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New Axion Isocurvature

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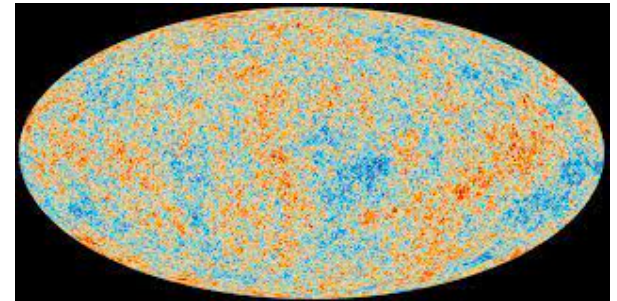
NPKI 23

Adiabatic Perturbations

- The primordial perturbations are measured by the CMB to be adiabatic to good precision.

$$\delta f_i(x, t) = \frac{df_i(t)}{dt} \delta t(x, t)$$

Any thermodynamic quantity \nearrow \nwarrow Perturbation of the common clock



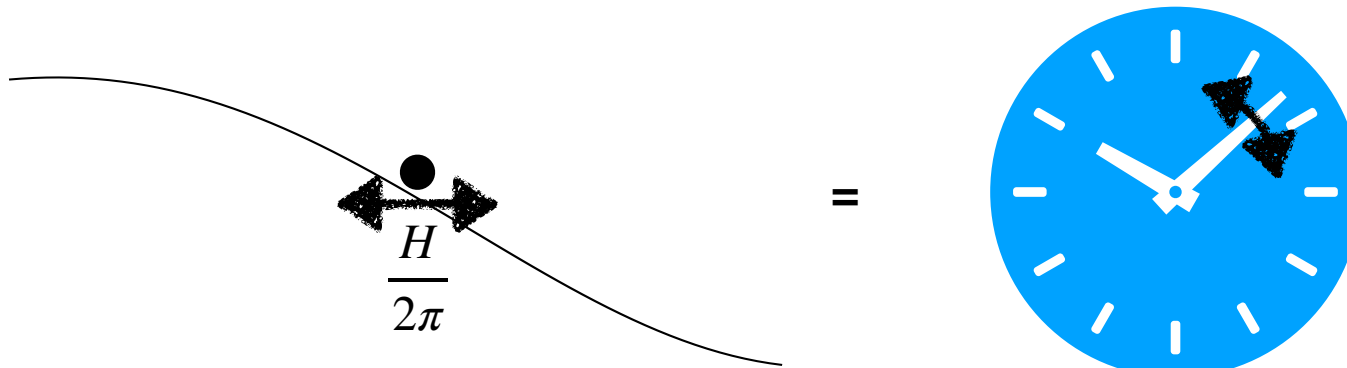
- Any perturbations that doesn't satisfy this is "isocurvature perturbation".

$$S = \frac{\delta(n_a/s)}{n_a/s} \quad (\text{Late times})$$

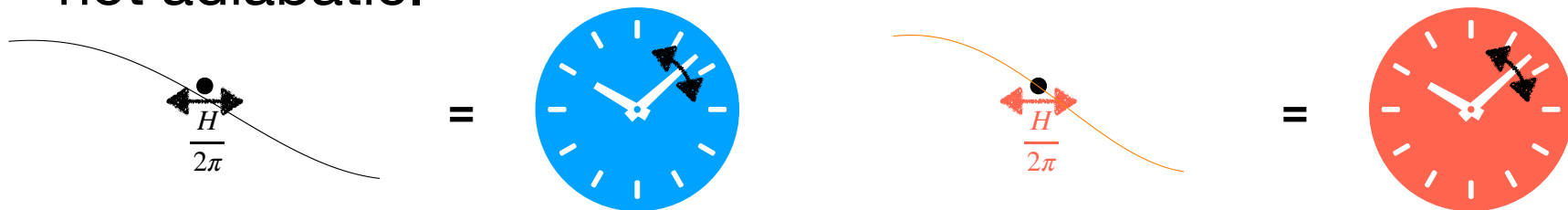
- CMB puts a constraint on the power spectrum of S.

Production of Isocurvature

- For a single field, slow roll inflation, only adiabatic modes are produced.



- For multi-field inflation, any perturbation of a single field is not adiabatic.



Adiabatic mode: $\frac{\delta\varphi_1}{\dot{\varphi}_1} = \frac{\delta\varphi_2}{\dot{\varphi}_2} = \dots = \delta t = \text{Moving both clocks together}$

Axion Isocurvature

- Standard Axion Isocurvature: during inflation, axions get dS-fluctuations

$$\langle \delta\phi_a, \delta\phi_a \rangle = \frac{H^2}{4\pi^2}$$

- These fluctuations cannot be removed by the perturbations of the clock.
- In other words:

$$\frac{\delta\phi_a}{\dot{\phi}_a} \neq \frac{\delta\phi_i}{\dot{\phi}_i}$$

The axion The inflation

Axion Isocurvature

- We can estimate the isocurvature

$$S = \frac{\delta(n_a/s)}{n_a/s} \approx 2 \frac{\delta a}{a} \sim \frac{H_{inf}}{f}$$

- From CMB

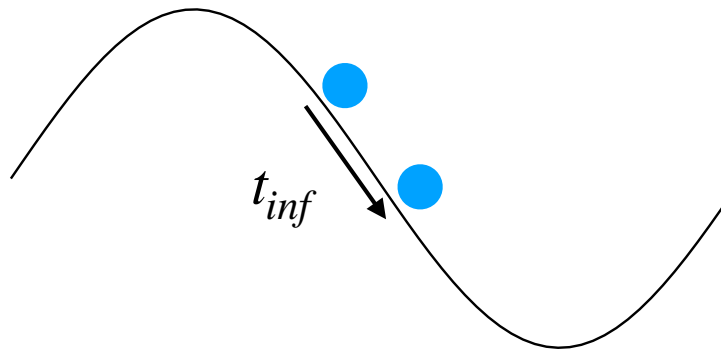
$$\alpha \sim \frac{S^2}{R^2} \lesssim 10^{-2} \quad \text{(uncorrelated)}$$

- Bound on high scale inflation

$$H_{inf} \lesssim 10^{-6} f$$

Our Idea

- Neglect the axion field perturbations
- Assume that the axion can move a bit during inflation:



- The length of inflation is given by the inflation field, which has perturbations, i.e. t_{inf} is inhomogeneous
- **Result:** The axion at the end of inflation is inhomogeneous.

Estimating the Effect

- The distance the axion moves is

$$\Delta\phi_a = \frac{d\phi_a}{dN} N_{inf}$$

- The adiabatic perturbation can be written as

$$R = \delta N_{inf}$$

- Therefore the axion perturbation is

$$\delta\phi_a a = \frac{d\phi_a}{dN} \delta N_{inf} \longrightarrow S = 2 \frac{\delta\phi_a}{\phi_a} \sim \frac{m^2}{H_{inf}^2} R$$

Low Scale Inflation

- Isocurvature perturbation:

$$S \sim \frac{m^2}{H_{inf}^2} R$$

- The CMB is sensitive to

$$\alpha \sim \frac{S^2}{R^2} < 10^{-3} \longrightarrow \begin{array}{l} S \sim 0.03R \\ H_{inf} \sim 5m_a \end{array}$$

(anti-correlated)

- Very low scale of inflation,

$$H_{inf} \lesssim \text{eV} \longrightarrow \Lambda_{inf} \lesssim 100 \text{ TeV}$$

Isocurvature Constraint

- Doing the calculation with \neq less squiggly lines

$$\alpha \equiv \frac{\langle |\mathcal{S}(k)|^2 \rangle}{\langle |\mathcal{S}(k)|^2 \rangle + \langle |\mathcal{R}(k)|^2 \rangle} \simeq \frac{4 V_{ak}'^2}{9 H_k^4 \phi_{ak}^2}$$

- Constraint

$$H \gtrsim 4.6 \text{ eV} \left(\frac{m_a}{\text{eV}} \right) \left(\frac{10^{-3}}{\alpha} \right)^{1/4}$$
$$\Lambda_{\text{inf}} = \left(\frac{3H^2 M_{\text{pl}}^2}{8\pi} \right)^{1/4} \gtrsim 140 \text{ TeV} \left(\frac{m_a}{\text{eV}} \right)^{1/2} \left(\frac{10^{-3}}{\alpha} \right)^{1/8}$$



That's all Folks!

Wait...
Where are the equations?



Solving the Perturbations during inflation

- During inflation, taking the newtonian gauge

$$ds^2 = (1 + 2\Phi)dt^2 - a(t)^2(1 - 2\Psi)d\vec{x} \cdot d\vec{x}$$

- For all the fields during inflation:

$$\dot{\Phi} + H\Phi = 4\pi G \sum_{j=1}^N \dot{\phi}_j \delta\phi_j,$$

$$\delta\ddot{\phi}_j + 3H\delta\dot{\phi}_j + \left(\frac{k^2}{a^2} + V_j''\right)\delta\phi_j = 4\dot{\phi}_j\dot{\Phi} - 2V_j'\Phi,$$

- Assuming slow roll and superhorizon

$$\Phi = -C_1 \frac{\dot{H}}{H^2} - H \frac{d}{dt} \left(\frac{\sum_j d_j V_j}{\sum_j V_j} \right),$$

$$\frac{\delta\phi_i}{\dot{\phi}_i} = \frac{C_1}{H} - 2H \left(\frac{\sum_j d_j V_j}{\sum_j V_j} - d_i \right), \quad i, j = 1, \dots, \tilde{N}.$$

Adiabatic and Isocurvature in Inflation

- The solution can be written as

$$\frac{\delta\phi_i}{\dot{\phi}_i} = \frac{C_1}{H} + 2HC_3 \frac{V_a}{V_a + V_i}$$

$$\frac{\delta\phi_a}{\dot{\phi}_a} = \frac{C_1}{H} - 2HC_3 \frac{V_i}{V_a + V_i}$$

$$\Phi = -C_1 \frac{\dot{H}}{H^2} + \frac{C_3}{3} \frac{V_i V_a'^2 + V_a V_i'^2}{(V_a + V_i)^2}$$

- Where the constants are determined by initial conditions (Bunch-Davies)

$$\delta\phi_j(k, t) = \frac{H(t_k)}{\sqrt{2k^3}} e_j(\mathbf{k}) \quad \xrightarrow{\substack{V_a' \gg V_i' \\ V_i \gg V_a}} \quad \begin{aligned} C_1 &\simeq -\frac{8\pi G H_k}{\sqrt{2k^3}} \frac{V_i}{V_i'} e_i(\mathbf{k}) = -\frac{3H_k^3}{\sqrt{2k^3 V_i'}} e_i(\mathbf{k}) \\ C_3 &\simeq -\frac{3H_k}{2\sqrt{2k^3 V_i'}} e_i(\mathbf{k}), \end{aligned}$$

$$\langle e_j(\mathbf{k}) e_{j'}^*(\mathbf{k}') \rangle = \delta_{jj'} \delta^{(3)}(\mathbf{k} - \mathbf{k}')$$

Transition to Hot Big Bang

- After inflation, the C_3 contribution to Φ vanishes. And so

$$C_1 = \text{adiabatic}, C_3 = \text{isocurvature}$$

(Assumes axion vacuum energy is subdominant during inflation)

- Together the two modes give

$$\begin{aligned} \delta_\gamma &= -2\Phi, & \swarrow \text{Adiabatic} \\ \delta_b &= \frac{3}{2}\delta_\nu = \frac{3}{4}\delta_\gamma \\ \delta_a &= S + \frac{3}{4}\delta_\gamma, & \nearrow \text{Isocurvature} \end{aligned}$$

Primordial Initial Conditions

- For the CMB calculation, S and Φ are the initial conditions.
- Adiabatic mode

$$\frac{1}{4}\delta_\gamma = \frac{1}{4}\delta_\nu = \frac{1}{4}\delta_b = \frac{1}{3}\delta_a = R \simeq C_1 \quad (R \simeq const)$$

$$\mathcal{R} \simeq -\frac{8\pi G H_k}{\sqrt{2k^3}} \sum_j \frac{V_j}{V_j'} e_j(\mathbf{k}) \simeq -\frac{3H_k^3}{\sqrt{2k^3} V_i'} e_i(\mathbf{k})$$

- Isocurvature mode

$$S = 2 \left. \frac{\delta\phi_a}{\phi_a} \right|_{C_1 \rightarrow 0} = \frac{4}{3} \frac{V'(\phi_a)_k C_3}{\phi_{ak}} = -2 \frac{V'(\phi_a)_k}{\phi_{ak}} \frac{H_k}{\sqrt{2k^3} V'(\phi_i)_k} e_i(\mathbf{k}) \quad \left(\frac{\delta\phi_a}{\phi_a} \simeq const \right)$$

Results



Power Spectra

- Power spectra

$$\Delta_{\mathcal{R}}^2(k) = \frac{k^3}{2\pi^2} \langle \mathcal{R}^2 \rangle = \frac{H_k^2}{8M_{Pl}^2 \pi^2 \epsilon_k}$$

$$\Delta_S^2(k) = \frac{k^3}{2\pi^2} \langle S^2 \rangle = \frac{V'(\phi_a)_k^2 H_k^2}{\pi^2 V'(\phi_i)_k^2 \phi_{ak}^2} = \frac{4}{9} \frac{V'(\phi_a)_k^2}{H_k^4 \phi_{ak}^2} \Delta_{\mathcal{R}}^2$$

$$\Delta_{S\mathcal{R}}^2(k) = \frac{k^3}{2\pi^2} \langle S \mathcal{R} \rangle = \frac{3H_k^4 V'(\phi_a)_k}{2\pi^2 V'(\phi_i)_k^2 \phi_{ak}} = \frac{2V'(\phi_a)_k}{3H_k^2 \phi_{ak}} \Delta_{\mathcal{R}}^2$$

- Fully correlated! (Not surprising - only inflaton perturbation)

Spectral Indices

- Definition

$$\Delta_{\mathcal{R}}^2(k) = A^2 \left(\frac{k}{k_0} \right)^{n_{\text{ad}}-1}$$

$$\Delta_S^2(k) = B^2 \left(\frac{k}{k_0} \right)^{n_{\text{iso}}-1}$$

- Results

$$\begin{aligned} n_{\text{ad}} - 1 &= \frac{k_0}{A^2} \left. \frac{d\Delta_{\mathcal{R}}^2(k)}{dk} \right|_{k=k_0} \\ &= \frac{k_0}{A^2} \frac{1}{8M_{\text{Pl}}^2 \pi^2} \left. \frac{d(H_k^2 / \epsilon_k)}{dk} \right|_{k=k_0} \\ &\simeq -6\epsilon_i + 2\eta_i - 4\epsilon_a \simeq 2\eta_i, \end{aligned}$$

$$\begin{aligned} n_{\text{iso}} - 1 &= \frac{k_0}{B^2} \left. \frac{d\Delta_S^2(k)}{dk} \right|_{k=k_0} \\ &= \frac{k_0}{\pi^2 B^2} \left. \frac{d(H_k^2 V'(\phi_a)_k / V'(\phi_i)_k^2 \phi_a^2)}{dk} \right|_{k=k_0} \\ &\simeq 2(\eta_i - \epsilon_a - \epsilon_i) \sim (n_{\text{ad}} - 1), \end{aligned}$$

Constraints

- The isocurvature ratio

$$\alpha \equiv \frac{\langle |\mathcal{S}(k)|^2 \rangle}{\langle |\mathcal{S}(k)|^2 \rangle + \langle |\mathcal{R}(k)|^2 \rangle} \simeq \frac{4 V_{ak}'^2}{9 H_k^4 \phi_{ak}^2}$$

- The bounds on (anti-)correlated isocurvature from Planck

$$\alpha_{\text{low-scale}} \lesssim 10^{-3}$$

$$H \gtrsim 4.6 \text{ eV} \left(\frac{m_a}{\text{eV}} \right) \left(\frac{10^{-3}}{\alpha} \right)^{1/4}$$
$$\Lambda_{\text{inf}} = \left(\frac{3H^2 M_{\text{pl}}^2}{8\pi} \right)^{1/4} \gtrsim 140 \text{ TeV} \left(\frac{m_a}{\text{eV}} \right)^{1/2} \left(\frac{10^{-3}}{\alpha} \right)^{1/8}$$

Relic Abundance

- There is a second bound: if the axion moves - it can roll to the bottom!
- Model dependent - how long inflation lasts before the CMB modes exit the horizon.
- Taking the conservative approach of the shortest possible inflation:

$$\theta_I = \theta_i e^{\frac{-m_a^2 N}{3H^2}}$$

- And so we get a bound around

$$H \gtrsim 1.2 m_a \left(\frac{N}{30}\right)^{1/2}$$

- Slightly weaker, but the isocurvature bound can be improved in the future!

Conclusion

- Bounds on the scale of inflation not only from above, but also from below.



- A new source of fully anti-correlated axion isocurvature
- The mechanism is general for any production mechanism before inflation.