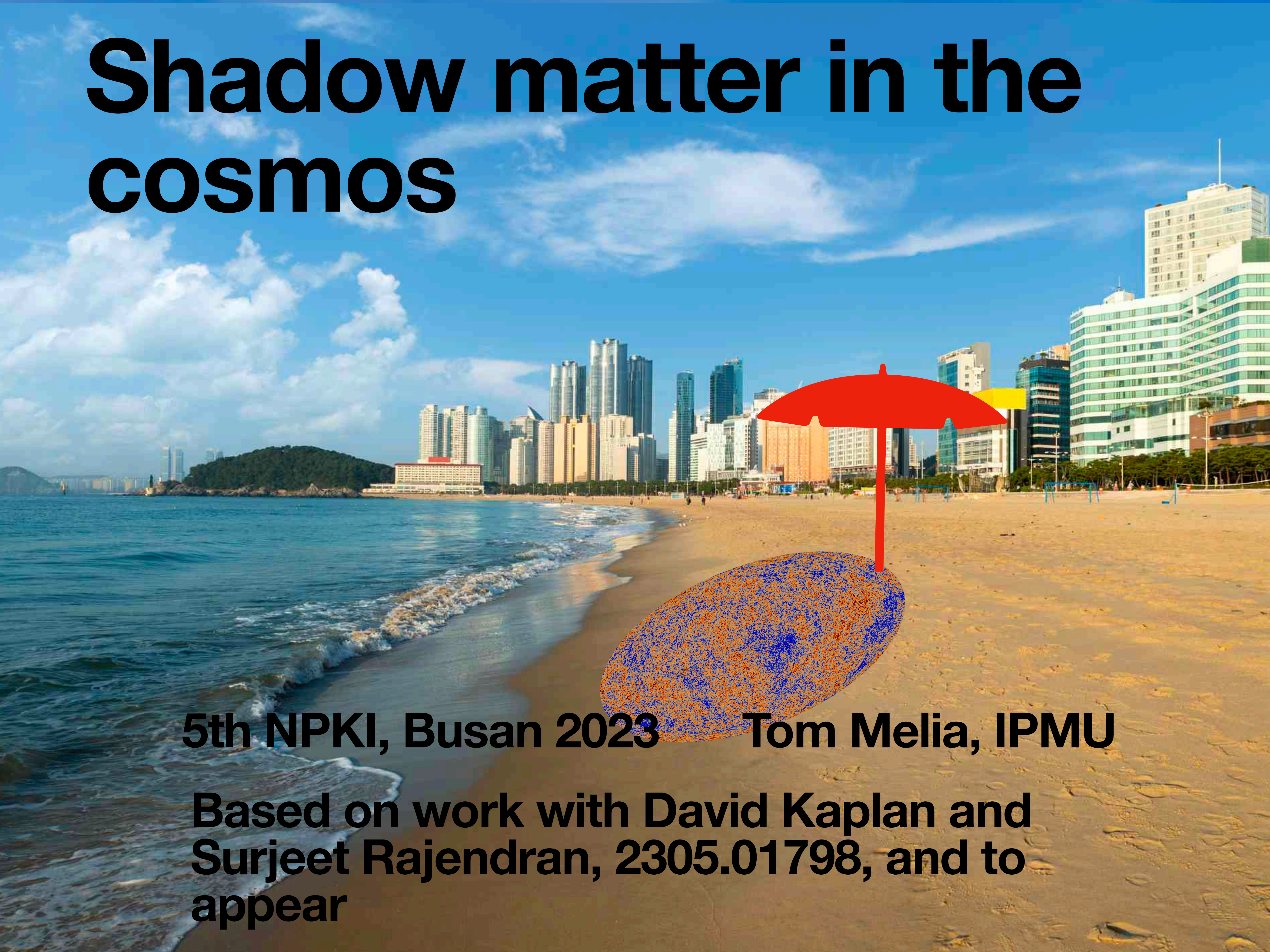
Shadow matter in the cosmos



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Based on work with David Kaplan and Surjeet Rajendran, 2305.01798, and to appear

Classical physics asserts

Maxwell $$\partial_\mu F^{\mu\nu} = J^\nu$$

Einstein $$G^{\mu\nu} = T^{\mu\nu}$$

Quantum physics described by Schrodinger equation

$$i\frac{\partial}{\partial t}|\psi\rangle = H|\psi\rangle$$

From this equation classical physics follows…

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From this equation classical physics follows…

$$\partial_t\langle\hat{\phi}\rangle = i\left\langle\left[\hat{H},\hat{\phi}\right]\right\rangle = \left\langle\frac{\partial\hat{H}}{\partial\hat{\pi}}\right\rangle$$

$$\partial_t\langle\hat{\pi}\rangle = i\left\langle\left[\hat{H},\hat{\pi}\right]\right\rangle = -\left\langle\frac{\partial\hat{H}}{\partial\hat{\phi}}\right\rangle$$

..in expectation value

A subtlety for gauge theories

$$\left\langle \frac{\delta S}{\delta \phi} \right\rangle = \left\langle \frac{\delta \mathcal{L}}{\delta \phi} - \partial_\mu \frac{\delta \mathcal{L}}{\delta \left(\partial_\mu \phi \right)} \right\rangle = 0$$

Fewer d.o.f. than fields

1. **Fewer 2nd order equations for evolution**

2. **Additional constraint equations on the dof**

Compare

$$\partial_\mu F^{\mu\nu} = J^\nu$$

$$i\frac{\partial}{\partial t}|\psi\rangle = H_{EM}|\psi\rangle$$

$$G^{\mu\nu} = T^{\mu\nu}$$

$$i\frac{\partial}{\partial t}|\psi\rangle = H_{GR}|\psi\rangle$$

Both dynamics and Constraints:

Dynamics only

$$\partial_\mu F^{\mu 0} = J^0$$

$$G^{0\mu} = T^{0\mu}$$

& Initial conditions

$$|\psi(0)\rangle$$

Compare

$$\partial_\mu F^{\mu\nu} = J^\nu$$

$$i\frac{\partial}{\partial t}|\psi\rangle = H_{EM}|\psi\rangle$$

$$G^{\mu\nu} = T^{\mu\nu}$$

$$i\frac{\partial}{\partial t}|\psi\rangle = H_{GR}|\psi\rangle$$

Both dynamics and Constraints:

Dynamics only

Gauss' law $\nabla \cdot \mathbf{E} = \rho_{ch}$

e.g. First Friedman $\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho$

& Initial conditions

$$|\psi(0)\rangle$$

What do we believe?

$$G^{\mu\nu} = T^{\mu\nu}$$

Full set of classical equations

$$i\frac{\partial}{\partial t}|\psi\rangle = H_{GR}|\psi\rangle$$

Schrodinger equation

Initial conditions $|\psi(0)\rangle$

What do we believe?

$$G^{\mu\nu} = T^{\mu\nu}$$

$$i\frac{\partial}{\partial t}|\psi\rangle = H_{GR}|\psi\rangle$$

Full set of classical equations

Schrodinger equation

Most fundamental

Initial conditions $|\psi(0)\rangle$

Why should they be just so?

Summary of the talk

Quantum mechanics only guarantees that, in the classical limit

$$\partial_t \left(\nabla \cdot \mathbf{E} - \rho_{ch}\right) = 0 \qquad \nabla_\mu \left(\sqrt{-g}\left(G^{\mu 0} - T^{\mu 0}\right)\right) = 0$$

Summary of the talk

Quantum mechanics only guarantees that, in the classical limit

$$\partial_t \left(\nabla \cdot \mathbf{E} - \rho_{ch}\right) = 0$$

$$\Downarrow$$

$$\nabla \cdot \mathbf{E} = \rho_{ch} + \rho_{dark}(\mathbf{x})$$

Dark charge

$$\nabla_\mu \left(\sqrt{-g}\left(G^{\mu 0} - T^{\mu 0}\right)\right) = 0$$

$$\Downarrow$$

$$G^{\mu\nu} = T^{\mu\nu} + T^{\mu\nu}_{aux}$$

Shadow Matter

$$T^{\mu\nu}_{\text{aux}} = \frac{1}{\sqrt{-g}} \begin{pmatrix} \mathbb{H} & \mathbb{P}^1 & \mathbb{P}^2 & \mathbb{P}^3 \\ \mathbb{P}^1 & 0 & 0 & 0 \\ \mathbb{P}^2 & 0 & 0 & 0 \\ \mathbb{P}^3 & 0 & 0 & 0 \end{pmatrix}$$

Summary of the talk

Quantum mechanics only guarantees that, in the classical limit

$$\partial_t \left(\nabla \cdot \mathbf{E} - \rho_{ch} \right) = 0 \qquad \nabla_\mu \left(\sqrt{-g} \left(G^{\mu 0} - T^{\mu 0} \right) \right) = 0$$

$$\Downarrow \qquad \Downarrow$$

$$\nabla \cdot \mathbf{E} = \rho_{ch} + \rho_{dark}(\mathbf{x}) \qquad G^{\mu\nu} = T^{\mu\nu} + T^{\mu\nu}_{aux}$$

Dark charge ***Shadow Matter***

Not adding any dof to the theory.
Just the choice of quantum state

$$T^{\mu\nu}_{\rm aux} = \frac{1}{\sqrt{-g}} \begin{pmatrix} \mathbb{H} & \mathbb{P}^1 & \mathbb{P}^2 & \mathbb{P}^3 \\ \mathbb{P}^1 & 0 & 0 & 0 \\ \mathbb{P}^2 & 0 & 0 & 0 \\ \mathbb{P}^3 & 0 & 0 & 0 \end{pmatrix}$$

Familiar from EM

In Weyl gauge $A_0 = 0 \qquad \Pi_j = \frac{\partial \mathcal{L}_{\mathcal{EM}}}{\partial A_j} = -E_j$

Comm. $$[\hat{A}_j(\mathbf{x}), \hat{E}_{j'}(\mathbf{x}')] = -i\,\delta(\mathbf{x}-\mathbf{x}')\,\delta_{jj'}$$

Ham. $$\hat{H}_W = \int d^3\mathbf{x}\left(\frac{1}{2}\left(\hat{\vec{E}}\cdot\hat{\vec{E}} + \hat{\vec{B}}\cdot\hat{\vec{B}}\right) + \hat{\vec{J}}\cdot\hat{\vec{A}} + \hat{H}_J\right)$$

SE $$i\frac{\partial|\Psi\rangle}{\partial t} = \hat{H}_W|\Psi\rangle$$

Supplemented with

$$\left(\vec{\nabla}\cdot\hat{\vec{E}} - \hat{J}_0\right)|\Psi_{EM}\rangle = 0$$

Familiar from EM

In Weyl gauge $A_0 \;=\; 0 \qquad \Pi_j = \frac{\partial \mathcal{L}_{\mathcal{EM}}}{\partial A_j} = -E_j$

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SE $i\frac{\partial|\Psi\rangle}{\partial t} = \hat{H}_W|\Psi\rangle$

Supplemented with

Gauss' law operator commutes with H

$$\left(\vec{\nabla}\cdot\hat{\vec{E}}-\hat{J}_0\right)|\Psi_{EM}\rangle = \;0$$

Familiar from EM

Instead consider

$$\left(\vec{\nabla} \cdot \hat{\vec{E}} - \hat{J}_0\right) |\Psi_{EM}\rangle = J_0^d\left(\mathbf{x}\right) |\Psi_{EM}\rangle$$

This is just like adding a background classical charge density

$$\tilde{\mathcal{L}}_{\mathcal{EM}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + A^\mu J_\mu + A^\mu J_\mu^d + \mathcal{L}_J \qquad J_\mu^d = \left(J_0^d\left(\mathbf{x}\right), 0, 0, 0\right)$$

But, …

Familiar from EM

Instead consider

$$\left(\vec{\nabla}\cdot\hat{\vec{E}}-\hat{J}_0\right)|\Psi_{EM}\rangle = J_0^d\left(\mathbf{x}\right)|\Psi_{EM}\rangle$$

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$$\tilde{\mathcal{L}}_{\mathcal{EM}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + A^{\mu}J_{\mu} + A^{\mu}J_{\mu}^{d} + \mathcal{L}_J \qquad J_{\mu}^{d} = \left(J_0^d\left(\mathbf{x}\right),0,0,0\right)$$

- **No additional microphysics**
- **Simply a choice of EM quantum state**

Minisuperspace

$$g_{\mu\nu} \rightarrow ds^2 = -N\left(t\right)^2 dt^2 + a\left(t\right)^2\left(dx^2 + dy^2 + dz^2\right)$$

00 component of metric, lapse function

$$S_{ms} = \int dt \sqrt{-g}\left(M_{pl}^2 R + \mathcal{L}_{matter}\right)$$

$$= \int dt \left(-6M_{pl}^2 \frac{a(t)\dot{a}(t)^2}{N(t)} + \frac{a(t)^3\dot{\phi}(t)^2}{2N(t)} - N(t)a(t)^3 V(\phi)\right)$$

Minisuperspace

$$S_{ms} = \int dt \left(-6M_{pl}^2 \frac{a(t)\dot{a}(t)^2}{N(t)} + \frac{a(t)^3 \dot{\phi}(t)^2}{2N(t)} - N(t)a(t)^3 V(\phi) \right)$$

The equations of motion naively follow

$$\frac{\delta S_{ms}}{\delta N} = a^3 \left(6M_{pl}^2 \frac{\dot{a}^2}{N^2 a^2} - \frac{\dot{\phi}^2}{2N^2} - V(\phi) \right) = 0$$

$$\frac{\delta S_{ms}}{\delta a} = 3Na^2 \left(4M_{pl}^2 \frac{\ddot{a}}{N^2 a} + 2M_{pl}^2 \frac{\dot{a}^2}{N^2 a^2} - 4M_{pl}^2 \frac{\dot{a}\dot{N}}{N^3 a} + \frac{\dot{\phi}^2}{2N^2} - V(\phi) \right) = 0$$

$$\frac{\delta S_{ms}}{\delta \phi} = -\frac{a^3}{N} \left(\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} - \frac{\dot{N}\dot{\phi}}{N} + N^2 \frac{\partial V(\phi)}{\partial \phi} \right) = 0$$

Minisuperspace

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But this variation is suspect!

$$\frac{\delta S_{ms}}{\delta N} = a^3 \left(6M_{pl}^2 \frac{\dot{a}^2}{N^2 a^2} - \frac{\dot{\phi}^2}{2N^2} - V(\phi) \right) = 0$$

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Minisuperspace

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$$\frac{\delta S_{ms}}{\delta N} = a^3 \left(6M_{pl}^2 \frac{\dot{a}^2}{N^2a^2} - \frac{\dot{\phi}^2}{2N^2} - V(\phi) \right) = 0$$

If $N(t) \rightarrow N(t) + \delta N(t)$

Redefine $dt' = (1 + \frac{\delta N}{N})dt$

Then $S_{ms}[N + \delta N, a, \phi] = S_{ms}[N, a, \phi]$

Minisuperspace

$$S_{ms} = \int dt \left(-6M_{pl}^2 \frac{a(t)\dot{a}(t)^2}{N(t)} + \frac{a(t)^3\dot{\phi}(t)^2}{2N(t)} - N(t)a(t)^3 V(\phi) \right)$$

For the quantum theory

$$\pi_N = \frac{\delta \mathcal{L}}{\delta \dot{N}} = 0 \qquad \pi_a = \frac{\delta \mathcal{L}}{\delta \dot{a}} = -12M_{pl}^2 \frac{a\dot{a}}{N} \qquad \pi_\phi = \frac{\delta \mathcal{L}}{\delta \dot{\phi}} = \frac{a^3}{N}\dot{\phi}$$

Hamiltonian

$$H = \left[\pi \dot{a} + \pi_\phi \dot{\phi} - \mathcal{L}\right]_{\dot{a}=\cdots,\dot{\phi}=\cdots} = -\frac{N}{24M_{pl}^2 a}\pi^2 + \frac{N}{2a^3}\pi_\phi^2 + Na^3 V(\phi)$$

$$= -N\frac{\delta S_{ms}}{\delta N} \equiv N\tilde{H}$$

Minisuperspace

Schrodinger equation

$$i\partial_t|\psi\rangle = N(t)\hat{\tilde{\tilde{H}}}|\psi\rangle$$

Just choose N = choice of time coordinate

Only get the 'a' hamiltonian eqns in gravitational sector

$$\partial_t\langle\hat{a}\rangle = -i\langle\left[\hat{\tilde{\tilde{H}}},\hat{a}\right]\rangle$$

$$\partial_t\langle\hat{\pi}\rangle = -i\langle\left[\hat{\tilde{\tilde{H}}},\hat{\pi}\right]\rangle$$

Minisuperspace

First Friedman $\langle\hat{\tilde{\hat{H}}}\rangle = 0$ **not a consequence of the quantum theory**

QM only guarantees

$$\partial_t\langle\hat{\tilde{\hat{H}}}\rangle = i\langle\left[\hat{\tilde{\hat{H}}},\hat{\tilde{\hat{H}}}\right]\rangle = 0$$

i.e.

$$\langle\hat{\tilde{\hat{H}}}\rangle = \mathbb{H}_0$$

c.f. Wheeler-DeWitt

$$\hat{H}|\psi\rangle = 0$$

Minisuperspace

First Friedman $\langle\hat{\tilde{H}}\rangle = 0$ **not a consequence of the quantum theory**

QM only guarantees

$$\partial_t\langle\hat{\tilde{H}}\rangle = i\langle\left[\hat{\tilde{H}},\hat{\tilde{H}}\right]\rangle = 0$$

i.e. $$\langle\hat{\tilde{H}}\rangle = \mathbb{H}_0$$

$$6M_{pl}^2\frac{\dot{a}^2}{a^2} - \frac{\dot{\phi}^2}{2} - V(\phi) = \frac{\mathbb{H}_0}{a^3}$$

Could **be zero, but generally contributes like dark matter**

General(er) relativity

$$S = \int d^4x \sqrt{-g}\left(M_{pl}^2 R + \mathcal{L}_{matter}\right)$$

The equations of motion naively follow

$$\frac{\delta S}{\delta g_{\mu\nu}} = \sqrt{-g}\left(M_{pl}^2 G^{\mu\nu} - T^{\mu\nu}\right) = 0$$

For the quantum theory

$$\pi^{ij} \equiv \frac{\delta L_{grav}}{\delta g_{ij,0}}$$

$$\pi^{i} \equiv \frac{\delta L_{grav}}{\delta g_{0i,0}} = 0$$

$$\pi \equiv \frac{\delta L_{grav}}{\delta g_{00,0}} = 0$$

General(er) relativity

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The equations of motion naively follow

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For the quantum theory

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$$\pi^{i} \equiv \frac{\delta L_{grav}}{\delta g_{0i,0}} = 0$$

$$\pi \equiv \frac{\delta L_{grav}}{\delta g_{00,0}} = 0$$

The corresponding variations are suspect!

'Gauge fix' $g_{00} = -1 \qquad g_{0i} = 0$

General(er) relativity

$$S = \int d^4x \sqrt{-g} \left(M_{pl}^2 R + \mathcal{L}_{matter} \right)$$

The equations of motion that actually follow from the quantum theory

$$\begin{aligned} G^{00} &= 8\pi G_N T^{00} + 8\pi G_N \frac{\mathbb{H}}{\sqrt{-g}} \\ G^{0i} &= 8\pi G_N T^{0i} + 8\pi G_N \frac{\mathbb{P}^i}{\sqrt{-g}} \\ G^{ij} &= 8\pi G_N T^{ij} \end{aligned}$$

An auxiliary 'shadow matter' tensor

$$T^{\mu\nu}_{\text{aux}} = \frac{1}{\sqrt{-g}} \begin{pmatrix} \mathbb{H} & \mathbb{P}^1 & \mathbb{P}^2 & \mathbb{P}^3 \\ \mathbb{P}^1 & 0 & 0 & 0 \\ \mathbb{P}^2 & 0 & 0 & 0 \\ \mathbb{P}^3 & 0 & 0 & 0 \end{pmatrix}$$

$$0 = \nabla_\mu T^{\mu\nu}_{\text{aux}}$$

$$\Longrightarrow \quad \begin{aligned} \partial_0 \mathbb{H} &= -\partial_i \mathbb{P}^i \\ \partial_0 \left(\gamma_{ij} \mathbb{P}^j \right) &= 0 \end{aligned}$$

Shadow matter in the cosmos

$$ds^2 = -dt^2 + a(t)^2(\delta_{ij} + h_{ij})dx^i dx^j$$

$$h_{ij} = h\delta_{ij} + D_{ij}\eta + (\partial_i w_j + \partial_j w_i) + s_{ij})$$

$$\tilde{D}_{ij} = \hat{k}_i \hat{k}_j - \delta_{ij}/3.$$

Linear perturbation theory of scalars

(And working with background a~t^(2/3)

1. $G^i_j = 0 \quad \Longrightarrow$

$$\ddot{\tilde{h}} + 3\frac{\dot{a}}{a}\dot{\tilde{h}} - 2\frac{k^2}{a^2}\tilde{\eta} = 0$$

$$\ddot{\tilde{h}} + 3\frac{\dot{a}}{a}\dot{\tilde{h}} + 6\left(\ddot{\tilde{\eta}} + 3\frac{\dot{a}}{a}\dot{\tilde{\eta}}\right) - 2\frac{k^2}{a^2}\tilde{\eta} = 0$$

$$\tilde{\eta} \propto \text{const} \qquad \tilde{h} \propto a$$

Shadow matter in the cosmos

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$$\tilde{D}_{ij} = \hat{k}_i\hat{k}_j - \delta_{ij}/3.$$

Linear perturbation theory of scalars

2. $\mathbb{H} \equiv \mathbb{H}_0 + \delta\mathbb{H}$ $\mathbb{P}^i$

In pert theory

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}G_N\frac{\mathbb{H}_0}{a^3}$$

Zeroth order

$$\begin{aligned}\frac{\dot{a}}{a}\dot{\tilde{h}} - 2\frac{k^2}{a^2}\tilde{\eta} &= 8\pi G_N\frac{1}{a^3}\left[-\frac{1}{2}\tilde{h}\mathbb{H}_0 + \delta\mathbb{H}\right] \\ -2i\frac{k}{a^2}\dot{\tilde{\eta}} &= 8\pi G_N\frac{\mathbb{P}_\parallel}{a^3}\,.\end{aligned}$$

First order

Shadow matter in the cosmos

$$ds^2 = -dt^2 + a(t)^2(\delta_{ij} + h_{ij})dx^i dx^j$$

$$h_{ij} = h\delta_{ij} + D_{ij}\eta + (\partial_i w_j + \partial_j w_i) + s_{ij})$$

$$\tilde{D}_{ij} = \hat{k}_i\hat{k}_j - \delta_{ij}/3.$$

Linear perturbation theory of scalars

2. $\mathbb{H} \equiv \mathbb{H}_0 + \delta\mathbb{H}$ $\quad \mathbb{P}^i$

In pert theory

$$\delta\mathbb{H} \propto \text{const} + 1/a^{1/2} \qquad \mathbb{P}^i \propto 1/a^2$$

Consistent with

$$\partial_0\delta\mathbb{H} = -ik\mathbb{P}_{\|}$$

$$\partial_0\left(a^2\mathbb{P}_{\|}\right) = 0$$

To summarise

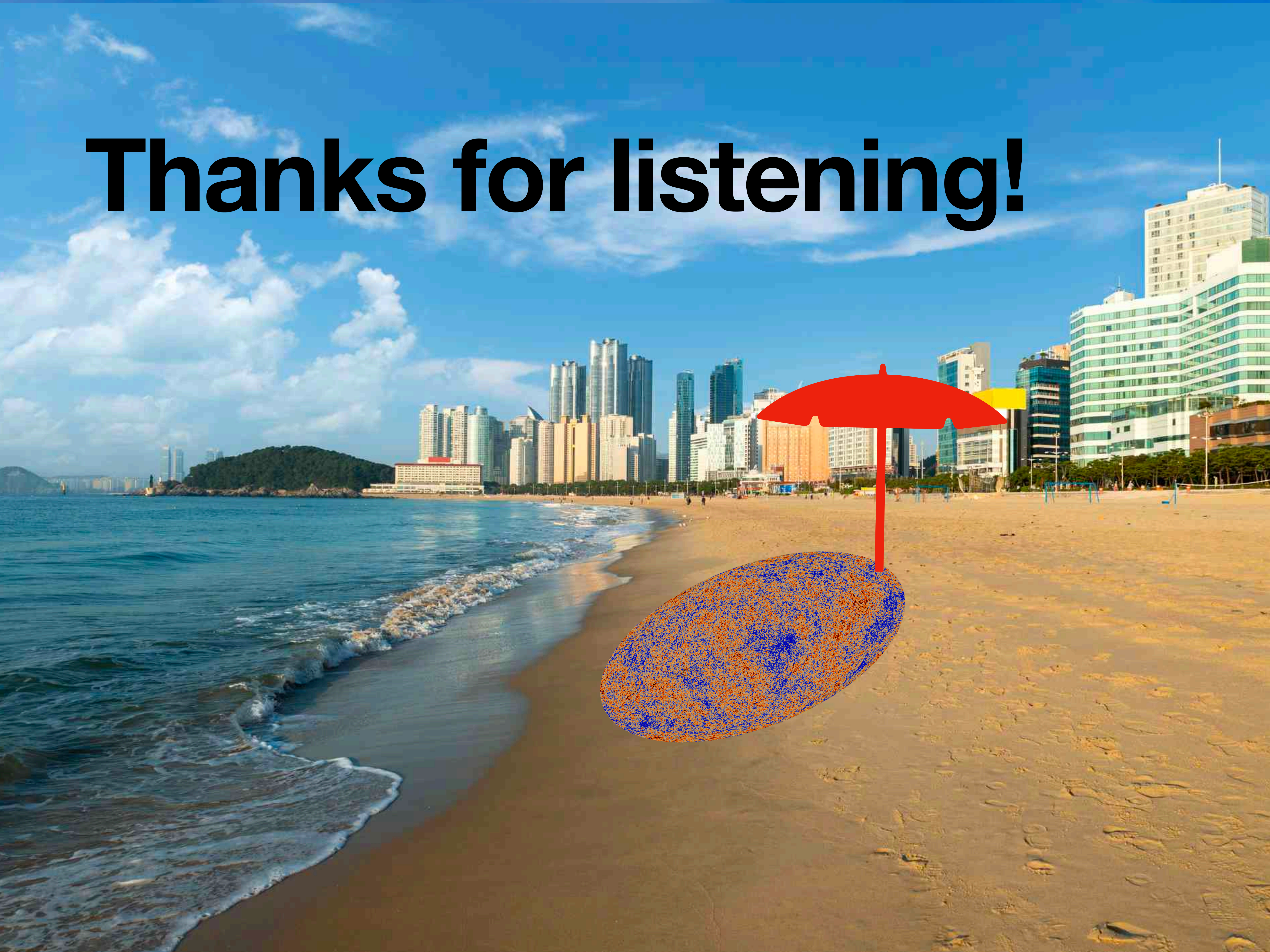
Classical physics is a limit of quantum mechanics

Certain classical equations are not guaranteed by the quantum theory

One should consider a broader class of states in GR

Some observations to conclude with

- Shadow matter sources linear growth in curvature perturbations, like dark matter
- Could have either sign, trivial violation of NEC
- Detailed study of cosmological evolution under way, and the nonlinear regime.
- Inflation dynamically drives to conventional GR


Thanks for listening!