Gegenbauer naturalness

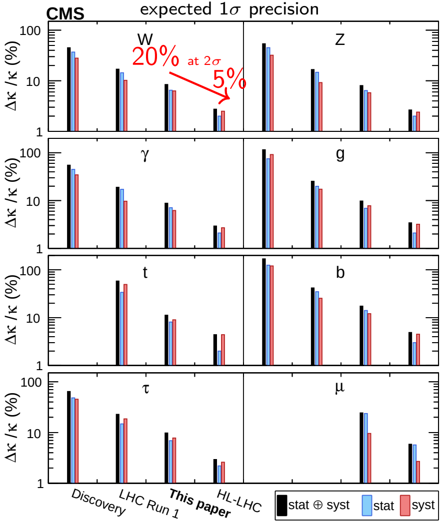
Gauthier Durieux
(CERN)

Gegenbauer Goldstones, JHEP 01 (2022) 076, [2110.06941]
Gegenbauer's Twin, JHEP 05 (2022) 140, [2202.01228]
Charting the Higgs self-coupling boundaries, JHEP 12 (2022) 148, [2209.00666]

with Matthew McCullough and Ennio Salvioni

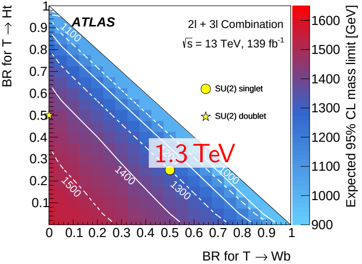
NPKI – Busan – 7 June 2023

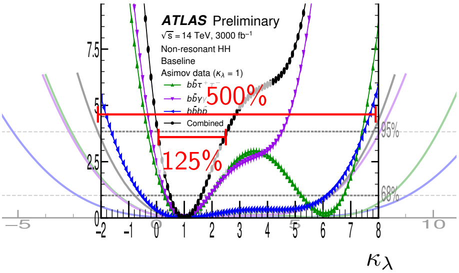
LHC data across decades



[CMS-HIG-22-001]

[EXOT-2018-58]





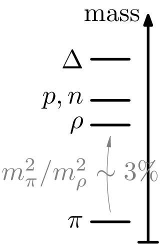
[HDBS-2022-03]
[ATL-PHYS-PUB-2022-053]

Naturalness exacerbated

$$\frac{m_h^2}{M_X^2} \sim \%$$

$$\delta\kappa_V \sim \%$$

Composite Higgs



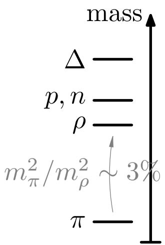
realise the Higgs as the
pseudo-Nambu-Goldstone boson (pNGB)
of a new strong sector

e.g. global SO(5) $\rightarrow$ SO(4) spontaneous breaking
at scale f

small mass obtained from the
explicit breaking of SO(5)
by e.g. the SM

small $\delta\kappa_V$ implies $v^2/f^2 \ll 1$
and requires fine-tuning in minimal models

Composite Higgs



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e.g. global SO(5) $\to$ SO(4) spontaneous breaking
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small m … the
e …)
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small(ish) m_h^2/M_X^2!

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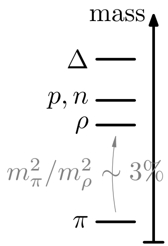
Composite Higgs



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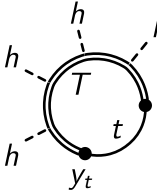
s… 1
and requ… models

no small $\delta\kappa_V$ or v/f!

Minimal composite Higgs

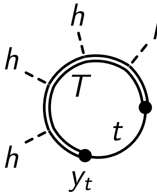
$V(h) \sim$ 

$+ \cdots \sim \boldsymbol{\kappa}\, \frac{y_t^2 N_c}{16\pi^2} f^2 M_T^2 \left(-\sin^2\frac{h}{f} + \boldsymbol{\delta}\, \sin^4\frac{h}{f} \right)$

$\rightarrow \frac{v^2}{f^2} = \sin^2\frac{\langle h\rangle}{f} = \frac{1}{2\delta}$ vs. $-\delta\kappa_V \simeq \frac{v^2}{2f^2} \lesssim 5\%$

$\rightarrow \frac{m_h^2}{M_T^2} = \kappa\, \underbrace{\frac{4y_t^2 N_c}{16\pi^2}\left(1 - \frac{1}{2\delta}\right)}_{\sim 7\%}$ vs. $M_T \gtrsim 1.5$ TeV

Minimal composite Higgs

$V(h) \sim$ 

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$$\rightarrow \frac{v^2}{f^2} = \sin^2\frac{\langle h\rangle}{f} = \frac{1}{2\delta}$$

$1/\delta \lesssim 0.20$

$-\delta\kappa_V \simeq \frac{v^2}{2f^2} \lesssim 5\%$

$$\rightarrow \frac{m_h^2}{M_T^2} = \kappa\, \underbrace{\frac{4y_t^2 N_c}{16\pi^2}\left(1 - \frac{1}{2\delta}\right)}_{\sim 7\%}$$

$\kappa \lesssim 0.10$

$M_T \gtrsim 1.5$ TeV

Few percent fine-tuning wrt $\delta \lesssim 1$, $\kappa \simeq 1$ expectation

Naturally small vev

radiatively stable deepest minimum
close to the origin

in the low-energy pNGB potential

Radiatively stable SO($N+1$) $\to$ SO(N) potentials

$\vec{\phi} \equiv (\frac{\vec{h}}{h} \sin\frac{h}{f},\ \cos\frac{h}{f}),\ \ h \equiv |\vec{h}|$

Linear one-loop correction to $V(\frac{h}{f})$:

$$\frac{\Lambda^2}{32\pi^2 f^2}\left(\underline{V'' + (N-1)\cot\frac{h}{f}\ V'}\right)$$

Radiative stability at one-loop and linear order order if $\propto V$

Differential equation of Gegenbauer polynomials

$$V(\tfrac{h}{f}) \propto G_n^{(N-1)/2}(\cos\tfrac{h}{f})$$

Radiatively stable SO($N+1$) → SO(N) potentials

Explicit SO($N+1$) → SO(N) breaking by an irrep spurion K:

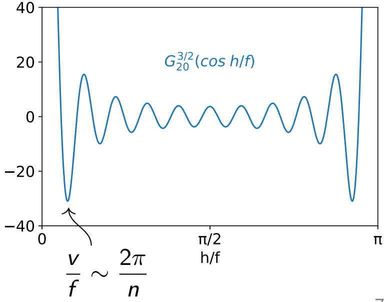
$$K^{i_1 \dots i_n}\, \phi_{i_1} \cdots \phi_{i_n} \quad \text{(symmetric traceless)}$$

$\vec{\phi} \equiv (\frac{\vec{h}}{h} \sin \frac{h}{f},\ \cos \frac{h}{f}),\ \ h \equiv |\vec{h}|$

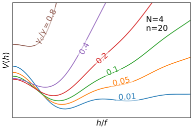
No other invariant, linear in K, can be constructed,
so all-loop linear renormalisation can only be multiplicative.

Obtain Gegenbauer polynomials:

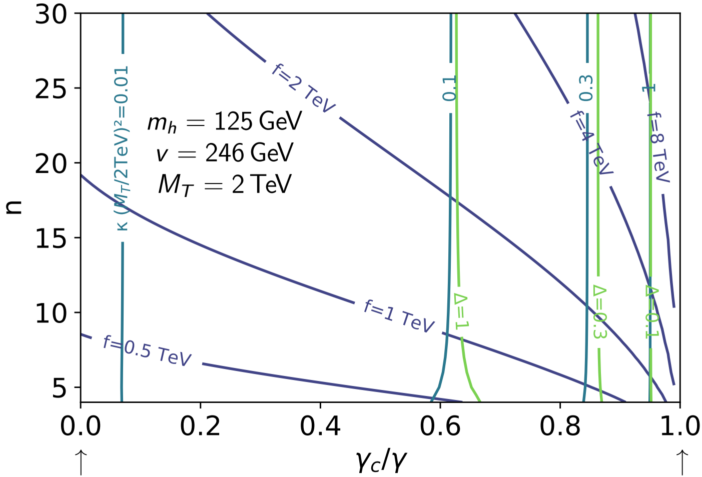
$$K^{i_1 \dots i_n}\, \phi_{i_1} \cdots \phi_{i_n} \;\propto\; G_n^{(N-1)/2}(\cos \tfrac{h}{f})$$



The Gegenbauer Higgs



$$V(h) = \boldsymbol{\kappa} \frac{N_c y_t^2}{16\pi^2} f^2 M_T^2 \left[\sin^2 \frac{h}{f} + \gamma\, G_n^{3/2}\left(\cos \frac{h}{f} \right) \right]$$



↑ pure Gegenbauer $v/f \approx 5.1/n$

$\gamma_c \sim 10^{-3}$

↑ no EWSB $v/f \to 0$

The Gegenbauer Higgs

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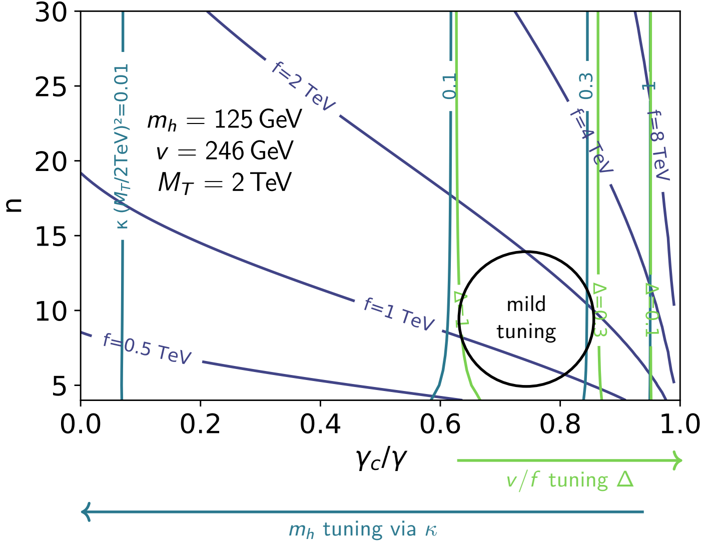

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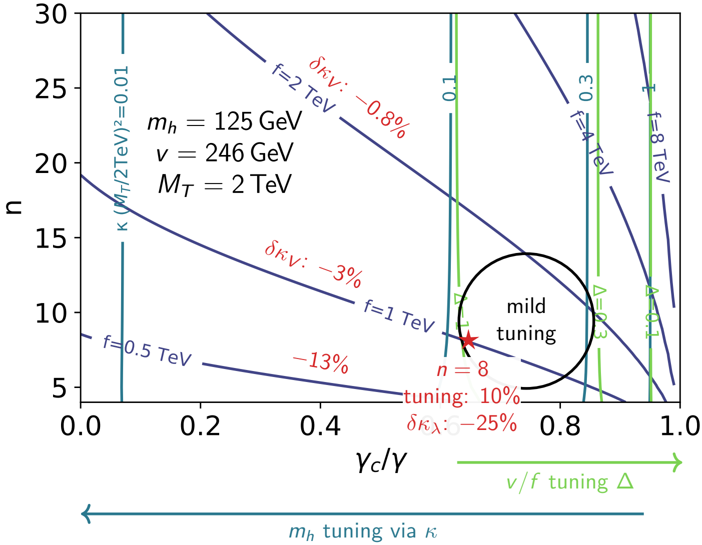

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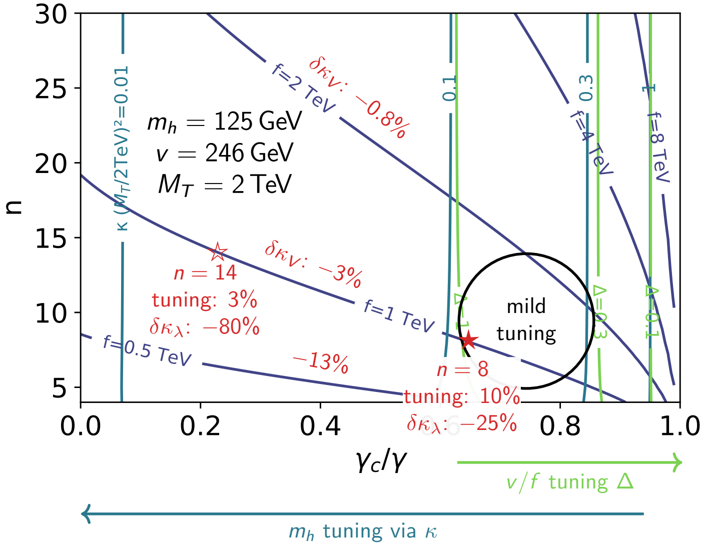
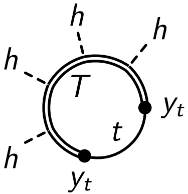
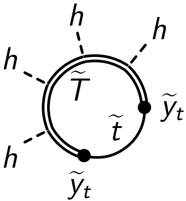


Naturally smaller mass

[Chacko, Goh, Harnik '05]
[Barbieri, Greco, Rattazzi, Wulzer '15]



$+$

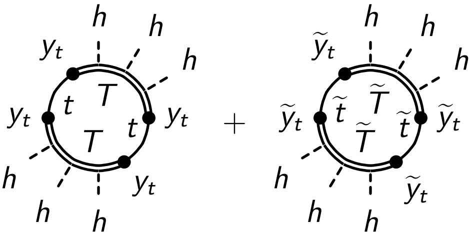


$$\frac{N_c y_t^2}{16\pi^2}\, f^2 M_T^2\, \sin^2\frac{h}{f} \quad + \quad \frac{N_{\widetilde{c}}\widetilde{y}_t^2}{16\pi^2}\, f^2 M_{\widetilde{T}}^2\, \cos^2\frac{h}{f}$$

if twin parity enforces $y_t = \widetilde{y}_t$ and $M_T = M_{\widetilde{T}}$
no M_T^2 sensitivity

Structurally smaller mass

[Chacko, Goh, Harnik '05]
[Barbieri, Greco, Rattazzi, Wulzer '15]



$$\frac{N_c y_t^4}{16\pi^2}\, f^4\, \sin^4\frac{h}{f}\, \log M_T \quad + \quad \frac{N_c \tilde{y}_t^4}{16\pi^2}\, f^4\, \cos^4\frac{h}{f}\, \log M_{\tilde{T}}$$

retaining $\log M_T$ sensitivity only

Gegenbauer's Twin

- global $SO(8) \supset SO(4) \times \widetilde{SO(4)}$

Leopold B. Gegenbauer
1849–1903

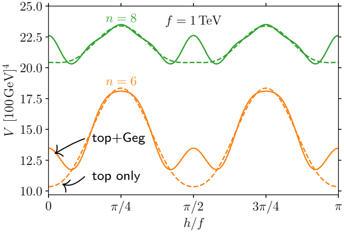
- spontaneous $\quad SO(8) \to SO(7)$

  7 NGBs
  6 eaten by $W^\pm, Z$ and $\widetilde{W}^\pm, \widetilde{Z}$
  1 Higgs: $\vec{\phi} = (\vec{0}_3,\ \sin\frac{h}{f};\ \ \vec{0}_3,\ \cos\frac{h}{f})^T$ in unitary gauge

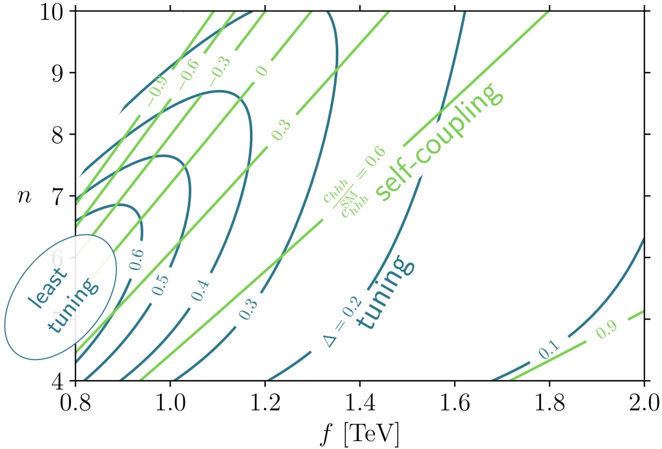
- minimal explicit breaking is insufficient

- explicit $\quad SO(8) \to SO(4) \times \widetilde{SO(4)}$

  radiative stability from irrep spurion
  $G_n^{3/2}(\cos\frac{2h}{f})$ potential

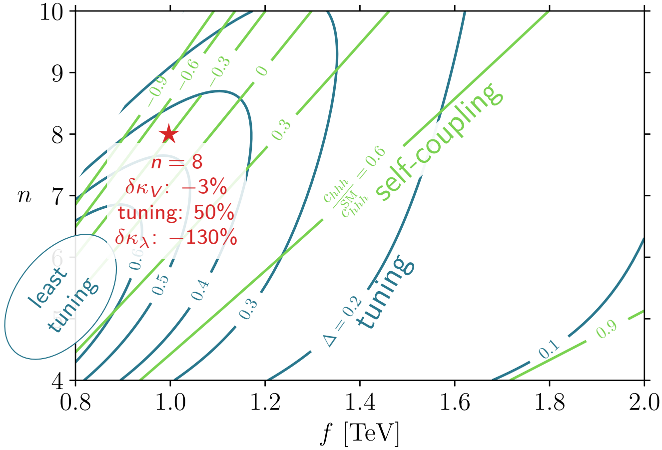


Gegenbauer's Twin



(and possibly large M_T, with unitarity violating H scattering towards 6 TeV)

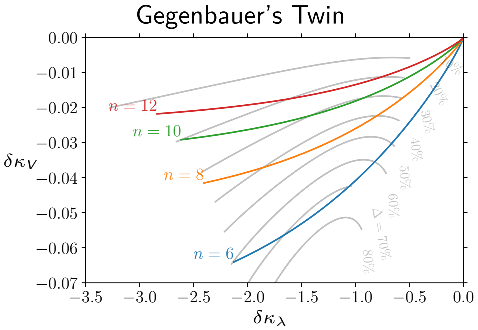
Gegenbauer's Twin

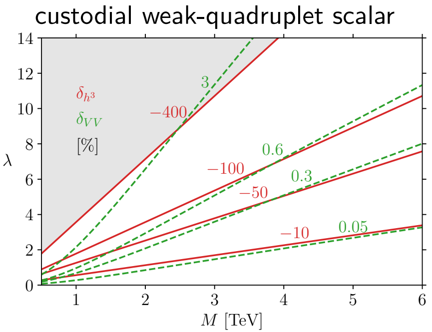


(and possibly large M_T, with unitarity violating H scattering towards 6 TeV)

Naturally large $\delta\kappa_\lambda / \delta\kappa_V$

see also: [Di Luzio, Gröber, Spannowsky '17] [Gupta, Rzehak, Wells '13] [Falkowski, Rattazzi '19] [Logan, Rentala '15] [Chala, Krause, Nardini '18] [etc.]





$$\lambda\, H^* H^* (\epsilon H)\Phi + \lambda \frac{1}{\sqrt{3}} H^* H^* H^* \widetilde{\Phi}$$

- a loop factor allowed dimensionally (or v^2/M_X^2 if dim-6/dim-8)
- dim≫6 operators may be very relevant
- vacuum stability as limiting constraint

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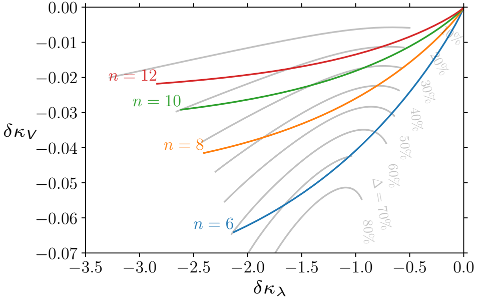
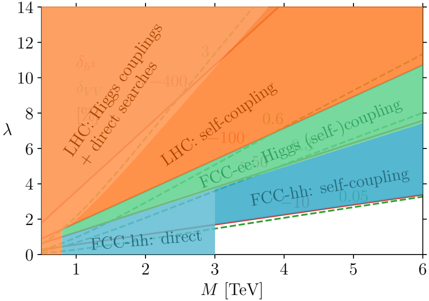


custodial weak-quadruplet scalar



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Gegenbauer naturalness

The linear renormalisation of pNGB potentials admits (remarkable) eigenfunctions.

An example theory exists with natural $\frac{m_h^2}{M_X^2} \sim \%$

natural $\delta\kappa_V \sim \%$

natural $\frac{\delta\kappa_V}{\delta\kappa_\lambda} \sim \%$ (bonus)

So LHC still probes symmetry-based naturalness, and $\delta\kappa_\lambda$ could be the first signal.

Key is a non-minimal breaking of the global pNGB-Higgs symmetry.

Could you UV-complete it?