
Working Example - Scalar Theory

Matching the $\partial^{2n}\phi^4$ interactions @ 1-loop

Working Example - Scalar Theory

Matching the $\partial^{2n}\phi^4$ interactions @ 1-loop

$$c_n g_4^2 \frac{s^n}{M^{2n}} = \frac{1}{2\pi i}\int_{\frac{M^2}{s}}^{\infty} \frac{dz}{z^{n+1}} \text{Disc}\, \hat{\mathscr{A}}^{(1)}_{\text{UV}}(\phi\phi \to \phi\phi)\Big|_{z=\frac{M^2}{s}}\Big|_{g_4^2} = \frac{g_4^2}{2\pi}\int_{\frac{M^2}{s}}^{\infty}\frac{dz}{z^{n+1}}\int d\text{LIPS}$$

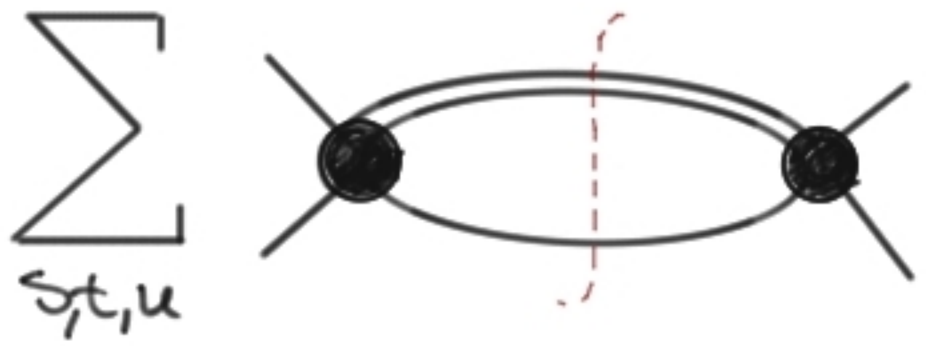


Working Example - Scalar Theory

Matching the $\partial^{2n}\phi^4$ interactions @ 1-loop

$$c_n g_4^2 \frac{s^n}{M^{2n}} = \frac{1}{2\pi i}\int_{\frac{M^2}{s}}^{\infty} \frac{dz}{z^{n+1}} \text{Disc}\, \hat{\mathcal{A}}^{(1)}_{\text{UV}}(\phi\phi \to \phi\phi)\Big|_{z=\frac{M^2}{s}} \Big|_{g_4^2} = \frac{g_4^2}{2\pi}\int_{\frac{M^2}{s}}^{\infty} \frac{dz}{z^{n+1}} \int d\text{LIPS}$$

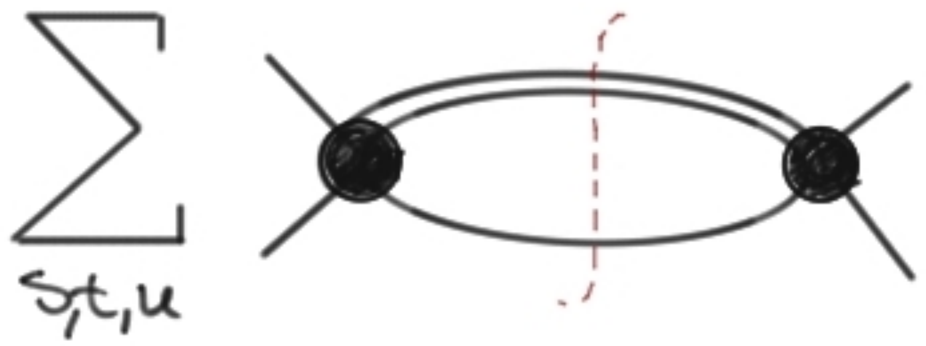
$$\Delta c_n = \frac{1}{16\pi^2}\frac{1}{n(n+1)} \qquad n \geq 1$$



Working Example - Scalar Theory

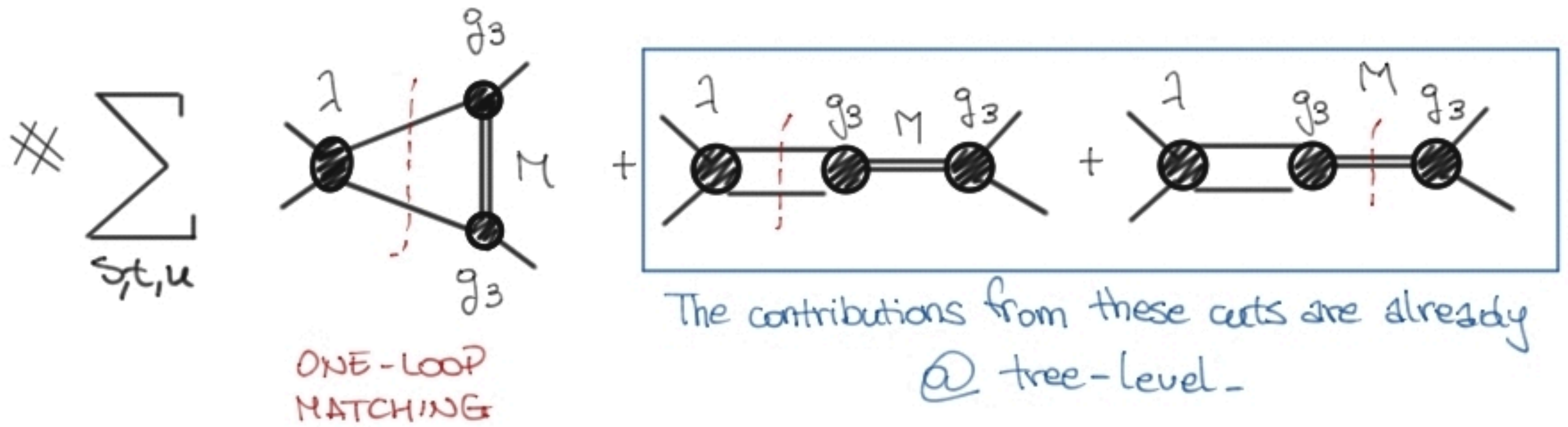
Matching the $\partial^{2n}\phi^4$ interactions @ 1-loop

$$c_n g_4^2 \frac{s^n}{M^{2n}} = \frac{1}{2\pi i}\int_{\frac{M^2}{s}}^{\infty} \frac{dz}{z^{n+1}} \text{Disc}\, \hat{\mathscr{A}}^{(1)}_{\text{UV}}(\phi\phi \to \phi\phi)\Big|_{z=\frac{M^2}{s}} \Big|_{g_4^2} = \frac{g_4^2}{2\pi}\int_{\frac{M^2}{s}}^{\infty} \frac{dz}{z^{n+1}} \int d\text{LIPS} \qquad \Delta c_n = \frac{1}{16\pi^2}\frac{1}{n(n+1)} \quad n \geq 1$$



We can now consider a more subtle contribution @ $\mathcal{O}(\lambda g_3^2)$

$$\frac{\lambda g_3^2}{2\pi}\int_0^{\infty}\frac{dz}{z^{n+1}}\int d\text{LIPS}\left(-\frac{2}{(l-\sqrt{z}p_3)^2 - M^2}\right) + \frac{\lambda g_3^2}{2\pi}\int_0^{\infty}\frac{dz}{z^{n+1}}\left(-\frac{1}{zs - M^2 + i\epsilon}\right)\int d\text{LIPS} + +\lambda g_3^2 \frac{s^n}{M^{2n+2}} B(M^2 - i\epsilon; 0,0)$$

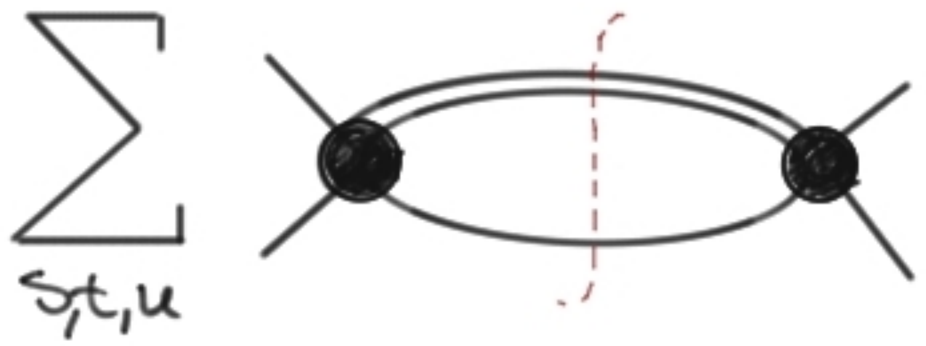


Working Example - Scalar Theory

Matching the $\partial^{2n}\phi^4$ interactions @ 1-loop

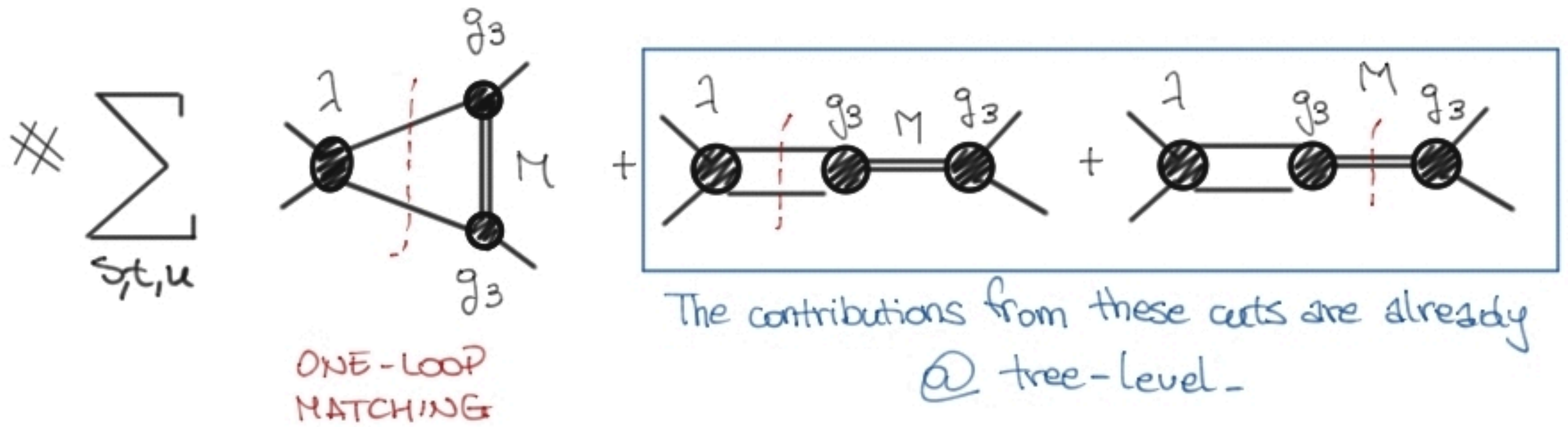
$$c_n g_4^2 \frac{s^n}{M^{2n}} = \frac{1}{2\pi i}\int_{\frac{M^2}{s}}^{\infty}\frac{dz}{z^{n+1}}\mathrm{Disc}\,\hat{\mathscr{A}}^{(1)}_{\mathrm{UV}}(\phi\phi\to\phi\phi)\Big|_{z=\frac{M^2}{s}}\Big|_{g_4^2} = \frac{g_4^2}{2\pi}\int_{\frac{M^2}{s}}^{\infty}\frac{dz}{z^{n+1}}\int d\mathrm{LIPS}$$

$$\Delta c_n = \frac{1}{16\pi^2}\frac{1}{n(n+1)} \qquad n \geq 1$$



We can now consider a more subtle contribution @ $\mathcal{O}(\lambda g_3^2)$

$$\frac{\lambda g_3^2}{2\pi}\int_0^{\infty}\frac{dz}{z^{n+1}}\int d\mathrm{LIPS}\left(-\frac{2}{(l-\sqrt{z}p_3)^2-M^2}\right) + \frac{\lambda g_3^2}{2\pi}\int_0^{\infty}\frac{dz}{z^{n+1}}\left(-\frac{1}{zs-M^2+i\epsilon}\right)\int d\mathrm{LIPS} + +\lambda g_3^2\frac{s^n}{M^{2n+2}}B(M^2-i\epsilon;0,0)$$

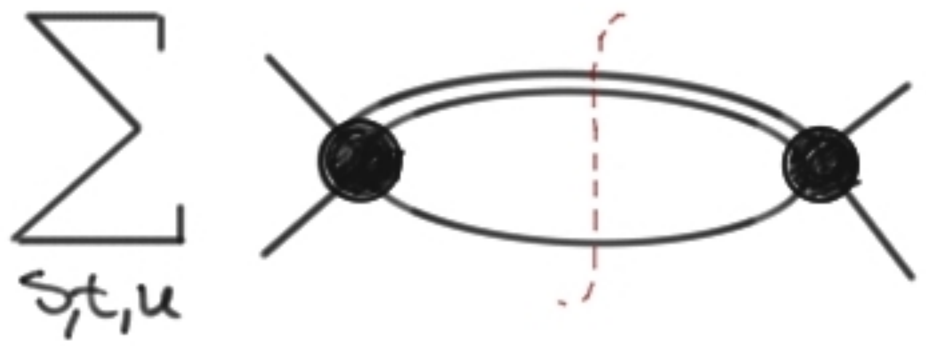


Working Example - Scalar Theory

Matching the $\partial^{2n}\phi^4$ interactions @ 1-loop

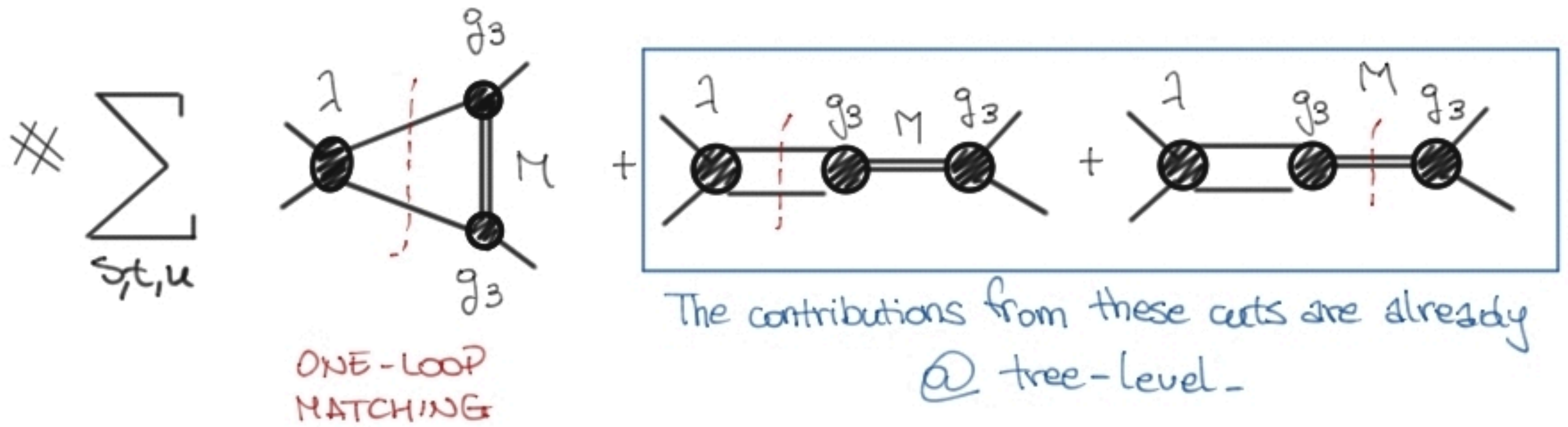
$$c_n g_4^2 \frac{s^n}{M^{2n}} = \frac{1}{2\pi i}\int_{\frac{M^2}{s}}^{\infty}\frac{dz}{z^{n+1}}\underset{z=\frac{M^2}{s}}{\text{Disc}}\,\hat{\mathcal{A}}^{(1)}_{\text{UV}}(\phi\phi\to\phi\phi)\Big|_{g_4^2} = \frac{g_4^2}{2\pi}\int_{\frac{M^2}{s}}^{\infty}\frac{dz}{z^{n+1}}\int d\text{LIPS}$$

$$\Delta c_n = \frac{1}{16\pi^2}\frac{1}{n(n+1)} \qquad n \geq 1$$



We can now consider a more subtle contribution @ $\mathcal{O}(\lambda g_3^2)$

$$\frac{\lambda g_3^2}{2\pi}\int_0^{\infty}\frac{dz}{z^{n+1}}\int d\text{LIPS}\left(-\frac{2}{(l-\sqrt{z}p_3)^2-M^2}\right) + \frac{\lambda g_3^2}{2\pi}\int_0^{\infty}\frac{dz}{z^{n+1}}\left(-\frac{1}{zs-M^2+i\epsilon}\right)\int d\text{LIPS} + +\lambda g_3^2\frac{s^n}{M^{2n+2}}B(M^2-i\epsilon;0,0)$$



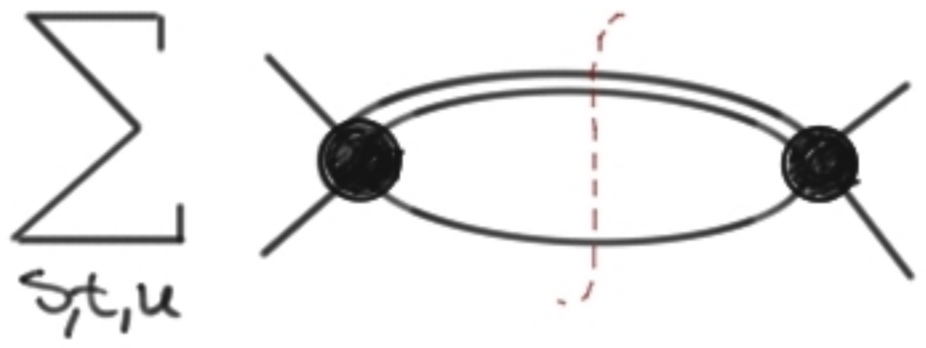
$$\Delta c_n = \lambda g_3^2\,\frac{(-1)^{n+1}}{16\pi^2(n+1)}\left(\frac{1}{\bar{\epsilon}} + H_{n+1} + \mathcal{O}(\epsilon)\right)$$

Working Example - Scalar Theory

Matching the $\partial^{2n}\phi^4$ interactions @ 1-loop

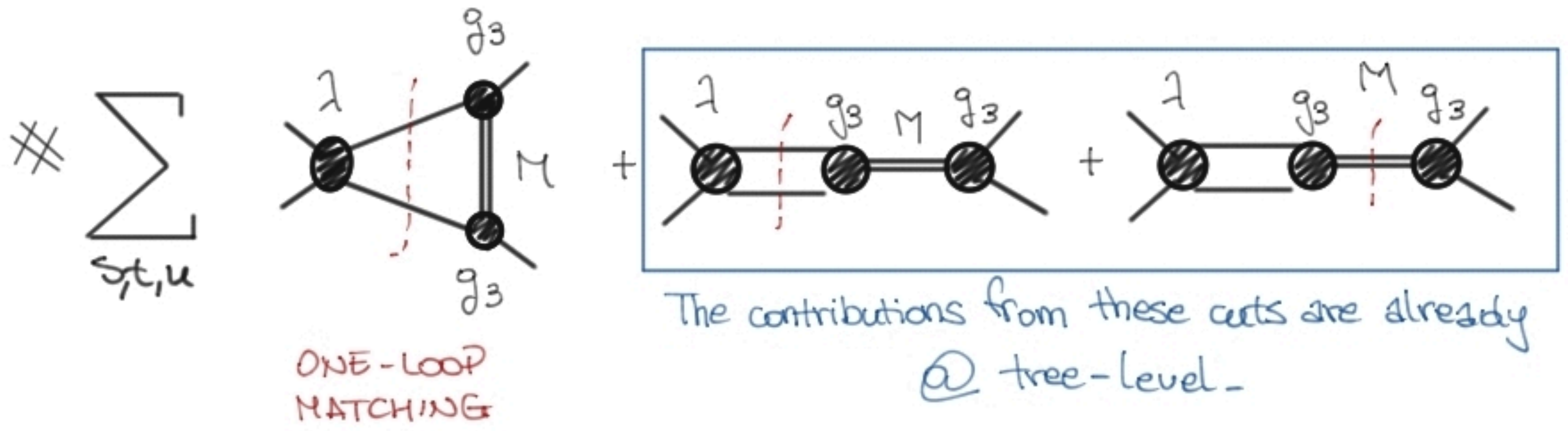
$$c_n g_4^2 \frac{s^n}{M^{2n}} = \frac{1}{2\pi i}\int_{\frac{M^2}{s}}^{\infty} \frac{dz}{z^{n+1}} \text{Disc}\,\hat{\mathscr{A}}^{(1)}_{\text{UV}}(\phi\phi \to \phi\phi)\Big|_{z=\frac{M^2}{s}}\Big|_{g_4^2} = \frac{g_4^2}{2\pi}\int_{\frac{M^2}{s}}^{\infty}\frac{dz}{z^{n+1}}\int d\text{LIPS}$$

$$\Delta c_n = \frac{1}{16\pi^2}\frac{1}{n(n+1)} \qquad n \geq 1$$



We can now consider a more subtle contribution @ $\mathcal{O}(\lambda g_3^2)$

$$\frac{\lambda g_3^2}{2\pi}\int_0^{\infty}\frac{dz}{z^{n+1}}\int d\text{LIPS}\left(-\frac{2}{(l-\sqrt{z}p_3)^2 - M^2}\right) + \frac{\lambda g_3^2}{2\pi}\int_0^{\infty}\frac{dz}{z^{n+1}}\left(-\frac{1}{zs - M^2 + i\epsilon}\right)\int d\text{LIPS} + +\lambda g_3^2 \frac{s^n}{M^{2n+2}} B(M^2 - i\epsilon; 0,0)$$



$$\Delta c_n = \lambda g_3^2 \frac{(-1)^{n+1}}{16\pi^2(n+1)}\left(\frac{1}{\bar{\epsilon}} + H_{n+1} + \mathcal{O}(\epsilon)\right)$$

Reminder: we have to subtract the IR loops (which are scaleless). Then the IR divergences of the UV completions are traded for UV divergences of the EFT.

Outlook

Outlook

- Positivity bounds for higher-point contact terms.

Outlook

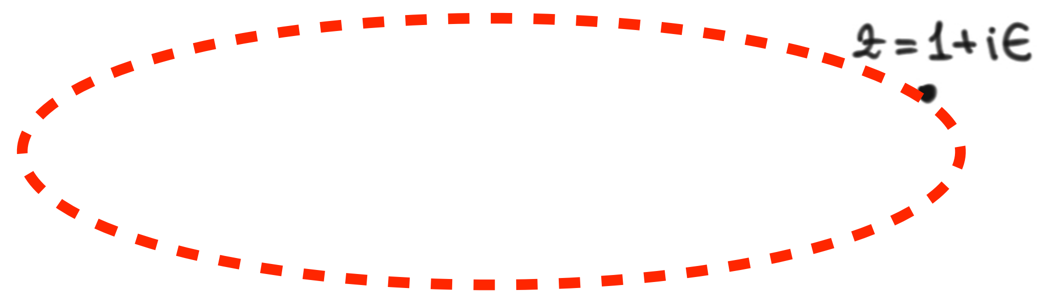
- Positivity bounds for higher-point contact terms.
- Generic constraints from the UV to the IR:
  - Structural properties of UV completions, e.g. supersymmetry in the UV
  - Magic zeros as selection rules?

Outlook

- Positivity bounds for higher-point contact terms.
- Generic constraints from the UV to the IR:
  - Structural properties of UV completions, e.g. supersymmetry in the UV
  - Magic zeros as selection rules?
- Efficiency improvements and software implementation (?)
  - Systematic approaches to dLIPS integration and conciliation with region expansion.

Thank you!

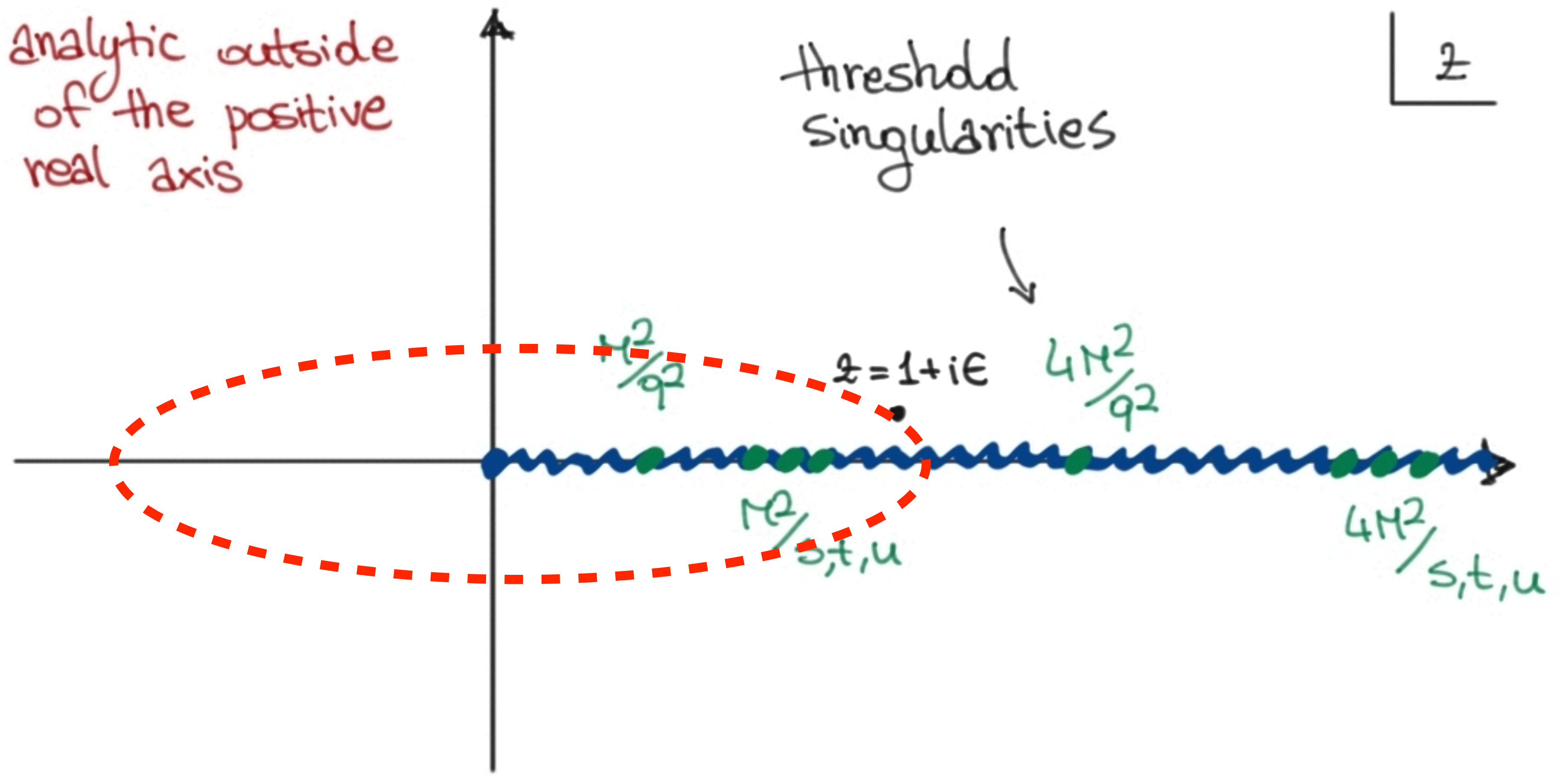
Anomalous dimension from the S-matrix



Anomalous dimension from the S-matrix

- *We can perform a complex rotation in* $z = e^{i(\pi - 2\epsilon)}$:

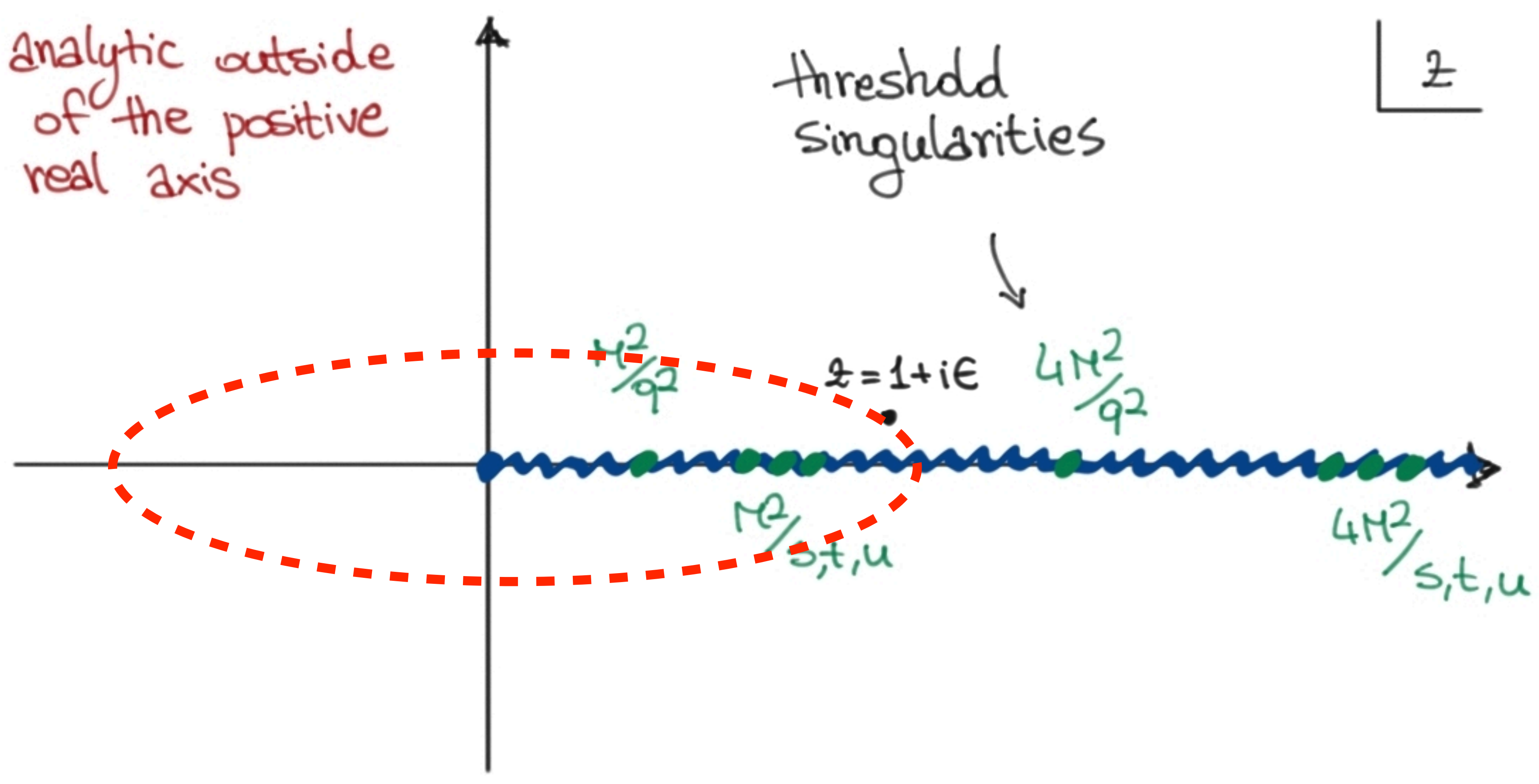
$$F_{\mathcal{O}}(\vec{m}; e^{i\pi}) = e^{i\pi D} F_{\mathcal{O}}(\vec{m}; 1 + i\epsilon) = F_{\mathcal{O}}(\vec{m}; 1 - i\epsilon) = F^*_{\mathcal{O}}(\vec{m}; 1 + i\epsilon), \text{ where } D = \sum_i p_i^\mu \frac{\partial}{\partial p_i^\mu}$$



Anomalous dimension from the S-matrix

- *We can perform a complex rotation in* $z = e^{i(\pi - 2\epsilon)}$*:*

$$F_{\mathcal{O}}(\vec{m}; e^{i\pi}) = e^{i\pi D} F_{\mathcal{O}}(\vec{m}; 1 + i\epsilon) = F_{\mathcal{O}}(\vec{m}; 1 - i\epsilon) = F^*_{\mathcal{O}}(\vec{m}; 1 + i\epsilon), \text{ where } D = \sum_i p_i^\mu \frac{\partial}{\partial p_i^\mu}$$



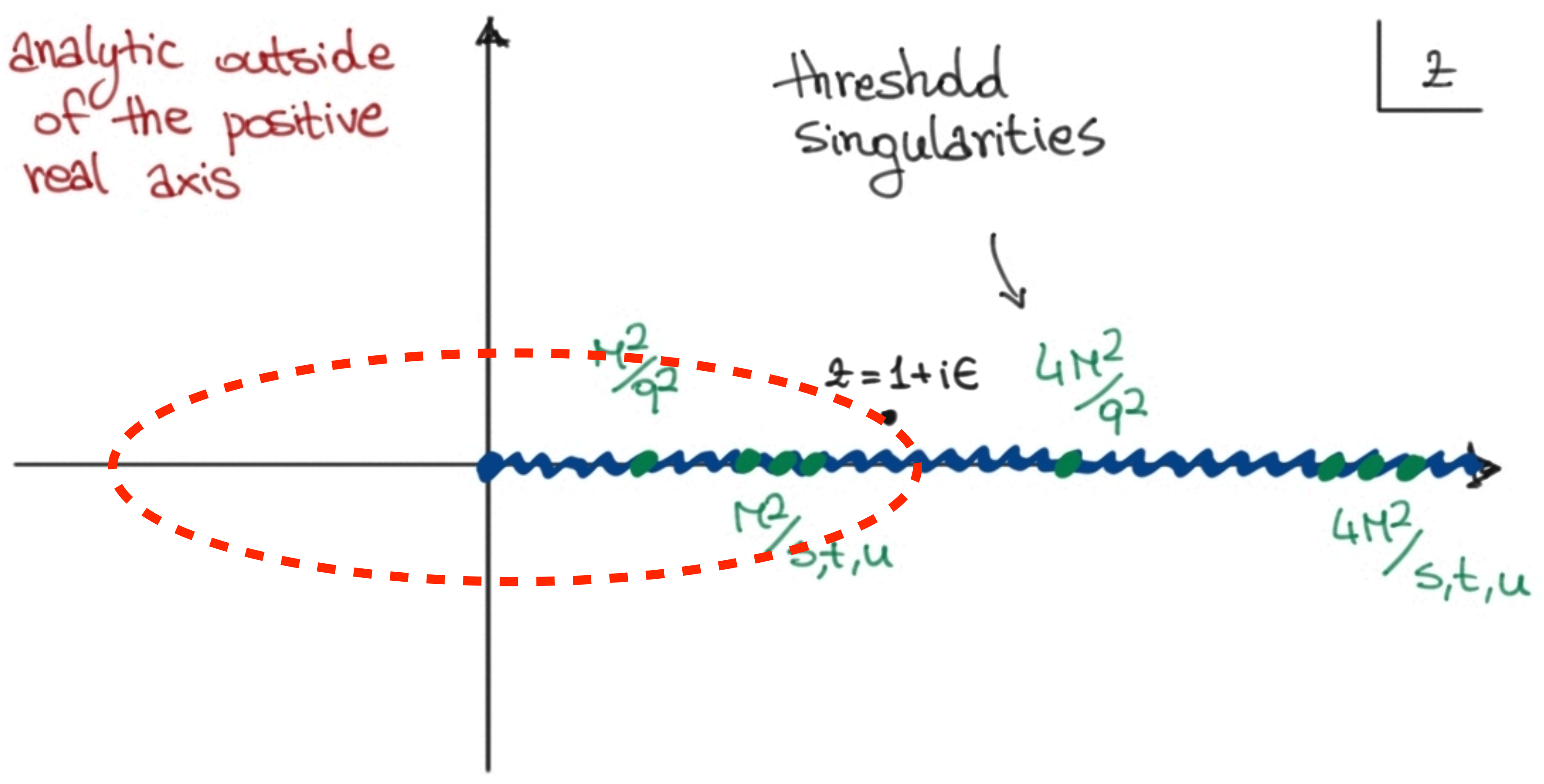
- $F_{\mathcal{O}}(\vec{m}; 1 \pm i\epsilon)$ *are related by unitarity:*

$$F_{\mathcal{O}}(\vec{m}; 1 + i\epsilon) = e^{-i\pi D} F_{\mathcal{O}}(\vec{m}; 1 - i\epsilon) = \sum_n {}_{\text{out}}\langle \psi_{\vec{m}} | \psi_{\vec{n}} \rangle_{\text{in}} \, F_{\mathcal{O}}(\vec{n}; 1 - i\epsilon)$$

Anomalous dimension from the S-matrix

- *We can perform a complex rotation in* $z = e^{i(\pi - 2\epsilon)}$:

$$F_{\mathcal{O}}(\overrightarrow{m}; e^{i\pi}) = e^{i\pi D} F_{\mathcal{O}}(\overrightarrow{m}; 1 + i\epsilon) = F_{\mathcal{O}}(\overrightarrow{m}; 1 - i\epsilon) = F^*_{\mathcal{O}}(\overrightarrow{m}; 1 + i\epsilon), \text{ where } D = \sum_i p_i^\mu \frac{\partial}{\partial p_i^\mu}$$



- $F_{\mathcal{O}}(\overrightarrow{m}; 1 \pm i\epsilon)$ *are related by unitarity:*

$$F_{\mathcal{O}}(\overrightarrow{m}; 1 + i\epsilon) = e^{-i\pi D} F_{\mathcal{O}}(\overrightarrow{m}; 1 - i\epsilon) = \sum_n {}_{\text{out}}\langle \psi_{\overrightarrow{m}} | \psi_{\vec{n}} \rangle_{\text{in}} \, F_{\mathcal{O}}(\vec{n}; 1 - i\epsilon)$$

- *The mass dimension of the form factor is* $\dim \mathcal{O} - m$:

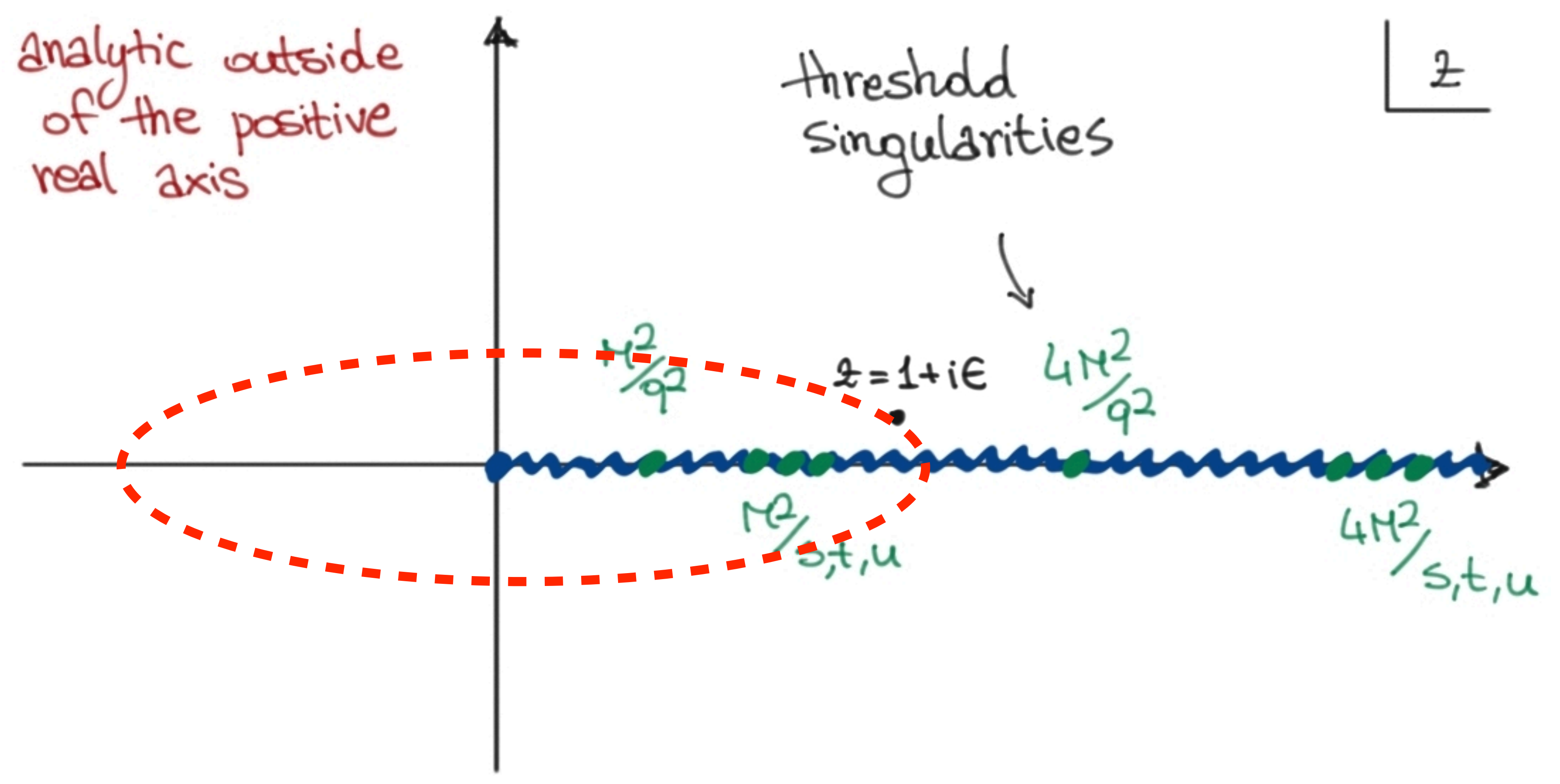
Homogeneity in the mass dimension tells us that we can rewrite D *in terms of the renormalisation scale:*

$$D = \dim \mathcal{O} - \#_m - \sum_{m_i} m_i \frac{\partial}{\partial m_i} - \sum_{g_j} [g_j] \, g_j \frac{\partial}{\partial g_j} - \mu \frac{\partial}{\partial \mu}$$

Anomalous dimension from the S-matrix

- *We can perform a complex rotation in* $z = e^{i(\pi - 2\epsilon)}$:

$$F_{\mathcal{O}}(\vec{m}; e^{i\pi}) = e^{i\pi D} F_{\mathcal{O}}(\vec{m}; 1 + i\epsilon) = F_{\mathcal{O}}(\vec{m}; 1 - i\epsilon) = F^{*}_{\mathcal{O}}(\vec{m}; 1 + i\epsilon), \text{ where } D = \sum_i p_i^{\mu} \frac{\partial}{\partial p_i^{\mu}}$$



- $F_{\mathcal{O}}(\vec{m}; 1 \pm i\epsilon)$ *are related by unitarity:*

$$F_{\mathcal{O}}(\vec{m}; 1 + i\epsilon) = e^{-i\pi D} F_{\mathcal{O}}(\vec{m}; 1 - i\epsilon) = \sum_n {}_{\text{out}}\langle \psi_{\vec{m}} | \psi_{\vec{n}} \rangle_{\text{in}} \, F_{\mathcal{O}}(\vec{n}; 1 - i\epsilon)$$

- *The mass dimension of the form factor is* $\dim \mathcal{O} - m$:

Homogeneity in the mass dimension tells us that we can rewrite D in terms of the renormalisation scale:

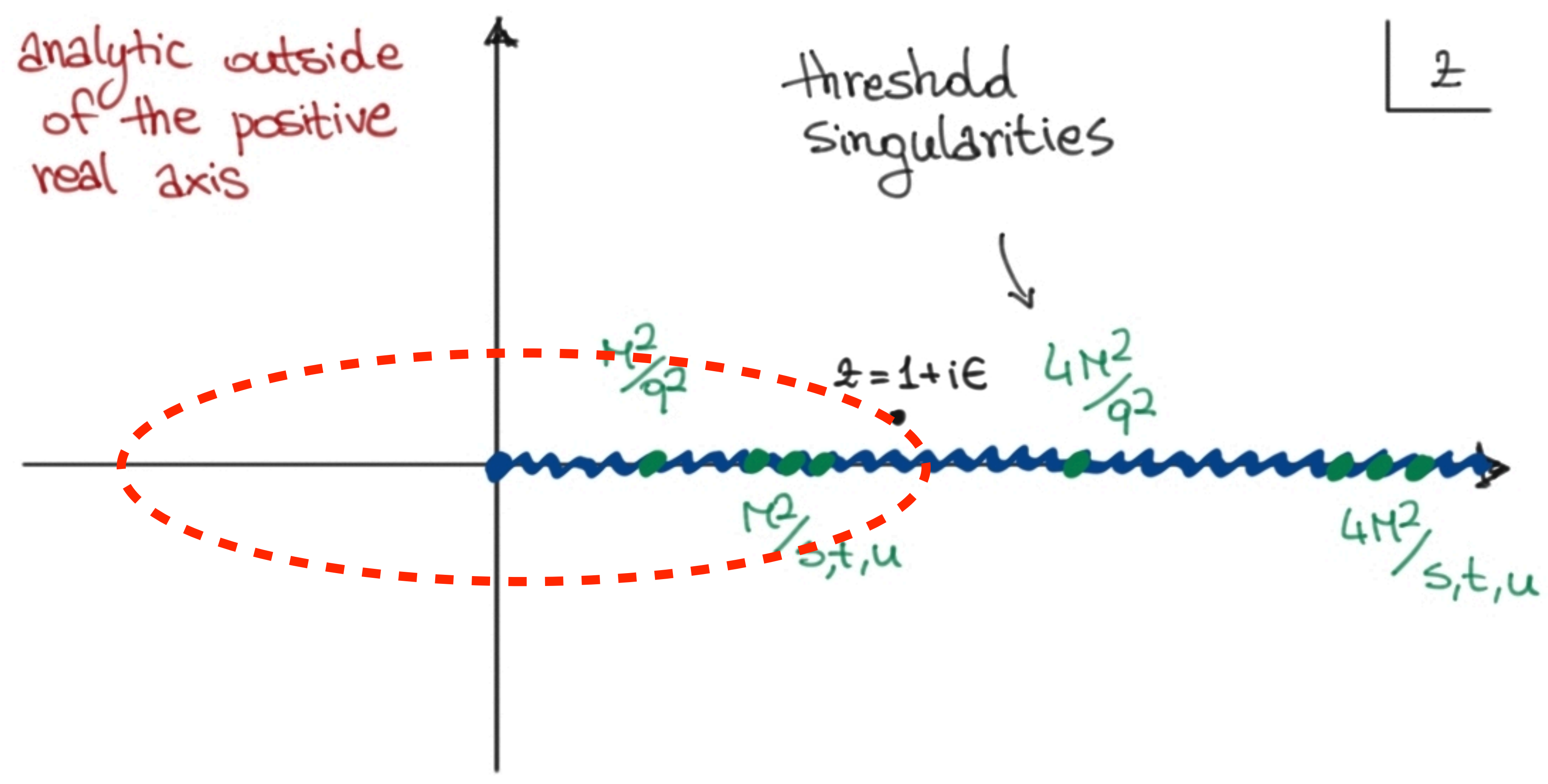
$$D = \dim\mathcal{O} - \#_m - \sum_{m_i} m_i \frac{\partial}{\partial m_i} - \sum_{g_j} [g_j]\, g_j \frac{\partial}{\partial g_j} - \mu \frac{\partial}{\partial \mu}$$

The Callan-Symanzik equation gives the anomalous dimensions!

Anomalous dimension from the S-matrix

- *We can perform a complex rotation in $z = e^{i(\pi-2\epsilon)}$:*

$$F_{\mathcal{O}}(\vec{m}; e^{i\pi}) = e^{i\pi D} F_{\mathcal{O}}(\vec{m}; 1+i\epsilon) = F_{\mathcal{O}}(\vec{m}; 1-i\epsilon) = F^*_{\mathcal{O}}(\vec{m}; 1+i\epsilon), \text{ where } D = \sum_i p_i^\mu \frac{\partial}{\partial p_i^\mu}$$



- *$F_{\mathcal{O}}(\vec{m}; 1 \pm i\epsilon)$ are related by unitarity:*

$$F_{\mathcal{O}}(\vec{m}; 1+i\epsilon) = e^{-i\pi D} F_{\mathcal{O}}(\vec{m}; 1-i\epsilon) = \sum_n {}_{\text{out}}\langle \psi_{\vec{m}} | \psi_{\vec{n}} \rangle_{\text{in}} F_{\mathcal{O}}(\vec{n}; 1-i\epsilon)$$

- *The mass dimension of the form factor is $\dim \mathcal{O} - m$:*

Homogeneity in the mass dimension tells us that we can rewrite D in terms of the renormalisation scale:

$$D = \dim\mathcal{O} - \#_m - \sum_{m_i} m_i \frac{\partial}{\partial m_i} - \sum_{g_j} [g_j]\, g_j \frac{\partial}{\partial g_j} - \mu \frac{\partial}{\partial \mu}$$

The Callan-Symanzik equation gives the anomalous dimensions!

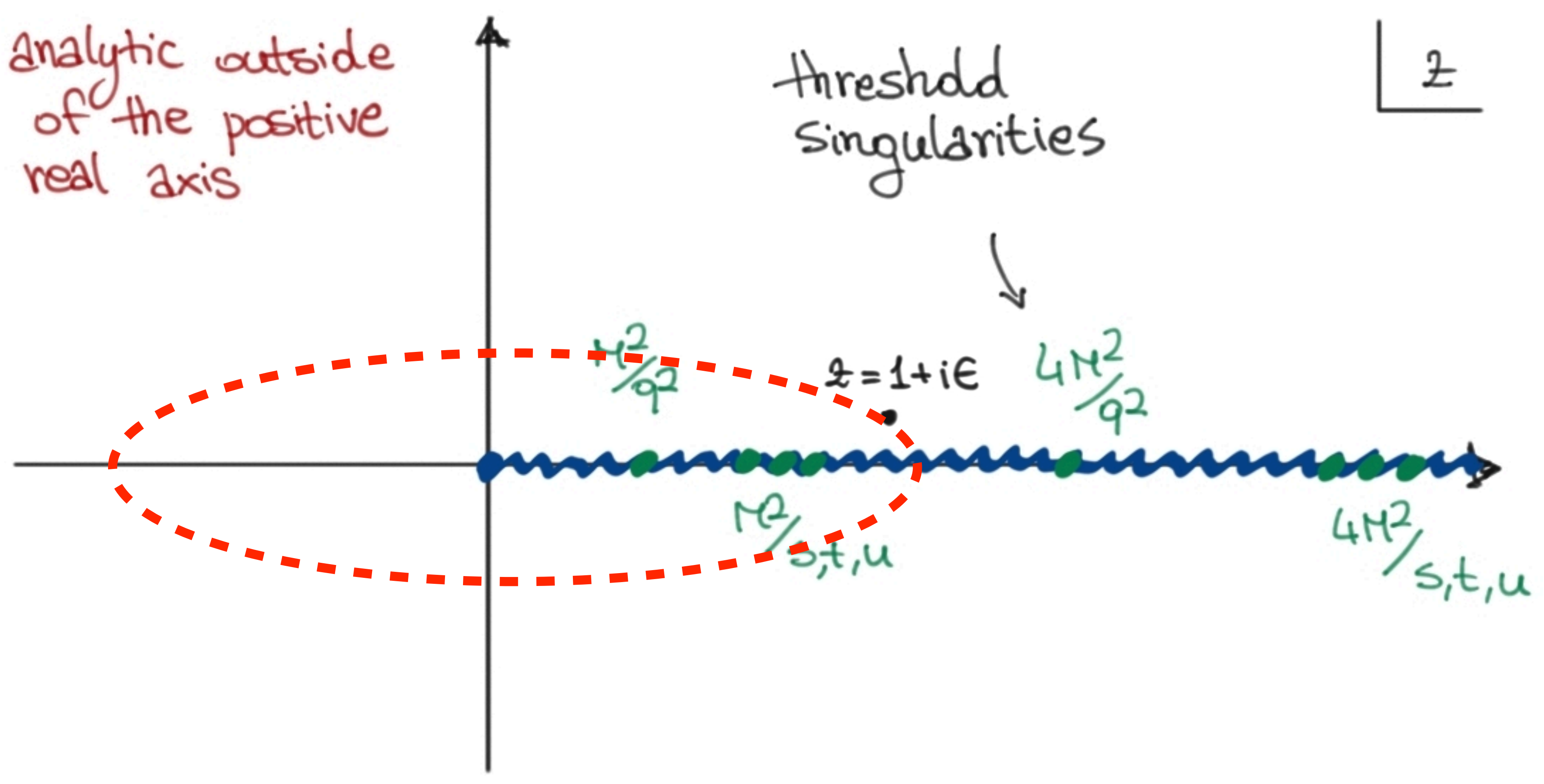
- *Refined version of the Decoupling Subtraction scheme:*

Massive modes contribute to the anomalous dimensions if the kinematics is above threshold!

Anomalous dimension from the S-matrix

- *We can perform a complex rotation in* $z = e^{i(\pi - 2\epsilon)}$:

$$F_{\mathcal{O}}(\vec{m}; e^{i\pi}) = e^{i\pi D} F_{\mathcal{O}}(\vec{m}; 1 + i\epsilon) = F_{\mathcal{O}}(\vec{m}; 1 - i\epsilon) = F^*_{\mathcal{O}}(\vec{m}; 1 + i\epsilon), \text{ where } D = \sum_i p_i^\mu \frac{\partial}{\partial p_i^\mu}$$



- $F_{\mathcal{O}}(\vec{m}; 1 \pm i\epsilon)$ *are related by unitarity:*

$$F_{\mathcal{O}}(\vec{m}; 1 + i\epsilon) = e^{-i\pi D} F_{\mathcal{O}}(\vec{m}; 1 - i\epsilon) = \sum_n {}_{\text{out}}\langle \psi_{\vec{m}} | \psi_{\vec{n}} \rangle_{\text{in}} F_{\mathcal{O}}(\vec{n}; 1 - i\epsilon)$$

- *The mass dimension of the form factor is* $\dim \mathcal{O} - m$:

Homogeneity in the mass dimension tells us that we can rewrite D *in terms of the renormalisation scale:*

$$D = \dim\mathcal{O} - \#_m - \sum_{m_i} m_i \frac{\partial}{\partial m_i} - \sum_{g_j} [g_j]\, g_j \frac{\partial}{\partial g_j} - \mu \frac{\partial}{\partial \mu}$$

The Callan-Symanzik equation gives the anomalous dimensions!

- *Refined version of the Decoupling Subtraction scheme:*

Massive modes contribute to the anomalous dimensions if the kinematics is above threshold!

The decoupling of heavy modes is manifest in the renormalisation of the couplings.